Exercise 5 – COVID-19

The goal with this exercise is to use mathematical modelling to solve a practical problem: Model the impact of COVID-19 on Denmark. You should:

- Identify issues relevant for the modelling
- Formulate mathematical models, which solves these issues
- Implement the model in a computer program if necessary
- Use the implemented model for analysis of data
- Report the results from the analysis and discuss the results as well as the validity of the model

The report for this exercise must be 5 pages at most, including graphs, tables, and images (excluding front page, references, and appendices). In this exercise a number of questions are asked. The report, however, should not be a list of answers, but instead be a coherent documentation and discussion of the analysis performed.

If you write the report as a group, **you must individualise the report**. I.e. clearly state who has done what. It is possible to only collaborate on parts of the reports, e.g. formulate models and code together, but write the actual reports individually. This balance is up to you. You just need to clearly state **who has collaborated and on what**.

Problem

We want an optimal allocation of ventilators to the hospitals in Denmark. To answer this question we will need to

- Estimate how many of the infected will be hospitalised and need a ventilator
- Estimate how many will be infected
- Estimate the allocations based on the demographics of the population in the different Danish regions

Further, we will look ahead to a future, where herd immunity has been achieved, and model that situation.

A: Classification

COVID-19 shows a very large variance in how it affects people. Some are asymptomatic, while others need intensive care. Based on the epidemiological reports by Statens Serum Institut and data from Statistics Denmark, the following probabilities of hospitalisations have been calculated, see table 1. It is under the assumption that the real number of cases is 10 times higher, than what is reported (note that this number is very uncertain).

Age [y]	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90+
No comorbidities	0,71	0,23	0,38	0,50	0,90	1,28	1,70	2,53	2,84	2,27
With comorbidities	0,71	0,23	5,53	8,48	6,53	7,78	9,94	15,94	11,98	7,23

Table 1: Probability in % of infection leading to hospitalisation for different age groups, with and without comorbidities. The comorbidities are hospitalisation due to: cancer, chronic lung disease, diabetes, haematological disease, and cardio-vascular diseases within the last 5 years. The numbers are under the assumption, that the real number of cases is 10 times higher than the reported.

(A1): *Given the probabilities in table 1 find the number of hospital beds needed pr. 1.000 infected assuming equal infection rate across the age groups.*

Find the number of people with comorbidities, by summing all patient hospitalisations the last 5 years due to the "comorbidity"[1] diseases.

Use demographic data from INDP01 and FOLK1A in StatBank Denmark.

(A2): *Find the number of beds needed pr. 1.000 infected if at-risk groups are totally isolated. I.e. no infection in people with co-morbidities and people 70+ years old.*

(A3): *What is the average age of a hospitalised person in the two scenarios?*

B: SIR model

A simple mathematical model for an epidemic is the SIR-model: Susceptible, Infected, Removed (recovered or dead). The model was originally formulated in 1927 by Kermack and McKendrick [3].

The model is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta SI}{N}, \tag{1}$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta SI}{N} - \gamma I, \tag{2}$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I, \tag{3}$$

where N is the total population, S the susceptible, I the infected, R the removed, β is the effective contact rate, how many an infected individual comes into contact with, and γ is the mean recovery rate. The basic reproduction number is given by $R_0 = \frac{\beta}{\gamma}$. R_0 is how many people a single infected will infect on average in a completely susceptible population.

(B1): *When is* $\frac{dI}{dt} = 0$ *and what are the implications of this?*

We now want to model the 5 Danish regions. The current number of infected in the five regions are given in table 2.

Region	Confirmed COVID-19 cases	Population	Cumulative incidense (per 100.000)
Hovedstaden	3.186	1.846.023	172,6
Sjælland	825	837.359	98,5
Syddanmark	704	1.223.105	57,6
Midtjylland	933	1.326.340	70,3
Nordjylland	307	589.936	52,0
In total	5.955	5.822.763	102,3

Table 2: Infection statistics the 11th of April. Modified from [1], table 6

Further, due to differences in population density, cultural differences, etc., we will assume different transmission rates given in table 3 (The actual numbers are very uncertain, and change due to the policies in place and the behaviour of the population). Further, we will assume, that the number of recovered is initially zero.

(B2): *Perform a numerical simulation for 365 days for each of the regions, using the current infection numbers as the initial conditions and find the infection numbers over time, as well as the hospitalisation numbers over time. The latter based on the previously calculated hospitalisation rates.*

[1] The following are comorbidities: "MALIGNANT NEOPLASM, TOTAL", "Diabetes mellitus", "DISEASES OF BLOOD AND BLOOD-FORMING ORGANS, TOTAL", "DISEASES OF THE CIRCULATORY SYSTEM, TOTAL", "Bronchitis, emphysema and asthma", "Other diseases of upper respiratory tract", "Other diseases of the respiratory tract"

Region	β	γ
Hovedstaden	0.13	$1/14$
Sjælland	0.11	$1/14$
Syddanmark	0.1	$1/14$
Midtjylland	0.11	$1/14$
Nordjylland	0.09	$1/14$

Table 3: Assumed SIR parameters for the five regions.

C: Allocation of resources

As we have seen already in Italy and Spain, a crucial resource is the number of ventilators and the personnel to operate them. There are 1260 ventilators available in Denmark [2]. So far about 20 % of the hospitalised have required a ventilator [1].

(C1): *Using the numerical simulations of the different regions, calculate the number of ventilators needed, and when they are needed.*

Assume initially, that the ventilators can be moved freely between the regions. Due to political considerations, it is not possible to have a region with a lot of missing capacity compared to the others.

(C2): *Optimize the allocations through a Mixed Integer Linear Program, with the objective of minimising the sum over all days of the maximum number of patients uncovered in any region (see [4]). Sub sample the data using every 7th day, due to the size of the problem.*[2]

(C3): *Add constraints, e.g. minimise the number of transfers of ventilators between regions, a delay when a ventilator is transferred between regions in which the ventilator cannot be used, only allow a limited number of transfers etc.*

Consider in your discussion what is most crucial for adequate capacity. Isolation of at-risk groups, reducing transmission rate, buying more ventilators, or something else within the models we have formulated here.

D: Perspectives for the future: Modelling secondary waves

We look ahead to the future, where a state of herd immunity has nearly been obtained. How might waves of infection spread across a spatial region?

A 1-D model of infection waves in a closed spatial region considers a density $\rho = \rho(\theta, t)$ of contagious individuals within the population.

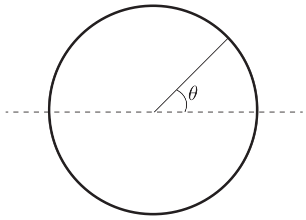
The closed spatial region (e.g. a nation with almost completely sealed-off borders) is modelled as a unit circle[3]. Alternativly, one can regard functions on the real axis between $-\pi$ and π where the extension to 2π-periodic funtions is differentiable, and where exiting the interval at π means entering the interval at $-\pi$. The angle θ measures distance along the circle or equivalently along the axis. Along the circle there is a density ρ of the infected individuals. These individuals are modelled as moving with a local velocity $u(\theta, t)$. Thus, the greater the velocity, the less time any individual spends in any given spatial region, and the less likely they are to infect susceptible individuals. .

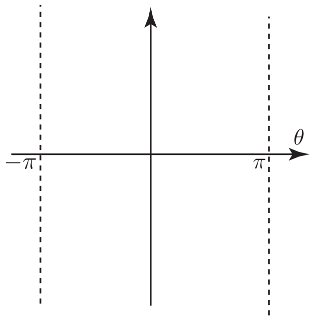
This model will rely on an approximate and empirical relation between the density of infected individuals and the velocity (and in turn infection probability). This relation states that

$$u(\rho) = (1 - \rho)^{0.3}$$

[2] Consider setting a time-limit when running the optimisation.

[3] Note that it is the circle line, not the disc





The product $\rho(\theta, t)u(\theta, t)$ is flux of infected individuals at a location θ at the time t. We will assume a situation where there is nearly a balance between new infected individuals and new recovered individuals (in other words that the situation is almost at herd immunity balance). Under this assumption the conservation law of infected individuals takes the form:

$$\frac{\partial}{\partial t}\rho(\theta, t) + \frac{\partial}{\partial \theta}\left(\rho(\theta, t)u(\rho)\right) = 0. \tag{4}$$

Suppose a small localised increase ρ_1 in the the density of infected individuals occurs at time $t = 0$.

In this model (4) the localised increase in infections will spread across the spatial region (will move to different values of θ).

(D1): *Find, based on the value of the herd immunity density ρ_0 and the velocity-density relation, the time for this wave to travel the entire region and return to where the increase was at $t = 0$. This will be experienced as a secondary infection wave.*

(D2): *Find the criterion for backward spread (against the flow of infected individuals) of such an infection wave.*

Hand in The report is uploaded to DTU Inside. The code must be included in an appendix in the report, and as separate files.

Poul Hjorth and Anders Nymark Christensen, April 2020

References

[1] Covid-19 i danmark epidemiologisk overvågningsrapport, 11-04-2020. https://www.ssi.dk/-/media/ssi-files/covid19-overvaagningsrapport-11042020-ednk.pdf?la=da. Accessed: 2020-04-20.

[2] Håndtering af covid-19: Prognose og kapacitet i danmark for intensiv terapi. https://www.sst.dk/-/media/Nyheder/2020/ITA_COVID_19_220320.ashx?la=da&hash=633349284353F4D8559B231CDA64169D327F1227. Accessed: 2020-04-20.

[3] William Ogilvy Kermack and Anderson G McKendrick. A contribution to the mathematical theory of epidemics. Proceedings of the royal society of london. Series A, Containing papers of a mathematical and physical character, 115(772):700–721, 1927.

[4] J Cole Smith and Z Caner Taskın. A tutorial guide to mixed-integer programming models and solution techniques. 2007.