PRACTICE PAPER

MATHEMTICS Extended Part

Module 2 (Algebra and Calculus)

Question-Answer Book

Time allowed: 1.5 hours

Name:______/100

School:______/200

Instructions

- 1. This paper must be answered in English.
- 2. Unless otherwise specified, all working must be clearly shown.
- 3. Unless otherwise specified, numerical answers must be exact.
- 4. This paper is for **internal use** only.
- 5. All questions are collected from AL/CE/DSE past papers, reference site: https://www.dse.life/ppindex/m2/

1. (1997-CE-A MATH 2 #07(Modified)) Let $T_n = (n^2 + 1)(n!)$ for any positive integer n. Prove, by mathematical induction, that

$$\sum_{k=1}^{n} T_k = n[(n+1)!]$$

for any positive integer n .	(6 marks)

(8 marks)

2. (1988-HL-GEN MATHS #07(Modified)) Let

$$A_n = 1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1}n^2$$

and

$$B_n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

where n is a positive integer.

- (a) Show, by mathematical induction, that $A_n = (-1)^{n-1}B_n$ for all positive integer n.
- (b) Hence, or otherwise, find $\sum_{n=1}^{2m} A_n$ and $\sum_{n=1}^{2m+1} A_n$.

F4 Level 5 M2 mock paper	Time limit: 1 hr 30 mins

3.	(2017-DSE-MATH-EP(M2) #02) Let $(1+ax)^8 = \sum_{k=0}^8 \lambda_k x^k$ and $(b+x)^9 = \sum_{k=0}^9 \mu_k x^k$, where a and b are constants. It is given that			
	$(b+x)^3 = \sum_{k=0}^{\infty} \mu_k x^k$, where a and b are constants. It is given that $\lambda_2 : \mu_7 = 7 : 4$ and $\lambda_1 + \mu_8 + 6 = 0$. Find a .	(6 marks)		

- 4. (1989-HL-GEN MATHS #05(Modified))
 - (a) Find the solution of $\sin x \sin 2x + \sin 3x = 0$ for $0 < x < 2\pi$.
 - (b) Let $f(\theta) = \sin 2\theta + \sin \theta + \cos \theta$.
 - i. Express $f(\theta)$ in terms of p, where $p = \sin \theta + \cos \theta$.
 - ii. Using (i) and the method of completing the square, find the smallest value of $f(\theta)$. For $0 < \theta < \pi$, find also the value of θ such that $f(\theta)$ attains its smallest value.

(18 marks)

F4 Level 5 M2 mock paper	Time limit: 1 hr 30 mins

5. (1984-HL GEN MATHS #05(Modified))

- (a) Express $\cot 4\theta$ in terms of $\cot \theta$. Hence solve the equation $x^4 4x^3 6x^2 + 4x + 1 = 0$. (Give your answers in terms of π .)
- (b) i. If $\cos \theta \cos \phi = a$ and $\sin \theta \sin \phi = b$ $(b \neq 0)$, show that

$$\frac{1}{2}(2-a^2-b^2) = \cos\theta - \phi \text{ and } \frac{-a}{b} = \tan\frac{\theta + \phi}{2}.$$

ii. Solve the system of equations

$$\begin{cases} \cos \theta - \cos \phi = 1\\ \sin \theta - \sin \phi = \sqrt{3} \end{cases}$$

where $0 \le \theta \le 2\pi$ and $0 \le \phi \le 2\pi$.

(18 marks)

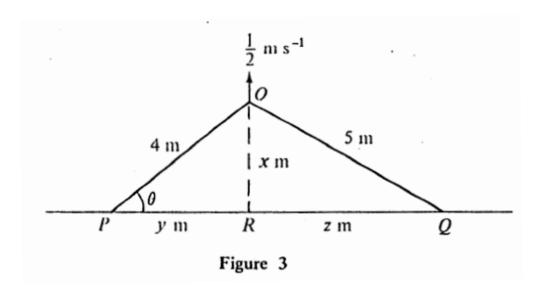
F4 Level 5 M2 mock paper	Time limit: 1 hr 30 mins

erms of h . Hence, find $f'(1)$ from first principles.	(6 mar

7.	(PP-DSE-MATH-EP(M2) #07) Let $f(x) = e^x(\sin x + \cos x)$.		
	(a) Find $f'(x)$ and $f''(x)$.		
	(b) Solve for x when $f''(x) - f'(x) + f(x) = 0$, where x is real	number.	
		(8 marks)	

8. (1996-CE-A MATH 1 #06) Find the equations of the two tangents to the curve $C: y = \frac{6}{x+1}$ which are parallel to the line x + 6y + 10 = 0. (7 marks)

9. (1994-CE-A MATH 1 #12)



In figure 3, two rods OP and OQ are hinged at O. The lengths of OP and OQ are 4 m and 5 m respectively. The end O is pushed upwards at a constant rate of $\frac{1}{2}$ ms⁻¹ along a fixed vertical axis, and the ends P and Q move along a horizontal rail. R is the projection of O on the rail. At time t seconds, OR = x m and $\angle OPQ = \theta$ where $0 < \theta < \frac{\pi}{2}$.

- (a) i. Express x in terms of θ .
 - ii. Hence find the rate of change of θ with respect to t in terms of θ .
- (b) Let PR = y m, RQ = z m.
 - i. Express $\frac{dy}{dt}$ and $\frac{dz}{dt}$ in terms of θ .
 - ii. Hence find the rate of change of PQ with respect to t when $\theta = \frac{\pi}{6}$, giving your answer correct to 3 significant figures.
- (c) i. Find the value of θ such that the area of $\triangle OPR$ is a maximum.
 - ii. By considering the value of $\angle OQR$, find the value of θ such that the area of $\triangle ORQ$ is a maximum, giving your answer correct to 3 significant figures.

(23 marks)

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