PRACTICE PAPER
MATHEMTICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

Time allowed: 1.5 hours

Name:_____ Marks:_____/100

School:_____

Instructions

1. This paper must be answered in English.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. This paper is for **internal use** only.
5. All questions are collected from AL/CE/DSE past papers, reference site: https://www.dse.life/ppindex/m2/

1. (1990-HL-GEN MATHS #05)

   (a) Prove by mathematical induction that for any positive integer n,

   $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

   (b) Find $1^3 - 2^3 + 3^3 - 4^3 + \cdots + (-1)^{r+1}r^3 + \cdots - (2n)^3$.

(13 marks)

2. (2010-CE-A MATH #05) The sum of the coefficients of x and x^2 in the expansion of $(1+4x)^n$ is 180, where n is a positive integer. Find the value of n and the coefficient of x^3. (10 marks)

3. (2014-DSE-MATH-EP(M2) #01) In the expansion of $(1-4x)^2(1+x)^n$, the coefficient of x is 1.

   (a) Find the value of n.

   (b) Find the coefficient of x^2.

(9 marks)

4. (2015-DSE-MATH-EP(M2) #07)

   (a) Prove that $\sin^2 x \cos^2 x = \dfrac{1 - \cos 4x}{8}$.

   (b) Let $f(x) = \sin^4 x + \cos^4 x$.

      i. Express $f(x)$ in the form of $A \cos Bx + C$, where A, B and C are constants.

      ii. Solve the equation $8f(x) = 7$, where $0 \leq x \leq \frac{\pi}{2}$.

(12 marks)

5. (2015-DSE-MATH-EP(M2) #08)

   (a) Using mathematical induction, prove that

$$\sin\frac{x}{2}\sum_{k=1}^{n}\cos kx = \sin\frac{nx}{2}\cos\frac{(n+1)x}{2}$$

   for all positive integers n.

   (b) Using (a), evaluate $\displaystyle\sum_{k=1}^{567}\cos\frac{k\pi}{7}$.

(13 marks)

6. (2016-DSE-MATH-EP(M2) #02)Prove that $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+h}} = \frac{h}{(x+h)\sqrt{x} + x\sqrt{x+h}}$. Hence, find $\frac{d}{dx}\sqrt{\frac{3}{x}}$ from first principles. (10 marks)

7. (1994-CE-A MATH 1 #04) Let $y = \tan\dfrac{1}{x}$.

   (a) Show that $x^2\dfrac{dy}{dx} + (y^2 + 1) = 0$.

   (b) Hence show that $\dfrac{d^2y}{dx^2} + \dfrac{2(x+y)}{x^2}\dfrac{dy}{dx} = 0$.

(11 marks)

8. (2004-CE-A MATH #09(Modified)) Let $P(a, b)$ be a point on the curve $C : y = x^3$ such that the tangent to C at P passes through the point $(0, 2)$.

   (a) Show that $b = 3a^3 + 2$.

   (b) Find the value of a and b.

(11 marks)

9. (1992-CE-A MATH 1 #05) The curve $(x-2)(y^2+3) = -8$ cuts the y-axis at two points. Find

   (a) the coordinates of the two points.

   (b) the slope of the tangent to the curve at each of the two points.

   (11 marks)