Electromagnetic Scattering and Radiation by Surfaces of Arbitrary Shape in Layered Media, Part I: Theory

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Abstract—**An accurate and general procedure is developed for the analysis of electromagnetic radiation and scattering by perfectly conducting objects of arbitrary shape embedded in a medium consisting of an arbitrary number of planar, dielectric layers. The key step in this procedure is the formulation of the so-called mixed-potential electric field integral equation (MPIE) that is amenable to an existing advanced solution technique developed for objects in free space, and which employs the method of moments in conjunction with a triangular-patch model of the arbitrary surface. Hence, our goal is to immediately increase analysis capabilities in electromagnetics, yet remain compatible with the large existing base of knowledge concerning the solution of surface integral equations. In this first part of a two-part paper, three alternative forms of the MPIE in plane-stratified media are developed and their properties are discussed. In the second part, one of the developed MPIEs is implemented to analyze scatterers and antennas of arbitrary shape that penetrate the interface between contiguous dielectric half-spaces.**

I. Introduction

SIMPLE AND EFFICIENT method of moments (MM) [1] procedures have been recently developed for the solution of the electromagnetic scattering and radiation problems involving objects of arbitrary shape in free space [2]–[6]. These procedures are based on the so-called mixed-potential form of the electric field integral equation (EFIE)—so named, because it involves both the vector and scalar potentials; the former expressed in terms of the induced current, and the latter in terms of the induced charge. In the case of perfectly electrically conducting (PEC) objects, the EFIE is more general than the magnetic field integral equation (MFIE) [7], since it is applicable to both closed and open surfaces [8]. The mixed-potential EFIE (MPIE) is preferable to the several other possible variants of the EFIE because it requires only the potential forms of the Green's functions, which are less singular than their derivatives needed in the other forms of the EFIE [4]. In the MM technique, originally developed by Rao, Wilton, and Glisson [5], the surface of the PEC object is modeled in terms of triangular patches and especially designed basis functions defined on pairs of adjacent triangles are used, which yield a surface current representation free of line or point charges at subdomain boundaries. This technique also employs a testing scheme where the derivatives of the scalar potential are in effect replaced by finite differences. More recently, Schaubert, Wilton, and Glisson [9] extended this procedure to volume integral equations for penetrable bodies, which they modeled in terms of tetrahedral elements.

The procedures described above were originally developed for antennas and scatterers residing in a homogeneous space. Although this restriction is not severe in some aerospace applications where the effect of the environment can be neglected, it does exclude many problems of practical interest where the proximity of the earth must be taken into account. Indeed, in many cases the influence of the ground or the ocean, which often can be adequately represented by a model consisting of one or more planar, dielectric layers, is the dominant effect in the problem. Therefore, in this paper we seek to develop, for arbitrarily shaped objects in layered media, an MPIE formulation that is amenable to the MM procedures originated by Wilton and his co-workers, which culminated in the development of the triangle-patch code [5], [10], [11] and its thin-wire counterpart, MININEC [12]. Since a great amount of effort went into the development of these codes, which can currently handle only objects in free space or over a PEC ground, it is desirable to have a formulation that would allow one to easily extend them to the case of bodies in stratified media. For simplicity's sake, we limit attention to PEC objects and surface integral equations, however, our formulation can, if desired, be easily extended to dielectric bodies in conjunction with either volume [9] or surface [6] integral equations.

Unlike in free space, or even in some complex environments with PEC boundaries, it is not a trivial task to formulate an MPIE for objects of arbitrary shape in a layered medium. This is due to the fact that in the layered medium the vector and scalar potentials are not unique [13] and the scalar potentials of point charges associated with horizontal and vertical dipoles are not, in general, identical [14]. Additional complications arise when the objects penetrate one or more of the interfaces between dielectric layers.

The advantages of the MPIE over the other forms of the EFIE are even more pronounced in a layered medium than in free space. This is due to the fact that the Green's functions in layered media comprise Sommerfeld-type integrals [15], which are extremely laborious to evaluate. Since the MPIE only involves the potential forms of the Green's functions,

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rather than the field forms, the Sommerfeld integrals it requires converge faster than those present in any other form of the EFIE [4].

Several researchers have previously recognized the advantages of the mixed-potential formulation in solving antenna problems in layered media. Mosig and Gardiol [16], [17] have applied the MPIE in conjunction with Glisson and Wilton's [4] MM procedure to a rectangular microstrip antenna. Johnson [18] has used a similar approach to solve the problem of a vertical cylinder penetrating the interface between contiguous half-spaces. Wilton and Singh [19] have applied an MPIE to analyze a periodic array of slots in a conducting screen backed with a layered dielectric. Michalski, Smith, and Butler [20], [21] have used an MPIE to solve the problem of a horizontal two-element wire array above and on opposite sides of the interface between two media. As was pointed out in [22], [23], the success of these efforts can be attributed to the fact that the structures considered could only support either vertical or horizontal components of the current. To our knowledge, an MPIE for arbitrarily shaped objects in a layered medium was first published in [23]. However, it was assumed in that paper that the antenna or scatterer was confined to a single layer. In a two-dimensional case, an MPIE has been derived by Xu [24].

Numerous papers have been published on the subject of antennas and scatterers in layered media, but—with the exception of the geophysics literature—most of them deal only with planar geometries, such as microstrip antennas, transmission lines, etc. Since the emphasis here is on objects of *arbitrary* shape, we reference only a few of these papers, to conserve space. The problem of arbitrarily shaped thin wires that are near to or penetrate an interface between contiguous half-spaces has been solved by Burke and Miller [25]. However, their approach, which is implemented in the powerful Numerical Electromagnetics Code (NEC) [26], is not easily extendable to arbitrary surfaces. From the many works devoted to electromagnetic modeling of buried inhomogeneities in the context of geophysical prospecting, we only mention the recent representative papers by Hohmann [27] and Wannamaker, Hohmann, and SanFilipo [28]. These authors use the volume integral equation technique in conjunction with a rather crude—but entirely adequate in the quasi-static regime—MM procedure employing piecewise constant current expansion and point-matching [1]. To overcome the problems associated with the singular behavior of the electric Green's function, Hohmann [27] employed a mixed-potential formulation, but only to the primary (or whole-space) component of the kernel; the part comprising the Sommerfeld integrals was left in the slowly convergent field form. Mention should also be made of the work by Karlsson and Kristensson [29], who employed the extended boundary condition integral equation in conjunction with the T-matrix approach to compute the field scattered by obstacles buried in a stratified ground. However, this method is only applicable to closed, smooth bodies and is only practical for simple shapes.

The outline of Part I of this paper is as follows. Section II contains the statement of the problem and a general discussion of various forms of the EFIE in a layered medium.

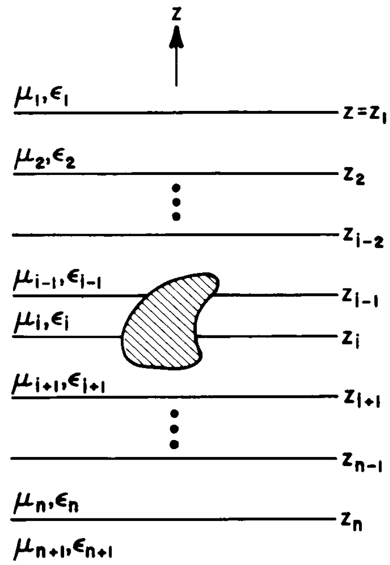

Fig. 1. Scatterer of arbitrary shape embedded in a layered dielectric medium.

The main results are in Section III, where we develop in detail three alternative mixed-potential formulations for arbitrarily shaped PEC objects in layered media. Some tedious details concerning the derivation of the Green's function for a stratified medium are relegated to Appendices I and II. The discussion of the various formulations is given in Section IV, and the conclusion in Section V.

In Part II of this paper [30], one of the developed MPIEs is implemented to analyze PEC scatterers and antennas of arbitrary shape that penetrate the interface between contiguous dielectric half-spaces.

II. Preliminaries

Statement of the Problem

Consider a medium consisting of $n+1$ dielectric layers separated by n planar interfaces parallel to the xy plane of a Cartesian coordinate system and located at $z = z_l$, $l = 1, 2, \cdots, n$, as illustrated in Fig. 1. The medium of the ith layer is characterized by permeability μ_i and permittivity ϵ_i, which may be complex if the medium is lossy. Let the PEC object (or collection of objects) in Fig. 1 occupy p layers with indices $l_1, l_2, \cdots, l_p$, where $1 \leq p \leq n+1$. For later convenience, define the ordered set of indices $L = \{l_1, l_2, \cdots, l_p\}$ in which $l_k < l_{k+1}$. Let S_i denote the surface of the object(s) in the ith layer and let $\hat{\mathbf{n}}_i$ be a unit vector normal to S_i. The quantity of interest is the surface current density $\mathbf{J}(\mathbf{r})$ excited on the object(s) by a given time-harmonic incident electric $\mathbf{E}^{\text{inc}}$. The $e^{j\omega t}$ time variation is assumed and suppressed.

Electric Field Integral Equation

The EFIE for the current density $\mathbf{J}$ on the surface S of the PEC object(s) embedded in a layered medium is obtained by enforcing the boundary condition [31]

$$-\hat{\mathbf{n}}_m \times \mathbf{E}_m^s(\mathbf{r}) = \hat{\mathbf{n}}_m \times \mathbf{E}_m^{\text{inc}}(\mathbf{r}), \quad \mathbf{r} \text{ on } S_m, \quad m \in L \qquad (1)$$

where $\mathbf{r}$ is the position vector defined with respect to a global coordinate origin, $\mathbf{E}_m^{inc}$ is the "incident" electric field (i.e., the field in the absence of the object) in the mth layer, and $\mathbf{E}_m^s$ is the scattered field in the mth layer. For the structure of Fig. 1, $\mathbf{E}_m^s$ and the scattered magnetic field $\mathbf{H}_m^s$ can be expressed as

$$-\mathbf{E}_m^s(\mathbf{r}) = \sum_{i\in L}[j\omega\mathbf{A}^{mi}(\mathbf{r}) + \nabla\phi^{mi}(\mathbf{r})] \quad (2)$$

$$\mathbf{H}_m^s(\mathbf{r}) = \frac{1}{\mu_m}\nabla\times\sum_{i\in L}\mathbf{A}^{mi}(\mathbf{r}) \quad (3)$$

where $\mathbf{A}^{mi}$ is the magnetic vector potential in the mth layer due to the current density $\mathbf{J}$ in the ith layer, and is given as

$$\mathbf{A}^{mi}(\mathbf{r}) = \int_{S_i}\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,dS' \quad (4)$$

and $\phi^{mi}(\mathbf{r})$ is the corresponding scalar potential which is related to $\mathbf{A}^{mi}(\mathbf{r})$ through the Lorentz condition

$$\phi^{mi}(\mathbf{r}) = \frac{j\omega}{k_m^2}\nabla\cdot\mathbf{A}^{mi}(\mathbf{r}) \quad (5)$$

where $k_m^2 = \omega^2\epsilon_m\mu_m$. In (4), $\bar{\mathbf{G}}_A^{mi}$ is the dyadic Green's function which represents the magnetic vector potential in region m due to a unit-strength, arbitrarily oriented current dipole in region i. $\bar{\mathbf{G}}_A^{mi}$ can be found by solving the inhomogeneous Helmholtz equation

$$(\nabla^2 + k_m^2)\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}') = -\mu_m\bar{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}') \quad (6)$$

where $\bar{\mathbf{I}}$ is the idemfactor, subject to the condition that the tangential component of $\mathbf{E}_m^s$ and $\mathbf{H}_m^s$ be continuous across the interfaces between dielectric layers. As is well known [15], for a horizontal, say, x-directed dipole, two components of the vector potential are required to satisfy the boundary conditions at the interfaces. Traditionally [32], the z component has been selected in addition to the x component. The Green's function in this case takes the form [23]

$$\bar{\mathbf{G}}_A^{mi} = (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})G_{xx}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{x}}G_{zx}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{y}}G_{zy}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{z}}G_{zz}^{mi}. \quad (7)$$

However, one may as well postulate the y component of the vector potential to accompany the primary x component [13]. This strategy leads to an alternative form of the Green's function,

$$\bar{\mathbf{G}}_A^{mi} = \hat{\mathbf{x}}\hat{\mathbf{x}}G_{xx}^{mi} + \hat{\mathbf{y}}\hat{\mathbf{y}}G_{yy}^{mi} + (\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}})G_{xy}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{z}}G_{zz}^{mi}. \quad (8)$$

We note that, except for G_{zz}^{mi}, the corresponding components in (7) and (8) are different, even though the same notation is used. The entries of the dyadics in (7) and (8) in the Fourier transform domain are listed in Appendix II. Their inversion in accord with Appendix I leads to improper integrals of the Sommerfeld [15] type.

Still other Green's functions, in addition to those in (7) and (8), are possible. For example, one can obtain another form of $\bar{\mathbf{G}}_A^{mi}$ by postulating that the y and z components of the vector potential, instead of the x and z or x and y components, accompany an x-directed dipole. However, this and other forms of $\bar{\mathbf{G}}_A^{mi}$ are less attractive for our purpose than (7) and (8), and are not considered here.

Substitution of (2) into (1) yields

$$\hat{\mathbf{n}}_m\times\sum_{i\in L}[j\omega\mathbf{A}^{mi}(\mathbf{r}) + \nabla\phi^{mi}(\mathbf{r})] = \hat{\mathbf{n}}_m\times\mathbf{E}_m^{\mathrm{inc}}(\mathbf{r}), \qquad \mathbf{r}\text{ on }S_m,\quad m\in L \quad (9)$$

which, by invoking (4) and (5), can be further transformed to

$$\frac{j\omega}{k_m^2}\hat{\mathbf{n}}_m\times(\nabla\nabla\cdot + k_m^2)\sum_{i\in L}\int_{S_i}\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,dS' = \hat{\mathbf{n}}_m\times\mathbf{E}_m^{\mathrm{inc}}(\mathbf{r}), \qquad \mathbf{r}\text{ on }S_m,\quad m\in L. \quad (10)$$

This equation is referred to as vector-potential EFIE [33], since it involves only the magnetic vector potential. Although (10) has often been used in MM analyses of straight wire antennas or planar scatterers, the presence of the mixed tangential derivatives makes it less suitable for objects of arbitrary shape. By introducing the differential operator under the integral sign in (10), we may obtain another form of the EFIE, in which the dyadic kernel is the Green's function for the electric field. However, this EFIE is not attractive because the kernel is highly singular (a distributional interpretation of the kernel is required), which makes the evaluation of the integrals required by the MM procedure difficult when the observation point is within the integration interval [34]. Also, the required differentiation of the Sommerfeld-type integrals adversely affects their convergence. These difficulties can be avoided if only *one* of the operators nabla, the divergence, is introduced inside the integral of (10) and then transferred, by a series of transformations, to act on the current. The result is the mixed-potential EFIE discussed below.

Mixed-Potential EFIE (MPIE)

We note that (9) would be in the desired mixed-potential form if the scalar potential were expressed in terms of the surface charge density $q(\mathbf{r})$. With this goal in mind, we substitute (4) into (5) and introduce the operator nabla under the integral signal (this step can be justified [35], [36]) to obtain

$$\phi^{mi}(\mathbf{r}) = \frac{j\omega}{k_m^2}\int_{S_i}[\nabla\cdot\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}')]\cdot\mathbf{J}(\mathbf{r}')\,dS'. \quad (11)$$

Obviously, our objective would be achieved if we transferred the divergence operator to act on the current, in view of the equation of continuity, $\nabla\cdot\mathbf{J} = -j\omega q$. It is shown below that this can only be accomplished if a scalar function G_ϕ^{mi} can be found, such that

$$\frac{j\omega}{k_m^2}\nabla\cdot\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}') = \frac{1}{j\omega}\nabla' G_\phi^{mi}(\mathbf{r}|\mathbf{r}'). \quad (12)$$

In a homogeneous medium, where G_ϕ^{mi} may be interpreted as the Green's function for the scalar potential, this is a quite trivial task. If the medium is stratified, however, G_ϕ^{mi} satisfying (12) does not in general exist [23], which can be attributed to the fact that the scalar potentials of point charges associated with horizontal and vertical current dipoles in a layered medium are in general different [14]. Hence, in order to achieve our goal, we follow the procedure proposed in [22], [23], and introduce a scalar function K_ϕ^{mi} and a vector function $\mathbf{P}^{mi}$ according to

$$\frac{j\omega}{k_m^2}\nabla\cdot\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}')=\frac{1}{j\omega}\nabla' K_\phi^{mi}(\mathbf{r}|\mathbf{r}')+j\omega\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}'). \quad (13)$$

We note that (13) would have the desired form (12) if it were not for the "correction term" comprising $\mathbf{P}^{mi}$. However, it is shown below that this term may be incorporated into the vector potential kernel. We also note that the choice of K_ϕ^{mi} and $\mathbf{P}^{mi}$ in (13) is not unique, giving rise to many possible formulations. Three particularly useful choices are discussed in detail in Section III.

Upon substituting (13) into (11) and using a vector identity [37, p. 487] and the Gauss theorem [37, p. 503], we can express the scalar potential as

$$\begin{aligned}\phi^{mi}(\mathbf{r})=&\int_{S_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')q(\mathbf{r}')\,dS'\\&+j\omega\int_{S_i}\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,dS'\\&+\frac{1}{j\omega}\left[\oint_{C_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{\mathbf{u}}_i\,dC'\right.\\&\left.-\oint_{C_{i-1}}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{\mathbf{u}}_{i-1}\,dC'\right]\end{aligned} \quad (14)$$

where C_i and C_{i-1} are the contours formed by the intersection of the surface S_i with the interfaces at $z=z_i$ and $z=z_{i-1}$, respectively, and $\hat{\mathbf{u}}_i$ and $\hat{\mathbf{u}}_{i-1}$ are the unit vectors perpendicular at $\mathbf{r}'$ to C_i and C_{i-1}, respectively, in the planes tangent to S_i Fig. 2. Substituting (14) into (9), we finally obtain

$$\begin{aligned}\hat{\mathbf{n}}_m\times\sum_{i\in L}\Bigg\{&j\omega\int_{S_i}\bar{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,dS'\\&+\nabla\int_{S_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')q(\mathbf{r}')\,dS'\\&+\frac{\nabla}{j\omega}\left[\oint_{C_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{\mathbf{u}}_i\,dC'\right.\\&\left.-\oint_{C_{i-1}}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{\mathbf{u}}_{i-1}\,dC'\right]\Bigg\}\\=\;&\hat{\mathbf{n}}_m\times\mathbf{E}_m^{\mathrm{inc}}(\mathbf{r}),\qquad \mathbf{r}\text{ on }S_m,\quad m\in L\end{aligned} \quad (15)$$

where we have introduced the dyadic kernel

$$\bar{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}')=\bar{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}')+\nabla\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}'). \quad (16)$$

We note that (15) would be in the desired mixed-potential form [4], [5] if it were not for the presence of the term contributed by the contour integrals, which occur when the object penetrates one or more of the interfaces. In Section III we show that with a proper choice of $\bar{\mathbf{G}}_A^{mi}$ and K_ϕ^{mi} in (13) the contour integrals cancel out. We note, however, that even if a formulation is chosen in which the contour integrals persist, the MM procedures developed in [4], [5] can be extended to accommodate these terms. As a matter of fact, the "correction term" $\nabla\mathbf{P}^{mi}$ could also be handled in this manner, instead of being incorporated into the vector potential kernel via (16).

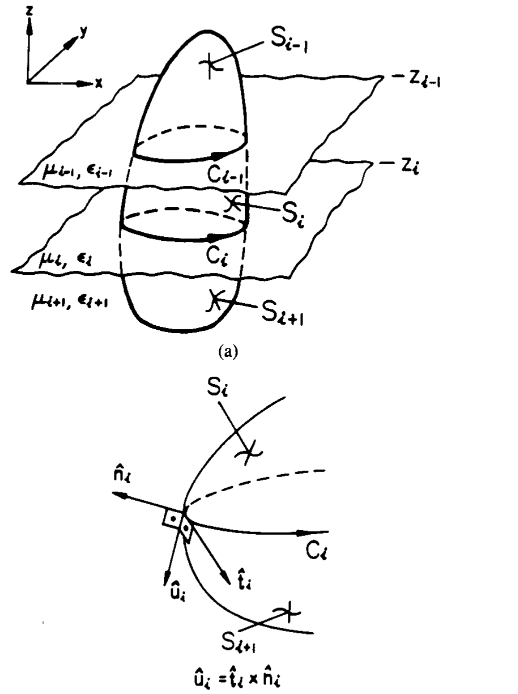

Fig. 2. (a) Arbitrarily shaped surface penetrating two adjacent interfaces. (b) Detail of contour C_i with unit vectors $\hat{\mathbf{u}}_i$, $\hat{\mathbf{t}}_i$, and $\hat{\mathbf{n}}_i$.

III. Integral Equation Formulation

In this section we present three different mixed-potential EFIEs for the problem of Fig. 1. Most of the derivations are done in the Fourier transform domain, which greatly simplifies the algebra. The Fourier transform pair employed is defined in Appendix I. Some useful related formulas are also given there. Our development necessarily begins with the vector potential Green's functions (7) or (8). The elements of these dyadics are listed in Appendix II, where only the outline of the derivation is given.

As mentioned in Section II, the decomposition embodied in (13) is nonunique, which means that an infinite number of different MPIEs is possible. We observe, however, by referring to (16), that the presence of the "correction" function $\mathbf{P}^{mi}$ has the undesirable effect of introducing new elements in the dyadic kernel $\bar{\mathbf{K}}_A^{mi}$, in addition to those already present in $\bar{\mathbf{G}}_A^{mi}$. Ideally, of course, $\mathbf{P}^{mi}$ should be zero, which, unfortunately, is only possible in a few special cases [14]. Hence, the best we can do is to develop MPIEs for which one or two of the components of $\mathbf{P}^{mi}$ are zero. It is shown below that for the vector Green's functions (7) and (8) the x and y components of $\mathbf{P}^{mi}$ are not independent, thus leaving us, for each $\bar{\mathbf{G}}_A^{mi}$, with only two degrees of freedom: either $P_x^{mi}=P_y^{mi}=0$ and $P_z^{mi}\neq 0$, or $P_x^{mi}\neq 0$, $P_y^{mi}\neq 0$, and $P_z^{mi}=0$. Of the four possible "minimal" formulations, only three lead to distinct

MPIEs. These three are discussed in detail below, where they are referred to as Formulations A, B, and C.

Formulation A

In this formulation, we employ the alternative form of $\bar{\mathbf{G}}_A^{mi}$ given in (8). In the Fourier transform domain (see Appendix I), the x, y, and z components of (13) become

$$\frac{j\omega}{k_m^2}(-jk_x\tilde{G}_{xx}^{mi} - jk_y\tilde{G}_{xy}^{mi}) = \frac{1}{j\omega}jk_x\tilde{K}_\phi^{mi} + j\omega\tilde{P}_x^{mi} \quad (17)$$

$$\frac{j\omega}{k_m^2}(-jk_x\tilde{G}_{xy}^{mi} - jk_y\tilde{G}_{yy}^{mi}) = \frac{1}{j\omega}jk_y\tilde{K}_\phi^{mi} + j\omega\tilde{P}_y^{mi} \quad (18)$$

$$\frac{j\omega}{k_m^2}\frac{\partial}{\partial z}\tilde{G}_{zz}^{mi} = \frac{1}{j\omega}\frac{\partial}{\partial z'}\tilde{K}_\phi^{mi} + j\omega\tilde{P}_z^{mi}. \quad (19)$$

Using (85)–(87) of Appendix II in (17) and (18), one finds that $\tilde{P}_x^{mi}$ and $\tilde{P}_y^{mi}$ are related by

$$\tilde{P}_y^{mi} = \frac{k_y}{k_x}\tilde{P}_x^{mi}. \quad (20)$$

In Formulation A, we choose $\tilde{P}_x^{mi} = \tilde{P}_y^{mi} = 0$, in which case K_ϕ^{mi} can be interpreted as the scalar potential of a point charge associated with a horizontal dipole [14]. Solving (17) or (18) for $\tilde{K}_\phi^{mi}$, one obtains

$$\tilde{K}_\phi^{mi} = -j\omega\frac{\tilde{G}_{mi}^{V_e}}{k_{zm}^2}, \quad (21)$$

which can be substituted into (19) to yield

$$\tilde{P}_z^{mi} = \frac{\mu_i\epsilon_i - \mu_m\epsilon_m}{\epsilon_i k_{zm}^2}\tilde{I}_{mi}^{V_e} \quad (22)$$

where we have introduced for later convenience

$$\tilde{I}_{mi}^{V_q} = \begin{cases} \overleftarrow{Z}_{i-1}^q\tilde{G}_{ii}^{I_q}(z_{i-1}, z')\overleftarrow{T}_{mi}^{V_q}(z), & i-1 \geq m \geq 1 \\ -\overrightarrow{Z}_i^q\tilde{G}_{ii}^{I_q}(z_i, z')\overrightarrow{T}_{mi}^{V_q}(z), & n+1 \geq m > i+1 \end{cases} \quad (23)$$

in which the subscript q stands for e or h. In the above, the notation of Appendix II is employed. Observe that $\tilde{P}_z^{mi} \equiv 0$ when $m = i$.

Substituting (84)–(87) of Appendix II and (22) into the Fourier domain counterpart of (16), and using the relations given in Appendix I, one obtains the dyadic kernel

$$\bar{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}') = \hat{\mathbf{x}}\hat{\mathbf{x}}K_{xx}^{mi} + \hat{\mathbf{y}}\hat{\mathbf{y}}K_{yy}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{z}}K_{zz}^{mi} + (\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}})K_{xy}^{mi} + \hat{\mathbf{x}}\hat{\mathbf{z}}K_{xz}^{mi} + \hat{\mathbf{y}}\hat{\mathbf{z}}K_{yz}^{mi} \quad (24)$$

with the elements given by

$$K_{xx}^{mi} = G_{xx}^{mi} = \frac{1}{2j\omega}\left\{k_m^2 S_0\left(\frac{1}{k_{zm}^2}\tilde{G}_{mi}^{V_e}\right) + S_0\left(\tilde{G}_{mi}^{V_h}\right) + \cos 2\zeta S_2\left[\frac{1}{k_\rho^2}\left(\tilde{G}_{mi}^{V_h} - \frac{k_m^2}{k_{zm}^2}\tilde{G}_{mi}^{V_e}\right)\right]\right\} \quad (25)$$

$$K_{yy}^{mi} = G_{yy}^{mi} = \frac{1}{2j\omega}\left\{k_m^2 S_0\left(\frac{1}{k_{zm}^2}\tilde{G}_{mi}^{V_e}\right) + S_0\left(\tilde{G}_{mi}^{V_h}\right) - \cos 2\zeta S_2\left[\frac{1}{k_\rho^2}\left(\tilde{G}_{mi}^{V_h} - \frac{k_m^2}{k_{zm}^2}\tilde{G}_{mi}^{V_e}\right)\right]\right\} \quad (26)$$

$$K_{xy}^{mi} = G_{xy}^{mi} = \frac{1}{2j\omega}\sin 2\zeta S_2\left[\frac{1}{k_\rho^2}\left(\tilde{G}_{mi}^{V_h} - \frac{k_m^2}{k_{zm}^2}\tilde{G}_{mi}^{V_e}\right)\right] \quad (27)$$

$$K_{xz}^{mi} = \frac{\partial}{\partial x}P_z^{mi} = -\cos\zeta S_1(\tilde{P}_z^{mi}) \quad (28)$$

$$K_{yz}^{mi} = \frac{\partial}{\partial y}P_z^{mi} = -\sin\zeta S_1(\tilde{P}_z^{mi}) \quad (29)$$

$$K_{zz}^{mi} = G_{zz}^{mi} + \frac{\partial}{\partial z}P_z^{mi} = \frac{\mu_i}{j\omega\epsilon_m}S_0(\tilde{G}_{mi}^{I_e}). \quad (30)$$

Finally, the Fourier inversion of $\tilde{K}_\phi^{mi}$ in (21) yields the scalar potential kernel

$$K_\phi^{mi} = -j\omega S_0\left(\frac{\tilde{G}_{mi}^{V_e}}{k_{zm}^2}\right). \quad (31)$$

We observe from (25) through (31) that in this formulation, when $m \neq i$, the effect of $\nabla\mathbf{P}^{mi}$ in (16) is to introduce two new entries, K_{xz}^{mi} and K_{yz}^{mi}, and to modify G_{zz}^{mi}. However, when the object is confined to a single layer ($m = i$), we simply have $\bar{\mathbf{K}}_A^{mi} = \bar{\mathbf{G}}_A^{mi}$, so no modification of the Green's function is required.

A useful property of Formulation A is the cancellation of the contour integrals in (15), which is the result of the continuity with respect to the z' coordinate of the scalar potential kernel at the ith interface: $K_\phi^{mi}(z' = z_i + 0) = K_\phi^{m,i+1}(z' = z_i - 0)$. We note, however, that a similar continuity with respect to the z coordinate does not hold, i.e., $K_\phi^{mi}(z = z_m + 0) \neq K_\phi^{m+1,i}(z = z_m - 0)$.

Formulation B

In this formulation, as in Formulation A, we employ the alternative form (8) of $\bar{\mathbf{G}}_A^{mi}$. However, rather than choosing $\tilde{P}_x^{mi} = \tilde{P}_y^{mi} = 0$, we select $\tilde{P}_z^{mi} = 0$ in (17)–(19). In this case K_ϕ^{mi} can be interpreted as the scalar potential of a point charge associated with a vertical dipole [14]. From (19) and (84) of Appendix II, we find

$$\tilde{K}_\phi^{mi} = -j\omega\frac{\tilde{G}_{mi}^{V_e}}{k_{zi}^2} \quad (32)$$

which can be substituted into (17) to yield

$$\tilde{P}_x^{mi} = \omega k_x\frac{\mu_m\epsilon_m - \mu_i\epsilon_i}{k_{zi}^2 k_{zm}^2}\tilde{G}_{mi}^{V_e} \quad (33)$$

which, in view of (20), also specifies $\tilde{P}_y^{mi}$.

Referring to (16) and proceeding as in Formulation A, we obtain the dyadic kernel

$$\bar{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}') = \hat{\mathbf{x}}\hat{\mathbf{x}}K_{xx}^{mi} + \hat{\mathbf{y}}\hat{\mathbf{y}}K_{yy}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{z}}K_{zz}^{mi} + (\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}})K_{xy}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{x}}K_{zx}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{y}}K_{zy}^{mi} \quad (34)$$

with the elements given by

$$K_{xx}^{mi} = G_{xx}^{mi} + \frac{\partial}{\partial x} P_x^{mi} = \frac{1}{2j\omega}\left\{k_i^2 S_0\left(\frac{1}{k_{zi}^2}\tilde{G}_{mi}^{V_e}\right) + S_0\left(\tilde{G}_{mi}^{V_h}\right) + \cos 2\zeta S_2\left[\frac{1}{k_\rho^2}\left(\tilde{G}_{mi}^{V_h} - \frac{k_i^2}{k_{zi}^2}\tilde{G}_{mi}^{V_e}\right)\right]\right\} \tag{35}$$

$$K_{yy}^{mi} = G_{yy}^{mi} + \frac{\partial}{\partial y} P_y^{mi} = \frac{1}{2j\omega}\left\{k_i^2 S_0\left(\frac{1}{k_{zi}^2}\tilde{G}_{mi}^{V_e}\right) + S_0\left(\tilde{G}_{mi}^{V_h}\right) - \cos 2\zeta S_2\left[\frac{1}{k_\rho^2}\left(\tilde{G}_{mi}^{V_h} - \frac{k_i^2}{k_{zi}^2}\tilde{G}_{mi}^{V_e}\right)\right]\right\} \tag{36}$$

$$K_{xy}^{mi} = G_{xy}^{mi} + \frac{\partial}{\partial x} P_y^{mi} = \frac{1}{2j\omega}\sin 2\zeta S_2\left[\frac{1}{k_\rho^2}\left(\tilde{G}_{mi}^{V_h} - \frac{k_i^2}{k_{zi}^2}\tilde{G}_{mi}^{V_e}\right)\right] \tag{37}$$

$$K_{zx}^{mi} = \frac{\partial}{\partial z} P_x^{mi} = \cos \zeta S_1(\tilde{R}^{mi}) \tag{38}$$

$$K_{zy}^{mi} = \frac{\partial}{\partial z} P_y^{mi} = \sin \zeta S_1(\tilde{R}^{mi}) \tag{39}$$

$$K_{zz}^{mi} = G_{zz}^{mi} = \frac{\mu_m}{j\omega\epsilon_i} S_0(\tilde{G}_{mi}^{I_e}) \tag{40}$$

where $\tilde{R}^{mi}$ is nonzero only for $m \neq i$ and is given by

$$\tilde{R}^{mi} = \frac{\mu_i\epsilon_i - \mu_m\epsilon_m}{\epsilon_m k_{zi}^2}\tilde{I}_{mi}^{I_e} \tag{41}$$

in which $\tilde{I}_{mi}^{I_e}$ can be obtained from $\tilde{I}_{mi}^{V_e}$ given in (23) by replacing in the latter the characteristic impedances Z_i^e by their reciprocals.

Finally, the Fourier inversion of $\tilde{K}_\phi^{mi}$ in (32) gives the scalar potential kernel

$$K_\phi^{mi} = -j\omega S_0\left(\frac{\tilde{G}_{mi}^{V_e}}{k_{zi}^2}\right). \tag{42}$$

We note that in Formulation B two new entries (K_{zx}^{mi} and K_{zy}^{mi}), not present in $\bar{\mathbf{G}}_A^{mi}$, are introduced to the dyadic kernel $\bar{\mathbf{K}}_A^{mi}$. Also, extra terms are added to G_{xx}^{mi}, G_{yy}^{mi}, and G_{xy}^{mi}. In the case where the object is confined to a single layer ($m = i$), $\bar{\mathbf{K}}_A^{mi} = \bar{\mathbf{G}}_A^{mi}$, and Formulation B becomes identical to Formulation A.

The continuity properties of the scalar potential kernel in Formulation B are complementary to those in Formulation A; that is, in the present case $K_\phi^{mi}(z' = z_i + 0) \neq K_\phi^{m,i+1}(z' = z_i - 0)$ and $K_\phi^{mi}(z = z_m + 0) = K_\phi^{m+1,i}(z = z_m - 0)$. As a result, the contour integrals in (15) do exist when the objective penetrates one or more of the interfaces.

Formulation C

The point of departure in this formulation is the traditional form of $\bar{\mathbf{G}}_A^{mi}$ given in (7). Using it in (13), we obtain in the Fourier transform domain

$$\frac{j\omega}{k_m^2}\left(-jk_x\tilde{G}_{xx}^{mi} + \frac{\partial}{\partial z}\tilde{G}_{zx}^{mi}\right) = \frac{1}{j\omega}jk_x\tilde{K}_\phi^{mi} + j\omega\tilde{P}_x^{mi} \tag{43}$$

$$\frac{j\omega}{k_m^2}\left(-jk_y\tilde{G}_{xx}^{mi} + \frac{\partial}{\partial z}\tilde{G}_{zy}^{mi}\right) = \frac{1}{j\omega}jk_y\tilde{K}_\phi^{mi} + j\omega\tilde{P}_y^{mi} \tag{44}$$

$$\frac{j\omega}{k_m^2}\frac{\partial}{\partial z}\tilde{G}_{zz}^{mi} = \frac{1}{j\omega}\frac{\partial}{\partial z'}\tilde{K}_\phi^{mi} + j\omega\tilde{P}_z^{mi}. \tag{45}$$

From (43) and (44), and referring to (81)–(83) of Appendix II, we find that (20) still holds. Since G_{zz}^{mi} in the present case is of the same as that in the "alternative" Green's function (8), specifying $\tilde{P}_z^{mi} = 0$ results in an MPIE identical to that in Formulation B. We therefore set $\tilde{P}_x^{mi} = \tilde{P}_y^{mi} = 0$ in (43) and (44), which yields the scalar potential kernel

$$\tilde{K}_\phi^{mi} = \frac{j\omega}{k_\rho^2}(\tilde{G}_{mi}^{V_e} - \tilde{G}_{mi}^{Vh}). \tag{46}$$

From the above and (45), there results

$$\tilde{P}_z^{mi} = \frac{1}{j\omega}\left[\frac{\mu_m}{k_m^2\epsilon_i}\frac{\partial}{\partial z}\tilde{G}_{mi}^{I_e} + \frac{1}{k_\rho^2}\frac{\partial}{\partial z'}(\tilde{G}_{mi}^{V_h} - \tilde{G}_{mi}^{V_e})\right]. \tag{47}$$

Substituting (81)–(84) of Appendix II and (47) into the Fourier domain counterpart of (16), and using the relations given in Appendix I, we obtain the dyadic kernel

$$\bar{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}') = (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})K_{xx}^{mi} + \hat{\mathbf{x}}\hat{\mathbf{z}}K_{xz}^{mi} + \hat{\mathbf{y}}\hat{\mathbf{z}}K_{yz}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{x}}K_{zx}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{y}}K_{zy}^{mi} + \hat{\mathbf{z}}\hat{\mathbf{z}}K_{zz}^{mi} \tag{48}$$

with the elements given by

$$K_{xx}^{mi} = G_{xx}^{mi} = \frac{1}{j\omega}S_0(\tilde{G}_{mi}^{V_h}) \tag{49}$$

$$K_{xz}^{mi} = \frac{\partial}{\partial x}P_z^{mi} = -\frac{\mu_i}{j\omega\epsilon_m}\cdot\cos\zeta S_1\left[\frac{1}{k_\rho^2}\left(\tilde{W}_{mi}^{V_e} - \frac{k_m^2}{k_{zm}^2}\tilde{W}_{mi}^{V_h}\right)\right] \tag{50}$$

$$K_{yz}^{mi} = \frac{\partial}{\partial y}P_z^{mi} = -\frac{\mu_i}{j\omega\epsilon_m}\cdot\sin\zeta S_1\left[\frac{1}{k_\rho^2}\left(\tilde{W}_{mi}^{V_e} - \frac{k_m^2}{k_{zm}^2}\tilde{W}_{mi}^{V_h}\right)\right] \tag{51}$$

$$K_{zx}^{mi} = G_{zx}^{mi} = -\frac{1}{j\omega}\cos\zeta S_1\left[\frac{1}{k_\rho^2}\left(\frac{k_m^2}{k_{zm}^2}\tilde{W}_{mi}^{I_e} - \tilde{W}_{mi}^{I_h}\right)\right] \tag{52}$$

$$K_{zy}^{mi} = G_{zy}^{mi} = -\frac{1}{j\omega}\sin\zeta S_1\left[\frac{1}{k_\rho^2}\left(\frac{k_m^2}{k_{zm}^2}\tilde{W}_{mi}^{I_e} - \tilde{W}_{mi}^{I_h}\right)\right] \tag{53}$$

TABLE I
SUMMARY OF THE PROPERTIES OF THE THREE MIXED-POTENTIAL FORMULATIONS

Form of $\bar{\mathbf{G}}_A^{mi}$	Choice of K_ϕ^{mi}	Form of $\bar{\mathbf{K}}_A^{mi}$	Properties of $\bar{\mathbf{K}}_A^{mi}$ for $m=i$	Continuity of K_ϕ^{mi} at the interfaces	Contour integrals	Formulation and authors
Alternative: $\begin{bmatrix} G_{xx}^{mi} & G_{xy}^{mi} & 0 \\ G_{xy}^{mi} & G_{yy}^{mi} & 0 \\ 0 & 0 & G_{zz}^{mi} \end{bmatrix}$	K_ϕ^{mi} associated with horizontal dipole: $P_x^{mi}=P_y^{mi}=0$ $P_z^{mi}\neq 0$	$\begin{bmatrix} K_{xx}^{mi} & K_{xy}^{mi} & K_{xz}^{mi} \\ K_{xy}^{mi} & K_{yy}^{mi} & K_{yz}^{mi} \\ 0 & 0 & K_{zz}^{mi} \end{bmatrix}$	$\bar{\mathbf{K}}_A^{mi}=\bar{\mathbf{G}}_A^{mi}$	$K_\phi^{mi}(z'=z_i^+)=K_\phi^{m,i+1}(z'=z_i^-)$ $K_\phi^{mi}(z=z_m^+)\neq K_\phi^{m+1,i}(z=z_m^-)$	No	A: Johnson [18] (vertical tube); Michalski and Smith [21] (horizontal wires)
	K_ϕ^{mi} associated with vertical dipole: $P_x^{mi}\neq 0$, $P_y^{mi}\neq 0$ $P_z^{mi}=0$	$\begin{bmatrix} K_{xx}^{mi} & K_{xy}^{mi} & 0 \\ K_{xy}^{mi} & K_{yy}^{mi} & 0 \\ K_{zx}^{mi} & K_{zy}^{mi} & K_{zz}^{mi} \end{bmatrix}$	$\bar{\mathbf{K}}_A^{mi}=\bar{\mathbf{G}}_A^{mi}$ $K_{zx}^{mi}=K_{zy}^{mi}=0$	$K_\phi^{mi}(z'=z_i^+)\neq K_\phi^{m,i+1}(z'=z_i^-)$ $K_\phi^{mi}(z=z_m^+)=K_\phi^{m+1,i}(z=z_m^-)$	Yes	B: Michalski and Zheng [38] (thin wire)
Traditional: $\begin{bmatrix} G_{xx}^{mi} & 0 & 0 \\ 0 & G_{xx}^{mi} & 0 \\ G_{zx}^{mi} & G_{zy}^{mi} & G_{zz}^{mi} \end{bmatrix}$	K_ϕ^{mi} associated with horizontal dipole: $P_x^{mi}=P_y^{mi}=0$ $P_z^{mi}\neq 0$	$\begin{bmatrix} K_{xx}^{mi} & 0 & K_{xz}^{mi} \\ 0 & K_{xx}^{mi} & K_{yz}^{mi} \\ K_{zx}^{mi} & K_{zy}^{mi} & K_{zz}^{mi} \end{bmatrix}$		$K_\phi^{mi}(z'=z_i^+)=K_\phi^{m,i+1}(z'=z_i^-)$ $K_\phi^{mi}(z=z_m^+)=K_\phi^{m+1,i}(z=z_m^-)$	No	C: Mosig and Gardiol [16] (planar patch); Michalski [22, 23] (single layer); Michalski *et al.* [20] (horizontal wires); Wilton and Singh [19] (periodic planar slots)

$$K_{zz}^{mi} = G_{zz}^{mi} + \frac{\partial}{\partial z} P_z^{mi} = \frac{\mu_m}{j\omega\epsilon_i} \cdot S_0 \left[\tilde{G}_{mi}^{I_e} - \frac{k_i^2}{k_\rho^2} \left(\frac{k_{zm}^2}{k_m^2} \tilde{G}_{mi}^{I_e} - \tilde{G}_{mi}^{I_h} \right) \right] \tag{54}$$

where we have introduced

$$\tilde{W}_{ii}^{V} = \frac{jk_{zi}}{2Z_iD_i} \{ \vec{\Gamma}_i e^{-jk_{zi}[(z+z')-2z_i]} - \overleftarrow{\Gamma}_{i-1} e^{-jk_{zi}[2z_{i-1}-(z+z')]} + 2j\vec{\Gamma}_i \overleftarrow{\Gamma}_{i-1} e^{-j2\psi_i} \sin[k_{zi}(z-z')] \}, \qquad m = i \tag{55}$$

$$\tilde{W}_{mi}^{V} = -\frac{jk_{zm}}{Z_m} \tilde{I}_{mi}^{V}, \qquad m \neq i \tag{56}$$

with $\tilde{I}_{mi}^{V}$ given in (23). For notational simplicity, we have suppressed in (55) and (56) the superscripts e or h. We note that one can obtain $\tilde{W}_{mi}^{I}$ from $\tilde{W}_{mi}^{V}$ by replacing in the latter the characteristic impedances Z_i by their reciprocals.

Finally, the Fourier inversion of (46) gives

$$K_\phi^{mi} = j\omega S_0 \left[\frac{1}{k_\rho^2} (\tilde{G}_{mi}^{V_e} - \tilde{G}_{mi}^{V_h}) \right]. \tag{57}$$

As in Formulation A, this K_ϕ^{mi} may be interpreted as the scalar potential of a point charge associated with a horizontal dipole [14]. However, these two potentials are not identical, since each corresponds to a different form of the vector potential Green's function.

We observe from (49)–(54) that Formulation C introduces two new entries (K_{xz}^{mi} and K_{yz}^{mi}), not present in $\bar{\mathbf{G}}_A^{mi}$, to the dyadic kernel. Also, extra terms are added to G_{zz}^{mi}. In contrast to Formulations A and B, these modifications occur even if the object is confined to a single layer. As in Formulation A, the scalar potential kernel in the present case has the continuity property that $K_\phi^{mi}(z' = z_i + 0) = K_\phi^{m,i+1}(z' = z_i - 0)$, which causes the contour integrals in (15) to cancel. Formulation C also shares with Formulation B the useful property that $K_\phi^{mi}(z = z_m + 0) = K_\phi^{m+1,i}(z = z_m - 0)$.

IV. Discussion

The properties of three MPIEs are summarized for easy reference in Table I. Each of the three formulations, called A, B or C, can be derived from either the alternative or the traditional form of the dyadic Green's function for the vector potential. These two dyadics are shown in matrix form in the first column of Table I. We note that Formulation A is derived from the alternative forms of $\bar{\mathbf{G}}_A^{mi}$, Formulation C from the traditional form, and Formulation B from either of the two (this is due to the fact that both forms of $\bar{\mathbf{G}}_A^{mi}$ share the same G_{zz}^{mi}). The distinguishing feature of each of the three formulations is the choice of the scalar potential kernel K_ϕ^{mi}, which also specifies the "correction" vector $\mathbf{P}^{mi}$ according to (13). Although, as the second column of Table I indicates, the scalar potential kernel in Formulations A and C are both associated with a horizontal dipole, they are different, because they correspond to different vector potential Green's functions. In Formulation B, K_ϕ^{mi} is that associated with a vertical dipole. Actually, by properly choosing K_ϕ^{mi}, one can also derive Formulation A from the traditional form of $\bar{\mathbf{G}}_A^{mi}$, and Formulation C from the alternative form.

The forms of the dyadic vector potential kernel are shown for each of the three formulations in column three. Comparing this column with column one, we observe that in all three formulations two new entries, in addition to those already present in $\bar{\mathbf{G}}_A^{mi}$, appear in $\bar{\mathbf{K}}_A^{mi}$, which is of course undesirable. We note, however (column four), that in Formulations A and B the number of entries in $\bar{\mathbf{K}}_A^{mi}$ is not increased over that in $\bar{\mathbf{G}}_A^{mi}$, when the object is confined to a single layer ($m = i$). We should point out that the correspondence between the number of entries in $\bar{\mathbf{K}}_A^{mi}$ and the number of distinct Sommerfeld-type integrals that need be evaluated is not one-to-one. In fact, we can show by referring to Section III that in the general case only four distinct Sommerfeld integrals are required in all three formulations. When $m = i$, the number of distinct integrals in Formulations A and B reduces to three.

As mentioned at the end of Section II, one may leave the $\mathbf{P}^{mi}$ term as a part of the scalar potential (cf. (15)), thus avoiding the modification of the vector potential kernel as in (16). However, since this would constitute a departure from the standard form of the MPIE [4], [5], we prefer to proceed according to (16). Although in our formulations some of the terms introduced to $\bar{\mathbf{K}}_A^{mi}$ by $\nabla \mathbf{P}^{mi}$ in (16) become singular when $\mathbf{r}$ and $\mathbf{r}'$ coincide on an interface, they are no more singular than the terms already present in $\bar{\mathbf{G}}_A^{mi}$, and can be handled in a similar way as the latter.

The continuity properties of the scalar potential kernels across the interfaces are summarized in column five of Table I. In formulations A and C, K_ϕ^{mi} is continuous with respect to z', which results in the cancellation of the undesirable contour integrals in (15) (cf. column six). In Formulations B and C, K_ϕ^{mi} is continuous with respect to z, which results in considerable simplifications in the numerical procedure when the object penetrates one or more interfaces. This last point can be fully appreciated only after we have discussed the details of the MM procedure in Part II [30].

We conclude from the above summary that when the object is confined to a single layer, Formulations A and B become identical and are preferable to Formulation C, because they have fewer terms in $\bar{\mathbf{K}}_A^{mi}$. In the general case, Formulation C enjoys a clear advantage over Formulations A and B, because it does not have contour integrals and because its scalar potential kernel is continuous at the interfaces with respect to z, which results in the simplifications mentioned above. If we had to choose between Formulations A and B, we would prefer the latter, because the advantages of having the scalar potential kernel continuous with respect to z more than compensate for the complications caused by the presence of the contour integrals. This point is further elaborated upon in Part II [30].

The previous works related to the mixed-potential EFIE and reviewed in Section I can be classified as shown in the last column of Table I.

V. Conclusion

In this paper, three particularly useful mixed-potential integral equations for PEC scatterers or antennas of arbitrary shape embedded in a plane-stratified dielectric medium are derived and their properties discussed. One of the three MPIEs (Formulation C) is shown to be simpler than the others in the case where the object penetrates one or more of the interfaces, and appears to be especially well suited for the application of the numerical solution techniques developed in [4], [5]. The computer implementation of the MPIE for the case of a two-layer medium is discussed in Part II of this paper [30].

Appendix I

Fourier Transform Relations

Since the layered medium is invariant along the x and y coordinates and all quantities depend on $x - x'$ and $y - y'$, for example, $G_\phi^{mi}(\mathbf{r}|\mathbf{r}') = G_\phi^{mi}(x - x', y - y', z, z')$, it is convenient to introduce the "shifted" Fourier transform pair

$$\mathfrak{F}\{f(x-x', y-y')\} = \tilde{f}(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-x', y-y') \cdot e^{j[k_x(x-x')+k_y(y-y')]}\, dx\, dy \quad (58)$$

$$\mathfrak{F}^{-1}\{\tilde{f}(k_x, k_y)\} = f(x-x', y-y') = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}(k_x, k_y)\, e^{-j[k_x(x-x')+k_y(y-y')]}\, dk_x\, dk_y. \quad (59)$$

By changing the polar coordinates in both the transform and space domains according to

$$x - x' = \xi \cos \zeta, \quad y - y' = \xi \sin \zeta \quad (60)$$

$$k_x = k_\rho \cos \alpha, \quad k_y = k_\rho \sin \alpha \quad (61)$$

where

$$\xi = \sqrt{(x-x')^2 + (y-y')^2}, \quad \zeta = \arctan\left(\frac{y-y'}{x-x'}\right) \quad (62)$$

$$k_\rho = \sqrt{k_x^2 + k_y^2}, \quad \alpha = \arctan\left(\frac{k_y}{k_x}\right) \quad (63)$$

we can conveniently express various inverse Fourier integrals that arise in terms of the Sommerfeld-type [15] integrals of the form

$$S_n[\tilde{g}(k_\rho)] = \frac{1}{2\pi}\int_0^\infty \tilde{g}(k_\rho) J_n(k_\rho \xi) k_\rho^{n+1}\, dk_\rho \quad (64)$$

where J_n is the Bessel function of order n, if we recognize that

$$\mathfrak{F}^{-1}\{\tilde{g}(k_\rho)\} = S_0[\tilde{g}(k_\rho)] \quad (65)$$

$$\mathfrak{F}^{-1}\{jk_x \tilde{g}(k_\rho)\} = \cos \zeta S_1[\tilde{g}(k_\rho)] \quad (66)$$

$$\mathfrak{F}^{-1}\{jk_y \tilde{g}(k_\rho)\} = \sin \zeta S_1[\tilde{g}(k_\rho)] \quad (67)$$

$$\mathfrak{F}^{-1}\{k_x^2 \tilde{g}(k_\rho)\} = -\frac{1}{2}\{\cos 2\zeta S_2[\tilde{g}(k_\rho)] - S_0[k_\rho^2 \tilde{g}(k_\rho)]\} \quad (68)$$

$$\mathfrak{F}^{-1}\{k_y^2 \tilde{g}(k_\rho)\} = \frac{1}{2}\{\cos 2\zeta S_2[\tilde{g}(k_\rho)] + S_0[k_\rho^2 \tilde{g}(k_\rho)]\} \quad (69)$$

$$\mathfrak{F}^{-1}\{k_x k_y \tilde{g}(k_\rho)\} = -\frac{1}{2}\sin 2\zeta S_2[\tilde{g}(k_\rho)]. \quad (70)$$

Appendix II

Vector Potential Green's Functions in a Layered Medium

The Green's functions for the layered medium of Fig. 1 are most easily determined in the Fourier transform domain, where the original problem is in effect reduced to that of solving an equivalent transmission-line network along the z coordinate [39], [40]. In this equivalent model, a uniform transmission line section corresponds to each layer of the medium. It can be shown that in general two dual transmission-line networks must be considered, one for the TE waves, and one for the TM waves. In what follows, quantities pertaining to the TE and TM transmission lines will be distinguished by superscripts h and e, respectively.

Let $\tilde{G}_{mi}^{V_q}(z, z')$, where q stands for e or h, denote the voltage at a point z in section m due to a unit-strength current source at a point z' in section i of the respective transmission line network. Similarly, let $\tilde{G}_{mi}^{I_q}(z, z')$ be the current at z in the mth transmission line section due to a unit-strength voltage source at z' in the ith section. Then, the standard transmission line analysis shows that

$$\tilde{G}_{mi}^{V}(z, z') = \begin{cases} \tilde{G}_{ii}^{V}(z_{i-1}, z')\overrightarrow{T}_{mi}^{V}(z), & i-1 \geq m \geq 1 \\ \tilde{G}_{ii}^{V}(z_{i}, z')\overleftarrow{T}_{mi}^{V}(z), & n+1 \geq m \geq i+1 \end{cases} \tag{71}$$

where

$$\tilde{G}_{ii}^{V}(z, z') = \frac{Z_i}{2}[e^{-jk_{zi}|z-z'|} + \tilde{Q}_i^V(z, z')] \tag{72}$$

with

$$\begin{aligned} \tilde{Q}_i^V(z, z') = \frac{1}{D_i}\{&\overleftarrow{\Gamma}_i e^{-jk_{zi}[(z+z')-2z_i]} \\ &+ \overrightarrow{\Gamma}_{i-1} e^{-jk_{zi}[2z_{i-1}-(z+z')]} \\ &+ 2\overleftarrow{\Gamma}_i\overrightarrow{\Gamma}_{i-1} e^{-j2\psi_i} \cos[k_{zi}(z-z')]\} \end{aligned} \tag{73}$$

$$D_i = 1 - \overleftarrow{\Gamma}_i\overrightarrow{\Gamma}_{i-1} e^{-j2\psi_i}, \tag{74}$$

in which $\psi_i = k_{zi}d_i$, $d_i = z_{i-1} - z_i$. In (72), Z_i denotes the characteristic impedance of the ith transmission line section and is given as

$$Z_i^e = \frac{k_{zi}}{\omega\epsilon_i} \text{ or as } Z_i^h = \frac{\omega\mu_i}{k_{zi}} \tag{75}$$

for the TM- and TE-type transmission lines, respectively, where $k_{zi}^2 = k_i^2 - k_\rho^2$, with $\mathrm{Im}(k_{zi}) \leq 0$ and $\mathrm{Re}(k_{zi}) \geq 0$. For notational simplicity, in (71)–(74) and henceforth in this Appendix we omit, where there is no danger of confusion, the superscripts e and h. The other quantities in (71)–(74) are defined as follows. $\overrightarrow{\Gamma}_k$ and $\overleftarrow{\Gamma}_k$ are the reflection coefficients looking to the right (i.e., in the $+z$ direction) and to the left at the kth interface, and are given by

$$\overrightarrow{\Gamma}_k = \frac{\overrightarrow{Z}_k - Z_{k+1}}{\overrightarrow{Z}_k + Z_{k+1}}, \quad \overleftarrow{\Gamma}_k = \frac{\overleftarrow{Z}_k - Z_k}{\overleftarrow{Z}_k + Z_k}, \qquad k = 1, 2, \cdots, n, \tag{76}$$

with $\overrightarrow{\Gamma}_0 = \overleftarrow{\Gamma}_{n+1} = 0$. $\overrightarrow{Z}_k$ and $\overleftarrow{Z}_k$ are the total impedances looking to the right and left, respectively, at the kth interface, and are given by the recursion formulas

$$\overrightarrow{Z}_k = Z_k \frac{\overrightarrow{Z}_{k-1} + jZ_k t_k}{Z_k + j\overrightarrow{Z}_{k-1} t_k}, \qquad k = 2, 3, \cdots, n \tag{77}$$

$$\overleftarrow{Z}_k = Z_{k+1} \frac{\overleftarrow{Z}_{k+1} + jZ_{k+1} t_{k+1}}{Z_{k+1} + j\overleftarrow{Z}_{k+1} t_{k+1}}, \qquad k = n-1, n-2, \cdots, 1 \tag{78}$$

where $t_k = \tan\psi_k$, $\overrightarrow{Z}_1 = Z_1$ and $\overleftarrow{Z}_n = Z_{n+1}$. Finally, $\overrightarrow{T}_{mi}^V$ and $\overleftarrow{T}_{mi}^V$ in (71) are given by

$$\overrightarrow{T}_{mi}^V(z) = \frac{e^{-jk_{zm}(z-z_m)}}{1 + \overrightarrow{\Gamma}_{m-1} e^{-j2\psi_m}}[1 + \overrightarrow{\Gamma}_{m-1} e^{-j2k_{zm}(z_{m-1}-z)}] \cdot \prod_{k=m}^{i-2} \frac{(1 + \overrightarrow{\Gamma}_k) e^{-j\psi_{k+1}}}{1 + \overrightarrow{\Gamma}_k e^{-j2\psi_{k+1}}} \tag{79}$$

and

$$\overleftarrow{T}_{mi}^V(z) = \frac{e^{-jk_{zm}(z_{m-1}-z)}}{1 + \overleftarrow{\Gamma}_m e^{-j2\psi_m}}[1 + \overleftarrow{\Gamma}_m e^{-j2k_{zm}(z-z_m)}] \cdot \prod_{k=i+1}^{m-1} \frac{(1 + \overleftarrow{\Gamma}_k) e^{-j\psi_k}}{1 + \overleftarrow{\Gamma}_k e^{-j2\psi_k}}. \tag{80}$$

Here, $\overrightarrow{T}_{mi}^V(z)$ denotes the voltage transfer function between a point z in the mth section and the right terminal of the source section i, where $i > m$. Similarly, $\overleftarrow{T}_{mi}^V(z)$ is the voltage transfer function between a point z in the mth section and the left terminal of the source section i, where $i < m$.

The function $\tilde{G}_{mi}^I$ are duals of the functions $\tilde{G}_{mi}^V$ and are obtained by replacing in the latter all characteristic impedances Z_i by their reciprocals, Y_i. (As a result of this operation, all reflection coefficients change signs.) It turns out that $\tilde{G}_{mi}^I$ and $\tilde{G}_{mi}^V$ depend on k_x and k_y only through k_ρ (cf. (63) of Appendix I).

The electric and magnetic fields anywhere in the layered medium can be easily found in terms of the transmission-line Green's functions. Hence, expressing $\tilde{\mathbf{E}}^{mi}$ first in terms of G_{mi}^V and $\tilde{G}_{mi}^I$, and then in terms of $\tilde{\mathbf{G}}_A^{mi}$ given by either (7) or (8), we can easily solve for the elements of the vector-potential Green's function in both the traditional and alternative forms. In this manner, the elements of the "traditional" Green's function (7) are found in the Fourier transform domain as [41]

$$\tilde{G}_{xx}^{mi} = \tilde{G}_{yy}^{mi} = \frac{1}{j\omega}\tilde{G}_{mi}^{V_h} \tag{81}$$

$$\tilde{G}_{zx}^{mi} = -\frac{k_x}{\omega k_\rho^2}\left(\frac{k_m^2}{k_{zm}^2}\frac{\partial}{\partial z}\tilde{G}_{mi}^{V_e} - \frac{\partial}{\partial z}\tilde{G}_{mi}^{V_h}\right) \tag{82}$$

$$\tilde{G}_{zy}^{mi} = -\frac{k_y}{\omega k_\rho^2}\left(\frac{k_m^2}{k_{zm}^2}\frac{\partial}{\partial z}\tilde{G}_{mi}^{V_e} - \frac{\partial}{\partial z}\tilde{G}_{mi}^{V_h}\right) \tag{83}$$

$$\tilde{G}_{zz}^{mi} = \frac{1}{j\omega}\frac{\mu_m}{\epsilon_i}\tilde{G}_{mi}^{I_e}. \tag{84}$$

Similarly, the elements of the "alternative" Green's function in (8) have the Fourier domain counterparts

$$\tilde{G}_{xx}^{mi} = \frac{1}{j\omega k_\rho^2}\left(\frac{k_m^2 k_x^2}{k_{zm}^2}\tilde{G}_{mi}^{V_e} + k_y^2\tilde{G}_{mi}^{V_h}\right) \tag{85}$$

$$\tilde{G}_{xy}^{mi} = \tilde{G}_{yx}^{mi} = \frac{k_x k_y}{j\omega k_\rho^2}\left(\frac{k_m^2}{k_{zm}^2}\tilde{G}_{mi}^{V_e} - \tilde{G}_{mi}^{V_h}\right) \tag{86}$$

$$\tilde{G}_{yy}^{mi} = \frac{1}{j\omega k_\rho^2}\left(\frac{k_m^2 k_y^2}{k_{zm}^2}\tilde{G}_{mi}^{V_e} + k_x^2\tilde{G}_{mi}^{V_h}\right). \tag{87}$$

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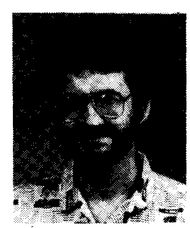

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