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A1: MATH REVIEW AND PRACTICE

Mathematics is the art of giving the same name to different things. —Henri Poincare

Course: CS 5006
Summer 2018
Due: 18 May 2018, 5pm

OBJECTIVES

After you complete this assignment, you will be comfortable with:

- Logarithms
- Summation
- Factorials
- Big-O
- Growth of functions
- Run time (seconds) of various Big-O vals
- Understand Big-O, Big-Theta, Big-Omega

RELEVANT READING

- Kleinberg and Tardos, Chapter 1 and 2

SO YOU THOUGHT YOU KNEW ENOUGH MATH…

Problem 1: Order the functions (2 points)

Order the following functions in ascending order of growth:

4_______ $f_a(n) = 3^n$

2_______ $f_b(n) = n^{1/4}$

5_______ $f_c(n) = n^n$

1_______ $f_d(n) = \log_2 n$

3_______ $f_e(n) = n \times \log_2 n$

Problem 2: Simplify factorials (6 points)

Simplify the following factorial expressions, showing your work.

(a)

$$\frac{(n+1)!}{(n-1)!}$$

$$\frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}} = n(n+1) = n^2 + n$$

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(b)

$$\frac{(n-1)!}{(n+2)!}$$

$$\frac{(n-1)!}{(n+2)!} = \frac{\cancel{(n-1)!}}{(n+2)(n+1)(n)\cancel{(n-1)!}} = \frac{1}{(n+2)(n+1)n} = \frac{1}{(n^2+n+2n+2)n}$$
$$= \frac{1}{(n^2+3n+2)n} = \frac{1}{n^3+3n^2+2n}$$

(c)

$$\frac{(n-r)!}{(n-r-2)!}$$

$$\frac{(n-r)!}{(n-r-2)!} = \frac{(n-r)(n-r-1)\cancel{(n-r-2)!}}{\cancel{(n-r-2)!}} = (n-r)(n-r-1)$$
$$= (n^2 - rn - n - rn + r^2 + r) = n^2 + r^2 - 2rn - n + r$$

Problem 3: Order of functions (3 points)

Find the least integer n such that $f(x)$ is $O(x^n)$ for each of the following functions.

(a) $f(x) = 2x^2 + x^3 \log x$

(a) 4

(b) $f(x) = 3x^5 + (\log x)^4$

(b) 6

(c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

(c) 2

Problem 4: Big-O (4 points)

Specify true/false if the following function is $O(x^2)$.

(a) $f(x) = 17x + 11$

(a) True

(b) $f(x) = x^2 + 1000$

(b) True

(c) $f(x) = x \log x$

(c) True

(d) $f(x) = x^4/2$

(d) False

Problem 5: Big-O (3 points)

Give a big-O estimate for each of the following functions.

(a) $n \log(n^2 + 1) + n^2 \log n$

(a) $n^2 \log n$

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(b) $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1) = (n \log n)^2 + 2n \log n + 1 + n^2 \log n + \log n + n^2 + 1$

(c) $n^{2^n} + n^{n^2}$

(b) $(n \log n)^2$

(c) n^{n^2}

Problem 6: Asymptotic Notation (7 points)

For each of the following pairs of functions, indicate whether $f = O(g)$, $f = \Omega(g)$ or $f = \Theta(g)$

(a) (1 point) $f(n) = n - 100$
$g(n) = n - 200$

Assume constant $c_1 = 3$, $n - 100 \le 3(n - 200), n - 100 \le 3n - 600$. As n goes toward infinity, g(n) will be bigger than f(n), so big O. There is no c greater than 1 that you could multiply g(n) by that would make f(n) ≥ g(n). so.

$$f = O(g)$$

(b) (1 point) $f(n) = 100n + \log n$
$g(n) = 10n \log 10n$

f(n) = 100n + log n = n as n increases. $g(n) = 10n \log 10n = n \log 10n$ as n gets closer to infinity. n will always be less than $n \log 10n$. So

$$f = O(g)$$

(c) (1 point) $f(n) = \log 2n$
$g(n) = \log 3n$

$f(n) = \log 2n = \log_{10} 2n$ could also be written as $10^x = 2n$. $g(n) = \log 3n = \log_{10} 3n$ could also be written as $10^x = 3n$. As n increases the amount you multiply n by matters less. Since as n increase the coefficient matters less, if you multiply g(n) by any constant > 1, it will be bigger than f(n) so:

$$f = O(g)$$

(d) (1 point) $f(n) = \sqrt{n} -$
$g(n) = (\log n)^3$

Log n has a lower growth rate than n. Log n doesn't grow that fast, but $\sqrt{n}$ goes towards infinity, it will surpass log n. With g(n) = $(\log n)^3$, if you pick a high enough value for c, $f(n) \le c * g(n)$ but if the value of c is too low $f(n) \ge c * g(n)$.

$$f = \Theta(g)$$

(e) (1 point) $f(n) = n2^n$
$g(n) = 3^n$

$f = \Omega(g(n))$ because as n increases, the n f(n) is going to be bigger than g(n) regardless of the constant you multiply g(n) by because 2^n is multiplied by and even increasing n.

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(f) (1 point) $f(n) = n!$
$g(n) = 2^n$

$n!$ Grows faster than 2^n. Regardless of the constant you multiple 2^n by, n! will be larger the closer n gets to infinity.

$$f = \Omega(g)$$

(g) (1 point) $f(n) = 100n + \log n$
$g(n) = n + (\log n)^2$

$f(n) = 100n + \log n = 100n = n$ as n gets closer to infinity. $g(n) = n + (\log n)^2 = n$ as n gets closer to infinity. Since both equations get closer to n as n gets closer to infinity, with a large enough constant, g(n) will be larger than f(n). Also, as you head towards infinity $(\log n)^2 > \log n$.

$$f = O(g)$$

Problem 7: Logarithm (3 points)

Solve each of the following equations.

(a) (1 point) $\log_4(x^2 - 2x) = 3$ – has a typo

$$\log_4(x^2 - 2x) = 3,$$
$$4^3 = x^2 - 2x,$$
$$64 = x^2 - 2x$$

(b) (1 point) $\log(6x) - \log(4 - x) = \log(3)$

$\log(6x) - \log(4 - x) = \log 3,\ \log\left(\frac{6x}{4-x}\right) = \log 3,\ 10^{\log 3} = \frac{6x}{4-x},$
$3 = \frac{6x}{4-x},\ 12 - 3x = 6x,\ 12 = 9x$

$\frac{4}{3} = x$

(c) (1 point) $\ln(x) + \ln(x + 3) = \ln(20 - 5x)$

$\ln(x) + \ln(x + 3) = \ln(20 - 5x),\ \ln(x * (x + 3)) = \ln(20 - 5x),\ \ln(x^2 + 3x) = \ln(20 - 5x)$
$e^{\ln(20-5x)} = x^2 + 3x,\ 20 - 5x = x^2 + 3x,\ x^2 + 8x - 20 = 0,\ (x - 2)(x + 10) = 0$
x is either 2 or -10, Type equation here.

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Problem 8: Summation (4 points)

(a) (1 point)

$$\sum_{k=1}^{4} k^3$$

$$1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 9 + 27 + 64 = 36 + 64 = 100$$

(b) (1 point)

$$\sum_{i=1}^{5} \sum_{j=1}^{4} ij$$

$$1*1+1*2+1*3+1*4+2*1+2*2+2*3+2*4+3*1+3*2+3*3+3*4+4*1+4*2+4*3+4*4+5*1+5*2+5*3+5*4 = 1+2+3+4+2+4+6+8+3+6+9+12+4+8+12+16+5+10+15+20 = 150$$

(c) (1 point)

$$\sum_{i=1}^{3} \sum_{j=1}^{4} ij$$

$$1*1+1*2+1*3+1*4+2*1+2*2+2*3+2*4+3*1+3*2+3*3+3*4 = 1+2+3+4+2+4+6+8+3+6+9+12 = 60$$

(d) (1 point)

$$\sum_{s\in\{2,4,6\}} s$$

$$2 + 4 + 6 = 12$$

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Question	Points	Score
Order the functions	2	
Simplify factorials	6	
Order of functions	3	
Big-O	4	
Big-O	3	
Asymptotic Notation	7	
Logarithm	3	
Summation	4	
Total:	32	

SUBMISSION DETAILS

Things to submit:

- Submit your assignment in your Github repo.
  - The written parts of this assignment as a .pdf named "CS5006 [lastname] A1.pdf". For example, my file would be named "CS5001-Slaughter-A1pdf". (There should be no brackets around your name).
  - Make sure your name is in the document as well (e.g., written on the top of the first page).
  - Make sure your assignment is in the A1 folder in your Github repo.

HELPFUL HINTS

- Start early!
- Ask clarification questions on Piazza.
- If you have code that isn't working, send a private note to the Instructors (only Instructors!) on Piazza. Note: in the "To" field, type "Instructors", not a specific instructor, to send a message to all instructors.