Decision-Making under Uncertainty: Individual Assignment

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This is the individual assignment for the DTU course *Decision-Making under Uncertainty* for July 2022.

Bibliography

In the following, we will analyze the methodology and results of the following paper:
Alvarez, Pamela P.; Vera, Jorge (2011): *Application of Robust Optimization to the Sawmill Planning Problem*; Annals of Operations Research — 2011, pp. 1-19.

Summary of planning problem

The paper centres around maximizing profits for a sawmill company in Chile. The sawmill makes money by purchasing logs, sawing the logs into different boards, possibly drying the boards to enhance the quality, and selling the boards either to a local market or to where they are needed around the World. In this process, the planning team has to decide how many logs to buy and when, how to cut the logs, if they should be dried or not, and finally, when to sell them. Notice that the team has restrictions on sawing-, warehouse-, and drying capacity. Notice also that there everything comes at some cost—it costs money to store logs and/or boards in the inventory, there is some conversion rate, $R < 1$, when cutting the logs etc. Finally, it is possible to *reprocess* some boards, i.e., cut them into different types, depending on the original type.
The planning problem is quite complex, with many variables and different approaches. In order to maximize profits, the planning team therefore applies OR to help solve this. Solving the deterministic problem, where all parameters are assumed to be known exactly, has been done before.

Summary of methodology

The paper discusses how robust optimization, specifically budgeted robust optimization, can be used to improve the performance and reliability of solutions from a linear programming model in a sawmill operation. They allow conversion rates, R, to be stochastic, and investigate the impact this has on the solution and optimal objective value when assuming varying degrees of variability and/or uncertainty budgets. The main advantage of this approach is that it allows for the tradeoff between robustness requirements and the loss in optimality to be studied. The simulation results show that the robust solutions computed preserved feasibility in a high percentage and that the principal decisions on the optimal robust solutions do not present significant changes in their structure. This reinforces the applicability of this methodology to problems associated with the Chilean Forest Supply Chain.

Correct optimization methodology?

In this paper, the authors chose to use robust optimization to investigate the effect uncertainty in the conversion rate of logs to boards has on the solution to the milling problem. In general, robust optimization has the advantage of needing little to no information about the distribution of the stochastic variable. This makes the methodology good in cases where it is very hard/expensive to get more detailed information about the stochastic variables. Without knowing much about forestry, it seems unlikely that it would be particularly hard/expensive to get more information about how the conversion rate varies in real life. It is therefore my opinion that it would have been better to use real data to generate scenarios and perform stochastic programming—instead of choosing seemingly arbitrary deviations.

There is one caveat; if the issue is that the problem would be too large to solve using stochastic programming, the authors might have been forced to use robust optimization, since it generally requires less computing power. Since they do not mention anything in this regard, it seems unlikely that this is the case.

Modelling of uncertainty

In this paper, the authors loosen the assumption on the conversion rate, R, such that it becomes stochastic. This corresponds to saying that there is some uncertainty regarding the yield of boards you can get by cutting e.g. 1 ton of logs. In this paper, they choose to apply budgeted robust optimization. Specifically, they assume that the *true* conversion rate, $\tilde{R}$, lies within $\tilde{R} \sim [R \pm \sigma_R \cdot R]\,, \quad \sigma_R \in \{5\%\ 10\%\ 15\%\ 20\%\}$. Then they use different budgets of uncertainty, $\Gamma \in [0, 104]$, indicating the number of parameters (104 total) that are allowed to vary by $|R| \cdot \sigma_R$.

Data handling for the uncertainty

In this paper, the testing of the methodology is done using Monte-Carlo simulations. Specifically, they have simulated 600 parameters for the uncertain parameter, $\tilde{R}$. 300 of which the parameters are drawn from a uniform distribution for each of the cases for σ_R. In the remaining 300 simulations the parameters are drawn from a normal distribution defined such that 95% of the probability mass lies with the previously specified interval (notice that this can lead to unfeasible solutions). The robust solutions are then compared to the solutions where the uncertain parameter is assumed known. The paper does not take offset in real data—there is nothing to indicate the choices of σ_R are reasonable. Therefore, the results of this paper are not directly applicable in real life; it is more a theoretical scenario where the conversion follows either a uniform or a normal distribution with the variabilities previously discussed. Since the paper does not include any real data, nothing is done in terms of analysing performance in- and out-of-sample and/or comparing methods out of sample.

Analysis of numerical experiments

In this paper, the only numerical analysis done is a comparison of budgeted robust optimization assuming different uncertainty set widths with wait-and-see solutions. All data is generated arbitrarily for the paper. For the selected uncertainties the primary focus is the effect the budgets have on the objective value. The authors conclude that the robust solutions are *close* to the wait-and-see solutions and hence argue that it would be useful. However, they fail to put their chosen uncertainties into relation to real data and the profitability of Chilean forestry. It is therefore hard to determine both if the solution reflects something that could resemble real life and what *close* means in this context.

Further, the uncertain data in the model (conversion rates) are also present in the objective function as well as in several constraints. This means that there is an implicit relationship between the constraints and the objective function. This relation is simply ignored, and it is difficult to determine what effect this has on the final result.

Overall, the paper could be significantly improved by taking real data into account. Both give an estimate of the profitability of a real sawing mill, such that the change to objective value can be seen in relation to this. Additionally, real data ought to be used on uncertainties of conversion rates, either to support the choices of uncertainties or to use directly by stochastic programming or, alternatively, in the evaluation.