A Brief Table of Integrals

Basic Forms

1.
$$\int k \, dx = kx + C$$
 (any number k)

3.
$$\int \frac{dx}{x} = \ln|x| + C$$

5.
$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$$

$$7. \int \cos x \, dx = \sin x + C$$

$$9. \int \csc^2 x \, dx = -\cot x + C$$

11.
$$\int \csc x \cot x \, dx = -\csc x + C$$

$$13. \int \cot x \, dx = \ln|\sin x| + C$$

$$15. \int \cosh x \, dx = \sinh x + C$$

17.
$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

19.
$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C \quad (a > 0)$$

2.
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$4. \int e^x dx = e^x + C$$

$$\mathbf{6.} \int \sin x \, dx = -\cos x + C$$

$$8. \int \sec^2 x \, dx = \tan x + C$$

10.
$$\int \sec x \tan x \, dx = \sec x + C$$

12.
$$\int \tan x \, dx = \ln |\sec x| + C$$

$$14. \int \sinh x \, dx = \cosh x + C$$

16.
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

18.
$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C$$

20.
$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\frac{x}{a} + C \quad (x > a > 0)$$

Forms Involving ax + b

21.
$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

22.
$$\int x(ax+b)^n dx = \frac{(ax+b)^{n+1}}{a^2} \left[\frac{ax+b}{n+2} - \frac{b}{n+1} \right] + C, \quad n \neq -1, -2$$

23.
$$\int (ax+b)^{-1} dx = \frac{1}{a} \ln |ax+b| + C$$

25.
$$\int x(ax+b)^{-2} dx = \frac{1}{a^2} \left[\ln|ax+b| + \frac{b}{ax+b} \right] + C$$
 26. $\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln\left| \frac{x}{ax+b} \right| + C$

27.
$$\int (\sqrt{ax+b})^n dx = \frac{2}{a} \frac{(\sqrt{ax+b})^{n+2}}{n+2} + C, \quad n \neq -2$$
 28. $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}}$

24.
$$\int x(ax+b)^{-1} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$$

26.
$$\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln \left| \frac{x}{ax+b} \right| + C$$

28.
$$\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}}$$

29. (a)
$$\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C$$
 (b) $\int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax-b}{b}} + C$

(b)
$$\int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax-b}{b}} + C$$

30.
$$\int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} + C$$

30.
$$\int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} + C$$
 31. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax+b}} + C$

Forms Involving $a^2 + x^2$

32.
$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

33.
$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C$$

34.
$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C = \ln \left(x + \sqrt{a^2 + x^2} \right) + C$$

35.
$$\int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln\left(x + \sqrt{a^2 + x^2}\right) + C$$

36.
$$\int x^2 \sqrt{a^2 + x^2} \, dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln \left(x + \sqrt{a^2 + x^2} \right) + C$$

37.
$$\int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$$

38.
$$\int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \ln\left(x + \sqrt{a^2 + x^2}\right) - \frac{\sqrt{a^2 + x^2}}{x} + C$$

39.
$$\int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \ln \left(x + \sqrt{a^2 + x^2} \right) + \frac{x\sqrt{a^2 + x^2}}{2} + C$$

40.
$$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2+x^2}}{x} \right| + C$$
 41.
$$\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$$

41.
$$\int \frac{dx}{x^2 \sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$$

Forms Involving $a^2 - x^2$

42.
$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x + a}{x - a} \right| + C$$

43.
$$\int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left| \frac{x + a}{x - a} \right| + C$$

44.
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

45.
$$\int \sqrt{a^2 - x^2} \ dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

46.
$$\int x^2 \sqrt{a^2 - x^2} \ dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2 - x^2} (a^2 - 2x^2) + C$$

47.
$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$
48.
$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2 - x^2}}{x} + C$$

49.
$$\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2 - x^2} + C$$
 50.
$$\int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

50.
$$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

51.
$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

Forms Involving $x^2 - a^2$

52.
$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

53.
$$\int \sqrt{x^2 - a^2} \ dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$

54.
$$\int \left(\sqrt{x^2 - a^2}\right)^n dx = \frac{x\left(\sqrt{x^2 - a^2}\right)^n}{n+1} - \frac{na^2}{n+1} \int \left(\sqrt{x^2 - a^2}\right)^{n-2} dx, \quad n \neq -1$$

55.
$$\int \frac{dx}{\left(\sqrt{x^2 - a^2}\right)^n} = \frac{x\left(\sqrt{x^2 - a^2}\right)^{2-n}}{(2 - n)a^2} - \frac{n - 3}{(n - 2)a^2} \int \frac{dx}{\left(\sqrt{x^2 - a^2}\right)^{n-2}}, \quad n \neq 2$$

56.
$$\int x \left(\sqrt{x^2 - a^2} \right)^n dx = \frac{\left(\sqrt{x^2 - a^2} \right)^{n+2}}{n+2} + C, \quad n \neq -2$$

57.
$$\int x^2 \sqrt{x^2 - a^2} \ dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 - a^2}| + C$$

58.
$$\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C$$

59.
$$\int \frac{\sqrt{x^2 - a^2}}{x^2} dx = \ln|x + \sqrt{x^2 - a^2}| - \frac{\sqrt{x^2 - a^2}}{x} + C$$

60.
$$\int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + \frac{x}{2} \sqrt{x^2 - a^2} + C$$

61.
$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C = \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C$$
 62.
$$\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$$

62.
$$\int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$$

Trigonometric Forms

$$63. \int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\mathbf{64.} \ \int \cos ax \ dx = \frac{1}{a} \sin ax + C$$

65.
$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

66.
$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

67.
$$\int \sin^n ax \, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx$$

68.
$$\int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$$

69. (a)
$$\int \sin ax \cos bx \, dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$$

(b)
$$\int \sin ax \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

(c)
$$\int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$70. \int \sin ax \cos ax \, dx = -\frac{\cos 2ax}{4a} + C$$

71.
$$\int \sin^n ax \cos ax \, dx = \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$$

72.
$$\int \frac{\cos ax}{\sin ax} dx = \frac{1}{a} \ln|\sin ax| + C$$

73.
$$\int \cos^n ax \sin ax \, dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$$

74.
$$\int \frac{\sin ax}{\cos ax} dx = -\frac{1}{a} \ln|\cos ax| + C$$

75.
$$\int \sin^n ax \cos^m ax \, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax \, dx, \quad n \neq -m \quad (\text{reduces } \sin^n ax)$$

76.
$$\int \sin^n ax \cos^m ax \, dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax \, dx, \quad m \neq -n \quad (\text{reduces } \cos^m ax)$$

77.
$$\int \frac{dx}{b+c\sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right] + C, \quad b^2 > c^2$$

78.
$$\int \frac{dx}{b + c \sin ax} = \frac{-1}{a\sqrt{c^2 - b^2}} \ln \left| \frac{c + b \sin ax + \sqrt{c^2 - b^2} \cos ax}{b + c \sin ax} \right| + C, \quad b^2 < c^2$$

79.
$$\int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C$$

80.
$$\int \frac{dx}{1-\sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$$

81.
$$\int \frac{dx}{b + c \cos ax} = \frac{2}{a\sqrt{b^2 - c^2}} \tan^{-1} \left[\sqrt{\frac{b - c}{b + c}} \tan \frac{ax}{2} \right] + C, \quad b^2 > c^2$$

82.
$$\int \frac{dx}{b + c\cos ax} = \frac{1}{a\sqrt{c^2 - b^2}} \ln \left| \frac{c + b\cos ax + \sqrt{c^2 - b^2}\sin ax}{b + c\cos ax} \right| + C, \quad b^2 < c^2$$

$$83. \int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$$

$$84. \int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$$

$$85. \int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$$

$$86. \int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax + C$$

87.
$$\int x^n \sin ax \, dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$
88.
$$\int x^n \cos ax \, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

88.
$$\int x^n \cos ax \, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

89.
$$\int \tan ax \, dx = \frac{1}{a} \ln |\sec ax| + C$$

$$90. \int \cot ax \, dx = \frac{1}{a} \ln |\sin ax| + C$$

91.
$$\int \tan^2 ax \, dx = \frac{1}{a} \tan ax - x + C$$

92.
$$\int \cot^2 ax \, dx = -\frac{1}{a} \cot ax - x + C$$

93.
$$\int \tan^n ax \, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \, dx, \quad n \neq 0$$

93.
$$\int \tan^n ax \, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \, dx$$
, $n \neq 1$ **94.** $\int \cot^n ax \, dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax \, dx$, $n \neq 1$

95.
$$\int \sec ax \, dx = \frac{1}{a} \ln |\sec ax + \tan ax| + C$$

$$\mathbf{96.} \int \csc ax \, dx = -\frac{1}{a} \ln \left| \csc ax + \cot ax \right| + C$$

$$97. \int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

98.
$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

99.
$$\int \sec^n ax \, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx, \quad n \neq 1$$

100.
$$\int \csc^n ax \, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax \, dx, \quad n \neq 1$$

101.
$$\int \sec^n ax \tan ax \, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$$

102.
$$\int \csc^n ax \cot ax \, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$$

Inverse Trigonometric Forms

103.
$$\int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1 - a^2 x^2} + C$$

103.
$$\int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1 - a^2 x^2} + C$$
104.
$$\int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1 - a^2 x^2} + C$$

105.
$$\int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \ln (1 + a^2 x^2) + C$$

106.
$$\int x^n \sin^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1-a^2 x^2}}, \quad n \neq -1$$

107.
$$\int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1-a^2 x^2}}, \quad n \neq -1$$

108.
$$\int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{1 + a^2 x^2}, \quad n \neq -1$$

Exponential and Logarithmic Forms

109.
$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

110.
$$\int b^{ax} dx = \frac{1}{a} \frac{b^{ax}}{\ln b} + C, \quad b > 0, b \neq 1$$

111.
$$\int xe^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) + C$$

112.
$$\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

113.
$$\int x^n b^{ax} dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} dx, \quad b > 0, b \neq 1$$

114.
$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

115.
$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$$
 116. $\int \ln ax \, dx = x \ln ax - x + C$

116.
$$\int \ln ax \, dx = x \ln ax - x + C$$

117.
$$\int x^n (\ln ax)^m \, dx = \frac{x^{n+1} (\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx, \quad n \neq -1$$

118.
$$\int x^{-1} (\ln ax)^m dx = \frac{(\ln ax)^{m+1}}{m+1} + C, \quad m \neq -1$$
 119. $\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$

119.
$$\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$$

Forms Involving $\sqrt{2ax - x^2}$, a > 0

$$120. \int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1}\left(\frac{x - a}{a}\right) + C$$

121.
$$\int \sqrt{2ax - x^2} \ dx = \frac{x - a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x - a}{a} \right) + C$$

122.
$$\int \left(\sqrt{2ax - x^2}\right)^n dx = \frac{(x - a)\left(\sqrt{2ax - x^2}\right)^n}{n + 1} + \frac{na^2}{n + 1} \int \left(\sqrt{2ax - x^2}\right)^{n - 2} dx$$

123.
$$\int \frac{dx}{\left(\sqrt{2ax-x^2}\right)^n} = \frac{(x-a)\left(\sqrt{2ax-x^2}\right)^{2-n}}{(n-2)a^2} + \frac{n-3}{(n-2)a^2} \int \frac{dx}{\left(\sqrt{2ax-x^2}\right)^{n-2}}$$

124.
$$\int x\sqrt{2ax - x^2} \ dx = \frac{(x+a)(2x-3a)\sqrt{2ax-x^2}}{6} + \frac{a^3}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

125.
$$\int \frac{\sqrt{2ax - x^2}}{x} dx = \sqrt{2ax - x^2} + a \sin^{-1} \left(\frac{x - a}{a} \right) + C$$

126.
$$\int \frac{\sqrt{2ax - x^2}}{x^2} dx = -2\sqrt{\frac{2a - x}{x}} - \sin^{-1}\left(\frac{x - a}{a}\right) + C$$

127.
$$\int \frac{x \, dx}{\sqrt{2ax - x^2}} = a \sin^{-1} \left(\frac{x - a}{a} \right) - \sqrt{2ax - x^2} + C$$
 128.
$$\int \frac{dx}{x \sqrt{2ax - x^2}} = -\frac{1}{a} \sqrt{\frac{2a - x}{x}} + C$$

Hyperbolic Forms

$$129. \int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$$

$$130. \int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$$

131.
$$\int \sinh^2 ax \, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$$

133.
$$\int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, \quad n \neq 0$$

134.
$$\int \cosh^n ax \, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, \quad n \neq 0$$

135.
$$\int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$$

137.
$$\int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$$

139.
$$\int \tanh ax \, dx = \frac{1}{a} \ln \left(\cosh ax \right) + C$$

$$141. \int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C$$

143.
$$\int \tanh^n ax \, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax \, dx, \quad n \neq 1$$

144.
$$\int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax \, dx, \quad n \neq 1$$

145.
$$\int \operatorname{sech} ax \, dx = \frac{1}{a} \sin^{-1} (\tanh ax) + C$$

$$147. \int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C$$

149.
$$\int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx, \quad n \neq 1$$

150.
$$\int \operatorname{csch}^{n} ax \, dx = -\frac{\operatorname{csch}^{n-2} ax \operatorname{coth} ax}{(n-1)a} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax \, dx, \quad n \neq 1$$

151.
$$\int \operatorname{sech}^n ax \tanh ax \, dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$$
152.
$$\int \operatorname{csch}^n ax \coth ax \, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$$

153.
$$\int e^{ax} \sinh bx \, dx = \frac{e^{ax}}{2} \left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

154.
$$\int e^{ax} \cosh bx \, dx = \frac{e^{ax}}{2} \left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

Some Definite Integrals

155.
$$\int_0^\infty x^{n-1} e^{-x} dx = \Gamma(n) = (n-1)!, \quad n > 0$$
 156. $\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}, \quad a > 0$

$$\left(\frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot n} \cdot \frac{\pi}{2}\right)$$

$$\mathbf{157.} \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (n-1)}{2 \cdot 4 \cdot 6 \cdot \dots \cdot n} \cdot \frac{\pi}{2}, & \text{if } n \text{ is an even integer } \ge 2\\ \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (n-1)}{3 \cdot 5 \cdot 7 \cdot \dots \cdot n}, & \text{if } n \text{ is an odd integer } \ge 3 \end{cases}$$

135.
$$\int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$$
 136.
$$\int x \cosh ax \, dx = \frac{x}{a} \sinh ax - \frac{1}{a^2} \cosh ax + C$$

137.
$$\int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$$
138.
$$\int x^n \cosh ax \, dx = \frac{x^n}{a} \sinh ax - \frac{n}{a} \int x^{n-1} \sinh ax \, dx$$

140.
$$\int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C$$

$$142. \int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C$$

146.
$$\int \operatorname{csch} ax \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C$$

$$148. \int \operatorname{csch}^2 ax \, dx = -\frac{1}{a} \coth ax + C$$

52.
$$\int \operatorname{csch}^n ax \operatorname{coth} ax \, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$$

$$\int_{-\pi}^{\infty} e^{-ax^2} dx = 1 \sqrt{\pi}$$

if n is an even integer
$$> 2$$