

**HKSYCIA WONG TAI SHAN MEMORIAL COLLEGE**  
**SECOND TERM EXAMINATION (2019 – 2020)**  
**FORM 4**  
**MATHEMATICS Compulsory Part Paper I**  
**TIME ALLOWED: 2 HOURS 15 MINS.**  
**SETTER: KSY**

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Name: _____ (      )	Class: F4 _____	Mark
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1. This paper totally consists of **20** pages.
  2. After the announcement of the start of the examination, you should first write your Name, Class and Class Number in the space provided on ALL ODD pages.
  3. This paper consists of **THREE** sections, A(1), A(2) and B.
  4. Answer **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
  5. Unless otherwise specified, all working must be clearly shown.
  6. Unless otherwise specified, numerical answers should be exact or correct to 3 significant figures.
  7. The diagrams in this paper are not necessarily drawn to scale.
  8. No extra time will be given to candidates for writing Name, Class and Class Number after the “Time is up” announcement.
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## Section A(1) (35 marks)

1. Simplify  $\frac{(ab^{-1})^{64}}{a^{-25}}$  and express your answer with positive indices.

(3 marks)

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2. Factorize

(a)  $a^4 + 2a^2b^2 + b^4$

(b)  $a^4 + a^2b^2 + b^4$

(4 marks)

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3. The lengths of the sides of a right-angled triangle are  $(6 - x)$ ,  $(13 - x)$  and  $(14 - x)$  respectively.

- (a) Find the value of  $x$ .  
(b) Find the area of the triangle.

(4 marks)

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4. In the first term, the ratio of the numbers of boys to girls in S4 is 3 : 2. If the number of boys is increased by 14% and that of the girls is decreased by 18% in second term, what is the percentage change in the total number of students in S4.

(4 marks)

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5. In a polar coordinates system,  $O$  is the pole. The polar coordinates of points  $A$  and  $B$  are  $(7, 22.5^\circ)$  and  $(3, 112.5^\circ)$  respectively.  $C$  is a point such that  $O$  is the mid-point of  $AC$ .

- (a) Describe the geometric relationship between  $OB$  and  $AC$ . Explain your answer.  
(b) Find the area of  $\triangle ABC$ .

(4 marks)

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6. The sum and the difference of two numbers are 18 and 12 respectively. Without finding the numbers, find the product of the two numbers.

(4 marks)

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7. Given that  $h(x) = x^2 + kx + 1$ . If  $h(-2) = h(5)$ .

(a) Find the value of  $k$ .

(b) Define  $g(x) = h(x - 1) - 5$ . By considering the sum of roots of  $g(x) = 0$ , Shing Wai claims that the axis of symmetry of the graph of  $g(x)$  is  $x = 5$ . Do you agree? Explain your answer.

(6 marks)

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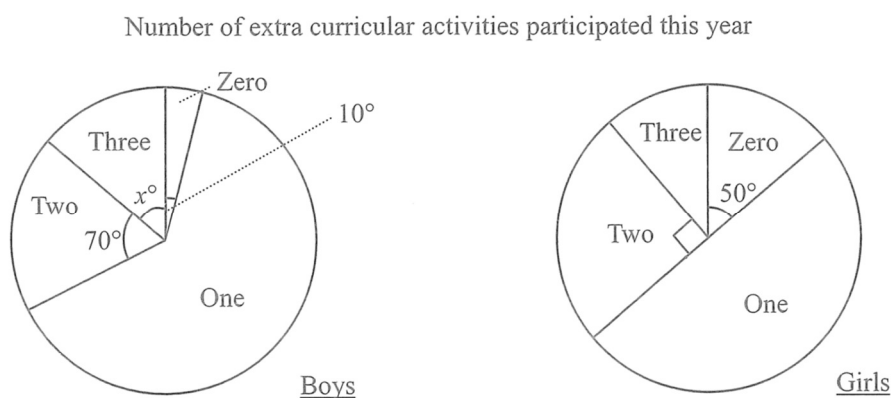
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8. There are 180 students in S5. The following pie charts show the distribution of the numbers of extra curricular activities that the boys and girls participated respectively this year.

It is given that half of the girls in S5 participated in 1 extra curricular activity only and the boys participated in 1.5 extra curricular activities on average.



- (a) Find  $x$ .
- (b) A S5 student is selected randomly. The probability of choosing a boy participating in 3 extra curricular activities is the same as that of choosing a girl participating in 3 extra curricular activities. Find the number of girls in S5.

(6 marks)

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**Section A(2) (35 marks)**

9. The base radius and the height of a right circular cone are  $3r$  and  $3r + 15$  respectively.
- (a) Find the volume of the cone in terms of  $r$ .
- (b) If the volume of the cone is  $54\pi$ , find the value(s) of  $r$ .

(5 marks)

[illegible]

10. It is given that  $h(x)$  is partly constant and partly varies as  $x$ . Suppose that  $h(721) = 1443$  and  $h(831) = 1663$ .

- (a) Find  $h(x)$ .
- (b) Solve the equation  $h(x) = x^2$ . Leave your answer in surd form.
- (c) If  $g(x) = x^2 - h(x)$ , using the result of (b), find the minimum value of  $g(x)$ .

(6 marks)

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11.  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 2kx - (3 - k) = 0$ . If  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{2}$ , find the value of

- (a)  $k$ ,  
(b)  $\beta^2 - 2\alpha$ .

(6 marks)

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12. The coordinates of the point  $A$  are  $(2,3)$ .  $A$  is rotated anticlockwise about the origin through  $270^\circ$  to point  $B$ . Meanwhile,  $A$  is reflected with respect to the  $y$ -axis first, and then translated vertically downward by 1 unit and then horizontally to the right by 1 unit to point  $C$ .

- (a) Write down the coordinates of  $B$  and  $C$ .
- (b) Find the equation of the line passing through  $B$  and  $C$ .
- (c) Let  $L$  be the line passing through  $A$  and perpendicular to  $BC$ . Find the equation of  $L$ .

Hence, or otherwise, find the area of  $\triangle ABC$ .

(8 marks)

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13. Let  $f(x) = 2x^3 - 5x^2 - x + 4k$ , where  $k$  is a constant. When  $f(x)$  is divided by  $x - k$ , the quotient is  $g(x)$  and the remainder is  $k$ .

- (a) Find the value(s) of  $k$ .
- (b) Suppose  $k$  is a positive integer. Solve the equation  $f(x) = k$ .
- (c) Suppose  $0 < k < 1$ . Billy Claims that the graph of  $y = g(x)$  intersects the line  $y = 4$  at two distinct points. Do you agree? Explain your answer.

(10 marks)

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## Section B (35 marks)

14. (a) Express  $\frac{5}{\sqrt{2}+\sqrt{3}i}$  in form  $a + bi$  where  $a$  and  $b$  are real numbers.

(b) Suppose  $p$  and  $q$  are real numbers such that  $\frac{5}{\sqrt{2}+\sqrt{3}i}$  is one of the roots of the equation

$$x^2 - px - q = 0. \text{ Find } p \text{ and } q.$$

(5 marks)

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- (5 marks)

Answers written in the margins will not be marked.

16. Let  $f(x) = 2x^2 - 4x + 11$ .

- Solve  $f(x) = 0$ , and express your answer in the form of  $a + bi$  if necessary.
- Using the method of completing square, write down the coordinates of vertex and the axis of symmetry.

(7 marks)

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17. (a) Solve the equation  $2u^2 - u - 1 = 0$ .

(b) Using (a), solve the following equations,

(i)  $2(x - 1)^2 - x = 0$

(ii)  $2^{2x+1} - 2^x = 1$

(iii)  $4(\log x)^2 - \log x^2 - \log 100 = 0$

(10 marks)

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18. Suppose that  $\alpha$  is a common root of the following quadratic equations

$$\begin{cases} x^2 - 4x \sin \theta - 2 = 0 \\ x^2 - 4x \cos \theta + 2 = 0 \end{cases}$$

- (a) Show that  $\alpha = \frac{1}{\cos \theta - \sin \theta}$

- (b) Hence, show that  $\sin^2 \theta = \frac{1}{4}$ .

- (c) If  $\sin \theta > 0$ , find the minimum value of  $y = x^2 - 4x \sin \theta - 2$  and the corresponding value of  $x$ .

(8 marks)

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