Formální a experimentální sémantika I

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PS 2025

Goals, completion, literature, content

- Goals
  - introductory course on:
    a) formal aspects of meaning in natural language
    b) experimental methods
    c) data from Slavic languages (case system, aspect system)
  - prerequisites: 0 (elementary logic)
  - systematic introduction

- slides available at (for approximately one month):

Formal Requirements

- plan: propositional logic, predicate logic, formal semantics
- mix of logic and formal semantics
- approximately in 1/3 of the classes: task for independent (short) logic/semantic homework related to proof verification, logical types, ...
  - in-class task review
  - tasks: 30% of the credit
- written exam: 2/3 threshold (- homework)

References

1) Portner, Paul H. 2005. What Is Meaning: Fundamentals of Formal Semantics (Fundamentals in Linguistics). Oxford: Blackwell.
2) Cann, Ronnie. 1993. Formal Semantics: An Introduction. Cambridge University Press.
3) Chierchia, Gennaro, and Sally McConnell-Ginet. 1990. Meaning and Grammar. An Introduction to Semantics. Cambridge, Mass.: MIT Press.
4) Lepore, Ernest, and Sam Cumming. *Meaning and argument: an introduction to logic through language*. John Wiley & Sons, 2009.
5) articles, ...

Content

- Topics:

1. Propositional and predicate logic
2. Meanings – concepts, truth conditions
3. Compositionality of meaning – reference, predication, compositionality, syntax/semantics interface
4. Types of predicates: AP, PP, relative clauses, event semantics, sets and functions
   a) semantic roles and cases (morphology-semantics, syntax-semantics)
5. Modifiers: Adjectives in NP, relative clauses, adverbs
6. Referring expressions: definite NPs, indefinite NPs, theory of reference, plurals, kinds
7. Quantifiers: generalized quantifiers, NPI, NP conjunction
   a) negation: negative polarity items vs. n-words (experiments)

7. Extension vs. intension: time, modality
8. Aspectual system of natural languages
    a) Slavic aspectual system
    b) experiments

Logic and natural language

Intro

Arguments

- not proofs, but symbolization of Czech/English arguments into a formal language;
- shallow analysis – Quine;
- the path from the simplest, insufficient but pedagogical form to more complex formalizations;
- the contextual nature of language, not automatisms;
- conversational implicatures and their distinction from the meaning of natural language – Sperber & Wilson;

Basic Concepts

Arguments

(1) Example 1: If Mirek Dušín met Červenáček in the clubhouse, then Červenáček was in the clubhouse. If Červenáček was there, then Červenáček is not sweeping the sidewalk at home. But if Červenáček's father came home, then Červenáček is sweeping the sidewalk. It follows that if MD met Červenáček in the clubhouse, then Červenáček's father did not come home.

(2) Anyone who considers alternative courses of action believes that they are free. Because everyone considers alternative courses of action, it follows that we all consider ourselves free.

What is a statement

- indicative, true/false;

Premises and Conclusion

- premise indicators: *for the following reasons; because; it is given; let's assume; facts:*
- conclusion indicators: *therefore; it follows that; then; as a consequence, ...*

Arguments in Standard Form

1. identify premises and conclusion;
2. place premises first;
3. place conclusion last;
4. write implicit premises / conclusion.

(3) a. Anyone who considers alternative courses of action believes that they are free.
b. Everyone considers alternative courses of action.
c. Conclusion: We all consider ourselves free.

Multiple Conclusions

(4) a. All people are mortal and rational.
b. Peter is a person.
c. Conclusion1: Peter is rational.
d. Conclusion2: Peter is mortal.

Deductive Validity

- A deductively valid argument is one in which it is not possible for the premises to be true and the conclusion false at the same time.

(5) a. The current prime minister will win the next presidential election.
    b. If the current prime minister wins the presidential election, then the Czech Republic will prosper.
    c. Conclusion: The Czech Republic will prosper.

(6)
a. All badgers eat with cutlery.
b. All entities that eat with cutlery can fly.
c. Conclusion: All badgers can fly.

- invalid:

(7)
a. If God exists, then the world is perfect.
b. God does not exist.
c. Conclusion: The world is not perfect.

- true x false statement;
- valid x invalid argument.

Exercises:

(8)
  a. Marie is either swimming or singing.
  b. She is not swimming.
  c. Conclusion: So she is singing.

(9)
  a. Peter will either swim or sing.
  b. He will swim.
  c. Conclusion: So he will not sing.

(10) a. Peter had schnitzel for dinner today.
b. So, Peter had dinner today.

(11) a. Everyone loves someone.
b. Someone is loved by everyone.

(12) a. There is evil in this world.
b. If God existed, then evil would not exist.
c. God does not exist.

- exercise: table;

Sound Argument

- true premises;

Missing Premises and Conclusion

(13) a. The Svratka flows into the Danube.
     b. So, the Bobrava flows into the Danube.

- Sokal!

(14) a. Only children are allowed free entry.
     b. So, none of our employees are allowed free entry.

Exercises:

(15) Either I pay Peter, or he will publish those photos; but if I pay him, I will have nothing left. So, either I will be completely broke, or I will lose my job.

Forms of Arguments and Propositional Logic

Formal Validity

(16) a. It is raining and Peter is sleeping.
b. $\vDash$ It is raining

(17) a. If Karel died, then his children are sad.
b. Karel died.
c. $\vDash$ His children are sad.

(18) a. Either you are a dualist, or you are a monist.
b. You are not a dualist.
c. $\vDash$ You are a monist.

- intuitive validity is not enough for formal validity

(19) a. Peter is a bachelor.
 b. $\not\models$ Peter is unmarried.

- formal validity is a matter of form, not lexical content

Quotation Marks: use/mention

(20) a. 'Robert Zimmermann' is the name of Bob Dylan.
b. A mouse is a mammal.
c. 'Mouse' has five letters.

- I use names to refer to objects;

Metalinguistic Variables

- in propositional logic: small letters starting with p;
- or Greek alphabet;

(21) a. $p \to q$
b. p
c. $\models q$

Logic: the science of reasoning

(22) a. If the sun is shining, then it is daytime.
b. It is not daytime.
c. $\models$ The sun is not shining.

```python
import ttg

# Create a truth table for the expression

print(ttg.Truths(['p', 'q'], ['((p => q) and ~q)','~p']))
```

```
## +-----+-----+----------------------+------+
## |  p  |  q  |   ((p => q) and ~q)  |  ~p  |
## |-----+-----+----------------------+------|
## |  1  |  1  |          0           |  0   |
## |  1  |  0  |          0           |  0   |
## |  0  |  1  |          0           |  1   |
## |  0  |  0  |          1           |  1   |
## +-----+-----+----------------------+------+
```

- but this is exactly where all LLMs used to fail (before trained by underpaid people):

https://g.co/gemini/share/af61bad89636

https://g.co/gemini/share/52b1b4c1adb7

- not work anymore but here:

https://g.co/gemini/share/234dd13d4662

and:
https://www.theguardian.com/technology/2025/sep/11/google-gemini-ai-training-humans

- good intuitive online proover:

https://www.umsu.de/trees/

Nonformal Validity

(23) a. Peter is an old bachelor.
b. Peter is not married.

(24) a. Something is red.
b. Something is colored.

(25) a. Peter knows that it is raining.
b. It is raining.

- logical expressions;

1.5.5 Propositional Logic: Connectives

- conjunction, negation, disjunction, and conditional;
- symbolic notation: a simple proposition is symbolized by a

Type/Token

(26) a. Rychlonožka
b. Being a smoker is unhealthy.

- if a simple propositional type occurs more than once in an argument, then each token of that type must be symbolized by the same propositional variable in all its occurrences;

(27) If Peter goes to the store, he can buy beer.

(27) Peter went to the store.

(27) $\models$ Peter can buy beer.

- if two propositional types can be considered paraphrases (relative to the context of the argument in which they occur), then each of them should be symbolized by the same propositional variable;

Conjunction

(28) a. Peter slept, but Karel sang.

b. Peter slept while Karel sang.

c. Peter slept, although Karel sang.

d. Peter slept, even though Karel sang.

e. Peter slept, and yet Karel sang.

f. Peter slept; Karel sang.

(30) Proposition θ is a logical conjunction if the truth of its parts (propositions) α and β implies the truth of θ; and the truth of θ implies the truth of α and β.

- table;

```python
import ttg

# Create a truth table for the expression

print(ttg.Truths(['p', 'q'], ['p and q']))
```

```
## +-----+-----+-----------+
## |  p  |  q  |  p and q  |
## |-----+-----+-----------|
## |  1  |  1  |     1     |
## |  1  |  0  |     0     |
## |  0  |  1  |     0     |
## |  0  |  0  |     0     |
## +-----+-----+-----------+
```

Deductive and Conversational Aspects of Conjunction

(31) a. Peter is tired, but Karel is not tired.
b. Peter is tired and Karel is not tired.
c. Peter is tired while Karel is not tired.

(32) a. Marie got married and had a child.
b. Marie had a child and got married.

- contrast;
- temporal sequence – both are missing in the truth table;

- the question is whether the logical connective is truly adequate for capturing the conjunction in natural language;
- yes, but it is strictly limited to truth-conditional meanings of propositions;
- i.e., table, everything else is derived differently;
- contrast is derived as a presupposition;

Presupposition

- Truth: property of sentences – correspondence theory of truth;
- we know the meaning of a sentence even if we do not know its truth value, but it is enough to know the truth conditions;
- how the world must look for the sentence *The door is open* to be true;
- for constructing truth conditions, connectives are essential: *The door is open and the sun is shining outside*;

- entailment: one sentence entails another if the truth of the first guarantees the truth of the second and the falsity of the second guarantees the falsity of the first;
- from a+b follows c, from c follows a and b;
- d is a contradiction;

(33) a. Peter has black hair.
b. Peter is a linguist.
c. Peter is a black-haired linguist.
d. Peter is a black-haired linguist, but Peter does not have black hair.

- Similarly:

(34) a. Some children sang and danced.
     b. Some children sang.

- From a) follows b) and 0 b) guarantees the falsity of a);
- it is a semantic, not a syntactic property, sentences that are syntactically identical do not imply:

(35) a. Peter is a supposed linguist.
     b. Peter is supposed and Peter is a linguist.

(36) a. Few children danced and sang.
     b. Few children sang and few children danced.

- Entailment is logically based on implication;
- presupposition is quite similar to entailment;
- it is a kind of implication: if A presupposes B, then A not only implies B, but also implies that the truth of B is taken as given or uncontroversial;
- background information shared by the speaker and the addressee;

(37) a. Peter stopped smoking.
     b. Peter smoked.

- How to distinguish presupposition from entailment?

- Embedding an expression that carries a presupposition often preserves the presupposition:

(38) a. Paul stopped smoking.
b. Paul did not stop smoking.
c. Did Paul stop smoking?
d. I regret that Paul stopped smoking.

- Negation and questions preserve presuppositions;

- this cannot be true for entailment, because it preserves truth conditions;

(39)
a. It was Peter who brought the cake.
b. Was it Peter who brought the cake?
c. Someone brought the cake.

- a presupposes and entails c, while b only presupposes c;

- entailment is non-cancelable, presupposition is cancelable: *Paul did not stop smoking* can be taken as a metalinguistic negation that cancels the presupposition;
- an example where there is entailment but no presupposition:

(40) a. Peter bought a cake.
     b. Someone bought a cake.

- Classic test for presupposition: if A presupposes B, then A implies B and nonB;

- entailment: preservation of truth values, presupposition: background information that guarantees the appropriateness of the statement in a given context;
- both are based on implication, but entailment has nothing to do with background information and moreover presuppositions survive in certain embedded contexts;
- in questions and negation:

(41) a. Did you show her the letter? ‘It was assumed that you were supposed to show her the letter’
b. Were you showing her the letter? ‘Is it true that you showed her the letter?’

Implicature

Turning scales in implicatures:

(43) Some students passed the exam.

a. implicature: not all

(44) Most students did not pass the exam.

a. implicature: some did

(45) Peter is tired, but Karel is not tired and there is nothing surprising about it.

- The contrast in *but* is taken as a presupposition:

(46) a. It is not true that Peter is singing, but Alík is sleeping.
b. Peter is singing, but Alík is sleeping?

- Temporal sequence is taken as an implicature: maxims of cooperative conversation: it recommends speakers to order events by time;
- if we have two options for ordering events, then the choice of one of them is meaningful in the given context;
- similarly *but*: if we use it instead of *and*, then we presuppose that there is a contrast between the two propositions;

Logical Conjunction of Phrases

(47) a. Peter slept and snored.
b. Peter was and will be a smoker.
c. Peter speaks quickly and incomprehensibly.
d. Peter read a book and a magazine.
e. Peter and Pavel read a book.

- This is strictly speaking a problem: conjunction is defined for

- atomic individuals are not enough, but plural individuals are;

(50) a. Romeo loves Juliet and Juliet loves Romeo.
b. Romeo and Juliet are lovers.

(51) a. Romeo is a lover.
b. Juliet is a lover.

Does not guarantee "Romeo and Juliet are lovers."

- is *and* ambiguous?

(52) a. Tom moved the piano and Karel moved the piano.
b. Tom and Karel moved the piano.

- distributive and collective reading;

- similarly from *Tom moved the piano. Karel moved the piano.* It does not follow *Tom and Karel moved the piano together.*
- it is possible at least for the reading:

(53) Tom and Karel moved the piano each separately.

- to say that it is a logical conjunction? No. Because *Tom and Karel moved the piano.* is neutral towards distributive/collective reading.
- similarly:

(54) The red and blue flag is hanging on the pole.

- problematic constructions: compound words, idioms, quantifiers:

(55) Some trains stop in Brno and in Jihlava.

- from this follows: *Some trains stop in Brno; Some trains stop in Jihlava.* But not vice versa. Scope problem.

Collective and Distributive Predication

- traditional division:

1) purely distributive predicates: *have blue eyes*, *run*, ...
2) purely collective predicates: *meet*, *be a good team*
3) mixed predicates: *write a letter*, *play a song*

(56) a. Peter and Marie have blue eyes. D
b. Peter and Marie met. C
c. Peter and Marie wrote a letter. D/C

- Aristotle's example:

(57) 2 and 3 are 5.

- usual solution: *and*: distributive ($\wedge$) and collective (forming a set)

(58) Peter and Marie wrote a letter.

a. Write_Letter(Peter) $\wedge$ Write_Letter(Marie)

b. Write_Letter({Peter,Marie})

- *a*, *i*, *s*

(59) a. Peter i Marie mají modré oči. D

b. ??Peter i Marie se sešli. C

c. Peter i Marie napsali dopis. D

(60) a. Peter s Marií mají modré oči. ?C
b. Marie s Klárou porodili ??holčičku.
c. Peter s Marií se sešli. C
d. Peter s Marií napsali dopis. C

(61) a. The ducks flew and quacked.
b. The ducks flew and swam.

Exercises

(62) a. Peter read RŠ and thought about the distinction between reality and fantasy. P $\wedge$ R
b. Rychlonožka and Červenáček are an advanced unit. P
c. Peter and Jana are married. Ambiguous: P $\wedge$ R / P
d. No barber shaves and cuts at the same time. P

(63)
a. Peter and Pavel are brothers. Ambiguous: P $\wedge$ R / can Peter / Pavel refer to a woman, then also Q $\wedge$ S???
b. A man and a woman got married yesterday. Ambiguous: P $\wedge$ R / just like brothers;
c. Peter and Pavel traveled on one ticket. Ambiguous: P $\wedge$ R / P
d. Lines a and b are parallel. P
e. Peter and Pavel are cousins. Ambiguous: P $\wedge$ R / P

(64) a. Red and blue flags were hanging on the poles.
Ambiguous: $P \wedge R$ / P

b. No one except you can come to my party. $P \wedge \neg R$

c. I hate everyone except Karel. $P \wedge \neg R$

d. Peter and Pavel bought an apartment together. P

e. I believe that Karel and Peter will be there. P Not believing parts of the conjunction is irrational, but not illogical. I can have psychological lapses.

f. Peter and Karel sang and then danced. $P \wedge Q \wedge R \wedge S$

(65)
a. Postmodern art and premodern art are interesting. $P \wedge Q$
b. I live between Brno and Prague. P
c. Peter's sweater is black and white. P
d. Members of the American Congress are Republicans and Democrats. P

1.7 Negace

- explicitně: *Není pravda, že ...*

(66) Není pravda, že Brno je větší než Praha.

- obvykle: Brno není větší než Praha.
- na auxiliáru nebo na verbu:
- nebo na participiu:

(67) a. Petr nekouří.
b. Brno je neporazitelné.

(68) Výrok α je logickou negací v případě, že je analyzovatelný na podvýrok β takový, že α je pravdivý tehdy a jen tehdy, když β je nepravdivý.

```
import ttg

print(ttg.Truths(['p'], ['~p']))
```

```
## +-----+------+
## |  p  |  ~p  |
## |-----+------|
## |  1  |  0   |
## |  0  |  1   |
## +-----+------+
```

Negation and information structure

(69) John nezabil soudce stříbrným kladivem.

a. $\neg$p ... John zabil soudce stříbrným kladivem

b. John zabil soudce $\neg$ stříbrným kladivem

(70) Peter didn't come because Marie invited him.

a. [Because Marie invited him]$_T$, Peter didn't come.

b. [Peter came]$_T$ not:, because Marie invited him.

Negation and IS

- It is important to realize what the negated sentences are negations of: *Peter did not read The Idiot* is not a negation of the sentence *Peter read* but of the sentence *Peter read The Idiot*;
- however, the truth conditions of negation with a wide scope are almost vacuous:

(71) Peter did not eat the candies.

a. $\neg\exists e\exists x(\text{Agent(Peter,e)} \wedge \text{eating(e)} \wedge \text{Theme(x,e)} \wedge \text{candies(x)})$

b. … in a universe with one fish and nothing else

Czech examples

- negation naturally creates truth-conditional effects depending on the information structure
- Hajičovský example

(72) Babička nespí kvůli únavě.

a. Říkám o babičce, že není pravda, že spí kvůli únavě. ..., vždyť ona je vzhůru.

b. Říkám o babiččině spaní, že není pravda, že je kvůli únavě. ..., ale kvůli tomu, že je nemocná

c. Říkám o babiččině nespaní, že je to kvůli únavě. ..., ne kvůli tomu rámusu

- but this is purely a distinction based on topic and focus and

Other negated expressions

(75) nikdo, nic, nikde, nikdy

- what is *nikdo*:

(76) Nikdo nepřišel =? Není pravda, že každý přišel.

- it is not a contradiction;
- contradiction = 1-0, 0-1; *Peter died / Peter did not die.*
- *nikdo – každý* – kontrární: they cannot both be true, but they can both be false;

Other negated expressions

(77) a. nikdo = ne někdo
b. nic = ne něco
c. nikde = ne někde
d. nikdy = ne někdy

- negative concord in Czech: a negated argument must have negation on the verb, does not apply to non-arguments:

(78) Brno není neporazitelné. = Brno je porazitelné.

(79) Nikdo není dokonalý x= Někdo je dokonalý.= Nikdo je dokonalý.

Symbol

$\neg$, $\sim$

Ambiguity and the need for parentheses

(80) a. 6:3x2 = 1 ?
b. 6:(3x2) = 1
c. (6:3)x2 = 4

(81) It is not true that Peter is tall and Karel is tall.

(82) a. $\neg(P \wedge K)$
b. $\neg P \wedge K$

(83) Note: $\neg(P \wedge K) \neq \neg P \wedge \neg K$

Without

(84) He finished driving school without passing the tests.

(85) $A \wedge\neg T$

- but:

(86) Without cardamom, you cannot cook curry.

- it is a conditional: If you do not have cardamom, then you cannot cook curry;

Arguments

- two statements (premise and conclusion) in one sentence:

(87) I fired him because he asked for it.

(88) We will win the next election because we are the most popular.

(89) a. He asked for it.
b. $\models$ I fired him.

Arguments

- more complicated:

(90) Bohouš was not both a member of the RŠ and the BKK. He was a member of the BKK, so he was not a member of the RŠ.

(91) $\neg(R \wedge B), B \models \neg R$

- deductively valid: the first premise is true if R or B is false (to make the conjunction false and thus the negation true), the second premise states the truth of B, so only the falsity of R remains, qed;

Other problems with negation

(92) a. It is not true that some dogs are hairy.
b. Some dogs are not hairy.
c. Peter did not see Marie leave.
d. Peter saw Marie not leave.
e. It is not true that Peter is a good mushroom picker.
f. Peter is a bad mushroom picker.

- cannot change the scope of negation and other logical operators: quantifiers, psychological verbs, ...

Well-formed formulas

- How many tokens of a simple expression are in the notation:

(93) $(P \wedge Q) \wedge \neg P$

- 2*P, Q, P & Q, $\sim$P, whole: 6; e.g. & $\sim$P is not a WFF;
- examples: which of the following VL expressions are WFF, what kind of statements are they, create Czech sentences that can be symbolized by them:

(94)
a. $P \wedge \neg(T \wedge (R \wedge S))$
b. $\neg T \wedge \neg R$
c. $\neg(T \wedge Q$
d. $\neg\wedge P\ Q$

Scope

(95) The scope of a token of an expression is the shortest well-formed formula in which this expression occurs. E.g.:

(96) $\neg(P \wedge R)$

- for conjunction, it is the inside of the parentheses;
- relative scope: wider and narrower;

(97) The scope of the token of expression alpha is wider than the scope of the token of expression beta within some well-formed formula gamma if the scope of expression beta is a proper part of the scope of expression alpha.

Scope

- exercise:

(98) a. What is the scope of the second token of conjunction: $(P \wedge Q) \wedge (\neg R \wedge T)$

b. First token of negation: $\neg\ P \wedge (Q \wedge \neg R)$

c. Third token of conjunction: $\neg(P \wedge Q) \wedge (R \wedge \neg S)$

Main connective

- the connective with the widest scope;
- determines the logical status of the statement;

Analysis of statements using truth tables

- three permissible forms of a deductively valid argument: true premises, true conclusion, one or more false premises + true conclusion, one or more false premises and false conclusion, the only forbidden: true premises + false conclusion;
- truth tables provide us with the overall value (T/F) for any complex statement depending on the truth of individual components;

(99) Červenáček is a member of RŠ, but it is not true that at the same time Štětináč raises rats and Rychlonožka is brave.

(100) $R \wedge \neg(S \wedge T)$

- advice: a truth table for n simple statements has 2N rows;
- to ensure we do not miss any possibility or have duplicates: in each subsequent column, the number of $1/0$ doubles;
- the result is under the main connective of the statement;

Arguments:

(101) a. It is not true that at the same time MD is brave and Bohouš is truthful.

b. MD is brave.

c. $\models$ Bohouš is not truthful.

(102) a. $\neg(S \wedge P)$

b. S

c. $\models \neg P$

(103) $\neg(S \wedge P), S \models \neg P$

```
import ttg

print(ttg.Truths(['p', 'q'], ['~(p and q)','p','~q']))
```

```
## +-----+-----+--------------+-----+------+
## |  p  |  q  |  ~(p and q)  |  p  |  ~q  |
## |-----+-----+--------------+-----+------|
## |  1  |  1  |      0       |  1  |  0   |
## |  1  |  0  |      1       |  1  |  1   |
## |  0  |  1  |      1       |  0  |  0   |
## |  0  |  0  |      1       |  0  |  1   |
## +-----+-----+--------------+-----+------+
```

- deductively valid;
- if the truth table of an argument in every row, where the premises are evaluated as 1 under the main connective, is evaluated as 1 under the conclusion, it is a truth table of a deductively valid argument;

- invalid:

(104) a. It is not true that at the same time Rychlonožka is in the clubhouse and Červenáček is knitting nets.
b. Rychlonožka is not in the clubhouse.
c. $\not\models$ Červenáček is not knitting nets.

(105) a. $\neg(S \land P)$

b. $\neg S$

c. $\vDash \neg P$

```
import ttg

print(ttg.Truths(['p', 'q'], ['~(p and q)','~p','~q']))
```

```
## +-----+-----+--------------+------+------+
## |  p  |  q  |  ~(p and q)  |  ~p  |  ~q  |
## |-----+-----+--------------+------+------|
## |  1  |  1  |       0      |  0   |  0   |
## |  1  |  0  |       1      |  0   |  1   |
## |  0  |  1  |       1      |  1   |  0   |
## |  0  |  0  |       1      |  1   |  1   |
## +-----+-----+--------------+------+------+
```

- exercise:

(106) a. Rychlonožka, Červenáček, and Bohouš ran a race at Masná enclosure.

b. It is not true that Rychlonožka and Štětináč ran a race at Masná enclosure.

c. $\vDash$ It is not true that Bohouš and Štětináč ran a race at Masná enclosure.

(107) a. Mažňák is defeatable.

b. $\vDash$ Mažňák is not undefeatable.

(108) $M \vDash \neg\neg M$

(109) a. Petr finished driving school without passing the final tests.

b. $\models$ Petr did not pass the final tests.

(110) a. $A \wedge \neg Z$

b. $\models \neg Z$

(111) A Z $A \wedge \neg Z \models \neg Z$

Logical disjunction

- expressions: “either – (or)”, “or”;
- syntactically: disjunction of lexical verbs, auxiliaries, adverbs, objects, subjects, NP modifiers;

(112) Def.: Statement C is a logical disjunction if it can be analyzed into components A and B, and from the truth of either A or B follows the truth of C and vice versa.

- table:

```
import ttg

print(ttg.Truths(['p', 'q'], ['p or q']))
```

```
## +-----+-----+----------+
## |  p  |  q  |  p or q  |
## |-----+-----+----------|
## |  1  |  1  |    1     |
## |  1  |  0  |    1     |
## |  0  |  1  |    1     |
## |  0  |  0  |    0     |
## +-----+-----+----------+
```

- vee - v/$\vee$

Disjunction and Negation

(113) Neither Rychlonožka nor Červenáček was in the clubhouse.

- this is not a disjunction, but a negation;
- if it were a disjunction of two negations, then it would be enough for the truth if one boy was not in the clubhouse, but the sentence is true only in one case: 0 0 (i.e., negation of disjunction): $\neg$(R v C)
- $\neg$(R v C) is not the same as $\neg$R v $\neg$C
- truth: 0 0   1 0, 0 1, 0 0

- similarly:

(114) Peter is neither a violinist nor a football player. $\neg$(H v F)

- Logical form: different concepts (analytical philosophy: inferential potential of the sentence – Frege, epistemological potential of the sentence – Russell, ontological state of the world – Wittgenstein) X LF in generative grammar is purely a linguistic level of representation, which determines what transformations are allowed: QR, binding of anaphors, islands, ...;

- but:

(115) Either Karel is not a violinist, or he is not a football player. $\neg$H v $\neg$F

(116) Karel left and Marie stayed or Klára stayed.

(117) a. Either Karel left and Marie stayed, or Klára stayed.
b. Karel left and either Marie stayed, or Klára stayed.

(118) a. (K & M) v L
b. K & (M v L)

- this is ungrammatical in VL: K & M v L;
- rule: Every WFF in VL can have only one connective with the widest scope: the main connective;
- we relax this for multiple disjunctions and conjunctions;

Exercise

A) symbolize and if ambiguous, then all meanings;

(119) Give me freedom or I will die. Not a statement.

(120) Either Peter ate meat, or Karel ate fish. P v K

(121) I don't want freedom and I don't want to die. $\neg$S & $\neg$Z

(122) Neither coffee nor tea is free with the menu. $\neg$(K v C)

(123) Tonight I am going for a beer with Klára or with Bára or with Karel. K v B v L

(124) I did not feel well in England or in France. $\neg$(A v F)

(125) It was neither my first nor my last disappointment. $\neg$(P v

B) symbolize arguments:

(126) I will become either a professor or a rich man. I will not become rich. So I will become a professor.

(127) $P \vee B, \neg B \models P$

(128) Karel is not a plumber because neither he nor Peter is a plumber.

(129) $\neg(K \vee P) \models \neg K$

(130) Because neither Karel nor Peter went to the exhibition, it is not true that Karel and Pavel went to the exhibition.

(131) $\neg(K \vee P) \models \neg(K \mathbin{\&} R)$

(132) Karel or Peter, but not Marie, went on a trip. Karel did not go on a trip, so Peter went on a trip.

(133) $(K \vee P) \& \neg M, \neg K \models P$

C) Truth tables for the last four arguments

(134) $P \vee B, \neg B \models P$

- deductively follows;

(135) $\neg(K \vee P) \models \neg K$

- deductively follows;

(136) $\neg(K \vee P) \models \neg(K \mathbin{\&} R)$

- deductively follows;

(137) $(K \vee P) \mathbin{\&} \neg M, \neg K \models P$

- deductively follows;

Inclusive or Exclusive Disjunction

- according to some logicians, disjunction in natural language is as ambiguous as "kose" in Czech;
- examples:

(138) You can pay by card or in cash.

(139) You can have beer, wine, or rum.

(140) I am going for a beer with Karel or with Pavel today.

(141) Today is either Monday or Tuesday.

(142) Rychlonožka was born either on the Other Side or in Dvorce.

- exclusive table;
- there is indeed a good reason: e.g., 3: I am going with K. and we meet P. by chance, is then 3 false? Hardly.
- 4 is not an argument at all;
- 7 can get a spanking for something else (incomplete induction);
- at least unconvincing;
- especially a problem for inferences:

(145) a. Karel ate soup or spinach.
b. Karel ate soup.
c. $\models$ Karel did not eat spinach.

- rather not;
- pragmatic justification: conjunction is logically stronger than disjunction, so by asserting disjunction we presuppose the negation of conjunction – just like with quantifiers: Some students took the test presupposes that not all of them took it;

Problems

(146) Either Karel wants a cat, or he wants a pig.

(147) Karel wants a cat or a pig.

(148) Either Klára weighs less than Bára, or she weighs less than Pavel.

(149) Klára weighs less than Bára or Pavel.

- 1 and 3 are disjunctions, but 2 and 4 are ambiguous:

(150) a. Karel wants a cat or a pig, but I don't know which.
b. Karel wants a cat or a pig, but he doesn't care which.

- from b) it does not follow that Karel wants a cat. nor Karel wants a pig. – not a logical disjunction;

- if connectives are in the scope of psychological words, then they do not behave like usual connectives;
- similarly 4: from a) Klára weighs less than Bára. b) Klára weighs less than Pavel. it does not follow 4, because Klára can weigh less than Bára without weighing less than Bára or Pavel.

Free choice reading of disjunction

(151) a. Karel can be in Paris or in London. $\rightarrow$ Karel can be in Paris and Karel can be in London.

b. Karel is in Paris or in London. $\not\rightarrow$ Karel is in Paris and Karel is in London.

c. Karel must be in Paris or in London. $\not\rightarrow$ Karel must be in Paris and Karel must be in London.

- parallel to the distribution of *-koliv*:

(152) a. Karel can go to any city.

b. #Karel went to any city.

c. #Karel must go to any city.

- standard analysis of *or* + standard analysis of modal verbs cannot explain FCI interpretation
- *can* ... $\exists$ across possible worlds, *must* ... $\forall$ across possible worlds:

(153) a. We can invite Karel or Marie to the party. $\rightarrow$ We can invite Karel to the party and we can invite Marie to the party.

b. $MAY(\phi(k) \vee \phi(m)) \not\rightarrow$ $MAY(\phi(k)) \wedge MAY(\phi(m))$

Material Implication

(154) If Karel sings, then Alík growls.

- antecedent and consequent;
- other ways to express it: *provided that, on the condition that, if, when, in case*;
- if the antecedent is 1 and the consequent is 0, then the conditional is 0 and vice versa;

- sometimes it can be expressed not only in the form of a subordinate clause:

(155) Karel's singing leads to/has the result of/causes Alík's growling.

(156) Every student loves their teachers.

- definition:

(157) Statement A is a logical conditional only if statement A can be analyzed into two sub-statements: B and C, where B (antecedent) is a condition for the truth of C (consequent), and A as a whole is 0 only if B is 1 and C is 0.

```python
import ttg

print(ttg.Truths(['p', 'q'], ['p => q']))
```

```
## +-----+-----+----------+
## |  p  |  q  |  p => q  |
## |-----+-----+----------|
## |  1  |  1  |    1     |
## |  1  |  0  |    0     |
## |  0  |  1  |    1     |
## |  0  |  0  |    1     |
## +-----+-----+----------+
```

- this is the so-called material implication (conditional);
- common usage does not seem to commit to B=0, but allowing this into the system would have fatal consequences for entailment;
- so we accept that material implication is a reasonable formalization of the condition;

Symbol

- $\rightarrow$, $\supset$;
- need for parentheses:

(158) a. If it is not true that Špilberk is open, then Petrov is open.

b. It is not true that if Špilberk is open, then Petrov is open.

Necessary and Sufficient Condition

(160) Writing the final test is a sufficient condition for getting credit.

(161) Writing the final test is a necessary condition for getting credit.

- both are conditionals;

(162) Sufficient condition: if A is a sufficient condition for B, then if A occurs, B will occur.

- $A \rightarrow B$
- for the necessary condition, it is the opposite: writing the test does not guarantee 1 for the consequent, so it is not an implication (in this direction);
- conversely, yes: 1 in the second part of the necessary condition guarantees 1 in the first part;

(163) Necessary condition: if A is a necessary condition for B, then if B occurs, A will also occur.

Only if

(164) Karel will get a driver's license only if he passes the tests.

- is it a necessary or sufficient condition?
- it is a necessary but not a sufficient condition;
- only reverses the order:

(165) A, only if B $=$ A $\rightarrow$ B

- we consider it as a conditional, but it is not equivalent to a conditional in all respects:

(166) If water boils, then it evaporates.

(167) If my pulse rises, then I am riding my bike fast.

- should be the same as:

(168) Water boils only if it evaporates.

(169) My pulse rises only if I am riding my bike fast.

- intuitively, conditionals are much weaker than only if because they do not assert anything about the necessity of the given relationship;
- it is the difference between a focus operator (*only*) and a simple implication (*if ..., then ...*):

(170) Only Peter collected enough old paper.

Unless

(171) Peter will die unless he is operated on.

- conditional, but what kind?
- the only thing it asserts is that $\mathsf{A} \rightarrow \mathsf{B} = \neg\mathsf{B} \rightarrow \mathsf{A}$;

- examples:

(172) Peter will go to the demonstration unless the police arrest him.

(173) Peter will not become president unless Karel is elected prime minister again.

Because, given that, since

- so far we have considered these expressions as indicators of premises, which we will not change

(174) I will leave the window open because it is not raining.

- both statements must be 1 (if the whole is 1), not so for conditionals:

(175) I will leave the window open if it is not raining.

Conditionals and Parentheses

(176) a. If oxygen is sufficient for the fire to start, then the alarm will ring.

b. If there is oxygen, then if a fire starts, the alarm will ring.

(177) a. $(O \rightarrow F) \rightarrow A$

b. $O \rightarrow (F \rightarrow A)$

- $A \rightarrow B \rightarrow C$ is a poorly formed formula;
- for others, it worked: A v B v C ... conditional is asymmetric;

If and Only If

- biconditional; sometimes has its own symbol $\equiv$

Strengthening Conditionals

- in many cases, material implication seems too weak (in terms of truth conditions) to capture the meaning of conditionals in natural language
- examples of so-called *conditional perfection* (Geis & Zwicky 1971):

(178) a. If you mow the lawn, I'll give you five dollars.
b. If John leans out of that window any further, he'll fall.
c. If you disturb me tonight, I won't let you go to the movies tomorrow.
d. If you heat iron in a fire, it turns red.

- rather than $p \to q$: $p \to q \wedge \neg p \to \neg q$
- biconditional

p	q	$\rightarrow$	$\equiv$
1	1	1	1
0	1	1	0
1	0	0	0
0	0	1	1

- counterexamples (see Von Fintel, Kai. "Conditional strengthening." Unpublished manuscript (2001).):

(179) a. If it doesn't say 'Goodyear', it isn't Polyglas.
b. If this cactus grows native to Idaho, then it's not an Astrophytum.
c. If you scratch on the eight-ball, then you lost the game.
d. If the axioms aren't consistent with each other, then every WFF in the system is a theorem.

Validity

- validity is a property of arguments;
- repetition;
- for an argument:

(180) $\neg A \vee B, \neg(A \wedge B) \models B \rightarrow A$ invalid

(181) $E \rightarrow F \models \neg F \rightarrow \neg E$ valid

(182) $D \rightarrow (B \wedge C) \models B \rightarrow (C \rightarrow D)$ invalid

(183) $(P \rightarrow Q) \rightarrow R \models P \rightarrow (Q \rightarrow R)$ valid

Contradiction, Tautology, Contingency

- contradiction = a statement that is necessarily false;
- tautology = a statement that is necessarily true;
- contingent statement = a statement that can be either true or false;

(184) Either Brno is the most beautiful city in the world or Brno is not the most beautiful city in the world.

- table;

```python
import ttg

print(ttg.Truths(['p'], ['p or ~p']))
```

```
## +-----+-----------+
## |  p  |  p or ~p  |
## |-----+-----------|
## |  1  |     1     |
## |  0  |     1     |
## +-----+-----------+
```

- logical truths = necessary truths = tautologies;

(185) Starobrno is a good beer, but Starobrno is not a good beer.

```
import ttg

print(ttg.Truths(['p'], ['p and ~p']))
```

```
## +-----+-------------+
## |  p  |  p and ~p   |
## |-----+-------------|
## |  1  |      0      |
## |  0  |      0      |
## +-----+-------------+
```

- an argument that has a contradiction as one of its premises is necessarily deductively valid, which is why contradictions are so problematic;
- conversely, if the conclusion is a tautology, then the argument is also necessarily deductively valid;

(186) $P \vee \neg Q$

(187) $(P \wedge Q) \wedge (\neg P \wedge Q)$ contradiction

(188) $(P \rightarrow Q) \wedge (\neg P \rightarrow Q)$ contingency

(189) $P \wedge (P \rightarrow (\neg P \wedge Q))$ contradiction

(190) $P \wedge (Q \rightarrow Q)$ contingency

Consistency

- if someone is inconsistent, then not everything they say is true;
- i.e., in the truth table, there must be at least one row where all statements are true;

(191) $\{P \rightarrow Q, Q \rightarrow (R \wedge \neg S), \neg S \rightarrow \neg P, P\}$ inconsistent

(192) $\{(P \wedge Q) \vee \neg R, R \rightarrow (\neg Q \rightarrow P)\}$ consistent

(193) $\{P \rightarrow (R \vee \neg T), \neg P \rightarrow (\neg R \vee T), R \wedge \neg T\}$ consistent

```python
import ttg

print(ttg.Truths(['p', 'q', 'r', 's'], ['p => q', 'q => (r and ~s)', '~s => ~p', 'p']))
```

```
## +-----+-----+-----+-----+----------+------------------+------------+-----+
## |  p  |  q  |  r  |  s  |  p => q  |  q => (r and ~s)  |  ~s => ~p  |  p  |
## |-----+-----+-----+-----+----------+------------------+------------+-----|
## |  1  |  1  |  1  |  1  |    1     |        0         |     1      |  1  |
## |  1  |  1  |  1  |  0  |    1     |        1         |     0      |  1  |
## |  1  |  1  |  0  |  1  |    1     |        0         |     1      |  1  |
## |  1  |  1  |  0  |  0  |    1     |        0         |     0      |  1  |
## |  1  |  0  |  1  |  1  |    0     |        1         |     1      |  1  |
## |  1  |  0  |  1  |  0  |    0     |        1         |     0      |  1  |
## |  1  |  0  |  0  |  1  |    0     |        1         |     1      |  1  |
## |  1  |  0  |  0  |  0  |    0     |        1         |     0      |  1  |
## |  0  |  1  |  1  |  1  |    1     |        0         |     1      |  0  |
## |  0  |  1  |  1  |  0  |    1     |        1         |     1      |  0  |
## |  0  |  1  |  0  |  1  |    1     |        0         |     1      |  0  |
## |  0  |  1  |  0  |  0  |    1     |        0         |     1      |  0  |
## |  0  |  0  |  1  |  1  |    1     |        1         |     1      |  0  |
## |  0  |  0  |  1  |  0  |    1     |        1         |     1      |  0  |
## |  0  |  0  |  0  |  1  |    1     |        1         |     1      |  0  |
## |  0  |  0  |  0  |  0  |    1     |        1         |     1      |  0  |
## +-----+-----+-----+-----+----------+------------------+------------+-----+
```

```
import ttg

print(ttg.Truths(['p', 'q', 'r'], ['(p and q) or ~r', 'r => (~q => p)']))
```

```
## +-----+-----+-----+-----------------+-----------------+
## |  p  |  q  |  r  |  (p and q) or ~r  |  r => (~q => p)  |
## |-----+-----+-----+-----------------+-----------------|
## |  1  |  1  |  1  |        1        |        1        |
## |  1  |  1  |  0  |        1        |        1        |
## |  1  |  0  |  1  |        0        |        1        |
## |  1  |  0  |  0  |        1        |        1        |
## |  0  |  1  |  1  |        0        |        1        |
## |  0  |  1  |  0  |        1        |        1        |
## |  0  |  0  |  1  |        0        |        0        |
## |  0  |  0  |  0  |        1        |        1        |
## +-----+-----+-----+-----------------+-----------------+
```

Logical Equivalence

- it is a relationship between two statements;

(194) Two statements are logically equivalent if their truth tables are identical.

(195) It is not true that Peter and Karel came. $=$? Either Peter did not come, or Karel did not come.

(196) $\neg(\mathrm{p} \wedge \mathrm{q}) =?\ \neg\mathrm{p} \vee \neg\mathrm{q}$

(197) $(A \to B) \to (A \to C)$, $A \to (B \to C)$ equivalent

(198) $(P \vee Q) \vee R$, $\neg(\neg P \vee \neg Q) \wedge \neg R$

(199) $A \to (B \to C)$, $\neg C \to \neg A$

(200) $(P \wedge Q) \to L$, $P \to (Q \to L)$

```python
import ttg

print(ttg.Truths(['p', 'q'], ['~(p or q)', '~p and ~q']))
```

```
## +-----+-----+-------------+-------------+
## |  p  |  q  |  ~(p or q)  |  ~p and ~q  |
## |-----+-----+-------------+-------------|
## |  1  |  1  |      0      |      0      |
## |  1  |  0  |      0      |      0      |
## |  0  |  1  |      0      |      0      |
## |  0  |  0  |      1      |      1      |
## +-----+-----+-------------+-------------+
```

Exercise

(201) If a fetus is a person, then the fetus has a right to life. If the fetus has a right to life, then it is not true that there is a right to end its life. However, if abortions are moral, someone has the right to end the life of the fetus. It follows: if fetuses are persons, then abortions are immoral.

(202) $P \rightarrow R, R \rightarrow \neg S, A \rightarrow S \vDash P \rightarrow \neg A$

Reference