1.

$$
\begin{aligned}
f'(\pi) &= \lim_{h\to 0} \frac{f(\pi+h)-f(\pi)}{h} \\
&= \lim_{h\to 0} \frac{e^{\sin \pi + h} - e^{\sin \pi}}{h} \\
&= \lim_{h\to 0} \frac{e^{-\sin h} - 1}{h} \\
&= \lim_{h\to 0} \frac{e^{-\sin h} - 1}{-\sin h} \lim_{h\to 0} \frac{-\sin h}{h} \\
&= (1)(-1) \\
&= -1
\end{aligned}
$$

2. (a) $f'(0) = \pi \cos \pi \cdot 0 = \pi$.

   (b) We have

$$
\begin{aligned}
g_1(0) = 0, g_1'(0) = \pi \\
g_2(0) = 0, g_2'(0) = \pi^2 \\
g_3(0) = 0, g_3'(0) = \pi^3
\end{aligned}
$$

   so on and so forth. We shall claim $g_n'(0) = \pi^n$ and prove it by induction.

3. $f'(x) = 2x + a$ and $f”(x) = 2$. Existence of extrema by derivative solvable and minima by second derivative test.

4. $f'(x) = 3ax^2 + 2bx + c$. The quadratic function has no solution when $b^2 < 3ac$, otherwise, if $f'(x) = 0$, we have $f''(x) = 0$ so that the point is not an extrema, which yields $b^2 = 3ac$. Afterall, the condition required is $b^2 \leq 3ac$.

5. (a) Suppose f is nonzero at every point. First case: either left or right limit $\neq 0$, this immediately discontinuous. Second case: both left and right limit equal 0, but f is nonzero, then f is discontinuous. Then f must at some point equal 0.

   (b) i. $y - f(c) = f'(c)(x - c)$.

   ii. $x = c - \frac{f(c)}{f'(c)}$.

(c) i.

$$\begin{aligned}|x_{n+1} - x_n| &= |x_n - \frac{f(x_n)}{f'(x_n)} - x_{n-1} + \frac{f(x_{n-1})}{f'(x_{n-1})}| \\ &\leq |x_n - x_{n-1}| + |\frac{f(x_n)}{f'(x_n)} - \frac{f(x_{n-1})}{f'(x_{n-1})}| \\ &\to |x_n - x_{n-1}|\end{aligned}$$

ii. $f(x_n) = 0 \implies x_{n+1} = x_n$.

(d) 3,3.142547,3.141593,3.141593.

Bonus (a) $y - f(t) = f'(t)(x - t)$.

(b) $y - f(t) = -\frac{1}{f'(t)}(x - t)$.

(c) i. There is some point c such that $f'(c) = 0$, then g' is not entirely finite.

ii. $f'(x) = e^x \implies g'(x) = e^{-x} \implies g(x) = -e^{-x}$.

iii. $(t + \frac{e^t - e^{-t}}{e^t + e^{-t}}, \frac{2}{e^t + e^{-t}})$