Using coordinate system to construct a circle, we find that if a circle is centered at (h, k) with radius r, then by Pythagoras theorem (or distance formula), we have the following thought:

Let P be a moving point on the circumference of that circle, the distance between P and the center of the circle is fixed by radius r. Therefore, the equation of the circle (in fact circumference) is given by the following equation:

$$(x-h)^2+(y-k)^2=r^2$$

By expansion and rearrangement of terms of the equation, we have the equation becomes

$$x^2+y^2+Dx+Ex+F=0$$

where D, E, F are real coefficients computed by $D=-2h, E=-2k, F=h^2+k^2-r^2$.

Moreover, we call circles of the same center **concentric circles**.

Challange

Given a circle $C_0 : x^2+y^2=R^2$, where $R>0$. Let $P(h, k)$ be a moving point on C_0. We construct another circle C_P centered at P with radius r, where $0<r<R$. Let F and G be the closest point from the origin to C_P and the farthest point from the origin to C_P respectively. Denote the locus of F and G by C_F and C_G respectively.

1. Describe the geometric relationship between O, F and G.
2. Find the equation of straight line for OP in terms of h, k and R.
3. Find the coordinates of F and G in terms of h, k, R and r.
4. Prove that C_F and C_G are concentric circles.
5. Define the tangent lines of C_P passing through the origin to be L_1 and L_2. Find the area enclosed by L_1, L_2, C_F, C_G and C_P.