STAT210/410

Review of Simple Linear Regression

You may wish to refer to the text &/or the Powerpoint slides for Chapter 3.

One of the more challenging problems confronting the water pollution control field is presented by the tanning industry. Tannery wastes are chemically complex. They are characterised by high values of biochemical oxygen demand, volatile solids and other pollution measures. The experimental data were obtained from 33 samples of chemically treated waste. Readings on SRP, the percent reduction in total solids, and ODP, the percent reduction in chemical oxygen demand for the 33 samples were recorded.

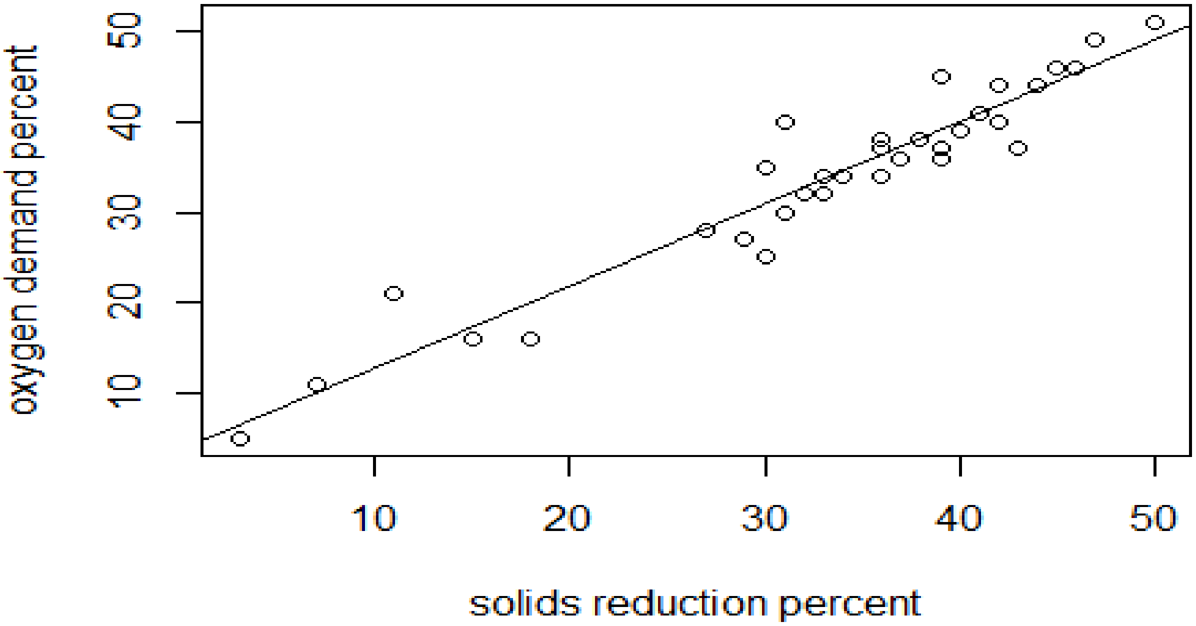
Source: Walpole R.E, and Myers R.H., (1989), *Probability and Statistics for Engineers and Scientists*, 4th ed., Macmillan, New York, page 359.

The data have been analysed and the R code and resultant output are given below.

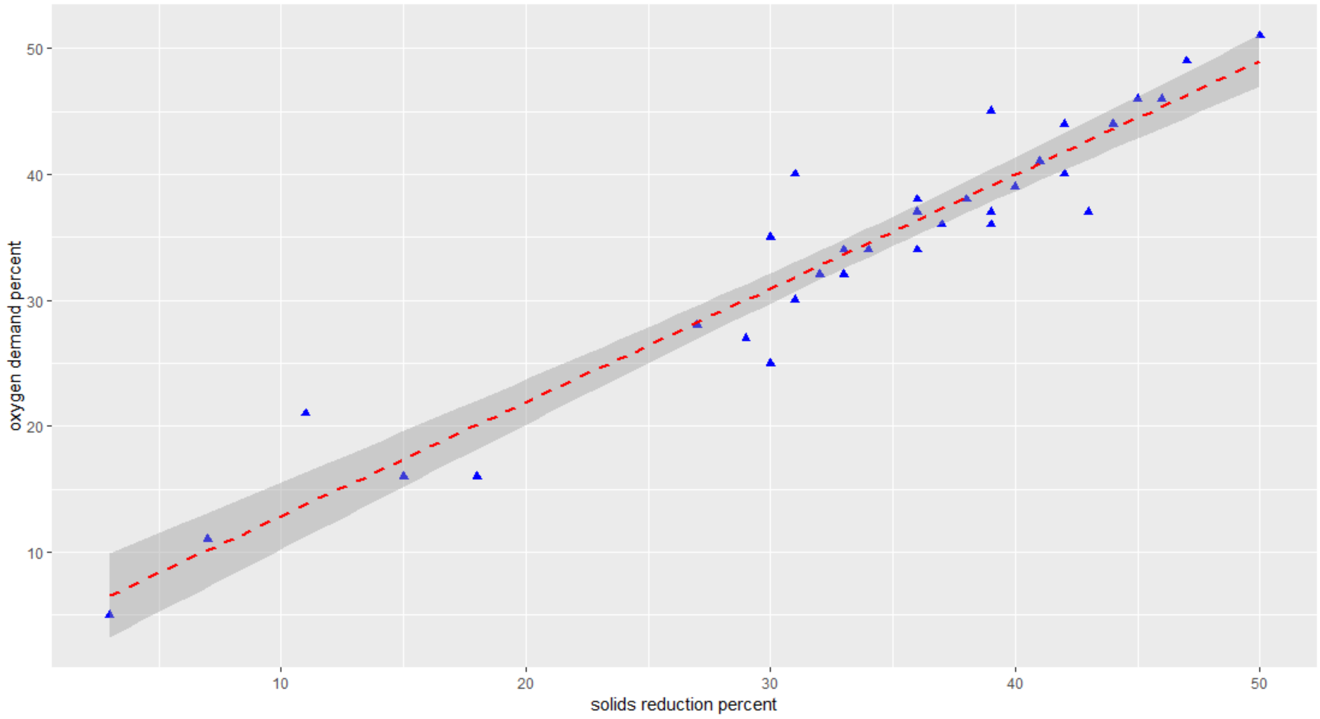
- Work through the code and output, ensuring that you understand the process and the results. Ask for help as required.
- Answer the questions that are embedded in the output below.

1. Read in data and produce exploratory plot

```
options(digits=3,show.signif.stars=F) # set no. of significant figures


# read in data

dat1 <- read.table("CH03_SLREx.txt",header=T)

# fit simple linear regression model
xy.lm <- lm(ODP~SRP, data=dat1)

#plot data and fitted line
plot(ODP~SRP,data=dat1, ylab = "oxygen demand percent", xlab="solids reduction per
cent")
abline(xy.lm)
```



```
# alternative plot using package ggplot2
install.packages("ggplot2")
library(ggplot2)
ggplot(data=dat1, aes(x=SRP, y=ODP)) +
  geom_point(pch=17, color="blue", size=2) +
  geom_smooth(method="lm", color="red", linetype=2) +
  labs(title="", x="solids reduction percent", y="oxygen demand percent")
```



Q1: Describe the association between the two variables, stating the form, direction and strength of the association.

There appears to be a positive linear association between ODP and SRP. The association is reasonably strong (quantified by r in Q4 below). The greatest variability in the response (scatter) appears at around SRP~30

2. Simple linear regression analysis to obtain estimates of regression coefficients, their std. errors and 95% CIs

```
# Table of regression coefficients and analysis of variance table
print(summary(xy.lm))

##
## Call:
## lm(formula = ODP ~ SRP, data = dat1)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.939 -1.783 -0.228  1.506  8.157
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.8296     1.7684    2.17    0.038
## SRP           0.9036     0.0501   18.03   <2e-16
##
## Residual standard error: 3.23 on 31 degrees of freedom
## Multiple R-squared:  0.913,  Adjusted R-squared:  0.91
## F-statistic:  325 on 1 and 31 DF,  p-value: <2e-16

print(anova(xy.lm))

## Analysis of Variance Table
##
## Response: ODP
##           Df Sum Sq Mean Sq F value Pr(>F)
## SRP        1   3391    3391     325 <2e-16
## Residuals 31    323      10

#CI for regression coefficients
betaCI(xy.lm)

##             Estimate Std. Error 2.5 % 97.5 %
## (Intercept)    3.830     1.7684 0.223   7.44
## SRP            0.904     0.0501 0.801   1.01
```

With reference to the output above:

Q2: Give the equation of the line of best fit: *ODP= 3.83 + 0.904 SDP*

Q3: Is SRP a useful predictor of ODP? Justify your response.

SRP is a useful predictor. This can be seen from the summary table of coefficients, where the p-value for test of slope (coefficient of SRP): H_0: β_1=0 is <2 x10^{-16}. The same p-value is found in the anova table as well. Consequently we can reject the null and conclude that there is a statistically significant (p<0.05), positive ($\widehat{\beta_1} > 0$) linear association between SRP and ODP.

Q4: What is the value of the correlation coefficient, r?

From the table of coefficients we see that R-squared is 0.91, and consequently, taking the square root, and noting the sign of the slope is positive, we obtain r = 0.95.

Q5: State and interpret the value of the coefficient of determination, r^2.

With R-squared =0.91, we can state that 91% of the observed variability in the response ODP, is explained by the linear relationship with SRP.

Q6: What is the estimate of σ^2? Find this value from two different parts of the output.

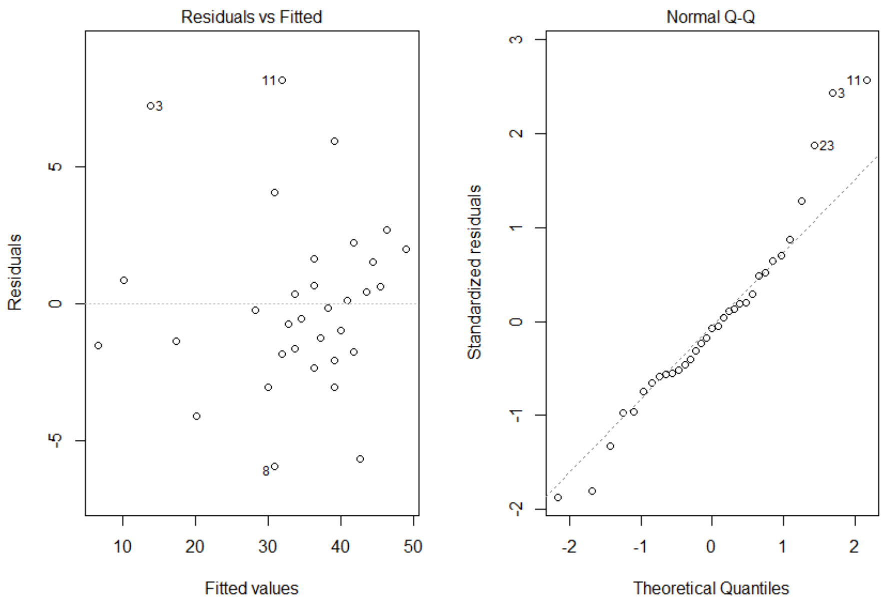
1. *Summary table of coefficients: $\hat{\sigma}$= residual std error = 3.23 , so 3.23^2 = 10.*
2. *Alternatively, from the ANOVA table: $\hat{\sigma}^2$ = MSE Residuals (or MS Error) =10.*

Q7: Give an informative interpretation of the 95% CI for β_1.

The 95% CI for the slope of the regression line, β_1, is (0.801, 1.01). We can state with 95% confidence, that for every 1% increase in SRP, the ODP will increase by between 0.801% and 1.01%.

3. Check Model Assumptions

```
#diagnostic plots
par(mfrow=c(1,2)) # 2 plots to the page (1 row, 2 columns)
plot(xy.lm,which=1:2,add.smooth=F)
```



```
#Shapiro Wilk's Test of normality
print(shapiro.test(xy.lm$residuals))

##
##  Shapiro-Wilk normality test
##
## data:  xy.lm$residuals
## W = 1, p-value = 0.1
```

Q8: State the assumptions of the linear model.

$\varepsilon \sim N(0,\sigma^2)$

Residuals

- *are independent*
- *follow a normal distribution*
- *are centred around 0 (i.e. observations are centred around the line of best fit)*
- *have constant variance with respect to X.*

NB: We can also use the QQ plot to detect possible outliers.

With reference to the output above:

Q9: Summarise the information available from the two residuals plots.

The residuals appear to be randomly scattered about 0, which suggests that a straight line model is appropriate, and that the assumption of constant variance appears valid, apart from perhaps one or two extreme values (obs 3 and 11). From the QQ plot, we see that most of the points are in the straight line with some deviation in the tails of the plot. This is due to some of the extreme values in the data. Overall, the normal QQ plot suggests that residuals are approximately normally distributed. Furthermore, the Shapiro Wilk's test (null hypothesis: residuals are normally distributed) is not significant (p=0.1), so we have no reason to reject the null – the assumption of normality appears reasonable.

Q10: What does the Shapiro-Wilk's test imply?

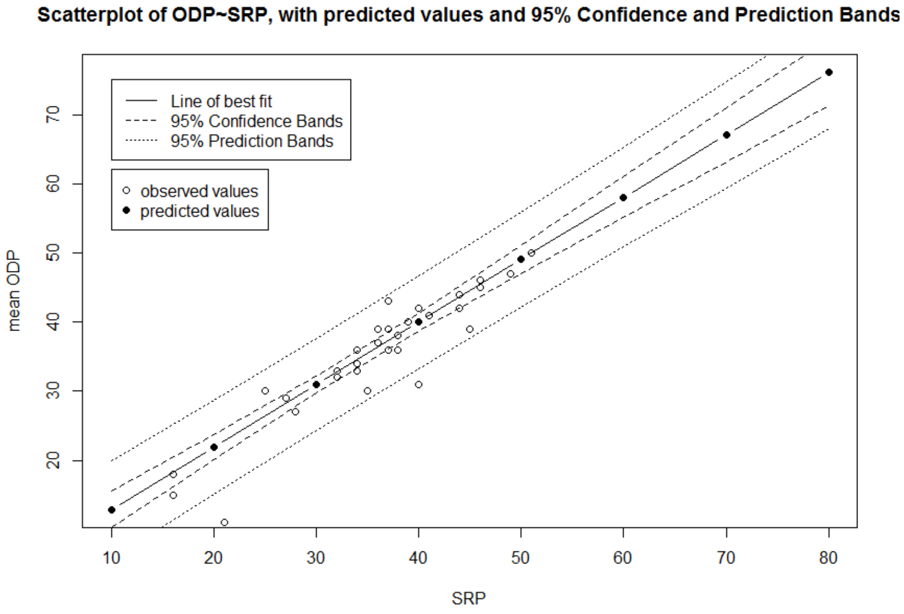
See above.

4. Predicting from the model

```
# # predictions for 8 data points: SRP = 10,20,30,40,50,60,70,80
# create new data frame of 8 observations

pred.df <- data.frame(SRP=c(10,20,30,40,50,60,70,80))
CI <- predict(xy.lm,interval="confidence",newdata=pred.df,level=0.95)
PI<- predict(xy.lm,interval="predict",newdata=pred.df,level=0.95)


with(pred.df, cbind(SRP, CI, PI))
```

```
  SRP  fit  lwr  upr  fit   lwr  upr
   10 12.9 10.2 15.5 12.9  5.76 20.0
   20 21.9 20.1 23.7 21.9 15.08 28.7
   30 30.9 29.7 32.1 30.9 24.24 37.6
   40 40.0 38.6 41.3 40.0 33.26 46.7
   50 49.0 47.0 51.1 49.0 42.12 55.9
   60 58.0 55.1 61.0 58.0 50.83 65.3
   70 67.1 63.2 71.0 67.1 59.43 74.7
   80 76.1 71.2 81.0 76.1 67.92 84.3
```

```
par(mfrow=c(1,1)) # 1 plot per page
plot(pred.df$SRP,CI[,1],type="b", pch=16, xlab="SRP", ylab="mean ODP", main="Scatt
erplot of ODP~SRP, with predicted values and 95% Confidence and Prediction Bands")
points(dat1$ODP, dat1$SRP)
legend(10,50, lty=c(1, 2,3),  legend=c("Line of best fit", "95% Confidence Bands",
"95% Prediction Bands"))
legend(10,42, pch=c(1,16), legend = c("observed values", "predicted values"))
lines(pred.df$SRP,CI[,2],lty=2) # lty indicates line type, lty= 2 -> dashed line
lines(pred.df$SRP,CI[,3],lty=2)
lines(pred.df$SRP,PI[,2],lty=3) # lty=3 produces a dotted line
lines(pred.df$SRP,PI[,3],lty=3)
```



Q11: With reference to the CI and PI for the predicted ODP for each of the eight values of SRP (previous page), and the plot shown above, comment on the reliability of using the SLR model for prediction.

First, note that:

- *the confidence and prediction bands get wider as we move away from the point of means (i.e. mean ODP, SRP). This is evident in the curvature of the bands.*
- *the prediction intervals for individual responses are much wider than the confidence intervals for mean responses. Means are less variable than individual responses.*

With reference to the plot, it appears that the regression model does a reasonably good job of predicting, with most observations falling within the bands.

R script file

Note that your script file should be structured in the following way:

- Comment giving brief summary of analysis, name of author
- Read in the data file
- Produce exploratory plots
- Fit various models using appropriate model selection method.
- Obtain relevant output (e.g. estimates of coefficients, CIs etc)
- Check model assumptions using diagnostic plots
- Use the model for prediction, inference etc.

```
#########################################################

# fit simple linear regression model to tannery data.
# Code developed by Jackie Reid

# options() allows the user to set and examine a variety of
# global options which affect the way in which
# R computes and displays its results.
# In the options command below, we set the number of digits to
print # but it is a suggestion only and not strictly adhered to.
# show.signif.stars=F removes some outdated display in
# output associated with p-values
options(digits=3,show.signif.stars=F)

# read in data and attach dataframe to search path
# the first place that is searched for data frames and variables
# header=T assigns variable names to the first row of the dataset
# dat1 is the name of the dataframe that contains the data, which
is # read from the text file "CH03-SLREx.txt"
dat1 <- read.table("CH03_SLREx.txt",header=T)

# fit simple linear regression model using lm(y~x, data= …)
# lm stands for linear model
xy.lm <- lm(ODP~SRP, data=dat1)

# plot data and fitted line using plot(y~x) or plot(x,y)
plot(ODP~SRP,data=dat1, ylab = "oxygen demand percent",
xlab="solids reduction percent")
# add the line of best fit to the existing plot
abline(xy.lm)
```

```
# alternative plot using package ggplot2

library(ggplot2)
ggplot(data=dat1, aes(x=SRP, y=ODP)) +
  geom_point(pch=17, color="blue", size=2) +
  geom_smooth(method="lm", color="red", linetype=2) +
  labs(title="", x="solids reduction percent", y="oxygen demand
percent")

# Table of regression coefficients, std errors, t-tests and p-values;
# and aov table
print(summary(xy.lm))
print(anova(xy.lm))


# CI for regression coefficients
confint(xy.lm)

# diagnostic plots - check model assumptions
# par(mfrow=c(1,2)) sets up plotting window
# to allow one row and 2 columns for the plots (ie 2 plots in
window) par(mfrow=c(1,2))


# plot(model) produces diagnostic (residuals plots)
#which=1:2 plots the first 2 of 5 possible residuals plots
# the residuals vs fitted and the Normal QQ plot
# add.smooth=F removes a smoothing curve from the plot,
# which can sometimes be distracting or misleading.
plot(xy.lm,which=1:2, add.smooth=F)


# Shapiro Wilk's Test of normality
# Null hypothesis is that residuals come from a normal distribution
print(shapiro.test(xy.lm$residuals))


# predictions for 8 data points: SRP = 10,20,30,40,50,60,70,80
# create new data frame of 8 observations
pred.df <- data.frame(SRP=c(10,20,30,40,50,60,70,80))
# predict mean values and 95% CI for those predictions
CI <-
predict(xy.lm,interval="confidence",newdata=pred.df,level=0.95) #
predict mean values and 95% prediction intervals
PI<- predict(xy.lm,interval="predict",newdata=pred.df,level=0.95)


# print SRP, fitted values 95%CIs and 95% PIs in columns (cbind)
# rbind() would print them in rows
# note use of with to identify dataframe used
with(pred.df, cbind(SRP,CI,PI))

# par(mfrow=c(1,1)) sets up plotting window
# to allow one row and 1 column for the plot(s)
# one plot in this case

par(mfrow=c(1,1))

# plot predicted values against SRP
# note use of $ sign to identify that SRP comes from pred.df data
# frame (as distinct from SRP in dat1 data frame)

# CI[,1] is the first column in the object labelled CI
# It contains the predicted or “fitted values”

# type=”b” plots both the points and lines joining the points
# other options include type=”l” (lines only), type=”p” (points only)

# pch = 16 plots the points as dots, pch=1 plots as an open circle

plot(pred.df$SRP,CI[,1],type="b", pch=16, xlab="SRP", ylab="mean
ODP", main="Scatterplot of ODP~SRP, with predicted values and 95%
Confidence and Prediction Bands")

# points() command adds points to an existing plot

points(dat1$ODP, dat1$SRP)

# legend(x,y,..) places legend on plot at coordinates (x,y)
# lty=c(1,2,3) puts 3 lines in the legend
# lty=1 is a solid line; lty=2 is a dashed line;
# lty=3 is a dotted line

legend(10,75, lty=c(1, 2,3),  legend=c("Line of best fit", "95%
Confidence Bands", "95% Prediction Bands"))
legend(10, 62, pch=c(1,16), legend = c("observed values", "predicted
values"))

# lines() adds lines to an existing plot
# CI[,2] gives the lower bound of the CI;
# CI[,3] contains the upper bound
# similarly for PI

 lines(pred.df$SRP,CI[,2],lty=2)
 lines(pred.df$SRP,CI[,3],lty=2)
 lines(pred.df$SRP,PI[,2],lty=3)
 lines(pred.df$SRP,PI[,3],lty=3)
```