DANMARKS TEKNINSKE UNIVERSITET

ADVANCED TIME SERIES ANALYSIS
Course number: 02427

Computer Exercise 3

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Introduction

This is answer to "Computer Exercise 3" in the course in advanced times series analysis at DTU. The exercise is divided into 2 parts that will be answered independently. The subject of the exercise is parameter estimation for various non-linear models. Whenever there is a referral to *the lecture notes*, the notes in mention is *Modelling Non-Linear and Non-Stationary Time Series*, by Henrik Madsen and Jan Holst, December 2006.

Part 1

Question 1a

Phase plot 1

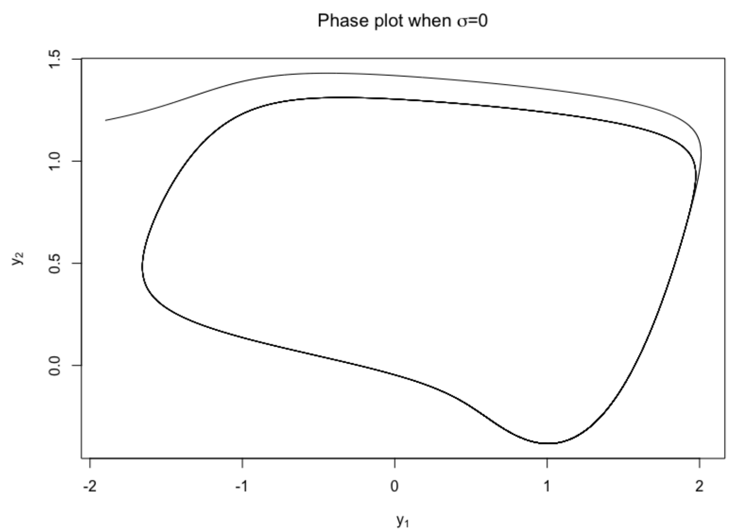

Figure 1: Phase plot when $\sigma = 0$

Phase plot 2

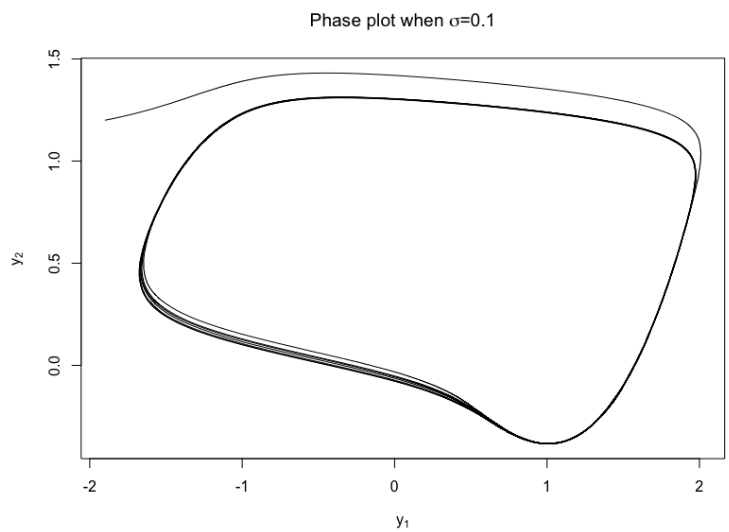

Figure 2: Phase plot when $\sigma = 0.1$

Phase plot 3

Figure 3: Phase plot when $\sigma = 0.2$

Phase plot 4

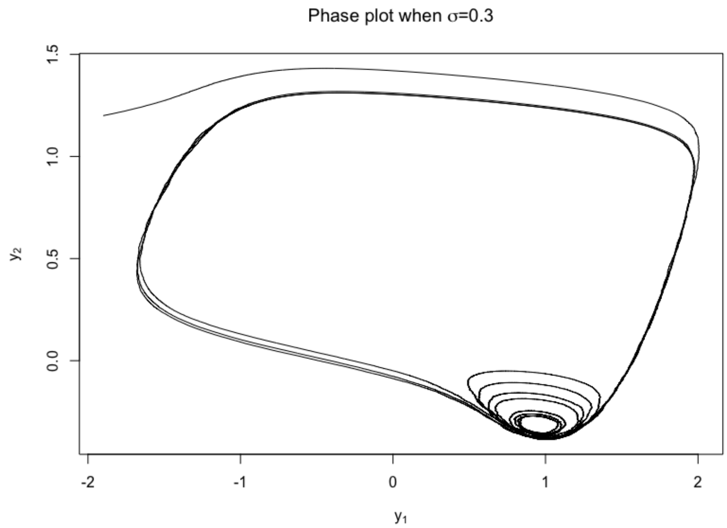

Figure 4: Phase plot when $\sigma = 0.3$

Phase plot 5

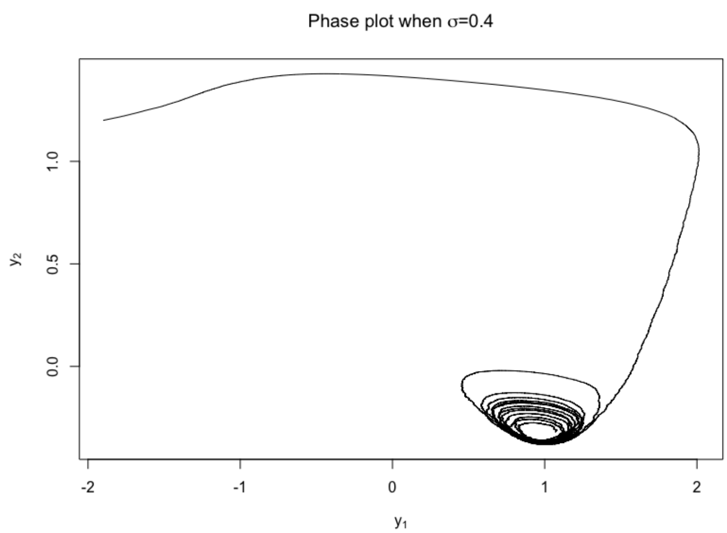

Figure 5: Phase plot when $\sigma = 0.4$

From the plots it is evident that adding noise generates the possibility of the process deviating from the deterministic solution (as given in the first plot where $\sigma = 0$). It seems that when the noise is small, the process deviates somewhat but only locally, the general trend of the phase is completely intact. As the noise increases the possibility of making a large deviation which results in a different cycles increases. This is evident in the plots where $\sigma \geq 0.2$. We see the phase in the neighborhood of the "corner" at $\approx (1, -0.2)$ has a possibility of completely changing the cycle to one that is significantly smaller, if the noise happens to make it deviate enough at the right time. For $\sigma = 0.4$ we in fact see that it does so in all the cycles, effectively changing the phase plot completely.

Question 1b

We are asked to produce a 3-dimensional plot that shows the frequency together with the trajectory of the phase plot.

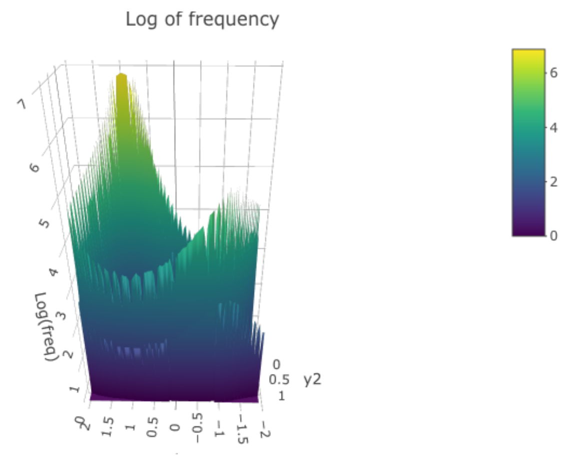

Figure 6: 3-dimensional histogram of phase plot.

We have done a log transformation of the frequency values, in order to ensure the visibility of places the trajectory only passes through few times. We discussed the plot in the group and determined that it was quite difficult to determine what the trajectory of the process using this 3-dimensional plot. We preferred using a 2-dimensional heatmap instead, which allows us to see the trajectory, but also gives us the sense of frequency at any given point.

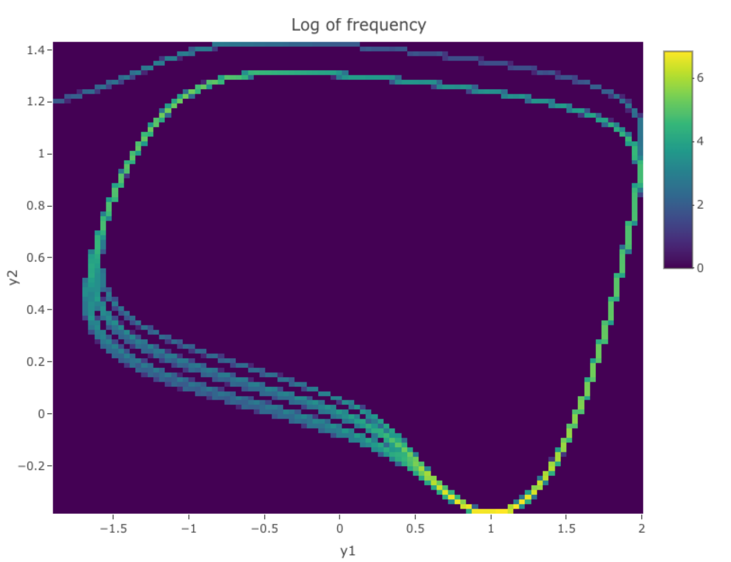

Figure 7: 2-dimensional heat plot of the phase when $\sigma = 0.1$.

We immediately recognize the trajectory from the plot shown in exercise 1a). Now we are able to see that the frequency around (1,-0.25) is significantly higher than the rest of the plot (remembering we are looking at the log transformed data). This gives us some indication that there is some form of stability around this point.
We can repeat this process for all the simulated σ values, and get:

$\sigma = 0.2$.

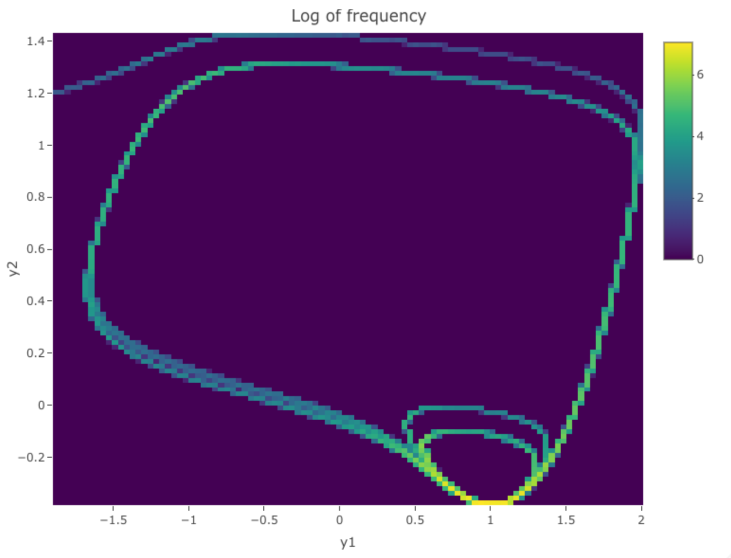

Figure 8: 2-dimensional heat plot of the phase when $\sigma = 0.2$.

$\sigma = 0.3$.

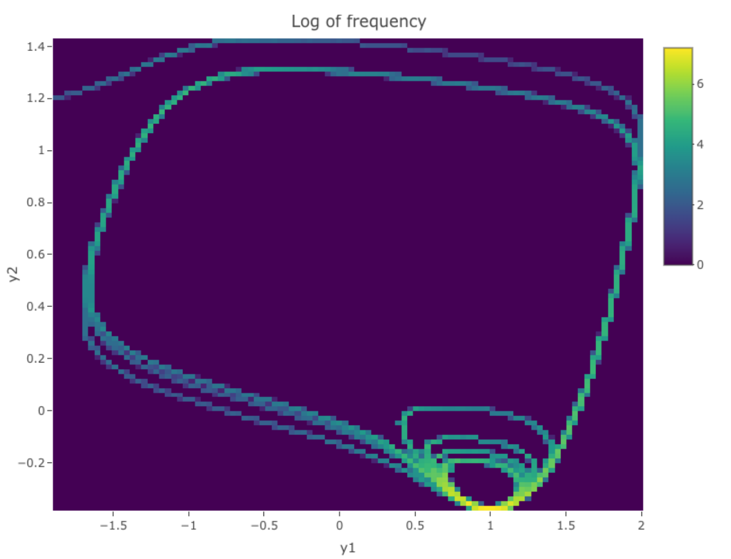

Figure 9: 2-dimensional heat plot of the phase when $\sigma = 0.3$.

$\sigma = 0.4$.

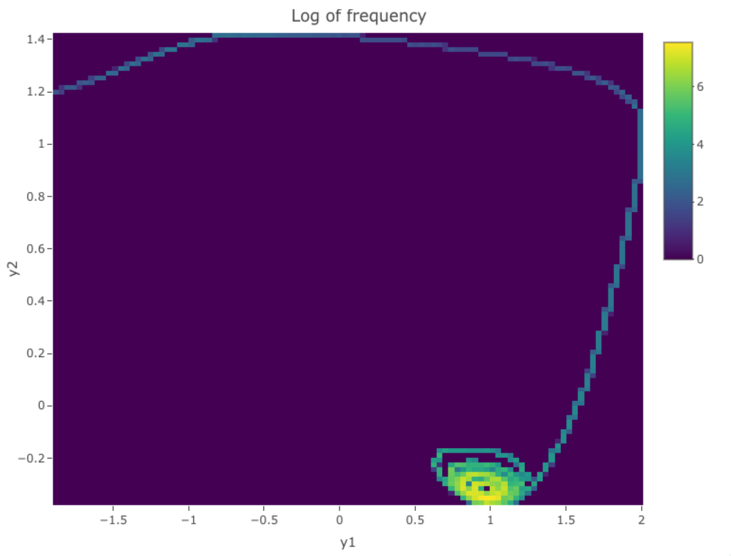

Figure 10: 2-dimensional heat plot of the phase when $\sigma = 0.4$.

We can see that the trend in all the plots is that the frequency is high close to the area of (1,-0.25). This gives us a strong indication that there is some form of stability around that point.
To investigate whether the trajectory does converge towards (1,-0.25) or not, we tried to plot the trajectory when $\sigma = 0.2$ as a 3d line plot.

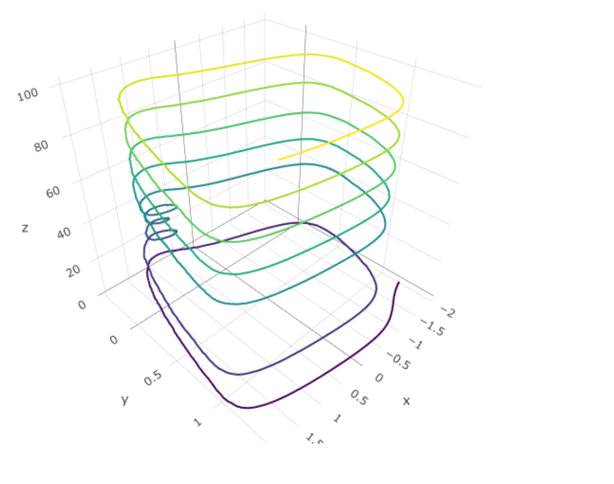

Figure 11: 3-dimensional phase plot when $\sigma = 0.2$.

We can clearly see from this plot that the process is not stable in the region around (1,-0.25) since it can "jump" back to the trajectory with the larger diameter. In turn we can only conclude that the process seems to be stable in the region of $y_1 \in [-2, 2], \quad y_2 \in [-1, 1.5]$.

Part 2

Question 2a

We are asked to use splines to model the window areas of the rooms. The physical consideration are that when the sun rotates around the earth, the angle of the sun through the windows change, effectively changing the window area over time.
Let us use room 1 as an example. The windows in room 1 are facing in the north-northeasten direction. We would therefore expect its effective window area to be largest during the morning, and gradually become smaller until it reaches zero around noon. One approach could now be to model this effect directly and in turn compute the window area as a function of time, and use this instead of a constant when fitting the model. Another approach could be to generate multiple basic splines and use a weighted sum of these to model the window area. This way allows the model to optimize on the weighting of the basic splines, and effectively choose the optimal parameters by itself. We will use the latter approach, we model the splines to consist of 5 basic splines functions, and starting from 7 in the morning until 21 in the evening. Outsize the time period the value of the splines will equal zero, implying that the effective window area after sunset is 0.

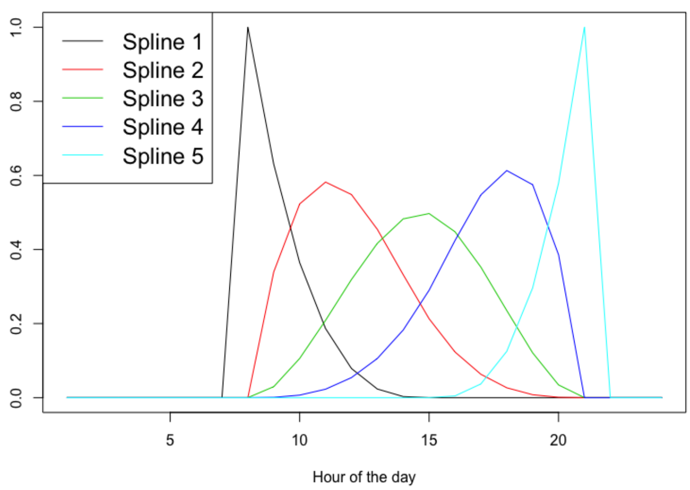

Figure 12: Spline functions.

We can see that these 5 spline functions each represent roughly what we would expect the window area from 5 different directions to look like. In the case of room 1, we would somehow expect the fit to estimate significant non-zero values for the weighting of the first 2 or 3 splines, while we would expect spline 4 and 5 to be non-significant. We get the following estimates for the weighting of the basis spline functions:

Table 1: Estimates of weights of spline functions.

	Estimate	Std Error	P-value
Spline 1	2.0237e+02	3.1945e+00	0.0000e+00
Spline 2	2.7061e-04	7.5441e-04	7.1984e-01
Spline 3	6.5749e+00	1.3359e+00	9.0340e-07
Spline 4	1.7900e+00	1.0845e+00	9.8933e-02
Spline 5	3.2060e-01	1.1479e+00	7.8003e-01

We see that as we expected spline 4 and 5 are in fact not significant with a confidence level of 5%. A bit to our surprise spline 2 also proved not significant, however, we expect this may be due to the fact that spline 1 and 3 together "cover" the time period, were spline 2 could have been significant.

Question 2b

We will look further into the temperature in room one. The first thing we can do is look closer into how we model the window area using splines. We tried using 7 splines instead of 5. For comparison we will use BIC (Bayes information criterion).

$$BIC_{12345} = 126.6373$$
$$BIC_{1234567} = 142.7225$$

We can see that according to the BIC there is no reason to include all 7 splines compared to only 5. We will not estimate the parameters with more than 7 splines, as it is computationally too demanding.
We will now look into the possibility of removing the non-significant basis spline functions of the 5 basis spline functions. We found non-significance at spline 2, 4, and 5. Removing these yielded:

$$BIC_{13} = 105.3246$$

In turn we found that the optimal model is to generate 5 splines, and use spline 1 and 3 of these.
The residuals of these three model proposals are:

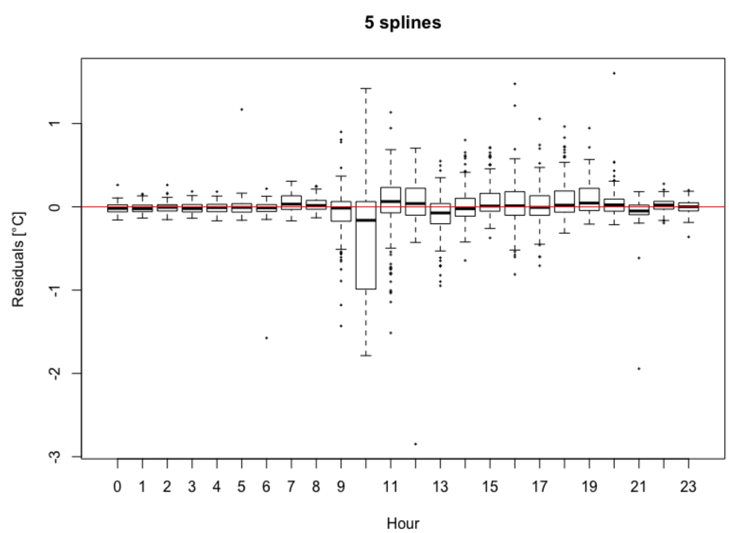

Figure 13: Residuals using 5 splines.

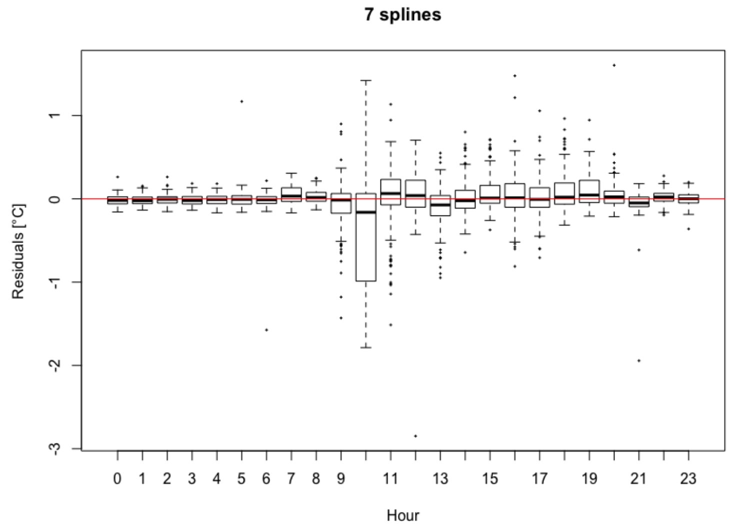

Figure 14: Residuals using 7 splines.

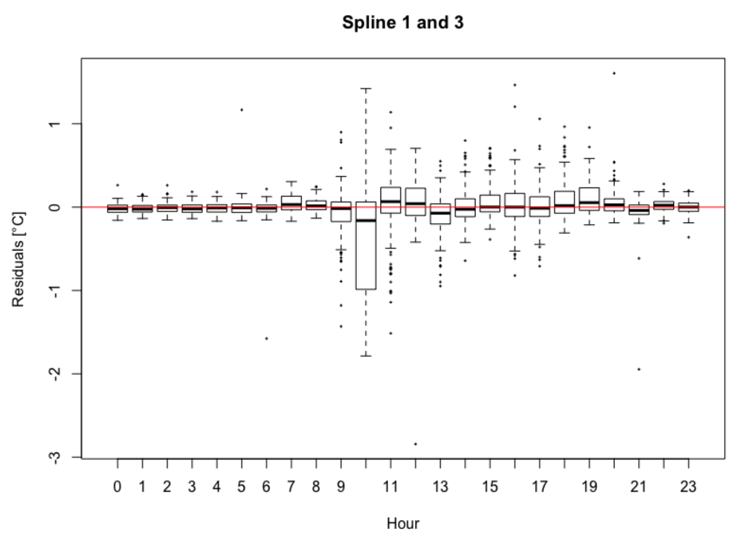

Figure 15: Residuals when selecting spline 1 and 3 from the 5 splines.

From the plots we can see that there is no real difference between them, we will therefore continue with the simplest model, i.e. using spline basis function 1 and 3.

The next improvement to the model could be to let some of the heat enter directly into the thermal mass. This is done by adding a scaling factor, $\alpha \in [0, 1]$, to the drift term for the indoor air temperature, and its opposite in the drift term for the thermal mass such that:

$$
\begin{aligned}
dT_{i1} &= \frac{1}{C_i}\left(\frac{1}{R_{ia}}\left(T_a - T_{i1}\right) + \frac{1}{R_{im}}\left(T_m - T_{i1}\right) + \alpha\Phi + \left(A_{w1}B_{s1} + A_{w3}B_{s3}\right) + G_v\right)dt + \sigma_1 dw_1 \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}\left(T_{i1} - T_m\right) + (1-\alpha)\Phi\right)dt + \sigma_2 dw_2 \\
yT_{i1} &= T_{i1} + e_1
\end{aligned}
\tag{1}
$$

Fitting this model and computing the BIC-score yielded a score of 113.4, i.e. a higher score than not using this scaling factor. We will therefore not use this approach. We can instead try to use a heat-loss variable, ρ, to model potential heat loss in the indoor air temperature. This can be done by:

$$
\begin{aligned}
dT_{i1} &= \frac{1}{C_i}\left(\frac{1}{R_{ia}}(T_a - T_{i1}) + \frac{1}{R_{im}}(T_m - T_{i1}) + \rho\Phi + (A_{w1}B_{s1} + A_{w3}B_{s3}) + G_v\right)dt + \sigma_1 dw_1 \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}(T_{i1} - T_m)\right)dt + \sigma_2 dw_2 \\
yT_{i1} &= T_{i1} + e_1
\end{aligned}
\tag{2}
$$

Fitting this model and computing the BIC-score yielded a score of 113.4 (again), i.e. a higher score than not using this heat loss variable. We will therefore not use this approach either.

We can now try to use the neighboring room as a boundary condition. We start by assuming the indoor air temperature measurement for room 2 is sufficient, and thereby use the model given by (note that yT_{i2} is the measurement for room 2):

$$
\begin{aligned}
dT_{i1} &= \frac{1}{C_{i1}}\left(\frac{1}{R_{ia}}(T_a - T_{i1}) + \frac{1}{R_{im}}(T_m - T_{i1}) + \frac{1}{R_{i1i2}}(T_{i1} - yT_{i2}) + \Phi + (A_{w1}B_{s1} + A_{w3}B_{s3}) + G_v\right)dt + \sigma_{11} dw_{11} \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}(T_{i1} + T_{i2} - 2T_m)\right)dt + \sigma_2 dw_2 \\
yT_{i1} &= T_{i1} + e_1
\end{aligned}
\tag{3}
$$

Fitting this model and computing the BIC-score yielded a score of -51.9, which is a lot lower than the previous models. It therefore seems to be a very good idea to include the neighboring room into the equation.
We can expand this idea by including room 2 as a new state, remembering that our goal is to model room 1 and not room 2, the model for room 2 does therefore not have to be perfect at this point.

$$
\begin{aligned}
dT_{i1} &= \frac{1}{C_{i1}}\left(\frac{1}{R_{ia}}(T_a - T_{i1}) + \frac{1}{R_{im}}(T_m - T_{i1}) + \frac{1}{R_{i1i2}}(T_{i1} - T_{i2}) + \Phi + (A_{w1}B_{s1} + A_{w3}B_{s3}) + G_v\right)dt + \sigma_{11} dw_{11} \\
dT_{i2} &= \frac{1}{C_{i2}}\left(\frac{1}{R_{im}}(T_m - T_{i2}) + \frac{1}{R_{i1i2}}(T_{i2} - T_{i1}) + \Phi\right)dt + \sigma_{12} dw_{12} \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}(T_{i1} + T_{i2} - 2T_m)\right)dt + \sigma_2 dw_2 \\
yT_{i1} &= T_{i1} + e_1 \\
yT_{i2} &= T_{i2} + e_2
\end{aligned}
\tag{4}
$$

Fitting this model and computing the BIC-score yielded a score of -5635.3, which again is a lot lower than before. However, we must remember that we are now including a new room into the model and not just its temperature, this will naturally lead to a much higher log-likelihood, which makes it difficult to compare the model using only the BIC score. Since we want to model room 1 we can look at the RSS for the 3 models.

$$
\begin{aligned}
RSS_{model1} &= 209.4 \\
RSS_{model2} &= 196.9 \\
RSS_{model3} &= 240.3
\end{aligned}
\tag{5}
$$

As we feared the RSS of room 1 is not consistent with the BIC score. Since our goal is to find the best model for room 1 and not for both room 1 and 2, we will not include room 2 as a new hidden state in the model. Looking at the parameters for model2 we find the thermal resistance between room 1 and 2, R_{i1i2}, has a p-value of 5.5377e-01, i.e. not significant. Before we conclude what model to use, we will therefore take a careful look at the residuals.

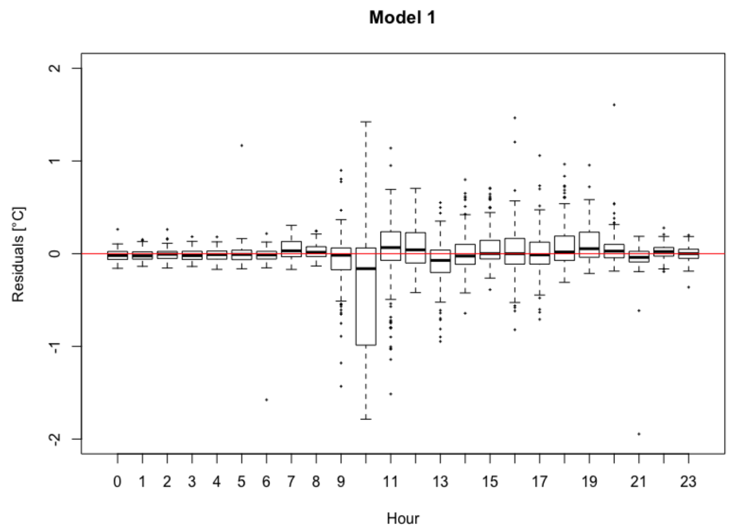

Figure 16: Residuals using model 1.

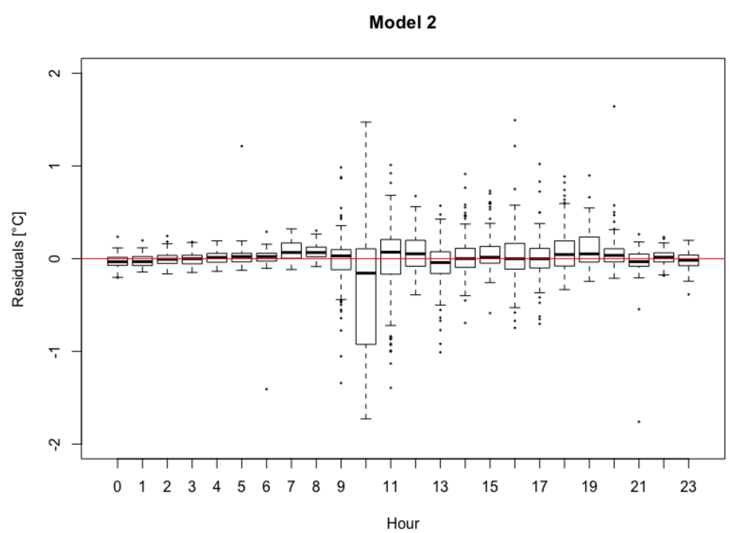

Figure 17: Residuals using model 2.

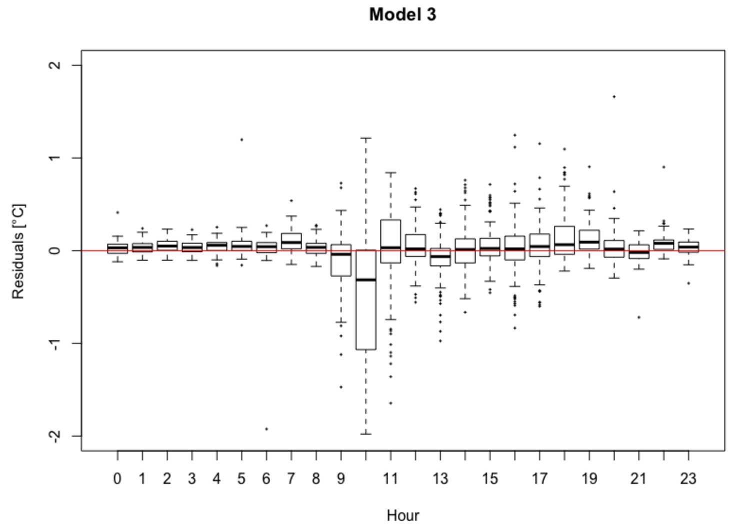

Figure 18: Residuals using model 3.

From the plots, we see no significant improvement in the residuals when including the neighboring room into the equation, almost on the contrary. In order to keep our model as simple as possible, we will therefore not include the neighboring room into our model.
Before we go any further let us take a look at the residuals as a function of the measurements.

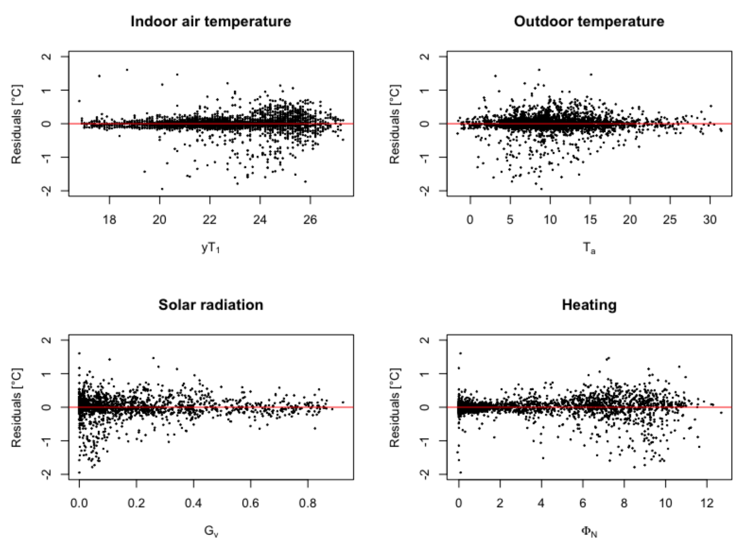

Figure 19: Residuals as a function of measurements.

We cannot immediately see any dependency between the measurements and the residuals. We will therefore consider using a more "black-box" approach to further optimize the model. We suspect some issues regarding differences between how the temperature behaves on weekdays vs weekends.

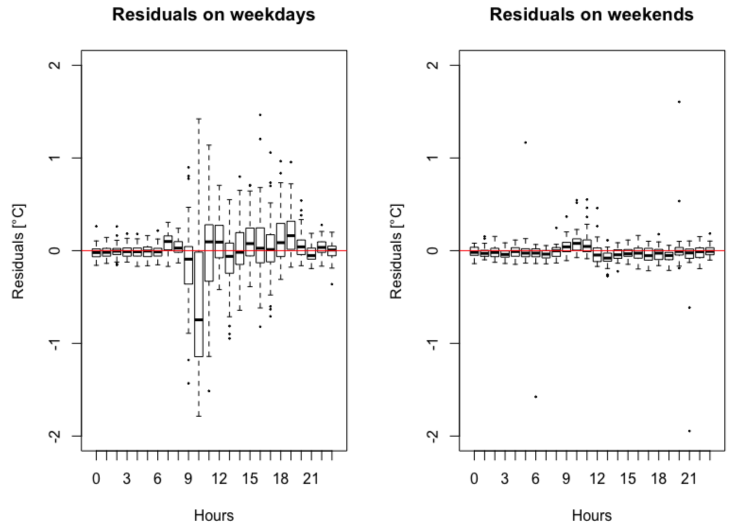

Figure 20: Residuals for weekdays and weekends.

We can clearly see that the model is having issues determining the temperature on weekdays compared to weekends. More specifically it seems to systematically overestimate the temperature around 9am. To counteract this we can add some variables into the model simply letting it know that the time is 7, 8, 9, and 10, and it is a weekday.

$$\begin{aligned}
dT_{i1} &= \frac{1}{C_i}\left(\frac{1}{R_{ia}}(T_a - T_{i1}) + \frac{1}{R_{im}}(T_m - T_{i1}) + \Phi + (A_{w1}B_{s1} + A_{w3}B_{s3}) + G_v + \gamma_7 J_7 + \gamma_8 J_8 + \gamma_9 J_9 + \gamma_{10} J_{10}\right)dt + \sigma_1 dw_1 \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}(T_{i1} - T_m) + \Phi\right)dt + \sigma_2 dw_2 \\
yT_{i1} &= T_{i1} + e_1
\end{aligned} \tag{6}$$

Where

$$J_i = \begin{cases} 1 & Hour = i \;\&\; weekday \\ 0 & otherwise \end{cases} \tag{7}$$

Fitting this model and computing the BIC-score yielded a score of -437.1, and RSS of 169.8, which is the lowest we have seen so far.

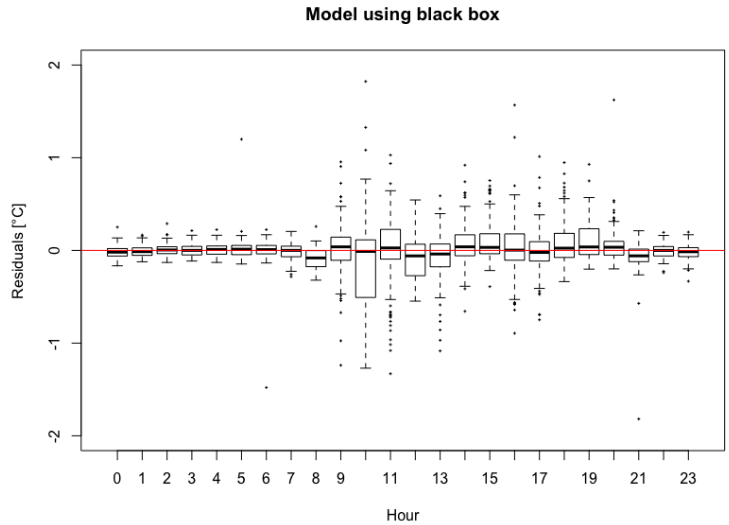

Figure 21: Residuals when using black-box approach.

We can see an improvement already. We can check if it in fact had the wanted effect on the systematic error on weekdays.

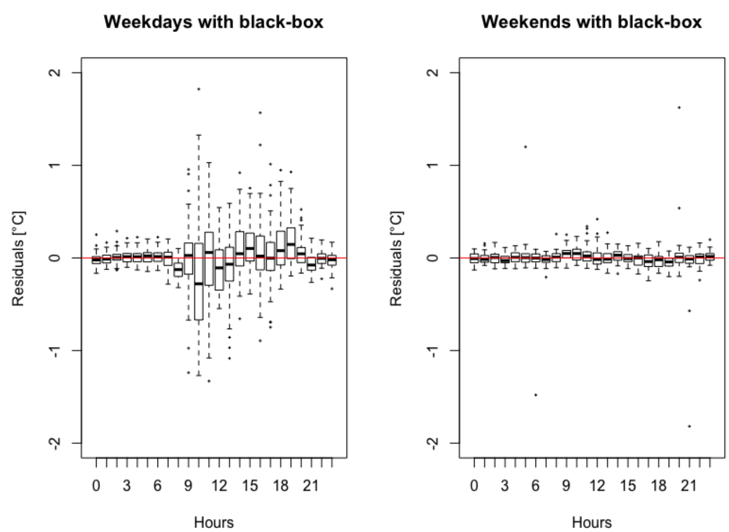

Figure 22: Residuals when using black-box approach.

We can see that it did significantly reduce the systematic error in the mentioned time interval.
We have chosen to focus much on our time on the drift term. It is important to mentioned, that the diffusion term can also be tweaked to optimize the model. We tried allowing for higher variance on weekdays around 9am, and saw that we got a much higher likelihood. However, when we plotted the residuals we saw that the model simply did not fit the data around the period with a systematic error this way. In the end we chose not to do this, and added a "black-box" term instead, which we believe yielded a more accurate model of room 1.

Question 3a

Before attempting to create a model, the floor plan is evaluated, to get a good understanding of the rooms and how the interplay. As we are modelling the temperature in continues time, the temperature in a room is only dependant on the neighboring rooms. All thermal mass is considered a one unit.

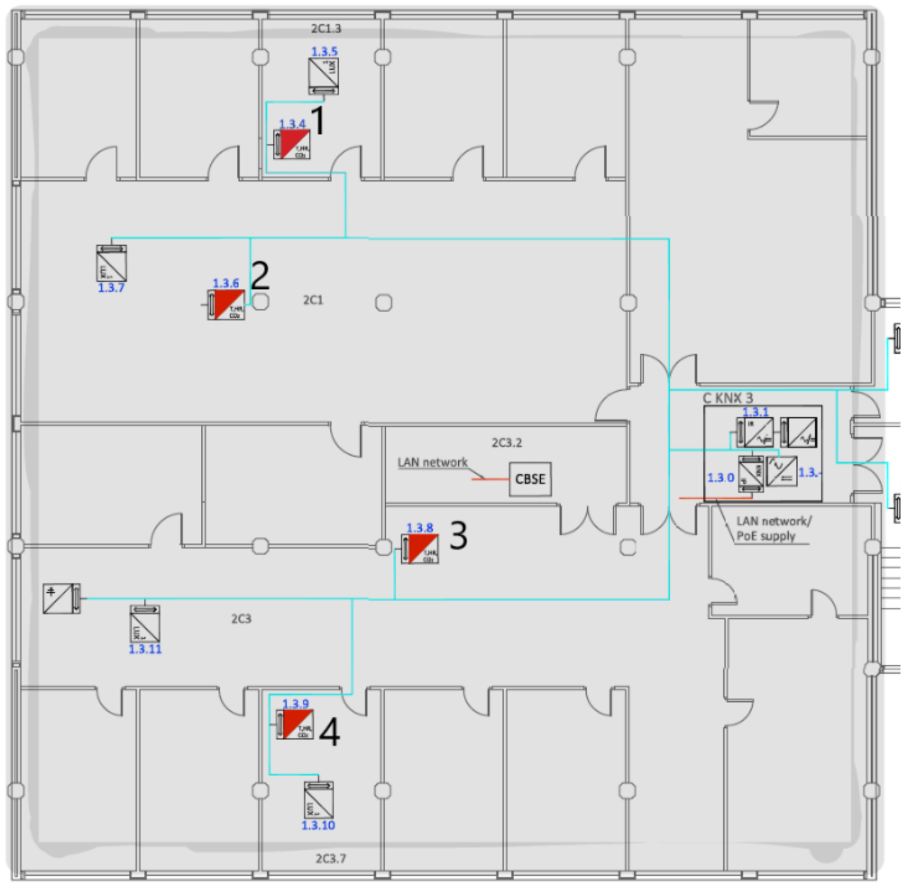

Figure 23: North facing floor plan of the second floor of the east-most wing.

Room 1

Room one has a window facing north-northeast, why the solar radiation Gv should be taken into account. A large part of the wall area is towards the outside, why a heat transfer between the room and the outside is taken into account. It is heated from north heating circuit and is neighboring room 2, why a heat transfer must be accounted for.

Room 2

Room two is likewise heated from north heating circuit and is neighboring room 1, the heat transfer must be the opposite from 2 to 1, than from 1 to 2. As the room is largely internal in the building, the heat exchange to the outside is not considered. Likewise a number of rooms separate room three and two, thus no heat transfer between these is taken into account.

Room 3

Is modelled like room two, not taking into account relatively small window, using room four as neighbour and the southern heating circuit.

Room 4

Is modelled like room one, using room three as neighbour and the southern heating circuit.

First model

This is implemented in below equations.

$$
\begin{aligned}
dT_{i1} &= \frac{1}{C_{i1}} \left(\frac{1}{R_{i1a}} (T_a - T_{i1}) + \frac{1}{R_{i1i2}} (T_{i1} - T_{i2}) + \frac{1}{R_{im}} (T_m - T_{i1}) + \Phi_N + A_w G_v \right) dt + \sigma_1 dw_1 \\
dT_{i2} &= \frac{1}{C_{i2}} \left(-\frac{1}{R_{i1i2}} (T_{i1} - T_{i2}) + \frac{1}{R_{im}} (T_m - T_{i2}) + \Phi_N \right) dt + \sigma_2 dw_2 \\
dT_{i3} &= \frac{1}{C_{i3}} \left(\frac{1}{R_{i3i4}} (T_{i3} - T_{i4}) + \frac{1}{R_{im}} (T_m - T_{i3}) + \Phi_S \right) dt + \sigma_3 dw_3 \\
dT_{i4} &= \frac{1}{C_{i4}} \left(\frac{1}{R_{i4a}} (T_a - T_{i4}) - \frac{1}{R_{i3i4}} (T_{i3} - T_{i4}) + \frac{1}{R_{im}} (T_m - T_{i4}) + \Phi_S + A_w G_v \right) dt + \sigma_4 dw_4 \\
dT_m &= \frac{1}{C_m} \left(\frac{1}{R_{im}} (T_{i1} + T_{i2} + T_{i3} + T_{i4} - 4 \cdot T_m) \right) dt + \sigma_5 dw_5 \\
yT_{i1} &= T_{i1} + e_1 \\
yT_{i2} &= T_{i2} + e_2 \\
yT_{i3} &= T_{i3} + e_3 \\
yT_{i4} &= T_{i4} + e_4
\end{aligned}
\tag{8}
$$

From figure 24, the first model manages to predict the temperature fairly well for room two and three, with few times of day with high variance on the temperature. The result for room four is slightly worse, however room one is by far the worst. The model achieves a BIC-value = -7821.6.

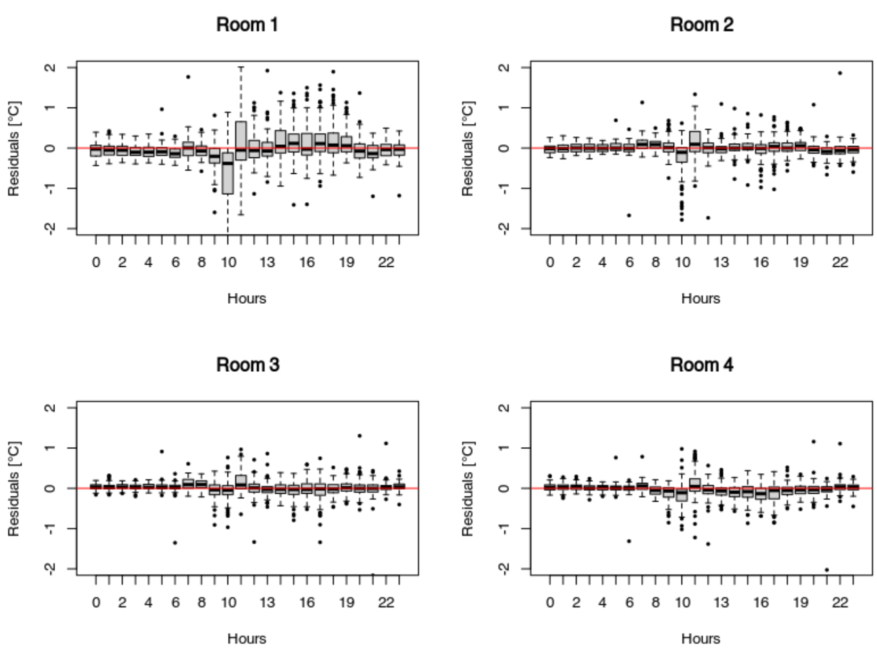

Figure 24: Residuals for each hour of the day

Second model

The second model add splines to room one and four.

$$
\begin{aligned}
dT_{i1} &= \frac{1}{C_{i1}}\left(\frac{1}{R_{i1a}}\left(T_a - T_{i1}\right) + \frac{1}{R_{i1i2}}\left(T_{i1} - T_{i2}\right) + \frac{1}{R_{im}}\left(T_m - T_{i1}\right) + \Phi_N + \left(\sum_{k=1}^{N} a_k bs_k(t)\right) G_v\right) dt + \sigma_1 dw_1 \\
dT_{i2} &= \frac{1}{C_{i2}}\left(-\frac{1}{R_{i1i2}}\left(T_{i1} - T_{i2}\right) + \frac{1}{R_{im}}\left(T_m - T_{i2}\right) + \Phi_N\right) dt + \sigma_2 dw_2 \\
dT_{i3} &= \frac{1}{C_{i3}}\left(\frac{1}{R_{i3i4}}\left(T_{i3} - T_{i4}\right) + \frac{1}{R_{im}}\left(T_m - T_{i3}\right) + \Phi_S\right) dt + \sigma_3 dw_3 \\
dT_{i4} &= \frac{1}{C_{i4}}\left(\frac{1}{R_{i4a}}\left(T_a - T_{i4}\right) - \frac{1}{R_{i3i4}}\left(T_{i3} - T_{i4}\right) + \frac{1}{R_{im}}\left(T_m - T_{i4}\right) + \Phi_S + \left(\sum_{k=1}^{N} a_k bs_k(t)\right) G_v\right) dt + \sigma_4 dw_4 \\
dT_m &= \frac{1}{C_m}\left(\frac{1}{R_{im}}\left(T_{i1} + T_{i2} + T_{i3} + T_{i4} - 4 \cdot T_m\right)\right) dt + \sigma_5 dw_5 \\
yT_{i1} &= T_{i1} + e_1 \\
yT_{i2} &= T_{i2} + e_2 \\
yT_{i3} &= T_{i3} + e_3 \\
yT_{i4} &= T_{i4} + e_4
\end{aligned}
\tag{9}
$$

As can be seen in figure 25, the model demonstrates much better performance across all rooms than model 1. The computations is very lengthy, and some parameters should be removed, prior to attempting further improvements to the model. A BIC-value = -12930.73 is achieved and, this model is used as the base for the next model.

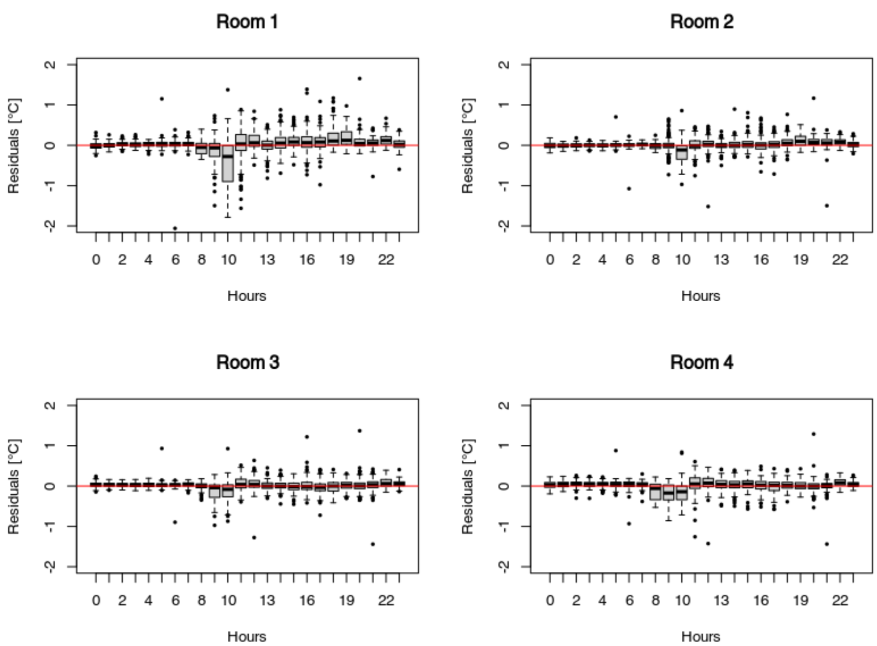

Figure 25: Residuals for each hour of the day

Third model

This model removes the five splines, with high P-values, as these are deemed insignificant. This resulted in a slightly better BIC-score, -12971.15.

Fourth model

In the four attempt it is assumed that lack of heating indicates weekends and holidays. The model implements parameters for indicator functions for 8, 9 and 10 o'clock in room one on days where the rooms are heated significantly. Similarly to what is done in the previous question. This way the activity of people in the building or the errors in the model to capture the heating correctly is improved, despite the source of the error. This improves the model and the parameter estimation perform slightly faster than the second model. This model scores a BIC-value = -13235.47. As can be seen in figure 26, the model is more precise and accurate at ten o'clock in room 1, which has been a pain point throughout the model building.

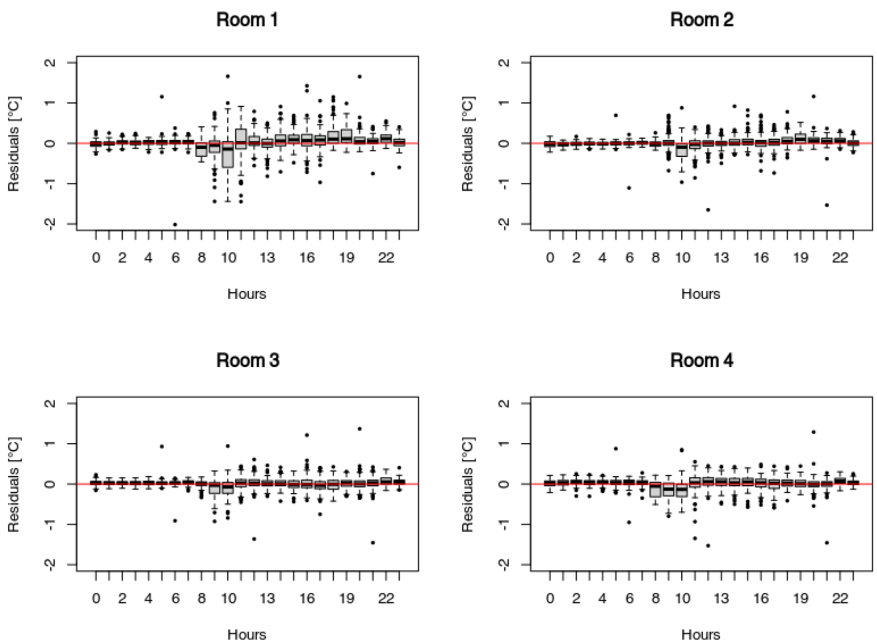

Figure 26: Residuals for each hour of the day

In order to evaluate the quality of the model, the residuals for days, where we assumed no people were present is plotted for each hour of the day in figure 27 . This reveals that we over estimate the effect of solar radiation in room 4 during the afternoon hours. The effect, does however contribute to a better fit on the remaining days, thus it is deemed unnecessary to correct this slight overestimation.

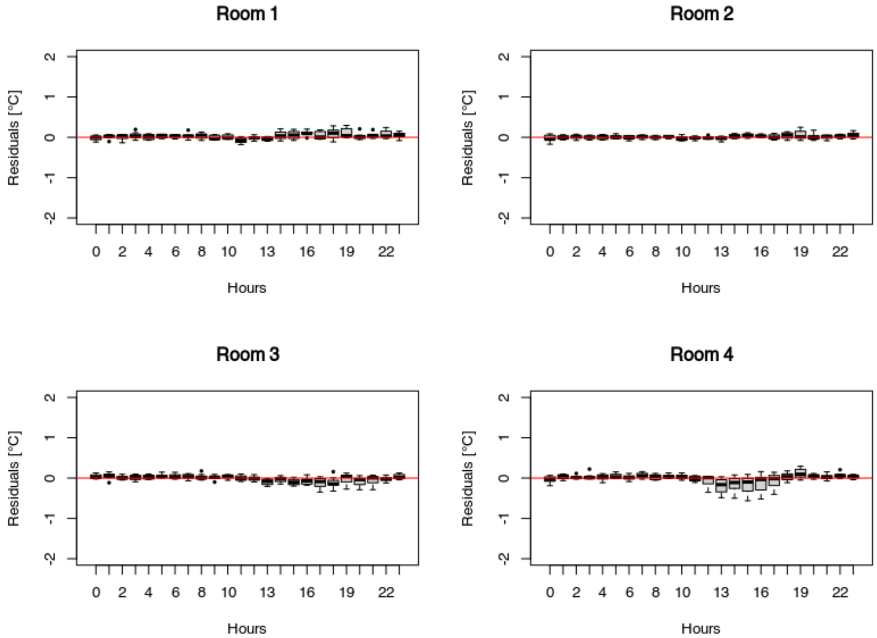

Figure 27: Residuals for each hour of the day, on days without heating

In conclusion the temperature is is modelled fairly well, though with a slight over estimation around 10 o'clock in the morning in room one. A trace of this is found in room two, most likely because of the dependency between the rooms. The computational demand of the model is on the limits of what is feasible to work effectively with, however the results are quite impressive.