

# Exploring Computer Science Concepts

Via ACSL Competitions

# Number Systems

- Decimal
  - base 10
- Binary
  - base 2
- Octal
  - base 8
- HexaDecimal
  - base 16

# Digits per base

- Decimal
  - Ten digits
  - 0 1 2 3 4 5 6 7 8 9
- Binary
  - Two Digits
  - 0 1
- Octal
  - 8 digits
  - 0 1 2 3 4 5 6 7
- Hexadecimal
  - 16 digits
  - 0 1 2 3 4 5 6 7 8 9 A B C D E F

# Counting in Decimal

- Increment the lowest place value ( right most digit)
- When last digit is reached
  - Set the current column to 0
  - Increment the column on the left by 1
- Lets count with decimal
  - 0 to 10
  - 95 to 105
- How frequently do we add a new digit ?

Decimal	Decimal
0	95
1	96
2	97
3	98
4	99
5	100
6	101
7	102
8	103
9	104
10	105

# Counting in other bases

- Counting in binary
  - We add a new digit frequently
    - At 2 , 4, 8, 16 ... decimal values
- Counting in Octal
  - We add a new digit at every
    - At 8, 64,128 ...
- Counting in HexaDecimal
  - We add a new digit at every
    - At 16, 256 ... values

Decimal	Binary	Octal	HexaDecimal
0	0	0	1
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10

# Place Values

- Decimal

- *We deal in power of 10s*

- $2452 = 2 * 1000 + 4 * 100 + 5 * 10 + 2 * 1$

- $101 = 1 * 100 + 0 * 10 + 1 * 1$

- Binary

- *We deal in power of 2s*

- $111 = 1 * 4 + 1 * 2 + 1 * 1$

- $1010 = 1 * 8 + 0 * 4 + 1 * 2 + 0 * 1$

# Place values

## Converting binary → decimal

Decimal	Binary
<p>2452 →</p> $\begin{aligned} 2 \times 10^3 &= 2000 \\ 4 \times 10^2 &= 400 \\ 5 \times 10^1 &= 50 \\ 2 \times 10^0 &= 2 \\ &= 2452 \end{aligned}$	<p>111 →</p> $\begin{aligned} 1 \times 2^2 &= 1 \times 4 = 4 \\ 1 \times 2^1 &= 1 \times 2 = 2 \\ 1 \times 2^0 &= 1 \times 1 = 1 \\ &= 7 \end{aligned}$
<p>101 →</p> $\begin{aligned} 1 \times 10^2 &= 100 \\ 0 \times 10^1 &= 0 \\ 1 \times 10^0 &= 1 \\ &= 101 \end{aligned}$	<p>1010 →</p> $\begin{aligned} 1 \times 2^3 &= 1 \times 8 = 8 \\ 0 \times 2^2 &= 0 \times 4 = 0 \\ 1 \times 2^1 &= 0 \times 2 = 2 \\ 0 \times 2^0 &= 0 \times 1 = 0 \\ &= 10 \end{aligned}$
<p>756 →</p>	<p>1000 →</p>

# Converting Binary to Decimal

128   64   32   16   8   4   2   1

$$1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 = 64 + 16 + 1 = 81$$

$$1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 = 128 + 4 + 2 + 1 = 135$$

$$1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 = ?$$



# Converting Decimal to Binary

128   64   32   16   8   4   2   1

= 22

= 33

= 130

# Exercises

- Convert following Decimal numbers to binary
  - $127_{10}$
  - $128_{10}$
  - $129_{10}$
  - $255_{10}$
  - $256_{10}$
- Convert following Binary numbers to Decimal
  - $101101_2$
  - $1110_2$
  - $1111_2$
  - $0110_2$

# Challenge

- How does binary addition and subtraction work ?

# Place values

## Converting octal, hexadecimal → decimal

Octal → Decimal	HexaDecimal → Decimal
<p>777 →</p> $\begin{aligned} 7 \times 8^2 &= 7 \times 64 = 448 \\ 7 \times 8^1 &= 7 \times 8 = 56 \\ 7 \times 8^0 &= 7 \times 1 = 7 \\ &= 511 \end{aligned}$	<p>2AB →</p> $\begin{aligned} 2 \times 16^2 &= 2 \times 256 = 512 \\ A \times 16^1 &= 10 \times 16 = 160 \\ B \times 16^0 &= 11 \times 1 = 11 \\ &= 683 \end{aligned}$
<p>137 →</p> $\begin{aligned} 1 \times 8^2 &= 1 \times 64 = 64 \\ 3 \times 8^1 &= 3 \times 8 = 24 \\ 7 \times 8^0 &= 7 \times 1 = 7 \\ &= 95 \end{aligned}$	<p>101 →</p> $\begin{aligned} 1 \times 16^2 &= 1 \times 256 = 256 \\ 0 \times 16^1 &= 0 \times 16 = 0 \\ 1 \times 16^0 &= 1 \times 1 = 1 \\ &= 257 \end{aligned}$
<p>756 →</p>	<p>4A3 →</p>

# Converting Decimal to Octal

$$(312)_{10} \rightarrow (?)_8$$

$$312 / 8 \rightarrow \text{quotient : } 39 \text{ , Remainder } 0$$

$$39 / 8 \rightarrow \text{quotient : } 4 \text{ , Remainder } 7$$

$$4 / 8 \rightarrow \text{quotient : } 0 \text{ , Remainder } 4$$

$$\mathbf{(470)_8}$$

$$(112)_{10} \rightarrow (?)_8$$

$$112 / 8 \rightarrow \text{quotient : } 14 \text{ , Remainder } 0$$

$$14 / 8 \rightarrow \text{quotient : } 1 \text{ , Remainder } 6$$

$$1 / 8 \rightarrow \text{quotient : } 0 \text{ , Remainder } 1$$

$$\mathbf{(160)_8}$$

# Exercises

- Convert following Decimal numbers to Octal
  - $111_{10}$
  - $88_{10}$
  - $511_{10}$
  - $512_{10}$
  - $513_{10}$
- Convert following Octal numbers to Decimal
  - $45_8$
  - $77_8$
  - $100_8$
  - $101_8$

# Converting Decimal to Hexadecimal

$$(312)_{10} \rightarrow (?)_8$$

$$312 / 16 \rightarrow \text{quotient: } 19, \text{ Remainder } 8$$

$$19 / 16 \rightarrow \text{quotient: } 1, \text{ Remainder } 3$$

$$1 / 16 \rightarrow \text{quotient: } 0, \text{ Remainder } 1$$

$$\mathbf{(138)}_{16}$$

$$(112)_{10} \rightarrow (?)_8$$

$$112 / 16 \rightarrow \text{quotient: } 7, \text{ Remainder } 0$$

$$7 / 16 \rightarrow \text{quotient: } 0, \text{ Remainder } 7$$

$$\mathbf{(70)}_{16}$$

# Hexadecimal → Octal

$(AC)_{16} \rightarrow (1010)_2 \times 16 + (1100)_2 \times 1$  // Replace hexa with binary  
→  $(1010\ 1100)_2$  // Convert to binary  
→  $(010\ 101\ 100)_2$  // group by 3s , added extra 0s in the front  
→  $(254)_8$  // Replace each group by octal value

$(1EF)_{16} \rightarrow (0001)_2 \times 256 + (1110)_2 \times 16 + (1111) \times 1$   
→  $(0001\ 1110\ 1111)_2$  // Convert to binary  
→  $(\cancel{000}\ 111\ 101\ 111)_2$  // group by 3 , removed 0s in the front  
→  $(757)_8$  // Replace each group by octal value



# Octal $\rightarrow$ Hexadecimal

- ( **757** )<sub>8</sub>  $\rightarrow$  ( ? )<sub>16</sub>
- ( **254** )<sub>8</sub>  $\rightarrow$  ( ? )<sub>16</sub>

# Exercises

- Convert following Decimal numbers to HexaDecimal/Octal
  - $255_{10}$
  - $256_{10}$
  - $257_{10}$
- Convert following HexaDecimal numbers to Decimal, Octal
  - $99_{16}$
  - $100_{16}$
  - $101_{16}$

# Recursion : programming

- Create factorial
  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = \mathbf{120}$
  - $2! = \mathbf{2}$
- Provide sum of Fibonacci numbers using recursion
  - fibonacci(13)
    - $0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 = \mathbf{33}$
  - fibonacci(3)
    - $0 + 1 + 1 + 2 + 3 = \mathbf{7}$