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1 CLRS 11.2-2

Consider a hash table with 9 slots and the hash function $h(k) = k \bmod 9$. Demonstrate what happens upon inserting the keys $5, 28, 19, 15, 20, 33, 12, 17, 10$ with collisions resolved by chaining.

假设hashtable为ht[1:9]

insert number	hash_number	is_collided	operation
5	5	false	ht[5] = Node(5)
28	1	false	ht[1] = Node(28)
19	1	true	N_19 := Node(19) N_19.next = ht[1] ht[1] = N_19
15	6	false	ht[6] = Node(15)
20	2	false	ht[2] = Node(20)
33	6	true	N_33 := Node(33) N_33.next = ht[6] ht[6] = N_33
12	3	false	ht[3] = Node(12)
17	8	false	ht[8] = Node(17)
10	1	true	N_10 := Node(10) N_10.next = ht[1] ht[1] = N_10

2 CLRS 11.4-1

Consider inserting the keys $10, 22, 31, 4, 15, 28, 17, 88, 59$ into a hash table of length $m = 11$ using open addressing. Illustrate the result of inserting these keys using linear probing with $h(k, i) = (k + i) \bmod m$ and using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \bmod (m - 1))$.

final result

hash_number	linear probing $h(k,i) = (k+i) \bmod m$	double hashing $h(k,i) = (h_1(k) + ih_2(k)) \bmod m$ $h_1(k) = k; h_2(k) = 1 + (k \bmod (m-1))$
0	22	22
1	88	/
2	/	59
3	/	17
4	4	4
5	15	15

6	28	28
7	17	88
8	59	/
9	31	31
10	10	10

3 CLRS 11.4-3 & 11.4-4

Consider an open-address hash table with independent uniform permutation hashing and no deletions.

1. Give upper bounds on the expected number of probes in an unsuccessful search and on the expected number of probes in a successful search when the load factor is $3/4$ and when it is $7/8$.
2. Show that the expected number of probes required for a successful search when $\alpha = 1$(that is, when $n = m$), is H_m, the mth harmonic number.

$$unsuccess: E(N) \leq \frac{1}{1-\alpha} = 4 \left(when\ \alpha\ is \frac{3}{4}\right) = 8 \left(when\ \alpha\ is \frac{7}{8}\right)$$

$$success: E(N) \leq \frac{1}{\alpha}\ln\left(\frac{1}{1-\alpha}\right) = \frac{8}{3}\left(when\ \alpha\ is \frac{3}{4}\right) = \frac{24}{7}\left(when\ \alpha\ is \frac{7}{8}\right)$$

$$H_m = \sum_{n=1}^{m} \frac{1}{n}$$

$$when\ success, N_i = \frac{1}{1 - \frac{i-1}{m}},$$

$$E[X] = \frac{1}{n}\sum_{i=1}^{n} N_i = \frac{1}{n}\sum_{i=1}^{n} \frac{m}{m-i+1} = \sum_{k=1}^{m} \frac{1}{k} = H_m$$