Locus and Circle: Homework

In this homework, we are going to solve a problem related to circle.

Given two points $A(10,0), B(-10,0)$ on the rectangular coordinate plane.

1. Find the equation of the perpendicular bisector L of the straight line AB, and the midpoint M of A and B. Show that M is on L.

2. Denote P and Q be points such that P is equidistant from A and L, and Q is equidistant from B and L.

   (a) Denote the locus of P by C_1 and locus of Q by C_2. Find the equation of C_1 and C_2.

   (b) Denote the closest point on C_1 to M by X, and that on C_2 to M by Y. What is the geometric relationship between A, B, X, Y, M?

   (c) Find the equation of circle with radius XM and center M.

3. Denote the circle in (2c) by C_0. Denote the intersection points of C_0 and L by H and K.

   (a) Find the equations of tangent to C_0 at H,K,X,Y.

   (b) Find the area enclosed by the above tangent lines and C_0.

功課：圓形與軌跡

在這份功課中，我們將會解決一條與圓相關的題目。

已知$A(10,0)$和 $B(-10,0)$ 為直角坐標平面上的兩點。

1. 求直線AB的垂直平分線L的方程，及AB的中點M。證明L通過M。

2. 設 P 和 Q 為兩點使得 P 與 A 和 L 等距，Q 與 B 和 L 等距。

   (a) 設C_1 為P的軌跡，設C_2 為Q的軌跡。求C_1 及C_2 的方程式。

   (b) 以X表C_1上與M的最接近點，以Y表C_2上與M的最接近點。求A、B、X、Y、M之間的幾何關係。

   (c) 求半徑為XM，圓心為M的圓方程。

3. 用C_0 表(2c)描述的圓，並用H及K表示C_0 和L的交點。

   (a) 求C_0在H、K、X、Y處的切線方程。

   (b) 求上述切線和C_0所圍成的面積。