Tarea 5 Algebra

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1) $A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ Sea $A \in \mathbb{C}^{4\times 4}$

$$\det(A - \lambda Id) = \det \begin{pmatrix} 2-\lambda & 0 & 0 & 0 \\ 0 & 1-\lambda & -1 & 0 \\ 0 & 2 & 1-\lambda & 0 \\ 0 & 0 & 0 & 2-\lambda \end{pmatrix}$$

des. por filas

$$(A - \lambda Id)(1|1) = \begin{pmatrix} 1-\lambda & -1 & 0 \\ 2 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix} - 0\,(1) = (*)$$

- Como el valor es 0 y lo multiplico por 0 es 0, salteo el desarrollo ya que sabemos el resultado.

$\rightarrow (*) = \det(A - \lambda Id)(1|1) - (*)$

Por regla de sarrus:

$$|B| = \begin{pmatrix} 1-\lambda & -1 & 0 \\ 2 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \\ 1-\lambda & -1 & 0 \\ 2 & 1-\lambda & 0 \end{pmatrix}$$

$$|B| = (1-\lambda)(1-\lambda)(2-\lambda) + 2\cdot 0\cdot 0 + 0(-1)0 - 0(1-\lambda)0 - (1-\lambda)00 - \ldots$$
$$- 2(-1)(2-\lambda) =$$
$$= (1-\lambda)^2(2-\lambda) + 2(2-\lambda)$$
$$= (2-\lambda)(\lambda^2 - 2\lambda + 3) = 2\lambda^2 - 4\lambda + 6 - \lambda^3 + 2\lambda^2 - 3\lambda =$$
$$= -\lambda^3 + 4\lambda^2 - 7\lambda + 6$$

$$(*) = (2-\lambda)(-\lambda^3 + 4\lambda - 7\lambda + 6) =$$
$$= -2\lambda^3 + 8\lambda^2 - 14\lambda + 12 + \lambda^4 - 4\lambda^3 + 7\lambda^2 - 6\lambda =$$
$$\boxed{= \lambda^4 - 6\lambda^3 + 15\lambda^2 - 20\lambda + 12}$$

El polinomio caract es $= \lambda^4 - 6\lambda^3 + 15\lambda^2 - 20\lambda + 12$

Asamblea

2) Autovalores? $\longrightarrow$ igualar el pol. caract a 0 y buscar raíces

$$\lambda^4 - 6\lambda^3 + 15\lambda^2 - 20\lambda + 12 = 0$$

$$(2-\lambda)(-\lambda^3 + 4\lambda^2 - 7\lambda + 6) = 0$$

$$(2-\lambda)(2-\lambda)(\lambda^2 - 2\lambda + 3) = 0$$

$$(-(-2+\lambda))(-(-2+\lambda))(\lambda^2 - 2\lambda + 3) = 0$$

$$(\lambda-2)^2(\lambda^2 - 2\lambda + 3) = 0 \longrightarrow$$

$$\boxed{\begin{aligned} &\frac{2 \pm \sqrt{(-2)^2 - 4\cdot 1\cdot 3}}{2\cdot 1} \\ &= \frac{2 \pm \sqrt{-8}}{2} \\ &= \frac{2 \pm 2\sqrt{2}\, i}{2} \\ &= 1 \pm \sqrt{2}\, i \\ &x_1 = 1 + \sqrt{2}\, i \\ &x_2 = 1 - \sqrt{2}\, i \end{aligned}}$$

$$(\lambda-2)^2(\lambda - 1 - \sqrt{2}\, i)(\lambda - 1 + \sqrt{2}\, i) = 0$$

Rta los autovalores son:

$\lambda = 2$ $\qquad \lambda_2 = 1 + \sqrt{2}\, i$

$\lambda_3 = 2$ $\qquad \lambda_4 = 1 - \sqrt{2}\, i$

3) Autovalores $= \{2,\ 1+\sqrt{2}i,\ 1-\sqrt{2}i\}$

- $(A - 2Id)\, X_2 = 0$

· Planteamos matriz ampliada para poder trabajar

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{F_3 + 2F_2} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{F_2 - \frac{1}{3}F_3}$$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[F_3/-3]{F_2/-1} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$X_2 = \left\{ s \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \;\middle|\; s, r \in \mathbb{C} \right\}$$

con $v_2 = v_3 = 0$ y v_1 y v_2 son variables libres

- $(A - (1+\sqrt{2}i)\, Id)\, X_{(1+\sqrt{2}i)} = 0$

$$\left(\begin{array}{cccc|c} 1-\sqrt{2}i & 0 & 0 & 0 & 0 \\ 0 & i\sqrt{2} & -1 & 0 & 0 \\ 0 & 2 & -i\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1-\sqrt{2}i & 0 \end{array}\right) \xrightarrow{F_3 + (-\sqrt{2}i)F_2} \left(\begin{array}{cccc|c} 1-\sqrt{2}i & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{2}i & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\sqrt{2}i & 0 \end{array}\right) \xrightarrow{F_2/-1}$$

$$\left(\begin{array}{cccc|c} 1-\sqrt{2}i & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-i\sqrt{2} & 0 \end{array}\right) \xrightarrow[F_4/1-\sqrt{2}i]{F_1/1-i\sqrt{2}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & i\sqrt{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$V = \begin{pmatrix} 0 \\ s \\ -\sqrt{2}i\,s \\ 0 \end{pmatrix} = s \begin{pmatrix} 0 \\ 1 \\ -\sqrt{2}i \\ 0 \end{pmatrix}$$

$$X_{1+\sqrt{2}i} = \left\{ s \begin{pmatrix} 0 \\ 1 \\ -\sqrt{2}i \\ 0 \end{pmatrix} \;\middle|\; s \in \mathbb{C} \right\}$$

● $A-(1-\sqrt{2})\,Id\; X_{(1-\sqrt{2})} = 0$

$$\left\{\begin{array}{cccc|c} 1+\sqrt{2}i & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}i & -1 & 0 & 0 \\ 0 & 2 & \sqrt{2}i & 0 & 0 \\ 0 & 0 & 0 & 1+\sqrt{2}i & 0 \end{array}\right\} \xrightarrow{F_3+\sqrt{2}i\,F_2} \left\{\begin{array}{cccc|c} 1+\sqrt{2}i & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}i & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\sqrt{2}i & 0 \end{array}\right\}$$

$$\xrightarrow[F_2/1+\sqrt{2}i]{F_1/1+\sqrt{2}i} \left\{\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}i & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right\} \xrightarrow{-1\cdot F_2} \left\{\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{2}i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right\}$$

$$V = \begin{pmatrix} 0 \\ s \\ \sqrt{2}i\,s \\ 0 \end{pmatrix} = s\begin{pmatrix} 0 \\ 1 \\ \sqrt{2}i \\ 0 \end{pmatrix}$$

$$X_{(1-\sqrt{2})} = \left\{ s\begin{pmatrix} 0 \\ 1 \\ \sqrt{2}i \\ 0 \end{pmatrix} \,\middle|\, s \in \mathbb{C} \right\} \rightarrow \text{con } v_1 = v_4 = 0 \text{ y } v_3 = \sqrt{2}i \cdot v_2$$