Post Phi Calibration

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1 Introduction

In my experiments with Janus particles i had difficulties determining their orientation due to in homogeneous illumination by the dark field condenser. So as an attempt to account for that afterwards i wrote an algorithm that creates a binned mapping of the tracked angels form raw images that has a minimal overall shift of the angels. The code is found in the last section.

2 Theory

A typical histogram for orientations ϕ (Fig:**??**) obtained by a threshold and center of mass based tracking algorithm is non uniform as it should if there is no external magnetic or electric fields, geometrical restriction, additional dynamic of the particle or else that could cause a preferred orientation. This non uniformity is caused by an ill illumination by the dark field condenser and can be seen in Fig**??**, an image of a spherical polystyrene particle.

The idea of the algorithm is to reshape the bins from the ϕ_{raw} histogram to the same height while preserving the integral over a bin i.e. preserving the counts and the overall order of the bins.

$$d_0 \cdot h_i = N_i \tag{1}$$

$$h_0 = <h> = \frac{\sum_i^n h_i}{n} \tag{2}$$

$$a_i = \frac{h_0}{h_i} \tag{3}$$

$$d_i = \frac{d_0}{a_i} \tag{4}$$

$$\tag{5}$$

where h_i is the initial height of a bin, d_0 is the initial bin size, a_i are coefficients and n is the number of bins. Note that

$$\sum_i^n d_0 = \sum_i^n d_i = 2\pi \tag{6}$$

Aligning these new bins by placing the center of the first new bin $\tilde{\phi}_i$ at θ leads to displacements of the bin centers $d\phi_i$

$$\tilde{\phi}_1 = \phi_1 + h \tag{7}$$

$$\tilde{\phi}_{i+1} = \tilde{\phi}_i + \frac{1}{2}(d_i + d_{i+1}) \tag{8}$$

ϕ_i are the initial bin centers. The mean squared displacement of those which can be minimized with respect to θ

$$\sigma(\theta) = \sum_i^n d\phi_i^2 \tag{9}$$

$$\left.\frac{\partial \sigma}{\partial \theta}\right|_{\theta_{min}} = 0 \tag{10}$$

For the minimization i used a function from the scipy packege called fmin. This new arrangement for the bins sets up the mapping where it is looked up in which initial bin some tracked raw ϕ is located and replaced at a distance from the bin centers new position that is relative to the old bins center scaled with the coefficient a_i of that bin.

$$\phi_{new} = \tilde{\phi}_i + \frac{(\phi - \phi_i)}{a_i} \tag{11}$$

where i is the corresponding bin index.