Using coordinate system to construct a circle, we find that if a circle is centered at (h, k) with radius r, then by Pythagoras theorem (or distance formula), we have the following thought:

Let P be a moving point on the circumference of that circle, the distance between P and the center of the circle is fixed by radius r. Therefore, the equation of the circle (in fact circumference) is given by the following equation:

$$(x-h)^2 + (y-k)^2 = r^2$$

By expansion and rearrangement of terms of the equation, we have the equation becomes

$$x^2 + y^2 + Dx + Ex + F = 0$$

where D, E, F are real coefficients computed by $D = -2h, E = -2k, F = h^2 + k^2 - r^2$.

Moreover, we call circles of the same center **concentric circles**.

Challange

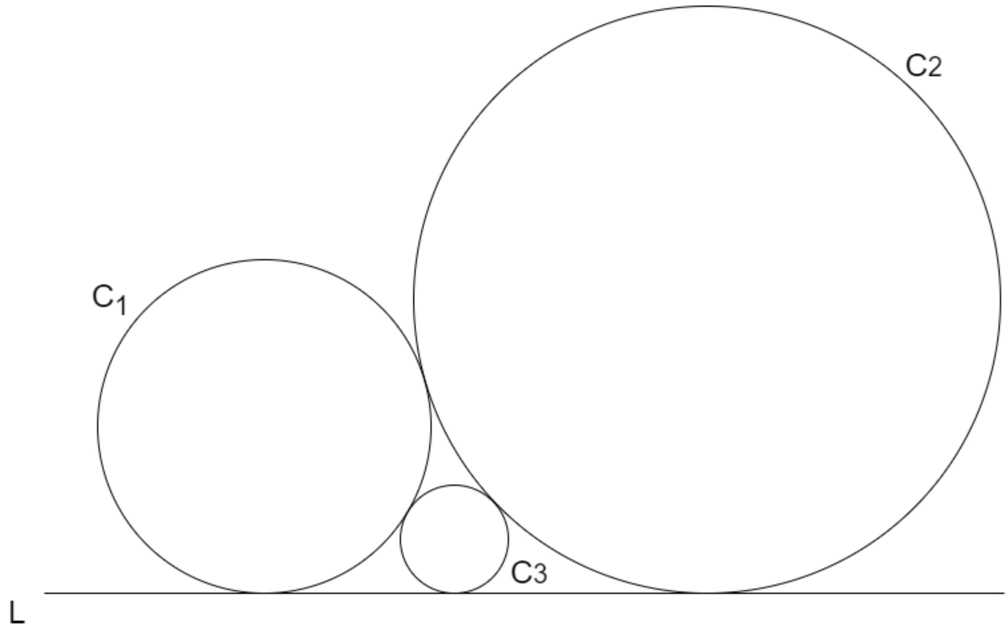
Given a circle $C_0 : x^2 + y^2 = R^2$, where $R > 0$. Let $P(h, k)$ be a moving point on C_0. We construct another circle C_P centered at P with radius r, where $0 < r < R$. Let F and G be the closest point from the origin to C_P and the farthest point from the origin to C_P respectively. Denote the locus of F and G by C_F and C_G respectively.

1. Describe the geometric relationship between O, F and G.
2. Find the equation of straight line for OP in terms of h, k and R.
3. Find the coordinates of F and G in terms of h, k, R and r.
4. Prove that C_F and C_G are concentric circles.
5. Define the tangent lines of C_P passing through the origin to be L_1 and L_2. Find the area enclosed by L_1, L_2, C_F, C_G and C_P.

Challenging problems set

1. Let $\triangle ABC$ be with arbitrary sides a, b, c. Compute the ratio of area of the triangle to its inscribed circle.

2. Refer to the following diagram:



Suppose as shown in the figure that three circles C_1, C_2, C_3 are all touching others at one point, and they share the same taangent line L. Denote the touching point of C_1 and L be X, the touching point of C_2 and L be Y, the touching point of C_3 and L be Z, the touching point of C_1 and C_2 be P, the touching point of C_1 and C_3 be Q, the touching point of C_2 and C_3 be R. If $XZ = 1$ and $YZ = 2$, find the area of $\triangle PQR$.