HKDSE MOCK EXAM PAPER
MATHEMATICS Compulsory Part
Question-Answer Book
Set 1

Time allowed: 2 hours 15 minutes

Name:____________________ Marks:____________________/105

Instructions

1. This paper must be answered in English.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. This paper is for **internal use** only.
5. All questions are constructed by Mok Owen.
6. The mock paper is composed of 3 parts, including Section A(1), Section A(2) and Section B. Each part consist of 35 marks each.

Section A(1) (35 marks)

1. Simplify $\dfrac{(m^3n^{-2})^3}{(m^{-1}n^7)^{-2}}$ and express your answer with positive indices. (3 marks)

2. Make a the subject of the formula $\dfrac{a+1}{a-1} = \dfrac{b+c}{d-c}$. (3 marks)

3. Factorize

   (a) $4x^2 + 4xy + y^2$,

   (b) $12x^2 + xz + z^2$,

   (c) $(4x^2 + 4xy + y^2) - (12x^2 + xz + z^2)$.

   (3 marks)

4. Given that $a : b = 5 : 6$ and $2b = 3c$.

   (a) Find $a : b : c$.

   (b) Find the value of $\dfrac{9a + 2b + 3c}{a + b + c}$.

   (3 marks)

5. Given a stock X in the market at \$x per unit at instance. It is known that a person could buy a certain amount of stock X at this price level. What is the percentage change in amount affordable for that person if the stock price is increased by 20% ? (4 marks)

6. Consider the compound inequality

$$\begin{cases} \dfrac{x}{x+1} \le 5 \\ 3x + 2 \le 0 \end{cases}$$

   (a) Solve the inequality system.

   (b) Write down the number of integers satisfying the inequality.

   (4 marks)

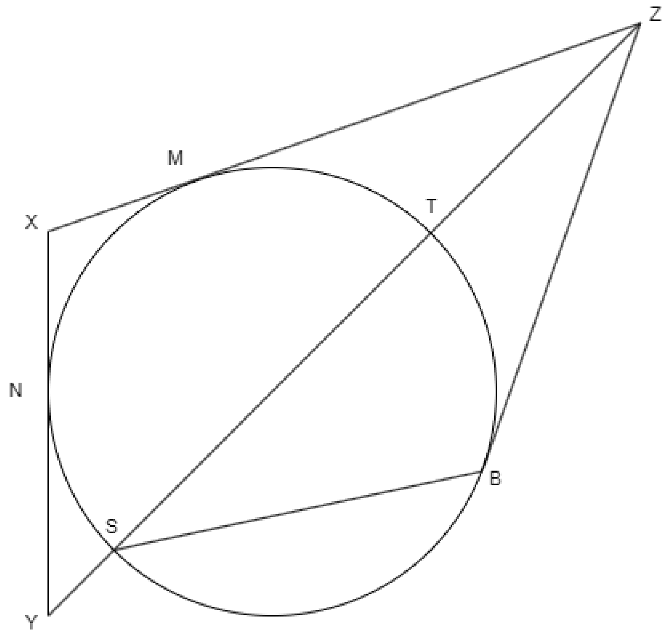
7. Let $f(x) = x^2 - kx - (k+1)$ has equal roots. Find

   (a) k,

   (b) the possible y-intercepts of $y = kf(x)$.

(5 marks)

8. Given the following figure



Suppose ST is the diameter of the circle passing through S,N,M,T,B. Assume XMZ and XNY are tangent to the circle. Further assume ZB is a tangent to the circle, while $\angle TSB = 30°$ and $\angle NYS = 45°$. Let the center of the circle be O. Find the value of $\angle MON$. (5 marks)

9. The following stem-and-leaf diagram shows the distribution of the ages of Tommy's girlfriends:

Stem	Leaf
0	6 6 7 8 9 9
1	0 0 1 5 5 5 5 7 8 9
2	2 3 7
3	0 b

It is known that the range of the distribution is over 30.

(a) Is it possible to have mean of the distribution to be less than 15.6? Explain your answer briefly.

(b) Suppose the current mean is maximized. Compute the standard deviation of the distribution.

(5 marks)

Section A(2) (35 marks)

10. It is given that $f(x)$ is composed of 3 parts. The first part varies as constant. The second part and the third part varies directly and inversely with x respectively. Suppose that $f(1) = f(-1)$, $2f(2) = 5$ and $4f(4) = 19$.

(a) Solve $f(x) = 1$. (3 marks)

(b) Define $g(x) = xf(x) - 1$ and denote the graph of $y = g(x)$ by G. Denote G's x-intercepts by X and Y, where X is on the left of Y, and the vertex of G by V. Find the area of $\triangle VXY$. (3 marks)

11. The table below shows the distribution of the range of years of sentence of prisoners in a certain jail.

Years of sentence	1 - 10	11 - 20	21 - 30	31 - 40
Number of prisoners	109	75	n	n - 10

It is given that the expected years of sentence in the jail is 17.5 years if a random prisoner is chosen.

(a) Find the median group and mode group of the distribution. (4 marks)

(b) Tommy was one of the prisoners in this jail with less than 10 years of sentence. During period of sentence, he murdered 3 prisoners whose years of sentence are below 10 years. He then received an extension of sentence. The mean of the new distribution is nearly unchanged. In what range of years of sentence should Tommy be extended to? (3 marks)

12. Given that $f(x)$ is a polynomial such that $(x-1)f(x)$ is divisible by x^2+x+1.

(a) Show that $f(x)$ is divisible by $x^2 + x + 1$. (3 marks)

(b) Consider $g(x)$ is a polynomial such that $f(x)$ is a factor of $g(x)$. Someone claims that if $(x-1)g(x)$ is divisible by x^3 then $g(x)$ is at least of degree 5. Is the claim correct? Explain your answer. (4 marks)

13. It is given that $A(-3,4)$ and $B(4,3)$ are two points and L is a straight line parallel to line AB in a rectangular plane. Let P be a moving point in the rectangular plane such that P is equidistant to A and B. Denote the locus of P by Γ.

    (a) Describe the geometric relationship between Γ and L.

    (b) Suppose L passes through the origin.

        i. Show that Γ also passes through the origin. Hence, find the distance between L and A.

        ii. Find the equation of inscribed circle of $\triangle OAB$.

14. The volume and the curved surface area of a frustum is 4875π cm^3 and 1365π respectively.

    (a) Find the height of the frustum.

    (b) It is given that the frustum is melted and recast into 2 similar frustums. The ratio of their base area is $1:16$. Suppose the upper base radius and the lower base radius of the smaller frustum is 5 and 3 respectively,

        i. Find the height of the smaller frustum.

        ii. Hence, find the total surface area of the greater frustum.

Section B (35 marks)

15. Given 10 different English books, 8 different Chinese books and 5 different Japanese books. You have to take 8 books from the given books to do reading class. Find the probability of

    (a) selecting at least 6 books of the same language.

    (b) giving the reading class with English books in the first 3 lessons, then Chinese books in the next 3 lessons, and 2 Japanese books in the last two lessons, given that none of the languages are selected with more than 5 books. The order of presentation should be considered.

16. Let $f(x,y) = x^2 + 2kxy + k^2y^2 + 2x + 2ky + 2$, where k is a real constant.

(a) When $k = 0$, by the method of completing the square, find the extremum value of $z = f(x, y)$, and indicate whether it is a maximum value or a minimum value.

(b) By considering $u = x + ky$ such that $g(u) = f(x, y)$, show that $z = f(x, y)$ has its global minimum as the same value of when $k = 0$.

(c) Define the set of real number pair (x, y) satisfying $z_0 = f(x, y)$ the z_0-level set of f.

i. Find the equation of the 10-level set of f in terms of k.

ii. Hence, or otherwise, find the area bounded by the 10-level set of f in terms of k.