Homework assignment 2 for course 01018.

Deadline: Wednesday 21st October 2020 before midnight.

Question 1

Consider the group $(G, \circ)$ of all the maps $f_{a,b} : \mathbb{R} \to \mathbb{R}$ defined by $f_{a,b}(x) := ax + b$, where $a, b \in \mathbb{R}$, $a \neq 0$ and $\circ$ is the composition operation. Let $\varphi : G \to \mathbb{R}^*$ with $\varphi(f_{a,b}) := a$. Here $\mathbb{R}^*$ is the set of all non-zero real numbers.

a) Let $a, b, c, d \in \mathbb{R}$ and suppose that $a \neq 0$ and $c \neq 0$. Show that $f_{a,b}^{-1} = f_{a^{-1}, -a^{-1}b}$ and $f_{a,b} \circ f_{c,d} = f_{ac, ad+b}$.

b) Define $T := \{f_{1,b} \mid b \in \mathbb{R}\} \subset G$. Show that T is a subgroup of $(G, \circ)$.

c) Show that for any $g \in G$ it holds that $gT = Tg$. Remark: if H is a subgroup of a group $(G, \cdot)$ such that for all $g \in G$ the left coset gH is equal to the right coset Hg, one calls H a *normal* subgroup of $(G, \cdot)$.

Question 2

Consider the permutations $g_1 = (1\,3\,2)$ and $g_2 = (1\,3\,4\,5)$ from S_5.

a) Compute the order and the sign of the permutation $g_1 \circ g_2$.

b) It is given that the set of permutations $H := \{\mathrm{id}, g_1, g_1^2\} \subset S_5$ is a subgroup of $(S_5, \circ)$. You may use this fact without proving it. Now compute the cosets $g_2 \circ H$ and $H \circ g_2$.

c) How many distinct left cosets does H have in S_5?