Testing a PPI analysis of superlative-modified numerals

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Abstract Superlative-modified numerals differ from comparative-modified numerals in that they trigger speaker ignorance inferences obligatorily, and are also degraded under negation. While the ignorance contrast has been widely studied, both experimentally and theoretically, the negation contrast has been largely neglected. Recently it has been proposed (Spector 2015) that the resistance of superlative-modified numerals to embedding under negation must not be studied in isolation, but rather by identifying superlative-modified numerals as part of a larger class of positive polarity items. According to Szabolcsi (2004), and more recently Nicolae (2012) and Spector (2014), while positive polarity items vary with respect to what types of downward-entailing operators anti-license them, they are all known to be (1) degraded in the immediate scope of negation but (2) acceptable in the antecedent of a conditional or the restriction of a universal. Furthermore, they are also known to be (3) acceptable in the immediate scope of negation if the negation itself is further embedded in another downward-entailing environment. We test these expectations for superlative-modified numerals, as well as predictions made by alternative accounts involving processing complexity of negation and valence matching. While we find straightforward support for predictions (1) and (2) of the PPI analysis, the patterns related to (3) seem to be complicated by these additional factors as well.

Keywords superlative-modified numerals; positive polarity items; anti-licensing; rescuing; processing negation; experimental data

1 Introduction

Comparative-modified numerals (*more/less than three*) and superlative-modified numerals (*at least/most three*) can express similar truth conditions but exhibit other interesting contrasts, including a reported contrast under negation. In this paper we experimentally investigate this contrast under negation in depth from the perspective of a positive polarity item analysis of superlative-modified numerals. While we find multiple sources of support for such an analysis, we also identify some challenges, and investigate some variations of the PPI analysis and alternative explanations for these challenges. In doing so, we contribute not just new data that support a better understanding of polarity sensitive items, but also a more articulated set of predictions for acceptability of these items in a wider variety of contexts.

1.1 Motivating data

A comparative-modified numeral such as *less than four* and a superlative-modified numeral such as *at most three* have equivalent truth conditions.

(1) **Truth conditions**

a. John has less than four diamonds.
→ John has 0, 1, 2, or 3 diamonds.

b. John has at most three diamonds.
→ John has 0, 1, 2, or 3 diamonds.

However, they differ in other ways. First, in an out-of-the-blue unembedded context the comparative-modified numeral may trigger speaker ignorance inferences, but these are optional; in contrast, for the superlative-modified numeral these inferences are obligatory (Geurts and Nouwen 2007, Nouwen 2010, Coppock and Brochhagen 2013, Kennedy 2015, Spector 2015, Cremers et al. 2017, a.o.), (2). Second, the comparative-modified numeral can easily take semantic scope under negation, but the superlative-modified numeral has a much harder time doing so (Nilsen 2007, Geurts and Nouwen 2007, Cohen and Krifka 2011, 2014, Coppock and Brochhagen 2013, Mayr 2013, Spector 2015), (3).

(2) **Ignorance**

a. John has less than four diamonds.
($\rightsquigarrow$ The speaker doesn't know how many diamonds John has.) (optional ignorance)

b. John has at most three diamonds.
$\rightsquigarrow$ The speaker doesn't know how many diamonds John has. (obligatory ignorance)

(3) **Embedding under negation**

a. John doesn't have less than four diamonds.
→ John has four or more diamonds. (easy)

b. John doesn't have at most three diamonds.
→ John has four or more diamonds. (hard)

The ignorance contrast has been extensively studied in the literature, both experimentally and theoretically (see the references above). In contrast, the negation contrast has been largely neglected. While it is sometimes acknowledged (see the references above), few theories have tried to derive it (only Cohen and Krifka 2011, 2014, Spector 2015), or do so in a way that integrates it with the ignorance contrast (only Spector 2015).

A recent proposal by Spector (2015) argues that the resistance of superlative-modified numerals to embedding under negation must not be studied in isolation but rather taken as a clue to diagnosing superlative-modified numerals more generally as positive polarity items. Once we recognize the similarity, a general recipe can be used that will derive not just the negation facts for a wide range of positive polarity items in general, but also the ignorance facts for superlative-modified numerals in particular.

Our goal in this paper is to investigate the empirical basis for a positive polarity item analysis of superlative-modified numerals, while considering extensions and variations of a PPI account and testing their predictions. We ask, therefore: to what extent do superlative-modified numerals behave as we would expect if they were positive polarity items? And, to the extent that other factors may be at play, what predictions do they make if superlative modified numerals are PPIs?

1.2 Background on positive polarity items

As their name suggests, polarity sensitive items are items whose felicity is sensitive to the polarity of their embedding environment. Negative polarity items (NPIs) are items that have to be in a negative environment, (4), and conversely positive polarity items (PPIs) are items that have to be in positive

environments, (5). For example, (4) is only good if the NPI *anything* is in the scope of a negation, while (5) is good only if the PPI *something* is interpreted above negation (*not > something).

(4) I *(don't) see anything. NPIs

(5) I (*don't) see something. PPIs

But this description is too simplistic. The rich literature on polarity sensitive items has shown that in fact *negative* must be understood more broadly as *downward-entailing*, and *positive* – as *upward-entailing*. For example, the PPI *some* is also bad in the scope of another downward-entailing operator such as *without*, (6).

(6) John came to the party without someone. *without > someone

Moreover, it has been shown that, whether 'negative' or 'positive', polarity sensitive items may be additionally sensitive to other features of the environment. For example, in the scope of *no* the PPI *someone* is bad, but the PPI *still* is fine:

(7) No one called someone. *no one > someone

(8) No student is still sleeping. ✓no student > still

Since the object of our interest is PPIs, let's leave NPIs aside and sum up what we have found out for PPIs: PPIs are items that are anti-licensed in downward-entailing environments. Moreover, individual PPIs may vary as to what type of downward-entailing environments anti-license them.

In spite of this variation, there are some features that all PPIs share (Szabolcsi 2004 and references therein; Spector 2014). (Below the square brackets indicate semantic scope, not clausal boundary.)

1. **All PPIs are anti-licensed in the immediate scope of negation.**[1] For example, although *someone* and *still* vary with respect to the types of downward-entailing operators that anti-license them as we saw above, they are both anti-licensed in the immediate scope of negation.

   (9) John didn't see someone. *not [someone]

   (10) John isn't still sleeping. *not [still]

2. **No PPI is anti-licensed in the antecedent of a conditional or the restriction of a universal.** For example, while both *someone* and *still* are bad in the immediate scope of negation, they are both good in the antecedent of a conditional or the restriction of a universal.

   (11) ✓if/every [someone][ ]

   a. If John saw someone, he should have let us know.
   b. Every student who saw someone should speak up.

   (12) ✓if/every [still][ ]

   a. If John is still sleeping, we'll leave without him.
   b. Every student who is still sleeping will be left behind.

[1] Many are fine in the scope of extraclausal negation; see Nicolae (2012). Some are however anti-licensed even at a distance; see Spector (2014).

3. **If the anti-licenser is itself in the scope of a downward-entailing operator, then the PPI is *rescued*, i.e., it can be interpreted in the scope of the anti-licenser.** For example, *someone* under negation becomes good when further embedded in the scope of another downward-entailing operator such as *doubt*, and *still* under negation becomes good when further embedded in the antecedent of a conditional.

(13) I doubt that John didn't call someone. ✓doubt [not [someone]]

(14) If John weren't still sleeping, we would leave. ✓if [not [still]][ ]

Recall that our purpose in discussing PPIs was to get a better understanding of what it means to say that superlative-modified numerals are PPIs. Despite the variation among PPIs, we have seen that they also have some things in common, namely, (1) all PPIs are anti-licensed in the immediate scope of negation, (2) no PPIs are anti-licensed in the antecedent of a conditional or the restriction of a universal, and (3) all PPIs exhibit rescuing if the anti-licensing operator is itself embedded in a downward-entailing environment. If superlative-modified numerals are PPIs, then we expect to find them exhibiting all these patterns.

1.3 Goal of this paper

In this paper we present three experiments where we test whether superlative-modified numerals exhibit the signature behaviors of a PPI. In Experiment 1 (§2) we test all of the following: (1) Anti-licensing in the immediate scope of an unembedded negation, (2) Licensing in the antecedent of a conditional and in the restriction of a universal, and (3) Rescuing if the anti-licensing operator (we test for negation) is itself embedded in a downward-entailing environment (we test for further embedding in the antecedent of a conditional and in the restriction of a universal). We find support for (1) and (2) but not for (3) – Rescuing. In two follow-up experiments we probe this pattern further by testing different related hypotheses of the negation contrast. In Experiment 2 (§3) we test whether Rescuing of superlative modified numerals might be conditioned on negativity match between the embedding antecedent of a conditional / restriction of a universal and the continuation. We find support for this for the superlative modifier *at least* but not for *at most*. In Experiment 3 (§4) we test whether Rescuing of superlative modified numerals is sensitive to the type of higher downward-entailing environment in which the anti-licenser is embedded. We find that that is indeed the case, as matrix negation does not seem to rescue. In sum, while superlative-modified numerals seem to behave like PPIs with respect to Anti-licensing and Licensing, they do not seem to behave the same with respect to Rescuing. We take this to indicate that a PPI analysis for superlative-modified numerals does make sense, but that the Rescuing expectation may need to be adjusted by other considerations.

1.4 General note on methodology

Task, instructions, stimuli, design In each of our three studies we presented participants with superlative-modified numerals in various linguistic contexts, using comparative-modified numerals as controls.

Some of our critical items in Experiment 1 involved a superlative-modified numeral in an unembedded context. As mentioned at the outset, in this type of context it has been observed that superlative-modified numerals require speaker ignorance. We did not want any contrasts we would find between

superlative- and comparative-modified numerals to be due to this separate factor. For that reason, taking advantage of the fact that in the same context comparative-modified numerals are observed to be compatible with ignorance, we decided to present our stimuli in a context that ensured speaker ignorance. To that end we adapted a card game from Cremers and Chemla (2017). For Experiments 2 and 3 it is not obvious that speaker ignorance about the exact number is crucial to the felicity of any of the critical items in the way it is for some of the critical items in Experiment 1. However, since they were follow-ups to Experiment 1, to facilitate comparison between the various experiments, we maintained a similar type of context and stimulus.

We were also faced with the question of how exactly to probe for the previously observed contrast between superlative- and comparative-modified numerals under negation, given the close relationships in this domain between syntax and semantics and our concerns about pre-judging the source of this contrast. In addition, our intuition is that the mathematical/logical nature of these numerical expressions means that with some concentration, one can compute the truth conditions in all cases, but that sometimes this involves much more difficulty for superlative-modified numerals. Therefore, we decided to ask for what we call *comprehensibility* judgments (*Do you think x will understand what y said?*). This directly builds into the probe the issue of processing (difficulty of parsing), but this is involved in standard acceptability judgments as well, just in a more indirect way. In addition, we will see that the results of these comprehensibility judgments do indeed show precisely the contrast under negation between superlative- and comparative- modified numerals that has been discussed in previous literature, which allows us to more carefully extend the inquiry to the other contexts predicted by a PPI analysis.

Finally, because the contrast we are interested in seems to be subtle and we wanted to maximize contrasts between items, and also because we had no particular concerns about conscious decision making affecting these judgments, we chose not to use any filler items. The questionnaires were prepared in Qualtrics (Qualtrics Labs 2016) such that each participant saw all the trials, in a separate random order for each participant.

Statistical analysis Results were analyzed in R (R Core Team 2015).

For logistic mixed effects models we used the *lme4* package (Bates et al. 2015), extracting the results of interest using the packages *effects* (Fox 2003) (predicted group mean probabilities and the confidence intervals associated with them) and *lsmeans* (Lenth 2016) (predicted odds ratios of interest and the confidence intervals associated with them). The p values for the planned comparisons were p-adjusted using the 'holm' method. In the results sections we abbreviate confidence intervals with 'CI' and odds ratios with 'OR'.

In all models we treated participants as random effects. We didn't treat items – the names of the card suits, i.e., diamonds, hearts, clubs, and spades – as random effects because we did not expect them to interact with comprehensibility judgments.

Plots were done with *ggplot2* (Wickham 2009). Throughout the paper we chose to plot model predictions, that is, the *predicted* means and the confidence intervals associated with them rather than the *raw* means and the confidence intervals associated with them. In general one good reason to do this is the fact that, if there are covariates, the means (and associated CIs) from the model will be adjusted for all these covariates. Another reason is for consistency: since odds ratios (and associated CIs) come out of the model, instead of reporting means (and associated CIs) from the raw data but then odds ratios (and associated CIs) from the model, we can report all the results from the same source, namely, the model. These considerations aside, note that for our data sets this choice does not make a difference for our pattern of results: as the reader can see in the Appendix, where we report the raw means (and their

associated CIs), the plots from the models and the plots from the raw data are similar. One difference is that the CIs from the model are sometimes narrower, but that is because the model takes into account the various interactions, and so it can yield more precision.

Our data and analysis scripts for each experiment are available online at [to be added later].

2 Experiment 1: (Anti-)licensing and rescuing

The goal of this experiment was to test if superlative-modified numerals do indeed exhibit the anti-licensing and rescuing patterns typical for a PPI. In particular, our critical items were sentences where the superlative-modified numeral was placed in six contexts: in a positive/negative declarative, in a positive/negative antecedent of a conditional, and in a positive/negative restriction of a universal.

2.1 Methods

Participants Participants were 27 self-reported native speakers of English, recruited via Amazon's Mechanical Turk and paid $2 for their participation. Two participants were excluded prior to analysis (one due to a coding error, the other due to multiple indications of not engaging with the task as instructed such as response times and response pattern).

Task and instructions Participants were introduced to the task as follows:

> In this survey you will answer questions about a group of friends playing a game. At the beginning of the game each player gets dealt a hand of seven cards. After taking a quick look at them, they must place the cards face down and try to remember their hands. Then they take turns giving clues about their hands to the other players in the form of statements describing their hands. You will see what a player remembers about his/her cards and the statement s/he makes, then you will be asked if you think the other players will understand what s/he said.
>
> Note: a  or  a means that the player doesn't remember if a particular card in his hand was a diamond or a heart, or a club or a spade, respectively.

Stimuli Participants then saw picture-sentence items presented as in Figure 1.

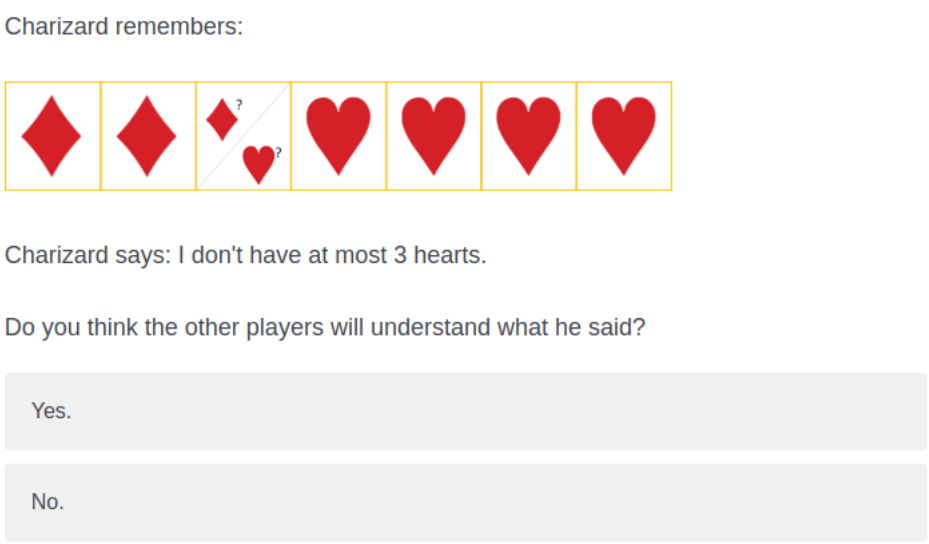

Figure 1: Example trial: Superlative-modified numeral in a negative declarative

Design Each participant saw 24 trials in total, obtained by crossing three factors: (1) Modifier Type (levels: Comparative, Superlative) / Modifier (levels: *more than*, *less than*, *at least*, *at most*); (2) Polarity (levels: Positive, Negative); and (3) embedding Environment (levels: Declarative (that is, no embedding), Antecedent of Conditional, Restriction of Universal). The names of the players were different for each item. The numeral was kept the same across items to minimize variations due to numeral complexity. The suits (diamonds, spades, hearts, clubs) were counterbalanced across items. See Table 1 for a summary.

Env	Polarity	Schematic structure of item
Decl	Positive	I have Comp/Sup 3 Y
	Negative	I don't have Comp/Sup 3 Y
AntCond	Positive	If you have Comp/Sup 3 Y, then we have something in common
	Negative	If you don't have Comp/Sup 3 Y, then we have something in common
RestUniv	Positive	Everyone who has Comp/Sup 3 Y has something in common with me
	Negative	Everyone who doesn't have Comp/Sup 3 Y has something in common with me

Table 1: Environment and polarity types, and the schematic structure of the sentence associated with them – where Comp ∈ {more than, less than} and Sup ∈ {at least, at most} and Y ∈ {diamonds, spades, hearts, clubs}.

2.2 Results

We wanted to see how the comprehensibility of ModType = SUP varies with Polarity and Environment. Thus, we fitted a logistic mixed effects model with a binary response variable indicating whether a sentence was judged comprehensible or not, with a 3-way interaction term of ModType, Polarity, and Environment as a fixed effect, and with participant as a random effect. Model predictions are shown in

Figure 2.

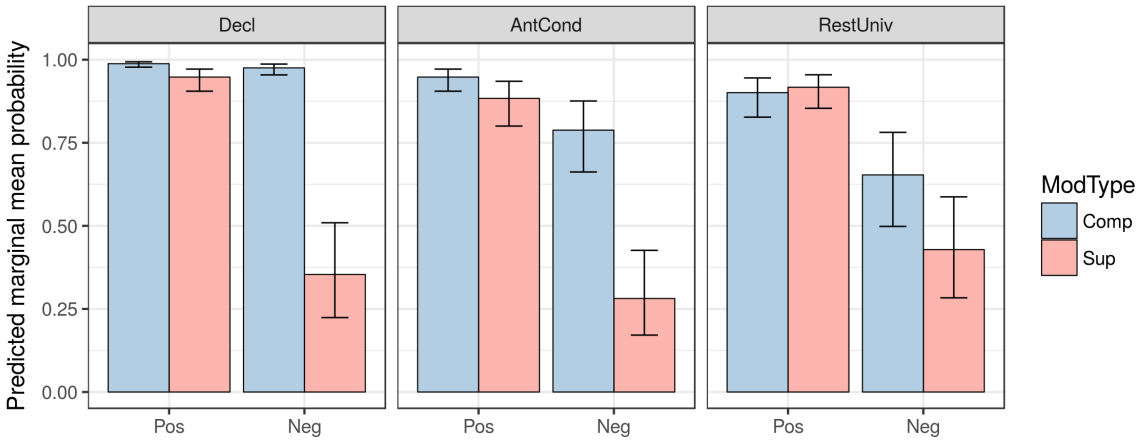

Figure 2: Exp 1 model predictions, by Modifier Type. Bars represent 95% binomial confidence intervals.

Taking COMP, POS, and DECL as reference levels for Modifier Type, Polarity, and Environment, we found a significant two-way interaction of Modifier Type = SUP and Polarity = NEG ($\beta = -2.76$, $z = -2$, $p = 0.045$). To get a better understanding of the patterns, we unpacked the model further.

First, we extracted the predicted marginal mean probabilities for each group and the confidence intervals associated with them. We found that COMP and SUP were both well rated in all the environments so long as the polarity was positive (DECL-POS: CI for COMP = [0.98, 0.99], CI for SUP = [0.91, 0.97]; ANTCOND-POS: CI for COMP = [0.91, 0.97], CI for SUP = [0.80, 0.94]; RESTUNIV-POS: CI for COMP = [0.83, 0.95], CI for SUP = [0.85, 0.95]). However, with negative polarity the patterns are different. In DECL-NEG COMP is rated highly (CI = [0.95, 0.99]) whereas SUP were much less so (CI = [0.22, 0.51]). In a negative antecedent of a conditional COMP continues to be fairly well-accepted (CI = [0.66, 0.88]) but SUP is judged much worse (CI = [0.17, 0.43]). And, finally, in a negative restriction both COMP and SUP are degraded, although COMP remains better (CI for COMP = [0.49, 0.78] vs. CI for SUP = [0.28, 0.59]).

Second, we extracted the predicted odds ratios for the contrasts between COMP and SUP in the various conditions, along with the confidence intervals associated with them. We found that chances to be judged comprehensible are higher for COMP than for SUP in all the conditions with negative polarity, although the magnitude of the difference varies by Environment, as follows: 72.25 times higher in DECL-NEG (OR = 72.25, CI = [8.37, 623.35], $z = 5.24$, $p < .0001$), 9.48 times higher in ANTCOND-NEG (OR = 9.48, CI = [2.52, 35.68], $z = 4.48$, $p < .0001$), and 2.51 times higher in RESTUNIV-NEG (OR = 2.51, CI = [0.75, 8.41], $z = 2.01$, $p = 0.1770$).

Are the differences between COMP and SUP that we saw above driven by just one or another of the modifiers (e.g. *at most* vs. *at least*), or are they characteristic of comparatives as a class vs. superlatives as a class? To determine that, we also fitted a similar model for Mod. The model did not converge, probably due to insufficient data per cell, so we cannot plot model predictions the way we did for ModType. However, in Figure 3 we show the raw means by Modifier. We can see that, while it is true that select members of each class, namely the downward-monotone modifiers *less than* and *at most*, are indeed consistently worse than their upward-monotone counterparts, in all the conditions with negative polarity comprehensibility decreased not just for the downward monotone superlative modifier *at most*

but also for the upward monotone superlative modifier *at least*.

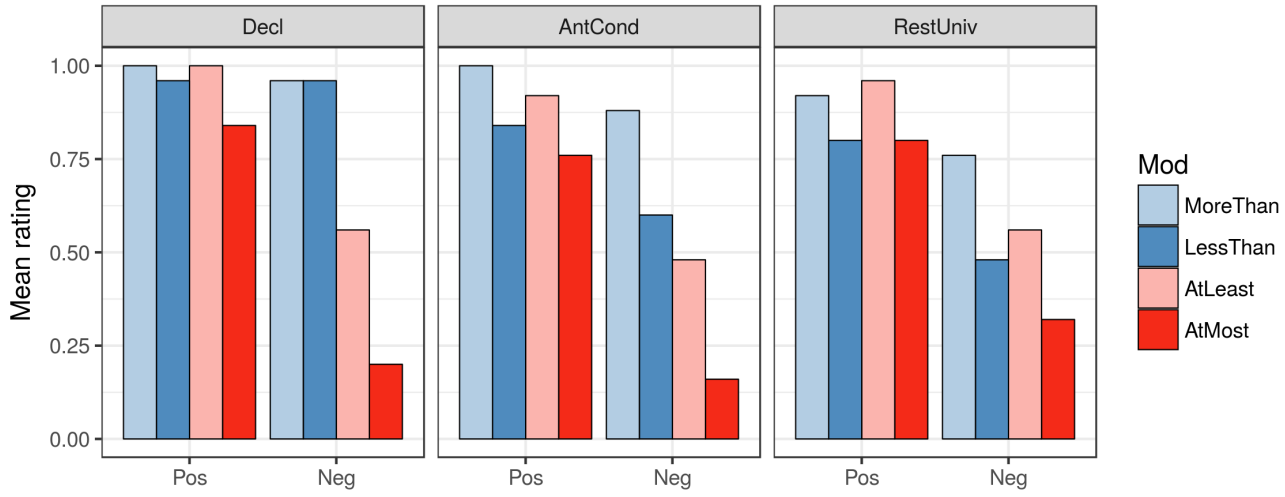

Figure 3: Exp 1 raw means by Modifier

2.3 Discussion

First, we found a significant, and qualitatively very large, contrast between COMP and SUP in DECL-NEG, matching observations that have been made in previous literature prompting a PPI analysis of superlative modifiers. Second, we found that both COMP and SUP were rated highly in both ANTCOND-POS and RESTUNIV-POS. Both these findings support the (1) anti-licensing and (2) licensing expectations of a PPI analysis. However, we also found contrasts between COMP and SUP in ANTCOND-NEG and RESTUNIV-NEG. These findings are surprising as they do not match the (3) rescuing expectation of a PPI analysis. In two follow-up experiments we investigate this further.

3 Experiment 2: Rescuing and valence

The goal of our second experiment was to further explore rescuing of superlative-modified numerals when negation is further embedded in the antecedent of a conditional or the restriction of a universal. In particular, it is possible that rescuing is further sensitive to other factors. For example, Cohen and Krifka (2014) note that superlative-modified numerals in the antecedent of a conditional / restriction of a universal are sensitive to whether there is positivity/negativity match between the antecedent / restriction and the continuation. In (15) below a superlative-modified numeral in a negative restriction of a universal (*Everyone who doesn't donate at least $10 . . .*) is felicitous with a continuation that is pragmatically perceived as negative (*. . . is a fool*) but not with a continuation that is pragmatically perceived as positive (*. . . will get a thank-you card*).

(15) Everyone who doesn't donate at least $10 is a fool / #will get a thank-you card.

Let us call this type of inferred positivity/negativity of the continuation its *valence*.

Now, note that in our first experiment the continuations in the conditional and universal conditions always had positive, or neutral valence (*. . . then we have something in common* / *. . . has something in common with me*). This means that every time the superlative-modified numeral was in a negative antecedent / restriction, there was a mismatch between the polarity of the antecedent / restriction and the valence of the continuation. If rescuing were contingent on this type of match, this would explain why superlative-modified numerals in those conditions weren't rescued.

In this experiment we thus retest the rescuing hypothesis in a way that allows us to check if it is contingent on polarity-valence match between the embedding antecedent / restriction and the continuation. Our critical items in this experiment were sentences where the superlative-modified numeral was placed in the antecedent of a conditional / restriction of a universal with polarity positive or negative (that is, with or without negation, as in Exp 1) and where the continuation had positive or negative valence.

3.1 Methods

Participants Participants were 45 self-reported native speakers of English, recruited via Amazon's Mechanical Turk and paid $1 for their participation. Five participants were excluded prior to analysis (they rated all the sentences the same, which suggests that they didn't understand, or ignored, the task).

Task and instructions This experiment differs from Exp 1 in that we want to vary the valence of the continuation. A natural way to adapt our setup from Exp 1 to this purpose was to have the players make up rules about winning and losing. Since the first argument of the conditional / universal would always have been a description of a hand of cards as remembered by the player uttering the sentence, and since it would have been odd to have a player make up a rule that would cause himself/herself to lose, in this experiment participants were given a slightly different context:

> In this survey you will answer questions about a group of friends playing a game. At the beginning of the game each player gets dealt a hand of seven cards. They are not allowed to see their own cards but they are allowed to take a quick look at their neighbor's hand. They try to remember their neighbor's hand as well as they can because in the next step they have to come up with a rule that would make that neighbor (and possibly other players too) lose or win. You will see what a player remembers about their neighbor's hand and the rule they make up, then you will be asked if you think the other players will understand what they said. Note, we're not asking you if it is a good rule or a bad rule, but whether it is a rule that is going to be understandable for the other players to follow.
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> Note: a  or  a means that the player doesn't remember if a particular card in his hand was a diamond or a heart, or a club or a spade, respectively.

Stimuli Participants saw picture-sentence items presented as in Figure 4.

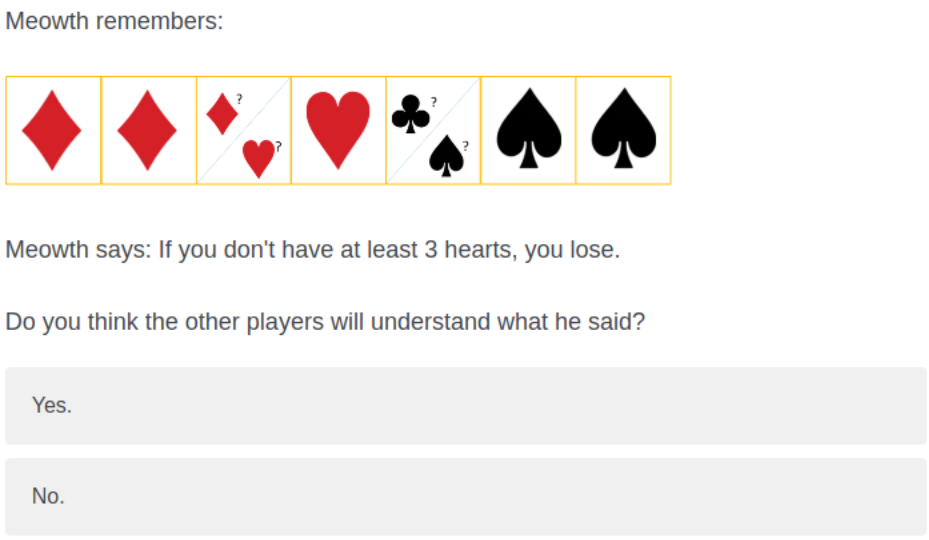

Figure 4: Example trial: Superlative-modified numeral under negation, embedded in the antecedent of a conditional, with negative valence in the continuation

Design Each participant saw 32 trials in total, obtained by crossing four factors: (1) Modifier Type (levels: Comparative, Superlative) / Modifier (levels: *more than*, *less than*, *at least*, *at most*); (2) Polarity of the embedding Environment (same as Polarity in Experiment 1; levels: Positive, Negative); (3) embedding Environment (levels: Antecedent of Conditional, Restriction of Universal); and (4) Valence of the continuation (levels: Positive, Negative). Just like in Experiment 1, the names of the players were different for each item, the numeral 3 was kept the same across items to minimize variations due to numeral complexity, and the suits (diamonds, spades, hearts, clubs) were counterbalanced across items. See Table 2 for a summary.

Environment	Polarity	Valence	Schematic structure of item
AntCond	Positive	Positive	If you have Comp/Sup 3 Y, you win
		Negative	If you have Comp/Sup 3 Y, you lose
	Negative	Positive	If you don't have Comp/Sup3 Y, you win
		Negative	If you don't have Comp/Sup 3 Y, you lose
RestUniv	Positive	Positive	Everyone who has Comp/Sup 3 Y wins
		Negative	Everyone who has Comp/Sup 3 Y loses
	Negative	Positive	Everyone who doesn't have Comp/Sup 3 Y wins
		Negative	Everyone who doesn't have Comp/Sup 3 Y loses

Table 2: Environment, polarity types, valence types, and the schematic structure of the sentence associated with them – where Comp ∈ {more than, less than} and Sup ∈ {at least, at most}, and Y ∈ {diamonds, spades, hearts, clubs}.

3.2 Results

We hypothesized that perhaps the rescuing of a superlative-modified numeral embedded under negation by further embedding in the antecedent of a conditional or the restriction of a universal is further conditioned on negativity match between the embedding antecedent / restriction and the continuation. We thus fitted a logistic mixed effects model with a binary response variable indicating whether a sentence was judged comprehensible or not, with a four-way interaction term of ModType, Polarity, Match (indicating whether the value of Valence matched that of Polarity; e.g., Polarity = YES and Valence = NO results in Match = NO), and Environment as a fixed effect, and with participant as a random effect.

Taking COMP, POS, YES, and ANTCOND as reference levels for ModType, Polarity, Match, and Environment, we found a significant main effect of Polarity = NEG ($\beta = -2.53$, $z = -4.46$, $p < 0.0001$) and a significant main effect of Modifier Type = SUP ($\beta = -1.24$, $z = -2.05$, $p = 0.0405$). Model predictions are shown in Figure 5. (For ease of exposition, we label conditions for Polarity and Valence rather than for Polarity and Match. Thus, a condition with positive polarity and negative valence will be labelled 'Pos.Neg' rather than 'Pos.No'. However, recall that the model did not have Valence as a separate factor.)

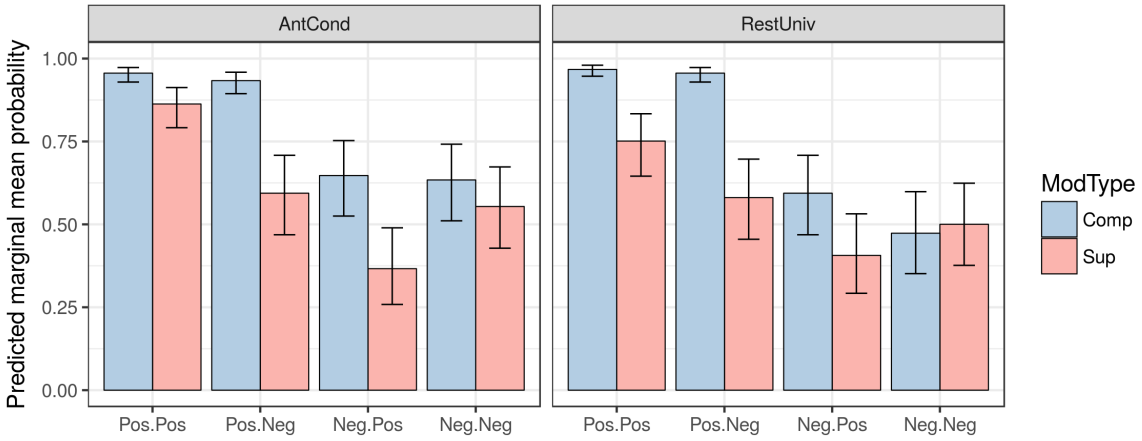

Figure 5: Exp 2 model predictions, by Modifier Type. Bars represent 95% binomial confidence intervals.

Regarding the two overall contexts of conditionals and universals, note that while the results were overall slightly worse in the universal case, the patterns were similar regardless of Environment. First, in the cases of positivity match (i.e., where both polarity and valence were positive) both Comp's and Sup's were rated highly (ANTCOND: CI for COMP = [0.93, 0.97], CI for SUP = [0.79, 0.91]; RESTUNIV: CI for COMP = [0.95, 0.98], CI for SUP = [0.65, 0.83]). (While the contrast between COMP and SUP in ANTCOND-POS.POS, especially in RESTUNIV-POS.POS, may be unexpected in light of Exp 1, where there seemed to be no such contrast, note that Sup's too are rated fairly highly also.) Second, in the cases of mismatch where Polarity = POS and Valence = NEG, COMP continues to be rated highly (ANTCOND: CI = [0.89, 0.96]; RESTUNIV: CI = [0.93, 0.97]), while SUP is rated much worse (ANTCOND: CI = [0.47, 0.71]; RESTUNIV: CI = [0.46, 0.70]). Last, in the cases where Polarity = NEG, both COMP and SUP seem to be degraded in both the mismatch and the match condition, but COMP seems to be better than SUP in the mismatch condition (ANTCOND-NEG.POS: CI for COMP = [0.53, 0.75], CI for SUP = [0.26, 0.49]; RESTUNIV-NEG.POS: CI for COMP = [0.47, 0.71], CI for SUP = [0.29, 0.53]) whereas they seem to be

almost on a par in the match condition (AntCond-Neg.Neg: CI for Comp = [0.51, 0.74], CI for Sup = [0.43, 0.67]; RestUniv-Neg.Neg: CI for Comp = [0.35, 0.60], CI for Sup = [0.38, 0.62]).

We also fitted a model by Modifier. Model predictions are shown in Figure 6.

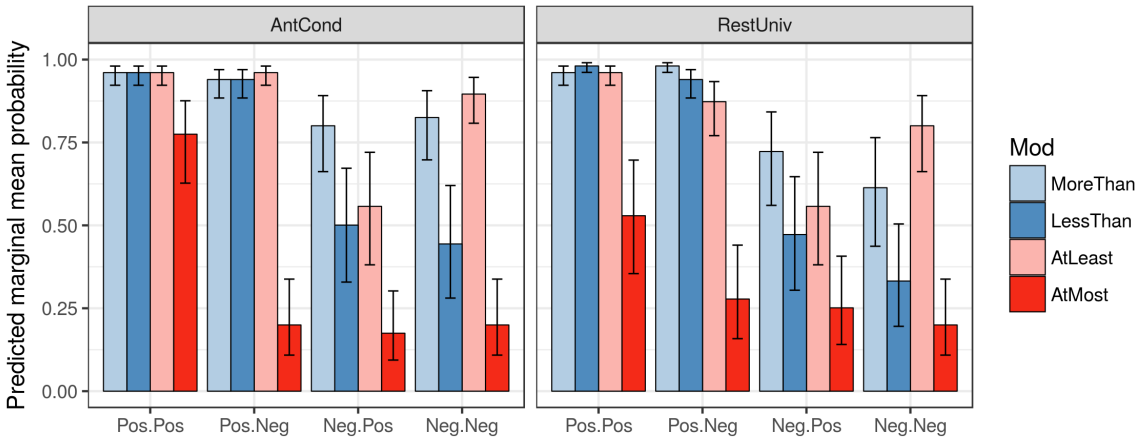

Figure 6: Exp 2 model predictions, by Modifier. Bars represent 95% binomial confidence intervals.

Taking MoreThan, Pos, Yes, and AntCond as reference levels for Mod, Polarity, Match, and Environment, we found a significant effect of Mod = AtMost ($\beta = -1.96$, $z = -2.36$, $p = 0.0180$), a weakly significant effect of Polarity = Neg ($\beta = -1.64$, $z = -1.95$, $p = 0.0508$), and a significant interaction between Modifier = AtMost and Match = No and ($\beta = -2.18$, $z = -1.97$, $p = 0.0484$).

Indeed, we notice that AtLeast and AtMost exhibit drastically different patterns. First, in the positivity match condition both are well-accepted, although, as in Experiment 1, AtMost is rated a little lower. Second, in the mismatch condition where Polarity = Pos and Valence = Neg, AtLeast is rated very highly (AntCond: CI = [0.92, 0.98]; RestUniv: CI = [0.77, 0.93]), but AtMost is rated very badly (AntCond: CI = [0.11, 0.34]; RestUniv: CI = [0.16, 0.44]). Last, in the conditions where Polarity = Neg, AtLeast improves in the match compared to the mismatch condition (AntCond: OR = 6.83, CI = [2.12, 22.06], $z = 3.21$, $p = 0.0052$; RestUniv: OR = 3.18, CI = [1.15, 8.83], $z = 2.22$, $p = 0.0789$) but AtMost doesn't (AntCond: OR = 1.18, CI = [0.38, 3.63], $z = 0.29$, $p = 1.000$; RestUniv: OR = 0.74, CI = [0.25, 2.17], $z = -0.54$, $p = 1.000$).

3.3 Discussion

Our analysis by Modifier Type seems to support the hypothesis of rescuing of superlative-modified numerals being contingent on polarity-valence match between the embedding antecedent of a conditional / restriction of a universal. However, our analysis by Modifier shows that this hypothesis is in fact supported only for *at least*, *at most* being at floor levels not just when the polarity of the embedding antecedent / restriction is negative, but also when the only negative element in the sentence is in the valence of the continuation. This contrast is surprising for the PPI analysis in which the negative element is expected to need to take scope above the PPI.

Would we get better data coverage if we said (cf. Cohen and Krifka 2014) that superlative-modified numerals in these environments are sensitive only to polarity-valence match? While that would capture the low rating of *at most* with positive Polarity but negative Valence, it would not explain why *at most*

remained bad even with a negative Valence even when the Polarity was also negative (there was no mismatch). It would also not explain why *at least* is penalized for mismatch only in the cases with negative *Polarity* (where it was under negation), but not in the cases with negative *Valence*.

Interestingly, Cohen and Krifka (2014:77) note that negative polarity items are sensitive to the valence of the continuation also. In the example below *any* in a positive antecedent of a conditional is felicitous only if the continuation has positive valence.

(16) If you eat any spinach, #I will give you $10 / I will whip you.

Although the current paper deals mainly with PPIs due to our focus on modified numerals, these results suggest future avenues of research for integrating polarity sensitivity with another type of polarity – valence.

4 Experiment 3: Rescuing and downward-entailing operator type

In our final experiment, we further probe patterns of rescuing for superlative modified numerals. As it stands, the PPI analysis predicts that superlative modified numerals should be rescued regardless of the type of downward-entailing environment in which the anti-licenser is embedded. Since our first two experiments only embedded the anti-licenser negation in conditionals and antecedents of relative clauses (which themselves tend to be supportive environments for PPIs), we pushed rescuing one step further in our third and final experiment. Our critical items are sentences where a superlative-modified numeral is in a clause with positive/negative polarity that is further embedded in the antecedent of a conditional (to compare directly with our first study) or (now, for something new) itself in the scope of a matrix negation.

4.1 Methods

Participants Participants were 45 self-reported native speakers of English, recruited via Amazon's Mechanical Turk and paid $1 for their participation. No participants in this study needed to be eliminated due to concerns about attending to the task.

Task and instructions This experiment differs from earlier experiments in that we needed to create a context in which characters may use embedded clauses, in order to embed one clausal negation under another. A natural way to adapt our setup for this purpose was to have a commentator discussing the mental states of players in the game, in our case always using the factive verb *know*. These mental states may also include partial knowledge, which continues to support an ignorance context for superlative modified numerals as in earlier experiments. Participants were thus introduced to the task as follows with this slightly different context:

> In this survey you will consider a commentator for a televised card-playing game, and answer questions about how understandable the commentator is.
>
> At the beginning of the game each player gets dealt seven cards, two of which are hidden. Then in each round some rule is issued, and players can choose whether or not to bet on their own hand. A commentator, who knows what the hidden cards are for each player, discusses the player's move.

You will see a player's hand and the commentator's comment, then you will be asked if you think the viewers will understand what the commentator said.

Note: In the hands that you will see, cards with a white background such as ♦ represent cards that are visible to the player, while cards with a grey background such as ♦ represent hidden cards, that is, cards that are not visible to the player but visible to the commentator.

Stimuli Participants saw picture-sentence pairs presented as in Figure 7.

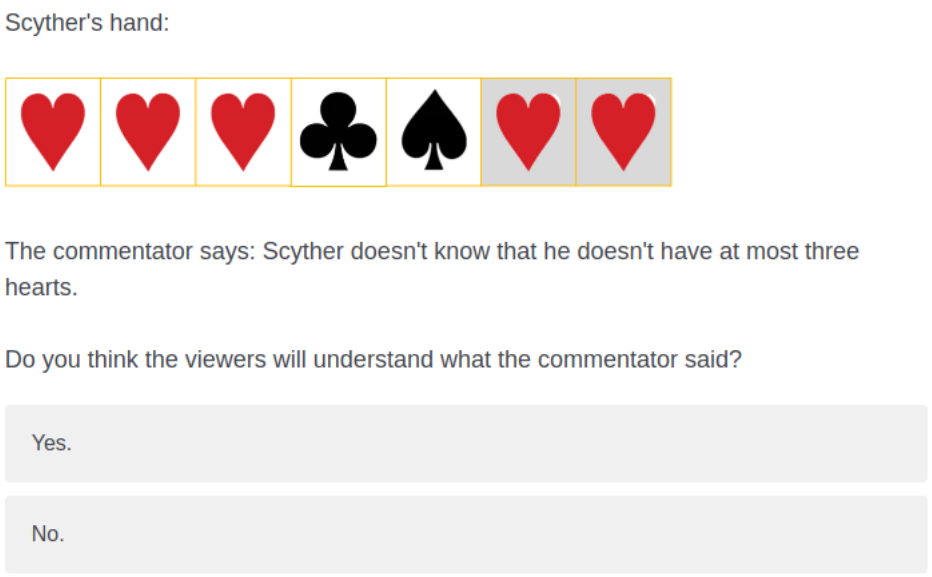

Figure 7: Example trial: Superlative-modified numeral under negation embedded in the scope of a matrix negation

Design Participants saw 16 trials, obtained by crossing three factors: (1) Modifier Type (levels: Comparative, Superlative) / Modifier (levels: *more than*, *less than*, *at least*, *at most*); (2) Polarity of the embedded clause (levels: Positive, Negative)[2]; and (3) matrix embedding Environment (levels: Scope of Negation, Antecedent of a Conditional). Just like in Experiments 1 and 2, the names of the players were different for each item, the numeral 3 was kept the same across items to minimize variations due to numeral complexity, and the suits (diamonds, spades, hearts, clubs) were counterbalanced across items. See Table 3 for a summary.

Environment	Polarity	Schematic structure of item
ScopeOfNeg	Positive	X doesn't know that s/he has Comp/Sup 3 Y
	Negative	X doesn't know that s/he doesn't have Comp/Sup3 Y

[2]This is similar to ANTCOND-POS/NEG in Experiments 1 and 2 in the sense that we are talking about whether a superlative-modified numeral in the immediate scope of negation is further embedded in the antecedent of a conditional. It is however also different in the sense that here the positive/negative clause does not make up the entire antecedent but is merely the complement of the verb *know*. This change is required in order to create a parallel structure to the case where the higher embedding environment is matrix negation.

Table 3 continued from previous page

AntCond	Positive	If X knew that s/he has Comp/Sup 3 Y, s/he would bet differently
	Negative	If X knew that s/he doesn't have Comp/Sup 3 Y, s/he would bet differently

Table 3: Matrix embedding environment and polarity of the embedded clause, and the schematic structure of the sentence associated with them – where X ∈ {[Pokemon names]}, Comp ∈ {more than, less than} and Sup ∈ {at least, at most} and Y ∈ {diamonds, spades, hearts, clubs}.

4.2 Results

We wanted to see whether the rescuing of a superlative-modified numeral embedded under negation is sensitive to the type of the additional downward-entailing environment. To that end, we fitted a logistic mixed effects model with a binary response variable indicating whether a sentence was judged comprehensible or not, with a three-way interaction term of ModType, Polarity of the embedded clause, and type of additional embedding downward-entailing Environment as a fixed effect, and with participant as a random effect. Model predictions are shown in Figure 8.

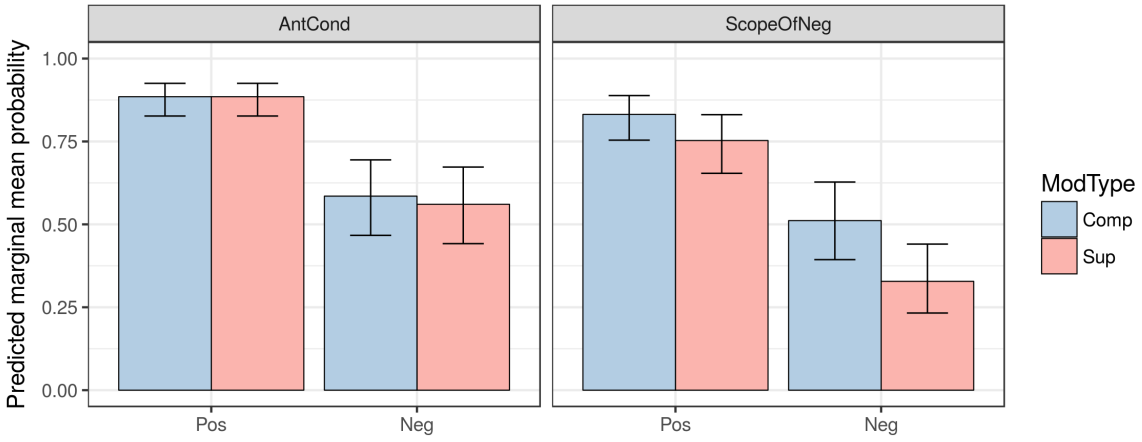

Figure 8: Exp 3 model predictions, by Modifier Type. Bars represent 95% binomial confidence intervals.

Taking COMP, POS, and ANTCOND as reference levels for ModType, Polarity, and Environment, we found a significant effect of Polarity = NEG ($\beta = 1.70$, $z = -4.33$, $p < 0.0001$).

As before, we unpacked the results further. In the conditions with Polarity of the embedded clause = POS we found that both COMP and SUP were rated similarly highly (ANTCOND: CI for COMP = [0.83, 0.93], CI for SUP = [0.83, 0.93]; SCOPEOFNEG: CI for COMP = [0.75, 0.89], CI for SUP = [0.65, 0.83]). In contrast, in the conditions with Polarity of the embedded clause = NEG, COMP was rated similarly to SUP in ANTCOND (OR = 1.11, CI = [0.50, 2.44], $z = 0.31$, $p = 1.000$) but better than SUP in SCOPEOFNEG (OR = 2.14, CI [0.96, 4.79], $z = 2.36$, $p = 0.0730$).

As usual, we also fitted a model by Modifier. Model predictions are shown in Figure 9.

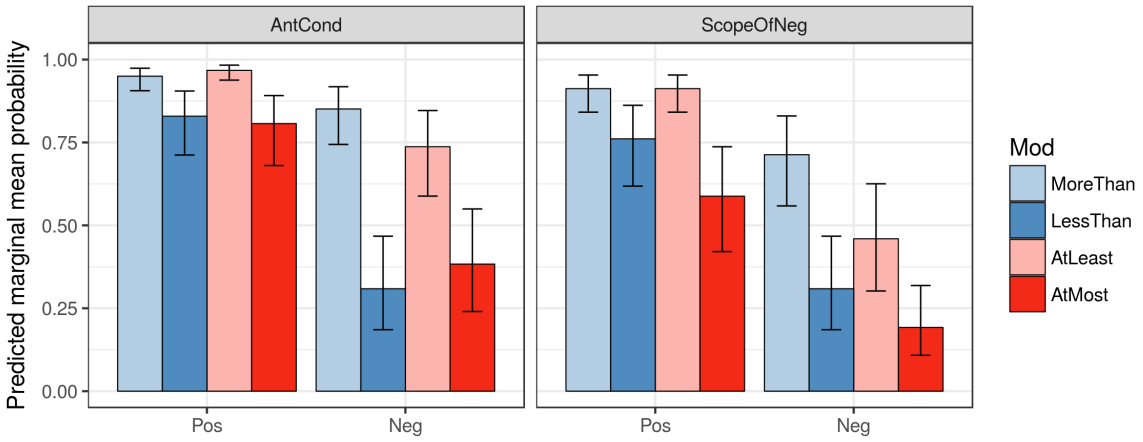

Figure 9: Exp 3 model predictions, by Modifier. Bars represent 95% binomial confidence intervals.

Taking MORETHAN, POS, and ANTCOND as reference levels for ModType, Polarity, and Environment, we found a weakly significant effect of Mod = LESSTHAN ($\beta = -1.36$, $z = -1.87$, $p = 0.614$) and a significant effect of Mod = ATMOST ($\beta = -1.51$, $z = -2.10$, $p = 0.0360$).

As for ATLEAST and ATMOST in the conditions with Polarity = NEG, their behaviors are parallel: both are better in ANTCOND than in SCOPEOFNEG (ATLEAST: OR = 3.30, CI = [1.30, 8.40], $z = 2.50$, $p = 0.0247$; ATMOST: OR = 2.61, CI = [0.98, 6.97], $z = 1.91$, $p = 0.0557$).

4.3 Discussion

We found evidence that rescuing may indeed be affected by the type of the additional embedding environment in that embedding the anti-licensing clausal negation in the antecedent of a conditional leads to rescuing but embedding it in the scope of a matrix negation does not. As in our first two experiments, support generally exists for a PPI analysis except where recuing is concerned, and where the pattern seems to be more complex than predicted under a straightforward application of such an analysis.

This experiment also revealed an interesting pattern for the comparative modified numeral *less than* in the conditions with Polarity = NEG. While LESSTHAN has generally tended to be worse than MORETHAN, now we seem to find evidence of a diverging behavior, just as we did for ATLEAST and ATMOST before. Since the PPI analysis has only been suggested for superlative modified numerals, this diverging behavior for a comparative remains unexplained under such an analysis.

Note that any comparisons between our findings for ANTCOND in this experiment versus Experiment 2 must be interpreted in the context where in Experiment 2 we were directly manipulating the polarity of the antecedent, whereas here in Experiment 3 we were manipulating the polarity of a clause embedded in the *know*-clause embedded in the antecedent. Moreover, the presence of *know* itself in Experiment 3 could affect the outcome. In the literature on PPIs (e.g., Nicolae 2012, 2017a,b, Spector 2014) factive verbs such as *know* are known to be intervenors. While this still wouldn't in itself explain the contrast we found between ANTCOND-NEG and SCOPEOFNEG-NEG, it could be one contributing factor.[3]

[3]For example, in theories such as Spector (2014) or Nicolae (2017a,b) where the PPI behavior of various items is derived via exhaustification and where exhaustification may be sensitive to presuppositions, the fact that *know* is a factive verb and thus contributes a presupposition could affect expectations related to Rescuing.

5 General discussion, conclusions, and outlook

We set out to investigate the reported contrast between superlative-modified numerals and comparative-modified numerals under negation, and in order to do so, we specifically tested the predictions of an analysis of superlative-modified numerals as PPIs. On the one hand, we found several results consistent with such an analysis: superlative-modified numerals were found to be degraded in the immediate scope of unembedded negation but accepted in a positive antecedent of a conditional / restriction of a universal (with a positive continuation) (Experiment 1). On the other hand, some results were surprising for a PPI analysis: *at most* was degraded in a positive antecedent / restriction with a negative continuation (Experiment 2) and rescuing was generally not supported except when made contingent on polarity-valence match, and only for *at least* (Experiment 2). Rescuing was also found to be sensitive to the type of the additional downward-entailing operator, being supported for the antecedent of a conditional but not for matrix negation (Experiment 3). This suggests that a PPI analysis is on the right track, but significant further refinements are needed, especially in the theory of rescuing.

One such refinement may already be present in the literature. For example, if, as in Spector (2014), the various patterns exhibited by PPIs, including Rescuing, are merely epiphenomena arising from attempts to exhaustify relative to certain alternatives in various types of structures, then it isn't surprising that particularities of the exhaustified element could affect the outcome. For example, if exhaustification takes into account presuppositional content also, then any presuppositional element in the structure could affect the result. Thus we urge caution in taking our experiments to prove that "superlative-modified numerals don't get rescued" but rather suggest that further refinements such as this are necessary.

Finally, we note that while more is needed for a PPI story, our results clearly also indicate problems for any simple account of this difference in terms of processing complexity of negation. If negation is merely difficult to process, leading to lower comprehensibility for superlative-modified numerals under negation in Experiment 1 and especially under two clausal negations in Experiment 3, what accounts for the difference between superlative- and comparative-modified numerals across these studies? It can't be enough overall to say that negation leads to lower comprehensibility. However, it may be the case that some conditions are indeed hard to process simple due to the presence of negation; these may be precisely the conditions under which we see comprehensibility of comparative-modified numerals dropping as well.

Overall, the patterns found in our results have proven much richer than predicted under a simple PPI analysis of superlative-modified numerals, or conversely by any accounts that might ignore polarity environments and focus on processing complexity of negation exclusively. By articulating and then testing several predictions of accounts of the distribution of these modifiers, we have arrived at a more thorough understanding not just of the phenomena in question but seemingly also of nuances of polarity more generally, and raised several new avenues for study, such as the relationship between polarity and valence. We hope that the patterns we have uncovered can more generally inform future theoretical work trying to identify the factors that may have an effect on the behavior of polarity sensitive items.

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Appendix

Throughout the paper the plots show model predictions, that is, the predicted means and the binomial confidence intervals associated with them, calculated from the model using the *effects* package in R (Fox 2003). Below we also show the raw data with raw means and the binomial confidence intervals associated with them, which we calculated using the *binom* package in R (the function `binom.confint` with the `wilson` method) (Dorai-Raj 2014).

Experiment 1 - raw means and the 95% confidence intervals associated with them

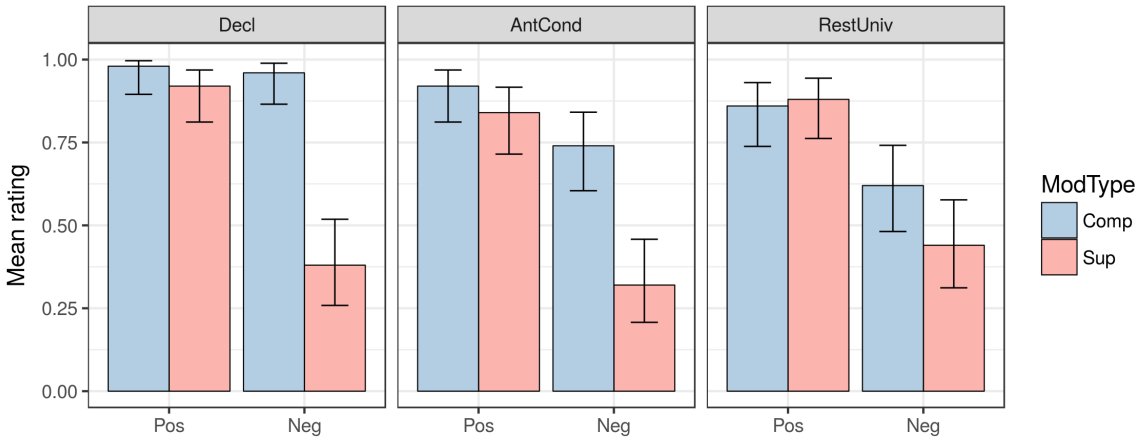

Figure 10: Exp 1 raw means, by Modifier Type. Bars represent 95% binomial confidence intervals.

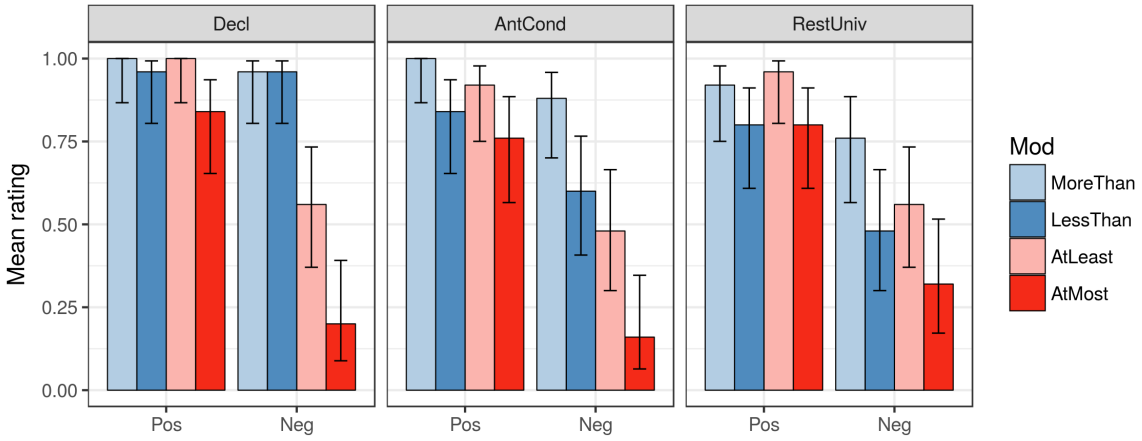

Figure 11: Exp 1 raw means, by Modifier. Bars represent 95% binomial confidence intervals.

Experiment 2 - raw means and the 95% confidence intervals associated with them

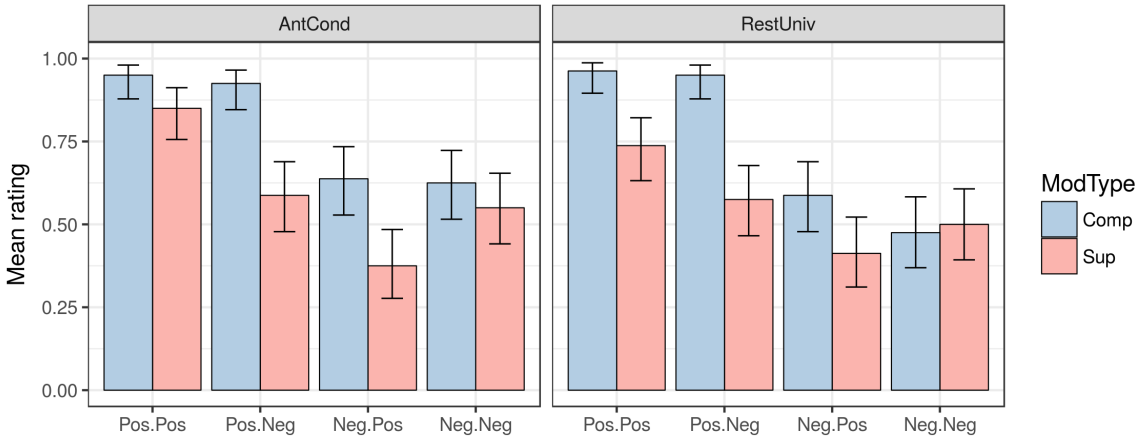

Figure 12: Exp 2 raw means, by Modifier Type. Bars represent 95% binomial confidence intervals.

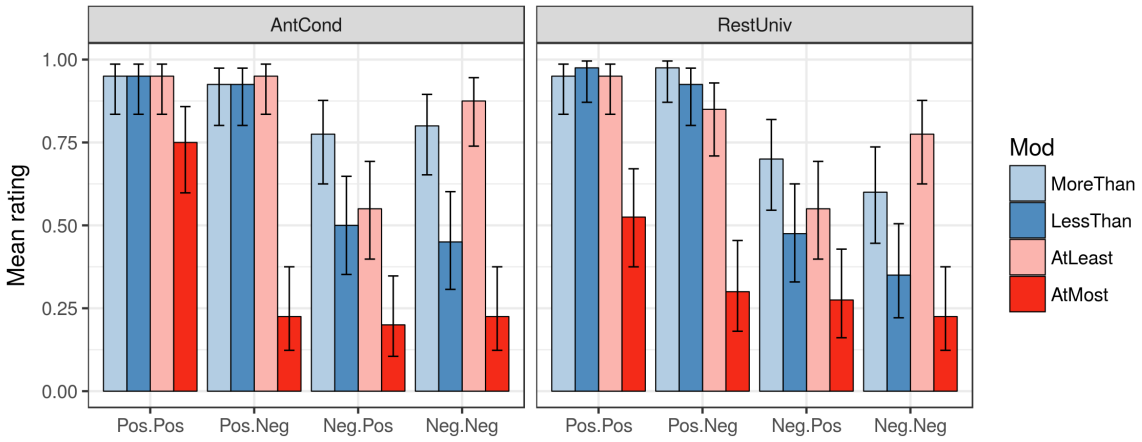

Figure 13: Exp 2 raw means, by Modifier. Bars represent 95% binomial confidence intervals.

Experiment 3 - raw means and the 95% confidence intervals associated with them

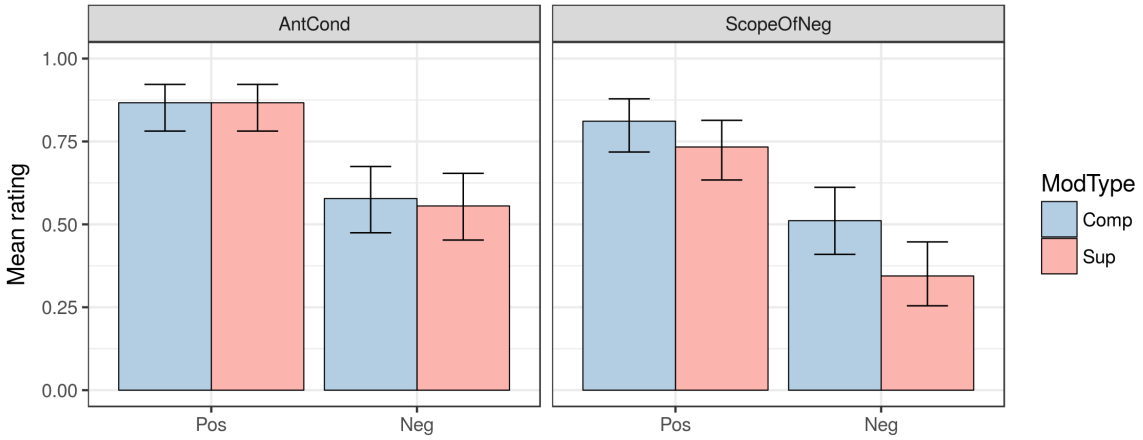

Figure 14: Exp 3 raw means, by Modifier Type. Bars represent 95% binomial confidence intervals.

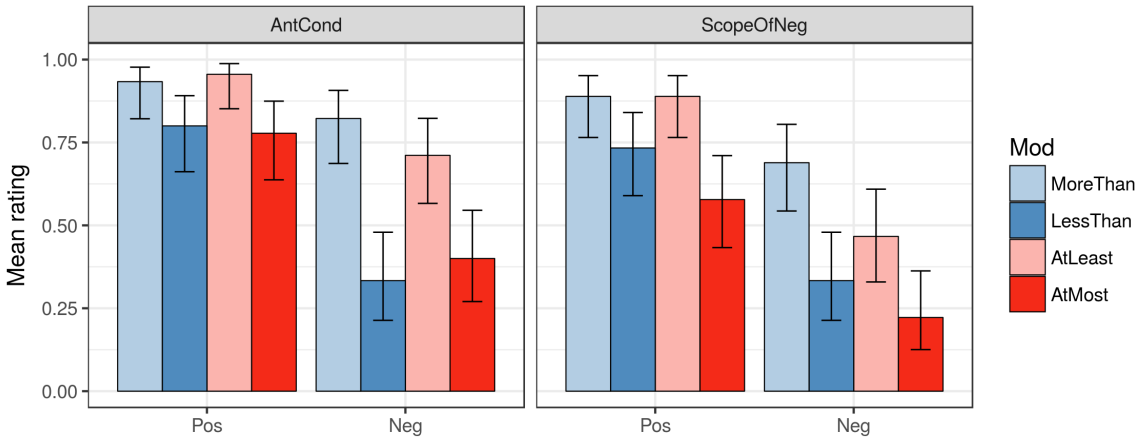

Figure 15: Exp 3 raw means, by Modifier. Bars represent 95% binomial confidence intervals.