1.

$$\begin{aligned} P(1) &: 1 = \frac{1(1+1)}{2} \\ P(n) &\implies P(n+1): \\ 1+2+3+\cdots+n+(n+1) &= \frac{n(n+1)}{2}+(n+1) \\ &= (n+1)(\frac{n}{2}+1) \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

2.

$$\begin{aligned} P(1) &: 1\times 2 = 1^2(1+1) \\ P(n) &\implies P(n+1): \\ 1\times 2+2\times 5+\cdots+n(3n-1)+(n+1)(3n+2) &= n^2(n+1)+(n+1)(3n+2) \\ &= (n+1)(n^2+3n+2) \\ &= (n+1)(n+1)(n+2) \\ &= (n+1)^2(n+2) \end{aligned}$$

3. (a) $(1+2x)^n = 1+2nx+4C_2^n x^2+8C_3^n+\cdots$.

(b)

$$\begin{aligned} (x-\frac{3}{x})^2(1+2x)^n &= (x^2-6+\frac{9}{x^2})(1+2nx+4C_2^n x^2+8C_3^n+\cdots) \\ -6+36C_2^n &= 210 \\ C_2^n &= 6 \\ n &= 4 \end{aligned}$$

4.

$$r\cos(\theta-\alpha) = r\cos\theta\cos\theta + r\sin\theta\sin\theta = \cos\theta+\sqrt{3}\sin\theta$$

$$\implies \begin{cases} r\cos\alpha &= 1 \\ r\sin\alpha &= \sqrt{3} \end{cases} \implies \begin{cases} r &= \sqrt{1^2+\sqrt{3}^2} = 2 \\ \alpha &= \arctan\sqrt{3} = \frac{\pi}{3} \end{cases}$$

5.

$$\begin{aligned} \frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta} &= \frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}} \\ &= \tan\frac{\alpha+\beta}{2} \end{aligned}$$

Note that

$$\begin{aligned}
3\sin\alpha - 4\cos\alpha &= 4\cos\beta - 3\cos\beta \\
3(\sin\alpha + \cos\beta) &= 4(\cos\alpha + \cos\beta) \\
\tan\left(\frac{\alpha+\beta}{2}\right) &= \frac{4}{3} \\
\tan(\alpha+\beta) &= \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)} \\
&= \frac{8/3}{1-16/9} \\
&= -\frac{24}{7}
\end{aligned}$$

6.

$$\begin{aligned}
\frac{d}{dx}(x^2) &= \lim_{h\to 0}\frac{(x+h)^2-x^2}{h} \\
&= \lim_{h\to 0}\frac{h(2x+h)}{h} \\
&= \lim_{h\to 0}(2x+h) \\
&= 2x
\end{aligned}$$

7. Differentiation

$$\begin{aligned}
\sin y + x\cos y\frac{dy}{dx} &= 0 \\
\frac{dy}{dx} &= -\frac{\tan x}{x} = -\frac{\sin x}{x\cos x}
\end{aligned}$$

8. (a) $\frac{dy}{dx} = x\cos x$, $\frac{d^2y}{dx^2} = \cos x - x\sin x$.

(b)

$$\begin{aligned}
x\cos x - x^2\sin x + kx\cos x + x(x\sin x + \cos x) &= 0 \\
k &= -2
\end{aligned}$$

9.

$$\begin{aligned}
y' &= 3x^2 \\
x^3 + 16 &= 3x^2(x-0) \\
x^3 &= 8 \\
x &= 2 \\
L : y &= 12x - 16
\end{aligned}$$