MTHS 100

TUTORIAL - Sample Solutions 3

First Hour

Question 1. Find the value of the following expressions

(a)

$$2^4 \div 2^2 \times 3 = 2^{4-2} \times 3 = 2^2 \times 3 = 4 \times 3 = 12.$$

(b)

$$5^{-2} \times 5^3 = 5^{-2+3} = 5^1 = 5.$$

(c)

$$(91^2 - 89)^0 = 1,$$

because for any number $a \neq 0$ $a^0 = 1$ by definition.

Question 2. Simplify

(a)

$$\frac{4m^2}{6m} = \frac{2 \times \cancel{2} \times m \times \cancel{m}}{\cancel{2} \times 3 \times \cancel{m}} = \frac{2m}{3}.$$

We note that we are not allowed to divide by zero, the initial expression makes no sense for $m = 0$. Hence we add to the the result that $m \neq 0$.

(b)

$$\frac{x^2 + x}{2x} = \frac{\cancel{x}(x+1)}{2\cancel{x}} = \frac{x+1}{2}, \ x \neq 0$$

(c)

$$\frac{2x}{x^2 + x} = \frac{2x}{x(x+1)} = \frac{2}{x+1}, \ x \neq 0 \text{ and } x \neq -1.$$

Question 3. Combine the like terms

(a)

$$\underline{v^2} - \underline{\underline{2v^3}} + \underline{3v^2} - 1 - \underline{\underline{v^3}} = v^2 + 3v^2 - 2v^3 - v^3 - 1 = 4v^2 - 3v^3 - 1.$$

(b)

$$x^{-1} + 4(x^{-1} + 3) = \underline{x^{-1}} + \underline{4x^{-1}} + 4 \times 3 = 5x^{-1} + 12.$$

(c)

$$a^2b - 5a(ab - 2 + b^2) - ab^2 = a^2b - 5a \times ab + 5a \times 2 - 5a \times b^2 - ab^2 =$$

$$\underline{a^2b} - \underline{5a^2b} + 10a - \underline{\underline{5ab^2}} - \underline{\underline{ab^2}} = -4a^2b - 6ab^2 + 10a.$$

(d)

$$5\sqrt{2} - 3(\sqrt{2} - 1) - (\sqrt{2} + 3)(\sqrt{2} - 3) = \underline{5\sqrt{2}} - \underline{3\sqrt{2}} + 3 - ((\sqrt{2})^2 - 3^2) =$$

$$= 2\sqrt{2} + 3 - 2 + 9 = 2\sqrt{2} + 10.$$

Question 4. Is it true or false?

(a) $(-1)^2 = 1$

True, because $(-1)^2 = (-1) \times (-1) = 1$.

(b) $(-1)^0 = 1$

True, because for any $a \neq 0$ $a^0 = 1$.

(c) $a^{99} \times a^{-99} = 0$, $a \neq 0$.

False because $a^{99} \times a^{-99} = a^{99-99} = a^0 = 1 \neq 0$.

(d) $a^{99} \times a^{-99} = 1$, $a \neq 0$.

True, see calculations in part (c).

(e) $-(-(-a)) = -a$

True, because $-(-(-a)) = -(a) = -a$, we used that $-(-a) = a$.

(f) $-(-(-a)) = a$

False, see part (e).

Question 5. Expand the brackets

(a)

$$t(5 + 7t^2) = 5t + 7t^3,$$

(b)

$$(2x + 4)^2 = (2x)^2 + 2 \times (2x) \times 4 + 4^2 = 4x^2 + 16x + 16.$$

(c)

$$(2x - 3)(2x + 3) = (2x)^2 - 3^2 = 4x^2 - 9.$$

(d)

$$(3 - ab)(a^2 - b) = 3a^2 - 3b - ab \times a^2 + ab \times b = 3a^2 - 3b - a^3b + ab^2.$$

Question 6. Factorize the following expressions

(a) $t^2 - t$.

The common factor is t, hence

$$t^2 - t = t(t-1).$$

(b) $t^3 - t$

The common factor is t, hence

$$t^3 - t = t(t^2 - 1) = t(t-1)(t+1).$$

(c) $3x^2y^3 - 6xy + 12xy^2$

The common factor is $3xy$, hence

$$3x^2y^3 - 6xy + 12xy^2 = 3xy(xy^2 - 2 + 4y).$$

(d) $88ab + 33a$

The common factor is $11a$, hence

$$88ab + 33a = 11a(8b + 3).$$

Question 7. Simplify by factorising and cancelling

(a)

$$\frac{x^2 - 1}{x - 1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x + 1,\ x \neq 1$$

(b)

$$\frac{x^2}{5x^4} = \frac{\cancel{x^2}}{5x^2 \times \cancel{x^2}} = \frac{1}{5x^2},\ x \neq 0$$

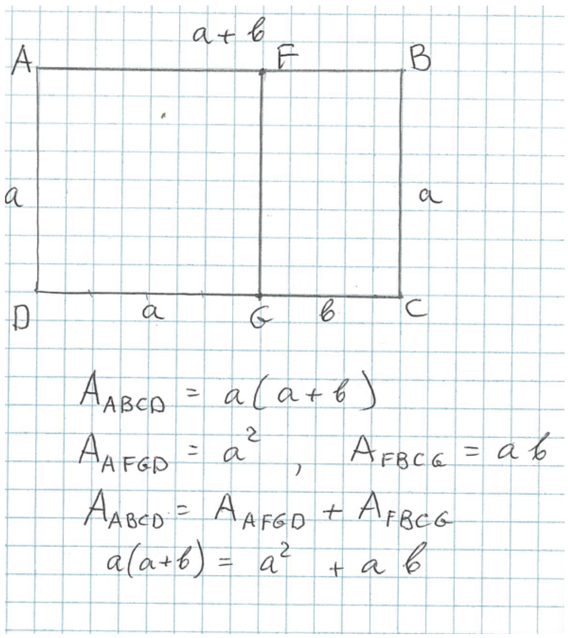
(c)

$$\frac{(a+b)^2}{a+b} = a + b,\ a + b \neq 0$$

(d)

$$\frac{t}{t(t^4+1)} = \frac{1}{t^4+1},\ t \neq 0.$$

Question 8. Illustrate that $a(a+b) = a^2 + ab$. Construct the corresponding rectangles and compare their areas. See the similar illustrations in the lecture notes Section 5.5.



Second Hour

Question 1. Solve the following linear equations

(a)

$$x - 6 = 0$$

add 6 to the left and to the right hand sides of the equation:

$$x - 6 + 6 = 0 + 6,$$

$$x = 6.$$

We check the solution. Substitute $x = 6$ into the left hand side of the equation $x - 6 = 0$. We get $6 - 6 = 0$, which is correct. We conclude that $x = 6$ solves the equation.

(b)

$$x + 32 = 108$$

$$x = 108 - 32 = 76.$$

Check the answer: $l.h.s. = 76 + 32 = 108 = r.h.s.$

(c)

$$2x + 32 = 108$$

$$2x = 108 - 32 = 76$$

$$2x \div 2 = 76 \div 2$$

$$x = 38.$$

Check the answer: $l.h.s. = 2 \times 38 + 32 = 76 + 32 = 108 = r.h.s.$

(d)

$$\frac{1}{7}x - 7 = 10 + \frac{1}{14}\,x$$

Find and combine like terms:

$$\frac{1}{7}x - \frac{1}{14}\,x = 10 + 7$$

$$\left(\frac{1}{7} - \frac{1}{14}\right) x = 17$$

$$\left(\frac{2}{14} - \frac{1}{14}\right) x = 17$$

$$\frac{1}{14}x = 17$$

Multiply both sides by 14:

$$\frac{1}{14} \times 14x = 17 \times 14$$

$$x = 238.$$

(e)

$$-\frac{2}{3}\,x - 2x - 9 = \frac{1}{3}\,x + 1$$

$$-\frac{2}{3}\,x - 2x - \frac{1}{3}\,x = 1 + 9$$

$$-\frac{2}{3}\,x - \frac{1}{3}\,x - 2x = 10$$

$$-3x = 10$$

$$x = -\frac{10}{3}.$$

Question 2. Solve the following equations

(a)

$$(x-1)(x+1)=0$$

There are two solutions:$x=1$ and $x=-1$.

We check the answer: substitute $x=1$ into the lhs of the equation.

$$(x-1)(x+1)=(1-1)(1+1)=0\times 2=0,$$

If $x=-1$, then

$$(x-(-1))(x+(-1))=(1+1)(1-1)=2\times 0=0.$$

(b)

$$-4x(x-\frac{5}{2})=0$$

The solutions are $x=0$ and $x=\frac{5}{2}$.

(c)

$$(x-771)^2=0$$

The solution is $x=771$.

Question 3. Factorise the left hand side and solve the equations

(a) $x^2+2x+1=0$

Note that we can use the formula $a^2+2ab+b^2=(a+b)^2$ for $a=x$ and $b=1$. Then

$$x^2+2x+1=(x+1)^2.$$
$$(x+1)^2=0$$

for $x=-1$.

(b) $4x^2-4x+1=0$

We use $a^2-2ab+b^2=(a-b)^2$ for $a=2x$ and $b=1$.

$$\begin{aligned} 4x^2-4x+1&=(2x-1)^2.\\ (2x-1)^2&=0 \text{ if}\\ 2x-1&=0, \text{ i.e.}\\ 2x&=1 \text{ and } x=\frac{1}{2}. \end{aligned}$$

(c) $16x^2 - 25 = 0$

Rewrite the equation as $(4x)^2 - 5^2 = 0$. Use the formula $a^2 - b^2 = (a-b)(a+b)$ for $a = 4x$ and $b = 5$. Then

$$(4x)^2 - 5^2 = (4x - 5)(4x + 5).$$

$(4x-5)(4x+5) = 0$ for $4x - 5 = 0$ or for $4x + 5 = 0$. The solutions are $x = \frac{5}{4}$ and $x = -\frac{5}{4}$.

Check the solutions!

(d) $x^4 - \frac{1}{2}x^3 = 0$. The common factor for the terms x^4 and $\frac{1}{2}x^3$ is x^3. Factoring out the common factor we get

$$x^4 - \frac{1}{2}x^3 = x^3(x - \frac{1}{2}).$$

This is equal to zero for $x = 0$ or for $x = \frac{1}{2}$.

Question 4. Is it true or false?

(a) The equation $x^2 = 1$ has one and only one solution;

False, the equation has two solutions. Subtracting 1 from both sides we get $x^2 - 1 = 0$, or $(x-1)(x+1) = 0$. The solutions are $x = 1$ and $x = -1$.

(b) The equation $x^2 = -1$ has two solutions;

False. x^2 is always nonnegative, can not be equal to -1.

(c) If $ab = 0$, then either $a = 0$ or $b = 0$, or both are zeroes;

True. If $a = 0$, then $0 \times b = 0$. If $b = 0$, then $a \times 0 = 0$. If $a = b = 0$, then $0 \times 0 = 0$. If both a and b are not equal to zero, their product is not the zero. It is one of the properties of real numbers.

(d) If $ab = 0$, then $a = 0$ and $b = 0$;

False. It could be that $a = 0$ but $b \neq 0$. Then their product is zero.

(e) If $a - b = 0$, then either $a = 0$ or $b = 0$;

False. Counter-example: $a = b = 6$. $a - b = 0$, but a and b are not zeros.

(f) If $a - b = 0$, then $a = b$.

True. Add b to both sides and get $a - b + b = 0 + b$, or $a = b$.

Question 5. Solve the equations

(a) $2x^2 - 6x + 9 = x^2 - 12x$.

First add $-x^2 + 12x$ to both sides.

$$2x^2 - 6x + 9 - x^2 + 12x = 0.$$

Combine like terms.

$$x^2 + 6x + 9 = 0$$

$$(x+3)^2 = 0$$

The solution is $x = -3$.

We check that $x = -3$ solves the equation.

$$\begin{aligned} 2(-3)^2 - 6 \times (-3) + 9 &= (-3)^2 - 12 \times (-3) \\ 18 + 18 + 9 &= 9 + 36 \\ 45 &= 45, \end{aligned}$$

which is correct!

(b) $\frac{7}{9}x^2 - 2x + 1 = \frac{2}{3}x^2 - 2x + 10$.

The same procedure as in previous part.

$$\begin{aligned} \frac{7}{9}x^2 - 2x + 1 - \frac{2}{3}x^2 + 2x - 10 &= 0 \\ \frac{1}{9}x^2 - 9 &= 0 \\ \left(\frac{1}{3}x - 3\right)\left(\frac{1}{3}x + 3\right) &= 0 \\ \frac{1}{3}x - 3 = 0 \text{ or } \frac{1}{3}x + 3 &= 0. \end{aligned}$$

There are two solutions, $x = 9$ and $x = -9$.