In this article we will examine the distance between a point and a line. By definition, the distance between two points on a $\mathbb{R}^2$ plane is

Definition 1 (Distance between two points)**.** *Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points on $\mathbb{R}^2$-plane. The distance between A and B is computed by the formula*

$$\operatorname{dist}(A, B) := \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Now let $L : ax + by + c = 0$ be a straight line and $P(x_0, y_0)$ be a point on $\mathbb{R}^2$-plane. Note that with the following axiom

Axiom. *The distance between a point P and a line L is defined by the shortest distance between P and a point on L*

$$\operatorname{dist}(P, L) := \inf\{\operatorname{dist}(P, Q) : Q \in L\}$$

we can choose the perpendicular displacement of P from L to define the distance. Using the line Γ perpendicular to L passing through P, we can find such Q by following computation:

$$\begin{cases} L : ax + by + c = 0 \\ \Gamma : bx - ay + (ay_0 - bx_0) = 0 \end{cases} \implies Q(\frac{b^2x_0 - aby_0 - ac}{a^2 + b^2}, \frac{a^2y_0 - abx_0 - bc}{a^2 + b^2})$$

Therefore, the distance between P an Q is

$$\begin{aligned} \operatorname{dist}(P, Q) &= \frac{1}{a^2 + b^2}\sqrt{[(a^2 + b^2)x_0 - (b^2x_0 - aby_0 - ac)]^2 + [(a^2 + b^2)y_0 - (a^2y_0 - abx_0 - bc)]^2} \\ &= \frac{1}{a^2 + b^2}\sqrt{(a^2x_0 + aby_0 + ac)^2 + (b^2y_0 + abx_0 + bc)^2} \\ &= \frac{\sqrt{a^2 + b^2}\sqrt{(ax_0 + by_0 + c)^2}}{a^2 + b^2} \\ &= \frac{\sqrt{(ax_0 + by_0 + c)^2}}{\sqrt{a^2 + b^2}} \\ &= \left|\frac{ax_0 + by_0 + c}{\sqrt{a^2 + b^2}}\right| \end{aligned}$$