PROJECT 1

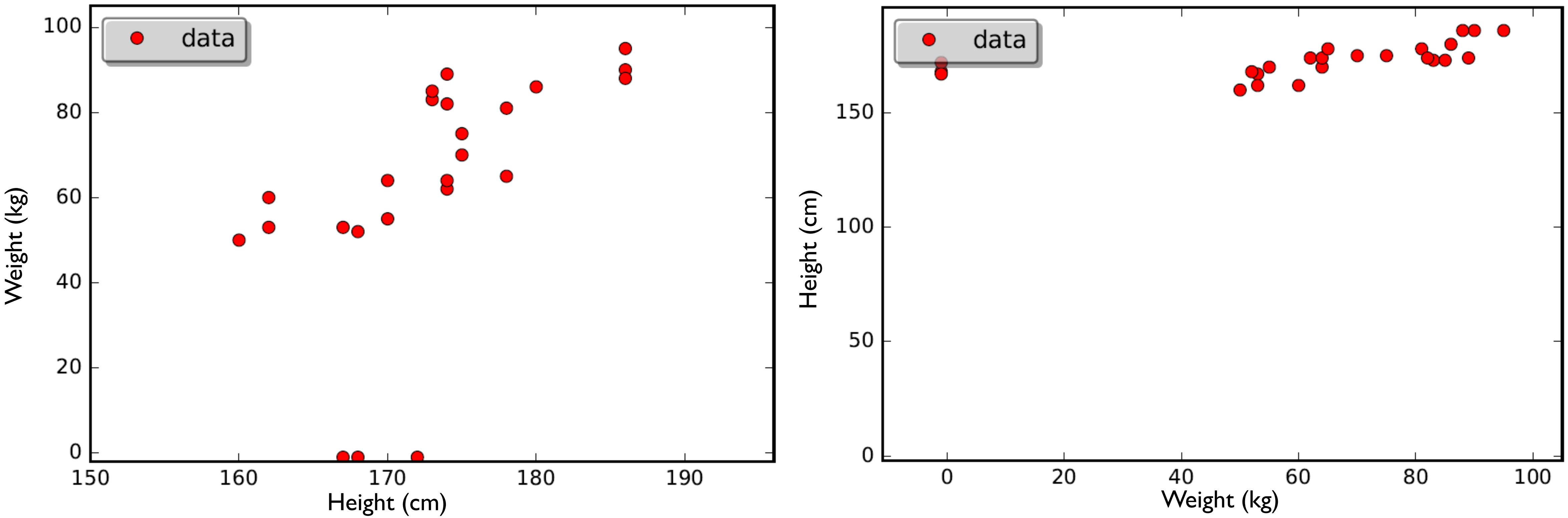
PATTERN RECOGNITION– WINTER 2017

Vatsala Sharma, 374843
Mayesha Tasnim, 374098
Chandan Acharya, 373970
Rahul Lao, 374021
Devendra Hupri, 374022
Rakesh Lagare, 374183
Vinay Pyati, 374182

TASKS

- 1.1 – Plotting Data without Outliers
- 1.2 – Fitting a Normal Distribution to 1D data
- 1.3 – Fitting a Weibull Distribution to 1D data
- 1.4 – Drawing Unit Circles
- 1.5 – Estimating the Dimension of Fractal Objects in an Image

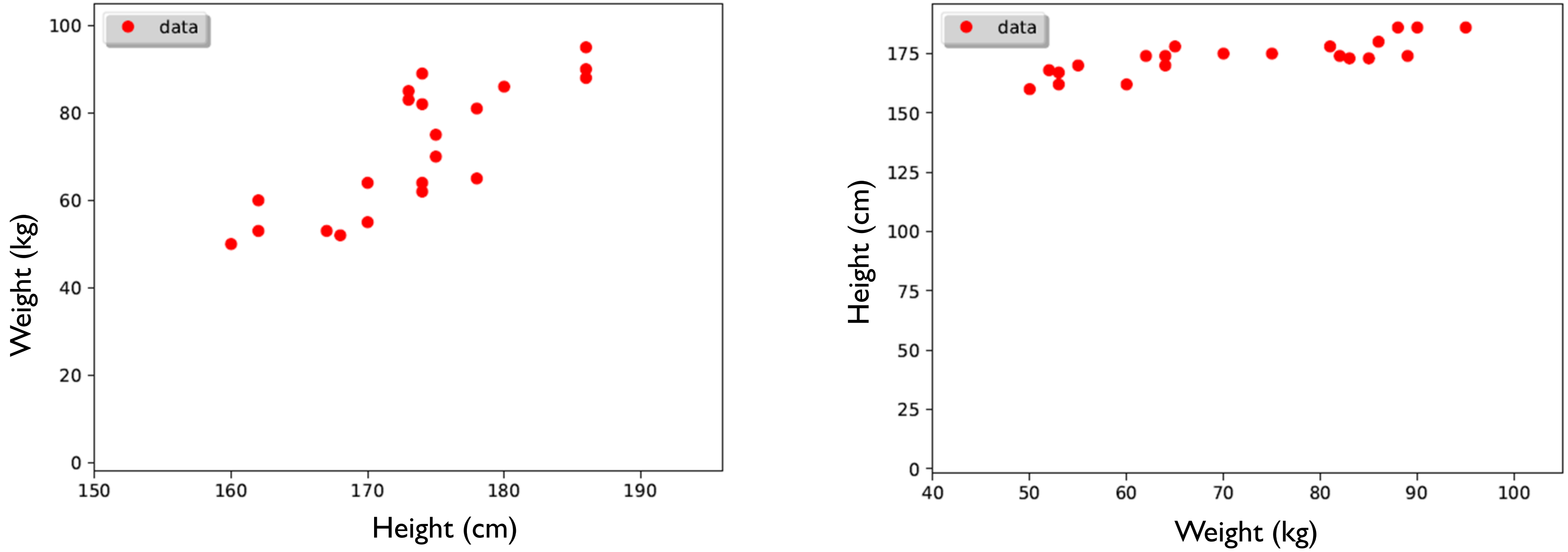
TASK 1.1 – PLOTTING DATA WITHOUT OUTLIERS



Weight and Height Data with Outliers

TASK 1.1 – PLOTTING DATA WITHOUT OUTLIERS

- Non negative values are copied from the data and stored in 1D arrays and then plotted



Weight and Height Data without Outliers

TASK 1.2 – FITTING A NORMAL DISTRIBUTION TO 1D DATA

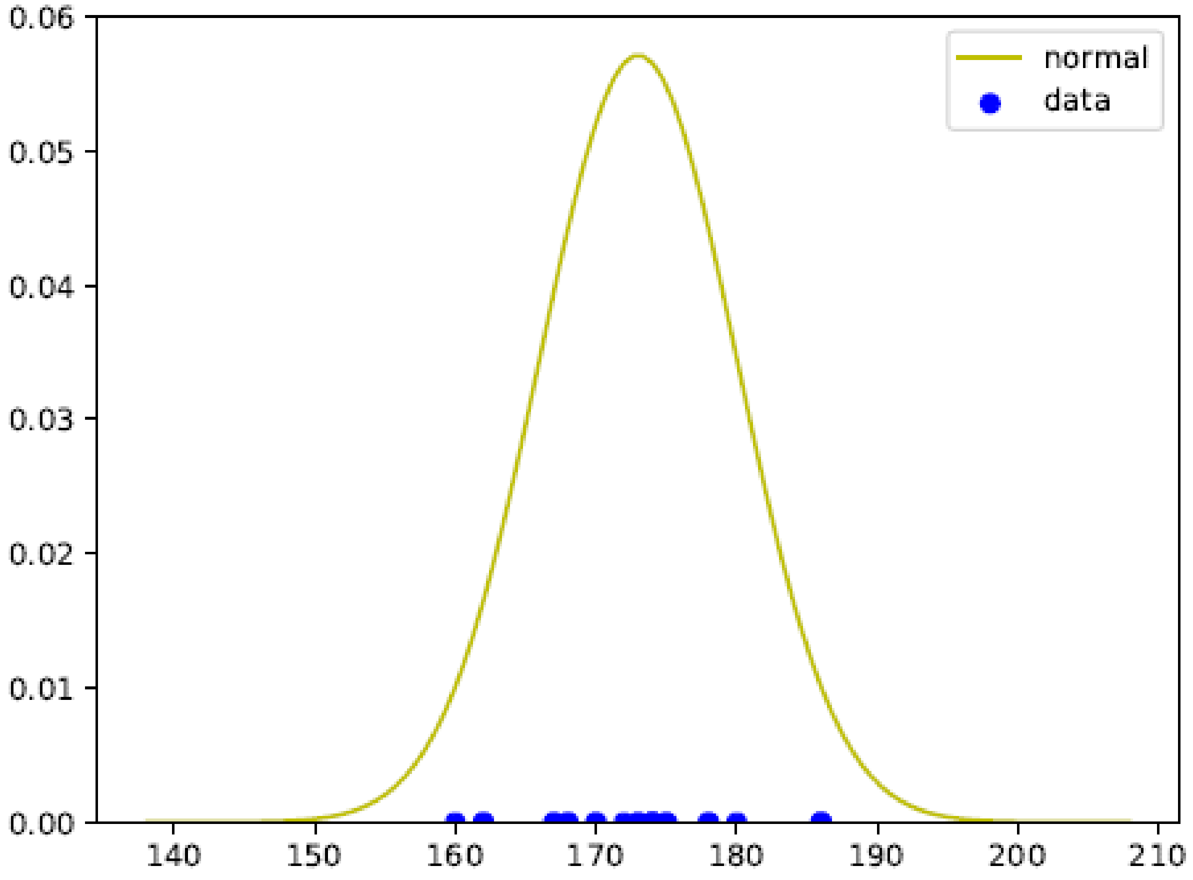
Plot the data and a normal distribution for body weight & height

- Calculated the Mean (μ) and Standard Deviation (σ) for data set.
- Computed probability density of normal distribution:

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

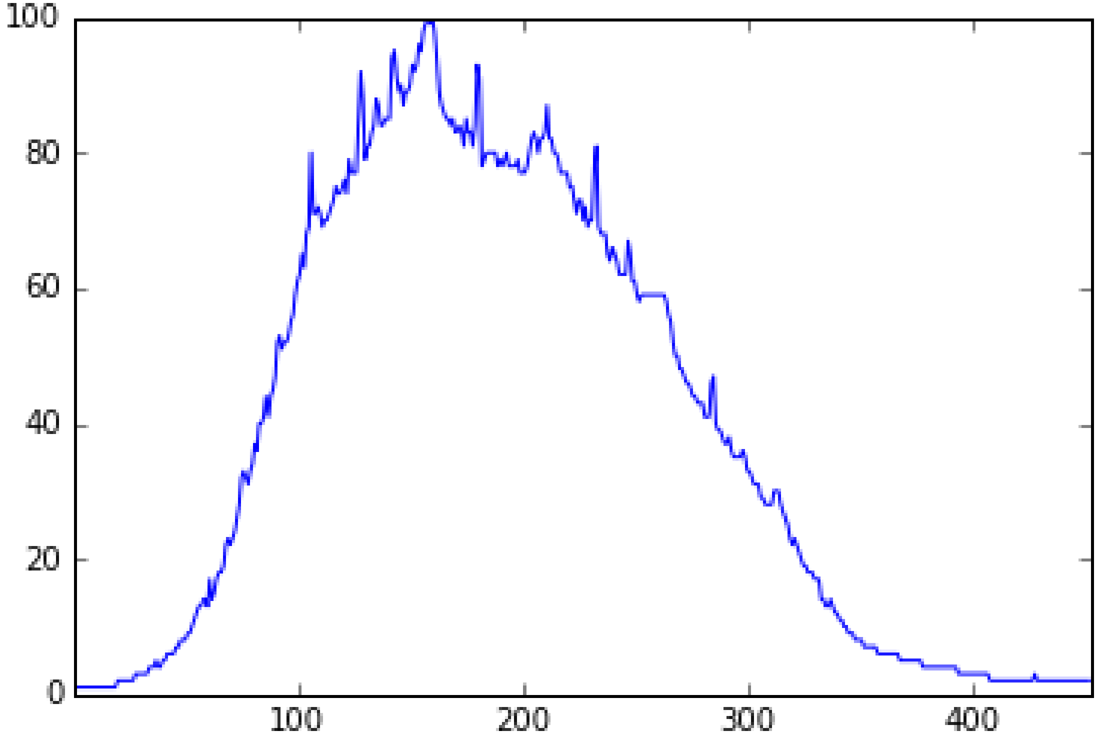
- Plotted the normal distribution characterising the density of data points

TASK 1.2 – FITTING A NORMAL DISTRIBUTION TO 1D DATA

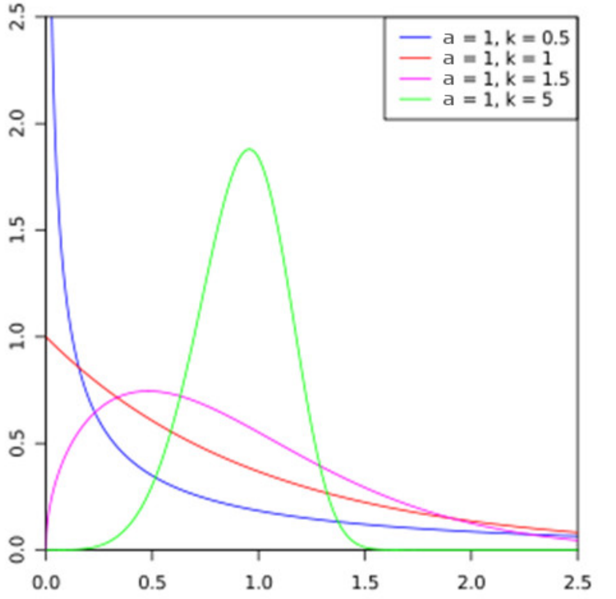


TASK 1.3 – FITTING A WEIBULL DISTRIBUTION TO 1-D DATA

Given 1-D data



Weibull Distribution



$$f(x \mid \kappa, \alpha) = \frac{\kappa}{\alpha}\left(\frac{x}{\alpha}\right)^{\kappa-1} e^{-\left(\frac{x}{\alpha}\right)^{\kappa}}$$

TASK 1.3 – FITTING A WEIBULL DISTRIBUTION TO 1-D DATA

Determining k and a iteratively:

$$\begin{bmatrix} \kappa^{\text{new}} \\ \alpha^{\text{new}} \end{bmatrix} = \begin{bmatrix} \kappa \\ \alpha \end{bmatrix} + \begin{bmatrix} \frac{\partial^2 L}{\partial \kappa^2} & \frac{\partial^2 L}{\partial \kappa \partial \alpha} \\ \frac{\partial^2 L}{\partial \kappa \partial \alpha} & \frac{\partial^2 L}{\partial \alpha^2} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\partial L}{\partial \kappa} \\ -\frac{\partial L}{\partial \alpha} \end{bmatrix}$$

where

$$\frac{\partial L}{\partial \kappa} = N/\kappa - N \log \alpha + \sum_i \log d_i - \sum_i (d_i/\alpha)^\kappa \log(d_i/\alpha)$$

$$\vdots$$

Summations over data points:

$$\mathrm{D} = \mathrm{d}_1, d_2, d_3, \ldots, d_N = \underbrace{x_1, x_1, x_1, \ldots,}_{h_1} \underbrace{x_2, x_2, x_2, \ldots\ldots,}_{h_2} \underbrace{x_n, x_n, x_n, \ldots}_{h_n}$$

$$\sum_{i=1}^{N} D = \sum_{j=1}^{n} h_j * x_j =< \boldsymbol{h}, \boldsymbol{x} >$$

TASK 1.3 – FITTING A WEIBULL DISTRIBUTION TO 1-D DATA

Initialized values:

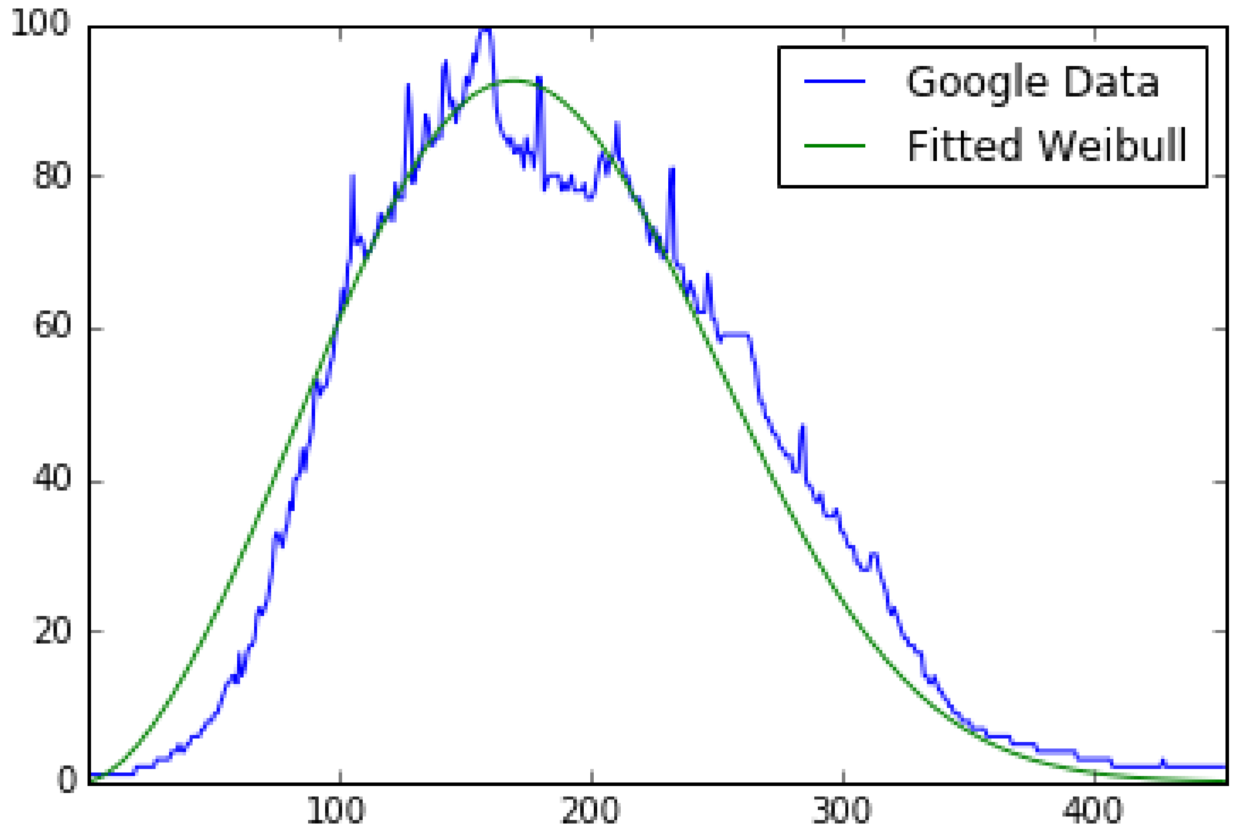
$$\kappa = 1.0$$

$$\alpha = 1.0$$

After 20 iterations:

$$\kappa = 2.72$$

$$\alpha = 202.30$$



1.4 – DRAWING UNIT CIRCLES

- Plot the R^2 unit circle considering the L_p norm for p=0.5
- L_p norm of x (when P >= 1):

$$\|\boldsymbol{x}\|_p = \left(\sum_{i=1}^{m} |x_i|^p\right)^{\frac{1}{p}}$$

- When 0 < p < 1

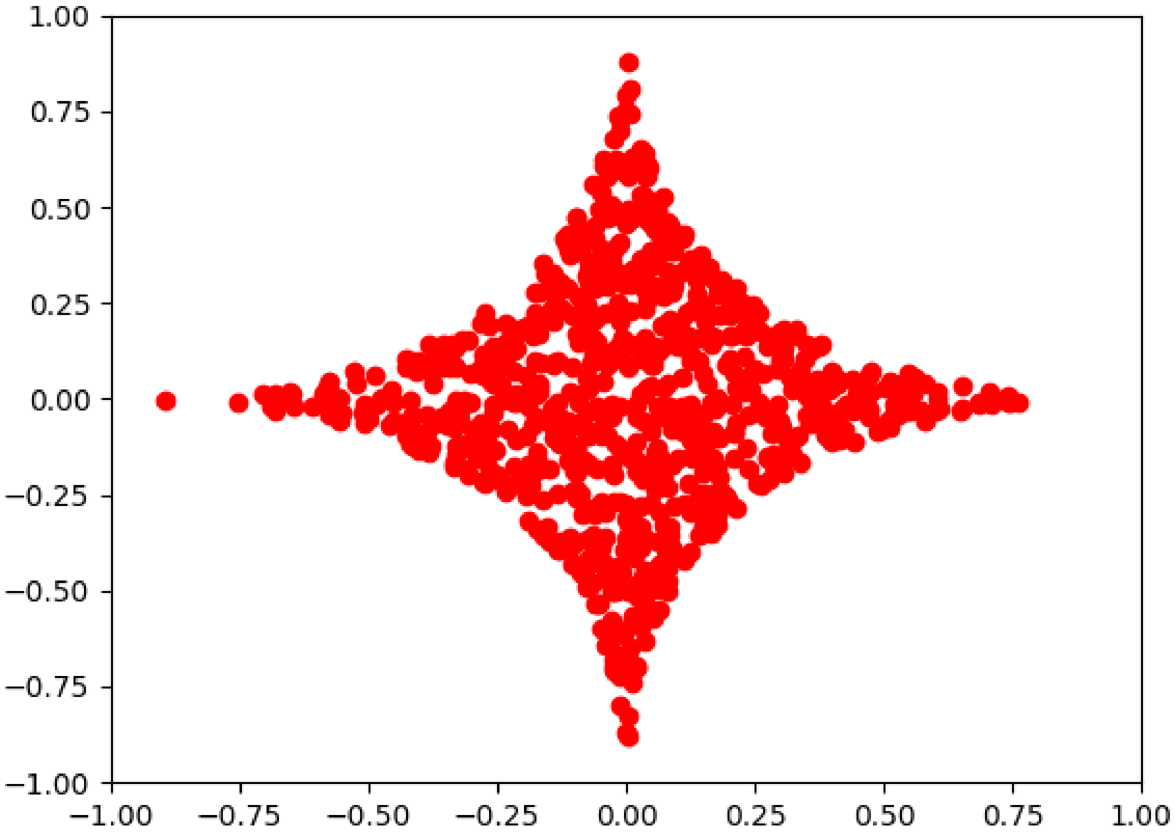
$$\Rightarrow |x_1|^p + |x_2|^p + \cdots + |x_n|^p$$ (Source: wikipedia)

- The metrics is given as

$$d_p(x, y) = \sum_{i=1}^{n} |x_i - y_i|^p$$

$$\Rightarrow \mathbf{1 = x^{1/2} + y^{1/2}}$$ $\because$ it is a unit circle with p =1/2

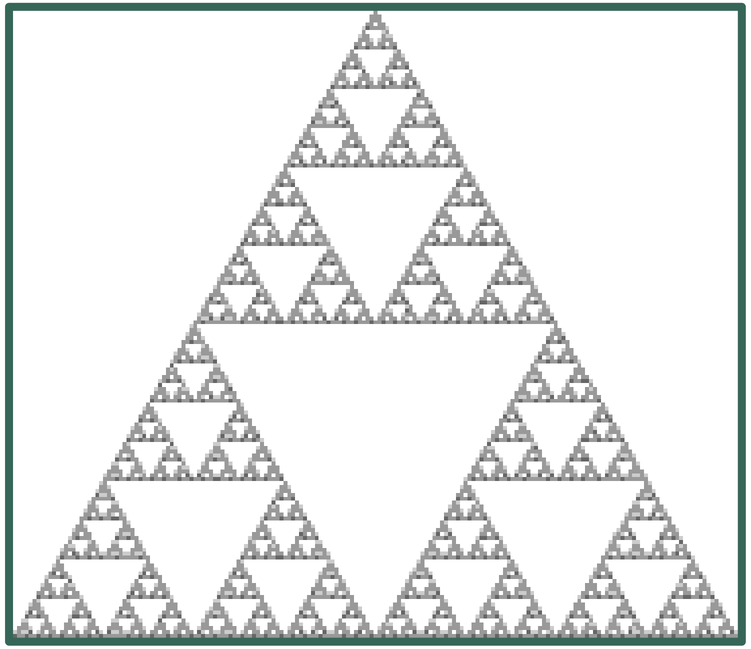
1.4 – DRAWING UNIT CIRCLES



Unit Circle with p=0.5

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE

A fractal is an abstract, mathematical object that exhibits similar patterns across all scales

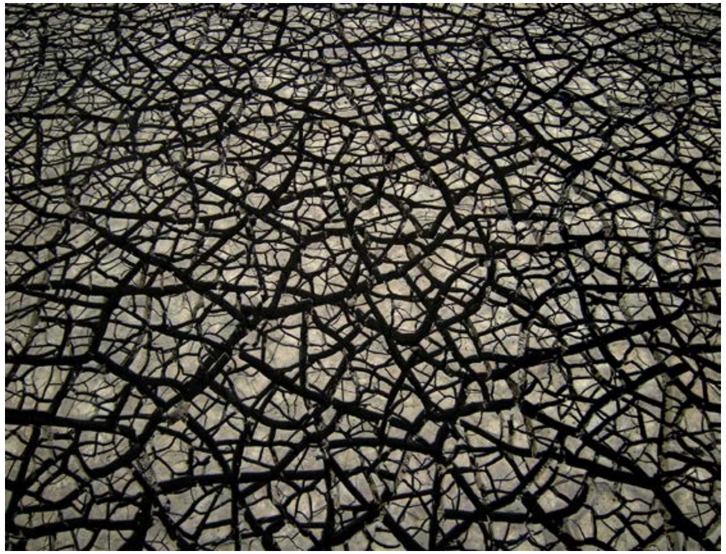


Sierpenski triangle

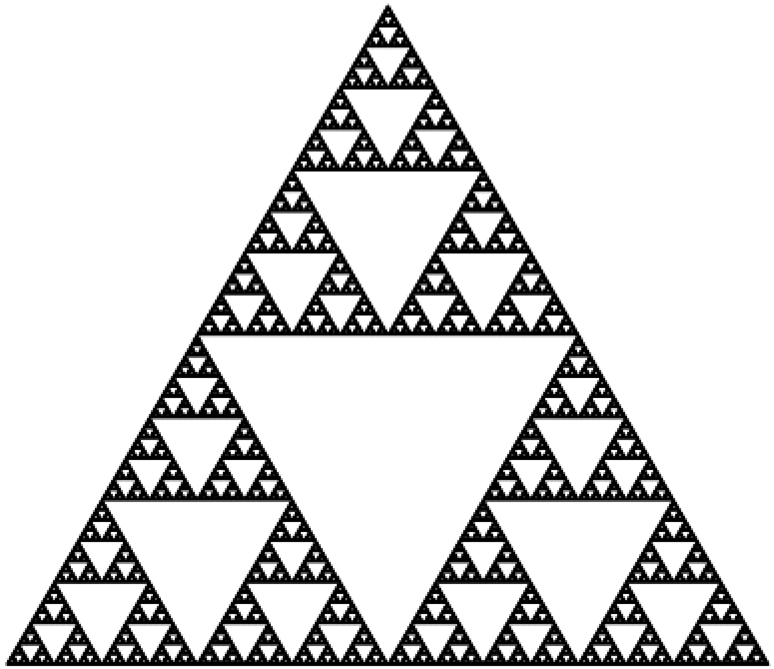
zooming in onto a fractal reveals similar structures

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE

Fractals in Nature



TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE



For a perfect fractal,

Fractal Dimension, $D = -\frac{\log n}{\log s}$

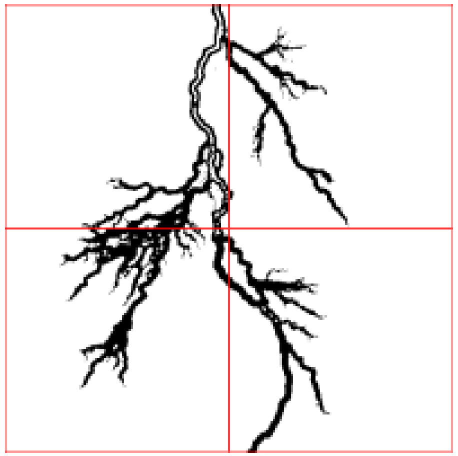
Where
s = scale factor of shrunken structure ($0 < s < 1$)
n = copies of shrunken structure needed to cover original

For Sierpenski triangle

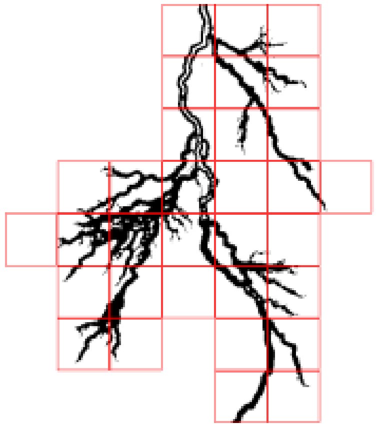
$D \approx 1.585$

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE

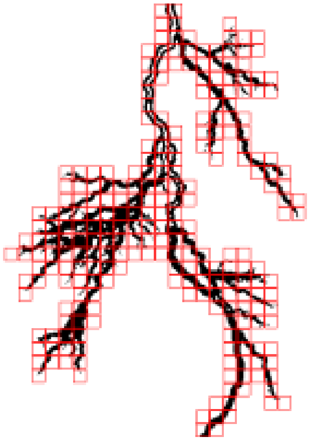
Box Counting method for calculating fractal dimension of an object inside an image,



$s_i = \frac{1}{2}, n_i = 4$



$s_i = \frac{1}{8}, n_i = 32$



$s_i = \frac{1}{32}, n_i = 254$

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE

Following the model for perfect fractals, box counting dimension of a fractal is

$$D \cdot \log \frac{1}{s_i} + b = \log n_i$$

We can re-imagine this equation as

$$y(x) = w_0 + w_1\, x$$

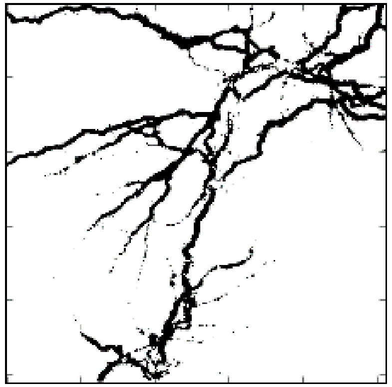
$$w_0 \equiv \text{offset}$$

$$w_1 \equiv \text{slope} \quad \approx \mathbf{D}$$

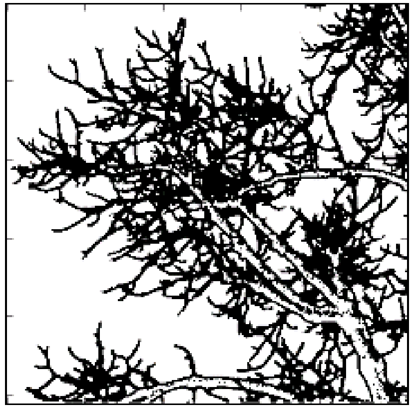
Given a set of $\log 1/s_i$ **and** $\log n_i$ **, we can fit a line through the data using**

$$\boldsymbol{w} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE



s_i	½	¼	1/8	1/16	1/32	1/64	1/127
n_i	4	14	45	147	404	1064	2905



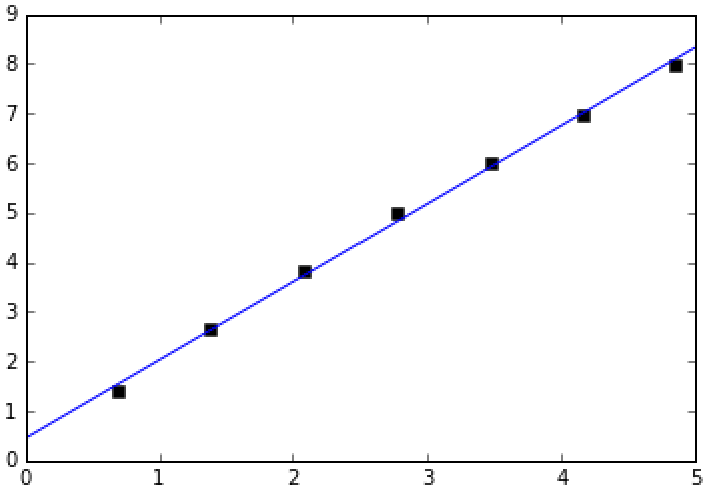
s_i	½	¼	1/8	1/16	1/32	1/64	1/127
n_i	4	16	61	218	777	2679	8685

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE

D ≈ 1.577

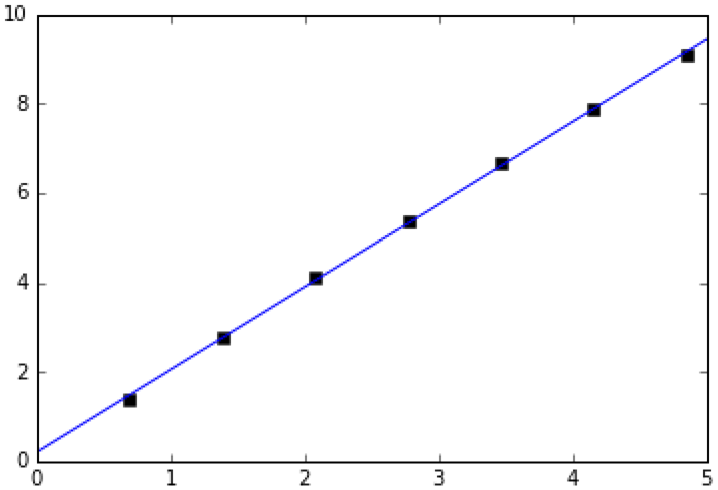
D ≈ 1.846

$\log n_i$



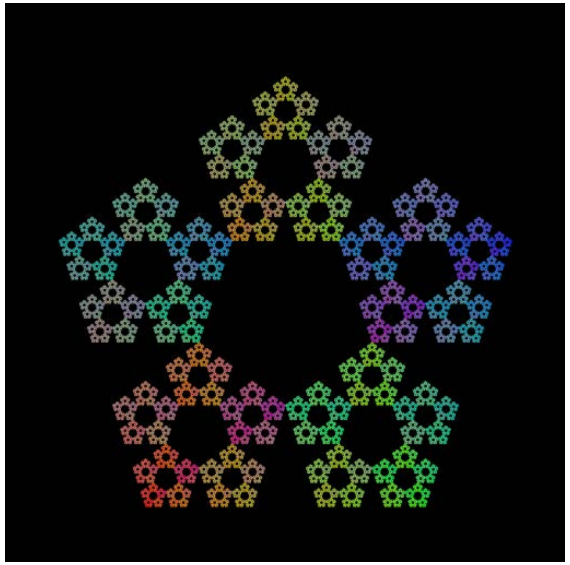
$\log 1/s_i$

$\log n_i$



$\log 1/s_i$

TASK 1.5 – ESTIMATING THE DIMENSION OF FRACTAL OBJECTS IN AN IMAGE



$D \approx 1.66$

$D \approx 1.842$

THANK YOU!