1. Let $f(x) = e^{\sin x}$. Compute $f'(\pi)$ from first principle.

2. Let $f(x) = \sin \pi x$.

   (a) Find $f'(0)$;

   (b) Suppose g_n is a sequence of functions defined by $g_1(x) = f(x)$ and $g_n(x) = f(g_{n-1}(x))$. Show that $g_n'(0) = 1$ for all positive integer n.

3. Define $f(x) = x^2 + ax + b$, where a, b are constants from x. Show that the extrema of f must exist, and discuss whether it is maxima of minima of f.

4. Give a condition for which $f(x) = ax^3 + bx^2 + cx + d$ has no extrema, where a, b, c, d are real numbers and $a \neq 0$.

5. In this question we will introduce the Newton's method of finding x-intercept. Define $f(x)$ to be a differentiable (and immediately continuous) function with respect to x.

   (a) Prove that if f is a continuous function, and there are points a and b satisfy $f(a) > 0$ and $f(b) < 0$ (or vice versa), then there exists a point c between a and b such that $f(c) = 0$. This is called the intermediate value theorem.

   (b) Suppose that $f(c) \neq 0$ and $f'(c) \neq 0$.

      i. Find the tangent line at $(c, f(c))$.

      ii. Compute the x-intercept of the tangent line.

   (c) In reference to (b), we take the form of x-intercept of tangent line, and define the recurrence relation by

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

      i. Show that $|x_{n+1} - x_n| \leq |x_n - x_{n-1}|$ for all integer n.

      ii. Show that if the sequence arrive at the desired x-intercept at x_k for some k, then $x_n = x_{n+1}$ for all $n \geq k$.

   (d) Approximate π correct to 6 decimal place by finding the x-intercept of $f(x) = \sin x$, showing the step of applying Newton's method started at $x = 3$.

Bonus Given a differentiable function f.

(a) Find the tangent line at $(t, f(t))$.

(b) Find the normal line at $(t, f(t))$.

(c) If a function g always satisfy $g'(x) = -\frac{1}{f'(x)}$, then we may call it the **normal function of** f.

i. Show that if f has extrema, then no entirely continuous functions (continuous at every real number) can be a normal function of f.

ii. Show that $f(x) = e^x$ has normal function.

iii. Derive the locus of the intersection of tangent lines of f and g.