Given the electromagnetic field $E(x,y,0)$ defined in the plane $z=0$, the field $E(x,y,z)$ in the plane z can be calculated using the Fresnel diffraction integral:

$$E(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \iint\limits_{-\infty}^{+\infty} E(x',y',0)\, \mathrm{e}^{\mathrm{i}\frac{k}{2z}\left((x-x')^2+(y-y')^2\right)}\, \mathrm{d}x'\mathrm{d}y' \tag{1}$$

$$E(x,y,z) = E(x,y,0) * h(x,y,z)\ , \tag{2}$$

where

$$h(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \mathrm{e}^{\mathrm{i}\frac{k}{2z}(x^2+y^2)} \tag{3}$$

$$(f*g)(x) = \int f(x)\, g(x-x')\, \mathrm{d}x' \tag{4}$$

Since $\mathcal{F}\{f*g\} = \mathcal{F}\{f\}\cdot\mathcal{F}\{g\}$

$$\check{E}(f_x,f_y,z) = \check{E}(f_x,f_y,0) H(f_x,f_y,z) \tag{5}$$

$$\begin{aligned} H(f_x,f_y,z) &= \mathcal{F}\{h(x,y,z)\} &(6)\\ &= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \mathcal{F}\left\{\mathrm{e}^{\mathrm{i}\frac{k}{2z}x^2}\right\} \mathcal{F}\left\{\mathrm{e}^{\mathrm{i}\frac{k}{2z}y^2}\right\} &(7) \end{aligned}$$

$$\mathcal{F}\left\{\mathrm{e}^{\mathrm{i}\alpha t^2}\right\} = \sqrt{\frac{\pi}{\alpha}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\, \mathrm{e}^{-\mathrm{i}\frac{(2\pi f)^2}{4\alpha}} \tag{8}$$

$$\begin{aligned} H(f_x,f_y,z) &= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \sqrt{\frac{2\pi z}{k}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\, \mathrm{e}^{-\mathrm{i}\frac{4\pi^2 f_x^2 2z}{4k}} \sqrt{\frac{2\pi z}{k}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\, \mathrm{e}^{-\mathrm{i}\frac{4\pi^2 f_y^2 2z}{4k}} &(9)\\ &= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z}\frac{2\pi z}{k}\, \mathrm{e}^{-\mathrm{i}\pi\lambda z f_x^2}\, \mathrm{e}^{-\mathrm{i}\pi\lambda z f_y^2} = \mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{-\mathrm{i}\pi\lambda z(f_x^2+f_y^2)} &(10) \end{aligned}$$

$$H(f_x,f_y,z) = \mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{-\mathrm{i}\pi\lambda z(f_x^2+f_y^2)} \tag{11}$$

$$\check{E}(f_x,f_y,z) = \mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{-\mathrm{i}\pi\lambda z(f_x^2+f_y^2)}\, \check{E}(f_x,f_y,0) \tag{12}$$

Propagate from the SLM plane $E_{\mathrm{SLM}}(x,y)$ to the lense:

$$E^-_{\mathrm{Lense}}(x,y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E_{\mathrm{SLM}}(x',y')\, \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x'^2+y'^2)} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{13}$$

$$E^+_{\mathrm{Lens}}(x,y) = E^-_{\mathrm{Lens}}(x,y)\mathrm{e}^{-\mathrm{i}\frac{k}{2f}(x^2+y^2)} \tag{14}$$

Propagate from $E^{+}_{\mathrm{Lense}}(x, y)$ to the focal plane:

$$E_{\mathrm{Focus}}(x, y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E^{+}_{\mathrm{Lens}}(x', y')\, \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x'^2+y'^2)} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{15}$$

...:

$$E_{\mathrm{Focus}}(x, y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E^{-}_{\mathrm{Lens}}(x', y')\, \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{16}$$

$$= \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\; 4\pi^2\, \check{E}^{-}_{\mathrm{Lens}}(f_x, f_y) \tag{17}$$

$$\check{E}^{-}_{\mathrm{Lens}}(f_x, f_y) = \mathrm{e}^{\mathrm{i}kf} \mathrm{e}^{-\mathrm{i}\pi\lambda f(f_x^2+f_y^2)}\, 4\pi^2\, \check{E}^{-}_{\mathrm{SLM}}(f_x, f_y) \tag{18}$$

$f_x = x/(\lambda f)$, $f_y = y/(\lambda f)$

$$\boxed{E_{\mathrm{Focus}}(x, y) = \frac{\mathrm{e}^{2\mathrm{i}kf}}{\mathrm{i}\lambda f}\, 4\pi^2\, \check{E}_{\mathrm{SLM}}(f_x, f_y)} \tag{19}$$

Let's consider a the relation between the field at the SLM plane and the field at the planes different but close to the focal we find:

$$E_{\mathrm{Focus}}(x, y, z) = \frac{\mathrm{e}^{\mathrm{i}k(f+z)}}{\mathrm{i}\lambda(f+z)} \mathrm{e}^{\mathrm{i}\frac{k}{2(f+z)}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E^{+}_{\mathrm{Lens}}(x', y')\, \mathrm{e}^{\mathrm{i}\frac{k}{2(f+z)}(x'^2+y'^2)} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda(f+z)}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{20}$$

$$\lim_{z\to 0} \frac{k}{2(f+z)} \approx \frac{k}{2f} - \frac{kz}{2f^2} + \ldots \tag{21}$$

$$E_{\mathrm{Focus}}(x, y, z) \approx \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E^{-}_{\mathrm{Lens}}(x', y')\, \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x'^2+y'^2)} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{22}$$

$$E_{\mathrm{Focus}}(x, y, z) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)} \mathcal{F}\left\{E^{-}_{\mathrm{Lens}}(x, y)\, \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} \tag{23}$$

$$\mathcal{F}\left\{E^{-}_{\mathrm{Lens}}(x, y)\, \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} = \mathrm{e}^{\mathrm{i}k(f+z)} \mathrm{e}^{-\mathrm{i}\pi\lambda(f+z)(f_x^2+f_y^2)}\, 4\pi^2\, \mathcal{F}\left\{E_{\mathrm{SLM}}(x, y)\, \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} \tag{24}$$

$$\mathrm{e}^{-\mathrm{i}\pi\lambda(f+z)(f_x^2+f_y^2)} = \mathrm{e}^{-\mathrm{i}\frac{\pi\lambda(f+z)}{\lambda^2 f^2}(x^2+y^2)} = \mathrm{e}^{-\mathrm{i}\frac{k}{2f}} \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}} \tag{25}$$

$$E_{\mathrm{Focus}}(x, y, z) \approx \frac{\mathrm{e}^{\mathrm{i}k(2f+z)}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)} \mathrm{e}^{-\mathrm{i}\frac{k}{2f}(x^2+y^2)} \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)} \mathcal{F}\left\{E_{\mathrm{SLM}}(x, y)\, \mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} \tag{26}$$

$$\boxed{E_{\mathrm{Focus}}(x,y,z) \approx \frac{\mathrm{e}^{\mathrm{i}k(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{-\mathrm{i}\frac{kz}{f^2}(x^2+y^2)}\mathcal{F}\left\{E_{\mathrm{SLM}}(x,y)\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\}} \tag{27}$$

In the following, we will consider the incoming beam to be a uniform plane wave, *i.e.*, $E_{\mathrm{Beam}}(x,y) = E_0$. The SLM is typically a two-dimensional pixellated device whose pixels have area d^2, row index $n_x = 1,\dots,N_x$ and column index $n_y = 1,\dots,N_y$. The complex amplitude imposed on the incoming beam by pixel (n_x,n_y) is $\alpha_{n_x,n_y} = \alpha\exp(\mathrm{i}\varphi_{n_x,n_y})$, where A is an attenuation constant and φ_{n_x,n_y} the phase shift. Therefore, the electric field after the SLM is $E_{n_x,n_y} = E_0\,\alpha\,\exp(\mathrm{i}\varphi_{n_x,n_y})$. From Eq. (27) we can now calculate the electric field in the trap volume around the focal plane of the objective as:

$$E_{\mathrm{Focus}}(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda}(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{-\mathrm{i}\frac{2\pi z}{\lambda f^2}(x^2+y^2)}d^2\sum_{n_x=1}^{N_x}\sum_{n_y=1}^{N_y}E_0\,\alpha\,\mathrm{e}^{\mathrm{i}\varphi_{n_x,n_y}-\mathrm{i}\frac{\pi z}{\lambda f^2}(x^2+y^2)-\mathrm{i}\frac{2\pi}{\lambda f}(x_{n_x,n_y}x+y_{n_x,n_y}y)} \tag{28}$$

$$E_{\mathrm{Focus}}(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda}(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{-\mathrm{i}\frac{2\pi z}{\lambda f^2}(x^2+y^2)}d^2\sum_{n_x=1}^{N_x}\sum_{n_y=1}^{N_y}E_0\,\alpha\,\mathrm{e}^{\mathrm{i}\left(\varphi_{n_x,n_y}-\Delta_{n_x,n_y}(x,y,z)\right)} \tag{29}$$

$$\Delta_{n_x,n_y}(x,y,z) = \frac{2\pi}{\lambda f}(x_{n_x,n_y}x+y_{n_x,n_y}y)+\frac{\pi z}{\lambda f^2}(x^2+y^2) \tag{30}$$

The power, *i.e.*, the total, time-averaged energy flux, right after the SLM is ($P_0 = I_0 A_{\mathrm{SLM}}$):

$$P_0 = \frac{c\varepsilon_0}{2}N_xN_y\,d^2\,|E_0|^2\,\alpha^2 \tag{31}$$

The time-averaged energy flux through a diffration-limited spot with area $f^2\lambda^2/(N_xN_yd^2)$ at (x,y,z) is:

$$P(x,y,z) = \frac{c\varepsilon_0}{2}\frac{f^2\lambda^2}{N_xN_yd^2}\left|E_{\mathrm{Focus}}(x,y,z)\right|^2 = \frac{c\varepsilon_0}{2}\frac{f^2\lambda^2}{N_xN_yd^2}\frac{d^4}{\lambda^2f^2}|E_0|^2\,\alpha^2\left|\sum_{n_x=1}^{N_x}\sum_{n_y=1}^{N_y}\mathrm{e}^{\mathrm{i}\left(\varphi_{n_x,n_y}-\Delta_{n_x,n_y}(x,y,z)\right)}\right|^2$$

We introduce the dimensionless variable:

$$\boxed{V(x,y,z) = \frac{1}{N_xN_y}\sum_{n_x=1}^{N_x}\sum_{n_y=1}^{N_y}\mathrm{e}^{\mathrm{i}\left(\varphi_{n_x,n_y}-\Delta_{n_x,n_y}(x,y,z)\right)}}\,, \tag{32}$$

defined as:

$$|V(x,y,z)|^2 = \frac{P(x,y,z)}{P_0}\,. \tag{33}$$