Abstract

It is quite straight forward to think about straight line and it seems unnecessary to discuss what does it mean by a straight line, as what we could relate when the word 'straight forward' appears, however, a straight line is not as trivial as a word phrase did in the study of Mathematics. I will introduce some interesting pure mathematical concepts in relation to straight line. It might sounds like a word game to realize such meaning and their importance to the discussion, but it worths defining such words to unsieze the mystery of Geometry.

1 Topological Space

A **topological space** is a basic setup for discussion in pure mathematics. For this discussion I hope we all have included the knowledge about Mathematical sets; otherwise, it is too young to debate.

Definition 1.1 (Topology). *A **topology** $\mathcal{T}_X$ on a set X is a collection of subsets that satisfies the following properties:*

- $\emptyset, X \in \mathcal{T}_X$*;*
- $\bigcup_{\alpha\in I} A_\alpha \in \mathcal{T}_X$ *if* $A_\alpha \in \mathcal{T}_X$ *with arbitrary index set* I*;*
- $\bigcap_{\alpha\in I} A_\alpha \in \mathcal{T}_X$ *for* $A_\alpha \in \mathcal{T}_X$ *with a finite index set* I*.*

A topology can be thought of as a structure presetting what sets are observable. The word 'topology' is a composite word formed by 'top' and 'logic', what I would like to interpret as 'a logic of topping sets up'. Therefore we have many works on sets.

If $A \in \mathcal{T}_X$ we shall call A an **open set** of the topology; and if $B \subset X$ and $B^c \in \mathcal{T}_X$, we shall call B a **closed set** of the topology. It is quite absurd to name the sets open in a topology as there is a similar term called open interval in usual discussion, which was puzzling when I first heard the term. We shall understand it in the following way: An **open set** of a topology is a set that is *directly observable* through the topology, and a closed set is *indirectly observable* through the topology, in which you can observe it by flipping it (e.g. taking its complement). If a set cannot be observed using any strategies, i.e. not related to the topology, we shall call it **neither open nor closed**; and if both the set and the complement of the set are able to be observed through the topology directly, we shall call it **both open and closed**.

To be simple, it is like if the set is opened to you, then it is an open set, vice versa.

Definition 1.2 (Topological Space)**.** *A* ***topological space*** *is a pair* $(X, \mathcal{T}_X)$ *consisting of a set* X *and a topology* $\mathcal{T}_X$ *on* X*.*

Example (Trivial topological space)**.** *Let* X *be a non-empty set and define* $\mathcal{T}_X = \{\emptyset, X\}$*. This is a minimal topology on* X*.*

Prove that trivial topology is indeed a topology.

To prove the case we need to verify all requirements. It is clear that $\emptyset, X \in \mathcal{T}_X$ by definition. We also find $\emptyset \cap X = \emptyset \in \mathcal{T}_X$ and $\emptyset \cup X = X \in \mathcal{T}_X$. The requirement is then satisfied. □

The achievement here is we have built a basic space contains only a structure, describing the containment of sets and objects. Could we consider any lines here? Not yet. We shall have no idea on lines if we are not describing geometrical objects.

2 Metric Space

3 Euclidean and Non-Euclidean Geometry

4 Curves

5 Curvature

6 Straight line