

1. (a)

$$e^{i2x} = \cos^2 x - \sin^2 x + i2 \sin x \cos x$$

$$e^{i3x} = 4 \cos^3 x - 3 \cos x + i(3 \sin x - 4 \sin^3 x)$$

(b) Remember to do both directions.

(c) i.

$$\begin{aligned} \sum_{k=0}^n \cos kx &= \operatorname{Re} \left\{ \sum_{k=0}^n e^{ikx} \right\} \\ &= \operatorname{Re} \left\{ \frac{1 - e^{i(n+1)x}}{1 - e^{ix}} \right\} \end{aligned}$$

ii.

$$\begin{aligned} \sum_{k=0}^n \sin kx &= \operatorname{Im} \left\{ \sum_{k=0}^n e^{ikx} \right\} \\ &= \operatorname{Im} \left\{ \frac{1 - e^{i(n+1)x}}{1 - e^{ix}} \right\} \end{aligned}$$

iii.

$$\begin{aligned} \sum_{k=0}^n r^k \cos kx &= \operatorname{Re} \left\{ \sum_{k=0}^n r^k e^{ikx} \right\} \\ &= \operatorname{Re} \left\{ \frac{1 - r^{n+1} e^{i(n+1)x}}{1 - r e^{ix}} \right\} \end{aligned}$$

iv.

$$\begin{aligned} \sum_{k=0}^n r^k \sin kx &= \operatorname{Im} \left\{ \sum_{k=0}^n r^k e^{ikx} \right\} \\ &= \operatorname{Im} \left\{ \frac{1 - r^{n+1} e^{i(n+1)x}}{1 - r e^{ix}} \right\} \end{aligned}$$

(d) i.

$$\begin{aligned} \sum_{k=0}^{\infty} r^k \cos kx &= \operatorname{Re} \left\{ \sum_{k=0}^{\infty} r^k e^{ikx} \right\} \\ &= \operatorname{Re} \left\{ \frac{1}{1 - r e^{ix}} \right\} \end{aligned}$$

ii.

$$\begin{aligned}\sum_{k=0}^{\infty} r^k \sin kx &= \operatorname{Im} \left\{ \sum_{k=0}^{\infty} r^k e^{ikx} \right\} \\ &= \operatorname{Im} \left\{ \frac{1}{1 - re^{ix}} \right\}\end{aligned}$$

2. (a) i. Taylor

ii. Taylor

iii. Taylor

(b) 弧長等於弧度角，并且弧長永遠大於垂直高度。

(c) 利用極限。

3. (a) $\lim_{x \rightarrow a} (f(x) - f(a)) = \lim_{x \rightarrow a} D_a f(x) \cdot \lim_{x \rightarrow a} (x - a) = 0$.

(b) 沒有一點有導數，則為離散函數。

4. (a) 0

(b) 代 $x = r \cos \theta, y = r \sin \theta$.