APPENDIX

”Intelligent Locomotion Planning with Guaranteed Posture Stability for Lower-Limb Exoskeletons”

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May 2021

1 Inertia matrix and gravity vector values:

The elements of inertia matrix M are

$$\begin{aligned}
M_{11}=&m_1l_{c1}^2+m_2(l_1+l_{c2})^2+m_3(l_1+l_{c3})^2+\\
&m_4(l_1+l_3+l_{c4})^2+I_1+I_2+I_3+I_4\\
M_{12}=&m_2l_{c2}(l_1+l_{c2})+m_3l_{c3}(l_1+l_{c3})+\\
&m_4(l_3+l_{c4})(l_1+l_3+l_{c4})+I_2+I_3+I_4\\
M_{13}=&m_3l_{c3}(l_1+l_{c3})+m_4(l_3+l_{c4})(l_1+l_3+l_{c4})+\\
&I_3+I_4\\
M_{14}=&m_4l_{c4}(l_1+l_3+l_{c4})+I_4\\
M_{22}=&m_2l_{c2}^2m_3l_{c3}^2+m_4(l_3+l_{c4})^2+I_2+I_3+I_4\\
M_{23}=&m_3l_{c3}^2+m_4(l_3+l_{c4})^2+I_3+I_4\\
M_{24}=&m_4l_{c4}(l_3+l_{c4})+I_4\\
M_{33}=&m_3l_{c3}^2+m_4(l_3+l_{c4})^2+I_3+I_4\\
M_{34}=&m_4l_{c4}(l_3+l_{c4})+I_4\\
M_{44}=&m_4l_{c4}^2+I_4
\end{aligned} \tag{A1}$$

and the elements of gravity matrix G are obtained as

$$\begin{aligned}
&G_{11}=3m_1gl_1+m_1gl_{c1}\\
&G_{12}=G_{22}=m_2gl_{c2}\\
&G_{13}=G_{23}=G_{33}=-m_3gl_3-m_3gl_{c3}\\
&G_{14}=G_{24}=G_{34}=G_{44}=-m_4gl_{c4}
\end{aligned} \tag{A2}$$

in which g is the gravitational acceleration.

2 DCM definition

Motion of the center of mass of the HES can be divided into the divergent and convergent parts as

$$\begin{aligned} C &= x_{CoM} - \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \\ D &= x_{CoM} + \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \end{aligned} \tag{A3}$$

where C and D are convergent and divergent parts of the motion and x_{CoM} and $\dot{x}_{CoM}$ denotes position and velocity of the center of mass. Also, α is a constant value defined in (8). Taking time derivative of (A3) and substituting (**??**) in the absence of control input (τ=0), the time derivative of the divergent and convergent parts of the motion are

$$\begin{aligned} \dot{C} &= -\sqrt{\alpha} C \\ \dot{D} &= \sqrt{\alpha} D \end{aligned} \tag{A4}$$

As seen, the convergent part of the motion (C) will converge to zero without need of any control effort. Therefore, controlling the divergent part of the motion (D) will guarantee convergence of the position of the center of mass to the desired position. Therefore, DCM is defined as

$$\zeta = x_{CoM} + \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \tag{A5}$$

and the paper focused on the control of the DCM by adjusting the control input (τ).