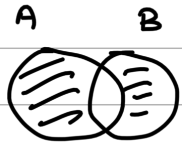
Assignment - 1

① $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B) = 0.2$,



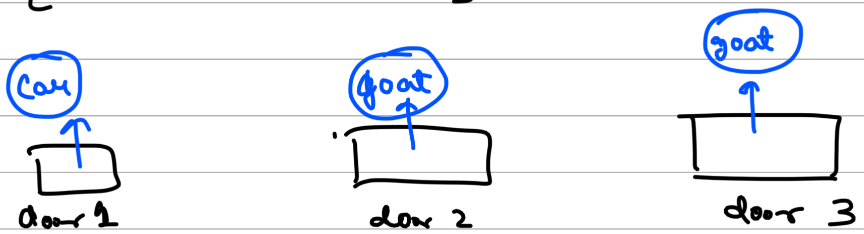
(a) → $(0.3 + 0.4 - 0.2 - 0.2)$

$= (0.3)$

(b) → $(A \cup B) \Rightarrow 0.3 + 0.4 - 0.2 = 0.5$

(c) → $[1 - 0.5 = 0.5]$

② →



Monty hall

0 Contestant

x_1 = Probability of winning Car if he switches the door.

$P(x_1) = P\left(\frac{x_1}{D_1}\right) \times P(D_1) + 2 \times P\left(\frac{x_1}{D_2}\right) \times P(D_2)$

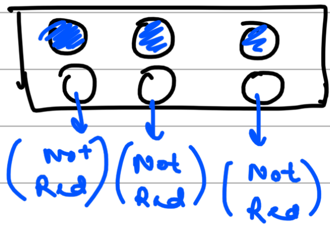
$P(x_1) = \left[\frac{1}{3} \times 0\right] + \frac{2}{3} \times 1 = \frac{2}{3}$

x_2 = ______ he does not switches the door

$P(x_2) = P\left(\frac{x_2}{D_1}\right) \times P(D_1) + 2 \times P\left(\frac{x_2}{D_2}\right) \times P(D_2)$

$P(x_2) = 1 \times \frac{1}{3} + [\cancel{\ }]\ 0 = \frac{1}{3}$

③ → $P\left(\frac{AR}{3R}\right) \times P(3R) = P(AR)$

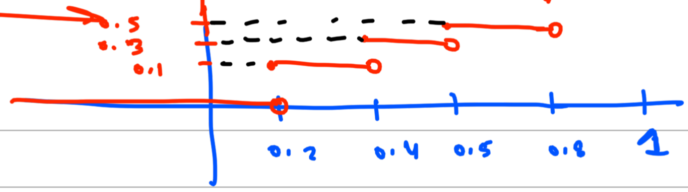


$P\left(\frac{AR}{3R}\right) \times ({}^6C_3) \times \cancel{\left(\frac{1}{2}\right)^3} \times \cancel{\left(\frac{1}{2}\right)^3} = \cancel{\left(\frac{1}{2}\right)^6}$

$P\left(\frac{AR}{3R}\right) = \frac{1}{({}^6C_3)} = \frac{\lfloor 3 \times \lfloor 3}{\lfloor 6} = \frac{\cancel{\lfloor 3} \times \cancel{3 \times 2}}{\cancel{6} \times 5 \times 4 \times \cancel{\lfloor 3}}$

$\boxed{P\left(\frac{AR}{3R}\right) = \frac{1}{20}}$

④ → $P_X(x) =$



$P(X \leq 0.5) = F(0.5^-) = 0.3$

$F(0.75^-) - F(0.25) = 0.5 - 0.1 = 0.4$

$P(X = 0.2 \mid X < 0.6) = \dfrac{P(X = 0.2)}{P(X \leq 0.6)} = \dfrac{0.1}{0.5} = 0.2$

⑤ $F \rightarrow$ ① RHC
② Non-decreasing

1 ↗

$\therefore$

$F(3) = F(3^+)$

$\dfrac{4c^2 - 9c + 6}{4} = 1$

$(4c^2 - 9c + 2) = 0$

$4c^2 - 8c - c + 2 = 0$

$4c(c-2) - 1(c-2) = 0$

$(4c-1)(c-2) = 0$

$\left[c = 2, \quad c = \frac{1}{4}\right]$

Now at

$F(1)$ if $c = 2$: $-5/6$

$F(1)$ if $c = 1/4$

$\dfrac{7 - 3/2}{6}$

$= \dfrac{11}{12}.$

Answer → $c = \dfrac{1}{4}$

2 ↗ → $F(2^-) - F(1) = \frac{11}{12} - \frac{11}{12} = 0$

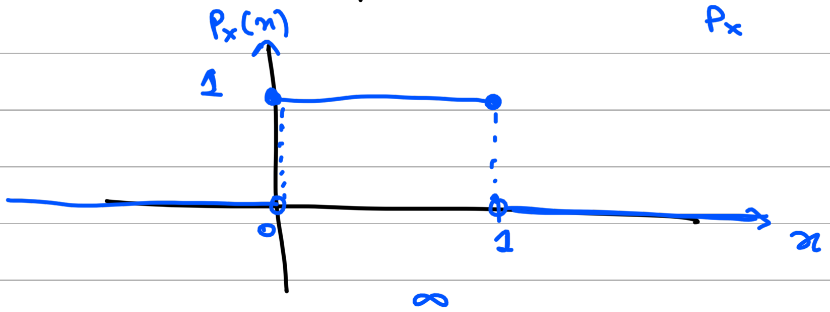
$F(3^-) - F(2^-) = 1 - \frac{11}{12} = \frac{1}{12}$

$F(1) - F(0) = \frac{11}{12} - \frac{2}{3} =$ ~~36~~ $\dfrac{11 - 8}{12} = \dfrac{3}{12} = \dfrac{1}{4}$

$F(2) - F(1^-) = \left(\frac{11}{12} - 2/3\right) = \dfrac{11 - 8}{12} = \dfrac{1}{4}$

$1 - \underbrace{F(3^-)}_{1} = 1 - 1 = 0$

⑥ → P_x $(x^2), (y^2)$



Now

~~E(x)~~ $E(g(x)) = \int_{-\infty}^{\infty} g(x)\, P_x(x)\, dx$

① $\left[E(x) = \int_0^1 x \times 1\, dx = \frac{1}{2} \right]$ $\boxed{E(x) = \frac{1}{2}}$

② $Var(X) = \underbrace{E(x^2)}_{\frac{1}{3}} - (E(x))^2 = \int_0^1 x^2\, dx - \left(\int_0^1 x\, dx\right)^2$

$= \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{3} - \frac{1}{4}$

$\boxed{Var(x) = \frac{1}{12}}$

③ ~~E(~~

$\underbrace{E(x^2)}_{\frac{1}{3}} + \underbrace{E(y^2)}_{\frac{2}{3}} = 1$

$E(y^2) - (E(y))^2 = 5/9 \longrightarrow 2/3 - (E(y))^2 = 5/9$

$\frac{6}{9} - \frac{5}{9} = (E(y))^2$

$= \boxed{\frac{1}{3} = E(y)}$

$\left[E(x+y) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \right]$