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Algorithms_sheet #1

[1] a) $f(x) = 10$

$\because 10 \leq 10x$
, $\boxed{C = 10}$, $\boxed{x_0 = 1}$

$\therefore$ 10 is $O(x)$ #

b) $f(x) = 3x + 7$

$\because 3x + 7 \leq 10x$
, $\boxed{C = 10}$, $\boxed{x_0 = 1}$

$\therefore$ $3x+7$ is $O(x)$ #

c) $f(x) = 5 \log x$

$\because 5 \log x \leq 5x$
, $\boxed{C = 5}$, $\boxed{x_0 = 1}$

$\therefore$ $5 \log x$ is $O(x)$ #

d) $f(x) = x^2 + x + 1$ should be $\leq C \cdot x$, $x \geq x_0$
$\Rightarrow$ Cannot find a value of C & x_0
$\therefore$ $x^2 + x + 1$ is not $O(x)$ #

[2] a) $f(x) = 17x + 11$

$$\lim_{x\to\infty} \frac{17x+11}{x^2} = \frac{17\frac{x}{x^2} + \frac{11}{x^2}}{\frac{x^2}{x^2}}$$

$$\lim_{x\to\infty} \frac{17x+11}{17x} = \underline{1}$$

$$= \lim_{x\to\infty} \frac{0}{1} = 0$$

$\therefore$ $17x+11$ is $O(x^2)$ $\therefore$ $17x+11$ not $\Omega(x^2)$ #
$17x+11$ is $\Theta(x)$ #

when $17x+11 \leq x^2$, $\boxed{C = 1}$, $\boxed{x_0 = 1}$

b) $f(x) = x^2 + 1000$ $\Rightarrow \lim_{x \to \infty} \frac{x^2/x^2 + 1000/x^2}{x^2/x^2} = \boxed{1}$

const.

$\therefore$ $x^2 + 1000$ is $\Theta(x)$ #

, $x^2 + 1000 \geq x^2$

when $\boxed{C=1}$, $\boxed{x_0 = 1}$

$\therefore$ $x^2 + 1000$ is $\Omega(x^2)$ #

c) $f(x) = x \log x$ $\lim_{x \to \infty} \frac{x \log x}{x^2} \Rightarrow \frac{\log x}{x} = \frac{1}{x \ln 2}$

$= \frac{1}{\ln 2} \lim_{x \to \infty} \frac{1}{x^2} = \underline{\text{zero}}$

$\therefore$ $x \log x$ is $O(x)$ $\underline{\text{not}}$ $\Omega(x)$. #

, $\because$ $x \log x \geq x^2$ $\rightarrow$ $\log x \geq x$, $\boxed{C=1}$ $\boxed{x_0 = 1}$

$\therefore$ $x \log x$ is not $\Theta(x)$. #

d) $f(x) = 2^x$ $2^x \geq C \cdot x^2$

$\boxed{C=1}$, $\boxed{x_0 \geq 4}$

$\therefore$ 2^x is $\Omega(x)$ not $\Theta(x)$ #

$\boxed{3}$ $f(x) = x^4 + 9x^3 + 4x + 7$ is $O(x^4)$?

Ans

$$x^4 + 9x^3 + 4x + 7 \leq 21x^4$$

$\boxed{C = 21}$, $\boxed{x_0 \geq 1}$

$\therefore$ $x^4 + 9x^3 + 4x + 7$ is $O(x^4)$ #

$\boxed{4}$ a) x^3 is $O(x^4)$ not $O(x^3)$

Ans

$x^3 \leq x^4$ when $\boxed{C = 1}$, $\boxed{x_0 \geq 1}$

$\therefore$ x^3 is $O(x^4)$ #

$x^4 > x^3$ $\therefore$ x^4 is not $O(x^3)$ #

→ by limits :
$$\lim_{x \to \infty} \frac{x^3/x^4}{x^4/x^4} = \frac{0}{1} = 0$$

$\therefore$ x^3 is $O(x^4)$ #

$$\lim_{x \to \infty} \frac{x^4/x^3}{x^3/x^3} = \frac{\infty}{1} = \infty$$

$\therefore$ x^4 is $\Omega(x^3)$ not $O(x^3)$ #

b) $x \log x$ is $O(x^2)$ but x^2 is not $O(x \log x)$.

Ans

$x \log x \leq x^2$, $c=1$, $x_0=1$

$\therefore x \log x$ is $O(x^2)$ #

$x^2 > x \log x$ $\therefore$ $\boxed{\cancel{x \log}}$ x^2 is $\Omega(x \log x)$

Not $O(x \log x)$ #

→ by Limits: $\lim_{x \to \infty} \frac{x \log x}{x^2} = \frac{\log x / x}{x / x} = \frac{0}{1} = 0$

$\therefore x \log x$ is $O(x^2)$ #

but $\lim_{x \to \infty} \frac{x^2}{x \log x} = \infty$

$\therefore x^2$ is Not $O(x \log x)$ #

c) $3x+7$ is $\Theta(x)$. Ans

$\lim_{x \to \infty} \frac{3x+7}{x} = \frac{3x/x + 7/x}{x/x} = \frac{3+0}{1} = 3$ → Const.

$\therefore 3x+7$ is $\Theta(x)$.

→ math def: $3x+7 \geq x$, $\boxed{c=1}$, $\boxed{x_0=1}$

$\therefore 3x+7$ is $\Theta(x)$ #

d) $2x^2+x-7$ is $\Theta(x^2)$.

Ans

$$\lim_{x\to\infty} \frac{2x^2+x-7}{x^2} \rightarrow \frac{\frac{2x^2}{x^2}+\frac{x}{x^2}-\frac{7}{x^2}}{\frac{x^2}{x^2}} = \frac{2+0+0}{1} = \boxed{2}$$

const.

$\therefore$ $2x^2+x-7$ is $\Theta(x^2)$

→ math dif: $2x^2+x-7 \geq 10x^2$

$\boxed{c=10}$, $\boxed{x_0=1}$

$\therefore$ $2x^2+x-7$ is $\Theta(x^2)$

e) $x+1$ is $\Theta(x)$ Ans

$$\lim_{x\to\infty} \frac{x+1}{x} \rightarrow \frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}} = \frac{1+0}{1} = \boxed{1}$$ const.

$\therefore$ $x+1$ is $\Theta(x)$

→ math dif. $x+1 > x$

$\boxed{c=1}$, $\boxed{x_0=1}$

$\therefore$ $x+1$ is $\Theta(x)$

$\boxed{5}$ a) $n\log(n^2+1) + n^2\log n$

$\because \quad n\log(n^2+1) \geq n^2\log n$

$\log(n^2+1) \geq n\log n$

is $O(n^2\log n)$ #

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b) $(n\log n+1)^2 + (\log n+1)(n^2+1)$

$(n\log n)^2 + 2n\log n + n^2\log n + n^2 + \log n + 2$

$(n\log n)^2 \geq n^2\log n$

$n^2(\log n)^{\cancel{2}} \geq \cancel{n^2}\log n$

$\log n \geq 1$ is $O((n\log n)^2)$ #

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c) $(n^3+n^2\log n)(\log n+1) + (17\log n+19)(n^3+2)$

$= n^3\log n + n^3 + n^2(\log n)^2 + n^2\log n + 17n^3\log n + 34\log n$
$+ 19n^3 + 38$

$= 20n^3 + 18n^3\log n + (n\log n)^2 + n^2\log n + 34\log n + 38$

$20n^3 \geq 18n^3\log n$

$20 \geq 18\log n$

is $O(20n^3)$ #

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d) $(2^n + n^2)(n^3 + 3^n)$

$\Rightarrow 2^n n^3 + 2^n 3^n + n^5 + 3^n n^2$

$= 2^n n^3 + 3^n n^2 + 6^n + n^5$

32	36	36	32	$\rightarrow 2$	— even
216	243	216	243	$\rightarrow 3$	— odd
1024	1296	1296	1024	$\rightarrow 4$	— even

$3^n n^2 \geq n^5$, $3^n n^2 \geq 6^n$

is $O(3^n n^2)$