DOUBLE TUTORIAL 8

First Hour

Question 1. Write the following expressions in the form of a power, i.e. x^n, where n is an integer or a fraction.

(a) $\sqrt{x}$, (b) $\dfrac{1}{x}$, (c) $\sqrt[6]{x^2}$, (d) $\dfrac{1}{\sqrt{x}}$, (e) $\dfrac{1}{x^4}$, (f) $\dfrac{1}{\sqrt[7]{x^5}}$.

Question 2. Which of the following functions are power functions?

(a) $y = x^2$, (b) $y = \sqrt[5]{x}$, (c) $y = \sqrt[5]{x^4}$, (d) $y = x^7$, (e) $y = x^{-3}$, (f) $y = x^{-3} + x$, (g) $y = x$, (h) $y = 1$, (i) $y = \dfrac{1}{x^2}$.

Question 3. Find the inverse of following functions. Assume that the range of these functions coincides with the codomain. Note that some of these functions are not invertible. Sketch the graphs and their inverses on the Cartesian plane.

(a) $y = x^2$ on the domain $[0, \infty)$.

(b) $y = x^2$ on the domain $(-\infty, \infty)$.

(c) $y = x^2$ on the domain $(-\infty, 0]$.

(d) $y = x^3$ on the domain $(-\infty, \infty)$.

(e) $y = \sqrt[4]{x}$ on the domain $[0, \infty)$.

Question 4. Simplify

(a) $4^5 \times 4^{-3}$, (b) $4^2 \div 2^4$, (c) $x^{100} x^{-99}$, (d) $\left(a^{100}\right)^{-2}$.

Question 5. Solve the equations, i.e. find *all* solution.

(a) $x^3 = 27$,

(b) $x^4 = 10$,

(c) $\sqrt{x} = 15$,

(d) $\sqrt[3]{x^2} = 9$,

(e) $2\sqrt{x} = 16 + \sqrt{x}$.

Question 6.   Sketch the graphs of the functions $y = x^3$ and $y = x^4$, defined on the interval $[-2, 2]$, in one Cartesian plane. For this find first the values of the functions at points $x = -2$, $x = -\frac{3}{2}$, $x = -1$, $x = -\frac{1}{2}$, $x = 0$, $x = \frac{1}{2}$, $x = 1$, $x = \frac{3}{2}$, $x = 2$.

(a) What is the range for each of the functions?

(b) Use the horizontal line test to find out if these functions are invertible?

(c) What are the coordinates of the point(s) of the intersection of these two graphs?

(d) Is each of the functions strictly monotone? Strictly increasing? Strictly decreasing?

(e) What is the global minimum/maximum of each function?

(f) Are the functions concave up / concave down on $[-2, 2]$?

(g) Are the functions concave up / concave down on $[-2, 0]$?

(h) Are the functions concave up / concave down on $[0, 2]$?

Second Hour

Question 1.   Is the following function a linear function, a quadratic function, a power function, or a polynomial function?

(a) $y = \sqrt[4]{x}$,

(b) $y = \dfrac{1}{x}$,

(c) $y = 2x^2 - 3x + 4$,

(d) $y = \dfrac{1 - x}{\sqrt{x}}$,

(e) $\dfrac{1}{7}x + \sqrt{7}$,

(f) $y = x$.

Question 2.   Solve the following equations for x.

(a) $x^2(x-3)(x+7) = 0$,

(b) $x^7 + x^5 = 0$

Question 3.   For each function from Q2 find the intervals where the function is negative/positive.

Question 4.   A polynomial function of order 3 has three zeros, $x = -4$, $x = 0$, $x = 1$. The leading coefficient $a_3 = 2$. Write out the polynomial in general form, $P_3(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$.