Assignment - Alternative B

Deadline: Tuesday, 26th July 2022, 23.59 CET

General information - READ CAREFULLY

- **Please choose between Alternative A or Alternative B**. You must submit only one report to either answering Alternative A or answering Alternative B!
- This assignment is part of the overall assessment of this course and, therefore, your answer counts for the final grade.
- This assignment must be solved and submitted as a group work before the above mentioned deadline closes.
- The assignment has to be submitted via DTU Learn using the according Assignment system. Use the entry `Assignments` in the course content and upload your files to the corresponding assignment. In case of technical problems with DTU Learn, please send your files to `dngk@dtu.dk` before the deadline.
- **The submission must consist of one pdf-document containing the answers to the tasks below. Furthermore, program code and scripts have to be uploaded as well**.
- You may use Julia or Python as the programming language to solve tasks. However, all input code is given in Julia only.
- **Name your report `Group<#>-Report-Alternative<A/B>.pdf`**
- **Name your codefiles `Group<#>-Task<#>.jl` (analog for python)**

Part 1 - Stochastic programming

In Task 1-4 you are optimizing the supply and production of the paper manufacturer *Paper2000*.

Task 1 - Scenario generation (10 points)

In the file `historic_demands.csv` you find the historic paper demands for the last 10 years on a weekly level. Use one of the bootstrapping variants from the course for scenario generation (and reduction via k-medoid clustering, if necessary) and generate 5 representative scenarios for the coming 52 weeks. Initialize the random number generator with `Random.seed!(1234)`.

1. Describe the steps of your scenario generation with short descriptions and plots. Report the probabilities of the 5 scenarios.
2. Briefly argue why the chosen method is an appropriate method for scenario generation in this case.

Task 2 - Two-stage Stochastic programming (35 points)

As input to the paper production, *Paper2000* uses wood from different suppliers. Your task is to help *Paper2000* with the selection of wood suppliers. Your model must ensure that *Paper2000* receives enough wood to be able to fulfill the market demand. The supplier selection is a tactical decision and we are looking at a planning horizon $\mathcal{T}$ of one year given in weekly periods, i.e, $\mathcal{T} = \{1, ..., 52\}$.
The market demand for paper $d_{s,t}$ (given in tons paper) during the next 52 periods is unknown and given in scenarios $\mathcal{S}$. Each scenario has a probability π_s.
Paper2000 prepared a set of potential suppliers is given by $\mathcal{W}$ from which the model should choose. Furthermore, consider the following requirements:

- The suppliers have to be selected at the beginning of planning horizon and the decision can not be revised later. The production of paper can be decided during the year.
- Each supplier can only be selected once.
- For a select supplier w, the delivery amount per week has to be decided now at the beginning of the planning horizon. The weekly delivery amount has a lower bound of $\underline{A}_w$ and an upper bound of $\overline{A}_w$ (given in tons wood). The amount can differ per week. For not selected suppliers, the delivery amount must be 0.
- The price per ton wood for a supplier w is given by c_w^D EUR.
- You can assume that η tons of wood lead to one ton of paper. For example, if $\eta = 2.6$ you need 2.6 tons of wood to produce one ton paper.
- The production capacity in each week is limited to U tons of wood that can be processed. The capacity is the same amount in every period.
- It is possible to store wood and paper from one period to the next without any losses.
- For paper there is a maximum storage capacity per period of L^P tons, it costs c^P EUR to store one ton of paper from one period to the next. The produced paper is going to the storage and from there to the demand sites.
- The storage for wood is more flexible, there are several storage sites $\mathcal{K}$ with different storage costs c_k^W EUR per ton wood and limits L_k^W (given in tons wood).
- The delivery amount of the suppliers are delivered to the storage sites $\mathcal{K}$. The delivery of one supplier can be split up into several sites.
- For both storages, paper and wood, it is possible to takeout some inventory in the same period it was placed into the storage. Assume empty storages at the beginning.
- Be aware that it could be the case that is not possible to cover the paper demand. Handles this approriately in your model to still get a solution. The missing paper demand is penalized with ϕ EUR per ton.
- Optimize the supplier selection by minimizing the cost for wood from the selected suppliers as well as storage costs for paper and wood.

In all following tasks, you have to define and describe all necessary sets, variables, parameters and constraints. The data is given in Julia file `TaskB2-data.jl` using the files `reduced-scenarios.csv` as input. Do not use your own scenarios from Task 1.

1. Formulate a general two-stage stochastic program to help the paper manufacturer. Define and describe all necessary sets, variables, parameters and constraints.
2. Solve your model with the provided data `TaskB2-data.jl`. Report the solution and present your observations regarding the results within the context of the planning problem.
3. Briefly answer: What are the most important decisions for the company in this planning problem (in terms of first-stage or second-stage) and why?

Task 3 - EVPI and VSS (12 points)

Evaluate your model from task 2 with respect to the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS). Use the same data file as in Task 2.

1. Calculate EVPI and VSS and interpret the two values in your own words also with respect to the problem setting (what do the numbers mean). State also the expected values of the wait-and-see solution and expected value solution that you used for calculation.
2. Answer briefly: Why are wait-and-see solutions theoretical solutions and rather irrelevant for application in practice?

Task 4 - Out-of-sample test (10 points)

1. Evaluate the stochastic solution and expected value solution in an out-of-sample test using the samples in `oos_samples.csv`. There are some lines of code included in `TaskB2-data.jl` that read these csv files and give the you the specific demand per sample and period.

2. Report your results in reasonable and concise manner including plots. Which method would you use in practice and why?

3. Why is important to run an out-of-sample test and not only rely on EVPI and VSS?

Part 2 - Robust optimization

In Task 5 you are optimizing the weekly production and capacity of Lovely Cards, which is a customer of *Paper2000*.

Task 5 - Robust optimization (25 points)

Lovely Cards is a customer of *Paper2000* and buys their paper to produce birthday cards. You want to help *Lovely Cards* to optimize their weekly production for the next 4 weeks $\mathcal{T} = \{1, 2, 3, 4\}$. For each pack of paper from *Paper2000*, *Lovely Cards* pays C^P EUR. The production targets (i.e. the amount of cards that we want to reach) for each week t is given by L_t. From each pack of paper, we can produce $\tilde{\epsilon}$ birthday cards, However, $\tilde{\epsilon}$ is uncertain and depends on the quality of the paper that *Paper2000* delivers. *Lovely Cards* has a very flexible production and the weekly production capacity can be decided freely for the next 4 weeks within the bounds $[\underline{K}, \overline{K}]$ (given in number of paper packs that can be processed) with a cost of C^K EUR per pack capacity. The capacity is the same for each period in the planning horizon. Your model aims at determining the capacity and purchasing of packs per week to minimize the costs while fulfilling the production targets.

1. Based on observations, we know that the number of birthday cards that we can produce from one pack of paper varies between $\epsilon \sim \mathtt{U}[400, 600]$ (uniform distributed).

   Formulate a <u>linear</u> robust optimization (RO) problem that provides a feasible solution for every possible realization of the uncertainty. Start with the robust counterpart and describe the steps that you used to reformulate the robust counterpart to its linear formulation including definition of the uncertainty set. State also the final linear formulation.

2. Solve the model for the given data set in `TaskB5-data.jl` and state the objective value, capacity and pack of paper purchased for each week (table or plot).

3. Extend your model from the previous task by the following requirements: *Lovely cards* can react to the actual quality of the paper by buying paper from another supplier *SuperPaper* to reach the weekly production targets. Each pack of paper from *SuperPaper* costs $\gamma = 3.5$ EUR and the number of packs is limited to $U = 20$ due to the short lead time. The paper at *SuperPaper* goes through an advanced quality assurance process, so that you are sure that you can produce $\sigma = 650$ birthday cards from each pack from *SuperPaper*. Note that the extra packs also need some capacity of the weekly production capacity.

   Formulate an adjustable robust optimization problem using linear decision rules for the problem setting described in this task. Define the linear decision rule and describe it. Start with the robust counterpart and describe the steps that you used to reformulate the robust counterpart to its linear formulation. State also the final linear formulation.

4. Solve the model for the given data set in `TaskB5-data.jl` and state the objective value, capacity and pack of paper purchased from each supplier for each week (table or plot).

5. Compare the RO and ARO solutions and describe your observations.

Task 6 - Reformulation to robust linear models (8 Points)

Consider the following basis model:

$$
\begin{array}{llllllll}
\text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\
\text{s.t.} & 5x_1 & + & 3x_2 & + & 3x_3 & \leq & 10 \\
 & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5 \\
\end{array}
$$
$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

In the following tasks some of the parameters are defined as uncertain parameters within given uncertainty sets. In each task you need to reformulate the model to a robust linear model that can be solved using a general purpose linear programming solver.
For each task you have to document the steps of reformulation (writing down just the final robust linear model is not enough).
You do not need to solve the models.

1. Parameters $\tilde{a}_2$ and $\tilde{a}_3$ are uncertain:

$$
\begin{array}{llllllll}
\text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\
\text{s.t.} & 5x_1 & + & \tilde{a}_2x_2 & + & \tilde{a}_3x_3 & \leq & 10 \\
 & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5 \\
\end{array}
$$
$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

   $\tilde{a}_2$ and $\tilde{a}_3$ can take values in the uncertainty set $U = \left\{\tilde{a}_2 + \tilde{a}_3 \leq 8, \tilde{a}_2 + \tilde{a}_3 \geq 2, \tilde{a}_2, \tilde{a}_3 \geq 0\right\}$.

2. Parameters $\tilde{a}_1$ $\tilde{a}_2$ and $\tilde{a}_3$ are uncertain:

$$
\begin{array}{llllllll}
\text{Max} & 10x_1 & + & 20x_2 & + & 15x_3 & & \\
\text{s.t.} & \tilde{a}_1x_1 & + & \tilde{a}_2x_2 & + & \tilde{a}_3x_3 & \leq & 10 \\
 & 7x_1 & - & 2x_2 & - & 2x_3 & \geq & 5 \\
\end{array}
$$
$$-2 \leq x_1 \leq 10$$
$$0 \leq x_2 \leq 15$$
$$-10 \leq x_3 \leq 10$$

   $\tilde{a}_1, \tilde{a}_2$ and $\tilde{a}_3$ are uniform distributed with $\tilde{a}_1 \sim \mathcal{U}(1, 8)$, $\tilde{a}_2 \sim \mathcal{U}(2, 9)$, $\tilde{a}_3 \sim \mathcal{U}(1, 5)$. At most two of the three parameters will deviate from their mean value.