

# ABOUT SOME WORDS

- **Relativity:** things such as
  - time of things
  - length of things
  - simultaneity of things
  - mass (kinetic energy) of things

are **relative to the speed of the observer** since (Einstein 1905).

- **Special Relativity:** Relativistic Physics in which acceleration and gravitation is ignored.
- **General Relativity:** Relativistic Physics in which acceleration and/or gravitation is not ignored.
- **Kinematics:** physics of moving (speed).
- **Dynamics:** physics of collisions (mass).

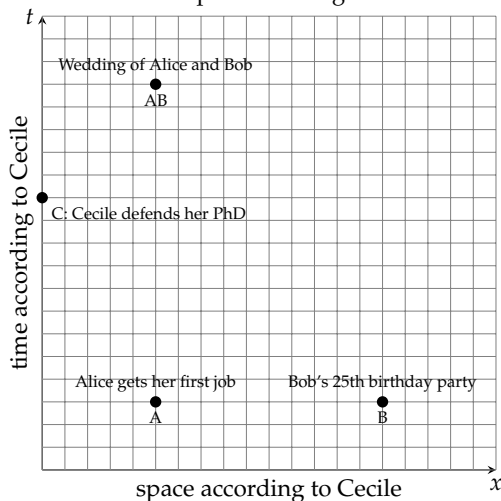
# Introduction into Special Relativity

# SPACETIME DIAGRAMS

## IMPORTANT EVENTS IN THE LIFE OF ALICE AND BOB

- AB happened later than A and B according to Alice.
- A and B happened *at the same time* according to Alice. In other words, A and B are *simultaneous*.
- Bob was not there when Alice got her first job.
- Bob had to travel to the left to his wedding since his 25th birthday.
- A and AB happened *at the same location* according to Alice.
- Cecile was present on her PhD defense, but she wasn't there when A, B, AB happened.

### 2D spacetime diagram

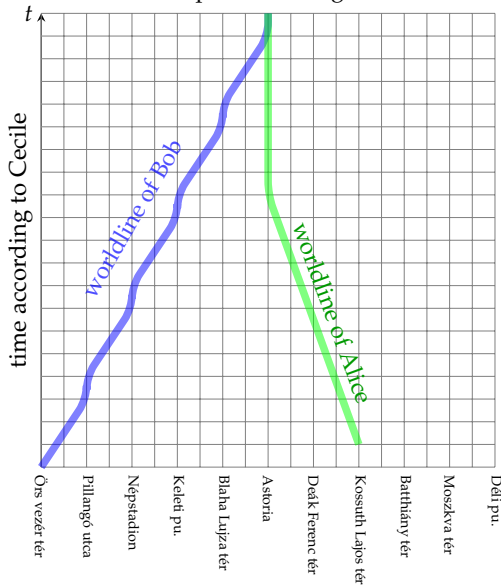


# SPACETIME DIAGRAMS

## RANDEVOUS OF ALICE AND BOB

- Bob probably travels on the subway, since he is always accelerating and decelerating as the subway stops at every station.
- Alice probably goes by bike since she did not get or ignored the red lamps during her journey.
- The subway waits 1 min at every station.
- Alice biked 11 min.
- Alice arrived to Astoria first, and then she waited 7 min for Bob.
- Alice arrived to Astoria at the same time when Bob left Keleti train station.

2D spacetime diagram

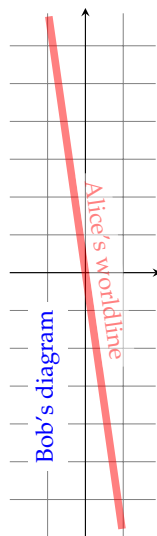
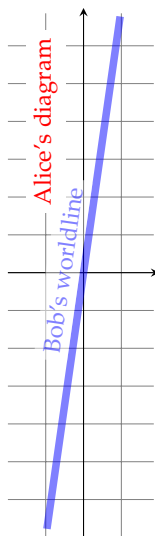


# SPACETIME DIAGRAMS

## MAIN RULES

ASSUMPTIONS concerning space time diagrams about the same reality/spacetime:

- **AxReals** We can use the real numbers to speak about spacetime diagrams
- **AxEvent** If an event is present in somebody's diagram, then it is present in everybody's diagram.
- **AxSelf** Every observer occupies the time axis of its spacetime diagram.
- **AxSym** Every observer uses the same measure system.

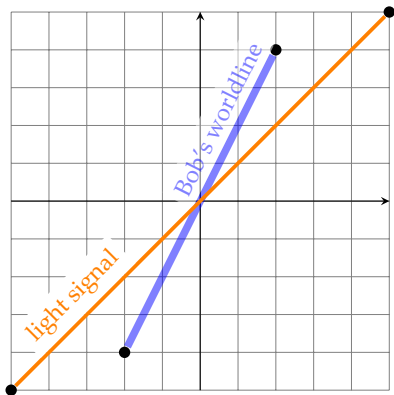


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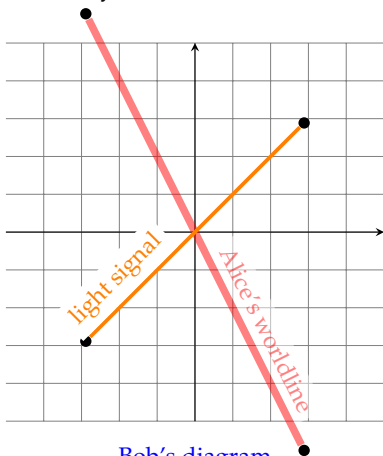
## ROLE OF LIGHT

EMPIRICAL FACT concerning physical reality

- **AxPhObs:** The speed of light is the same for every observer.



Alice's diagram



Bob's diagram

# Paradigmatic Relativistic Effects

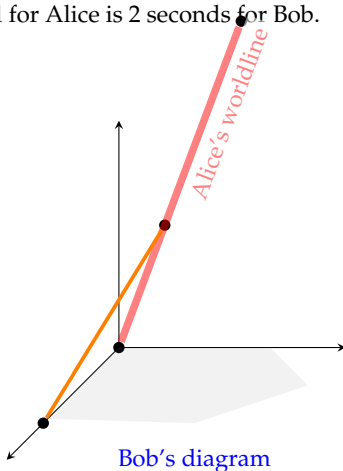
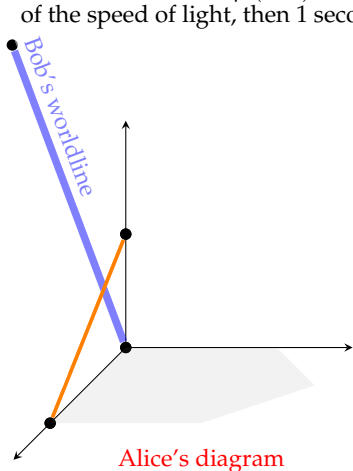
# PARADIGMATIC RELATIVISTIC EFFECTS

- **Asynchron:** Moving pairs of clocks get out of synchronism
- **Time Dilation:** Moving clocks slow down.
- **Length Contraction:** Moving meter rods shrink.



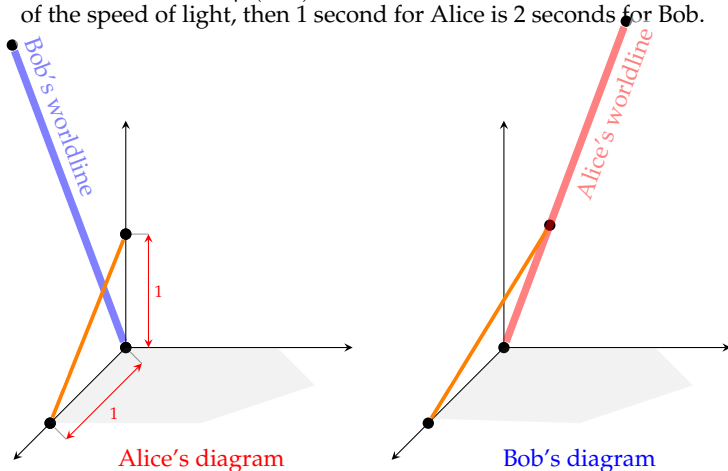
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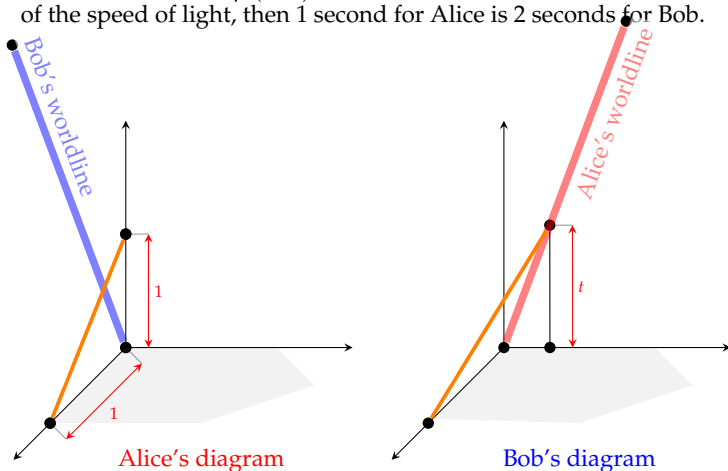
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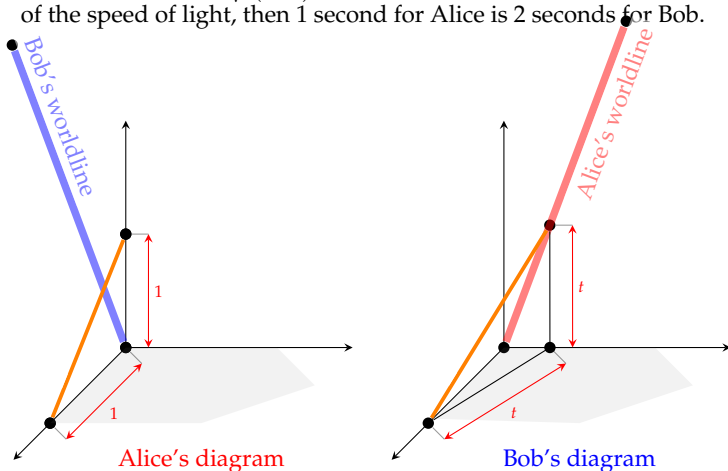
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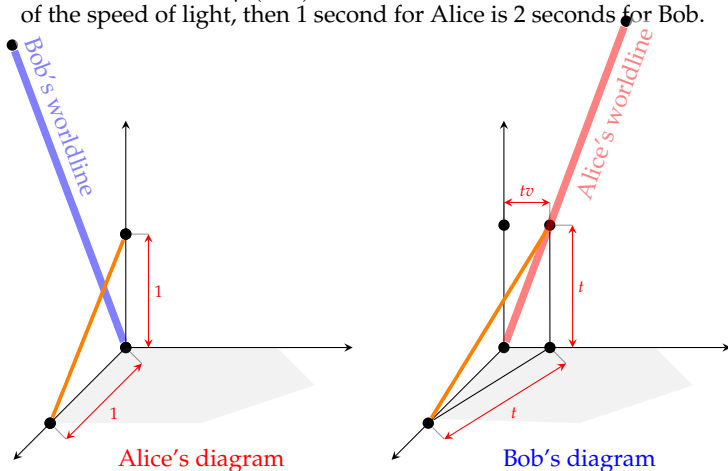
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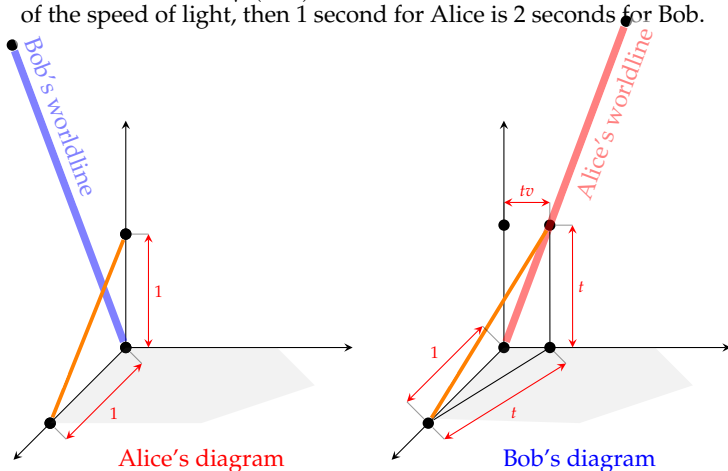
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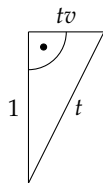
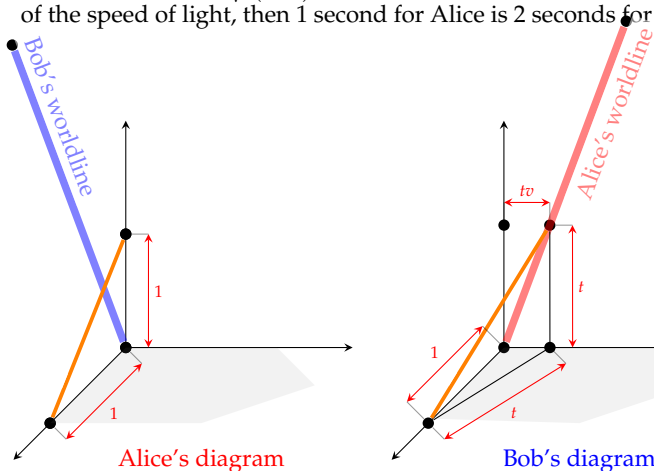
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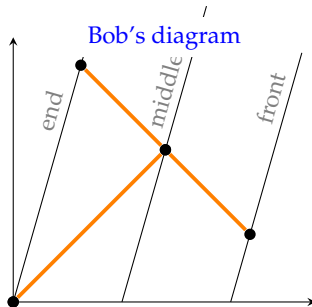
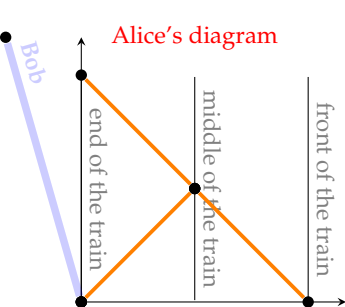
$$\begin{aligned}
 t^2 &= 1^2 + (tv)^2 \\
 t^2 &= 1 + t^2 v^2 \\
 t^2 - t^2 v^2 &= 1 \\
 t^2 (1 - v^2) &= 1 \\
 t^2 &= \frac{1}{(1 - v^2)} \\
 t &= \frac{1}{\sqrt{1 - v^2}}
 \end{aligned}$$

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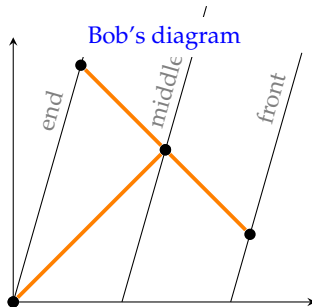
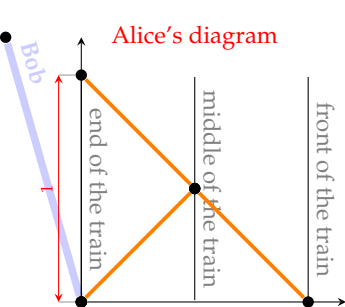


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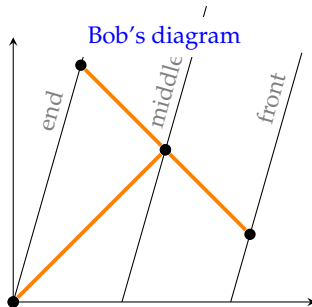
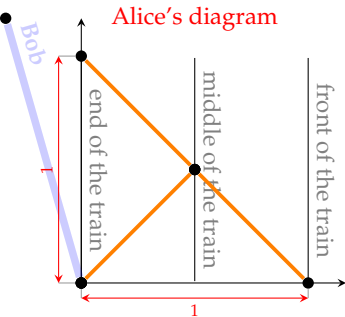


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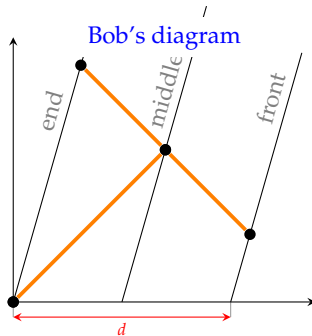
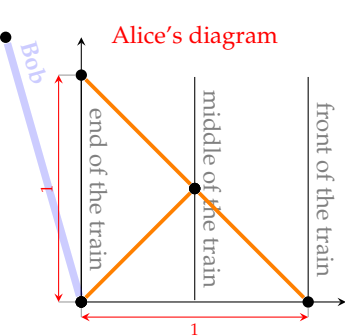


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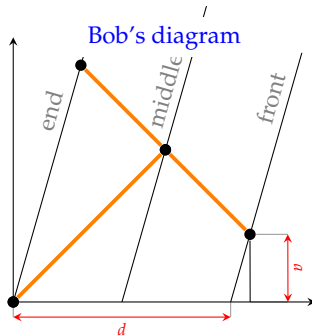
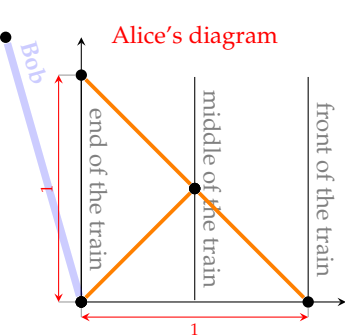


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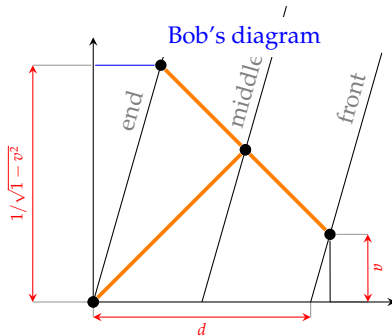
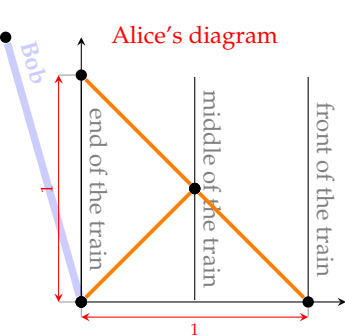


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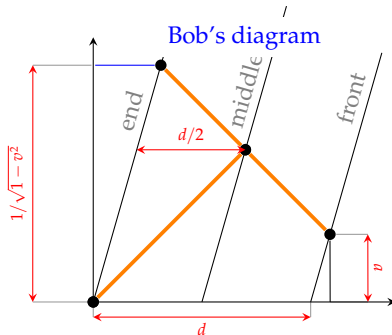
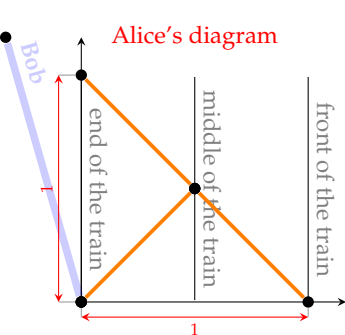


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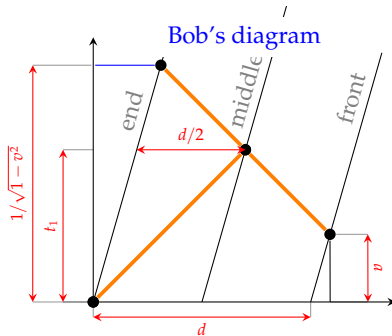
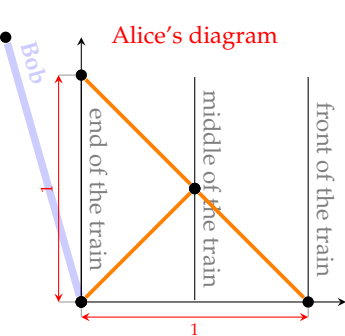


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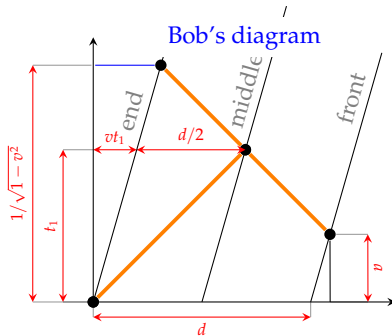
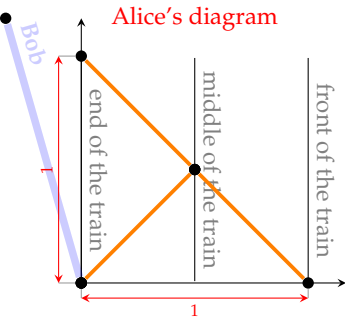


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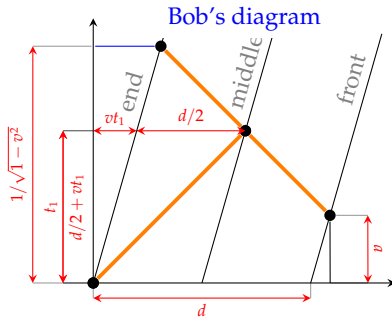
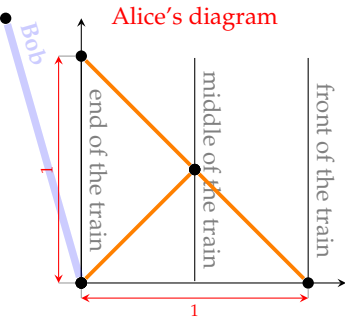


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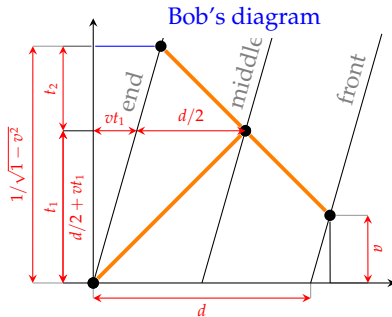
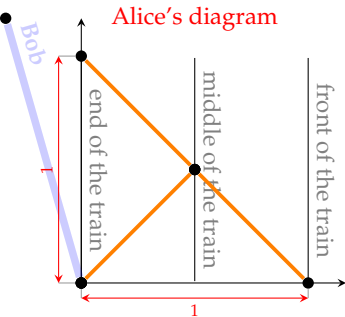


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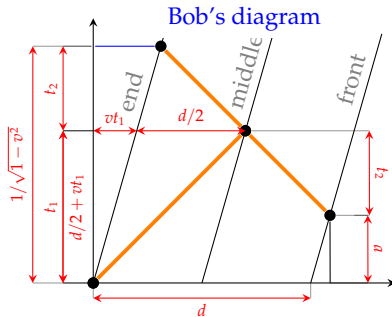
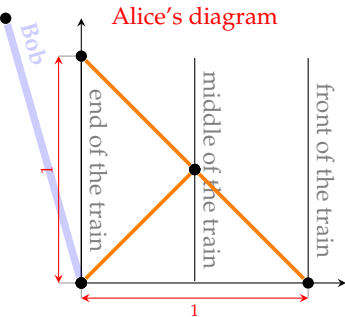


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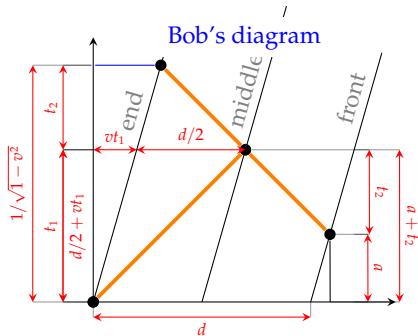
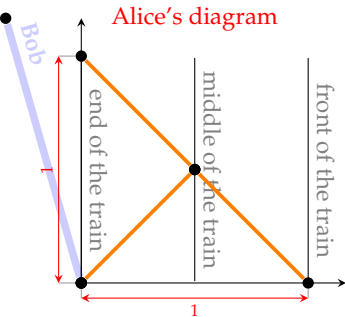


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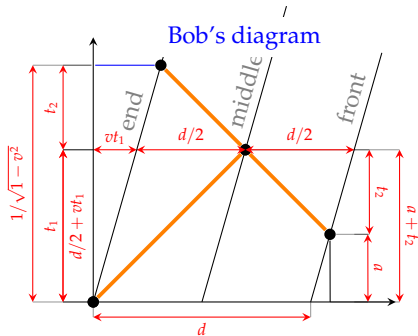
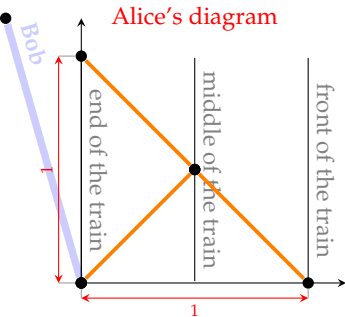
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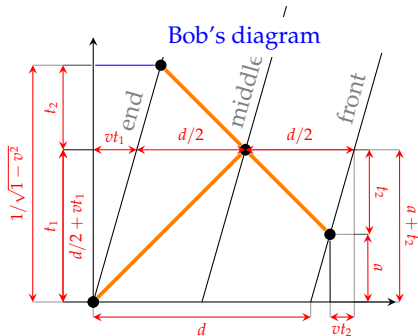
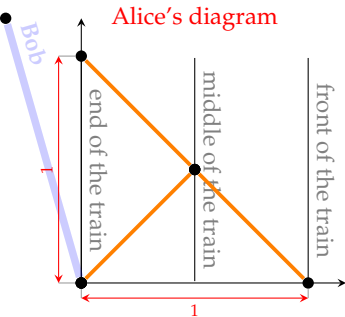


# LENGTH CONTRACTION AND ASYNCHRON

Alice is sitting in the 1-lightsecond long train's end, when two lightning struck into the end and the front of the ship at the same time according to Alice. If the train has a speed of  $v$  according to Bob, then

**Contraction:** 1 lightsecond for Alice is  $\sqrt{1-v^2}$  lightsecond for Bob. E.g., if Alice travels with the 87% of the speed of light, then the train is only 0.5 lightsecond long for Bob.

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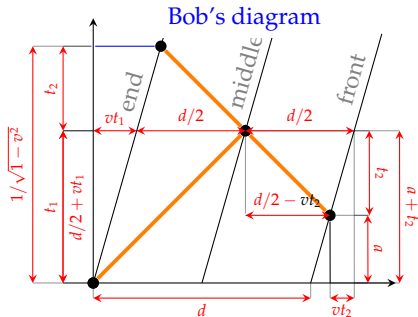
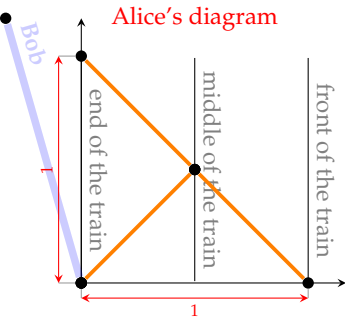


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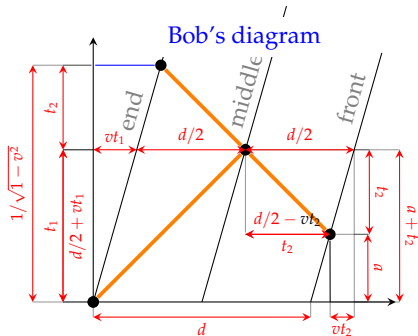
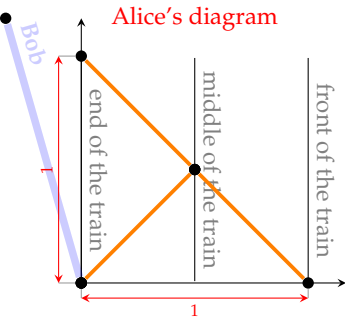


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$$t_1 = \frac{d}{2} + t_1 v$$

$$t_1 - t_1 v = \frac{d}{2}$$

$$t_1(1 - v) = \frac{d}{2}$$

$$t_1 = \frac{d}{2(1-v)}$$

$$t_2 = \frac{d}{c} - vt_2$$

$$\dots = \frac{d}{2(1+v)}$$

$$\frac{1}{\sqrt{1-v^2}} = t_1 + t_2$$

$$= \frac{d}{2(1-p)} + \frac{d}{2(1+p)}$$

$$= \frac{d(1+v) + d(1-v)}{2(1-v)(1+v)}$$

$$= \frac{d+dv+d-dv}{2(1-v)(1+v)}$$

$$= \frac{2d}{2(1-p)(1+p)}$$

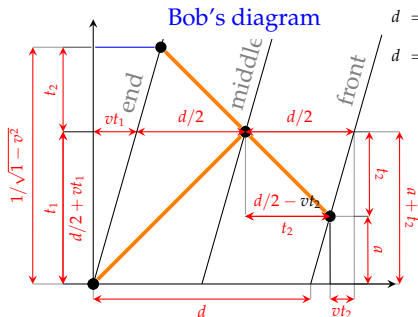
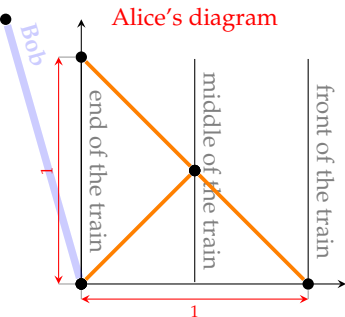
$$= \frac{d}{(1-v)(1+v)}$$

$$= \frac{d}{1-v^2}$$

$$\frac{d}{1-v^2} = \frac{1}{\sqrt{1-v^2}}$$

$$d = \frac{1-v^2}{\sqrt{1-v^2}}$$

$$d = \sqrt{1 - \epsilon^2}$$



$$a \equiv \frac{hw}{\sqrt{1-v^2}}$$

# Lorentz Transformation

# LORENTZ TRANSFORMATION

Assume that Bob moves in the direction of Alice's  $x$  axis.

(So Bob is in Alice's  $\overline{xt}$  plane)

Remember that every coordinate can be composed from the four unit vectors  $1_x, 1_y, 1_z, 1_t$ . A coordinate is just the story of how many  $1_x, 1_y, 1_z, 1_t$  vectors did you sum (respectively) to get the point in question. So just transform these vectors first, and then recompose them to get the result:

$$\begin{aligned}\ell(1_{\mathbf{x}}) &= \left( \frac{1}{\sqrt{1-v^2}}, 0, 0, \frac{v}{\sqrt{1-v^2}} \right), & \ell(1_{\mathbf{y}}) &= 1_{\mathbf{y}}, \\ \ell(1_{\mathbf{t}}) &= \left( \frac{v}{\sqrt{1-v^2}}, 0, 0, \frac{1}{\sqrt{1-v^2}} \right), & \ell(1_{\mathbf{z}}) &= 1_{\mathbf{z}}.\end{aligned}$$

Now if

$(x, y, z, t)$  : coordinates of an event  $e$  according to Alice

$v$ : Bob's speed according to Alice.

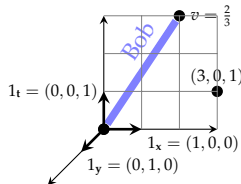
$(x', y', z', t')$  : coordinates of an event  $e$  according to **Bob**

The new coordinates are told by the Lorentz transformation:

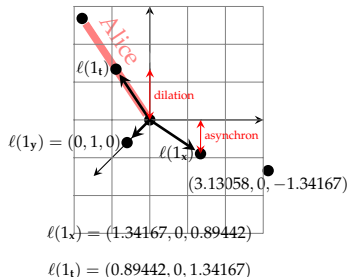
$$x' = \frac{x - vt}{\sqrt{1 - v^2}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - vx}{\sqrt{1 - v^2}}$$

For comparison: the old, Newtonian transformation was:  $x' = x - vt$ ,  $y' = y$ ,  $z' = z$ ,  $t' = t$

### Alice's diagram



### Bob's diagram

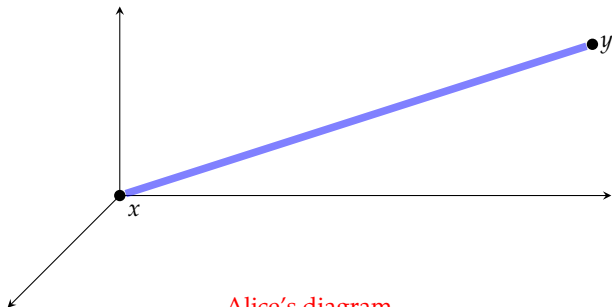


# NoFTL

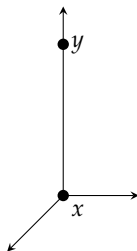
# NOFTL: NO FASTER THAN LIGHT **OBSERVERS**

Suppose indirectly, that Bob travels faster than the speed of light. Watch out for the following two assumptions:

- the dimension of spacetime is more than 2 – actually, there can be FTL observers in 2D spacetime.
- Bob can coordinatize the spacetime, i.e., he is an *observer*. Actually, FTL *particles* are possible as long as they cannot coordinatize their neighbourhood too well.



Alice's diagram

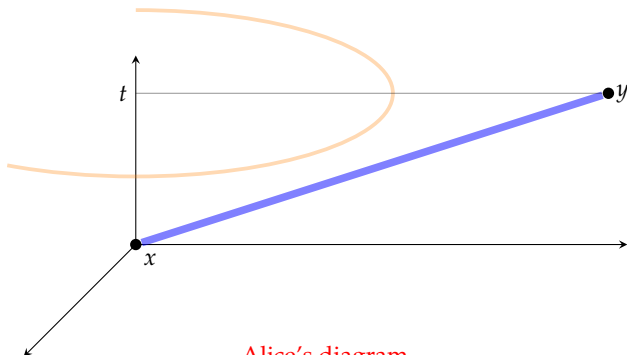


Bob's diagram

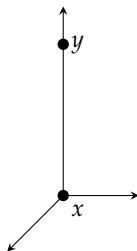
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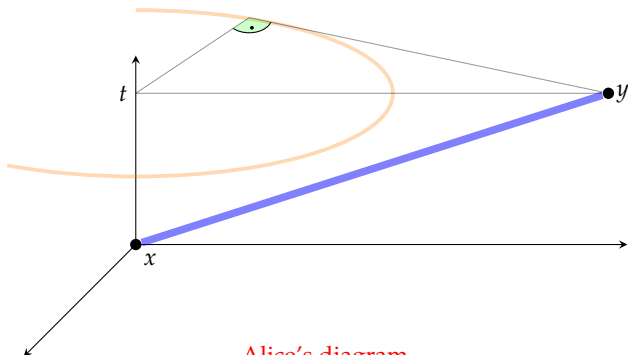


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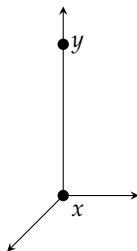
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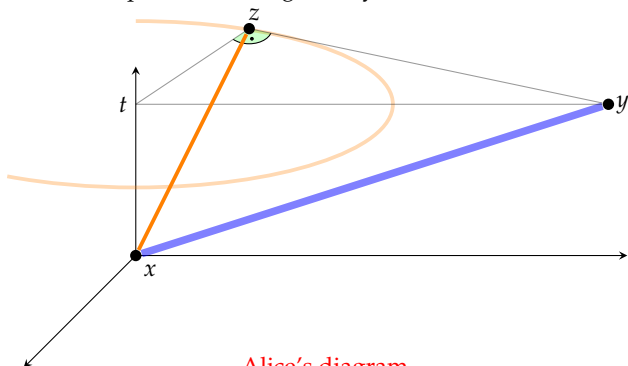


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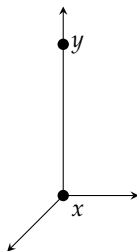
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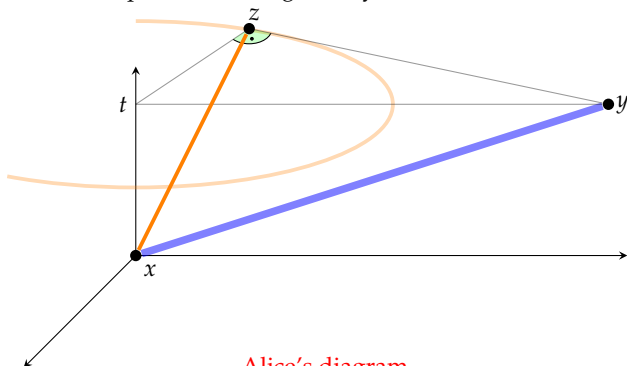
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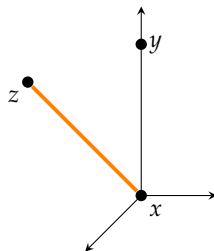
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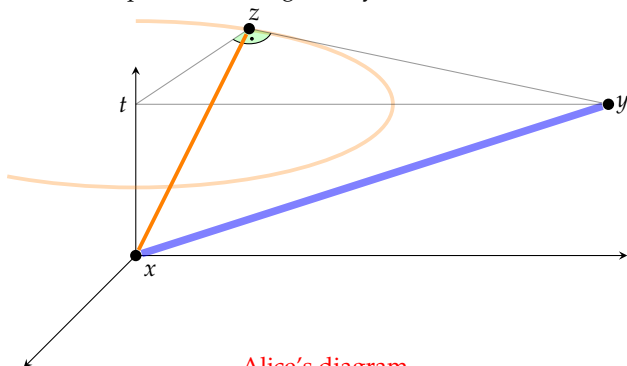


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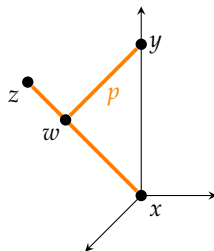
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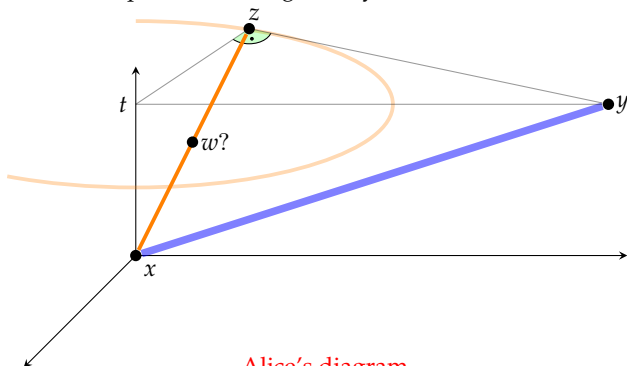


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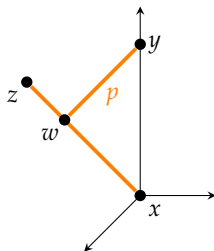
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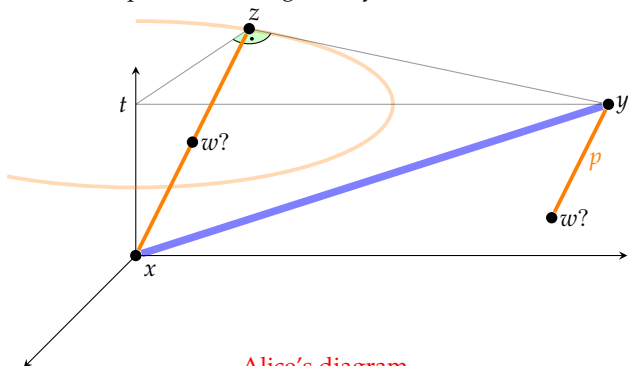


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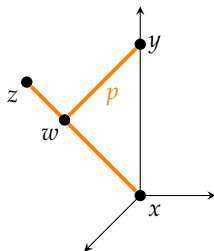
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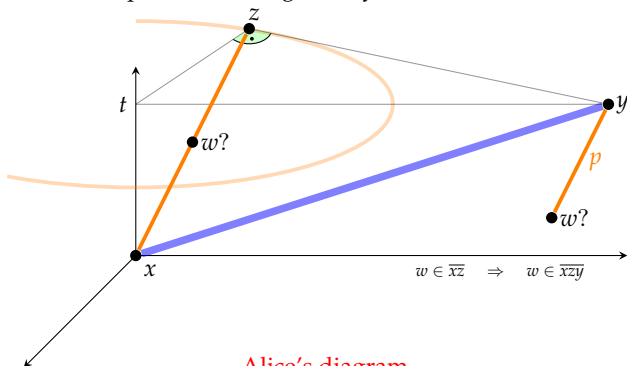


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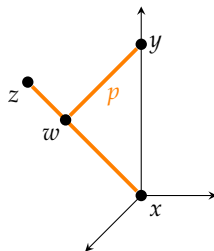
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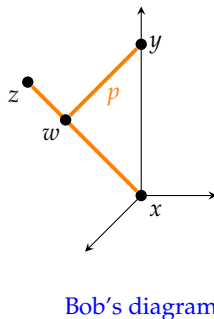
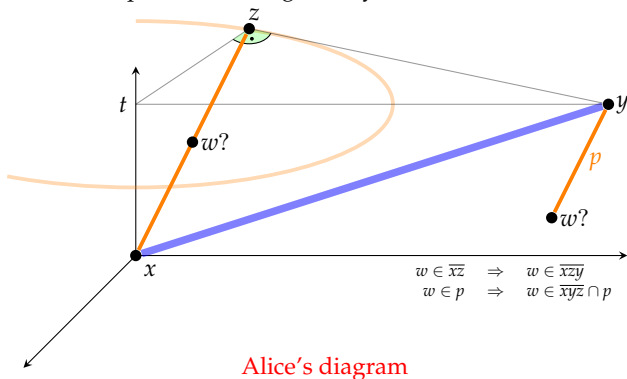


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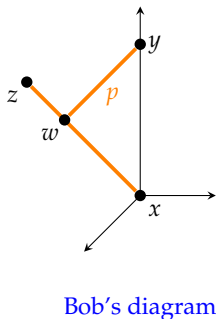
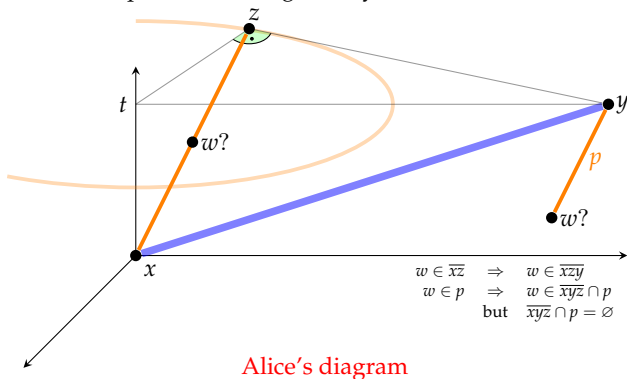
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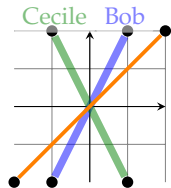
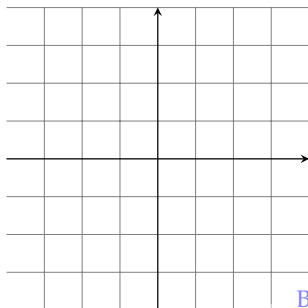


# SIMULTANEITIES

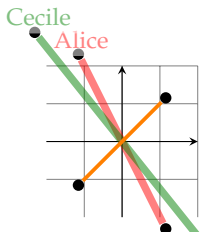
Two events are

- *timelike separated* iff there could be an observer between them.
- *lightlike separated* iff there could be a light-signal between them.
- *spacelike separated* iff they are simultaneous for an observer.

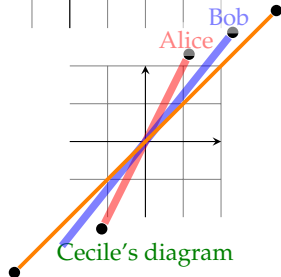
How the other coordinate systems look like?



Alice's diagram



Bob's diagram



Cecile's diagram

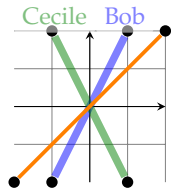
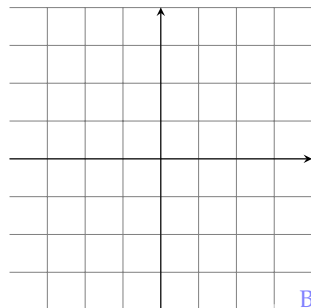


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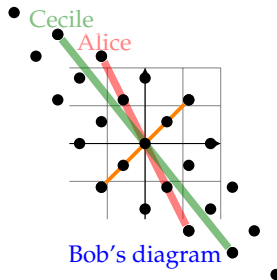
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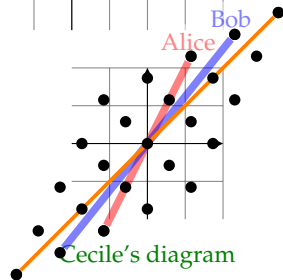
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Alice's diagram



Bob's diagram



Cecile's diagram

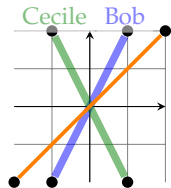
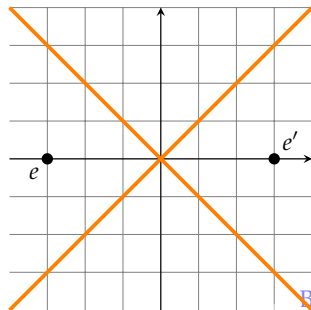
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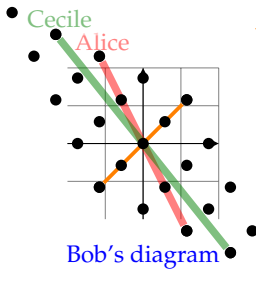
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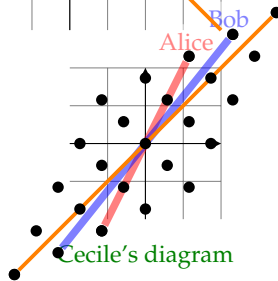
- $e$  and  $e'$  are simultaneous for me.



Alice's diagram



Bob's diagram



Cecile's diagram

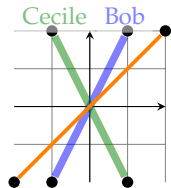
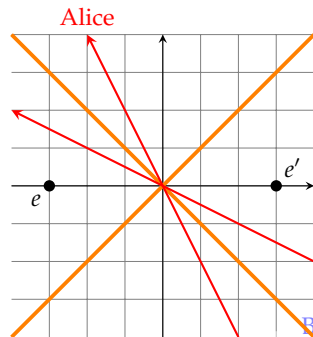
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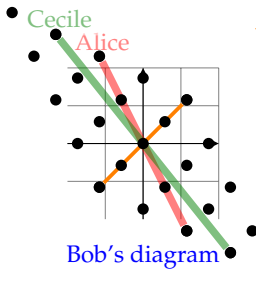
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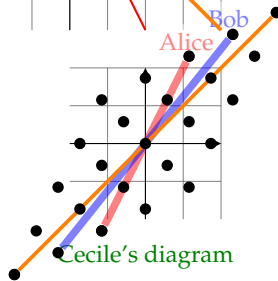
- $e$  and  $e'$  are simultaneous for me.
- $e$  happened before  $e'$  for **Alice**.



Alice's diagram



Bob's diagram



Cecile's diagram

# SIMULTANEITIES

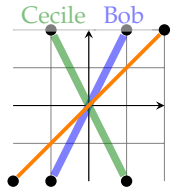
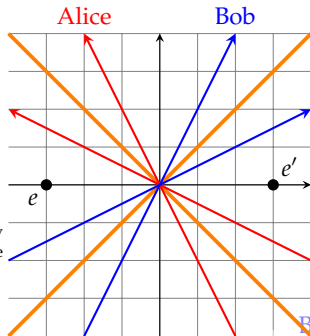
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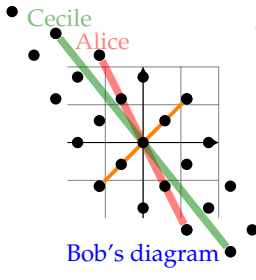
How the other coordinate systems look like?

- $e$  and  $e'$  are simultaneous for me.
- $e$  happened before  $e'$  for **Alice**.
- $e$  happened after  $e'$  for **Bob**.

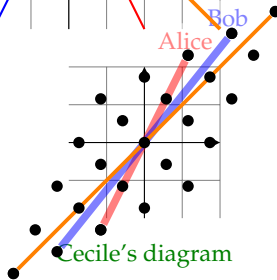
Notice that  $e$  and  $e'$  were *spacelike separated* for everybody! We can't play the same with timelike and lightlike separated events! (Why?) So all the observers agree on the order of time- and lightlike separated events.



Alice's diagram



Bob's diagram



Cecile's diagram

# CAUSALITY

- **NoFTL** theorem: There is no faster than light observer.
- **NoFTL** hypothesis: There is no faster than light particle/effect.

The latter is a hypothesis because we can not prove it, although it is very well confirmed. (No FTL particle was discovered yet.)

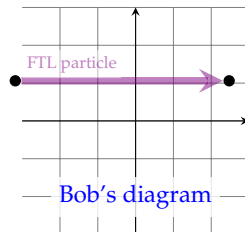
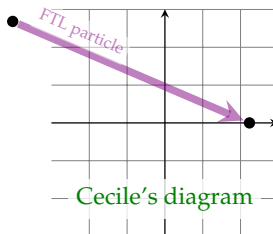
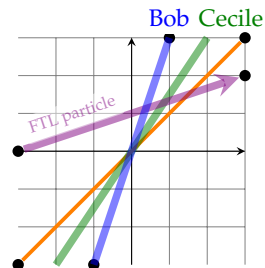
# CAUSALITY

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Also note that, if FTL particles exist, then funny things exists, like

- “infinite speed”, existence in multiple locations
- time travel



And time travel makes fun of Causality. Who sent the FTL message, Cecile or Alice? What caused what?

Basically by these reasons it is usual to exclude FTL particles, i.e., **to assume the NoFTL hypothesis**.

But never forget, that this is a new axiom, which is not implied by the axioms of special relativity.

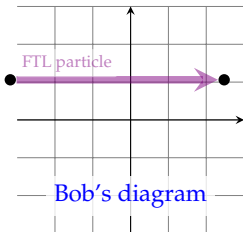
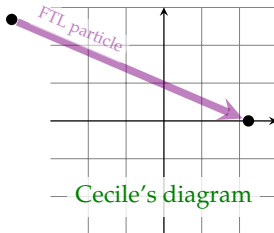
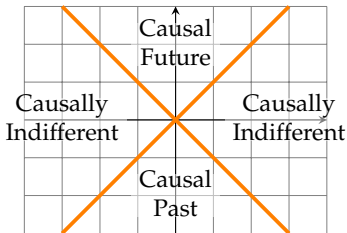
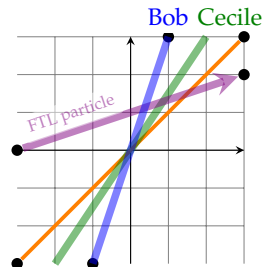
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# Minkowski distance



# MOTIVATING MINKOWSKI DISTANCE

We are looking for an *observer-independent* distance events.

MORAL OF PARADIGMATIC EFFECTS: Observers may not agree on the spatial distance and elapsed time between events.

BUT: proper relativity needs non-zero relative speed. Parallel moving observers agree on elapsed time and spatial distance.

IDEA: Every observer will have its own jurisdiction, and every observer has to ask the **competent** observer to decide Minkowski distance.

$k$  is **competent** for  $e$  and  $e'$  iff they are on the worldline of  $k$  or they both happened when  $k$ 's clock showed 0.

$$\mu_k(e, e') \stackrel{\text{def}}{=} \begin{cases} x & \text{if } m \text{ crosses } e \text{ and } e' \text{ (they happen 0 far away)} \\ & x \text{ measures the elapsed time between } e \text{ and } e' \text{ to be } x \\ 0 & \text{if there is a photon through } e \text{ and } e'. \text{ (Everybody agree!)} \\ -x & \text{if } m \text{ observes that } e \text{ and } e' \text{ happens at time 0} \\ & \text{and } m \text{ measures spatial distance between } e \text{ and } e' \text{ to be } x. \end{cases}$$

Therefore this notion is observer-independent. Also, the Minkowski distance of  $e$  and  $e'$  according to  $k$  happens to be

$$\mu_k(e, e') = \sqrt{(e_t - e'_t)^2 - (e_x - e'_x)^2 - (e_y - e'_y)^2 - (e_z - e'_z)^2}$$

where  $e$  and  $e'$  are coordinatized by  $k$  on  $(e_t, e_x, e_y, e_z)$  and  $(e'_t, e'_x, e'_y, e'_z)$ , respectively.

# $n$ -DIMENSIONAL MINKOWSKI SPACETIME

$n$  dimensional Minkowski spacetime is

$$(\mathbb{R}^n, \mu)$$

where

$$\mu(\vec{x}, \vec{y}) \stackrel{\text{def}}{=} \sqrt{(\vec{x}_1 - \vec{y}_1)^2 - \sum_{2 \leq i \leq n} (\vec{x}_i - \vec{y}_i)^2}$$

i.e., a spacetime diagram with its observer independent minkowski-distance.