PROLOG 2025 ELTE, LOGIC DEPARTMENT

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COURSE MATERIAL: LEARN PROLOG NOW!

Aim of the course: to advertise Prolog via programming in it.



One of the best introductory books. It does not cover much, but it is an excellent introduction **written by logicians**.

(for modal logicians: one of the authors is Patrick Blackburn!)

OUTLINE

Introduction (history, basic concepts, course material)

- history
- basic concepts
- course material

Syntax

- elements of the language
- facts, rules, knowledge base
- queries

Horn clauses

Unification

Introduction

Brief History

- 1972 Marseille (PROgramming in LOGic)
- 1978 PROLOG interpreter
- 1987 SWI (free; online & downloadable has better debugger).





- 1987 Jaffar and Lassez: Constraint logic programming. Powerful and beautiful clp packages are included since.
- 1990 SICStus Prolog (proprietary)

Used at

- Watson Q&A machine of IBM e.g. cancer
- NASA international space station (Clarissa)
- Ericsson Network Resource Manager (operator assistant)
- Logistics
- Data mining Rule Discovery System (RDS)

DECLARATIVE, LOGIC PROGRAMMING

Declarative programming: define what to solve, not how to solve it.

We describe the problem (instead of the way to the solution – algorithms), the program solves it using it's own strategies (e.g. backtracking)

Logic programming: syntax follows the syntax of logic (Horn logic).

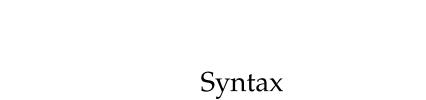
Program: logical formula(s)

Running the program: evaluation of the formula(s)

No classical variables (temporal substitution vs assignment)

Recursion instead of loops





LANGUAGE ELEMENTS

CONSTANTS

- Atoms
 - String beginning with lower case letter

```
dEPARTMENT, logic, l_o_g_i_c, year1988
```

• String in single quotes

```
'Mekis', '_String', 'WHAT3V3R'
```

Numbers: usual...

```
-1, 0, 1, 2, ..., integers, floats...
```

LANGUAGE ELEMENTS

VARIABLES

String starting with upper case letter

String starting with

The variable _ is very special, will treat it differently! (The string cannot contain any (,) or .)

Which of the following sequences of characters are **atom**s, which are **variable**s, and which are **neither**?

thisiaVariable

Which of the following sequences of characters are **atom**s, which are **variable**s, and which are **neither**?

thisiaVariable ELTE

atom

Which of the following sequences of characters are **atom**s, which are **variable**s, and which are **neither**?

thisiaVariable atom
ELTE variable
this_is_a_Variable

```
thisiaVariable atom
ELTE variable
this_is_a_Variable atom
'_John
```

```
thisiaVariable atom
ELTE variable
this_is_a_Variable atom
'_John neither
'John loves Mary'
```

```
thisiaVariable atom
ELTE variable
this_is_a_Variable atom
'_John neither
'John loves Mary' atom
prolog2017
```

thisiaVariable	atom
ELTE	variable
this_is_a_Variable	atom
'_John	neither
'John loves Mary'	atom
prolog2017	atom
Nimbus2000	

thisiaVariable	atom
ELTE	variable
this_is_a_Variable	atom
'_John	neither
'John loves Mary'	atom
prolog2017	atom
Nimbus2000	variable
John loves Mary	

thisiaVariable	atom
ELTE	variable
this_is_a_Variable	atom
'_John	neither
'John loves Mary'	atom
prolog2017	atom
Nimbus2000	variable
John loves Mary	neither
_John	

thisiaVariable	atom
ELTE	variable
this_is_a_Variable	atom
'_John	neither
'John loves Mary'	atom
prolog2017	atom
Nimbus2000	variable
John loves Mary	neither
_John	variable
'John'	

thisiaVariable	atom
ELTE	variable
this_is_a_Variable	atom
'_John	neither
'John loves Mary'	atom
prolog2017	atom
Nimbus2000	variable
John loves Mary	neither
_John	variable
'John'	atom

LANGUAGE ELEMENTS



TERMS

- Constants and variables are terms.
- If **f** is an atom and τ_1, \ldots, τ_n are **terms**, then

$$f(\tau_1, \tau_2, \ldots, \tau_n)$$

is a (complex) **term**.

```
grandmotherof(X, motherof(fatherof(Y)))
```

The number of contained terms are called the arities of the function/functor/predicate f. In PROLOG, if f is a ternary function we refer to that in the following way: f/3. This will be very important, because it is not uncommon in PROLOG to use different functions such as f/2, f/3 and f/5 in a parallel way. We will never do that though.

```
'loves (Vincent, Mia)'
```

```
'loves (Vincent, Mia)' atom
loves (Vincent Mia)
```

```
'loves(Vincent, Mia)' atom
loves(Vincent Mia) not term
or(super(M), bat(M))
```

Which of the following sequences of characters are **atoms**, which are **variables**, which are **complex terms**, and which are **not terms** at all? Give the functor and arity of each complex term.

```
'loves(Vincent,Mia)' atom
loves(Vincent Mia) not term
or(super(M),bat(M)) complex te
Butch(boxer)
```

atom
not term
complex term (or/2, super/1, bat/1)

Which of the following sequences of characters are **atoms**, which are **variables**, which are **complex terms**, and which are **not terms** at all? Give the functor and arity of each complex term.

```
'loves(Vincent, Mia)'
loves(Vincent Mia)
or(super(M), bat(M))
Butch(boxer)
boxer(Butch)
```

atom not term complex term (or/2, super/1, bat/1) neither

```
'loves(Vincent, Mia)'
loves(Vincent Mia) not term
or(super(M), bat(M)) complex term (or/2, super/1, bat/1)
Butch(boxer) neither
boxer(Butch) complex term (butch/1)
beats(Batman,'Superman')
```

```
'loves(Vincent, Mia)' atom
loves(Vincent Mia) not term
or(super(M), bat(M)) complex term (or/2, super/1, bat/1)
Butch(boxer) neither
boxer(Butch) complex term (butch/1)
beats(Batman,'Superman') complex term (butch/2)
_or(Super(M), Bat(M))
```

```
'loves (Vincent, Mia)'
                               atom
loves (Vincent Mia)
                               not term
                               complex term (or/2, super/1, bat/1)
or (super (M), bat (M))
                               neither
Butch (boxer)
boxer (Butch)
                               complex term (butch/1)
beats(Batman,'Superman')
                               complex term (butch/2)
_or(Super(M),Bat(M))
                               neither
or (super (man), bat (man))
                               complex term (or/2, super/1, bat/1)
(Batman beats Superman)
```

```
'loves(Vincent,Mia)'
loves(Vincent Mia)
or(super(M),bat(M))
Butch(boxer)
boxer(Butch)
beats(Batman,'Superman')
_or(Super(M),Bat(M))
or(super(man),bat(man))
(Batman beats Superman)
loves(Vincent,Mia
```

```
atom
not term
complex term (or/2, super/1, bat/1)
neither
complex term (butch/1)
complex term (butch/2)
neither
complex term (or/2, super/1, bat/1)
not term
```

```
'loves (Vincent, Mia)'
                               atom
loves (Vincent Mia)
                               not term
                               complex term (or/2, super/1, bat/1)
or (super (M), bat (M))
                               neither
Butch (boxer)
boxer (Butch)
                               complex term (butch/1)
beats(Batman,'Superman')
                               complex term (butch/2)
_or(Super(M),Bat(M))
                               neither
                               complex term (or/2, super/1, bat/1)
or (super (man), bat (man))
(Batman beats Superman)
                               not term
loves (Vincent, Mia
                               not term
```

LANGUAGE ELEMENTS

CONNECTIVES

```
and ,
or ;
if :-
not \+
```

The "not" \+ is **NOT** the negation you think of! Do not use it until we learn it how to use. It modifies the searching algorithm

of , and as such it is not a declarative command!

FACTS, RULES AND KNOWLEDGE BASE

If τ is a term that does not contain any variable, then τ is a **fact**. (Ending dot!)

```
loves(adam, motherof('Cain')).
```

- If σ and τ are terms, then $\sigma: -\tau$. is **a rule**.
- If ϱ . is a **rule** and τ is a term, then ϱ , τ . is a **rule** as well. (and)

(if)

(spaces and breaks do not matter)

A knowledge base is a collection of facts and rules.

KNOWLEDGE BASES

A **knowledge base** is collection of facts and rules.

```
loves(vincent, mia).
loves(marcellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
jealous(X, Y) :- loves(X, Z),loves(Y, Z).
```

QUERIES / PROOF SEARCHES

```
loves(vincent, mia).
loves(marcellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
jealous(X, Y) :- loves(X, Z),loves(Y, Z).
jealous(X, Y).
and will answer that...
```

```
man('Eomund').
woman('Theodwyn').
son('Eomer', 'Eomund').
daughter('Eowyn', 'Eomund').
daughter('Eowyn', 'Eomund').
man('Eomer').
woman('Eowyn').
son('Eomer', 'Theodwyn').
daughter('Eowyn', 'Theodwyn').
```

```
man('Eomund').
woman('Theodwyn').
son('Eomer', 'Eomund').
daughter('Eowyn', 'Eomund').
daughter('Eowyn', 'Eomund').
father(X,Y):= man(X), son(Y,X).
father(X,Y):= man(X), daughter(Y,X).
mother(X,Y):= woman(X), son(Y,X).
mother(X,Y):= woman(X), daughter(Y,X).
```

```
man('Eomund').
    woman('Theodwyn').
    son('Eomer', 'Eomund').
    daughter('Eowyn', 'Eomund').
    daughter('Eowyn', 'Eomund').

father(X,Y):= man(X), son(Y,X).
father(X,Y):= man(X), daughter(Y,X).
mother(X,Y):= woman(X), son(Y,X).
mother(X,Y):= woman(X), daughter(Y,X).
parent(X):= father(X,_); mother(X,_).
parent(X,Y):= father(X,Y); mother(X,Y).
```

```
man ('Eomund').
                                 man ('Eomer').
 woman ('Theodwyn').
                                 woman ('Eowyn').
 son('Eomer', 'Eomund').
                                 son('Eomer', 'Theodwyn').
 daughter ('Eowyn', 'Eomund'). daughter ('Eowyn', 'Theodwyn').
father (X,Y) := man(X), son(Y,X).
father (X,Y) := man(X), daughter (Y,X).
mother(X,Y) := woman(X), son(Y,X).
mother(X,Y) := woman(X), daughter(Y,X).
parent (X) := father(X, _); mother(X, _).
parent (X,Y):- father (X,Y); mother (X,Y).
sister (Y,Z):-parent (X), daughter (Y,X), daughter (Z,X), Z = Y.
sister(Y, Z):- parent(X), daughter(Y, X), son(Z, X).
brother (Y, Z) := parent(X), son(Y, X), son(Z, X), Z = Y.
brother (Y, Z) := parent(X), son(Y, X), daughter (Z, X).
```

```
man ('Eomund').
                                 man ('Eomer').
 woman ('Theodwyn').
                                 woman ('Eowyn').
 son('Eomer', 'Eomund').
                                 son('Eomer', 'Theodwyn').
 daughter('Eowyn', 'Eomund'). daughter('Eowyn', 'Theodwyn').
father (X,Y) := man(X), son(Y,X).
father (X,Y) := man(X), daughter (Y,X).
mother(X,Y) := woman(X), son(Y,X).
mother(X,Y) := woman(X), daughter(Y,X).
parent (X) := father(X, _); mother(X, _).
parent (X,Y):- father (X,Y); mother (X,Y).
sister (Y,Z):-parent (X), daughter (Y,X), daughter (Z,X), Z = Y.
sister(Y, Z):- parent(X), daughter(Y, X), son(Z, X).
brother (Y, Z) := parent(X), son(Y, X), son(Z, X), Z = Y.
brother (Y, Z) := parent(X), son(Y, X), daughter (Z, X).
sibling(X,Y):=brother(X,Y); sister(X,Y).
```

```
man ('Eomund').
                                 man ('Eomer').
 woman ('Theodwyn').
                                  woman ('Eowyn').
 son('Eomer', 'Eomund').
                                 son('Eomer', 'Theodwyn').
 daughter('Eowyn', 'Eomund'). daughter('Eowyn', 'Theodwyn').
father (X,Y) := man(X), son(Y,X).
father (X,Y) := man(X), daughter (Y,X).
mother(X,Y):-woman(X), son(Y,X).
mother(X,Y) := woman(X), daughter(Y,X).
parent (X) := father(X, _); mother(X, _).
parent (X,Y):- father (X,Y); mother (X,Y).
sister (Y,Z):-parent (X), daughter (Y,X), daughter (Z,X), Z = Y.
sister(Y, Z):- parent(X), daughter(Y, X), son(Z, X).
brother (Y, Z) := parent(X), son(Y, X), son(Z, X), Z = Y.
brother (Y, Z) := parent(X), son(Y, X), daughter (Z, X).
sibling (X,Y) := brother(X,Y); sister(X,Y).
ancestor (X,Y) := parent(X,Y).
ancestor (X,Y):- parent (X,Z), ancestor (Z,Y).
```

Horn clauses

HORN CLAUSES IN PROPOSITIONAL LOGIC

DEFINITION

A literal is an atomic sentence or a negation of an atomic sentence.

$$p, q, \ldots, \neg p, \neg q, \ldots$$

DEFINITION

A clause is a disjunction of literals

$$p \lor q \lor \neg p \lor \neg r \lor q$$

DEFINITION

A Horn clause is a clause with at most one positive (unnegated) literals in it.

$$\neg p \lor \neg q \lor \neg p \lor \neg r \lor q$$

Remember that $\neg A \lor B \iff A \to B$, therefore Horn clauses are sentences of form $(p_1 \land p_2 \land \cdots \land p_n) \to q$ or $p_1 \to (p_2 \to \cdots \to (p_n \to q) \cdots)$ And these sentences are very convenient to apply modus ponens...

HORN CLAUSES IN PROPOSITIONAL LOGIC

THEOREM

The satisfiability problem of a conjunction of Horn clauses is **P-complete** and solvable in **linear time**.

i.e., Horn-logic is super fast.

The cost is of course the expressive power. To use Horn-logic to solve problems the working logician sometimes has to be very tricky in choosing the primitives...

HORN CLAUSES IN PREDICATE LOGIC

In predicate logic, atomic sentences are predicates that may contain variables.

$$human(x) \to mortal(x)$$

All unquantified sentences are meant to be universally quantified (check the definition of $\mathfrak{M} \models \varphi$ compared to $\mathfrak{M}, \sigma \models \varphi$!)

$$\forall x (\text{human}(x) \rightarrow \text{mortal}(x))$$

The only way to play with the variables is to sometimes use the same variable again. The key to thinking in that logic is the so-called unification. Within a first-order environment, Horn logic gains so much expressive power that it torpedos P-completeness; the satisfiabilty problem of conjunction of Horn clauses is undecidable.



UNIFICATION: =/2

This is not equality. Equality is ==/2. Roughly:

"Two terms unify if they are the same term or if they contain variables that **can be** uniformly instantiated with terms in such a way that the resulting terms are equal."

Semantics of =/2.

- Constants: a=b is true iff they are the same atom or the same number, i.e., a=b.
- Variable + Term: $X=\tau$ is true, and $X \mapsto \tau$. (\mapsto : variable instantiation)
- Term + Variable: Similarly.
- Complex terms: $\sigma(\tau_1, \ldots, \tau_n) = \sigma'(\tau_1', \ldots, \tau_k')$ is true iff
 - $\sigma = \sigma'$, n = k,
 - $(\forall i \leq n)\tau_i = \tau'_i$, and
 - the variable instantiations are compatible.
 X → a, X → b ⇒ a = b ("For example, it is not possible to instantiate variable X to mia when unifying one pair of arguments, and to instantiate X to vincent when unifying another pair of arguments.")

Unification is intensional!

```
1 ?- mia = mia.
```

The symbol $?-\varphi$ is true iff \bigoplus finds a way to make φ true. Later we will see what is the backtracking algorithm that \bigoplus uses.

```
1 ?- mia = mia. yes
2 ?- mia = vincent.
```

```
1 ?- mia = mia.
2 ?- mia = vincent.
3 ?- 2 = 2.
4 ?- 'mia' = mia.
yes
```

1	?- mia = mia.	yes
2	?- mia = vincent.	yes
3	?- 2 = 2.	yes
4	?- 'mia' = mia.	yes
5	?- '2' = 2.	no
6	?- mia = X.	

```
1 ?- mia = mia.
                                                      yes
2 ?- mia = vincent.
                                                      yes
3 ? - 2 = 2.
                                                      yes
4 ?- 'mia' = mia.
                                                      yes
5 ?- '2' = 2.
                                                       no
6 ?- mia = X.
                                           X = mia yes
7 \ ?- \ X = Y.
                           X = _5071 Y = _5071
                                                      yes
8 ?- X = mia, X = vincent.
```

The symbol $?-\varphi$ is true iff $\ \stackrel{\frown}{\square}$ finds a way to make φ true. Later we will see what is the backtracking algorithm that $\ \stackrel{\frown}{\square}$ uses.

```
1 ?- mia = mia.
                                                             yes
2 ?- mia = vincent.
                                                             ves
3 ? - 2 = 2.
                                                             yes
4 ?- 'mia' = mia.
                                                             yes
5 ?- '2' = 2.
                                                              no
6 ?- mia = X.
                                                 X = mia yes
7 \ ?- \ X = Y.
                              X = _5071 Y = _5071
                                                             yes
8 ?- X = mia, X = vincent.
                                                              no
  "An instantiated variable isn't really a variable anymore: it has become what it
  was instantiated with."
```

9 : -k(s(g), Y) = k(X, t(k)).

The symbol $?-\varphi$ is true iff $\ \stackrel{\frown}{\square}$ finds a way to make φ true. Later we will see what is the backtracking algorithm that $\ \stackrel{\frown}{\square}$ uses.

```
1 ?- mia = mia.
                                                              yes
2 ?- mia = vincent.
                                                              ves
3 \ ?- \ 2 = 2.
                                                              yes
4 ?- 'mia' = mia.
                                                              yes
5 ?- '2' = 2.
                                                               no
6 ?- mia = X.
                                                 X = mia yes
7 \ ?- \ X = Y.
                               X = _5071 Y = _5071
                                                              yes
8 ?- X = mia, X = vincent.
                                                               no
  "An instantiated variable isn't really a variable anymore: it has become what it
  was instantiated with."
```

9 ?- k(s(g), Y) = k(X, t(k)). X = s(g) Y = t(k) yes 10 ?- k(s(g), t(k)) = k(X, t(Y)).

The symbol $?-\varphi$ is true iff $\ \stackrel{\frown}{M}$ finds a way to make φ true. Later we will see what is the backtracking algorithm that $\ \stackrel{\frown}{M}$ uses.

```
1 ?- mia = mia.
                                                             yes
2 ?- mia = vincent.
                                                            yes
3 ? - 2 = 2.
                                                            yes
4 ?- 'mia' = mia.
                                                            yes
5 ?- '2' = 2.
                                                             no
6 ?- mia = X.
                                                X = mia yes
7 \ ?- \ X = Y.
                              X = _5071 Y = _5071
                                                            yes
8 ?- X = mia, X = vincent.
                                                              no
  "An instantiated variable isn't really a variable anymore: it has become what it
  was instantiated with."
```

9 ?-
$$k(s(g),Y) = k(X,t(k))$$
. $X = s(g) Y = t(k)$ yes
10 ?- $k(s(g), t(k)) = k(X,t(Y))$. $X = s(g) Y = k$ yes
11 ?- loves(X,X) = loves(marcellus,mia).

The symbol $?-\varphi$ is true iff $\ \stackrel{\frown}{M}$ finds a way to make φ true. Later we will see what is the backtracking algorithm that $\ \stackrel{\frown}{M}$ uses.

```
1 ?- mia = mia.
                                                             yes
2 ?- mia = vincent.
                                                             yes
3 \ ?- \ 2 = 2.
                                                             yes
4 ?- 'mia' = mia.
                                                             yes
5 ?- '2' = 2.
                                                              no
6 ?- mia = X.
                                                 X = mia yes
7 \ ?- \ X = Y.
                              X = _5071 Y = _5071
                                                             yes
8 ?- X = mia, X = vincent.
                                                              no
  "An instantiated variable isn't really a variable anymore: it has become what it
  was instantiated with."
```

9 ?- k(s(g), Y) = k(X, t(k)). X = s(g) Y = t(k) yes 10 ?- k(s(g), t(k)) = k(X, t(Y)). X = s(g) Y = k yes 11 ?- loves(X,X) = loves(marcellus,mia).