

Given the electromagnetic field $E(x, y, 0)$ defined in the plane $z = 0$, the field $E(x, y, z)$ in the plane z can be calculated using the Fresnel diffraction integral:

$$E(x, y, z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x', y', 0) e^{i\frac{k}{2z}((x-x')^2 + (y-y')^2)} dx' dy' \quad (1)$$

$$E(x, y, z) = E(x, y, 0) * h(x, y, z) , \quad (2)$$

where

$$h(x, y, z) = \frac{e^{ikz}}{i\lambda z} e^{i\frac{k}{2z}(x^2 + y^2)} \quad (3)$$

$$(f * g)(x) = \int f(x) g(x - x') dx' \quad (4)$$

Since $\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$

$$\check{E}(f_x, f_y, z) = \check{E}(f_x, f_y, 0) H(f_x, f_y, z) \quad (5)$$

$$H(f_x, f_y, z) = \mathcal{F}\{h(x, y, z)\} \quad (6)$$

$$= \frac{e^{ikz}}{i\lambda z} \mathcal{F}\left\{e^{i\frac{k}{2z}x^2}\right\} \mathcal{F}\left\{e^{i\frac{k}{2z}y^2}\right\} \quad (7)$$

$$\mathcal{F}\left\{e^{i\alpha t^2}\right\} = \sqrt{\frac{\pi}{\alpha}} e^{i\frac{\pi}{4}} e^{-i\frac{(2\pi f)^2}{4\alpha}} \quad (8)$$

$$H(f_x, f_y, z) = \frac{e^{ikz}}{i\lambda z} \sqrt{\frac{2\pi z}{k}} e^{i\frac{\pi}{4}} e^{-i\frac{4\pi^2 f_x^2 2z}{4k}} \sqrt{\frac{2\pi z}{k}} e^{i\frac{\pi}{4}} e^{-i\frac{4\pi^2 f_y^2 2z}{4k}} \quad (9)$$

$$= \frac{e^{ikz}}{i\lambda z} \frac{2\pi z}{k} e^{-i\pi\lambda z f_x^2} e^{-i\pi\lambda z f_y^2} = e^{ikz} e^{-i\pi\lambda z (f_x^2 + f_y^2)} \quad (10)$$

$$H(f_x, f_y, z) = e^{ikz} e^{-i\pi\lambda z (f_x^2 + f_y^2)} \quad (11)$$

$$\check{E}(f_x, f_y, z) = e^{ikz} e^{-i\pi\lambda z (f_x^2 + f_y^2)} \check{E}(f_x, f_y, 0) \quad (12)$$

Propagate from the SLM plane $E_{\text{SLM}}(x, y)$ to the lense:

$$E_{\text{Lense}}^-(x, y) = \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2 + y^2)} \iint_{-\infty}^{+\infty} E_{\text{SLM}}(x', y') e^{i\frac{k}{2f}(x'^2 + y'^2)} e^{-i\frac{2\pi}{\lambda f}(xx' + yy')} dx' dy' \quad (13)$$

$$E_{\text{Lens}}^+(x, y) = E_{\text{Lens}}^-(x, y) e^{-i\frac{k}{2f}(x^2 + y^2)} \quad (14)$$

Propagate from $E_{\text{Lense}}^+(x, y)$ to the focal plane:

$$E_{\text{Focus}}(x, y) = \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} \iint_{-\infty}^{+\infty} E_{\text{Lense}}^+(x', y') e^{i\frac{k}{2f}(x'^2+y'^2)} e^{-i\frac{2\pi}{\lambda f}(xx'+yy')} dx' dy' \quad (15)$$

...:

$$E_{\text{Focus}}(x, y) = \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} \iint_{-\infty}^{+\infty} E_{\text{Lense}}^-(x', y') e^{-i\frac{2\pi}{\lambda f}(xx'+yy')} dx' dy' \quad (16)$$

$$= \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} 4\pi^2 \check{E}_{\text{Lense}}^-(f_x, f_y) \quad (17)$$

$$\check{E}_{\text{Lense}}^-(f_x, f_y) = e^{ikf} e^{-i\pi\lambda f(f_x^2+f_y^2)} 4\pi^2 \check{E}_{\text{SLM}}^-(f_x, f_y) \quad (18)$$

$$f_x = x/(\lambda f), f_y = y/(\lambda f)$$

$$E_{\text{Focus}}(x, y) = \frac{e^{2ikf}}{i\lambda f} 4\pi^2 \check{E}_{\text{SLM}}(f_x, f_y) \quad (19)$$

Let's consider a the relation between the field at the SLM plane and the field at the planes different but close to the focal we find:

$$E_{\text{Focus}}(x, y, z) = \frac{e^{ik(f+z)}}{i\lambda(f+z)} e^{i\frac{k}{2(f+z)}(x^2+y^2)} \iint_{-\infty}^{+\infty} E_{\text{Lense}}^+(x', y') e^{i\frac{k}{2(f+z)}(x'^2+y'^2)} e^{-i\frac{2\pi}{\lambda(f+z)}(xx'+yy')} dx' dy' \quad (20)$$

$$\lim_{z \rightarrow 0} \frac{k}{2(f+z)} \approx \frac{k}{2f} - \frac{kz}{2f^2} + \dots \quad (21)$$

$$E_{\text{Focus}}(x, y, z) \approx \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} e^{-i\frac{kz}{2f^2}(x^2+y^2)} \iint_{-\infty}^{+\infty} E_{\text{Lense}}^-(x', y') e^{-i\frac{kz}{2f^2}(x'^2+y'^2)} e^{-i\frac{2\pi}{\lambda f}(xx'+yy')} dx' dy' \quad (22)$$

$$E_{\text{Focus}}(x, y, z) = \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} e^{-i\frac{kz}{2f^2}(x^2+y^2)} \mathcal{F} \left\{ E_{\text{Lense}}^-(x, y) e^{-i\frac{kz}{2f^2}(x^2+y^2)} \right\} \quad (23)$$

$$\mathcal{F} \left\{ E_{\text{Lense}}^-(x, y) e^{-i\frac{kz}{2f^2}(x^2+y^2)} \right\} = e^{ik(f+z)} e^{-i\pi\lambda(f+z)(f_x^2+f_y^2)} 4\pi^2 \mathcal{F} \left\{ E_{\text{SLM}}(x, y) e^{-i\frac{kz}{2f^2}(x^2+y^2)} \right\} \quad (24)$$

$$e^{-i\pi\lambda(f+z)(f_x^2+f_y^2)} = e^{-i\frac{\pi\lambda(f+z)}{\lambda^2 f^2}(x^2+y^2)} = e^{-i\frac{k}{2f}} e^{-i\frac{kz}{2f^2}} \quad (25)$$

$$E_{\text{Focus}}(x, y, z) \approx \frac{e^{ik(2f+z)}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} e^{-i\frac{kz}{2f^2}(x^2+y^2)} e^{-i\frac{k}{2f}(x^2+y^2)} e^{-i\frac{kz}{2f^2}(x^2+y^2)} \mathcal{F} \left\{ E_{\text{SLM}}(x, y) e^{-i\frac{kz}{2f^2}(x^2+y^2)} \right\} \quad (26)$$

$$E_{\text{Focus}}(x, y, z) \approx \frac{e^{ik(2f+z)}}{i\lambda f} e^{-i\frac{kz}{f^2}(x^2+y^2)} \mathcal{F} \left\{ E_{\text{SLM}}(x, y) e^{-i\frac{kz}{2f^2}(x^2+y^2)} \right\} \quad (27)$$