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Algebra Tarea 8

$T: \mathbb{R}_3[x] \to \mathbb{R}_2[x]$ y $T(p(x)) = \dfrac{p(x) - p(1)}{x-1}$

a) Probar q T es trans. lineal

$P(x) = ax^2 + bx + c$

$$T(p(x)) = \frac{(ax^2 + bx + c) - (a+b+c)}{x-1}$$

$$= \frac{a(x^2-1) + b(x-1)}{x-1}$$

$$= a(x+1) + b \quad \bigstar$$

- $T(p(x) + \lambda p'(x)) = T((ax^2+bx+c) + \lambda(a'x^2+b'x+c'))$

$$= T\Big(x^2 \cdot \underbrace{(a+\lambda a')}_{a} + x\underbrace{(b+\lambda b')}_{b} + \underbrace{(c+\lambda c')}_{c}\Big)$$

$$\bigstar \; = (a+\lambda a')(x+1) + (b+\lambda b')$$

$$= a\cdot(x+1) + b + \lambda a'(x+1) + \lambda b'$$

$$= T(ax^2+b+c) + \lambda T(a'x^2+b'+c')$$

$$= T(P(x)) + \lambda T(p'(x))$$

T es trans. lineal.

b) Con $T: \mathbb{R}_{3[x]} \to \mathbb{R}_{2[x]}$

★ $T(ax^2+b+c) = a(x+1)+b$

$= ax + (a+b)$

$\forall w \in \operatorname{Im} T,\ w = \underbrace{ax}_{w_1} + \underbrace{(a+b)}_{w_2} \longrightarrow \begin{array}{l} a + 0b = w_1 \\ a+b = w_2 \end{array}$

$\longrightarrow \left(\begin{array}{cc|c} 1 & 0 & w_1 \\ 1 & 1 & w_2 \end{array}\right) \xrightarrow{F_2 - F_1} \left(\begin{array}{cc|c} 1 & 0 & w_1 \\ 0 & 1 & w_2 - w_1 \end{array}\right)$

- No se anula ninguna fila
- $\operatorname{Im} T = \mathbb{R}_2[x]$
- Y una base de $\mathbb{R}_2[x]$ es $\{x, 1\}$

c) $\forall v \in \operatorname{Nu} T,\ T(v) = 0$

$T(ax^2+bx+c) = \left[ax + (a+b) = \emptyset x + \emptyset\right]$

$\longrightarrow \begin{cases} a + \emptyset b = 0 \\ a + b = 0 \end{cases}$ Con esto planteo la matriz ampliada

$\longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}\right) \xrightarrow{F_2 - F_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$ $\boxed{\begin{array}{l} a = 0 \\ b = 0 \end{array}}$

$V = 0x^2 + 0x + c \longrightarrow V = \lambda_1 \cdot 0x^2 + \lambda_2 \cdot 0x + \lambda_3 c = \boxed{\lambda c}$

$\operatorname{Nu} T = \mathbb{R} \longrightarrow$ una base es $\{\lambda_3\}$