Exercise 4 – Filling the Qattara Depression

The goal with this exercise is to use mathematical modelling to solve a practical problem: Filling the Qattara Depression with water. You should:

- Identify significant issues
- Formulate mathematical models, which solves these issues
- Implement the model in a computer program
- Use the implemented model for analysis of data
- Report the results from the analysis and discuss the validity of the model

The report for this exercise must be 5 pages at most, including graphs, tables, and images (excluding frontpage and appendices). In this exercise a number of questions are asked. The report, however, should not be a list of answers, but instead be a coherent documentation and discussion of the analysis performed.

The exercise consists of several parts. The first is information seeking, to determine the relevance of the problem and putting it in perspective. We then investigate several variations of Mixed Integer Linear Programs, and their relevance.

To solve the exercise, we suggest you use JULIA. However, you may find it easier to use MATLAB or PYTHON for the initial data processing.

Part 1 - The Qattara depression

The Qattara depression is a large area below sea level in Egypt, see figure 1. The area is mostly desert, with a few oasis and oil fields. The economic importance is therefore minor. With the projected increase in sea levels due to global warming, a mitigation strategy could be to fill the depression with sea water. It would also change the local weather patterns due to the evaporation from the formed 'lake'. If just some of the evaporated water falls locally it would transform parts of the desert into green areas.

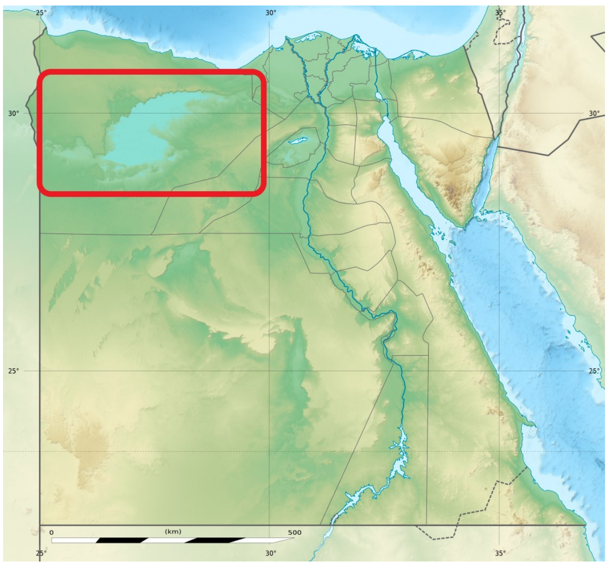

Figure 1: The Qattara depression is seen in the red box. Modified from: Wikipedia, Eric Gaba (Sting - fr:Sting) and NordNordWest / CC BY-SA (https://creativecommons.org/licenses/by-sa/3.0)

Problem 1 - volume What is the volume of the Qattara depression? How much would the global sea level decrease if it was filled to the sea level with water.

Part 2 - Digging a channel

Digging a channel from the depression to the sea will both be costly and time consuming. Large amounts of ordnance from the second world war will need to be removed. This is complicated by large mine fields from the same period.

Problem 2 - channel data We need to estimate how much dirt needs to be removed. The height over sea level is given at specific points in the file 'channel_data.txt', where each line is a latitude, longitude, and the height in meters over sea level. The locations are shown in figure 2. The first point is at sea level in the Mediterranean Sea, set that to distance zero. Calculate the distance between the points and Interpolate the height, such that there are 250 m between each point. Plot the distance from the Mediterranean Sea against the height

Figure 2: Sampled height locations. Google maps.

A much faster method than traditional excavation is by using nuclear bombs. As demonstrated by e.g. the Sedan and Chagan tests, large amounts of dirt can be removed quickly by nuclear detonations. More details on the effects can be found in [1]. However, due to problems with fallout - and the price of nuclear weapons - the total amount used should be minimised.

This can be formulated as an Mixed Integer Linear Problem[1].

$$\min \sum_{i=1}^{n} x_i \tag{1}$$

subject to

$$R_i \geq H_i + CHD \quad i = 1, \dots, n \tag{2}$$

$$R_i = K_2 \cdot x_{i-2} + K_1 \cdot x_{i-1} + K_0 \cdot x_i + K_1 \cdot x_{i+1} + K_2 \cdot X_{i+2} \quad i = 1, \dots, n \tag{3}$$

$$x_i \in \{0, 1\} \quad i = 1, \dots, n \tag{4}$$

where n is the total number of locations along the channel, x_i is a binary variable denoting whther a nuke is planted at position i or not, R_i is the amount of dirt removed at position i. CHD is the necessary channel depth and $K0$ is the amount of dirt removed by a nuke in a specific location, $K1$ is how much dirt it removes in the neighbouring positions, and $K2$ is how much dirt is removed 2 positions away. For now we will assume that the k-values are exact, that the dirt deposited by one explosion unto another locations is negligible, and that the necessary depth of the channel is ten meters: $CHD = 10$ m.

[1]The problem can also be formulated as a Dynamic Programming Problem, which is solvable in polynomial time. However, some of the constraint added below are not directly compatible with a Dynamic Program

The above formulation can be rewritten as

$$\min \sum_{i=1}^{n} x_i \tag{5}$$

subject to

$$\sum_{j\in S} R_{ij}\, x_j \geq H_i + CHD \quad i = 1, \ldots, n \tag{6}$$

$$x_i \in \{0, 1\} \quad i = 1, \ldots, n \tag{7}$$

where S is the set of locations

Problem 3 - MILP, direct solution Modify the the JULIA program 'nukeDemo.jl' to accept your interpolated height data, and find how many nukes are needed and their position. Plot them on the distance vs. height plot you made earlier.

To ensure a good flow in the channel it should be as smooth as possible. To enforce that, we can change the objective:

$$\min \sum_{i=1}^{n} |R_i - H_i - CHD| \tag{8}$$

You can find more information about how to handle non-linearities as this in [2] available on Inside.

Problem 4 - Smooth channel Implement the new objective function from equation 8. How does the result differ? Visualise the resulting depth of the channel for the two solutions and the number and placement of bombs used.

In practise the channel needs be dug in one single go, as the ground around it is rendering highly radioactive for several months. The engineers are concerned, that a slight mis-timing between the bombs, will mean that a bomb close to another bomb could be destroyed by the shock wave. To mitigate that risk, they do not want any bombs placed in locations directly next to each other.

Problem 5 - placement constraints Implement the constraint about no bombs being placed next to each other while still optimising channel smoothness. Compare to the previous solutions.

The design used for several types of nuclear bombs are "dial a yield", where the yield can be selected by the operator. See e.g. B61 WIKI. We will now investigate if we can get a smoother channel and use fewer nukes by having that option. The amount of dirt removed for each setting is given below:

Setting	k_0	k_1	k_2
1	300	140	40
2	500	230	60
3	1000	400	70

Problem 6 - Dial a yield Implement the possibility of using different yields in each location, while ensuring that no bombs are placed next to each other and optimising for channel smoothness. This can run for a very long time - instead you may consider

- What is the effect on run time by including only part of the channel?
  - What happens when a larger part of the channel is included?
  - Is it feasible to run for the full channel?

- Do we see a change in channel smoothness that makes it worth to consider?

- What is the effect on run time by using the Cbc vs. GLPK solver?
- What is the effect on run time by using only two of the settings instead of all three?

Alternatively, you may choose to report the bounds given by the solver if it does not find the optimal solution.

OPTIONAL - Different settings The settings used in Problem 6, might be far from the optimal options. Try out different settings, and see if you can get better results.

OPTIONAL - Depth of charge In practise the shape of the crater and amount of earth removed depends heavily on the depth at which the charge is placed. Find more information in [1] (available on Google Books) and include that in the simulations.

Reporting **Contents of the report**

1. Describe the problem and the background – what is modelled and why?
2. Describe data and experiments
3. Describe the mathematical model - how and why?
4. Describe results
5. Discuss results – how good are the results? To what degree do they reflect the true problem?
6. Conclude – what is the contribution of the analysis?

Hand in The report is uploaded to DTU Inside and to peergrade.io. The code must be included in an appendix in the report.

Anders Nymark Christensen 2020
Special thanks to Bernardo Martin Iradi and Stefan Røpke.

References

[1] S. Glasstone, P.J. Dolan, and United States. Department of Defense. The Effects of Nuclear Weapons. Castle House, 1980.

[2] J Cole Smith and Z Caner Taskın. A tutorial guide to mixed-integer programming models and solution techniques. 2007.