PRACTICE PAPER
MATHEMTICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

Time allowed: 1.5 hours

Name:_____ Marks:_____/100

School:_____

Instructions

1. This paper must be answered in English.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. This paper is for **internal use** only.
5. All questions are collected from AL/CE/DSE past papers, reference site: https://www.dse.life/ppindex/m2/

1. Prove, by mathematical induction, that for any positive integer n,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

(10 marks)

2. (2012-DSE-MATH-EP(M2) #03)Prove, by mathematical induction, that

$$1 \times 2 + 2 \times 5 + \cdots + n(3n-1) = n^2(n+1)$$

for all positive integers n. (10 marks)

3. (2004-CE-A MATH #02)

   (a) Expand $(1 + 2x)^n$ in ascending powers of x up to the term x^3, where n is a positive integer.

   (b) In the expansion of $(x - \frac{3}{x})^2(1 + 2x)^n$, the constant term is 210. Find the value of n.

(12 marks)

4. (1990-CE-A MATH 2 #06(a)) If $\cos\theta + \sqrt{3}\sin\theta = r\cos(\theta - \alpha)$, where $r > 0$ and $0° \leq \alpha \leq 90°$. Find r and α. (8 marks)

5. (2003-CE-A MATH #10) Given two acute angles α and β. Show that

$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\frac{\alpha+\beta}{2}$$

If $3\sin\alpha - 4\cos\alpha = 4\cos\beta - 3\sin\beta$, find the value of $\tan\alpha + \beta$. (16 marks)

6. (1996-CE-A MATH 1 #02) Find $\frac{d}{dx}(x^2)$ from first principles. (8 marks)

7. (2002-CE-A MATH #03) Let $x \sin y = 2002$. Find $\frac{dy}{dx}$. (8 marks)

8. (2015-DSE-MATH-EP(M2) #02) Let $y = x\sin x + \cos x$.

   (a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

   (b) Let k be a constant such that $x\dfrac{d^2y}{dx^2} + k\dfrac{dy}{dx} + xy = 0$ for all real values of x. Find the value of k.

(14 marks)

9. (2010-CE-A MATH #10) It is given that P is a point on the curve $C : y = x^3$. If the y-intercept of the tangent line L to C at P is -16, find the equation of L. (14 marks)