1. (a)

$$e^{i2x} = \cos^2 x - \sin^2 x + i2\sin x\cos x$$
$$e^{i3x} = 4\cos^3 x - 3\cos x + i(3\sin x - 4\sin^3 x)$$

   (b) Remember to do both directions.

   (c) i.

$$\sum_{k=0}^{n} \cos kx = \mathrm{Re}\left\{\sum_{k=0}^{n} e^{ikx}\right\} = \mathrm{Re}\left\{\frac{1-e^{i(n+1)x}}{1-e^{ix}}\right\}$$

   ii.

$$\sum_{k=0}^{n} \sin kx = \mathrm{Im}\left\{\sum_{k=0}^{n} e^{ikx}\right\} = \mathrm{Im}\left\{\frac{1-e^{i(n+1)x}}{1-e^{ix}}\right\}$$

   iii.

$$\sum_{k=0}^{n} r^k \cos kx = \mathrm{Re}\left\{\sum_{k=0}^{n} r^k e^{ikx}\right\} = \mathrm{Re}\left\{\frac{1-r^{n+1}e^{i(n+1)x}}{1-re^{ix}}\right\}$$

   iv.

$$\sum_{k=0}^{n} r^k \sin kx = \mathrm{Im}\left\{\sum_{k=0}^{n} r^k e^{ikx}\right\} = \mathrm{Im}\left\{\frac{1-r^{n+1}e^{i(n+1)x}}{1-re^{ix}}\right\}$$

   (d) i.

$$\sum_{k=0}^{\infty} r^k \cos kx = \mathrm{Re}\left\{\sum_{k=0}^{\infty} r^k e^{ikx}\right\} = \mathrm{Re}\left\{\frac{1}{1-re^{ix}}\right\}$$

ii.

$$\sum_{k=0}^{\infty} r^k \sin kx = \mathrm{Im}\left\{\sum_{k=0}^{n} r^k e^{ikx}\right\}$$
$$= \mathrm{Im}\left\{\frac{1}{1-re^{ix}}\right\}$$

2. (a) i. Taylor
   ii. Taylor
   iii. Taylor
   (b) 弧長等於弧度角，并且弧長永遠大於垂直高度。
   (c) 利用極限，$\lim_{x\to 0}\frac{\sin(e^x-1)-\sin(\cos x-1)}{x}=1$。
3. (a) $\lim_{x\to a}(f(x)-f(a)) = \lim_{x\to a} D_a f(x)\cdot \lim_{x\to a}(x-a)=0.$
   (b) 沒有一點有導數，則為離散函數。
4. (a) 0
   (b) 代$x=r\cos\theta, y=r\sin\theta$.最後可得$2\cot 2\theta$。$\theta\in\mathbb{R} \implies$ 發散。