

## Ejercicio 3

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$$a) f(x,y) = x^3 y^3 - 3xy$$

$$f_x = 3x^2 y^3 - 3y$$

$$= y \cdot (3x^2 y^2 - 3)$$

$$f_y = x^3 3y^2 - 3x$$

$$= x \cdot (x^2 3y^2 - 3)$$

• Para calcular los puntos, igualo a 0

$$0 = y \cdot (3x^2 y^2 - 3)$$

$$0 = x \cdot (x^2 3y^2 - 3)$$

• tengo dos casos en ambas

$$(I) 0 = y$$

$$(I) 0 = x$$

$$(II) 0 = 3x^2 y^2 - 3$$

$$(II) 0 = x^2 3y^2 - 3$$

$$3 = 3x^2 y^2$$

$$3 = x^2 3y^2$$

$$\rightarrow x = 0 \quad y = 0 \quad x = 1 \quad y = 1 \quad x = -1 \quad y = -1 \quad x = -1 \quad y = 1 \quad x = 1 \quad y = -1$$

$$\rightarrow P_1 = (0,0) \quad P_2 = (1,1) \quad P_3 = (-1,-1) \quad P_4 = (-1,1) \quad P_5 = (1,-1)$$

$$P_6 = (0,0)$$

$$f_{xx} = 6xy^3$$

$$f_{xx} = 6xy^3$$

$$f_{xx} = 9x^2 y^2 - 3$$

$$f_{xy} = 9x^2 y^2 - 3$$

$$f_{xy} = 6x^3 y$$

• Clasificamos

$$D = f_{xx}(x_0, y_0) \cdot f_{yy}(x_0, y_0) - [f_{xy}(x_0, y_0)]^2$$

$$D = 6x^4 y^3 \cdot 6x^3 y^4 - (9x^2 y^2 - 3)^2$$

$$= 36x^7 y^7 - 81x^4 y^4 + 18x^2 y^2 - 9$$

$$D = -45x^4 y^4 + 18x^2 y^2 - 9$$

• Valor D en los puntos para clasificarlos

$$D_{(-1,-1)} = -36$$

$$D_{(-1,1)} = -36$$

$$D_{(0,0)} = -9$$

$$D_{(1,-1)} = -36$$

• Al ser  $D < 0$  son punto silla

$$D_{(1,1)} = -36$$

Rta = Los puntos  $(0,0), (1,1), (-1,-1), (-1,1)$  y  $(1,-1)$

son todos puntos estacionarios clasificados como puntos silla

b)  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$   $f_{xx}, f_{yy}, f_{xy} = f_{yx}$  existen y son continuas

$$z(t) = f(t, e^t) \quad z''(t) = ?$$

$$\begin{aligned} z'(t) &= f_x(t, e^t) \cdot (t)' + f_y(t, e^t) \cdot (e^t)' \\ &= f_x(t, e^t) \cdot 1 + f_y(t, e^t) \cdot e^t \end{aligned}$$

$$\begin{aligned} (t)' &= 1 \\ (e^t)' &= e^t \end{aligned}$$

$$\begin{aligned} z''(t) &= f_{xx}(t, e^t) \cdot (t)' + f_{xy}(t, e^t) \cdot (e^t)' + [f_{yx}(t, e^t) \cdot (t)' + f_{yy}(t, e^t) \cdot (e^t)'] \cdot e^t + f_y(t, e^t) \cdot (e^t)' \\ &= f_{xx}(t, e^t) + f_{xy}(t, e^t) \cdot e^t + f_{yx}(t, e^t) \cdot e^t + f_{yy}(t, e^t) \cdot e^{2t} + f_y(t, e^t) \cdot e^t \end{aligned}$$

$$\boxed{z''(t) = f_{xx}(t, e^t) + f_{xy}(t, e^t) \cdot 2e^t + f_{yy}(t, e^t) e^{2t} + f_y(t, e^t) e^t}$$