Chapter Four: Synthesis of Array Antennas

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Outlines

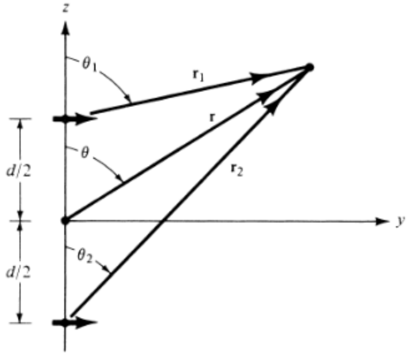
Introduction

- The radiation patterns of single-element antennas are relatively wide, i.e., they have relatively low directivity (gain).
- In long distance communications, antennas with high directivity are required
  - Possible to construct by enlarging the dimensions of the radiating aperture (maximum size much larger than λ)
  - To increase the electrical size of an antenna is to construct it as an assembly of radiating elements in a proper electrical and geometrical configuration called antenna array
- Usually, the array elements are identical. This is not necessary but it is practical and simpler for design and fabrication.
- The individual elements may be of any type (wire dipoles, loops, apertures, etc.)
- The total field of an array is a vector superposition of the fields radiated by the individual elements.

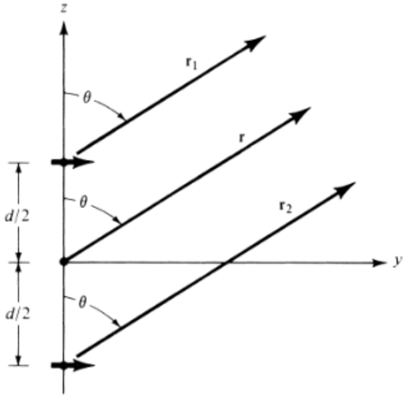
- To provide very directive pattern, it is necessary that the partial fields (generated by the individual elements) interfere constructively in the desired direction and interfere destructively in the remaining space.
- There are five basic methods to control the overall antenna pattern:
    1. The geometrical configuration of the overall array (linear, circular, spherical, rectangular, etc.),
    2. The relative placement of the elements,
    3. The excitation amplitude of the individual elements,
    4. The excitation phase of each element,
    5. The individual pattern of each element

- Assume we have an array of two infinitesimal horizontal dipoles positioned along the z -axis

Where the first element is excited by current $I_1 = I_0 e^{j\beta/2}$ and the second by $I_2 = I_0 e^{+j\beta/2}$



(a) Two infinitesimal dipoles



(b) Far-field observations

- The total field radiated by the two elements is

$$
\begin{aligned}
E_t &= E_1 + E_2 \\
&= a_\theta j\eta \frac{kI_0 l}{4\pi}\left(\frac{e^{-j(kr_1-\beta/2)}}{r_1}cos\theta_1 + \frac{e^{-j(kr_1-\beta/2)}}{r_2}cos\theta_2\right) \qquad (1)
\end{aligned}
$$

At far-field

$\theta_1 \simeq \theta_2 = \theta$

$r_1 \simeq r - \frac{d}{2}cos\theta$ for phase variation

$r_2 \simeq r - \frac{d}{2}cos\theta$ for phase variation

$\theta_1 \simeq \theta_2 = \theta$ for amplitude variation

$$E_t = a_\theta j\eta \frac{kI_0 l}{4\pi r}cos\theta\left(e^{+j(kdcos\theta+\beta)/2} + e^{-j(kdcos\theta+\beta)/2}\right)$$

OR

$$E_t = \underbrace{a_\theta j\eta \frac{kI_0 l}{4\pi r} cos\theta}_{\text{Element Factor}} \underbrace{2cos\left[\frac{1}{2}(kdcos\theta + \beta)\right]}_{\text{Array Factor}} \quad (2)$$

- Thus the total field of the array is equal to the product of the field created by a single element located at the origin and the array factor, AF :

$$AF = 2cos\left[\frac{1}{2}(kdcos\theta + \beta)\right] \quad (3)$$

In normalized form

$$AF = cos\left[\frac{1}{2}(kdcos\theta + \beta)\right] \quad (4)$$

- The far-zone field of a uniform two-element array of identical elements is the product of the field of a single element and the array factor of that array $E_t = [E(single\,element\,at\,reference\,point)] \times [array\,factor]$

- Pattern multiplication rule valid for arrays of identical elements.
    - This rule holds for any array consisting of decoupled identical elements, where the excitation magnitudes, the phase shift between the elements and the displacement between them are not necessarily the same.
- The total pattern can be controlled via the single-element pattern, or via the AF.
- Generally the AF depends on the:
    1. The number of elements,
    2. The mutual placement,
    3. The relative excitation magnitudes and phases.

N-Element Linear Array with Uniform Amplitude and Spacing

- Assume that each succeeding element has a β progressive phase lead current excitation relative to the preceding one
- An array of identical elements with identical magnitudes and with a progressive phase is called a uniform array.
- The AF of the uniform array can be obtained by considering the individual elements as point (isotropic) sources.
- The total field pattern can be obtained by simply multiplying the AF by the field pattern of the individual element (provided the elements are not coupled)

- The AF of an N -element linear array of isotropic sources is

$$AF = 1 + e^{j(kdcos\theta+\beta)} + e^{2j(kdcos\theta+\beta)} + \ldots + e^{j(N-1)(kdcos\theta+\beta)} \quad (5)$$

AF $= \sum_{n=1}^{N} e^{j(n-1)\psi}$
Where $\psi = kdcos\theta + \beta Moreconvenientforpatternanalysis:$

$$AF = e^{j[(N-1)/2]\psi} \left[\frac{sin\left(\frac{N}{2}\psi\right)}{sin\left(\frac{\psi}{2}\right)} \right]$$

- The phaser factor $e^{j[(N-1)/2]\psi}$
  - Not important unless the array output signal is further combined with the output signal of another antenna
  - It represents the phase shift of the array's phase center relative to the origin

- Neglecting the phase factor gives

$$AF = \left[\frac{sin\left(\frac{N}{2}\psi\right)}{sin\left(\frac{\psi}{2}\right)} \right] \tag{6}$$

- Normalizing the array factor we obtain

$$\mathrm{AF}_n = \frac{1}{N}\left[\frac{sin\left(\frac{N}{2}\psi\right)}{sin\left(\frac{\psi}{2}\right)} \right] Forsmallvaluesof\ \ \psi, \text{ it reduces to}$$

$$\mathrm{AF}_n \simeq \frac{1}{N}\left[\frac{sin\left(\frac{N}{2}\psi\right)}{\frac{\psi}{2}} \right]$$

Null - To find the nulls of the AF, equation (10) is set equal to zero
$\theta_n = cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$,
n = 1, 2, 3, . . . (n $\neq$
$0, N, 2N, 3N, \ldots) Theyarestudiedinordertodeterminethemaximum$

a: The maximum value of (10) occur when

$$\frac{\psi}{2} = \frac{1}{2}(kdcos\theta_m + \beta) = \pm m\pi$$

$$\theta_m = cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right], m = 0, 1, 2, \ldots$$

HPBW Calculated by setting the value of AF_n equal to $1/\sqrt{2}$ $\theta_h = cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right] Forasymmetricalpatternaround$ θ_m (the angle at which maximum radiation occurs), the HPBW is calculated as $\Theta_h = 2|\theta_h - \theta_m|$

Maxima of Minor Lobes: They are the maxima of AF_n , where $AF_n < 1$.

They occur approximately where the numerator attains a maximum and the AF is beyond its first null:

$$sin\left(\frac{N}{2}\psi\right) = \pm 1 \Rightarrow \frac{N}{2}(kdcos\theta_s + \beta) = \pm(2s+1)\frac{\pi}{2}$$

$\theta_s = cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2s+1}{N}\pi\right)\right]$,

s = 1, 2, 3, ...

It can be also written as

$$\theta_s = \frac{\pi}{2} - sin^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2s+1}{N}\pi\right)\right],$$
$$s = 1, 2, 3, \ldots$$

For large values of $d(d \ll \lambda)$, it reduces to

$$\theta_s = \frac{\pi}{2} - \left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2s+1}{N}\pi\right)\right],$$
$$s = 1, 2, 3, \ldots$$

The maximum of the first minor lobe occurs when $s = 1$

$$AF_N = 0.212 = -13.46dB$$

The maximum of the first minor lobe of the array factor is 13.46 dB down from the maximum at the major lobe.

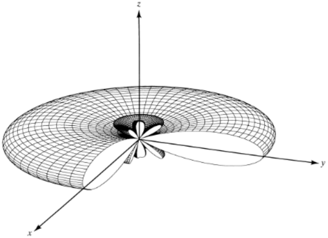
- An array, which has maximum radiation at $\theta = 90^\circ$ (normal to the axis of the array).
- The maximum of the AF occurs when $\psi = 0$. At $\theta = 90^\circ$ $\beta = 0$

The uniform linear array has its maximum radiation at $\theta = 90^\circ$, if all array elements have their excitation with the same phase ($\beta = 0$)

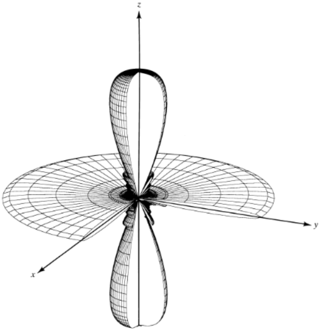
To ensure that there are no maxima in the other directions (called grating lobes), the separation between the elements should not be equal to multiples of a wavelength:

$$d \neq n\lambda, \quad n = 1, 2, 3, \dots \tag{4}$$

- For a uniform array with $\beta = 0$ and $d = n\lambda$, in addition to having the maxima of the array factor directed broadside ($\theta = 90^\circ$) to the axis of the array,
  - There are additional maxima directed along the axis ($\theta = 0^\circ$, 180°) of the array (end-fire radiation).
- One of the objectives in many designs is to avoid multiple maxima (grating lobes)
- To avoid any grating lobe, the largest spacing between the elements should be less than one wavelength ($d_{max} < \lambda$)



(a) Broadside ($\beta = 0$, $d = \lambda/4$)



(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

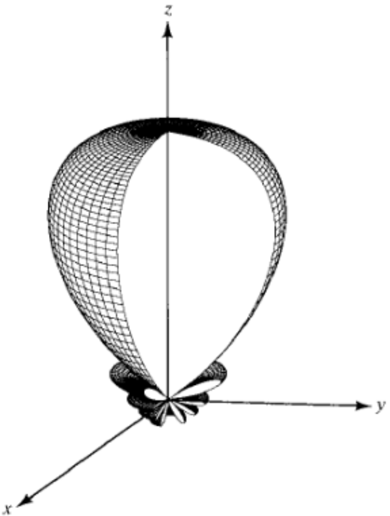
Ordinary End-Fire Array

- An array which has its maximum radiation along the axis of the array $(\theta = 0^\circ, 180^\circ\ )$.
- It may be required that the array radiates only in one direction- either $\theta = 0^\circ or 180^\circ\ $.

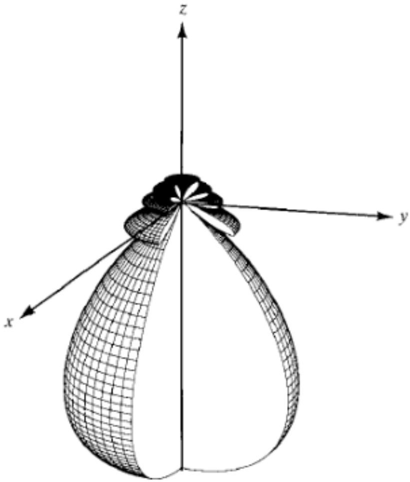
  For an AF maximum at $\theta = 0^\circ\ $,
  $\beta = kd Iftheelementseparationismultipleofawavelength$,d = n$\lambda$, then in ad- dition to the end-fire maxima there also exist maxima in the broadside directions.

- As with the broadside array, in order to avoid grating lobes, the maximum spacing between the element should be less than λ:

$$d_{max} < \lambda \tag{5}$$



(a) $\theta_0 = 0°$



(b) $\theta_0 = 180°$

Phased (Scanning) Arrays

- It is logical to assume that the maximum radiation can be oriented in any direction to form a scanning array.
- Let the maximum radiation of the array is required to be oriented at angle θ_0 $(0^\circ \leq \theta \leq 180^\circ)$. To accomplish this, the phase excitation β between the elements must be adjusted so that

$$\psi = kdcos\theta + \beta|_{\theta=\theta_0} = kdcos\theta_0 + \beta = 0$$

$\Rightarrow \beta = -kdcos\theta_0$

Hansen-Woodyard End-Fire Array

- To enhance the directivity of an end-fire array, Hansen and Woodyard proposed that the phase shift of an ordinary end-fire $\beta = \pm kd$ be increased for closely spaced elements of a very long array as

$$\beta = -\left(kd + \frac{2.94}{N}\right) \simeq -\left(kd + \frac{\pi}{N}\right), \textit{for max in } \theta = 0^\circ \qquad (6)$$

$$\beta = +\left(kd + \frac{2.94}{N}\right) \simeq +\left(kd + \frac{\pi}{N}\right), \textit{for max in } \theta = 0^\circ \qquad (7)$$

are known as the Hansen-Woodyard conditions for end-fire radiation. They follow from a procedure for maximizing the directivity.

Directivity of Broadside Array

$$U(\theta) = |AF_N|^2 = \left[\frac{sin\left(\frac{N}{2}kdcos\theta\right)}{\frac{N}{2}kdcos\theta} \right]^2 = \left[\frac{sinZ}{Z}\right]^2 \tag{8}$$

$$U_{max} = U(\theta = \frac{\pi}{2}) = 1 \tag{9}$$

the radiation intensity averaged over all directions is

$$P_{av} = \frac{1}{2}\int_0^{\pi} \left[\frac{sin\left(\frac{N}{2}kdcos\theta\right)}{\frac{N}{2}kdcos\theta} \right]^2 sin\theta d\theta \tag{10}$$

$$= \frac{1}{Nkd}\int_{-Nkd/2}^{Nkd/2} \left[\frac{sinZ}{Z}\right]^2 dZ = \frac{\pi}{Nkd} \tag{11}$$

The directivity becomes D = $2N\left(\frac{d}{\lambda}\right)$ $Thelengthofthearray$ L = (N - 1)d

$D_0 = 2\left(1 + \frac{L}{d}\right)\left(\frac{d}{\lambda}\right)$ (12)

For large arrays $(L \gg d)$

$$D_0 \simeq 2\left(\frac{L}{\lambda}\right) \tag{13}$$

Directivity of Ordinary End-Fire Array

In a similar way the directivity of an end-fire array becomes D = $\frac{U_{max}}{U_av} = 4N\left(\frac{d}{\lambda}\right)$

The length of the array $L = (N-1)d$

$$D_0 = 4\left(1+\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \tag{14}$$

For large arrays $(L \gg d)$

$$D_0 \simeq 4\left(\frac{L}{\lambda}\right) \tag{15}$$

Directivity of Hansen-Woodyard Array

The directivity of Hansen-Woodyard array is D = $1.805\left[4N\left(\frac{d}{\lambda}\right)\right]$ *Thelengthofthearray* L = (N - 1)d

$D_0 = 1.805\left[4\left(1+\frac{L}{d}\right)\left(\frac{d}{\lambda}\right)\right]$ (16)

For large arrays ($L \gg d$)

$$D_0 \simeq 1.805 \left[4 \left(\frac{L}{\lambda} \right) \right] \tag{17}$$

Exercise Given a linear uniform array of isotropic elements with $N = 10$, $d = \lambda/4$, find the directivity if:

1. $\beta = 0$ (broadside)
2. $\beta = kd$ (end-fire)
3. $\beta = kd\pi/N$ (Hansen-Woodyard)

[ans. a. 5 (=6.999 dB) b. 10 (10 dB) c. 17.89 (12.53 dB)]

Thank You!!!