PRACTICE QUESTIONS
MATHEMATICS Compulsory Part
Question-Answer Book

Instructions

1. This paper must be answered in English.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. This paper is for **internal use** only.
5. All questions are collected from AL/CE/DSE past papers, reference site: https://www.dse.life/ppindex/m2/

1. Let $f(x) = \frac{1}{2}x - \frac{1}{144}x^2 - 6$. Using the method of completing the sqaure, find the coordinates of the vertex of the graph of $y = f(x)$.

2. Let $C(k)$ be the curve $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$, where $k \neq -1$.

   (a) If $C(k)$ cuts the x-axis at two points at P and Q, and $PQ = 1$, find the value(s) of k.

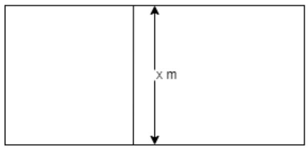
   (b) Find the range of values of k such that $C(k)$ does not cut the x-axis.

   (c) i. Find the points of intersection of the curves $C(1)$ and $C(2)$.

   ii. Show that $C(k)$ passes through the two points in (c)(i) fro all values of k.

3. (a) Let $f(x) = 36x - x^2$. Using the method of completing the sqaure, find the coordinates of the vertex of the graph of $y = f(x)$.

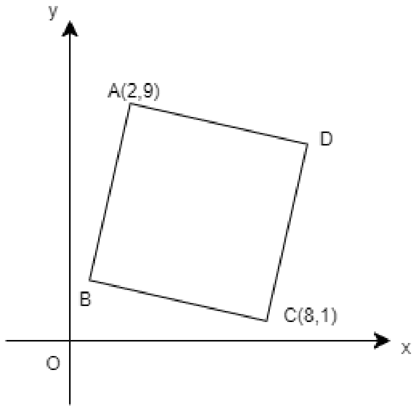
(b) The length of a piece of string is 108m. A guard cuts the string into two pieces. One piece is used to enclose a rectangular restricted zone of area A m^2. The other piece is used to divide this restricted zone into two rectangular regions as shown in the figure.



i. Express A in terms of x.

ii. The guard claims that the area of this restricted zone can be greater than 500 m^2. Do you agree? Explain your answer.

4. In the figure, $ABCD$ is a rhombus. The diagonals AC and BD cuts at E.



(a) Find

i. the coordinates of E,

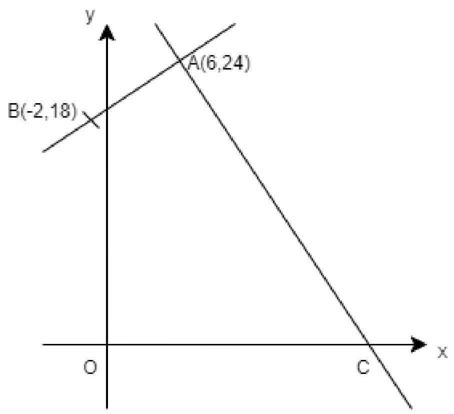
ii. the equation of BD.

(b) It is given that the equation of AD is $x + 7y - 65 = 0$. Find

i. the equation of BC,

ii. the length of AB.

5. In the figure, the straight line passing through A and B is perpendicular to the straight line passing through A and C, where C is a point lying on the x-axis.



(a) Find the equation of the straigh line passing through A and B.

(b) Find the coordinates of C.

(c) Find the area of $\triangle ABC$.

(d) A straight line passing through A and the line segment BC at D such that the area of $\triangle ABD$ is 90 square units. Let $BD : DC = r : 1$. Find the value of r.

6. (a) Factorize $a^4 - 16$ and $a^3 - 8$.
   (b) Find the L.C.M. of $a^4 - 16$ and $a^3 - 8$.

7. Factorize

   (a) $m^2 + 12mn + 36n^2$.
   (b) $m^2 + 12mn + 36n^2 - s^2$.

8. Factorize

   (a) $x^3 + x^2y - 7x^2$.

   (b) $x^3 + x^2y - 7x^2 - x - y + 7$.

9. Factorize

   (a) $4m^2 - 9$.

   (b) $2m^2n + 7mn - 15n$.

   (c) $4m^2 - 9 - 2m^2n - 7mn + 15n$

10. It is given that $f(x) = 2x^2 + ax + b$.

    (a) If $f(x)$ is divided by $(x - 1)$, the remainder is -5. If $f(x)$ is divided by $(x + 2)$, the remainder is 4. Find the values of a and b.

    (b) If $f(x) = 0$, find the values of x.

11. If $3x^2 - kx - 2$ is divisible by $x - k$, find the two values of k.

12. (a) Find the quotient when $5x^3 + 12x^2 - 9x - 7$ is divided by $x^2 + 2x - 3$.

(b) Let $g(x) = (5x^3 + 12x^2 - 9x - 7) - (ax + b)$, where a and b are constants. It is given that $g(x)$ is divisible by $x^2 + 2x - 3$.

i. write down the values of a and b.

ii. Solve the equation when $g(x) = 0$.

13. Simplify and express the following with positive indices

    (a) $\dfrac{x^3y^2}{x^{-3}y}$.

    (b) $x(\dfrac{x^{-1}}{y^2})^{-3}$.

    (c) $\dfrac{(m^5n^{-2})^6}{m^4n^{-3}}$.

14. Simplify $\sqrt{\dfrac{3^{5k+2}}{27^k}}$.

15. Simplify $\dfrac{\log\left(a^2\right)+\log\left(b^4\right)}{\log(ab^2)}$, where $a,b>0$.

16. Let $\log 2 = x, \log 3 = y$. Express the following in terms of x and y.

    (a) $\log 18$.

    (b) $\log 15$.

    (c) $\log \sqrt{12}$.

17. Solve the following without using calculator:

    (a) $3^x = \dfrac{1}{\sqrt{27}}$;

    (b) $\log x + 2\log 4 = \log 48$.

18. A researcher defined Scale A and Scale B to represent the magnitude of an explosion as shown in the table:

Scale	Formula
A	$M = \log_4 E$
B	$N = \log_8 E$

It is given that M and N are the magnitudes of an explosion on Scale A and Scale B respectively, while E is the relative energy released by the explosion. If the magnitude of an explosion is 6.4 on Scale B, find the magnitude of the explosion on Scale A.

19. Let a and b be constants. Denote the graph of $y = a + \log_b x$ by G. The x-intercept of G is 9 and G passes through the point $(243, 3)$. Express x in terms of y.

20. Solve the following:

    (a) $\begin{cases} 4^{x-y} = 4 \\ 4^{x+y} = 16 \end{cases}$, find x and y.

    (b) $3^{2x} + 3^x - 2 = 0$, find x.

    (c) $\log_3(x-3) + \log_3(x+3) = 3$, find x.

21. If $2\log_{10} x - \log_{10} y = 0$. Show that $y = x^2$.

22. Solve the following equations:

    (a) $1 - 2x = \sqrt{2 - x}$.

    (b) $x - \sqrt{x + 1} = 5$.

    (c) $x - 5\sqrt{x} - 6 = 0$.

23. Find the range of values of k for which the equation $2x^2 + x + 5 = k(x+1)^2$ has no real roots.

24. The quadratic equations $x^2 - 6x + 2k = 0$ and $x^2 - 5x + k$ have a common root α. (i.e. α is a root of both equations.) Show that $\alpha = k$ and hence find the value(s) of k.

25. Let α and β be the roots of $x^2 + kx + 1 = 0$, where k is a constant.

    (a) Find, in terms of k,

        i. $(\alpha + 2) + (\beta + 2)$,

        ii. $(\alpha + 2)(\beta + 2)$.

    (b) Suppose $\alpha + 2$ and $\beta + 2$ are roots of $x^2 + px + q = 0$, where p and q are constants. Find p and q in terms of k.

26. If $\dfrac{1}{m}+\dfrac{1}{n}=\dfrac{1}{a}$ and $m+n=b$, express the following in terms of a and b

    (a) mn,

    (b) m^2+n^2.

27. Suppose α andn β are roots of the equation $kx^2-4x+2k=0$, where k $(k\neq 0)$ is a constant. Express the following in terms of k:

    (a) $\alpha^2+\beta^2$,

    (b) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$.

28. Express $\dfrac{1}{1+2i}$ in the form of $a+bi$, where a and b are real numbers.

29. If $a:b=3:4$ and $a:c=2:5$, find

(a) $a:b:c$,

(b) the value of $\dfrac{ac}{a^2+b^2}$.

30. In a playground, the ratio of number of adults to the number of children is 13 : 6. If 9 adults and 24 children enter the playground, then the ratio of the number of adults to the number of children is 8 : 7. Find the original number of adults in the playground.

31. It is given that z varies directly as x^2 and inversely as y. If $x = 1$ and $y = 2$, then $z = 3$. Find z when $x = 2$ and $y = 3$.

32. A variable quantity y is the sum of two parts. The first part varies directly as another variable x, while the second part varies directly as x^2. When $x = 1$, $y = -5$; when $x = 2$, $y = -8$.

    (a) Express y in terms of x.

    (b) Hence, find the value of y when $x = 6$.

33. In a factory, the production cost of a carpet of perimeter s metres is $\$C$. It is given that C is a sum of two parts, one part varies as s and the second part varies as the square of s. When $s = 2$, $C = 356$; when $s = 5$, $C = 1250$.

    (a) Find the production cost of a carpet of perimeter 6 metres.

    (b) If the production cost of a carpet is \$539, find the perimeter of the carpet.

34. It is given that $h(x)$ is partly constant and partly varies as x. Suppose that $h(-2) = -96$ and $h(5) = 72$.

(a) Find $h(x)$.

(b) Solve the equation $h(x) = 3x^2$.

35. Simplify the following:

(a) $\dfrac{1-\cos^2 x}{\sin x}$.

(b) $\dfrac{\sin(180^\circ-\theta)}{\sin(90^\circ+\theta)}$.

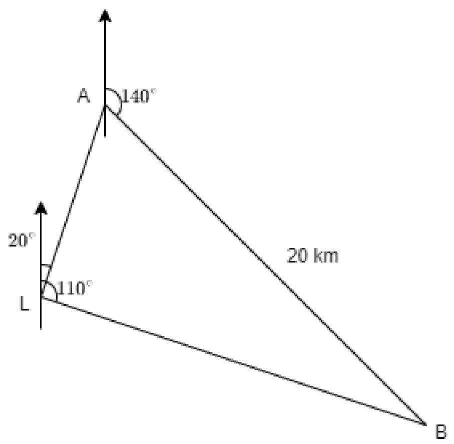
(c) $\sin^2(180^\circ-\phi)+\sin^2(270^\circ+\phi)$.

36. Solve the following with $0^\circ \le \theta < 360^\circ$. Give your answer in 3 significant figures if needed.

(a) $\sin^2\theta + 7\sin\theta = 5\cos^2\theta$.

(b) $\sin^2\theta - 3\cos\theta - 1 = 0$.

37. In the figure, the bearings of two ships A and B from a light house L are 020° and 110° respectively. B is 20 km and at a bearing of 140° from A.

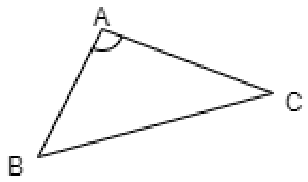


Find

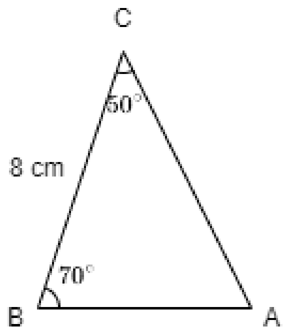
(a) the distance of L from B,

(b) the bearing of L from B.

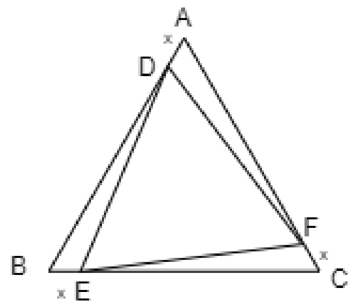
38. In the figure, $AB = 4$, $AC = 5$ and $BC = 7$. Calculate $\angle A$ to the nearest degree.



39. In the figure, find AB and the area of $\triangle ABC$.



40. In the figure, ABC is an equilateral triangle. $AB = 2$. D, E, F are points on AB, BC, CA respectively such that $AD = BE = CF = x$.



(a) By using the cosine formula or otherwise, express DE^2 in terms of x.

(b) Show that the area of $\triangle DEF = \dfrac{\sqrt{3}}{4}(3x^2 - 6x + 4)$.