Homework assignment 3 for course 01018.

Deadline: Monday 16th November 2020 before midnight.

Question 1

a) As usual, let $(\mathbb{Z}_n, +_n, \cdot_n)$ denote the ring of integers modulo n. Prove that the map $\psi : \mathbb{Z}_{12} \to \mathbb{Z}_4$ given by $\psi(i) = i \mod 4$, for $i \in \mathbb{Z}_{12}$ is a ring homomorphism.

b) Compute the image and the kernel of ψ.

c) Is the map $\varphi : \mathbb{Z}_{14} \to \mathbb{Z}_4$ given by $\varphi(i) = i \mod 4$, for $i \in \mathbb{Z}_{14}$ a ring homomorphism?

Question 2

Let $(D_4, \circ)$ be the dihedral group consisting of the symmetries of a square. As usual we write $D_4 = \{e, r, r^2, r^3, s, rs, r^2s, r^3s\}$. The symmetries r and s satisfy $r^{-1} = r^3$, $s^{-1} = s$ and $sr = r^{-1}s$. For a group homomorphism $\varphi : D_4 \to S_8$ it is given that $\varphi(r) = (1\,3\,5\,7)(2\,4\,6\,8)$ and $\varphi(s) = (1\,8)(2\,7)(3\,6)(4\,5)$.

a) Compute $\varphi(a)$ for all $a \in D_4$.

b) Determine a subgroup $H \subset S_8$ of $(S_8, \circ)$ such that $(D_4, \circ)$ is isomorphic to $(H, \circ)$.

c) What is the smallest integer n such that $(D_4, \circ)$ is isomorphic to a subgroup of $(S_n, \circ)$?