

Exploring Computer Science Concepts

Via ACSL Competitions

Number Systems

- Decimal
 - base 10
- Binary
 - base 2
- Octal
 - base 8
- HexaDecimal
 - base 16

Digits per base

- Decimal
 - Ten digits
 - 0 1 2 3 4 5 6 7 8 9
- Binary
 - Two Digits
 - 0 1
- Octal
 - 8 digits
 - 0 1 2 3 4 5 6 7
- Hexadecimal
 - 16 digits
 - 0 1 2 3 4 5 6 7 8 9 A B C D E F

Counting in Decimal

- Increment the lowest place value (right most digit)
- When last digit is reached
 - Set the current column to 0
 - Increment the column on the left by 1
- Lets count with decimal
 - 0 to 10
 - 95 to 105
- How frequently do we add a new digit ?

Decimal	Decimal
0	95
1	96
2	97
3	98
4	99
5	100
6	101
7	102
8	103
9	104
10	105

Counting in other bases

- Counting in binary
 - We add a new digit frequently
 - At 2 , 4, 8, 16 ... decimal values
- Counting in Octal
 - We add a new digit at every
 - At 8, 64,128 ...
- Counting in HexaDecimal
 - We add a new digit at every
 - At 16, 256 ... values

Decimal	Binary	Octal	HexaDecimal
0	0	0	1
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10

Place Values

- Decimal

- *We deal in power of 10s*

- $2452 = 2 * 1000 + 4 * 100 + 5 * 10 + 2 * 1$

- $101 = 1 * 100 + 0 * 10 + 1 * 1$

- Binary

- *We deal in power of 2s*

- $111 = 1 * 4 + 1 * 2 + 1 * 1$

- $1010 = 1 * 8 + 0 * 4 + 1 * 2 + 0 * 1$

Place values

Converting binary → decimal

Decimal	Binary
2452 → $2 \times 10^3 = 2000$ $4 \times 10^2 = 400$ $5 \times 10^1 = 50$ $2 \times 10^0 = 2$ = 2452	111 → $1 \times 2^2 = 1 \times 4 = 4$ $1 \times 2^1 = 1 \times 2 = 2$ $1 \times 2^0 = 1 \times 1 = 1$ = 7
101 → $1 \times 10^2 = 100$ $0 \times 10^1 = 0$ $1 \times 10^0 = 1$ = 101	1010 → $1 \times 2^3 = 1 \times 8 = 8$ $0 \times 2^2 = 0 \times 4 = 0$ $1 \times 2^1 = 0 \times 2 = 2$ $0 \times 2^0 = 0 \times 1 = 0$ = 10
756 →	1000 →

Converting Binary to Decimal

128 64 32 16 8 4 2 1

$$1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 = 64 + 16 + 1 = 81$$

$$1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 = 128 + 4 + 2 + 1 = 135$$

$$1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 = ?$$

Converting Decimal to Binary

128 64 32 16 8 4 2 1

= 22

= 33

= 130

Exercises

- Convert following Decimal numbers to binary
 - 127_{10}
 - 128_{10}
 - 129_{10}
 - 255_{10}
 - 256_{10}
- Convert following Binary numbers to Decimal
 - 101101_2
 - 1110_2
 - 1111_2
 - 0110_2

Challenge

- How does binary addition and subtraction work ?

Place values

Converting octal, hexadecimal → decimal

Octal → Decimal	Hexadecimal → Decimal
<p>777 →</p> $\begin{aligned} 7 \times 8^2 &= 7 \times 64 = 448 \\ 7 \times 8^1 &= 7 \times 8 = 56 \\ 7 \times 8^0 &= 7 \times 1 = 7 \\ &= 511 \end{aligned}$	<p>2AB →</p> $\begin{aligned} 2 \times 16^2 &= 2 \times 256 = 512 \\ A \times 16^1 &= 10 \times 16 = 160 \\ B \times 16^0 &= 11 \times 1 = 11 \\ &= 683 \end{aligned}$
<p>137 →</p> $\begin{aligned} 1 \times 8^2 &= 1 \times 64 = 64 \\ 3 \times 8^1 &= 3 \times 8 = 24 \\ 7 \times 8^0 &= 7 \times 1 = 7 \\ &= 95 \end{aligned}$	<p>101 →</p> $\begin{aligned} 1 \times 16^2 &= 1 \times 256 = 256 \\ 0 \times 16^1 &= 0 \times 16 = 0 \\ 1 \times 16^0 &= 1 \times 1 = 1 \\ &= 257 \end{aligned}$
<p>756 →</p>	<p>4A3 →</p>

Converting Decimal to Octal

$$(312)_{10} \rightarrow (?)_8$$

$$312 / 8 \rightarrow \text{quotient : } 39 \text{ , Remainder } 0$$

$$39 / 8 \rightarrow \text{quotient : } 4 \text{ , Remainder } 7$$

$$4 / 8 \rightarrow \text{quotient : } 0 \text{ , Remainder } 4$$

$$\mathbf{(470)_8}$$

$$(112)_{10} \rightarrow (?)_8$$

$$112 / 8 \rightarrow \text{quotient : } 14 \text{ , Remainder } 0$$

$$14 / 8 \rightarrow \text{quotient : } 1 \text{ , Remainder } 6$$

$$1 / 8 \rightarrow \text{quotient : } 0 \text{ , Remainder } 1$$

$$\mathbf{(160)_8}$$

Exercises

- Convert following Decimal numbers to Octal
 - 111_{10}
 - 88_{10}
 - 511_{10}
 - 512_{10}
 - 513_{10}
- Convert following Octal numbers to Decimal
 - 99_8
 - 77_8
 - 100_8
 - 101_8

Converting Decimal to Hexadecimal

$$(312)_{10} \rightarrow (?)_8$$

$$312 / 16 \rightarrow \text{quotient: } 19, \text{ Remainder } 8$$

$$19 / 16 \rightarrow \text{quotient: } 1, \text{ Remainder } 3$$

$$1 / 16 \rightarrow \text{quotient: } 0, \text{ Remainder } 1$$

$$\mathbf{(138)}_{16}$$

$$(112)_{10} \rightarrow (?)_8$$

$$112 / 16 \rightarrow \text{quotient: } 7, \text{ Remainder } 0$$

$$7 / 16 \rightarrow \text{quotient: } 0, \text{ Remainder } 7$$

$$\mathbf{(70)}_{16}$$

Hexadecimal → Octal

$(AC)_{16} \rightarrow (1010)_2 \times 16 + (1100)_2 \times 1$ // Replace hexa with binary
 $\rightarrow (1010\ 1100)_2$ // Convert to binary
 $\rightarrow (010\ 101\ 100)_2$ // group by 3 , added extra 0 in the front
 $\rightarrow (254)_8$ // Replace each group by octal value

$(1EF)_{16} \rightarrow (0001)_2 \times 256 + (1110)_2 \times 16 + (1111) \times 1$
 $\rightarrow (0001\ 1110\ 1111)_2$ // Convert to binary
 $\rightarrow (000\ 111\ 101\ 111)_2$ // group by 3 , added extra 0 in the front
 $\rightarrow (757)_8$ // Replace each group by octal value

Octal \rightarrow Hexadecimal

- (**757**)₈ \rightarrow (?)₁₆
- (**254**)₈ \rightarrow (?)₁₆

Exercises

- Convert following Decimal numbers to HexaDecimal/Octal
 - 255_{10}
 - 256_{10}
 - 257_{10}
- Convert following HexaDecimal numbers to Decimal, Octal
 - 99_{16}
 - 100_{16}
 - 101_{16}

Recursion : programming

- Create factorial
 - $5! = 5 \times 4 \times 3 \times 2 \times 1 = \mathbf{120}$
 - $2! = \mathbf{2}$
- Provide sum of Fibonacci numbers using recursion
 - fibonacci(13)
 - $0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 = \mathbf{33}$
 - fibonacci(3)
 - $0 + 1 + 1 + 2 + 3 = \mathbf{7}$