Homework assignment 4 for course 01018.

Deadline: Monday 7th December 2020 before midnight.

Question 1

Let $(\mathbb{R},+,\cdot)$ denote the field of real numbers and $(\mathbb{R}[X],+,\cdot)$ the ring of polynomials with coefficients in $\mathbb{R}$.

a) Let $I := \langle X^2, X+1\rangle$ be the ideal of $\mathbb{R}[X]$ generated by the polynomials X^2 and $X+1$. Determine whether or not $I = \mathbb{R}[X]$.

b) Let $J \subseteq \mathbb{R}[X]$ be the set of polynomials $f(X) \in \mathbb{R}[X]$ such that either $f(X) = 0$ or $\deg(f(X)) \geq 2$. Is J an ideal of $\mathbb{R}[X]$? Motivate your answer.

Question 2

As usual, the finite field with 3 elements is denoted by $(\mathbb{F}_3,+,\cdot)$, while $(\mathbb{F}_3[X],+,\cdot)$ denotes the ring of polynomials with coefficients in $\mathbb{F}_3$.

a) Use the extended Euclidean algorithm to compute the multiplicative inverse of the element $X + \langle X^4+X^3+X+2\rangle$ in the quotient ring $(\mathbb{F}_3[X]/\langle X^4+X^3+X+2\rangle,+,\cdot)$.

b) Determine the total number of zero-divisors in the quotient ring $(\mathbb{F}_3[X]/\langle X^4+X^3+X+2\rangle,+,\cdot)$. You may use that in $\mathbb{F}_3[X]$ it holds that $X^4+X^3+X+2 = (X^2+X+2)\cdot(X^2+1)$.