

Let us think of a scene. Suppose the Santa Claus was coming to town but he was missing direction. By asking the Government he was given a map with four coordinates. They are city $A(3, 6)$, city $B(-3, -6)$, city $C(2, -12)$ and city $D(8, 0)$.

You are going to help Santa Claus to figure out some important information about his route of distribution.

(F.1) Sketch the coordinates and the polygon $ABCD$ on a coordinate plane. You may label the plane by yourself.

(F.1) Find the area of the polygon $ABCD$.

(F.2) Find the mean of the coordinates, i.e. the mean value of x-coordinates and y-coordinates.

(F.3) Find the length of the sides of the polygon $ABCD$.

We then suppose the Santa Claus needs to distribute different cities with different weight of presents. For City A (Point A), it is 20kg; For City B (Point B), it is 30kg; For City C (Point C), it is 40kg; For City D (Point D), it is 50kg.

(F.4) Find the shortest route to reach every city.

(F.5) It is known that the energy required for each route (E) varies partly with m and partly with L^2 , where m is the weight of carrying presents and L is the length of the route. If the energy carrying all presents from A to B and carrying half of the presents from A to D are 1300 and 594 respectively. Find the energy needed for carrying all presents from A straightly to C .

(F.5) Find the total number of routes to go through all cities.

(F.5) Hence, find the route requiring the least energy to deliver all presents.