Test on Surds

Name:_____ Score:_____/50

1. (9marks, 3@) Simplify the following surds:

$a)\sqrt{20}$ $b)\sqrt{147}$ $c)\sqrt{3125}$

2. (12marks, 3@) Compute the following and present the answer in simplest form:

$a)\sqrt{20}+\sqrt{80}-\sqrt{125}$ $b)7\sqrt{3}-2\sqrt{27}-\dfrac{\sqrt{243}}{9}$

$c)2\sqrt{8}+\sqrt{99}+3\sqrt{11}$ $d)(1+\sqrt{2})^3$

3. (13marks, 3@ for (a)-(c), 4@ for (d)) Rationalize the following expressions:

$$a)\frac{1}{\sqrt{3}} \qquad b)\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$c)\frac{\sqrt{2}-1}{\sqrt{2}+1} \qquad d)\frac{1}{1+\sqrt{3}+\sqrt{5}}$$

4. (7marks) Solve x for the following equations:

   (a) (3marks) $\sqrt{x} = \sqrt{2} - 1$.

   (b) (4marks) $\sqrt{x^2 + 1} - x = 1$.

5. (9marks)Let $p = \sqrt{a^2+1} - a$.

   (a) (4marks) Rationalize $\dfrac{1}{\sqrt{a^2+1} - a}$.

   (b) (5marks) Solve a in the equation $p + \dfrac{1}{p} = \sqrt{8}$.

Suggested Solution

1. (a)

$$\begin{aligned}\sqrt{20} &= \sqrt{2^2 \times 5} && (1M)\\ &= 2\sqrt{5} && (1M+1A)\end{aligned}$$

(b)

$$\begin{aligned}\sqrt{147} &= \sqrt{3 \times 7^2} && (1M)\\ &= 7\sqrt{3} && (1M+1A)\end{aligned}$$

(c)

$$\begin{aligned}\sqrt{3125} &= \sqrt{5^5} && (1M)\\ &= 25\sqrt{5} && (1M+1A)\end{aligned}$$

2. (a)

$$\begin{aligned}\sqrt{20}+\sqrt{80}-\sqrt{125} &= 2\sqrt{5}+4\sqrt{5}-5\sqrt{5} && (1M)\\ &= \sqrt{5} && (1M+1A)\end{aligned}$$

(b)

$$\begin{aligned}7\sqrt{3}-2\sqrt{27}-\frac{\sqrt{243}}{9} &= 7\sqrt{3}-6\sqrt{3}-\frac{9\sqrt{3}}{9} && (1M)\\ &= 0 && (1M+1A)\end{aligned}$$

(c)

$$\begin{aligned}2\sqrt{8}+\sqrt{99}+3\sqrt{11} &= 4\sqrt{2}+3\sqrt{11}+3\sqrt{11} && (1M)\\ &= 4\sqrt{2}+6\sqrt{11} && (1M+1A)\end{aligned}$$

(d)

$$\begin{aligned}(1+\sqrt{2})^3 &= (1+\sqrt{2})^2(1+\sqrt{2})\\ &= (3+2\sqrt{2})(1+\sqrt{2}) && (1M)\\ &= 7+5\sqrt{2} && (1M+1A)\end{aligned}$$

3. (a)

$$\begin{aligned}\frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} && (1M)\\ &= \frac{\sqrt{3}}{3} && (1M+1A)\end{aligned}$$

(b)

$$\begin{aligned}\frac{2}{\sqrt{5}-\sqrt{3}} &= \frac{2}{\sqrt{5}-\sqrt{3}} \cdot \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} && (1M)\\ &= \frac{2(\sqrt{5}+\sqrt{3})}{5-3} \\ &= \sqrt{5}+\sqrt{3} && (1M+1A)\end{aligned}$$

(c)

$$\begin{aligned}\frac{\sqrt{2}-1}{\sqrt{2}+1} &= \frac{\sqrt{2}-1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} && (1M)\\ &= \frac{(\sqrt{2}-1)^2}{2-1} \\ &= 3-2\sqrt{2} && (1M+1A)\end{aligned}$$

(d)

$$\begin{aligned}\frac{1}{1+\sqrt{3}+\sqrt{5}} &= \frac{1}{(1+\sqrt{3})+\sqrt{5}} \cdot \frac{(1+\sqrt{3})-\sqrt{5}}{(1+\sqrt{3})-\sqrt{5}} && (1M)\\ &= \frac{1+\sqrt{3}-\sqrt{5}}{2\sqrt{3}-1} \cdot \frac{2\sqrt{3}+1}{2\sqrt{3}+1} && (1M)\\ &= \frac{7+3\sqrt{3}-\sqrt{5}-2\sqrt{15}}{11} && (1M+1A)\end{aligned}$$

4. (a)

$$\begin{aligned}\sqrt{x} &= \sqrt{2}-1 \\ x &= (\sqrt{2}-1)^2 && (1M)\\ &= 3-2\sqrt{2} && (1M+1A)\end{aligned}$$

(b)

$$\begin{aligned}
\sqrt{x^2+1}-x &= 1 \\
x^2+1 &= (x+1)^2 && (1M) \\
&= x^2+2x+1 \\
2x &= 0 \\
x &= 0 && (1M+1A)
\end{aligned}$$

5. (a)

$$\begin{aligned}
\frac{1}{\sqrt{a^2+1}-a} &= \frac{1}{\sqrt{a^2+1}-a}\cdot\frac{\sqrt{a^2+1}+a}{\sqrt{a^2+1}+a} && (2M) \\
&= \sqrt{a^2+1}+a && (1M+1A)
\end{aligned}$$

(b)

$$\begin{aligned}
p+\frac{1}{p} &= \sqrt{8} \\
\sqrt{a^2+1}-a+\sqrt{a^2+1}+a &= \sqrt{8} && (1M) \\
2\sqrt{a^2+1} &= \sqrt{8} \\
4(a^2+1) &= 8 && (1M) \\
a^2+1 &= 2 \\
a^2 &= 1 \\
a = 1 \text{ or } a &= -1 && (1M+2A)
\end{aligned}$$