Given the electromagnetic field $E(x, y, 0)$ defined in the plane $z = 0$, the field $E(x, y, z)$ in the plane z can be calculated using the Fresnel diffraction integral:

$$E(x, y, z) = \frac{e^{ikz}}{i\lambda z} \iint\limits_{-\infty}^{+\infty} E(x', y', 0)\, e^{i\frac{k}{2z}\left((x-x')^2 + (y-y')^2\right)}\, dx'dy' \tag{1}$$

$$E(x, y, z) = E(x, y, 0) * h(x, y, z) \,, \tag{2}$$

where

$$h(x, y, z) = \frac{e^{ikz}}{i\lambda z} e^{i\frac{k}{2z}(x^2+y^2)} \tag{3}$$

$$(f * g)(x) = \int f(x)\, g(x - x')\, dx' \tag{4}$$

Since $\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$

$$\check{E}(f_x, f_y, z) = \check{E}(f_x, f_y, 0) H(f_x, f_y, z) \tag{5}$$

$$H(f_x, f_y, z) = \mathcal{F}\{h(x, y, z)\} \tag{6}$$

$$= \frac{e^{ikz}}{i\lambda z} \mathcal{F}\left\{e^{i\frac{k}{2z}x^2}\right\} \mathcal{F}\left\{e^{i\frac{k}{2z}y^2}\right\} \tag{7}$$

$$\mathcal{F}\left\{e^{i\alpha t^2}\right\} = \sqrt{\frac{\pi}{\alpha}}\, e^{i\frac{\pi}{4}}\, e^{-i\frac{(2\pi f)^2}{4\alpha}} \tag{8}$$

$$= \frac{e^{ikz}}{i\lambda z} \sqrt{\frac{2\pi z}{k}}\, e^{i\frac{\pi}{4}}\, e^{-i\frac{4\pi^2 f_x^2 2z}{4k}} \sqrt{\frac{2\pi z}{k}}\, e^{i\frac{\pi}{4}}\, e^{-i\frac{4\pi^2 f_y^2 2z}{4k}} \tag{9}$$

$$= \frac{e^{ikz}}{i\lambda z} \frac{2\pi z}{k}\, e^{-i\pi\lambda z f_x^2}\, e^{-i\pi\lambda z f_y^2} = e^{ikz} e^{-i\pi\lambda z(f_x^2+f_y^2)} \tag{10}$$

$$\check{E}(f_x, f_y, z) = e^{ikz} e^{-i\pi\lambda z(f_x^2+f_y^2)}\, \check{E}(f_x, f_y, 0) \tag{11}$$

$$\check{E}(f_x, f_y, z) = e^{ikz} e^{-i\pi\lambda z(f_x^2+f_y^2)}\, \check{E}(f_x, f_y, 0) \tag{12}$$

Propagate from the SLM plane $E_{\text{SLM}}(x, y)$ to the lense:

$$E_{\text{Lense}}^{-}(x, y) = \frac{e^{ikf}}{i\lambda f} e^{i\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E_{\text{SLM}}(x', y')\, e^{i\frac{k}{2f}(x'^2+y'^2)} e^{-i\frac{2\pi}{\lambda f}(xx'+yy')}\, dx'dy' \tag{13}$$

$$E_{\text{Lens}}^{+}(x, y) = E_{\text{Lens}}^{-}(x, y) e^{-i\frac{k}{2f}(x^2+y^2)} \tag{14}$$

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Propagate from $E_{\text{Lense}}^{+}(x, y)$ to the focal plane:

$$E_{\text{Focus}}(x, y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E_{\text{Lens}}^{+}(x', y')\, \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x'^2+y'^2)} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{15}$$

...:

$$E_{\text{Focus}}(x, y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E_{\text{Lens}}^{-}(x', y')\, \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{16}$$

$$= \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\, 4\pi^2\, \check{E}_{\text{Lens}}^{-}(f_x, f_y) \tag{17}$$

$$\check{E}_{\text{Lens}}^{-}(f_x, f_y) = \mathrm{e}^{\mathrm{i}kf} \mathrm{e}^{-\mathrm{i}\pi\lambda f(f_x^2+f_y^2)}\, 4\pi^2\, \check{E}_{\text{SLM}}^{-}(f_x, f_y) \tag{18}$$

$f_x = x/(\lambda f),\ f_y = y/(\lambda f)$

$$E_{\text{Focus}}(x, y) = \frac{\mathrm{e}^{2\mathrm{i}kf}}{\mathrm{i}\lambda f}\, 4\pi^2\, \check{E}_{\text{SLM}}(f_x, f_y) \tag{19}$$

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