Analisis Parcial 2 | Molina Franco 44192153

1) a) Taylor $n=3$, $a=2$

- $f_{(x)} = \ln(x) \longrightarrow f_{(2)} = \ln(2)$

$f'_{(x)} = \frac{1}{x} \longrightarrow f'_{(2)} = \frac{1}{2}$

$f''_{(x)} = \frac{-1}{x^2} \longrightarrow f''_{(2)} = \frac{-1}{4}$

$f'''_{(x)} = \frac{2}{x^3} \longrightarrow f'''_{(2)} = \frac{1}{4}$

$f''''_{(x)} = \frac{-6}{x^4}$

- Polinomio de Taylor

$$T_{3,2}(x) = \frac{\ln(2)}{0!}(x-2)^0 + \frac{1}{2 \cdot 1!}(x-2)^1 - \frac{1}{4 \cdot 2!}(x-2)^2 + \frac{1}{4 \cdot 3!}(x-2)^3 =$$

$$\boxed{T_{3,2}(x) = \ln(2) + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3}$$

- Aproximo para $\ln(2,5)$

$$\boxed{T_{3,2}\left(\tfrac{5}{2}\right) = \ln(2) + \frac{1}{4} - \frac{1}{32} + \frac{1}{192}}$$

- Estimo el error (lagrange)

$$R_{n,a}(x) = \frac{f^{(n+1)}(t)}{(n+1)!}(x-a)^{n+1}$$

$$R_{3,2}\left(\tfrac{5}{2}\right) = \left| \frac{f^4(t)}{4!}\left(\frac{5}{2}-2\right)^4 \right|$$

$$= \frac{-6}{t^4} \cdot \frac{1}{24} \cdot \frac{1}{16} \qquad t \in \left(2, \tfrac{5}{2}\right)$$

- El error mas grande es 2

- $\left| \frac{-6}{384\, t^4} \right| < \left| \frac{-6}{2^4 \cdot 384} \right| = \boxed{\frac{1}{1024}}$

$\boxed{\text{• El resto es } \frac{1}{1024}}$

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1) b) $r(t) = (\ln(1-t^2)), (\sqrt{1+t}), (-e^{2t})$

Dominio? vect tang. $(t=0)$

$a(t) = \ln(1-t^2)$ $\qquad$ $b(t) = \sqrt{1+t}$

$Dom(a) = 1-t^2 > 0$ $\qquad$ $Dom(b) = 1+t \geq 0$

$= -1 < t < 1$ $\qquad$ $= t \geq -1$

$c(t) = -e^{2t}$

$Dom(c) = \mathbb{R}$

$Dom(r) = Dom(a) \cap Dom(b) \cap Dom(c)$

$= (-1, 1) \cap [-1, \infty) \cap (-\infty, \infty)$

$\boxed{Dom(r) = (-1, 1)}$

$r'(t)$ = derivar a, b, c

$r'(t) = (a'(t), b'(t), c'(t))$

$a'(t) = \dfrac{-2t}{1-t^2}$ $\qquad$ $b'(t) = \dfrac{1}{2\sqrt{1+t}}$ $\qquad$ $c'(t) = -e^{2t} \cdot 2$

Rta $\boxed{r'(t) = \left(\dfrac{-2t}{1-t^2}, \dfrac{1}{2\sqrt{1+t}}, -2e^{2t}\right)}$

$r'(0) = \left(\dfrac{-2 \cdot 0}{1-t^2}, \dfrac{1}{2\sqrt{1+0}}, -2e^{2 \cdot 0}\right)$

Rta $\boxed{r'(0) = \left(0, \dfrac{1}{2}, -2\right)}$