Parcial 1 A.M II

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Ejercicio 3

a) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}\,3^n} \cdot (3x-1)^n$ $\quad R=?\quad I=?$

$$= \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}\,3^n} \cdot \left(3\cdot\left(x-\frac{1}{3}\right)\right)^n$$

$$= \sum_{n=1}^{\infty} \frac{\cancel{3^n}}{\sqrt[4]{n}\cdot \cancel{3^n}} \cdot \left(x-\frac{1}{3}\right)^n$$

$$= \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}} \cdot \left(x-\frac{1}{3}\right)^n$$

(Por Crit. del cociente para Serie de potencias) $\quad L = \lim_{n\to\infty} \frac{\left|\frac{1}{\sqrt[4]{n+1}}\right|}{\left|\frac{1}{\sqrt[4]{n}}\right|} = \lim_{n\to\infty} \frac{\sqrt[4]{n}}{\sqrt[4]{n+1}}$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{1/4}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n\cdot 1+\frac{1}{n}}\right)^{1/4}$$

$$= \lim_{n\to\infty} \frac{1}{1+\underbrace{\frac{1}{n}}_{\to 0}} = \boxed{1} = L$$

Como $0 < L < \infty$

$\therefore R = \frac{1}{L}$

$$\boxed{R = 1}$$

① $A - R = -\frac{2}{3}$

② $A + R = \frac{4}{3}$

① $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}} \cdot \left(\frac{-2}{3} - \frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}} \cdot (-1)^n$ (Por criterio de series Alternantes)

$\lim_{n\to\infty} \frac{1}{\sqrt[4]{n}} = 0$ y es decreciente $\quad$ ① Converge

② $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}} \cdot \left(\frac{4}{3} - \frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}} \cdot 1^n$ (Por Serie de P)

$\frac{1}{\sqrt[4]{n}} = \frac{1}{n^{1/4}} \quad P = 1/4 \to P \leq 1$

② Diverge

$$I = \left\{ x \in \mathbb{R} : \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}} \left(x-\frac{1}{3}\right)^n \text{ converge} \right\}$$

$$\boxed{I = \left[\frac{-2}{3}, \frac{4}{3}\right) \quad R = 1}$$

b) $f(x) = \frac{1}{x^2}$

$f(x) = g'(x)$

$g(x) = \frac{-1}{x}$

$$\frac{-1}{x} = \frac{-1}{-3+3+x} = \frac{1}{3} \cdot \frac{1}{1-\frac{1}{3}\cdot(3+x)}$$

$$\frac{1}{x^2} = f(x) = g'(x) = \frac{1}{3}\left[\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n \cdot (3+x)^n\right]' = \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n \cdot n \cdot (3+x)^{n-1}$$

$$\Rightarrow \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{n+1} \cdot n \cdot (3+x)^{n-1}$$

$$\boxed{= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{n+2} \cdot (n+1) \cdot (3+x)^n}$$

(me salteo el +0 en la sumatoria para tener $(x-a)^n$)

(Por criterio del cociente)

$$L = \lim_{n\to\infty} \frac{\left(\frac{1}{3}\right)^{n+3} \cdot (n+2)}{\left(\frac{1}{3}\right)^{n+2} \cdot (n+1)} = \lim_{n\to\infty} \frac{(n+2)\cdot 3^{n+2}}{(n+1)\cdot 3^{n+3}}$$

$$= \lim_{n\to\infty} \frac{(n+2)\cdot \cancel{3^n}\cdot 3^2}{(n+1)\cdot \cancel{3^n}\cdot 3^3}$$

$$L = \frac{1}{3} \rightarrow 0 < L < \infty \qquad = \lim_{n\to\infty} \frac{n+2}{3n+3}$$

$$R = \frac{1}{L} \qquad = \lim_{n\to\infty} \frac{\cancel{n}\cdot\left(1+\overset{\to 0}{\frac{2}{n}}\right)}{\cancel{n}\cdot\left(3+\underset{\to 0}{\frac{3}{n}}\right)} = \frac{1}{3}$$

$$\boxed{R = 3}$$