

## APPENDIX

### ”Intelligent Locomotion Planning with Guaranteed Posture Stability for Lower-Limb Exoskeletons”

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# 1 Inertia matrix and gravity vector values:

The elements of inertia matrix  $M$  are

$$\begin{aligned}
M_{11} &= m_1 l_{c1}^2 + m_2 (l_1 + l_{c2})^2 + m_3 (l_1 + l_{c3})^2 + \\
&\quad m_4 (l_1 + l_3 + l_{c4})^2 + I_1 + I_2 + I_3 + I_4 \\
M_{12} &= m_2 l_{c2} (l_1 + l_{c2}) + m_3 l_{c3} (l_1 + l_{c3}) + \\
&\quad m_4 (l_3 + l_{c4}) (l_1 + l_3 + l_{c4}) + I_2 + I_3 + I_4 \\
M_{13} &= m_3 l_{c3} (l_1 + l_{c3}) + m_4 (l_3 + l_{c4}) (l_1 + l_3 + l_{c4}) + \\
&\quad I_3 + I_4 \\
M_{14} &= m_4 l_{c4} (l_1 + l_3 + l_{c4}) + I_4 \\
M_{22} &= m_2 l_{c2}^2 + m_3 l_{c3}^2 + m_4 (l_3 + l_{c4})^2 + I_2 + I_3 + I_4 \\
M_{23} &= m_3 l_{c3}^2 + m_4 (l_3 + l_{c4})^2 + I_3 + I_4 \\
M_{24} &= m_4 l_{c4} (l_3 + l_{c4}) + I_4 \\
M_{33} &= m_3 l_{c3}^2 + m_4 (l_3 + l_{c4})^2 + I_3 + I_4 \\
M_{34} &= m_4 l_{c4} (l_3 + l_{c4}) + I_4 \\
M_{44} &= m_4 l_{c4}^2 + I_4
\end{aligned} \tag{1}$$

and the elements of gravity matrix  $G$  are obtained as

$$\begin{aligned}
G_{11} &= 3m_1 g l_1 + m_1 g l_{c1} \\
G_{12} &= G_{22} = m_2 g l_{c2} \\
G_{13} &= G_{23} = G_{33} = -m_3 g l_3 - m_3 g l_{c3} \\
G_{14} &= G_{24} = G_{34} = G_{44} = -m_4 g l_{c4}
\end{aligned} \tag{2}$$

in which  $g$  is the gravitational acceleration.

## 2 DCM definition

Motion of the center of mass of the HES can be divided into the divergent and convergent parts as

$$\begin{aligned} C &= x_{CoM} - \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \\ D &= x_{CoM} + \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \end{aligned} \quad (3)$$

where  $C$  and  $D$  are convergent and divergent parts of the motion and  $x_{CoM}$  and  $\dot{x}_{CoM}$  denotes position and velocity of the center of mass. Also,  $\alpha$  is a constant value defined in (??). Taking time derivative of (4) and substituting (??) in the absence of control input ( $\tau=0$ ), the time derivative of the divergent and convergent parts of the motion are

$$\begin{aligned} \dot{C} &= -\sqrt{\alpha}C \\ \dot{D} &= \sqrt{\alpha}D \end{aligned} \quad (4)$$

As seen, the convergent part of the motion ( $C$ ) will converge to zero without need of any control input. Therefore, controlling the divergent part of the motion ( $D$ ) will guarantee convergence of the position of the center of mass to the desired position. Therefore, DCM is defined as

$$\zeta = x_{CoM} + \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \quad (5)$$

and the paper focused on the control of the DCM by adjusting the control input ( $\tau$ ).