

Ejercicio 1

a) $f(x) = (x^2 - 1)^2$ $g(x) = 1 - x^2$

$$f(0) = (0^2 - 1)^2 \quad f(1) = (1^2 - 1)^2$$

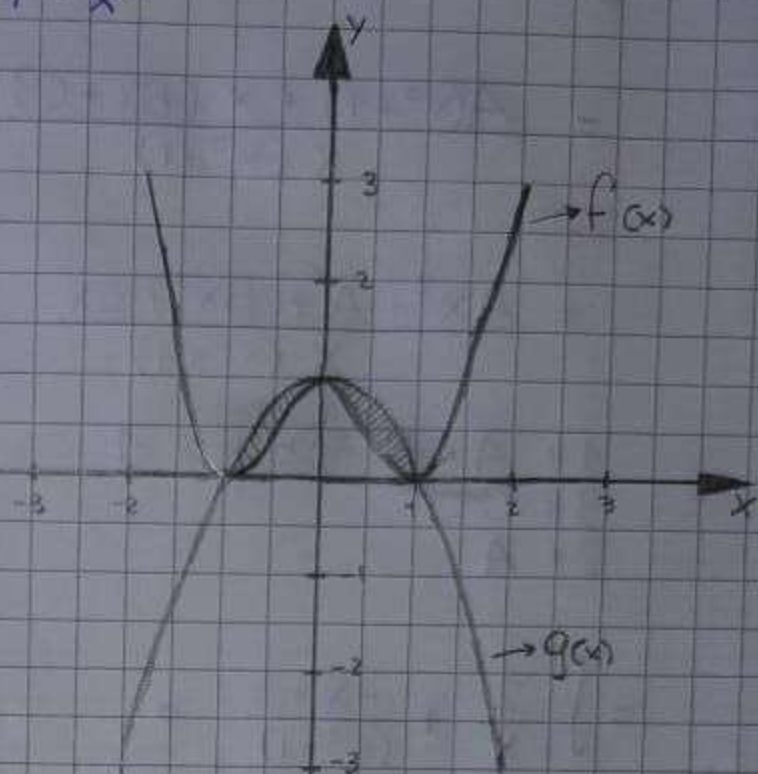
$$f(0) = 1 \quad f(1) = 0$$

$$g(0) = 1 - 0^2 \quad g(1) = 1 - 1^2$$

$$g(0) = 1 \quad g(1) = 0$$

$$f(-1) = ((-1)^2 - 1)^2 \quad g(-1) = 1 - (-1)^2$$

$$f(-1) = 0 \quad g(-1) = 0$$



$$\begin{aligned} A &= \int_{-1}^1 g(x) \, dx - \int_{-1}^1 f(x) \, dx \\ &= \int_{-1}^1 (1 - x^2) \, dx - \int_{-1}^1 (x^2 - 1)^2 \, dx \\ &= \left[x - \frac{x^3}{3} \right]_{-1}^1 - \left[\frac{x^5}{5} - \frac{2x^3}{3} + x \right]_{-1}^1 \end{aligned}$$

$$= \left[\left[1 - \frac{1}{3} \right] - \left[-1 - \left(-\frac{1}{3} \right) \right] \right] - \left[\left[\frac{3}{15} - \frac{10}{15} + 1 \right] - \left[\frac{-3}{15} + \frac{10}{15} - 1 \right] \right]$$

$$= \left[\frac{2}{3} + \frac{2}{3} \right] - \left[\frac{8}{15} + \frac{8}{15} \right]$$

$$= \frac{4}{3} - \frac{16}{15}$$

$$A = \frac{4}{15}$$

$$\begin{aligned} &\int (x^2 - 1)^2 \, dx \quad u = x \\ &= \int x^4 - 2x^2 + 1 \\ &= \frac{x^5}{5} - \frac{2x^3}{3} + x \end{aligned}$$

$$b) \int \frac{2+3x+x^2}{x \cdot (x^2+1)} dx = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$$

$$= \frac{A(x^2+1) + x \cdot (Bx+C)}{x \cdot (x^2+1)}$$

$$= \frac{Ax^2 + A + Bx^2 + Cx}{x \cdot (x^2+1)}$$

$$1 = A + B \rightarrow B = -1$$

$$3 = C$$

$$2 = A$$

$$= \int \frac{2}{x} + \frac{-x+3}{(x^2+1)} =$$

$$= \int \frac{2}{x} + \int \frac{-x+3}{x^2+1}$$

$$= \left[2 \int \frac{1}{x} \right] + \left[\frac{-1}{2} \int \frac{2x}{x^2+1} + 3 \cdot \int \frac{1}{x^2+1} \right]$$

(Busco los k_1 y k_2)

$$= 2 \ln|x| + \frac{-\ln|x^2+1|}{2} + 3 \cdot \frac{1}{1} \arctg(x) + C$$

$$= 2 \ln|x| - \frac{\ln|x^2+1|}{2} + 3 \arctg(x) + C$$