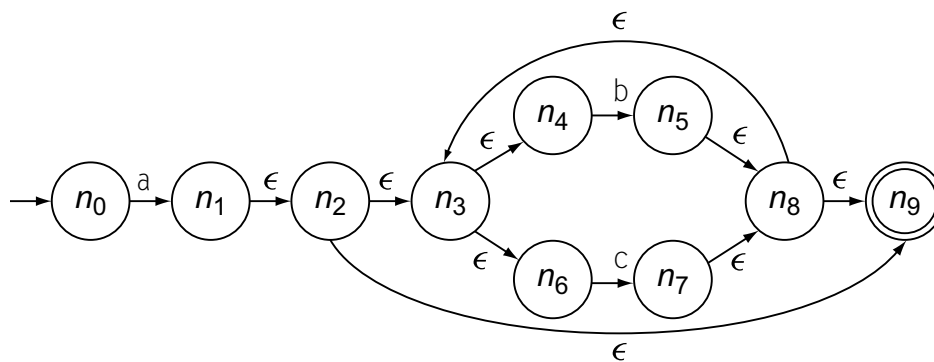


1. Build an NFA and a DFA from an NFA for $a(b|c)^*$

We start solving this problem by relying on the Thompson's construction to find a non-deterministic finite automaton (NFA):

- Create basic FA for each of the elements of the alphabet, in this case, letters a , b and c .
- To form a **concatenation**, connect the accepting state of one basic element with the initial state of another basic element using the ϵ -transition.
- To form an **alternation**, create a new initial state and connect that state to initial states of both basic elements using the ϵ -transition. Similarly, create a new accepting state, and connect it to the accepting states of both of the accepting states of the basic elements using the ϵ -transition.
- To form a **Kleene closure**, use the ϵ -transition to create one or more iterations over basic elements, and another ϵ -transition for an empty case.

The result for the given example is depicted in Figure 1 below.



(a) NFA for " $a(b | c)^*$ " (With States Renumbered)

Figure 1 NFA constructed using the Thompson construction for the given example.

To go from a NFA to a deterministic FA (DFA), we rely on the **Subset Construction**. The idea behind the Subset construction is to start from the initial state, and enumerate all of the other states that can be reached from that state using one specific alphabet item or the ϵ -transition. Each of those enumerations (clusters) is then examined in the same fashion. Once no transitions are possible, we have reached the end, and we have found the states of the DFA. ***The states of the DFA are the identified clusters.***

For the renumbered states in the given example, we start with the state n_0 , and we want to examine what are the states that we can reach from n_0 with either character a or the ϵ -transition.

Form n_0 , we can go to n_1 with a , and then from n_1 to n_2 with the ϵ -transition, and then to n_3 with the ϵ -transition. From n_3 , we can then go to n_4, n_6 or n_9 with the ϵ -transition. States $(n_1, n_2, n_3, n_4, n_6, n_9)$ become the first cluster, which we then examine in a similar fashion. The results are depicted in the Table below.

From the Table, we can easily construct the DFA by designating every cluster as one state of the new DFA. For the given example, the results are depicted in Figure 2.

Set Name	DFA States	NFA States	$\epsilon\text{-closure}(\text{Delta}(q, *))$		
			a	b	c
q_0	d_0	n_0	$\{n_1, n_2, n_3, n_4, n_6, n_9\}$	– none –	– none –
q_1	d_1	$\{n_1, n_2, n_3, n_4, n_6, n_9\}$	– none –	$\{n_5, n_8, n_9, n_3, n_4, n_6\}$	$\{n_7, n_8, n_9, n_3, n_4, n_6\}$
q_2	d_2	$\{n_5, n_8, n_9, n_3, n_4, n_6\}$	– none –	q_2	q_3
q_3	d_3	$\{n_7, n_8, n_9, n_3, n_4, n_6\}$	– none –	q_2	q_3

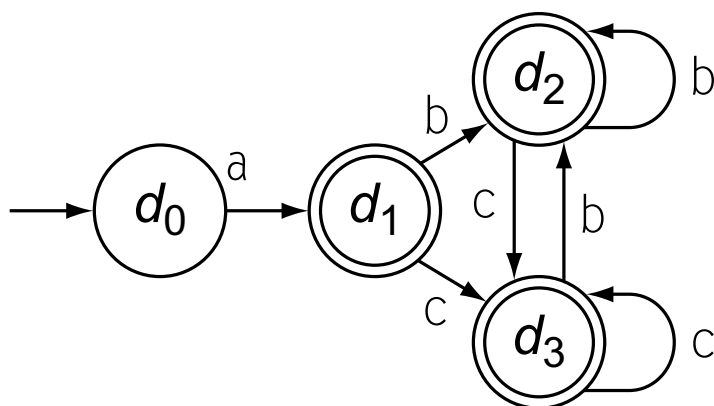


Figure 2 The resulting DFA.

Example credit: An example taken from the Cooper and Torczon book, Ch 2. P.