ASTR 4260: Problem Set #2

Due: Wednesday, February 12

Problem 1

Implement a higher-order integrator (either the trapezoidal rule or, preferably, Simpson's rule). Use this integrator to repeat problem 3 from last problem set, that is, to numerically compute the following integral:

$$\int_0^{\pi/2} \sin(x) dx$$

Again, report this for a range of step sizes, with the number of steps $N = 10, 10^2, 10^3, 10^4, 10^5$, and again, determine how the fractional error decreases with decreasing step size (increasing N). How does this differ from the method used in the previous problem set?

Problem 2

In problem set 1, you evaluated an analytic function which gave you the ratio of the obscured to unobscured flux of a star during a planetary transit. This was done for a star with a uniform brightness across its disk. However, most stars are actually brighter in their centers and dimmer at their edges – a feature known as limb darkening.[1] This limb darkening can be parameterized with the function $I(r)$, which is the intensity of the sun's surface as a function of radius r (or alternately angle θ – see Fig. 1a of Problem set 1).

This makes the transit calculation more complicated[2] and we need to evaluate the ratio of two integrals:

$$F(p,z) = \frac{\int_0^1 I(r)\,[1 - \delta(p,r,z)]\,2r\mathrm{dr}}{\int_0^1 I(r) 2r\mathrm{dr}} \tag{1}$$

where

$$\delta(p,r,z) = \begin{cases} 0 & r \geq z + p \quad \text{or} \quad r \leq z - p, \\ 1 & r + z \leq p \\ \pi^{-1} \arccos[(z^2 - p^2 + r^2)/(2zr)] & \text{otherwise} \end{cases} \tag{2}$$

Use the integrator from the first problem of this problem set to evaluate this function. Note that it will require two integrals for each evaluation of $F(p,z)$. In particular, I want you to evaluate $F(p,z)$ using no limb-darkening (i.e. $I(r) = 1$). Do this for the values $p = 0.2$ and $z = 0.9$ and, again, try using different numbers of steps to calculate the integral: $N = 10, 10^2, 10^3, 10^4, 10^5$. How does the fractional error decrease as N increases? Use the analytic formula in the first problem set to check your result.

[1] This is due to the fact that we actually see into the outer layers of a star, rather than a hard surface. When we look at the center (as opposed to the edge), we see deeper into the star, where the gas is hotter and therefore brighter

[2] See Mandel & Agol, 2002, ApJ, 580, L171 for more details.

Note: Since $I(r) = 1$ is basically the same as ignoring $I(r)$, you might be tempted to do just that and ignore it, but resist the temptation. In a future problem set, we will use this code with a more interesting limb-darkening function.

Problem 3

Use a Monte-Carlo integration to evaluate the same integral as in the last problem (again assuming $I(r) = 1$). To do this, generate N random x and y values that are each drawn from a uniform distribution from -1 to 1 (so that you are picking random points inside a box that covers the unit circle). Reject points that lie outside the unit circle (i.e. for which $x^2 + y^2 > 1$). Call the number of accepted points N_1. In addition, count how many of the accepted points lie within the eclipsing planet disk (i.e. for which $(x - z)^2 + y^2 < p^2$) and call that number N_2. Then an estimate for $F(p, z)$ is the ratio of points inside the star's disk that do not lie in the planet's disk to the number of points inside the star's disk (without the planet): $F(p, z) \approx (N_1 - N_2)/N_1$.

Evaluate this for the same p and z values as above. Again repeat for the same range of N values and determine how the error decreases with N. What "order" is this method?

To generate a random number from -1 to 1 in python[3], use:

```
import random
x = random.uniform(-1, 1)
```

[3] You could also use numpy.random.uniform to generate an array of random values.