PRACTICE PAPER
MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

Time allowed: 1.5 hours

Name:____________________ Marks:____________________/100

School:____

Instructions

1. This paper must be answered in English.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. This paper is for **internal use** only.
5. All questions are collected from AL/CE/DSE past papers, reference site: https://www.dse.life/ppindex/m2/

1. (1997-CE-A MATH 2 #07(Modified)) Let $T_n = (n^2+1)(n!)$ for any positive integer n. Prove, by mathematical induction, that

$$\sum_{k=1}^{n} T_k = n[(n+1)!]$$

for any positive integer n. (6 marks)

2. (1988-HL-GEN MATHS #07(Modified)) Let

$$A_n = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n-1}n^2$$

and

$$B_n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

where n is a positive integer.

(a) Show, by mathematical induction, that $A_n = (-1)^{n-1}B_n$ for all positive integer n.

(b) Hence, or otherwise, find $\sum_{n=1}^{2m} A_n$ and $\sum_{n=1}^{2m+1} A_n$.

(8 marks)

3. (2017-DSE-MATH-EP(M2) #02) Let $(1+ax)^8 = \sum_{k=0}^{8} \lambda_k x^k$ and $(b+x)^9 = \sum_{k=0}^{9} \mu_k x^k$, where a and b are constants. It is given that $\lambda_2 : \mu_7 = 7 : 4$ and $\lambda_1 + \mu_8 + 6 = 0$. Find a. (6 marks)

4. (1989-HL-GEN MATHS #05(Modified))

   (a) Find the solution of $\sin x - \sin 2x + \sin 3x = 0$ for $0 < x < 2\pi$.

   (b) Let $f(\theta) = \sin 2\theta + \sin\theta + \cos\theta$.

      i. Express $f(\theta)$ in terms of p, where $p = \sin\theta + \cos\theta$.

      ii. Using (i) and the method of completing the square, find the smallest value of $f(\theta)$. For $0 < \theta < \pi$, find also the value of θ such that $f(\theta)$ attains its smallest value.

(18 marks)

5. (1984-HL GEN MATHS #05(Modified))

   (a) Express $\cot 4\theta$ in terms of $\cot\theta$. Hence solve the equation $x^4 - 4x^3 - 6x^2 + 4x + 1 = 0$. (Give your answers in terms of π.)

   (b) i. If $\cos\theta - \cos\phi = a$ and $\sin\theta - \sin\phi = b$ ($b \neq 0$), show that

   $$\frac{1}{2}(2 - a^2 - b^2) = \cos\theta - \phi \text{ and } \frac{-a}{b} = \tan\frac{\theta+\phi}{2}.$$

   ii. Solve the system of equations

   $$\begin{cases} \cos\theta - \cos\phi = 1 \\ \sin\theta - \sin\phi = \sqrt{3} \end{cases}$$

   where $0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq 2\pi$.

(18 marks)

6. (2018-DSE-MATH-EP(M2) #01) Let $f(x) = (x^2 - 1)e^x$. Express $f(1+h)$ in terms of h. Hence, find $f'(1)$ from first principles. (6 marks)

7. (PP-DSE-MATH-EP(M2) #07) Let $f(x) = e^x(\sin x + \cos x)$.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) Solve for x when $f''(x) - f'(x) + f(x) = 0$, where x is real number.

(8 marks)

8. (1996-CE-A MATH 1 #06) Find the equations of the two tangents to the curve $C : y = \dfrac{6}{x+1}$ which are parallel to the line $x + 6y + 10 = 0$. (7 marks)

9. (1994-CE-A MATH 1 #12)

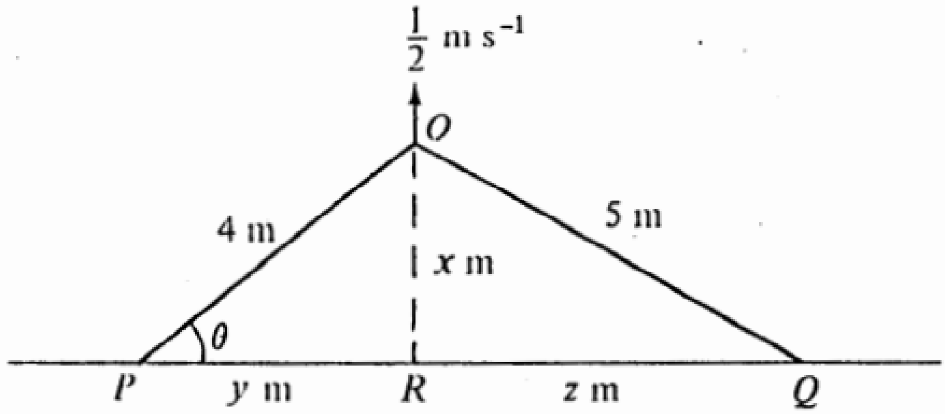

Figure 3

In figure 3, two rods OP and OQ are hinged at O. The lengths of OP and OQ are 4 m and 5 m respectively. The end O is pushed upwards at a constant rate of $\frac{1}{2}$ ms^{-1} along a fixed vertical axis, and the ends P and Q move along a horizontal rail. R is the projection of O on the rail. At time t seconds, $OR = x$ m and $\angle OPQ = \theta$ where $0 < \theta < \frac{\pi}{2}$.

(a) i. Express x in terms of θ.

ii. Hence find the rate of change of θ with respect to t in terms of θ.

(b) Let $PR = y$ m,$RQ = z$ m.

i. Express $\frac{dy}{dt}$ and $\frac{dz}{dt}$ in terms of θ.

ii. Hence find the rate of change of PQ with respect to t when $\theta = \frac{\pi}{6}$, giving your answer correct to 3 significant figures.

(c) i. Find the value of θ such that the area of $\triangle OPR$ is a maximum.

ii. By considering the value of $\angle OQR$, find the value of θ such that the area of $\triangle ORQ$ is a maximum, giving your answer correct to 3 significant figures.

(23 marks)