Quadratic equation: Homework

In this homework, we are going to solve some quadratic equation problem.
Given a quadratic equation $kx^2 + (k-6)x + 3k = 0$.

1. Find k such that the quadratic equation has only one root.

2. Suppose it has two distinct roots $\alpha < \beta$, find the following in terms of k:

   (a) $\alpha + \beta$;

   (b) $\alpha\beta$;

   (c) $\alpha^2 + \beta^2$;

   (d) $\beta - \alpha$;

   (e) $\beta^2 - \alpha^2$.

3. Form a quadratic equation with roots $(\alpha + \beta)^2$ and $(\beta - \alpha)^2$ in terms of k. Also, using the method of completing the square, find the vertex, the axis of symmetry, and the extreme value of the quadratic equation in terms of k.