The Big Picture of Design of Experiments

How, why and what for?

- You can find this presentation and all others at the following link: https://github.com/Mollinetti/Experiment-Design-R

- Any questions, send an e-mail to mmollinetti@gmail.com

Agenda

- What is Experiment Design?
- The Basic steps of Conducting an Experiment
- Foundation
- Point Estimators
- Normality
- Statistical Models
- Post-hoc Analysis
- Final Remarks
- Workshop

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What is Experiment Design?

- “Design of any task that aims to describe and explain the variation of information under conditions that are hypothesized to reflect the variation”
- **Validity, Reliability and Replicability** (when applicable)
- From devising the research question to the analysis of the statistical test and verification of Hypothesis
- A good design prevents wrong conclusions
- How many times have you read scientific papers with dubious results because the design was not well explained?

What is Experiment Design?

- The three basic principles (3+1):
  - Randomization
  - Replication
  - Blocking
  - Factorial principle

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The Basic Steps for conducting an experiment

1. Define your research question
2. Selection of response variable
3. List and classification of every variable that will be measured in the experiment as well as their levels and their nature
4. Determination of sample size, and desired confidence and power
5. Choose of experimental design based on the variables and research question
6. Performing the experiment
7. Statistical Analysis of the data
8. Conclusions and recommendations based on the results of the test and assumptions

The Basic Steps for conducting an experiment

Don't forget to keep these points in mind [Montgomery, 2012]:

1. Use your nonstatistical knowledge of the problem

2. Keep the design and analysis as simple as possible

3. Recognize the difference between practical and statistical significance

4. Experiments are usually iterative

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- What is Experiment Design?
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- Foundation
  - Research question
  - Factors and Levels
  - Qualitative and Quantitative variables
  - Dependent and Independent variables
  - Confidence and Power
  - Data Cleaning and Outliers
- Point Estimators
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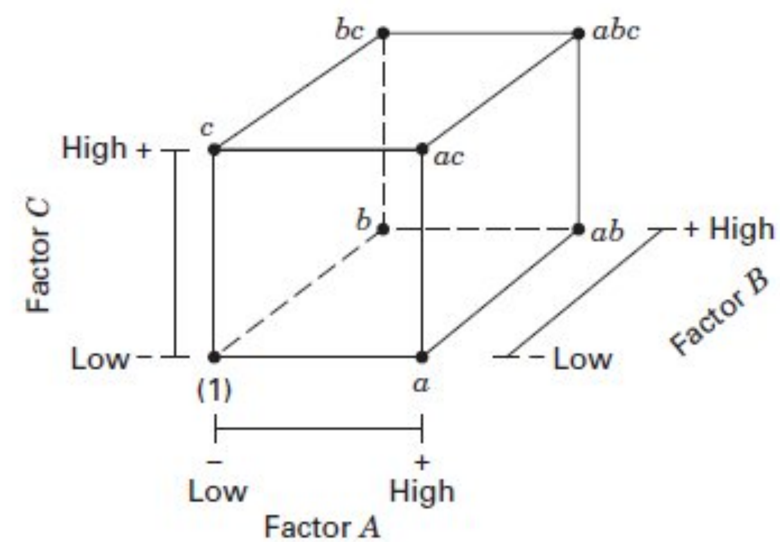
Research Question

- The main goal of the experiment that will be conducted
- A bad research question may invalidate the entire experiment
- The experiment answers the research question, **not the opposite**
- Before taking any step, always ask yourself: **“What is the research question?”**
- **Examples:**

  1. Is there a significant difference in the white blood cell levels between group A that took the medicine and group B that took a placebo?
  2. Based on the measurements of a patient that shows the signs of either Disease 1 or 2, which is the most probable disease that the patient has contracted?

Factors and levels

- It is crucial to decide the factors and levels of the experiment
- Factor: Variable measured
- Level: Degree of variation of the factor
- Example:
    - Sex (factor): Male/Female (Levels)
    - Chest pain (factor): None/ Mild/acute (Levels)



(a) Geometric view

	Factor		
Run	A	B	C
1	−	−	−
2	+	−	−
3	−	+	−
4	+	+	−
5	−	−	+
6	+	−	+
7	−	+	+
8	+	+	+

(b) Design matrix

Qualitative and Quantitative Variables

- Qualitative variables (numerical): a numerical continuous variable bounded within a range
- Quantitative variables (categorical): Attributes pertaining to a data point
- Levels of a qualitative variable are explicit, while levels of quantitative variables are implicit (usually requires binning)

Dependent and Independent variables

- Dependent variables (Response): Variable being tested and measured in a scientific experiment
  - Example: amount of Leucocytes/ brain wave activity
- Independent variables (Predictors): Variable that is changed or controlled in a scientific experiment
  - Example: Sleep phase/ treatment group/ Rh factor
- Some independent variables are difficult to control, hence the need of blocking

Confidence and power

- Confidence
  - Represented by α
  - Confidence level of any test is $100(1-\alpha)\,\%$
  - Associated to Type-I errors (False negative)
  - Easily controllable
- Power
  - Represented by β
  - Power of any test is $100(1-\beta)\,\%$
  - Associated to Type-II errors (false positive)
  - Hard to control

Data cleaning and outliers

- Errors in measurements, null values or values too abnormal for a given variable
- Most of the tests are sensitive to outliers
- Always check the consistency of the data before going further
- Check the leverage of the outliers
- Determine whether the outlier should be kept or eliminated

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  - Sample and Population estimators
  - Biased and Unbiased estimators
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Point Estimators

- Estimated value of maximum plausibility for a given population parameter θ
- A function $\widehat{\theta} = h(x)$ is called a point estimator of parameter θ and a value returned by this function is a point estimate
- Examples:
  - Mean
  - Standard Deviation
  - Variance
  - Standard Error

Sample and Population estimators

- Obtaining the true estimate of a population is almost impossible

  - What is the mean of the population of all the people that has a given disease?

- Therefore, a sample estimator is obtained by the point estimator of a given sample of population

- Sample mean $\neq$ Population mean

Biased and Unbiased estimators

- Unbiased estimator:
  - Lie close to the true estimator of the population
  - Minimum variance theorem
  - Example: mean, variance
- Biased Estimator:
  - Introduce some bias related to the true estimator of the population
  - Example: standard deviation

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    - Sampling Distributions
    - Gaussian Distribution
    - Non-Normal Data
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Sampling Distributions

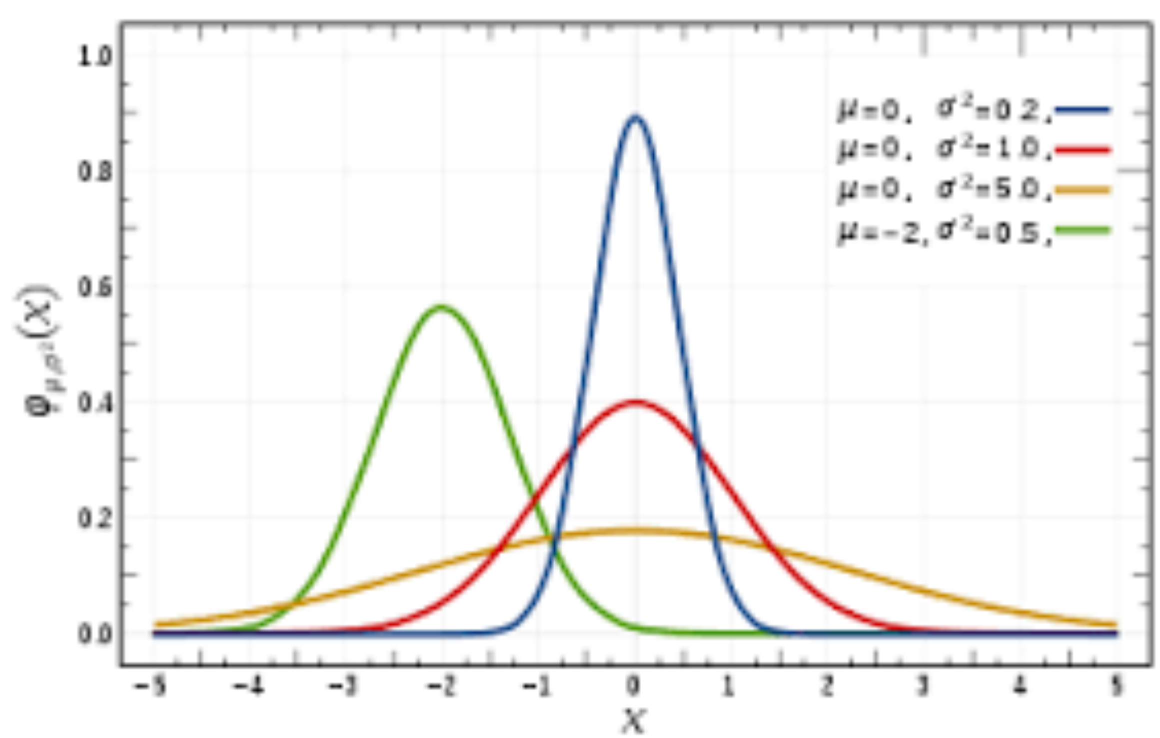
- Most of the methods studied assume that random samples are used
- The probability distribution of a statistic is called sampling distribution
- Useful distributions
  - Normal distribution
  - Chi-squared distribution
  - T-Distribution
  - F-distribution

Normality

- The great majority of the statistical tests in our classes require for the data to be normal
- What does that mean? The data has been sampled from a gaussian distribution with unknown mean and variable
- Verifying normality is a must for these tests (although most of the times it is overlooked)

Gaussian Distribution

- Parameters: mean μ and standard deviation σ^2
- The most natural random distribution
- Majority of tests in the field of statistics assume that variables assume distribution similar to the gaussian
- Bell-shaped curve
- Central Limit Theorem



Non-normal data

- What to do when your data does not comply to the assumption of normality?
  - Reduce/ Expand the number of samples
  - Transform the data (log, root, etc)
  - Use non-parametric tests

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- Normality
- Statistical Models
    - Hypothesis testing
    - Pick your statistical test
    - Assumptions of tests
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Statistical Tests

- After deciding on the design of the experiment, a statistical test must be done in order to answer the research question
- Choice of test depends on many things:
  - Number of variables (dependent and independent)
  - Sample size
  - Normality of data
  - Research question

Hypothesis Testing

- Many problems require to decide whether to accept or reject a statement about some parameter
- There are always two hypothesis:
  - H_0: Null Hypothesis, the baseline
  - H_1: Alternative Hypothesis, what is trying to be shown
- If the associated p-value is smaller than the cutoff (0.05 for 95% confidence) then H_0 is rejected
- Otherwise, we fail to reject H_0

Pick your test

- <u>**One dependent variable, One independent variable (2 groups)**</u>
  - T-test
  - Paired T-test
- ***One dependent variable, One independent variable (2+ groups)***
  - ANOVA (one or two-way)
  - ANCOVA
- <u>**2+ dependent variables, 2+ independent variables**</u>
  - MANOVA
  - MANCOVA
- <u>**One response, 2+ predictors (numerical)**</u>
  - Linear regression
  - Polynomial regression
  - Generalized Additive model (GAM)
  - Ridge Regression
  - LASSO
- <u>**One response, 2+ predictors (categorical)**</u>
  - Logistic Regression
  - Linear Discriminant Analysis (LDA)
  - Quadratic Discriminant Analysis (QDA)

Assumptions

- After choosing the most applicable statistical test, it is necessary to verify whether the data and the results are compliant to the test assumptions
- Each test has its own set of assumptions
- **The most overlooked detail when doing analysis of the results**
- Examples:
    - Normality
    - Heteroscedascity
    - Multicollinearity

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  - Power and sample size analysis
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Post-hoc Analysis

- Analysis done after conducting the statistical analysis of your experiment
- Purposes:
  - further analyze any intra or inter group relations and differences
  - Calculate the power
  - Define ideal sample size

Checking differences

- For some statistical models, like the MANOVA, MANCOVA and linear regression, further analysis of the differences can be done
- Examples of tests for checking differences:
  - Repeated one-way ANOVA
  - Repeated fitting linear models
  - Repeated t-tests
  - Tukey HSD test

Power and sample size analysis

- By defining the power of your test, it is possible to calculate the ideal sample size for a desired power
- Some statistical tests allow for direct power analysis by hypothesis testing:
  - Fligner-Killen test for t-tests
  - Power test for One-way ANOVA
- Other tests require repeated randomized sampling:
  - Bootstrapping
  - Monte Carlo simulations

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Final Remarks

- First and foremost: Remember your research question
- **P-values are not the final answer of anything**
- Do not answer your experiment with a research question
- Always be sure to verify the assumptions of the statistical test
- An unfavorable outcome might as well be a counter-example
- Remember: Experiment design is iterative and incremental!

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Workshop

- With this, our classes are over
- Bring your questions and data so they can be answered
- If there are no questions, I will show an example of real experiment data analysis using the R statistical language