

PRACTICE PAPER
MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

Time allowed: 1.5 hours

Name: _____

Marks: _____/100

School: _____

Instructions

1. This paper must be answered in English.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. This paper is for **internal use** only.
5. All questions are collected from AL/CE/DSE past papers, reference site:
<https://www.dse.life/ppindex/m2/>

- $$\sum_{k=1}^n T_k = n[(n+1)!]$$

(6 marks)

[illegible]

2. (1988-HL-GEN MATHS #07(Modified)) Let

$$A_n = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n-1}n^2$$

and

$$B_n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

where n is a positive integer.

(a) Show, by mathematical induction, that $A_n = (-1)^{n-1}B_n$ for all positive integer n .

(b) Hence, or otherwise, find $\sum_{n=1}^{2m} A_n$ and $\sum_{n=1}^{2m+1} A_n$.

(8 marks)

[illegible]

[illegible]

- $(b+x)^9 = \sum_{k=0}^9 \mu_k x^k$, where a and b are constants. It is given that

[illegible]

[illegible]

[illegible]

6. (2018-DSE-MATH-EP(M2) #01) Let $f(x) = (x^2 - 1)e^x$. Express $f(1 + h)$ in terms of h . Hence, find $f'(1)$ from first principles. (6 marks)

[illegible]

7. (PP-DSE-MATH-EP(M2) #07) Let $f(x) = e^x(\sin x + \cos x)$.

(a) Find $f'(x)$ and $f''(x)$.

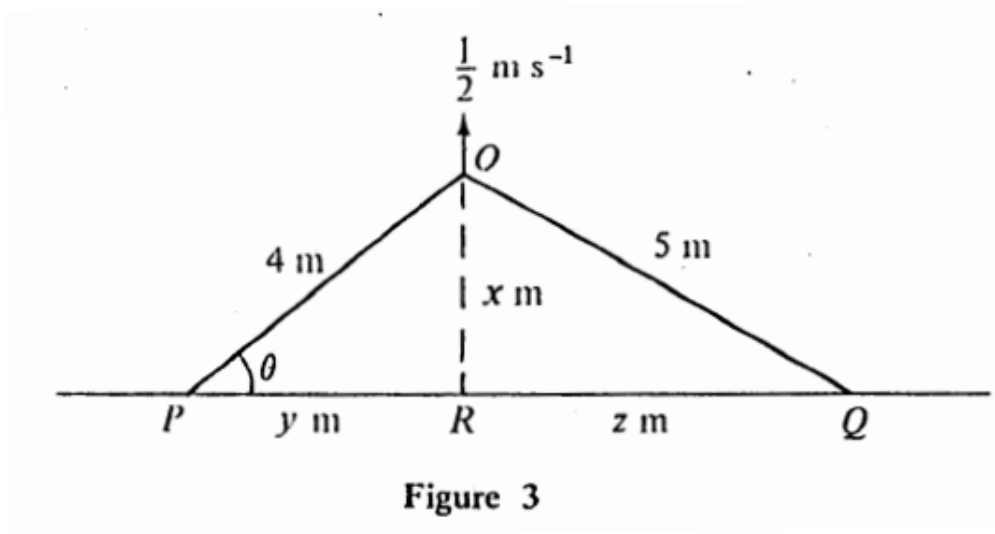
(b) Solve for x when $f''(x) - f'(x) + f(x) = 0$, where x is real number.

(8 marks)

8. (1996-CE-A MATH 1 #06) Find the equations of the two tangents to the curve $C : y = \frac{6}{x+1}$ which are parallel to the line $x + 6y + 10 = 0$. (7 marks)

[illegible]

9. (1994-CE-A MATH 1 #12)



In figure 3, two rods OP and OQ are hinged at O . The lengths of OP and OQ are 4 m and 5 m respectively. The end O is pushed upwards at a constant rate of $\frac{1}{2} \text{ m s}^{-1}$ along a fixed vertical axis, and the ends P and Q move along a horizontal rail. R is the projection of O on the rail. At time t seconds, $OR = x$ m and $\angle OPQ = \theta$ where $0 < \theta < \frac{\pi}{2}$.

- (a)
 - i. Express x in terms of θ .
 - ii. Hence find the rate of change of θ with respect to t in terms of θ .
- (b) Let $PR = y$ m, $RQ = z$ m.
 - i. Express $\frac{dy}{dt}$ and $\frac{dz}{dt}$ in terms of θ .
 - ii. Hence find the rate of change of PQ with respect to t when $\theta = \frac{\pi}{6}$, giving your answer correct to 3 significant figures.
- (c)
 - i. Find the value of θ such that the area of $\triangle OPR$ is a maximum.
 - ii. By considering the value of $\angle OQR$, find the value of θ such that the area of $\triangle ORQ$ is a maximum, giving your answer correct to 3 significant figures.

(23 marks)

[illegible]

[illegible]

[illegible]