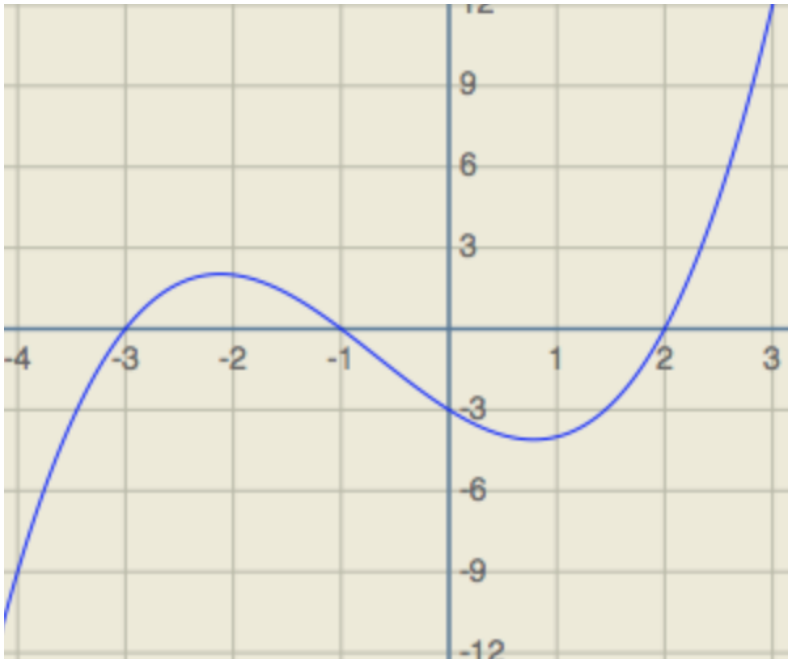
MTHS 100

DOUBLE TUTORIAL 6

First Hour

Question 1. A function is given by the graph below on the domain $[-4, 3]$. Answer the following questions.



(i) Find the coordinates of the points where the function is zero.

(ii) What are the intervals where the function is positive? negative?

(iii) What are the intervals where the function is increasing? decreasing?

(iv) Is the function concave up or concave down on the interval $[-4, -1]$?

(v) List the coordinates of all critical points of the function. Approximate the coordinates to one decimal place.

(vi) Which points are local maxima? local minima? Write down the coordinates of these points.

(vii) Which point is the global maximum? global minimum? Write down the coordinates of these points.

Question 2. Sketch a concave up function on the domain $[-5, 5]$ with a local minimum at the point $(1, -3)$, global maximum at the point $(5, 7)$, positive on the interval $(3, 5]$ and negative on the interval $(-5, 3)$, equal to zero at $x = -5$ and $x = 3$. Show the interval where the function is increasing. Show the interval where the function is decreasing.

Question 3. Function $f(x)$ is given by the formula $f(x) = \frac{1}{4}x - 1$ on the domain $\mathbb{R}$. Use the definition of increasing/decreasing functions and find out if $f(x)$ is increasing or decreasing.

Question 4. Is the function $f(x) = -5$ positive or negative? increasing? decreasing? strictly increasing? strictly decreasing? monotonic? strictly monotonic? Use the definitions to justify your answer.

Question 5*. Use formal definitions to answer the following questions.

(a) Is the function $f(x) = x^2$ increasing or decreasing on the interval $[0, \infty)$?

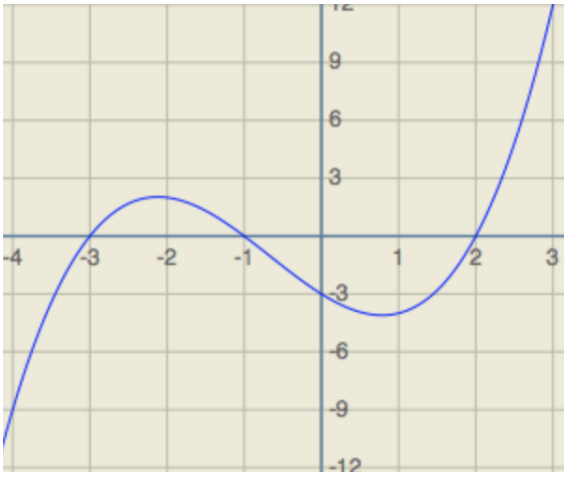
(b) Is the function $f(x) = x^2$ increasing or decreasing on the interval $(-\infty, 0]$?

(c) Is the function $f(x) = x^2$ monotonic on $\mathbb{R}$?

(d) Is the function $f(x) = x^2$ positive or negative on the interval $[1, 7]$?

Second Hour

Question 1. A function is given by the graph below on the domain $[-4, 3]$. Use the horizontal line test to find out if this function is invertible.



Restrict the domain and the codomain so that the function is invertible on the restricted domain. (There are many solutions for this question.)

Question 2. Veterinarians use the rule that one year of a dog's life is equivalent to seven years of human life. Write out the table for this rule for dog's life years $1, 2, 3, 4, 5, 6, 7$. What is the range of this function? Write out the table for the inverse function. What is the domain and the range of the inverse function?

Question 3. Sketch a graph of any invertible function. Use the horizontal line test to show that it is invertible. Show the domain and the codomain of this function.

Question 4. Sketch the graph of the function $y = 3$ on the domain $[-5, 5]$, with the codomain $[0, 3]$. What is the range of this function? Is it invertible? Why?

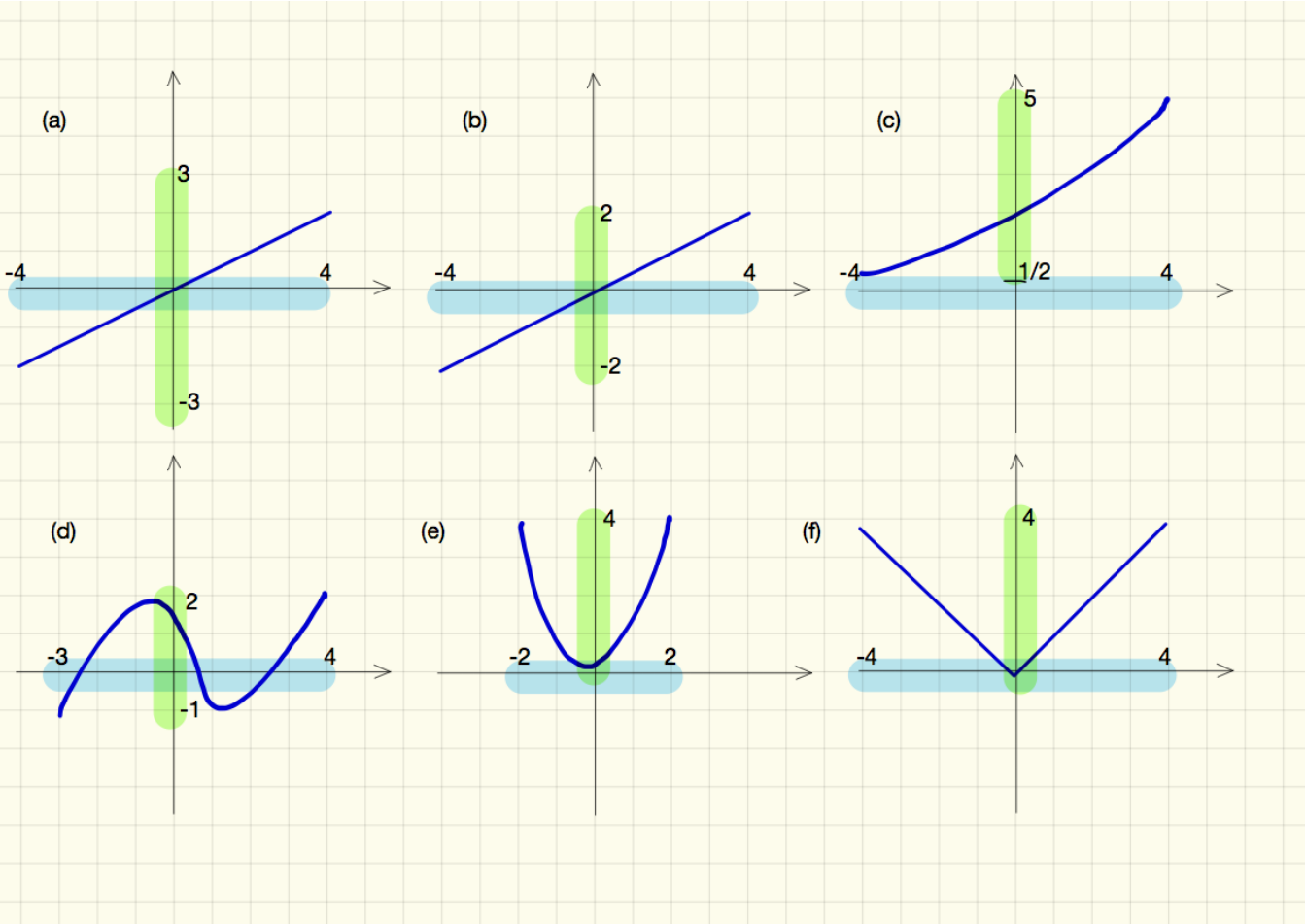
Question 5. Write out the table for the function given by $y = (x - 1)^2$ on the domain $x = -3, -2, -1, 0, 1, 2, 3$. What is the range of this function? Assume that the range coincides with the codomain of the function. Does this function has the inverse? Why?

Question 6. Is it true or false?

(a) If a function is monotonic, then it is invertible.

(b) If a function is strictly monotonic, then it is invertible.

Question 7. Decide if the functions given by the following graphs, domains and codomains, are invertible. If yes, sketch the inverse in the same Cartesian plane. What are domains and codomains for inverse functions?



(a) domain $= [-4, 4]$, codomain $= [-3, 3]$,

(b) domain $= [-4, 4]$, codomain $= [-2, 2]$,

(c) domain $= [-4, 4]$, codomain $= [\frac{1}{2}, 5]$,

(d) domain $= [-3, 4]$, codomain $= [-1, 2]$,

(e) domain $= [-2, 2]$, codomain $= [0, 4]$,

(f) domain $= [-4, 4]$, codomain $= [0, 4]$.