a·b – constant, ∀ a, ∃ b în f de a

a.î. $a \cdot b = 0$.

$b = 0$; $a \circ b = 4$

$b = \frac{4}{a}$

$\log_2 8 = 3$? $\log_2 2^3 = 3$

$\log_2 2^{?} = 10$ $2^{10} = 1024$

c) Calculați: $\log_2 1 \circ \log_2 2 \circ \log_2 3 \circ \dots \circ \log_2 2018$

$\log_2 ? = 10$.

$\log_2 1024 = 10$

$x \circ y = \frac{1}{10}(x-10)(y-10) + 10$.

$\log_2 10 \circ \log_2 2 = 0 \circ 1 = \frac{1}{10}(\cancel{-10} \cdot -9) + 10$ el abs

$= 19$

$\Rightarrow \varphi = 10$

Fie a-el absorbant

$x \circ a = a$, $\forall x \in \mathbb{R}$

$\Rightarrow \frac{1}{10}(x-10)(a-10) + 10 = a$

$\Leftrightarrow \frac{1}{10}(x-10)(a-10) = a - 10$

$\underbrace{1,2,3,4,5}_{a}$ b b

$a \cdot b = b$ | $\cdot 10$

$b = 0$.

$a - 10 = 0$

$a = 10$. (I)

$a \circ x = a$

$1/10(a-10)(y-10) + 10 = a$ | -10

$1/10(x-10)(a-10) = \frac{a-10}{0} \Rightarrow a = 10$ (II)

Din I, II $\Rightarrow a = 10$ – el abs

Determinați val reale ale lui x pt. care $x \circ x \le \frac{101}{10}$

$x \circ x = \frac{1}{10}(x-10)(x-10) + 10$

$= \frac{1}{10}(x-10)^2 + 10 \le 101/10$.

$\frac{1}{10}(x-10)^2 \le 101/10 - 10$

$1/10 (x-10)^2 \le 1/10$ | $\cdot 10$

$(x-10)^2 \le 1$.

$x - 10 = t$

$t^2 \le 1 \Rightarrow t \in [-1, 1]$

$-1 \le x - 10 \le 1$ | $+10$

$9 \le x \le 11$

$\boxed{x \in [9, 11]}$

$(x+2)^2 \le 25$

Rez: $x + 2 = t$

$\Rightarrow t^2 \le 25 \Rightarrow t \in [-\sqrt{25}, +\sqrt{25}]$

$t \in [-5, 5]$

$-5 \le \overbrace{x+2}^{t} \le 5$ | -2

$-7 \le x \le 3 \Leftrightarrow x \in [-7, 3]$

a) Determinați valorile lui m pt. care sist. e comp. nedet.

det $A = \begin{vmatrix} 1 & m & -1 \\ 2 & -1 & -2 \\ m & 2 & 1 \end{vmatrix} = -1 - 4 - 2m^2 \dots = 2m^2 + 3m - 1$

comp nedet. $\Rightarrow$ det $A = 0$.

$2m^2 + 3m - 1 = 0$.

$\Delta = b^2 - 4ac = 9 - 4 \cdot 2 \cdot (-1)$

$= 9 + 8 = 17$

$m = \left\{\frac{-3 \pm \sqrt{17}}{4}\right\}$, $m \in \mathbb{R}$

$A|B = \begin{pmatrix} 1 & m & -1 & 0 \\ 2 & -1 & -2 & 0 \\ m & 2 & 1 & 0 \end{pmatrix} = M$

It 1: ~~del~~ $c_1 \Rightarrow c_0 2 1$ (1)

$\Rightarrow M = \begin{vmatrix} m & -1 & 0 \\ -1 & -2 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 0 + 0 + 0 - 0 - 0 - 0$

$= 0$ ✓ (2)

Iterație 2: el c_2

$M = \begin{vmatrix} 1 & -1 & 0 \\ 2 & -2 & 0 \\ m & 1 & 0 \end{vmatrix}$ (2)

$= 0$ ✓

It 3: el c_3: $M = \begin{vmatrix} 1 & m & 0 \\ 2 & -2 & 0 \\ m & 1 & 0 \end{vmatrix} = 0$ (3)

It 4: el c_4

$\Leftrightarrow M = A$. (4)

Din (1,4) $\Rightarrow m = \left\{\frac{-3 \pm \sqrt{17}}{4}\right\}$ – sistem comp nedeterminat

Rezolvați m = ... rezolvați sist.

$A = \begin{pmatrix} 1 & -1 & -1 \\ 2 & -1 & -2 \\ -1 & 2 & 1 \end{pmatrix}$; det $A = 0 \Rightarrow$ { comp ned / incomp

$A|B = \begin{pmatrix} 1 & -1 & -1 & 0 \\ 2 & -1 & -2 & 0 \\ -1 & 2 & 1 & 0 \end{pmatrix} = M$

$\ne 0$, z.

α, β

It 1: el $c_1 \Rightarrow M = \begin{vmatrix} -1 & -1 & 0 \\ -1 & -2 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 0$

It 2: el $c_2 \Rightarrow M = \begin{vmatrix} 1 & -1 & 0 \\ 2 & -2 & 0 \\ -1 & 1 & 0 \end{vmatrix} = 0$

Analog It 3, 4

(1,4) sistem comp nedeterminat

$x = \alpha \in \mathbb{R}$, $y = \beta \in \mathbb{R}$

$-\alpha + 2\beta + z = 0$.

$z = 2\beta - \alpha$

$2\alpha - \beta - 2z = 0$.

$z = \frac{2\alpha - \beta}{2}$

$\alpha - \beta - z = 0 \Rightarrow z = \beta - \alpha$.

$2\alpha - (\alpha - z) - 2z = 0 \Rightarrow \alpha + z - 2z = 0$

$\Rightarrow \alpha - z = 0$

$\Rightarrow \boxed{z = \alpha}$

$-\alpha + 2y + \alpha = 0$.

$2y = 0 \Rightarrow y = 0$

$\boxed{x = z \in \mathbb{R}}$.

$\alpha = y + z \Rightarrow y = \alpha - z$

Soluție: $\{(\alpha, 0, \alpha) \mid \alpha \in \mathbb{R}\}$

– sistem de ec. nedeterminat

$\begin{cases} x + y = 4 \\ x + y = 5 \end{cases}$ $\Delta = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$A|B = \begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix} = 1 \ne 0$

$\Rightarrow$ inc

1. În $\mathbb{R}$ se definește $x \circ y = \frac{1}{10}xy - (x+y) + 20$.

a) Demonstrați că $x \circ y = \frac{1}{10}(x-10)(y-10) + 10$

a) $x \circ y = \frac{1}{10}xy - (x + y) + 20$.

$= \frac{1}{10}xy - \frac{1}{10}(10x + 10y) + 20$.

$= \frac{1}{10}(xy - 10x - 10y) + 20 = \frac{1}{10}(x(y - 10) - 10y) + 20$

$= \frac{1}{10}(x(y - 10) - 10y + 100) + 10$.

$10 : \frac{1}{10}$ $10 \cdot 10$

$= 1/10(x(y-10) - 10(y-10)) + 10 = 1/10(x-10)(y-10) + 10$

$\boxed{2x+5} = 2x + 4 + 1$

$= 2(x+2) + 1$

$\frac{x}{2} + 1 = \frac{x}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{1}{2}\left(x + \frac{\frac{1}{2}}{\frac{1}{2}}\right) + \frac{1}{2}$

$\frac{x}{2} = \frac{1}{2} \cdot x + 1$

$\frac{x}{3} + 1 = \frac{1}{3} \cdot x + 1$

$= \frac{1}{3} \cdot x + \frac{1}{3} + \frac{2}{3}$

$= \frac{1}{3}\left(x + \frac{\frac{1}{3}}{\frac{1}{3}}\right) + 2/3$

$= 1/3(x+1) + 2/3$

$10 : \frac{1}{10}$ | $x : \frac{1}{x} = \frac{x}{1} : \frac{1}{x}$

$= \frac{x}{1} \cdot \left(\frac{1}{x}\right)^{-1}$

$= \frac{x}{1} \cdot \frac{x}{1}$

$\boxed{x : \frac{1}{y} = x \cdot \left(\frac{1}{y}\right)^{-1} = xy.}$

37. Arătați că sistemul de ecuații liniare $\begin{cases} 2x - y + 3z = 1 \\ x + y + z = 2 \\ x - 2y + 2z = -1 \end{cases}$ are o infinitate de soluții.

$x = \alpha \in \mathbb{R}$

$2\alpha - y + 3z = 1$

$y = 2\alpha + 3z - 1$

$\alpha + 2\alpha + 3z - 1 + z = 2$

$\boxed{3\alpha + 4z = 3}$

$4z = 3 - 3\alpha \Rightarrow \boxed{z = \frac{3 - 3\alpha}{4}}$

S: $\left\{\left(\alpha, \frac{-\alpha+5}{4}, \frac{3-3\alpha}{4}\right), \alpha \in \mathbb{R}\right\}$

$y = 2\alpha + 3z - 1 = 2\alpha + \frac{3}{4}(3 - 3\alpha) - 1$

$y = \frac{8\alpha + 9 - 9\alpha - 4}{4} = \frac{-\alpha + 5}{4}$

Bacalaureat, 200

40. Se consideră sistemul $\begin{cases} x + my + 2z = 1 \\ x + (2m-1)y + 3z = 1 \\ x + my + (m-3)z = 2m - 1 \end{cases}$, $m \in \mathbb{R}$.

a) Determinați $m \in \mathbb{R}$ pentru care sistemul are soluție unică.

b) Determinați $m \in \mathbb{R}$ pentru care sistemul este compatibil nedeterminat.

a) $A = \begin{vmatrix} 1 & m & 2 \\ 1 & 2m-1 & 3 \\ 1 & m & m-3 \end{vmatrix} = (2m-1)(m-3) + 5m - 2(2m-1) - 3m - m(m-3) \ne 0$.

$\cancel{2m^2} - 7m + 3 + \cancel{2m} - 4m + 2 - \cancel{m^2} + 3m \ne 0$

$m^2 - 6m + 5 \ne 0$

$\Delta = 36 - 20 = 16$ | $m_{1,2} = \frac{6 \pm 4}{2}$ — 5, 1 — $\boxed{m \in \mathbb{R} \setminus \{1, 5\}}$

b) $A = 0 \Rightarrow m = \{1, 5\}$

A|B

Caz $\boxed{m = 1}$

$A|B = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & -2 & 1 \end{pmatrix}$

1 iterație caz 1

4 iterații caz 2

Caz 1

It 3 $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \ne 0 \Rightarrow m = 1$ – nu este sol pt sistem comp ned

37. Arătați că sistemul de ecuații liniare $\begin{cases} 2x - y + 3z = 1 \\ x + y + z = 2 \\ x - 2y + 2z = -1 \end{cases}$ are o infinitate de soluții.

$\begin{cases} x + y = 1 \\ x + y = 6 \end{cases}$

inf soluții $A = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 2 \end{vmatrix} = 4 - 6 - 1 - 3 + 2 + 2 \cdot 4$

$= 10 - 10 = 0$. → inc / comp ned.

$A|B = \begin{pmatrix} 2 & -1 & 3 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -1 \end{pmatrix} \ne 0$

$A = \{1, 2\}$ $B = \{2, 3\}$

$A \cap B = \{2\}$

$A \cup B = \{1, 2, 3\}$

$A \times B = \{(1,2), (1,3), (2,2), (2,3)\}$

$\boxed{A|B}$

$X_1 \times X_2 \times X_3 = X$

$x = (x_1, x_2, x_3)$

It 1: $\begin{pmatrix} -1 & 3 & 1 \\ 1 & 1 & 2 \\ -2 & 2 & -1 \end{pmatrix} = 0$

$A \setminus B = \{1\}$

sol unică = comp. determinat ($\Delta \ne 0$)

inf de sol = comp. nedeterminat ($\Delta = 0$ și $\exists M \in \mathcal{M}_{3,3}$, $M \subset A|B$, a.î. det $M \ne 0$)

nu admite soluție = incompatibil ($\Delta = 0$ și $\exists M \in \mathcal{M}_{3,3}$, $M \subset A|B$, det $M \ne 0$)