Given the electromagnetic field $E(x, y, 0)$ defined in the plane $z = 0$, the field $E(x, y, z)$ in the plane z can be calculated using the Fresnel diffraction integral:

$$E(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \iint\limits_{-\infty}^{+\infty} E(x',y',0)\, \mathrm{e}^{\mathrm{i}\frac{k}{2z}\left((x-x')^2+(y-y')^2\right)}\, \mathrm{d}x'\mathrm{d}y' \tag{1}$$

$$E(x,y,z) = E(x,y,0) * h(x,y,z)\ , \tag{2}$$

where

$$h(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \mathrm{e}^{\mathrm{i}\frac{k}{2z}(x^2+y^2)} \tag{3}$$

$$(f * g)(x) = \int f(x)\, g(x-x')\, \mathrm{d}x' \tag{4}$$

Since $\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$

$$\check{E}(f_x,f_y,z) = \check{E}(f_x,f_y,0) H(f_x,f_y,z) \tag{5}$$

$$\begin{align} H(f_x,f_y,z) &= \mathcal{F}\{h(x,y,z)\} \tag{6} \\ &= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \mathcal{F}\left\{\mathrm{e}^{\mathrm{i}\frac{k}{2z}x^2}\right\} \mathcal{F}\left\{\mathrm{e}^{\mathrm{i}\frac{k}{2z}y^2}\right\} \tag{7} \end{align}$$

$$\mathcal{F}\left\{\mathrm{e}^{\mathrm{i}\alpha t^2}\right\} = \sqrt{\frac{\pi}{\alpha}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\, \mathrm{e}^{-\mathrm{i}\frac{(2\pi f)^2}{4\alpha}} \tag{8}$$

$$\begin{align} H(f_x,f_y,z) &= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \sqrt{\frac{2\pi z}{k}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\, \mathrm{e}^{-\mathrm{i}\frac{4\pi^2 f_x^2 2z}{4k}} \sqrt{\frac{2\pi z}{k}}\, \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\, \mathrm{e}^{-\mathrm{i}\frac{4\pi^2 f_y^2 2z}{4k}} \tag{9} \\ &= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \frac{2\pi z}{k}\, \mathrm{e}^{-\mathrm{i}\pi\lambda z f_x^2}\, \mathrm{e}^{-\mathrm{i}\pi\lambda z f_y^2} = \mathrm{e}^{\mathrm{i}kz} \mathrm{e}^{-\mathrm{i}\pi\lambda z(f_x^2+f_y^2)} \tag{10} \end{align}$$

$$H(f_x,f_y,z) = \mathrm{e}^{\mathrm{i}kz} \mathrm{e}^{-\mathrm{i}\pi\lambda z(f_x^2+f_y^2)} \tag{11}$$

$$\check{E}(f_x,f_y,z) = \mathrm{e}^{\mathrm{i}kz} \mathrm{e}^{-\mathrm{i}\pi\lambda z(f_x^2+f_y^2)}\, \check{E}(f_x,f_y,0) \tag{12}$$

Propagate from the SLM plane $E_{\mathrm{SLM}}(x, y)$ to the lense:

$$E^{-}_{\mathrm{Lense}}(x,y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f} \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} E_{\mathrm{SLM}}(x',y')\, \mathrm{e}^{\mathrm{i}\frac{k}{2f}(x'^2+y'^2)} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\, \mathrm{d}x'\mathrm{d}y' \tag{13}$$

$$E^{+}_{\mathrm{Lens}}(x,y) = E^{-}_{\mathrm{Lens}}(x,y) \mathrm{e}^{-\mathrm{i}\frac{k}{2f}(x^2+y^2)} \tag{14}$$

Propagate from $E^+_{\mathrm{Lense}}(x,y)$ to the focal plane:

$$E_{\mathrm{Focus}}(x,y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f}\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\iint\limits_{-\infty}^{+\infty} E^+_{\mathrm{Lens}}(x',y')\,\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x'^2+y'^2)}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\,\mathrm{d}x'\mathrm{d}y' \tag{15}$$

...:

$$E_{\mathrm{Focus}}(x,y) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f}\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\iint\limits_{-\infty}^{+\infty} E^-_{\mathrm{Lens}}(x',y')\,\mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\,\mathrm{d}x'\mathrm{d}y' \tag{16}$$

$$= \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f}\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\;4\pi^2\,\check{E}^-_{\mathrm{Lens}}(f_x,f_y) \tag{17}$$

$$\check{E}^-_{\mathrm{Lens}}(f_x,f_y) = \mathrm{e}^{\mathrm{i}kf}\mathrm{e}^{-\mathrm{i}\pi\lambda f(f_x^2+f_y^2)}\,4\pi^2\,\check{E}^-_{\mathrm{SLM}}(f_x,f_y) \tag{18}$$

$f_x = x/(\lambda f),\ f_y = y/(\lambda f)$

$$\boxed{E_{\mathrm{Focus}}(x,y) = \frac{\mathrm{e}^{2\mathrm{i}kf}}{\mathrm{i}\lambda f}\,4\pi^2\,\check{E}_{\mathrm{SLM}}(f_x,f_y)} \tag{19}$$

Let's consider a the relation between the field at the SLM plane and the field at the planes different but close to the focal we find:

$$E_{\mathrm{Focus}}(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}k(f+z)}}{\mathrm{i}\lambda(f+z)}\mathrm{e}^{\mathrm{i}\frac{k}{2(f+z)}(x^2+y^2)}\iint\limits_{-\infty}^{+\infty} E^+_{\mathrm{Lens}}(x',y')\,\mathrm{e}^{\mathrm{i}\frac{k}{2(f+z)}(x'^2+y'^2)}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda(f+z)}(xx'+yy')}\,\mathrm{d}x'\mathrm{d}y' \tag{20}$$

$$\lim_{z\to 0}\ \frac{k}{2(f+z)} \approx \frac{k}{2f} - \frac{kz}{2f^2} + \ldots \tag{21}$$

$$E_{\mathrm{Focus}}(x,y,z) \approx \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f}\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\iint\limits_{-\infty}^{+\infty} E^-_{\mathrm{Lens}}(x',y')\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x'^2+y'^2)}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda f}(xx'+yy')}\,\mathrm{d}x'\mathrm{d}y' \tag{22}$$

$$E_{\mathrm{Focus}}(x,y,z) = \frac{\mathrm{e}^{\mathrm{i}kf}}{\mathrm{i}\lambda f}\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\mathcal{F}\left\{E^-_{\mathrm{Lens}}(x,y)\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} \tag{23}$$

$$\mathcal{F}\left\{E^-_{\mathrm{Lens}}(x,y)\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} = \mathrm{e}^{\mathrm{i}k(f+z)}\mathrm{e}^{-\mathrm{i}\pi\lambda(f+z)(f_x^2+f_y^2)}\,4\pi^2\,\mathcal{F}\left\{E_{\mathrm{SLM}}(x,y)\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} \tag{24}$$

$$\mathrm{e}^{-\mathrm{i}\pi\lambda(f+z)(f_x^2+f_y^2)} = \mathrm{e}^{-\mathrm{i}\frac{\pi\lambda(f+z)}{\lambda^2 f^2}(x^2+y^2)} = \mathrm{e}^{-\mathrm{i}\frac{k}{2f}}\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}} \tag{25}$$

$$E_{\mathrm{Focus}}(x,y,z) \approx \frac{\mathrm{e}^{\mathrm{i}k(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{\mathrm{i}\frac{k}{2f}(x^2+y^2)}\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\mathrm{e}^{-\mathrm{i}\frac{k}{2f}(x^2+y^2)}\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\mathcal{F}\left\{E_{\mathrm{SLM}}(x,y)\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\} \tag{26}$$

$$\boxed{E_{\text{Focus}}(x,y,z) \approx \frac{\mathrm{e}^{\mathrm{i}k(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{-\mathrm{i}\frac{kz}{f^2}(x^2+y^2)}\mathcal{F}\left\{E_{\text{SLM}}(x,y)\,\mathrm{e}^{-\mathrm{i}\frac{kz}{2f^2}(x^2+y^2)}\right\}} \tag{27}$$

In the following, we will consider the incoming beam to be a uniform plane wave, *i.e.*, $E_{\text{Beam}}(x,y) = E_0$. The SLM is typically a two-dimensional pixellated device whose pixels have area d^2, row index $n_x = 1,\ldots,N_x$ and column index $n_y = 1,\ldots,N_y$. The complex amplitude imposed on the incoming beam by pixel (n_x,n_y) is $A_{n_x,n_y} = A\exp(\mathrm{i}\varphi_{n_x,n_y})$, where A is an attenuation constant and φ_{n_x,n_y} the phase shift. Therefore, the electric field after the SLM is $E_{n_x,n_y} = E_0 A\exp(\mathrm{i}\varphi_{n_x,n_y})$. From Eq. (27) we can now calculate the electric field in the trap volume around the focal plane of the objective as:

$$E_{\text{Focus}}(x,y,z) \approx \frac{\mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda}(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{-\mathrm{i}\frac{2\pi z}{\lambda f^2}(x^2+y^2)}d^2\sum_{n_x}^{N_x}\sum_{n_y}^{N_y}E_0A\mathrm{e}^{\mathrm{i}\varphi_{n_x,n_y}-\mathrm{i}\frac{\pi z}{\lambda f^2}(x^2+y^2)-\mathrm{i}\frac{2\pi}{\lambda f}(x_{n_x,n_y}x+y_{n_x,n_y}y)} \tag{28}$$

$$E_{\text{Focus}}(x,y,z) \approx \frac{\mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda}(2f+z)}}{\mathrm{i}\lambda f}\mathrm{e}^{-\mathrm{i}\frac{2\pi z}{\lambda f^2}(x^2+y^2)}d^2\sum_{n_x}^{N_x}\sum_{n_y}^{N_y}E_0A\mathrm{e}^{\mathrm{i}\varphi_{n_x,n_y}-\Delta_{n_x,n_y}(x,y,z)} \tag{29}$$

$$\Delta_{n_x,n_y}(x,y,z) = \mathrm{i}\frac{2\pi}{\lambda f}(x_{n_x,n_y}x+y_{n_x,n_y}y)+\mathrm{i}\frac{\pi z}{\lambda f^2}(x^2+y^2) \tag{30}$$