

根數測驗

姓名: _____

得分: _____/50

1. (9分, 3@) 化簡以下根數:

a) $\sqrt{20}$

b) $\sqrt{147}$

c) $\sqrt{3125}$

2. (12分, 3@) 計算並化至最簡:

a) $\sqrt{20} + \sqrt{80} - \sqrt{125}$

b) $7\sqrt{3} - 2\sqrt{27} - \frac{\sqrt{243}}{9}$

c) $2\sqrt{8} + \sqrt{99} + 3\sqrt{11}$

d) $(1 + \sqrt{2})^3$

$$a) \frac{1}{\sqrt{3}}$$

$$b) \frac{2}{\sqrt{5} - \sqrt{3}}$$

$$c) \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$

$$d) \frac{1}{1 + \sqrt{3} + \sqrt{5}}$$

[illegible]

參考答案

1. (a)

$$\begin{aligned}\sqrt{20} &= \sqrt{2^2 \times 5} & (1M) \\ &= 2\sqrt{5} & (1M + 1A)\end{aligned}$$

(b)

$$\begin{aligned}\sqrt{147} &= \sqrt{3 \times 7^2} & (1M) \\ &= 7\sqrt{3} & (1M + 1A)\end{aligned}$$

(c)

$$\begin{aligned}\sqrt{3125} &= \sqrt{5^5} & (1M) \\ &= 25\sqrt{5} & (1M + 1A)\end{aligned}$$

2. (a)

$$\begin{aligned}\sqrt{20} + \sqrt{80} - \sqrt{125} &= 2\sqrt{5} + 4\sqrt{5} - 5\sqrt{5} & (1M) \\ &= \sqrt{5} & (1M + 1A)\end{aligned}$$

(b)

$$\begin{aligned}7\sqrt{3} - 2\sqrt{27} - \frac{\sqrt{243}}{9} &= 7\sqrt{3} - 6\sqrt{3} - \frac{9\sqrt{3}}{9} & (1M) \\ &= 0 & (1M + 1A)\end{aligned}$$

(c)

$$\begin{aligned}2\sqrt{8} + \sqrt{99} + 3\sqrt{11} &= 4\sqrt{2} + 3\sqrt{11} + 3\sqrt{11} & (1M) \\ &= 4\sqrt{2} + 6\sqrt{11} & (1M + 1A)\end{aligned}$$

(d)

$$\begin{aligned}(1 + \sqrt{2})^3 &= (1 + \sqrt{2})^2(1 + \sqrt{2}) \\ &= (3 + 2\sqrt{2})(1 + \sqrt{2}) & (1M) \\ &= 7 + 5\sqrt{2} & (1M + 1A)\end{aligned}$$

3. (a)

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad (1M)$$

$$= \frac{\sqrt{3}}{3} \quad (1M + 1A)$$

(b)

$$\frac{2}{\sqrt{5} - \sqrt{3}} = \frac{2}{\sqrt{5} - \sqrt{3}} \cdot \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \quad (1M)$$

$$= \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} \\ = \sqrt{5} + \sqrt{3} \quad (1M + 1A)$$

(c)

$$\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \cdot \frac{\sqrt{2} - 1}{\sqrt{2} - 1} \quad (1M)$$

$$= \frac{(\sqrt{2} - 1)^2}{2 - 1} \\ = 3 - 2\sqrt{2} \quad (1M + 1A)$$

(d)

$$\frac{1}{1 + \sqrt{3} + \sqrt{5}} = \frac{1}{(1 + \sqrt{3}) + \sqrt{5}} \cdot \frac{(1 + \sqrt{3}) - \sqrt{5}}{(1 + \sqrt{3}) - \sqrt{5}} \quad (1M)$$

$$= \frac{1 + \sqrt{3} - \sqrt{5}}{2\sqrt{3} - 1} \cdot \frac{2\sqrt{3} + 1}{2\sqrt{3} + 1} \quad (1M)$$

$$= \frac{7 + 3\sqrt{3} - \sqrt{5} - 2\sqrt{15}}{11} \quad (1M + 1A)$$

4. (a)

$$\sqrt{x} = \sqrt{2} - 1$$

$$x = (\sqrt{2} - 1)^2 \quad (1M)$$

$$= 3 - 2\sqrt{2} \quad (1M + 1A)$$

(b)

$$\sqrt{x^2 + 1} - x = 1$$

$$x^2 + 1 = (x + 1)^2 \quad (1M)$$

$$= x^2 + 2x + 1$$

$$2x = 0$$

$$x = 0 \quad (1M + 1A)$$

5. (a)

$$\frac{1}{\sqrt{a^2 + 1} - a} = \frac{1}{\sqrt{a^2 + 1} - a} \cdot \frac{\sqrt{a^2 + 1} + a}{\sqrt{a^2 + 1} + a} \quad (2M)$$

$$= \sqrt{a^2 + 1} + a \quad (1M + 1A)$$

(b)

$$p + \frac{1}{p} = \sqrt{8}$$

$$\sqrt{a^2 + 1} - a + \sqrt{a^2 + 1} + a = \sqrt{8} \quad (1M)$$

$$2\sqrt{a^2 + 1} = \sqrt{8}$$

$$4(a^2 + 1) = 8 \quad (1M)$$

$$a^2 + 1 = 2$$

$$a^2 = 1$$

$$a = 1 \text{ or } a = -1 \quad (1M + 2A)$$