SICP

God's Programming Book

Lecture-07 Recursion

Recursion

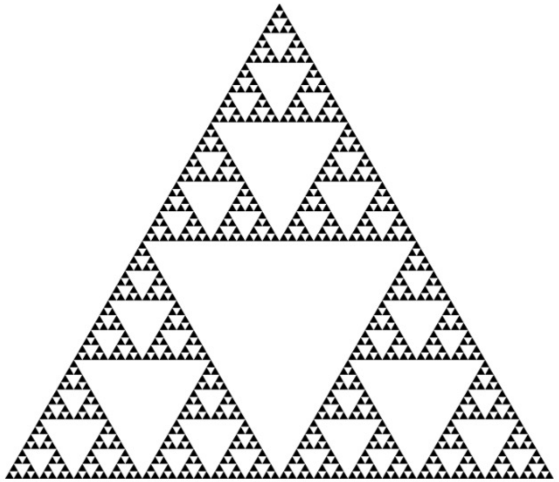
Slides Adapted from cs61a of UC Berkeley

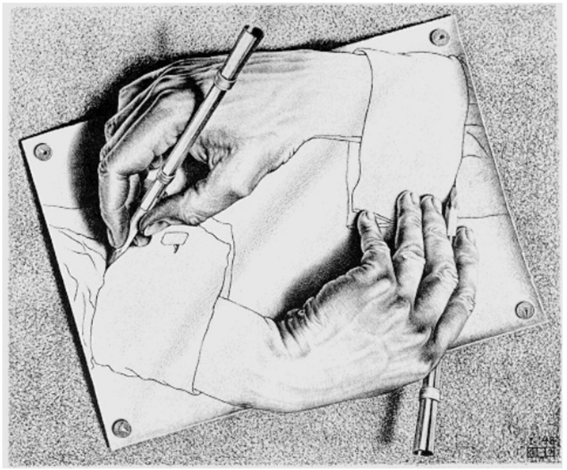
Recursive Functions

Recursive Functions

Definition: A function is called recursive if the body of that function calls itself, either directly or indirectly

Implication: Executing the body of a recursive function may require applying that function





Digit Sums

sum_digits(2019) = 2 + 0 + 1 + 9 = 12

The Problem Within the Problem

The sum of the digits of 6 is 6.

Likewise for any one-digit (non-negative) number (i.e., < 10).

The sum of the digits of 2019 is

`201` `9`

`Sum of these digits` + `This digit`

That is, we can break the problem of summing the digits of 2019 into a smaller instance of the same problem, plus some extra stuff.

We call this recursion

Sum Digits Without a While Statement

```
def split(n):
    """Split positive n into all but its last digit and its last digit."""
    return n // 10, n % 10


def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The def statement header is similar to other functions
- Conditional statements check for base cases
- Base cases are evaluated without recursive calls
- Recursive cases are evaluated with recursive calls

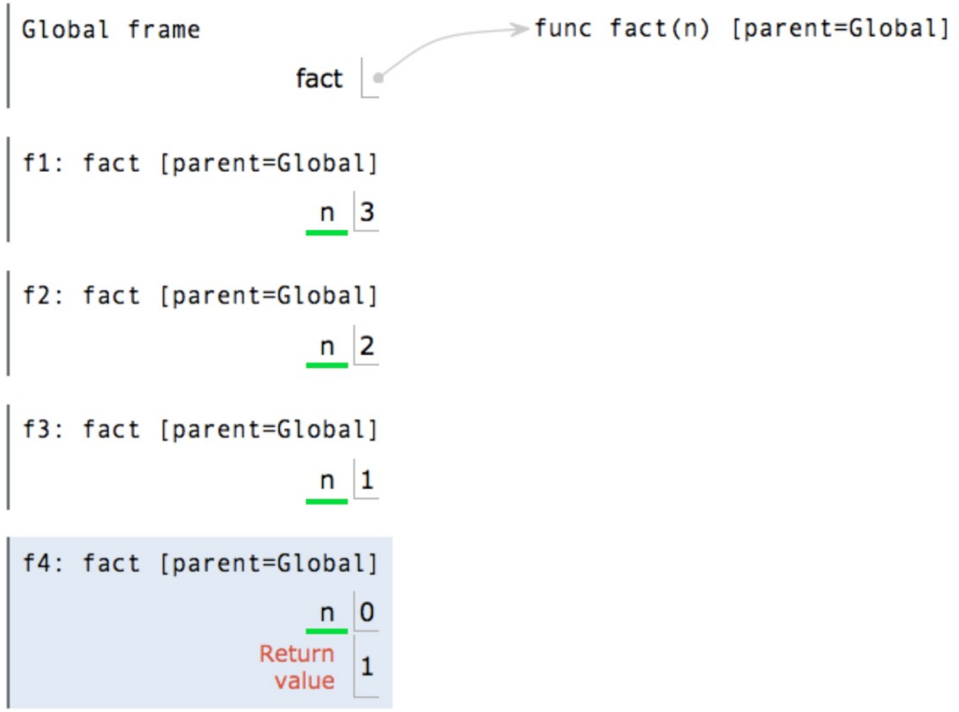
```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

Recursion in Environment Diagrams

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```



- The same function fact is called multiple times
- Different frames keep track of the different arguments in each call
- What n evaluates to depends upon the current environment
- Each call to fact solves a simpler problem than the last: smaller n

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using while:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

$$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Names:

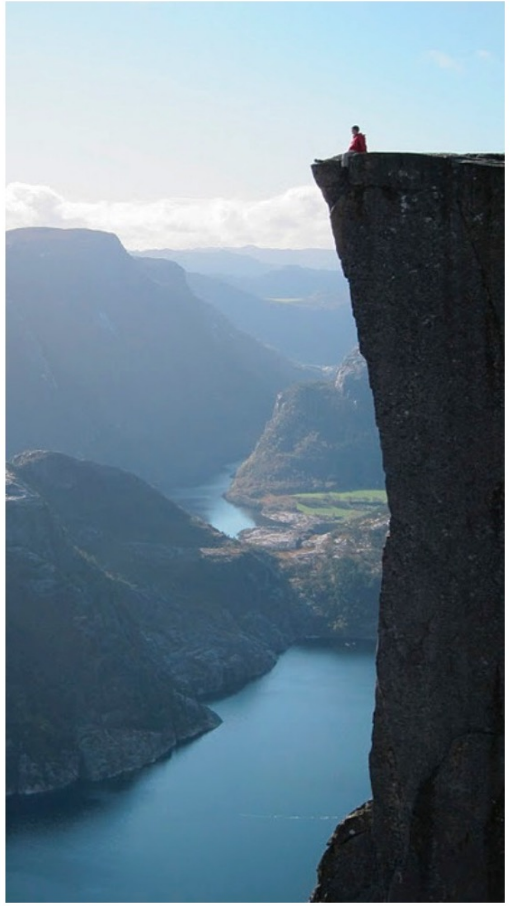
n, total, k, fact_iter

n, fact

Verifying Recursive Functions

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```



Is fact implemented correctly?

1. Verify the base case
2. Treat **fact** as a functional abstraction!
3. Assume that **fact(n-1)** is correct
4. Verify that **fact(n)** is correct

Mutual Recursion

(Demo)

Recursion and Iteration

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

A partial sum

What's left to sum

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Idea: The state of an iteration can be passed as arguments.

```
def sum_digits_iter(n):
    digit_sum = 0
    while n > 0:
        n, last = split(n)
        digit_sum = digit_sum + last
    return digit_sum
```

Updates via assignment become...

```
def sum_digits_rec(n, digit_sum):
    if n == 0:
        return digit_sum
    else:
        n, last = split(n)
        return sum_digits_rec(n, digit_sum + last)
```

...arguments to a recursive call

Thanks for Listening