Enhanced Multi-Platform Ebook Pricing Model

Your Name

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1 Problem Formulation

1.1 Decision Variables

Let $x_i \in [0, 1]$ be the normalized price variable for platform i, where $i = 1, 2, \ldots, n$ platforms.

The actual price is defined as:

$$p_i(x_i) = p_{\min} + x_i(p_{\max} - p_{\min}) \tag{1}$$

This normalization allows optimization over the unit interval while mapping to actual prices within the global bounds.

1.2 Given Parameters

- r_i: Royalty rate for platform i (as decimal, e.g., 0.70 for 70%)
- m_i: Market share/relative demand for platform i (normalized so $\sum_{i=1}^{n} m_i = 1$)
- $\bar{p}$: Average price from market data analysis
- λ_v: Weight parameter for price variance penalty
- λ_d: Weight parameter for deviation from market average penalty
- λ_c: Weight parameter for competitive advantage penalty
- α: Discount factor for your store (e.g., 0.85 for 15% discount advantage)

1.3 Enhanced Objective Function

The multi-objective optimization problem becomes:

$$\max_{x \in [0,1]^n} [\text{Revenue} - \lambda_v \cdot \text{Variance} - \lambda_d \cdot \text{Deviation} - \lambda_c \cdot \text{Competitive Penalty}] \tag{2}$$

Where:

Revenue Term:

$$\text{Revenue} = \sum_{i=1}^{n} r_i \cdot p_i(x_i) \cdot m_i \cdot f_i(x_i) \tag{3}$$

Price Variance Penalty:

$$\text{Variance} = \frac{1}{n} \sum_{i=1}^{n} (p_i(x_i) - \bar{p}_{\text{our}})^2 \tag{4}$$

where $\bar{p}_{\text{our}} = \frac{1}{n}\sum_{i=1}^{n} p_i(x_i)$ is the average price across all your platforms

Market Deviation Penalty:

$$\text{Deviation} = \left(\frac{1}{n}\sum_{i=1}^{n} p_i(x_i) - \bar{p}\right)^2 \tag{5}$$

Competitive Advantage Penalty:

$$\text{Competitive Penalty} = \max(0, p_1(x_1) - \alpha \cdot \min_{j\neq 1} p_j(x_j))^2 \tag{6}$$

1.4 Demand Function $f_i(x_i)$

To model price sensitivity, we use an exponential demand function:

$$f_i(x_i) = e^{-\beta_i(p_i(x_i) - p_{\text{ref}})} \tag{7}$$

Where:

- $\beta_i > 0$ is the price sensitivity parameter for platform i
- p_{ref} is a reference price (could be $\bar{p}$ or platform-specific)

2 Constraints

2.1 Essential Bounds

$$0 \leq x_i \leq 1 \quad \forall i \tag{8}$$

2.2 Optional Monotonicity (from original paper)

If you want to preserve format hierarchy:

$$x_{i+1} - x_i \geq \delta \quad \text{for ordered formats} \tag{9}$$

Only use this if platforms have clear value ordering (e.g., hardcover > paperback > ebook).

2.3 Maximum Price Spread (Optional)

To prevent extreme price differences:

$$\max_i p_i(x_i) - \min_i p_i(x_i) \leq \text{spread}_{\max} \tag{10}$$

3 Complete Mathematical Model

3.1 Enhanced Objective with Soft Constraints

Convert to minimization problem for standard solvers:

$$
\begin{aligned}
\min_{x} \Bigg[& -\sum_{i=1}^{n} r_i \cdot p_i(x_i) \cdot m_i \cdot e^{-\beta_i (p_i(x_i) - \bar{p})} \\
& + \lambda_v \sum_{i=1}^{n} (p_i(x_i) - \bar{p}_{\text{our}})^2 \\
& + \lambda_d \left(\bar{p}_{\text{our}} - \bar{p}\right)^2 \\
& + \lambda_c \cdot \max(0, p_1(x_1) - \alpha \cdot \min_{j \neq 1} p_j(x_j))^2 \Bigg]
\end{aligned} \tag{11}
$$

Subject to:

- $0 \leq x_i \leq 1 \quad \forall i$ (essential bounds only)
- Optional monotonicity constraints if format hierarchy exists
- Optional maximum spread constraint if needed

where $p_i(x_i) = p_{\min} + x_i(p_{\max} - p_{\min})$

4 Enhanced Seasonal Discount Strategy

We implement a sophisticated discount strategy that operates on multiple levels:

4.1 Multi-Level Discount Approach

Level 1: Competitive Positioning During discount periods, modify the competitive penalty:

Regular periods ($t \notin D$):

$$
\lambda_c \cdot \max(0, p_1(x_1) - \alpha \cdot \min_{j \neq 1} p_j(x_j))^2 \tag{12}
$$

Discount periods ($t \in D$):

$$
\lambda_c^{(D)} \cdot \max(0, p_1(x_1) - \alpha_{\text{discount}} \cdot \min_{j \neq 1} p_j(x_j))^2 \tag{13}
$$

where $\alpha_{\text{discount}} \in [0.7, 0.85]$ and $\lambda_c^{(D)} > \lambda_c$ to strengthen competitive pressure.

Level 2: Revenue Rebalancing Introduce discount-aware revenue weights:

$$
w_i^{(D)} = \begin{cases} w_i \cdot (1 + \gamma) & \text{if } i = 1 \text{ (your store)} \\ w_i & \text{otherwise} \end{cases} \tag{14}
$$

where $\gamma \in [0.2, 0.5]$ increases focus on your store's revenue during discount periods.

Level 3: Dynamic Market Share Adjustment During discount periods, adjust effective market share to reflect increased traffic:

$$
m_i^{(D)} = \begin{cases} m_i \cdot (1 + \theta \cdot d_i) & \text{where } d_i \text{ is discount depth} \\ m_i & \text{if no discount} \end{cases} \tag{15}
$$

4.2 Complete Discount Period Objective

During discount periods, the objective becomes:

$$\min_x \Bigg[-\sum_{i=1}^{n} w_i^{(D)} \cdot r_i \cdot p_i(x_i) \cdot m_i^{(D)} \cdot e^{-\beta_i(p_i(x_i)-\bar{p})} \\ + \lambda_v \sum_{i=1}^{n} (p_i(x_i) - \bar{p}_{\text{our}})^2 \\ + \lambda_d \left(\bar{p}_{\text{our}} - \bar{p}\right)^2 \\ + \lambda_c^{(D)} \cdot \max(0, p_1(x_1) - \alpha_{\text{discount}} \cdot \min_{j\neq 1} p_j(x_j))^2 \Bigg] \tag{16}$$

4.3 Discount Timing Strategy

Strategic Timing:

- **Spring Sale:** March-April (tax refund season)
- **Back-to-School:** August-September (educational content peak)

Discount Parameters by Period:

Period	α_{discount}	γ	Duration
Regular	$\alpha = 0.90$	0	10 months
Spring Sale	0.75	0.3	4 weeks
Back-to-School	0.80	0.4	6 weeks

4.4 Revenue Protection Mechanism

To ensure discounts don't hurt overall profitability:

Minimum Revenue Constraint:

$$\sum_{i=1}^{n} r_i \cdot p_i^{(D)}(x_i) \cdot m_i^{(D)} \geq \eta \cdot \sum_{i=1}^{n} r_i \cdot p_i^{(R)}(x_i) \cdot m_i \tag{17}$$

where $\eta \in [0.85, 0.95]$ ensures discount period revenue is at least 85-95% of regular period revenue, with the expectation that increased volume compensates.

5 Implementation Notes

5.1 Parameter Selection

- $\lambda_v \in [0.1, 1.0]$: Controls price variance (higher = more uniform pricing)
- $\lambda_d \in [0.1, 0.5]$: Controls deviation from market average
- $\lambda_c \in [0.1, 2.0]$: Controls competitive advantage strength
- β_i: Platform-specific elasticity (estimate from historical data)
- $\alpha \in [0.8, 0.95]$: Competitive discount (20% to 5% advantage)

5.2 Solver Recommendations

1. **SLSQP** (Sequential Least Squares Programming)
2. **Trust-Region Constrained**
3. **Interior Point Methods**

5.3 Multi-Objective Handling

Alternative approach using weighted sum:

$$\min_x [w_1 \cdot (-\text{Revenue}) + w_2 \cdot \text{Variance} + w_3 \cdot \text{Deviation} + w_4 \cdot \text{Competitive Penalty}] \tag{18}$$

Where $w_1 + w_2 + w_3 + w_4 = 1$ and weights reflect business priorities.

6 Expected Benefits

1. **Revenue Maximization:** Accounts for platform-specific royalty rates and market shares
2. **Price Consistency:** Minimizes variance across platforms while allowing differentiation
3. **Market Alignment:** Stays close to market average prices for consumer acceptance
4. **Competitive Advantage:** Encourages your store to maintain pricing advantage through soft constraints
5. **Flexibility:** Seasonal discount capabilities built into penalty structure
6. **Data-Driven:** Incorporates actual market data and platform characteristics

7 Extensions

1. **Dynamic Pricing:** Time-varying parameters based on sales data
2. **Cannibalization Effects:** Cross-platform demand interactions
3. **Bundle Pricing:** Multi-book pricing strategies
4. **A/B Testing Framework:** Experimental price variations for parameter estimation