1. Let $f(x) = e^{\sin x}$. Compute $f'(\pi)$ from first principle.

2. Let $f(x) = \sin \pi x$.

   (a) Find $f'(0)$;

   (b) Suppose g_n is a sequence of functions defined by $g_1(x) = f(x)$ and $g_n(x) = f(g_{n-1}(x))$. Show that $g_n'(0) = \pi^n$ for all positive integer n.

3. Define $f(x) = x^2 + ax + b$, where a, b are constants from x. Show that the extrema of f must exist, and discuss whether it is maxima of minima of f.

4. Give a condition for which $f(x) = ax^3 + bx^2 + cx + d$ has no extrema, where a, b, c, d are real numbers and $a \neq 0$.

5. In this question we will introduce the Newton's method of finding x-intercept. Define $f(x)$ to be a differentiable (and immediately continuous) function with respect to x.

   (a) Prove that if f is a continuous function, and there are points a and b satisfy $f(a) > 0$ and $f(b) < 0$ (or vice versa), then there exists a point c between a and b such that $f(c) = 0$. This is called the intermediate value theorem. [Hint: Suppose f is non-zero at every point, then we can pick some point to show the left limit does not equal to the right limit, or the left limit and the right limit both equal to 0 but the value is not 0, conclude it is not continuous.]

   (b) Suppose that $f(c) \neq 0$ and $f'(c) \neq 0$.

      i. Find the tangent line at $(c, f(c))$.

      ii. Compute the x-intercept of the tangent line.

   (c) In reference to (b), we take the form of x-intercept of tangent line, and define the recurrence relation by

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

      i. Show that $|x_{n+1} - x_n| \leq |x_n - x_{n-1}|$ for all integer n when x_n are near to the point of intersection.

      ii. Show that if the sequence arrive at the desired x-intercept at x_k for some k, then $x_n = x_{n+1}$ for all $n \geq k$.

(d) Approximate π correct to 6 decimal place by finding the x-intercept of $f(x) = \sin x$, showing the first 3 steps of applying Newton's method started at $x = 3$.

Bonus Given a differentiable function f.

(a) Find the tangent line at $(t, f(t))$.

(b) Find the normal line at $(t, f(t))$.

(c) If a function g always satisfy $g'(x) = -\frac{1}{f'(x)}$ without constant term, then we may call it the **normal function of** f.

i. Show that if f has extrema, then no entirely continuous functions (continuous at every real number) can be a normal function of f.

ii. Show that $f(x) = e^x$ has normal function.

iii. Derive the locus of the intersection of tangent lines of $f(x) = e^x$ and $g(x) = -e^{-x}$.