

1. (a)

$$O = \frac{f(x) - f(-x)}{2} \qquad E = \frac{f(x) + f(-x)}{2}$$

(b)

$$i) \sum_{k \text{ 偶數}} C_k^n O^{n-k} E^k \qquad ii) \sum_{k \text{ 奇數}} C_k^n O^{n-k} E^k$$

2.

$$\begin{aligned} (1+x)^n(1+x) &\geq (1+nx)(1+x) \\ &= 1 + (1+n)x + nx^2 \\ &\geq 1 + (n+1)x \end{aligned}$$

3. (a)

$$\begin{aligned} S_1 &= \sum_{i=1}^n i \\ \binom{n}{n-1, 1, 0} a &= \frac{n(n+1)}{2} \\ na &= \frac{n(n+1)}{2} \\ a &= \frac{(n+1)}{2} \\ S_2 &= \sum_{i=1}^n i^2 \\ \binom{n}{n-2, 2, 0} a^2 + \binom{n}{n-1, 0, 1} b &= \frac{n(n+1)(2n+1)}{6} \\ \frac{n(n-1)}{2} a^2 + nb &= \frac{n(n+1)(2n+1)}{6} \\ b &= \frac{(n+1)(-3n^2 + 8n + 7)}{24} \end{aligned}$$

(b)

$$\begin{aligned} a : b &= 3 : 1 \\ \frac{-3n^2 + 8n + 7}{12} &= \frac{1}{3} \\ -3n^2 + 8n + 3 &= 0 \\ (3n+1)(n-3) &= 0 \\ n &= 3 \end{aligned}$$