TUTORIAL Sample Solutions 4

First Hour

Question 1. One cup of flour is equal to about $125\,g$ of flour. How much flour is in 2.5 cups? Round the answer to integer.

Solution. Let x be the amount of flour in 2.5 cups. Then

$125\,g$ in one cup,

$x\,g$ in 2.5 cups,

which we can write as a proportion

$$\frac{1}{125} = \frac{2.5}{x}.$$

Multiplying both sides by $125x$ we get

$$x = 2.5 \times 125 = 312.5 \approx 313\,g.$$

Question 2. Highway speed limits in USA can range from an urban low of $35\,mph$ to a rural high of $85\,mph$. One mph (mile per hour) is approximately $1.6\,km/h$. Convert the speed limits, $35\,mph$ and $85\,mph$, into km/h.

Solution.

- First we convert $35\,miles$ into km. Let x_1 be $35\,miles$ converted into km.

  1 mile is $1,6\,km$,

  35 miles is $x_1\,km$.

  Then
  $$\frac{1.6}{1} = \frac{x_1}{35}.$$

  It follows that
  $$x_1 = 35 \times 1.6 = 56\,km.$$

  We conclude that
  $$35\,mph \approx 56\,km/h.$$

- In a similar way we find that $85\,mph$ converted to kilometres is

  $$x_2 = 85 \times 1.6 = 136\,km$$

  and

  $$85\,mph \approx 136\,km/h.$$

Question 3. The circumference of the circle C_1 is $c_1 = 3.5\,cm$. The radius of the circle C_2 is four times larger than the radius of the circle C_1 r_1, i.e. $r_2 = 4r_1$. Find the circumference of the circle C_2.

Solution.

All circles are similar and the proportion holds

$$\frac{c_2}{c_1} = \frac{r_2}{r_1}.$$

Given is that $r_2 = 4r_1$, we get that

$$\frac{c_2}{c_1} = \frac{4r_1}{r_1} = 4.$$

Multiplying both sides of the equation $\frac{c_2}{c_1} = 4$ by c_1 we get

$$c_2 = 4c_1 = 4 \times 3.5 = 14.$$

The circumference of the circle C_2 is $14\,cm$.

Question 4. Given two similar triangles, ABC and $A'B'C'$ with

$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}.$$

(a) Explain what does it mean that two triangles are similar.

Solution. Two triangles are similar if the corresponding angles are pairwise equal.

(b) Evaluate the length of the side $B'C'$ if $AC = 5\,cm$, $A'C' = 55\,cm$ and $BC = 7\,cm$.

Solution.
$$\frac{A'C'}{AC} = \frac{B'C'}{BC},$$
i.e.
$$\frac{55}{5} = \frac{B'C'}{7}.$$
Cancelling the fraction on the lhs and multiplying both sides by 7 we get

$$B'C' = 11 \times 7 = 77\,cm.$$

(c) Evaluate the constant of proportionality.

Solution. The constant of proportionality is

$$k = \frac{A'C'}{AC} = \frac{55}{5} = 11.$$

Question 5. On a trial production run, 50 parts were produced in 24 hours. At the same rate, how long should it take to produce 20000 parts? Express the answer first in hours, then in days and hours.

Solution.

50 parts in 24 hours

20 000 parts in t hours.

$$\frac{50}{20000} = \frac{24}{t}.$$

Multiplying both sides by $20000t$ yields

$$50t = 24 \times 20000.$$

Divide by 50 and get

$$t = \frac{24 \times 20000}{50} = \frac{24 \times 2000}{5} = 24 \times 400 = 9600 \, hours.$$

In days it will be

$$d = \frac{9600}{24} = 400 \, days.$$

Question 6. Is it true or false?

(a) Any two circles are similar to each other.

True. Similar figures have the same shape, but can have different size.

(b) Any two triangles are similar to each other.

False. Two triangles are similar if the corresponding angles are pairwise equal. A counter-example, with two not similar triangles is: one rectengular triangle with the angles 90^0, 30^0, and 60^0, and an equilateral triangle with the angles 60^0, 60^0, and 60^0.

(c) Any two squares are similar to each other.

True. All squares have a similar shape.

(d) If you walk a distance d twice faster, you need twice more time to walk the same distance.

False. Let v_1 be the initial velocity, and t_1 be the time needed to walk the distance d with the velocity v. Then

$$d = vt_1.$$

Twice faster means that the new velocity is $2v$. Let t_2 be the time needed to walk the same distance d with the velocity $2v$. Then

$$d = 2vt_2.$$

Hence

$$2vt_2 = vt_1,$$

dividing both sides by $2v$ we get

$$t_2 = \frac{t_1}{2}.$$

It means that we need twice less time to walk the same distance with the double velocity.

(e) If you walk a distance d twice faster, you need twice less time.

True. See part (d).

(f) If you walk twice faster, you will travel the twice distance in the same time.

True. Distance d_1 travelled in time t with velocity t is $d_1 = vt$. Distance d_2 travelled in time t with velocity $2v$ is $d_2 = 2vt$. Hence

$$\frac{d_2}{d_1} = \frac{2vt}{vt} = 2.$$

Question 7. You are preparing an injection of morphine. The order reads 60 mg. The stock morphine is 18 mg per 5 mL. How many mL will you need to draw up? Round your answer to the nearest mL.

Solution.

Let x be an amount of mL you need to draw up. There are

$18\,mg$ per $5\,mL$

$60\,mg$ per $x\,mL$.

$$\frac{5}{18} = \frac{x}{60}$$

We solve the equation for x.

$$x = \frac{5 \times 60}{18} = \frac{5 \times 10}{3} = \frac{50}{3} = 16\frac{2}{3} = 16.\bar{6} \approx 17\,mL.$$

Question 8.   To determine the number of deer in a forest, a forest ranger tags 265 and releases them back into the forest. Later, 880 deer are caught, out of which 105 of them are tagged. Estimate how many deer are in the forest. Round your answer to the nearest deer.

Solution. We assume that deer are distributed uniformly in the forest. Let n be a number of deer in the forest. Then

105 out of 880 are tagged,

265 out of n are tagged.

$$\frac{880}{105} = \frac{n}{265}.$$

$$n = \frac{880 \times 265}{105} \approx 2220.\overline{952380} \approx 2221.$$

There are approximately 2221 deer in the forest.

Second Hour

For all exercises in T8 we use the formula

$$\frac{\text{percentage}}{100\%} = \frac{\text{partial amount}}{\text{total amount.}}$$

Question 1.   What is $12\frac{1}{2}\%$ of 120?

Solution. Let x be the desired number (partial amount).

Then

$$\frac{12\frac{1}{2}}{100} = \frac{x}{120},$$

Multiplying both sides by 120 we get

$$x = \frac{12.5 \times 120}{100} = 15.$$

Question 2.   A copper compound contains 80% copper by weight. How much copper is there in a 1.95 gram sample of the compound?

Solution.

Let $x\,g$ be the amount of copper in the sample (partial amount).

$$\frac{80}{100} = \frac{x}{1.95},$$

$$x = \frac{80 \times 1.95}{100} = 1.56.$$

Therefore, the sample contains 1.56 gram of copper.

Question 3. In a class of 32 students, 30 passed the exam. What is percent of the student passed the exam in the class?

Solution

Let $p\%$ be the percentage of students who have passed the exam.

$$\frac{p}{100} = \frac{30}{32},$$

$$p = \frac{30 \times 100}{32} = 93.75\%.$$

Question 4. 30 is 15% of what number?

Solution. Let x be the desired number (total amount).

$$\frac{15}{100} = \frac{30}{x},$$

Multiplying both sides by x we get

$$\frac{15}{100} \times x = 30,$$

Multiplying both sides by $\frac{100}{15}$ yields

$$x = 30 \times \frac{100}{15} = 200.$$

Question 5. If the present length of a brass rod is $1.46\,m$, what will its length be after 0.3% expansion caused by heating?

Solution. Let l be the new length of the rod. Then the change in the length of the rod is $l - 1.46\,m$ (partial amount), and corresponds to 0.3%. The previous length $1.46\,m$ corresponds to 100%.

$$\frac{0.3}{100} = \frac{l - 1.46}{1.46},$$

$$l - 1.46 = \frac{0.3 \times 1.46}{100} = 0.00438.$$

Therefore the length after expansion is $l = 1.46 + 0.00438 = 1.46438\,m$.

Question 6. Harry bought a new bike for $180, which includes 10% GST. How much money was spent on the GST?

Solution. Le x be the price without GST.

Let x be the money spent on the GST. It corresponds to 10% of the original price. The price $180 with the GST corresponds to $100\% + 10\% = 110\%$. The proportion is

$$\frac{10}{110} = \frac{x}{180}.$$

$$x = \frac{180 \times 10}{110} = \frac{1800}{110} \approx 16.36.$$

Question 7. Write down the numbers in scientific notation. Which of these numbers is the smallest? The largest?

(a) $0.00000000001 = 10^{-11}$,

(b) $35,000,000 = 3.5 \times 10^7$,

(c) $78,950 = 7.895 \times 10^4$,

(d) $0.00467 = 4.67 \times 10^{-3}$,

(e) $0.000000000011 = 1.1 \times 10^{-11}$.

The smallest number is $0.00000000001 = 10^{-11}$, the largest is $35,000,000 = 3.5 \times 10^7$.

Question 8.

(a) How many centimetres are in one kilometre?

Solution: $1\,km = 1000\,m = 1000 \times 100\,cm = 10^5\,cm$.

(b) How many kilometres are in 450 metres?

Solution: $1\,m = 0.001\,km$, hence $450\,m = 0.45\,km$.

(c) The formula converting Celsius into Fahrenheit is $F = \frac{9}{5}C + 32$, where F is a temperature in degrees Fahrenheit, and C is a temperature in degrees Celsius. Convert $180^0 C$ into $^0 F$.

Solution: $F = \frac{9}{5} \times 180^0 + 32 = 356^0 F$

(d) It is necessary to travel 11km per second to escape the earth's gravity (escape velocity). Convert the velocity into km/h.

Solution: $1\,h = 60\,min = 60 \times 60\,sec = 3600\,sec$. Hence $1\,sec = 1/3600\,h$.

$$11\,km/sec = \frac{11\,km}{1\,sec} = \frac{11\,km}{1/3600\,h} = 11 \times 3600\,km/h = 39600\,km/h.$$

Question 9.

(a) How many square metres are in one square kilometre?

Solution: $1\,km^2 = 1\,km \times 1\,km = 1000\,m \times 1000\,m = 10^6\,m^2$, one million square metres.

(b) How many square kilometres are in $500\,m^2$?

Solution: $1\,km^2 = 10^6\,m^2$, hence $1\,m^2 = 10^{-6}\,km^2$, and

$$500\,m^2 = 500 \times 10^{-6}\,km^2 = 5 \times 10^2 \times 10^{-6}\,km^2 = 5 \times 10^{-4}\,km^2.$$

Question 10. Given that dm^3 of water is equal to one litre, and that $1\,dm = 10\,cm$, evaluate

(a) How many litres of water are in one cubic metre?

Solution:

$$1\,m = 100\,cm$$
$$1\,m = 10 \times 10\,cm$$
$$1\,m = 10\,dm$$
$$1\,m^3 = 10^3\,dm^3$$
$$1\,m^3 = 10^3\,l.$$

There are $1000\,l$ of water in one cubic metre.

(b) How many litres of water are in the swimming pool with the dimensions $5\,m \times 25\,m \times 1.80\,m$.

Solution:

The volume of the swimming pool is $V = 5 \times 25 \times 1.80\,m^3 = 225\,m^3$.

From part (a) we know that $1\,m^3 = 10^3\,l$. Hence $V = 225 \times 10^3\,l = 225,000\,l$, or $2.25 \times 10^5\,l$.