GROUP WORK PROJECT 1 M5 (Group ID: 8781)

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GROUP MEMBERS:

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0.0.1 Installing Dependancies

[44]:
```
!pip install nelson_siegel_svensson
!pip install yfinance
```

```
Requirement already satisfied: nelson_siegel_svensson in
c:\users\gast01\anaconda3\lib\site-packages (0.5.0)
Requirement already satisfied: scipy>=1.7 in c:\users\gast01\anaconda3\lib\site-
packages (from nelson_siegel_svensson) (1.9.1)
Requirement already satisfied: numpy>=1.22 in
c:\users\gast01\anaconda3\lib\site-packages (from nelson_siegel_svensson)
(1.24.4)
Requirement already satisfied: matplotlib>=3.5 in
c:\users\gast01\anaconda3\lib\site-packages (from nelson_siegel_svensson)
(3.5.2)
Requirement already satisfied: Click>=8.0 in c:\users\gast01\anaconda3\lib\site-
packages (from nelson_siegel_svensson) (8.0.4)
Requirement already satisfied: colorama in c:\users\gast01\anaconda3\lib\site-
packages (from Click>=8.0->nelson_siegel_svensson) (0.4.5)
Requirement already satisfied: kiwisolver>=1.0.1 in
c:\users\gast01\anaconda3\lib\site-packages (from
matplotlib>=3.5->nelson_siegel_svensson) (1.4.2)
Requirement already satisfied: pillow>=6.2.0 in
c:\users\gast01\anaconda3\lib\site-packages (from
matplotlib>=3.5->nelson_siegel_svensson) (9.2.0)
Requirement already satisfied: python-dateutil>=2.7 in
c:\users\gast01\anaconda3\lib\site-packages (from
matplotlib>=3.5->nelson_siegel_svensson) (2.8.2)
Requirement already satisfied: fonttools>=4.22.0 in
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matplotlib>=3.5->nelson_siegel_svensson) (4.25.0)
Requirement already satisfied: cycler>=0.10 in
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matplotlib>=3.5->nelson_siegel_svensson) (0.11.0)
Requirement already satisfied: pyparsing>=2.2.1 in
c:\users\gast01\anaconda3\lib\site-packages (from
matplotlib>=3.5->nelson_siegel_svensson) (3.0.9)
Requirement already satisfied: packaging>=20.0 in
c:\users\gast01\anaconda3\lib\site-packages (from
```

```
matplotlib>=3.5->nelson_siegel_svensson) (21.3)
Requirement already satisfied: six>=1.5 in c:\users\gast01\anaconda3\lib\site-
packages (from python-dateutil>=2.7->matplotlib>=3.5->nelson_siegel_svensson)
(1.16.0)
Requirement already satisfied: yfinance in c:\users\gast01\anaconda3\lib\site-
packages (0.2.55)
Requirement already satisfied: multitasking>=0.0.7 in
c:\users\gast01\anaconda3\lib\site-packages (from yfinance) (0.0.11)
Requirement already satisfied: platformdirs>=2.0.0 in
c:\users\gast01\anaconda3\lib\site-packages (from yfinance) (2.5.2)
Requirement already satisfied: pandas>=1.3.0 in
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c:\users\gast01\anaconda3\lib\site-packages (from
beautifulsoup4>=4.11.1->yfinance) (2.3.1)
Requirement already satisfied: python-dateutil>=2.8.1 in
c:\users\gast01\anaconda3\lib\site-packages (from pandas>=1.3.0->yfinance)
(2.8.2)
Requirement already satisfied: certifi>=2017.4.17 in
c:\users\gast01\anaconda3\lib\site-packages (from requests>=2.31->yfinance)
(2022.9.14)
Requirement already satisfied: charset-normalizer<4,>=2 in
c:\users\gast01\anaconda3\lib\site-packages (from requests>=2.31->yfinance)
(2.0.4)
Requirement already satisfied: idna<4,>=2.5 in
c:\users\gast01\anaconda3\lib\site-packages (from requests>=2.31->yfinance)
(2.10)
Requirement already satisfied: urllib3<3,>=1.21.1 in
c:\users\gast01\anaconda3\lib\site-packages (from requests>=2.31->yfinance)
(1.26.11)
Requirement already satisfied: six>=1.5 in c:\users\gast01\anaconda3\lib\site-
packages (from python-dateutil>=2.8.1->pandas>=1.3.0->yfinance) (1.16.0)
```

[45]:

```python
import pandas as pd
import matplotlib.pyplot as plt
```

```python
from scipy.interpolate import CubicSpline
from nelson_siegel_svensson.calibrate import calibrate_ns_ols
import numpy as np
from numpy import linalg as LA
import seaborn as sns
```

1 Task 2

1.0.1 Q2. a. and Q2. b.

We selected the country - India. The following dataset contains the daily yields of the Indian bonds with the following maturities from 2006 to 2025(current).

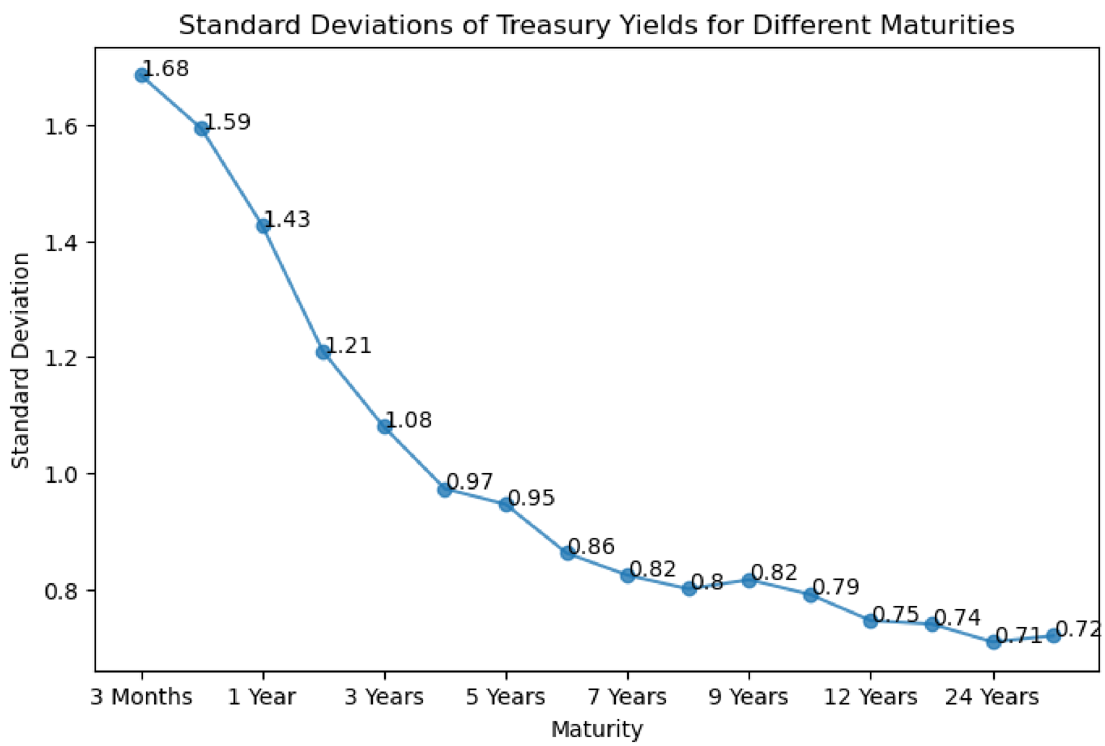
Maturity	Source
3 Months	India 3-Month Bond Yield
6 Months	India 6-Month Bond Yield
1 Year	India 1-Year Bond Yield
2 Year	India 2-Year Bond Yield
3 Year	India 3-Year Bond Yield
4 Year	India 4-Year Bond Yield
5 Year	India 5-Year Bond Yield
6 Year	India 6-Year Bond Yield
7 Year	India 7-Year Bond Yield
8 Year	India 8-Year Bond Yield
9 Year	India 9-Year Bond Yield
10 Year	India 10-Year Bond Yield
12 Year	India 12-Year Bond Yield
15 Year	India 15-Year Bond Yield
24 Year	India 24-Year Bond Yield
30 Year	India 30-Year Bond Yield

[46]:
```python
df = pd.read_csv('C:/Users/Gast01/WQU-GWP1/India Bond Yield Data.csv') # Change␣
↪the file path as the path in your device.

df.index = pd.to_datetime(df['Date'])

df = df.drop('Date', axis=1)
df = df.dropna()
```

[47]:
```python
def plot_std(df):
    """
    Plotting the standard deviation of the treasury yields for different␣
    ↪maturities
    """

    y_std = df.std()
```

```
    fig, ax = plt.subplots()
    y_std.plot(figsize = (8,5),marker='o', title='Standard Deviations of Treasury␣
↪Yields for Different Maturities', alpha=0.8)
    plt.xlabel("Maturity")
    plt.ylabel("Standard Deviation")
    for i in range(len(y_std)):
        ax.annotate(str(round(y_std.iloc[i],2)),xy=(i,y_std.iloc[i]))
    plt.show()
```

[48]:
```
plot_std(df)
```



[49]:
```
def plot_yield_curve(date, fig_n):
    """
    Plotting the yield curve for a given date
    """

    maturities = df.columns # Maturities
    fig, ax = plt.subplots(figsize=(6.15, 4))
    ax.plot(maturities, df.loc[date], marker='D', label='Yield Curve at ' +␣
↪date)

    ax.set_yticklabels(['{:.2f}%'.format(y) for y in ax.get_yticks()])
    ax.set_xticks(range(len(maturities)))
```

```
    labels = [m if i % 5 == 0 else '' for i, m in enumerate(maturities)]
    ax.set_xticklabels(labels)

    # Add labels and title
    ax.set_xlabel('Maturity')
    ax.set_ylabel('Yield')
    ax.set_title(fig_n+f'Treasury Yield Curve as of {date}')

    # Show the plot
    plt.grid(False)
    plt.show()
```

1.0.2 Q2. c.

Fitting a Nelson Siegel Model on all the maturities on a specific date and comparing it with the original yield curve as of that date

[50]:
```
def fit_ns(t, y, tau0=1.0):
    """
    Fitting the Nelson-Siegel model to the yield curve data for a given date.
    """

    curve, status = calibrate_ns_ols(t, y, tau0=1.0)  # starting value of 1.0 for␣
↪the optimization of tau
    assert status.success

    print(curve)

    return curve
```

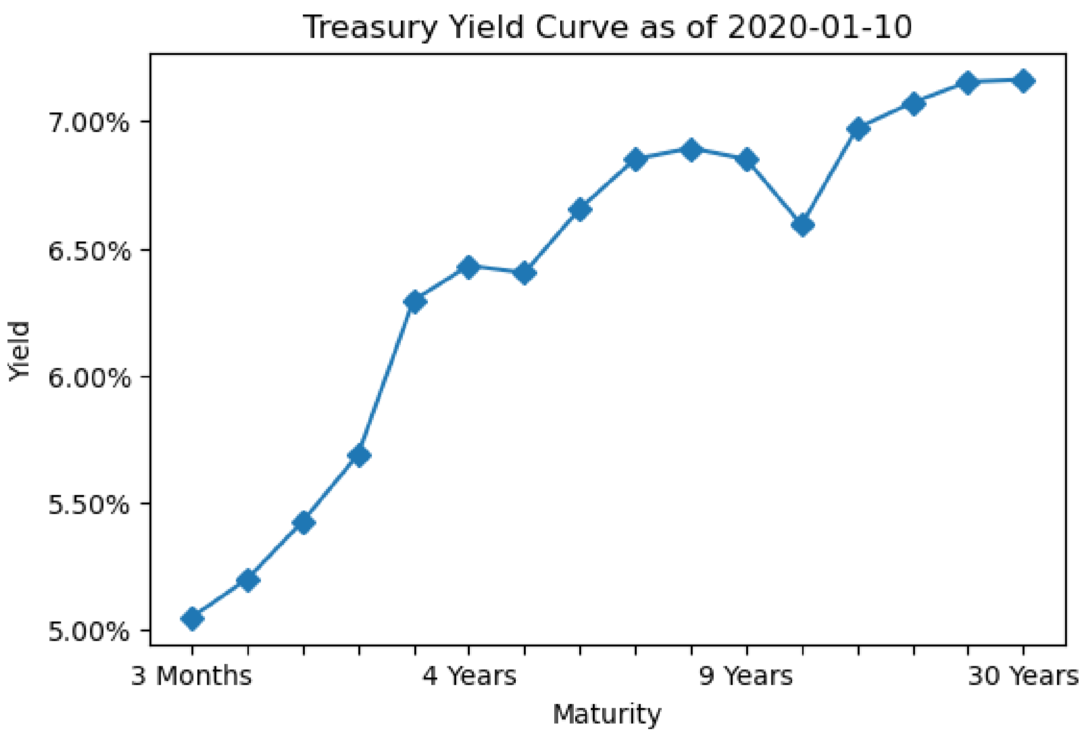
[51]:
```
def plot_ns(date, y_hat, t_hat):
    """
    Plotting the yield curve for a given date.
    """

    plt.plot(t_hat, y_hat(t_hat))
    plt.xlabel("Maturity")
    plt.ylabel("Yield")
    plt.title(f"NS Model Result as of {date}")

    plt.show()
```

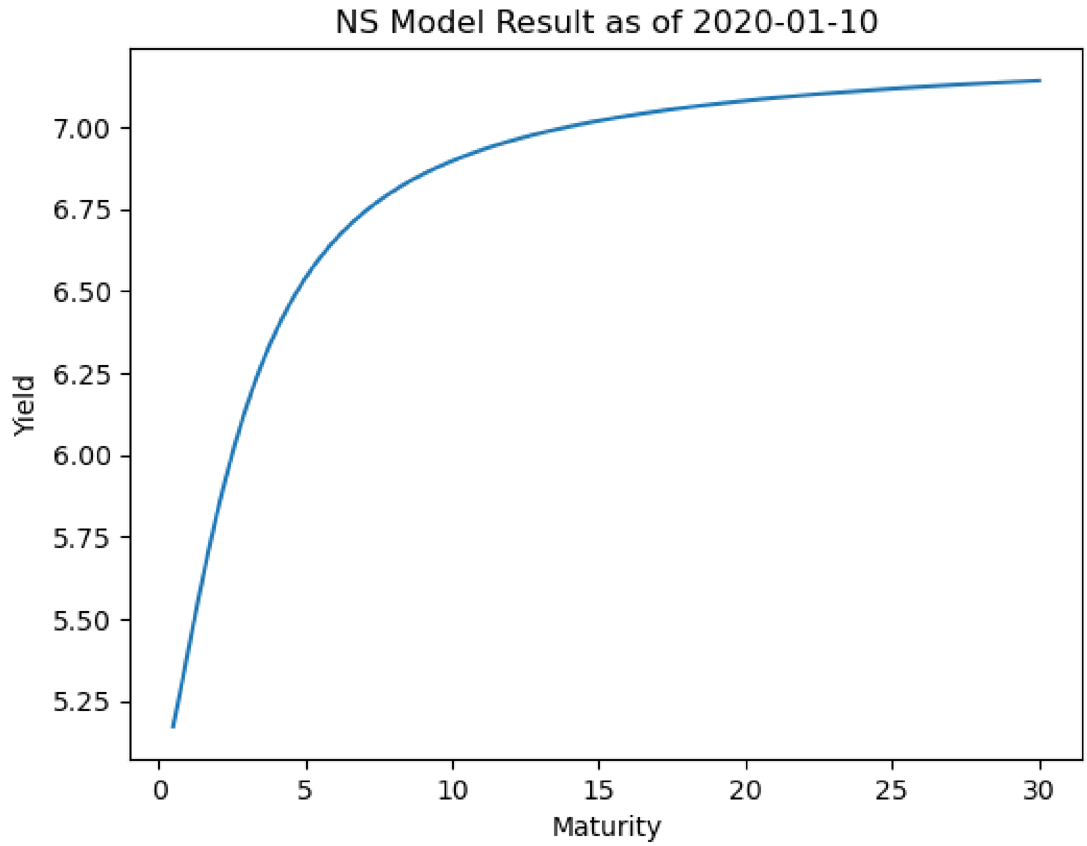
[52]:
```
date = "2020-01-10"
t = np.array([0.25,0.5,1,2,3,4,5,6,7,8,9,10,12,15,24,30])
y = np.array(df.loc[date])
curve = fit_ns(t, y, tau0=1.0)
y_hat = curve
```

```
t_hat = np.linspace(0.5,30,100)
plot_yield_curve(date,'')
plot_ns(date, y_hat, t_hat)
```

```
NelsonSiegelCurve(beta0=7.2625370213328555, beta1=-2.273671316630753,
beta2=-1.7792154742344135, tau=0.9062508587848979)
```

```
C:\Users\Gast01\AppData\Local\Temp\ipykernel_21940\2579923921.py:10:
UserWarning: FixedFormatter should only be used together with FixedLocator
  ax.set_yticklabels(['{:.2f}%'.format(y) for y in ax.get_yticks()])
```





1.0.3 Q2. f.

In the solution above, we have fit a Nelson Siegel Model on the yield curve as of 2020-01-10. The result `NelsonSiegelCurve(beta0=np.float64(7.262536960509667), beta1=np.float64(-2.273671047246356), beta2=np.float64(-1.779216868209846), tau=np.float64(0.9062504579008243))` indicates that the β_0 = 7.26 showing the level of the yield curve. β_1 = -2.27 shows the slope of the yield curve and the $\beta 2$ = -1.77 shows the shape or the curvature of the yield curve. the decay rate indicated by τ = 0.91 showing a slow rate of decay.

A level of 7.26 shows that the long term expectation of the yields to be around 7%.

A negative slope can be attributed to the fact that there are dips in the long term yields, for example the 10 year yield is smaller than the 6 years, 7years and 8 years yields.

A negative curvature indicates a concave behaviour. The yields started rising steep in the short run, but flatten over a longer period.

This interpretation is in line with the mid pandemic situation in 2020. Complete economic shutdowns has raised riskiness of the short term borrowings, hence the rise in yields sharply.

A smaller value of τ (close to 1) indicates that the effects of the slope and curvature parameters decay relatively slowly as maturity increases. This suggests that the short-to-medium-term rates have a strong influence on the shape of the curve, but as maturity increases, the curve flattens, and the level dominates.

1.0.4 Q2. d. and Q2. f.

Fitting a Cubic Spline Model on all the maturities on a specific date and comparing it with the original yield curve as of that date.

Since there are 16 maturities, there would be 15 splines that pass through the 16 points. The splines can be shown as follows:

$$f(x) = a_1x^3 + b_1x^2 + c_1x + d_1, \quad \text{when } 0.25 \leq x \leq 0.5$$

$$f(x) = a_2x^3 + b_2x^2 + c_2x + d_2, \quad \text{when } 0.5 \leq x \leq 1$$

$$f(x) = a_3x^3 + b_3x^2 + c_3x + d_3, \quad \text{when } 1 \leq x \leq 2$$

and so on...

From the above equations, we have (15 $\times$ 4 = 60) unknowns. Hence, we need 60 equations to solve for the parameters.

Thus plugging each boundary, we get 30 equations as shown below:

$$\begin{aligned}
a_1(0.25)^3 + b_1(0.25)^2 + c_1(0.25) + d_1 = 5.05 \quad (1)\\
a_1(0.5)^3 + b_1(0.5)^2 + c_1(0.5) + d_1 = 5.2 \quad (2)\\
a_2(0.5)^3 + b_2(0.5)^2 + c_2(0.5) + d_2 = 5.2 \quad (3)\\
a_2(1)^3 + b_2(1)^2 + c_2(1) + d_2 = 5.42 \quad (4)\\
a_3(1)^3 + b_3(1)^2 + c_3(1) + d_3 = 5.42 \quad (5)\\
a_3(2)^3 + b_3(2)^2 + c_3(2) + d_3 = 5.69 \quad (6)
\end{aligned}$$

Now since each interirior point is a part of 2 splines, their slopes and curvatures must be the same. Therefore, their first order derivatives and their second order derivatives should be the same. Since there are (16 - 2 = 14) interior points, we get 14 first order equations, and 14 second order equations.

The first order equations are as follows:

$$\begin{aligned}
3a_1(0.5)^2 + 2b_1(0.5) + c_1 = 3a_2(0.5)^2 + 2b_2(0.5) + c_2 \quad (31)\\
3a_2(1)^2 + 2b_2(1) + c_2 = 3a_3(1)^2 + 2b_3(1) + c_3 \quad (32)
\end{aligned}$$

The second order equations are as follows:

$$6a_1(0.5) + 2b_1 = 6a_2(0.5) + 2b_2 \quad (45)$$

$$6a_2(1) + 2b_2 = 6a_3(1) + 2b_3 \quad (46)$$

In total now we have 58 equations. Finally we assume the "Natural End Condition" and consider the secodn order derivatives of the exterior p[oints to be 0/ Since there are 2 exterior points, we get 2 equations, which gives us full 60 equations.

$$6a_1(0.25) + 2b_1 = 0 \quad (59)$$
$$6a_3(30) + 2b_3 = 0 \quad (60)$$

To calculate the cubic spline model, we use the `CubicSpline` function from the **SciPy** library, typically imported as `cs`.

The command is:

```
cs = CubicSpline(t, y, bc_type=((2, 0.0), (2, 0.0)))
```

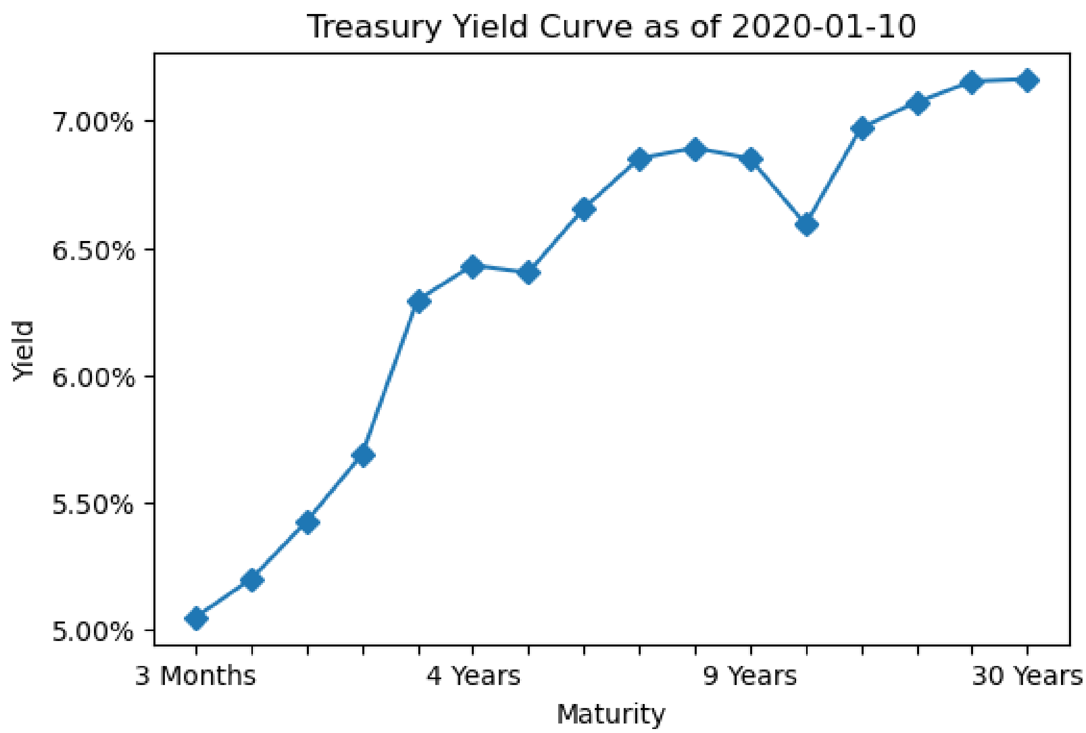
where:

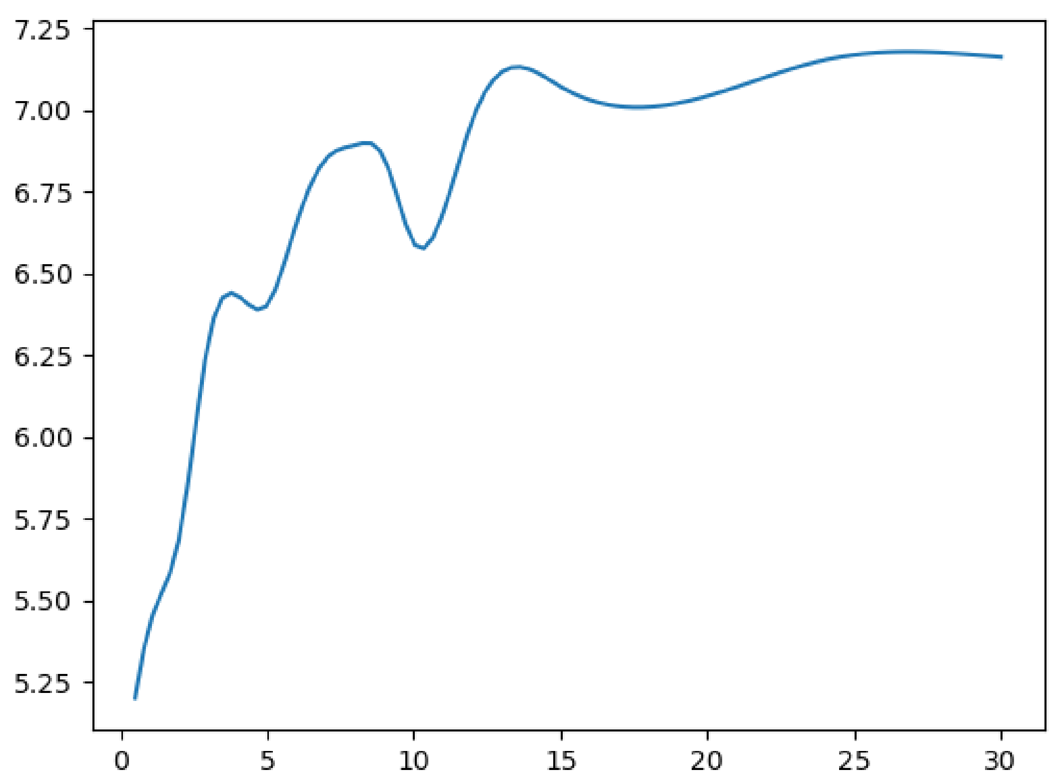
- `t` = maturities
- `y` = yields as of a specific date
- `bc_type=((2, 0.0), (2, 0.0))` sets the **natural end condition** by specifying that the second derivative at the two endpoints is zero.

[53]:
```python
def fit_cs(t, y, t_hat):
    """
    Fitting a Cubic Spline Model on all the maturities on a specific date.
    """
    cs = CubicSpline(t, y, bc_type=((2, 0.0), (2, 0.0)))
    interpolated_yields = cs(t_hat)
    plt.plot(t_hat, interpolated_yields)
```

[54]:
```python
date = "2020-01-10"
t = np.array([0.25,0.5,1,2,3,4,5,6,7,8,9,10,12,15,24,30])
y = np.array(df.loc[date])
t_hat = np.linspace(0.5,30,100)
plot_yield_curve(date,'')
fit_cs(t, y, t_hat)
```

```
C:\Users\Gast01\AppData\Local\Temp\ipykernel_21940\2579923921.py:10:
UserWarning: FixedFormatter should only be used together with FixedLocator
  ax.set_yticklabels(['{:.2f}%'.format(y) for y in ax.get_yticks()])
```


Treasury Yield Curve as of 2020-01-10
Yield
7.00%
6.50%
6.00%
5.50%
5.00%
3 Months
4 Years
9 Years
30 Years
Maturity


7.25
7.00
6.75
6.50
6.25
6.00
5.75
5.50
5.25
0
5
10
15
20
25
30

1.0.5 Q2. e.

Fit: - NS Model is a parametric model with defined parameters β_0, β_1 and β_2. The model smoothened the trend of the yield curve and just fit the general level witht the slope and curvature of the curve. It failed to capture the nuances of the mid term maturities. Features like a lower yield in the 10 year maturity did not get captured in the NS Model.

- Cubic Spline is a non parametric model which fits a spline between each point individually and thus captures a much more nuanced view of the yield curve. The fit is indeed much better than the NS Model.

Interpretation: - Since NS is a parametric approach, its parameters have a strict economic interpretation. - β_0 : Level - β_1 : Slope - β_2 : Curvature - τ : Decay Rate

It is helpful in making economic decision based on these parameters.

- Cubic Spline takes a non parametric approach and as a result does not have a direct economic interpretation. Its use cases are when one needs a data driven approach to mapping the yield movements, with a certain degree of smoothing involved. However, It is very succeptible to overfitting. Thus one must be careful towards the model complexity-overfitting tradeoff.

1.0.6 Q2. g.

Indeed Smoothing data can be considered unethical, as discussed in M2 L4. Howver, whether or not the NS model smoothing is unethical, depends on the specific use cases.

NS Model is a parametric approach to smoothen out the yield curve based on its broad economic parameters, $\beta_0, \beta_1, \beta_2$ and τ. It aims to provide a simpified picture of the economic conditions based on the yields, to provide insights on the expected trends in the market both in the short and long term. In this case the smoothing out is not done to hide variability, it is done to cancel out the noise to identify trends. It is not unethical to smoothen the curve via NS Model in this case.

However, say in a situation where the user does the simplification, but fails to disclose it, with a purpose of hiding variability, or hiding a sharp downturn in the yields, it becomes unethical.

2 Task 3

Q3. a & b) Generating 5 Gausian uncorrelated random variables and running Principal Components on Covariance Matrix

[55]:
```
np.random.seed(42)
sim_yld = np.random.normal(loc = 0, scale = 0.01, size = (200,5))
yld = pd.DataFrame(sim_yld, columns = ['3 Months', '1 Year', '3 Years', '10␣
 ↪Years', '30 Years'])
mean_yld = yld.mean()
std_yld = yld.std()
yc_standardised = (yld - mean_yld) / std_yld
std_data_cov = yc_standardised.cov()
eigenvalues, eigenvectors = LA.eig(std_data_cov)
```

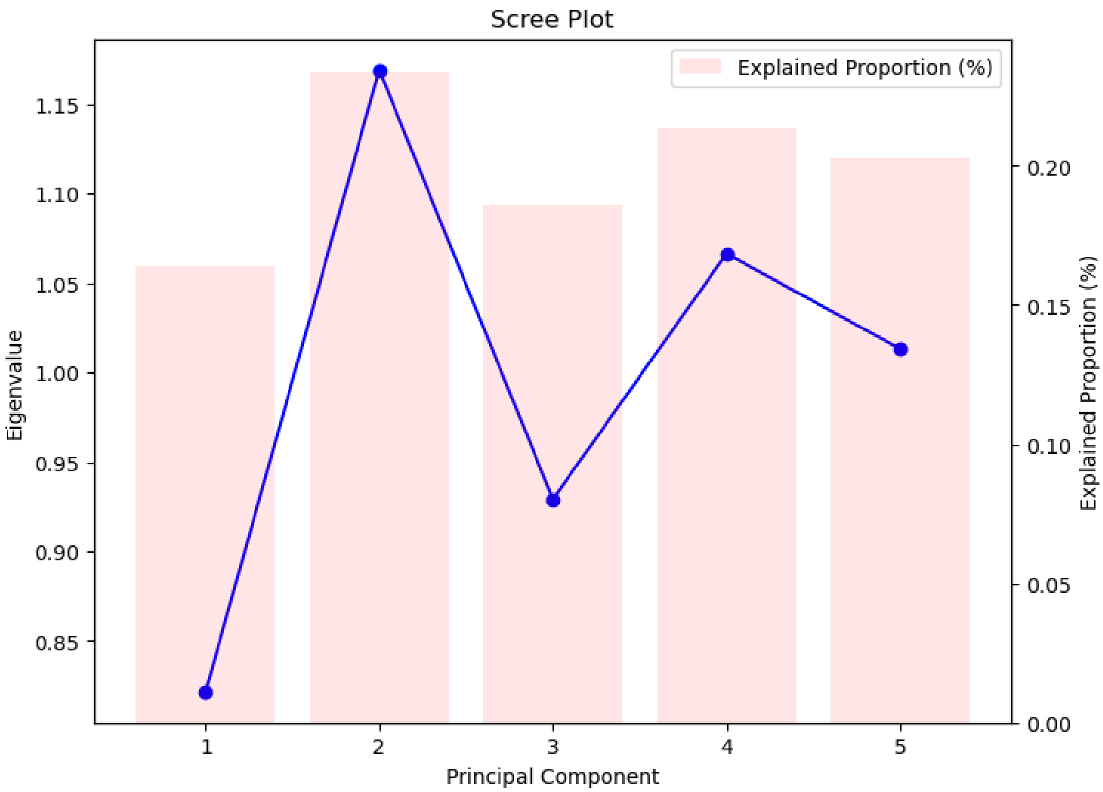
```
principal_components = yc_standardised.dot(eigenvectors)
principal_components.columns = ["PC_1","PC_2","PC_3","PC_4","PC_5"]
df_eigval = pd.DataFrame({"Eigenvalues":eigenvalues}, index=range(1,6))
df_eigval["Explained proportion"] = df_eigval["Eigenvalues"] / np.
 ↪sum(df_eigval["Eigenvalues"])
df_eigval.style.format({"Explained proportion": "{:.2%}"})
```

[55]: <pandas.io.formats.style.Styler at 0x1ea084207c0>

Q3. c) The above df shows the principal componentsw described by their eigen values and the amount of explanation of the variance they respectively do. As we can see that in case of random variables, which are uncorrelated, the 5 principal components more or less describe an equal proportion of the variance. This can be owed to the fact that the multivariate distributions are i.i.d across the observations.

Q3 d) Plotting the Scree plot of the variance explained by each component.

[56]:
```
# Plotting the Scree plot

plt.figure(figsize=(8, 6))
plt.plot(range(1, 6), df_eigval['Eigenvalues'], marker='o', linestyle='-',␣
 ↪color='b', label='Eigenvalues')
plt.title("Scree Plot")
plt.xlabel("Principal Component")
plt.ylabel("Eigenvalue")
plt.xticks(range(1, 6))

# Step 2: Plotting explained proportion as bars

plt.twinx()
plt.bar(range(1, 6), df_eigval['Explained proportion'], alpha=0.1, color='r',␣
 ↪label="Explained Proportion (%)")
plt.ylabel("Explained Proportion (%)")

plt.grid(False)
plt.legend(loc="upper right")
plt.show()
```



Here we can see that the explained variance does not reduce as the number of principal components increase. Infact they explain more or less similar proportions of the variance.

Q3. e & f) Loading actual data of government securities, for the past 6 months and converting yields into yield changes.

[57]:
```
df = pd.read_csv('C:/Users/Gast01/WQU-GWP1/India Bond Yield Data.csv')
df.index = pd.to_datetime(df['Date'])
df_sec = df[df.index >= (df.index.max() - pd.DateOffset(months=6))]

# Selecting 5 securities
df_sec = df_sec[['3 Months', '1 Year', '3 Years', '10 Years', '30 Years']]

# Converting yields into yield changes and dropping NaNs
df_pct_chg = df_sec.pct_change().dropna()
```

Q3. g) Running Principal Components on original yields, using covariance matrix.

[58]:
```
yld_chg_mean = df_pct_chg.mean()
yld_chg_std = df_pct_chg.std()
standardized_yld_chag = (df_pct_chg - yld_chg_mean) / yld_chg_std
std_yld_chg_cov = standardized_yld_chag.cov()
eigenvalues_yld, eigenvectors_yld = LA.eig(std_yld_chg_cov)
```

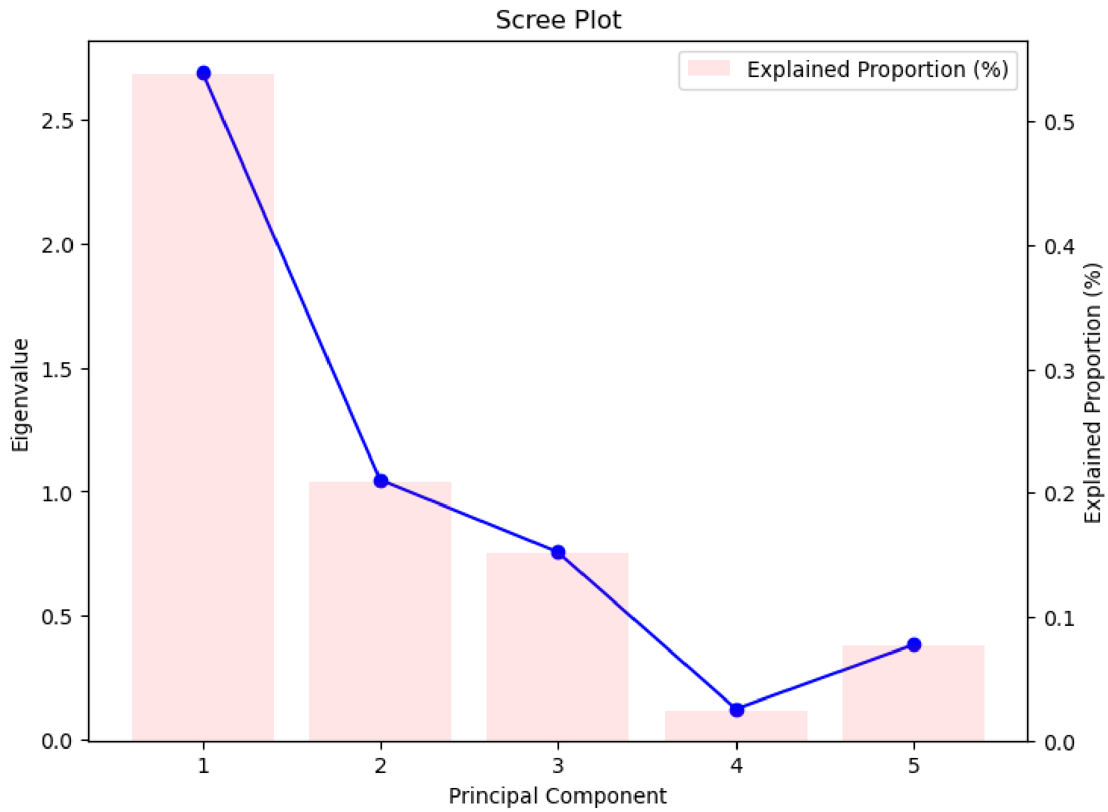
```
principal_components = standardized_yld_chag.dot(eigenvectors_yld)
principal_components.columns = ["PC_1","PC_2","PC_3","PC_4","PC_5"]
df_eigval_yld = pd.DataFrame({"Eigenvalues":eigenvalues_yld}, index=range(1,6))
df_eigval_yld["Explained proportion"] = df_eigval_yld["Eigenvalues"] / np.
 ↪sum(df_eigval_yld["Eigenvalues"])
df_eigval_yld.style.format({"Explained proportion": "{:.2%}"})
```

[58]: <pandas.io.formats.style.Styler at 0x1ea08d538e0>

Q3. h) From the above df we can see a different picture regarding the eigenvectors and their explanation of the variance. The first PC explains more than 50% of the variance in the yields. This is the "Level" of the yield curve. This shows the long term expectations of the yields. The secodn PC explains almost 21% of the variance. We can interpret this as the "Slope" of the yield curve. Finally the 3rd PC explains about 15% of the variance, and this can be attributed to the "Curvature" of the yield curve, or how fast the slope is changing. The last 2 PCs explain relatively lower proportion of the variance, since the level, slope and curvature are the 3 main principal components.

Q3. i) Producing Scree Plot

[59]:
```
# Plotting the Scree plot

plt.figure(figsize=(8, 6))
plt.plot(range(1, 6), df_eigval_yld['Eigenvalues'], marker='o', linestyle='-',␣
 ↪color='b', label='Eigenvalues')
plt.title("Scree Plot")
plt.xlabel("Principal Component")
plt.ylabel("Eigenvalue")
plt.xticks(range(1, 6))
plt.grid(False)

# Plotting explained proportion as bars
plt.twinx()
plt.bar(range(1, 6), df_eigval_yld['Explained proportion'], alpha=0.1,␣
 ↪color='r', label="Explained Proportion (%)")
plt.ylabel("Explained Proportion (%)")
plt.legend(loc="upper right")

# Show the plot
plt.show()
```



Q3. j.

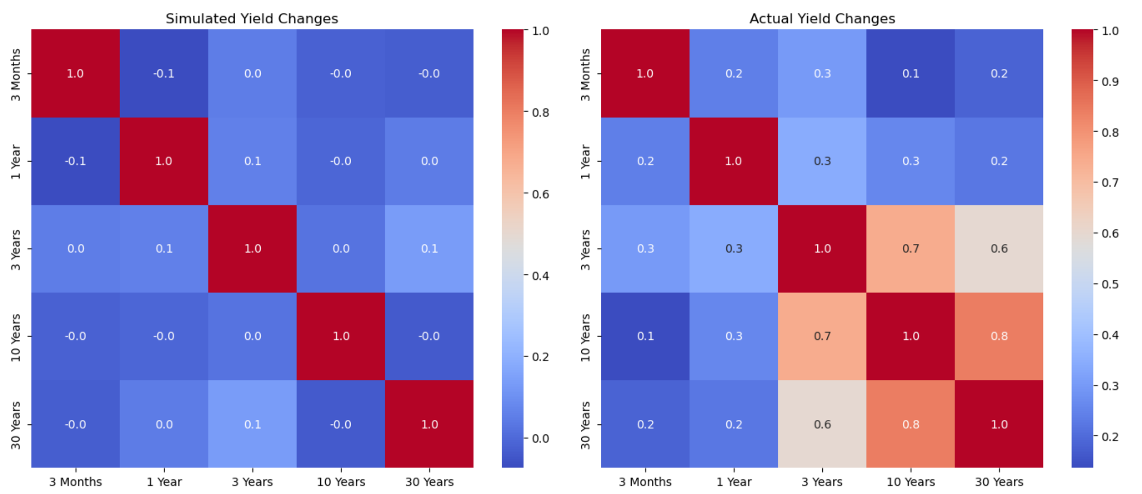
The above scree plot is more in line with our expectations. The plot can be read as an elbow method. The elbow forming when the first principal component is explaining the highest variance and as the number of PCs increase, they explain lesser and lesser of the variance of the yield curve. In our case, the first three principal components explain about 92% of the data.

This observation is different than the uncorrelated gausian random variables case because the data was generated such that the random variables had a same variance. As a result there is no dominant direction, and the variance is equally spread across the 5 eigenvalues, so the PC algorithm finds similar eigenvalues for the 5 PCs.

Since principal components are dimentionality reduction techniques, they work well when the variables are closely correlated. From the following figure we see that the yield changes from the actual data is more correlated than the gaussian data generation process. Hence PCA performed better on the real data.

[60]:
```
fig, axes = plt.subplots(1, 2, figsize=(14, 6))

# Simulated yield changes
sns.heatmap(yc_standardised.corr(), annot=True, cmap='coolwarm', fmt=".1f",␣
 ↪ax=axes[0])
axes[0].set_title('Simulated Yield Changes')
```

```
# Actual yield changes
sns.heatmap(standardized_yld_chag.corr(), annot=True, cmap='coolwarm', fmt=".
↪1f", ax=axes[1])
axes[1].set_title('Actual Yield Changes')

plt.tight_layout()
plt.show()
```



QUESTION 4

Empirical Analysis of ETFs

QUESTIONS 4(a):

ANSWER

For the group work, we decided to select a utility ETF called XLU, which has about 30 Holdings. The index includes securities of companies from the following industries: electric utilities; water utilities; multi-utilities; independent power and renewable electricity producers; and gas utilities. The fund is non-diversified.

[61]:
```
# Importing requied libraries

import datetime
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import seaborn as sns
import yfinance as yfin
import math
```

```python
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
from sklearn.decomposition import TruncatedSVD

from datetime import date

pd.options.display.float_format = "{:,.6f}".format
```

QUESTION 4(b):

ANSWER

One year of xlu data had been fetched from yfinance database as follows:

[62]:
```python
# Starting and end dates
start = datetime.date(2019, 1, 1)
end = datetime.date(2020, 1, 1)

xlu_tickers = [
    "NEE",   # NextEra Energy Inc.
    "SO",    # Southern Company
    "DUK",   # Duke Energy Corporation
    "AEP",   # American Electric Power Company Inc.
    "D",     # Dominion Energy Inc.
    "SRE",   # Sempra
    "EXC",   # Exelon Corporation
    "VST",   # Vistra Corp.
    "PEG",   # Public Service Enterprise Group Inc.
    "PCG",   # PG&E Corporation
    "XEL",   # Xcel Energy Inc.
    "ED",    # Consolidated Edison Inc.
    "AWK",   # American Water Works Company Inc.
    "WEC",   # WEC Energy Group Inc.
    "EIX",   # Edison International
    "ES",    # Eversource Energy
    "ATO",   # Atmos Energy Corporation
    "CMS",   # CMS Energy Corporation
    "NI",    # NiSource Inc.
    "PNW",   # Pinnacle West Capital Corporation
    "CNP",   # CenterPoint Energy Inc.
    "EVRG",  # Evergy Inc.
    "FE",    # FirstEnergy Corp.
    "NRG",   # NRG Energy Inc.
    "OGE",   # OGE Energy Corp.
    "AEE",   # Ameren Corporation
    "AES",   # The AES Corporation
    "LNT",   # Alliant Energy Corporation
    "UGI",   # UGI Corporation
```

```
    "IDA"    # IDACORP Inc.
]

# Get ETF data
df = yfin.download(xlu_tickers, start, end, auto_adjust = False)["Adj Close"]

# Convert DataFrame index to timezone-aware (UTC)
df.index = df.index.tz_localize('UTC')
```

```
[*********************100%***********************]  30 of 30 completed
```

Let us have a look at the first five rows of the daily data.

[63]:
```
df.head(5)
```

[63]:
```
Ticker                           AEE        AEP        AES        ATO        AWK  \
Date
2019-01-02 00:00:00+00:00  53.270638  58.418907  11.534039  77.131927  79.253159
2019-01-03 00:00:00+00:00  53.404800  58.282742  11.525903  77.715683  79.565666
2019-01-04 00:00:00+00:00  54.159462  58.819374  11.908201  78.711494  80.163826
2019-01-07 00:00:00+00:00  53.538963  58.490982  11.965140  78.136307  79.315666
2019-01-08 00:00:00+00:00  54.385868  59.211811  12.241695  79.157883  80.476303

Ticker                           CMS        CNP          D        DUK         ED  \
Date
2019-01-02 00:00:00+00:00  40.172016  23.255316  54.124615  65.633423  59.839413
2019-01-03 00:00:00+00:00  40.246857  23.388201  53.866066  65.610168  59.990711
2019-01-04 00:00:00+00:00  40.621044  23.820086  54.907814  66.152618  60.962288
2019-01-07 00:00:00+00:00  40.413158  23.911448  54.375534  65.873642  60.078316
2019-01-08 00:00:00+00:00  41.020199  24.368250  55.006664  66.702774  60.444637

Ticker                     …        OGE        PCG        PEG        PNW  \
Date                       …
2019-01-02 00:00:00+00:00  …  28.728699  23.682159  41.137554  63.742725
2019-01-03 00:00:00+00:00  …  28.983990  23.831417  41.129467  64.388847
2019-01-04 00:00:00+00:00  …  29.442030  24.279188  41.574585  65.181114
2019-01-07 00:00:00+00:00  …  29.284353  18.856174  41.663612  65.058044
2019-01-08 00:00:00+00:00  …  29.967651  17.473057  41.825474  66.034912

Ticker                            SO        SRE        UGI        VST        WEC  \
Date
2019-01-02 00:00:00+00:00  33.987129  43.674431  40.206642  19.215071  55.447842
2019-01-03 00:00:00+00:00  34.469097  43.874886  40.706722  19.034771  55.595539
2019-01-04 00:00:00+00:00  34.756737  45.188057  41.591496  19.807491  56.005760
2019-01-07 00:00:00+00:00  34.678986  45.179871  41.676102  20.142344  55.751431
2019-01-08 00:00:00+00:00  35.666267  46.443951  41.991550  20.150923  56.325752

Ticker                           XEL
```

```
Date
2019-01-02 00:00:00+00:00 39.967190
2019-01-03 00:00:00+00:00 39.809704
2019-01-04 00:00:00+00:00 40.199265
2019-01-07 00:00:00+00:00 40.025208
2019-01-08 00:00:00+00:00 40.489365

[5 rows x 30 columns]
```

Overview of the ETF Data

We can use the pandas *describe()* method to show summary stats of our data. We can see that all assets have same number of observations (count) since they all belong to the same portfolio. The other summary stats are relatively basic, like mean and standard deviation along with showing minimum, maximum, and a few quantiles.

[64]:
```
df.describe()
```

[64]:
```
Ticker         AEE         AEP         AES         ATO         AWK         CMS  \
count   252.000000  252.000000  252.000000  252.000000  252.000000  252.000000
mean     62.849829   71.180582   14.031441   90.896078  101.687288   49.038936
std       3.245782    5.109216    1.068596    5.416583    9.813964    3.909191
min      53.270638   58.282742   11.525903   77.131927   79.253159   40.172016
25%      60.837066   67.466366   13.314821   87.135111   93.918903   45.967132
50%      63.895058   72.929688   13.999062   92.650070  104.843307   49.257162
75%      65.094208   75.268444   14.701837   95.203959  110.105633   52.592812
max      68.704857   78.527794   16.838135   99.922478  117.211884   55.235107

Ticker         CNP           D         DUK          ED  …         OGE         PCG  \
count   252.000000  252.000000  252.000000  252.000000  …  252.000000  252.000000
mean     24.547177   59.984374   71.137369   69.934112  …   32.676920   14.893234
std       1.196643    3.548472    2.900219    4.330081  …    1.146570    5.269217
min      21.099733   51.683754   64.827538   59.289902  …   28.728699    3.781185
25%      24.072489   58.064060   69.467739   67.827507  …   32.078150   10.634584
50%      24.733862   59.510872   70.553871   70.953804  …   32.867311   17.030259
75%      25.513127   63.671884   72.968155   72.358782  …   33.288624   18.866125
max      26.322067   66.020317   77.733902   77.404655  …   35.019451   24.279188

Ticker         PEG         PNW          SO         SRE         UGI         VST  \
count   252.000000  252.000000  252.000000  252.000000  252.000000  252.000000
mean     48.428759   72.444121   44.384742   55.560342   39.517781   21.618953
std       2.487902    3.038539    4.653333    5.162168    2.706561    1.422285
min      41.072803   63.435036   33.987129   43.674431   32.437710   18.584152
25%      47.887321   70.596035   40.785181   51.892800   37.717456   20.698179
50%      48.764408   73.282551   44.307463   56.768843   40.448435   21.774379
75%      50.065635   74.517900   49.139080   59.809655   41.575356   22.810293
max      52.458244   77.214264   51.908264   63.737049   43.899555   23.848814
```

```
Ticker         WEC         XEL
count   252.000000 252.000000
mean     70.069637  49.470814
std       6.990331   3.973183
min      55.447842  39.809704
25%      64.181847  46.539083
50%      71.049866  50.785336
75%      76.504215  52.605733
max      81.964577  55.469280

[8 rows x 30 columns]
```

QUESTION 4(C) :

ANSWER Importance of Daily Returns

In financial analysis, the daily return of an asset, XLU-ETF in our case, is a fundamental metric used to measure its performance over a single trading day. It definately aids in quantifying the percentage change in the asset's value between the close of one trading day and the close of the subsequent day. To compute a daily return you have to use the closing prices of the asset on two consecutive trading days.

To account for dividend and stock splits, technically, one must use adjusted closing price to avoid impact on the individual prices of an ETF consitituents assets/holdings.

[65]:
```
# 1. Compute daily log returns
daily_returns = df.pct_change() # Used simple return to compute the daily␣
↪return (alternatively: log return can also be used, np.log(df) - np.log(df.
↪shift(1))
daily_returns = daily_returns.dropna()
daily_returns.head()
```

[65]:
```
Ticker                          AEE       AEP       AES       ATO       AWK  \
Date
2019-01-03 00:00:00+00:00  0.002519 -0.002331 -0.000705  0.007568  0.003943
2019-01-04 00:00:00+00:00  0.014131  0.009207  0.033169  0.012814  0.007518
2019-01-07 00:00:00+00:00 -0.011457 -0.005583  0.004782 -0.007308 -0.010580
2019-01-08 00:00:00+00:00  0.015818  0.012324  0.023113  0.013074  0.014633
2019-01-09 00:00:00+00:00 -0.007401 -0.007575 -0.001993 -0.016376 -0.012758

Ticker                          CMS       CNP         D       DUK        ED  \
Date
2019-01-03 00:00:00+00:00  0.001863  0.005714 -0.004777 -0.000354  0.002528
2019-01-04 00:00:00+00:00  0.009297  0.018466  0.019340  0.008268  0.016195
2019-01-07 00:00:00+00:00 -0.005118  0.003836 -0.009694 -0.004217 -0.014500
2019-01-08 00:00:00+00:00  0.015021  0.019104  0.011607  0.012587  0.006097
2019-01-09 00:00:00+00:00 -0.009528 -0.012270 -0.001659 -0.014754 -0.007905

Ticker                     …       OGE       PCG       PEG       PNW  \
```

```
Date                          …
2019-01-03 00:00:00+00:00  …  0.008886  0.006303 -0.000197  0.010136
2019-01-04 00:00:00+00:00  …  0.015803  0.018789  0.010822  0.012304
2019-01-07 00:00:00+00:00  … -0.005355 -0.223361  0.002141 -0.001888
2019-01-08 00:00:00+00:00  …  0.023333 -0.073351  0.003885  0.015015
2019-01-09 00:00:00+00:00  … -0.001897  0.015376 -0.007546 -0.013861

Ticker                           SO       SRE       UGI       VST       WEC  \
Date
2019-01-03 00:00:00+00:00  0.014181  0.004590  0.012438 -0.009383  0.002664
2019-01-04 00:00:00+00:00  0.008345  0.029930  0.021735  0.040595  0.007379
2019-01-07 00:00:00+00:00 -0.002237 -0.000181  0.002034  0.016905 -0.004541
2019-01-08 00:00:00+00:00  0.028469  0.027979  0.007569  0.000426  0.010301
2019-01-09 00:00:00+00:00 -0.008500 -0.009777 -0.010260 -0.006817 -0.005972

Ticker                          XEL
Date
2019-01-03 00:00:00+00:00 -0.003940
2019-01-04 00:00:00+00:00  0.009786
2019-01-07 00:00:00+00:00 -0.004330
2019-01-08 00:00:00+00:00  0.011597
2019-01-09 00:00:00+00:00 -0.007984

[5 rows x 30 columns]
```

Question 4(d):

ANSWER

[66]:
```
# Standardize stock returns dataset
daily_returns_means = daily_returns.mean()
daily_returns_stds = daily_returns.std()
standardized_returns = (daily_returns - daily_returns_means) /
 ↪daily_returns_stds
standardized_returns.head()
```

[66]:
```
Ticker                          AEE       AEP       AES       ATO       AWK  \
Date
2019-01-03 00:00:00+00:00  0.183467 -0.417136 -0.180848  0.763060  0.275034
2019-01-04 00:00:00+00:00  1.497990  0.948129  2.525046  1.371677  0.663016
2019-01-07 00:00:00+00:00 -1.398541 -0.801954  0.257450 -0.963011 -1.301299
2019-01-08 00:00:00+00:00  1.689014  1.316873  1.721822  1.401934  1.435291
2019-01-09 00:00:00+00:00 -0.939389 -1.037595 -0.283713 -2.015217 -1.537659

Ticker                          CMS       CNP         D       DUK        ED  \
Date
2019-01-03 00:00:00+00:00  0.074823  0.474428 -0.621087 -0.107677  0.196295
2019-01-04 00:00:00+00:00  0.908478  1.557150  2.049139  0.984270  1.857467
```

```
2019-01-07 00:00:00+00:00 -0.707965  0.314914 -1.165523 -0.596890 -1.873476
2019-01-08 00:00:00+00:00  1.550296  1.611321  1.192958  1.531243  0.630091
2019-01-09 00:00:00+00:00 -1.202526 -1.052565 -0.275864 -1.931300 -1.071818

Ticker                     …       OGE       PCG       PEG       PNW  \
Date                       …
2019-01-03 00:00:00+00:00  …  0.936019  0.058488 -0.115172  1.020420
2019-01-04 00:00:00+00:00  …  1.734425  0.196466  1.214933  1.250012
2019-01-07 00:00:00+00:00  … -0.707872 -2.479325  0.167047 -0.252975
2019-01-08 00:00:00+00:00  …  2.603594 -0.821695  0.377515  1.537098
2019-01-09 00:00:00+00:00  … -0.308622  0.158748 -1.002373 -1.520920

Ticker                           SO       SRE       UGI       VST       WEC  \
Date
2019-01-03 00:00:00+00:00  1.412395  0.330920  0.903831 -0.718700  0.143412
2019-01-04 00:00:00+00:00  0.751041  3.088817  1.559431  2.999621  0.675730
2019-01-07 00:00:00+00:00 -0.448120 -0.188322  0.170246  1.237130 -0.670014
2019-01-08 00:00:00+00:00  3.031574  2.876472  0.560524  0.011088  1.005713
2019-01-09 00:00:00+00:00 -1.157912 -1.232676 -0.696668 -0.527763 -0.831610

Ticker                          XEL
Date
2019-01-03 00:00:00+00:00 -0.582442
2019-01-04 00:00:00+00:00  0.959820
2019-01-07 00:00:00+00:00 -0.626204
2019-01-08 00:00:00+00:00  1.163311
2019-01-09 00:00:00+00:00 -1.036772

[5 rows x 30 columns]
```

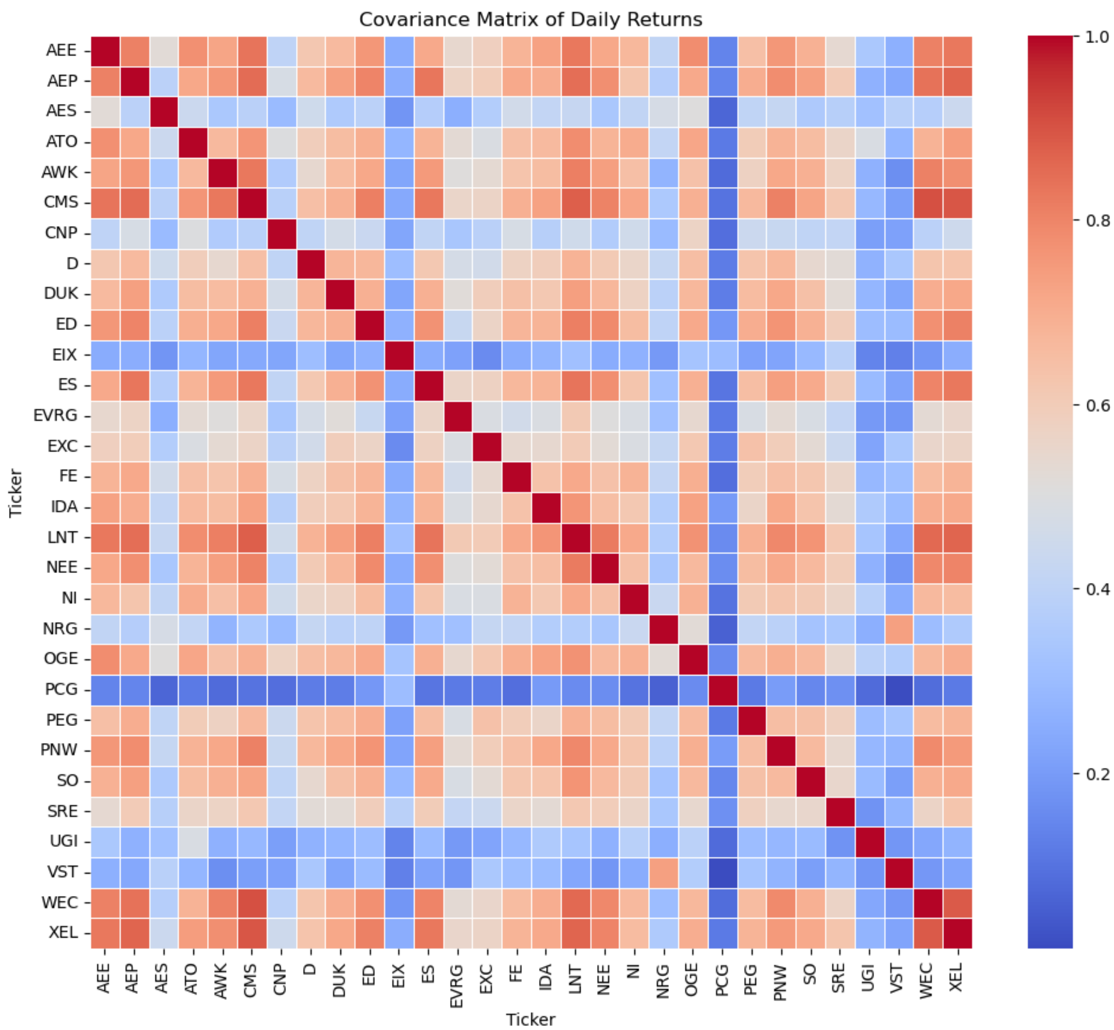
[67]:
```python
# Calculate covariance for standardized return matrix
standardized_returns_dvd_sqrt_n=(standardized_returns/math.
 ↪sqrt(len(standardized_returns)-1))
standardized_returns_cov = standardized_returns_dvd_sqrt_n.
 ↪T@standardized_returns_dvd_sqrt_n
standardized_returns_cov.head()
```

[67]:
```
Ticker       AEE       AEP       AES       ATO       AWK       CMS       CNP  \
Ticker
AEE     1.000000  0.806508  0.522380  0.771704  0.724853  0.834286  0.400583
AEP     0.806508  1.000000  0.389618  0.716670  0.757823  0.851062  0.479605
AES     0.522380  0.389618  1.000000  0.440486  0.343414  0.386723  0.291766
ATO     0.771704  0.716670  0.440486  1.000000  0.665883  0.763398  0.497982
AWK     0.724853  0.757823  0.343414  0.665883  1.000000  0.827256  0.360151

Ticker         D       DUK        ED  …       OGE       PCG       PEG       PNW  \
Ticker                                …
```

```
AEE     0.620388 0.662887 0.757681  … 0.781365 0.138429 0.646133 0.758155
AEP     0.661854 0.735293 0.800920  … 0.712112 0.143776 0.699679 0.782279
AES     0.451333 0.354717 0.390819  … 0.505821 0.070451 0.406286 0.421398
ATO     0.592838 0.651969 0.697263  … 0.719998 0.113870 0.600164 0.683686
AWK     0.540265 0.659178 0.713846  … 0.638205 0.082089 0.575092 0.716897

Ticker        SO       SRE       UGI       VST       WEC       XEL
Ticker
AEE     0.689450 0.538975 0.345584 0.258080 0.808670 0.827981
AEP     0.734961 0.604646 0.263487 0.235225 0.840927 0.866045
AES     0.350033 0.378617 0.316272 0.381779 0.376721 0.441355
ATO     0.652023 0.559600 0.485826 0.279780 0.686117 0.742385
AWK     0.691635 0.566795 0.261873 0.164898 0.810270 0.779340

[5 rows x 30 columns]
```

[68]:

```
plt.figure(figsize=(12, 10))
sns.heatmap(standardized_returns_cov, annot=False, cmap='coolwarm',␣
 ↪linewidths=0.5)
plt.title('Covariance Matrix of Daily Returns')
plt.show()
```



Question 4(e):

(compare and contrast PCA and SVD, explain what the eigenvectors, eigenvalues, singular values etc show us for the specific data, etc,)

ANSWER

Both PCA and SVD are dimensionality reduction technicque. They reduce dimensions of matrices but importantly they do retain substantial information or rather variance of a given financial data.

PCA plays a critical role of identifying key factors behind asset price movements whilst reducing dimensionality of risk-based models. Through PCA, one can establish a diversified portfolio based on the principal components. PCA can keep track of unique patterns and ambiguities in portfolio/ETF, which might not be captured in individual asset.

SVD stands for Singular Value Decomposition. It is a robust factorization technique that can effectively decompose any given matrix (Symmetric or not) into 3 matrices, namely, U, SIGMA and V^T. These matrices have mxn, mxn and nxn dimensions respectively. The magnitude of each singular value shows the importance of the corresponding dimension in the data.

Eigen vectors are extracted from PCA and they do represent the principal components which depict relative movements in daily returns of assets. On the other hand, eigenvalues, provides the amount of variance in the daily returns data related to principal components (eigenvector). The higher the eigenvalue, the better it gives information about the asset under analysis.

The following code solutions, shows how Eigenvalues and Eigenvectors help generate Principal Components based on ULX-ETF data under analysis. It is important to note that SVD can be applied to generate eigenvalues and eigenvectors.

[69]:
```
# Calculate eigenvectors and eigenvalues of the covariance matrix of␣
↪standardized dataset
eigenvalues, eigenvectors = np.linalg.eig(standardized_returns_cov)
eigenvalues
```

[69]:
```
array([16.94028168,  1.84467959,  1.26659788,  1.00338301,  0.85284461,
        0.78526915,  0.68313679,  0.62148507,  0.56747643,  0.54817586,
        0.50067043,  0.43553561,  0.39436812,  0.069993  ,  0.08276702,
        0.0974416 ,  0.1012309 ,  0.1310995 ,  0.36516125,  0.35232398,
        0.33001681,  0.2973241 ,  0.2820797 ,  0.16274718,  0.17443706,
        0.1935393 ,  0.2439289 ,  0.23422566,  0.21796411,  0.21981571])
```

[70]:
```
print(pd.DataFrame(eigenvectors).head())
```

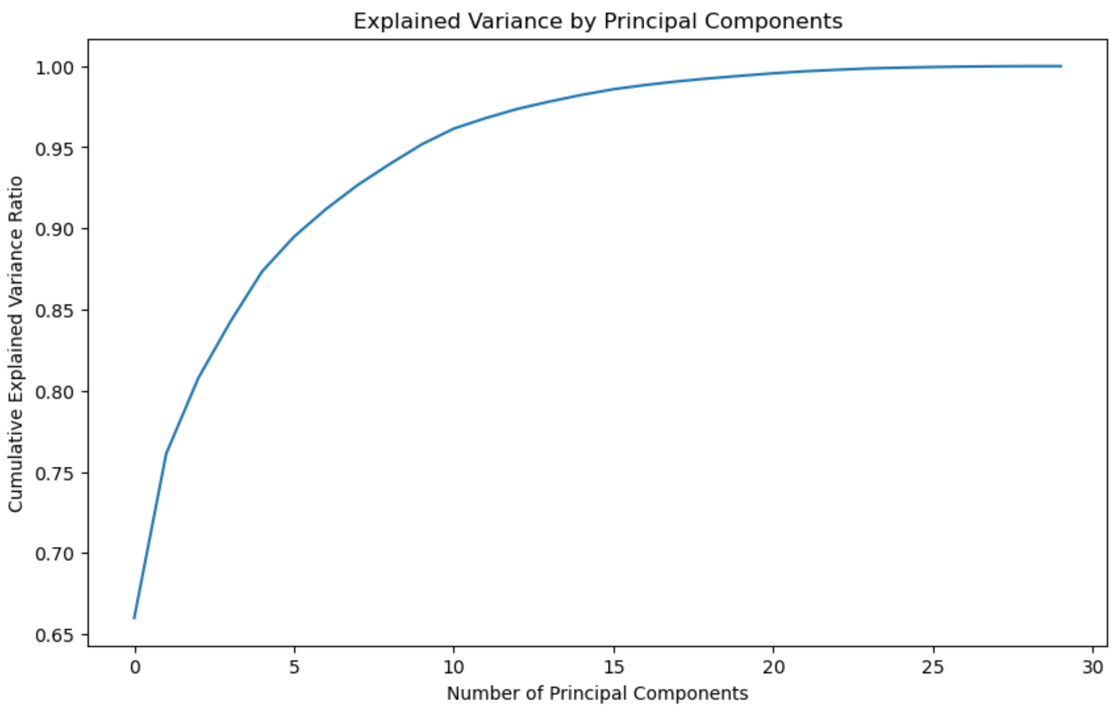
```
          0         1         2         3         4         5         6  \
0  0.213975  0.029619  0.048258 -0.097895  0.130893  0.023759  0.042472
1  0.219064  0.116063  0.034320  0.081349  0.006027 -0.016246  0.048883
2  0.127139 -0.328995  0.058619 -0.160036  0.144880  0.216880  0.387777
3  0.202650 -0.031362 -0.007037 -0.274623 -0.072982  0.025534 -0.076442
4  0.201149  0.183299  0.061811 -0.008564  0.061076  0.105053 -0.072020

          7         8         9  …        20        21        22        23  \
0 -0.207655  0.096317  0.051575  … -0.028912  0.341923 -0.220467 -0.238756
1  0.026483 -0.012475  0.066103  …  0.192394  0.086957 -0.160605  0.223547
2 -0.658972 -0.141537 -0.186651  … -0.012539 -0.139524  0.114498  0.021480
3  0.032477  0.135005  0.117306  … -0.166527  0.374332 -0.255891  0.304620
4  0.013562  0.152691 -0.084542  … -0.322111 -0.388178  0.055160  0.373393

         24        25        26        27        28        29
0 -0.037397  0.028031 -0.020502 -0.062437  0.291986  0.005186
1  0.065063 -0.054065  0.125431  0.118866  0.127292  0.178014
2 -0.085402 -0.007394  0.002390 -0.010983 -0.185881 -0.053939
3  0.157658  0.122317 -0.450398 -0.106012 -0.379782 -0.083003
4 -0.050249  0.051497 -0.031843 -0.431090  0.331946 -0.270047

[5 rows x 30 columns]
```

[71]:
```
# Transform standardized data with Loadings
principal_components = standardized_returns_cov.dot(eigenvectors)
principal_components.columns = ["PC_" + str(i) for i in range(1, 31)]
principal_components.head()
```

[71]:
```
            PC_1      PC_2      PC_3      PC_4      PC_5      PC_6      PC_7  \
Ticker
AEE     3.624796  0.054637  0.061123 -0.098226  0.111631  0.018657  0.029014
AEP     3.710999  0.214099  0.043469  0.081625  0.005140 -0.012757  0.033394
AES     2.153766 -0.606891  0.074247 -0.160577  0.123560  0.170309  0.264905
ATO     3.432954 -0.057853 -0.008913 -0.275552 -0.062242  0.020051 -0.052220
AWK     3.407518  0.338127  0.078290 -0.008592  0.052088  0.082495 -0.049200

            PC_8      PC_9     PC_10  …     PC_21     PC_22     PC_23  \
Ticker                                …
AEE    -0.129054  0.054657  0.028272  … -0.009542  0.101662 -0.062189
AEP     0.016459 -0.007079  0.036236  …  0.063493  0.025854 -0.045303
AES    -0.409541 -0.080319 -0.102317  … -0.004138 -0.041484  0.032298
ATO     0.020184  0.076612  0.064305  … -0.054957  0.111298 -0.072182
AWK     0.008429  0.086648 -0.046344  … -0.106302 -0.115415  0.015560

           PC_24     PC_25     PC_26     PC_27     PC_28     PC_29     PC_30
Ticker
AEE    -0.038857 -0.006523  0.005425 -0.005001 -0.014624  0.063642  0.001140
AEP     0.036382  0.011349 -0.010464  0.030596  0.027841  0.027745  0.039130
AES     0.003496 -0.014897 -0.001431  0.000583 -0.002572 -0.040515 -0.011857
ATO     0.049576  0.027501  0.023673 -0.109865 -0.024831 -0.082779 -0.018245
AWK     0.060769 -0.008765  0.009967 -0.007767 -0.100972  0.072352 -0.059361

[5 rows x 30 columns]
```

[72]:
```
# Visualization for PCA

x = standardized_returns_cov.values  # Convert DataFrame to NumPy array
x = StandardScaler().fit_transform(x)  # Standardize the data

# Apply PCA
pca = PCA(n_components=standardized_returns_cov.shape[1])
principalComponents = pca.fit_transform(x)
principalDf = pd.DataFrame(data=principalComponents, columns=['principal␣
↪component ' + str(i) for i in range(1, standardized_returns_cov.shape[1] +␣
↪1)], index=standardized_returns_cov.index)

# Get explained variance ratio
explained_variance_ratio = pca.explained_variance_ratio_

plt.figure(figsize=(10, 6))
```

```
plt.plot(np.cumsum(explained_variance_ratio))
plt.xlabel('Number of Principal Components')
plt.ylabel('Cumulative Explained Variance Ratio')
plt.title('Explained Variance by Principal Components')
plt.show()
```



Question 4(f):

ANSWER

[73]:
```
# Use SVD to calculate eigenvectors and eigenvalues of the covariance matrix of␣
↪standardized returns
U_st_return, s_st_return, VT_st_return = np.linalg.
↪svd(standardized_returns_dvd_sqrt_n)
print("\nSquared Singular values (eigenvalues):")
print(s_st_return**2)
print("\nMatrix V (eigenvectors)")
print(pd.DataFrame(VT_st_return.T).head())
```

```
Squared Singular values (eigenvalues):
[16.94028168  1.84467959  1.26659788  1.00338301  0.85284461  0.78526915
  0.68313679  0.62148507  0.56747643  0.54817586  0.50067043  0.43553561
  0.39436812  0.36516125  0.35232398  0.33001681  0.2973241   0.2820797
  0.2439289   0.23422566  0.21981571  0.21796411  0.1935393   0.17443706
```

```
  0.16274718  0.1310995   0.1012309   0.0974416   0.08276702  0.069993  ]

Matrix V (eigenvectors)
         0         1         2         3         4         5         6   \
0 -0.213975 -0.029619  0.048258 -0.097895  0.130893 -0.023759  0.042472
1 -0.219064 -0.116063  0.034320  0.081349  0.006027  0.016246  0.048883
2 -0.127139  0.328995  0.058619 -0.160036  0.144880 -0.216880  0.387777
3 -0.202650  0.031362 -0.007037 -0.274623 -0.072982 -0.025534 -0.076442
4 -0.201149 -0.183299  0.061811 -0.008564  0.061076 -0.105053 -0.072020

         7         8         9   …        20        21        22        23  \
0 -0.207655  0.096317  0.051575  … -0.005186 -0.291986  0.028031  0.037397
1  0.026483 -0.012475  0.066103  … -0.178014 -0.127292 -0.054065 -0.065063
2 -0.658972 -0.141537 -0.186651  …  0.053939  0.185881 -0.007394  0.085402
3  0.032477  0.135005  0.117306  …  0.083003  0.379782  0.122317 -0.157658
4  0.013562  0.152691 -0.084542  …  0.270047 -0.331946  0.051497  0.050249

         24        25        26        27        28        29
0  0.238756 -0.248860 -0.007813  0.621960  0.029028 -0.106674
1 -0.223547 -0.695047  0.198817 -0.371492  0.046595  0.098009
2 -0.021480 -0.033898  0.006948 -0.078480  0.070055  0.066760
3 -0.304620  0.069613 -0.078670 -0.166917  0.101919 -0.138824
4 -0.373393 -0.089398 -0.085108  0.056279 -0.084309 -0.087953

[5 rows x 30 columns]
```

[74]:
```python
# Presenting the result
print("ETF Returns Matrix Dimension:")
print(daily_returns.shape)
print("\nDimension of Matrix U:")
print(U_st_return.shape)
print("\nSingular values:")
print(s_st_return**2)
print("\nDimension of Matrix V^T:")
print(VT_st_return.shape)
```

```
ETF Returns Matrix Dimension:
(251, 30)

Dimension of Matrix U:
(251, 251)

Singular values:
[16.94028168  1.84467959  1.26659788  1.00338301  0.85284461  0.78526915
  0.68313679  0.62148507  0.56747643  0.54817586  0.50067043  0.43553561
  0.39436812  0.36516125  0.35232398  0.33001681  0.2973241   0.2820797
  0.2439289   0.23422566  0.21981571  0.21796411  0.1935393   0.17443706
  0.16274718  0.1310995   0.1012309   0.0974416   0.08276702  0.069993  ]
```

```
Dimension of Matrix V^T:
(30, 30)
```

[75]:
```
# Visualization for SVD
svd = TruncatedSVD(n_components=standardized_returns_cov.shape[1] - 1)
svd_components = svd.fit_transform(x)
svdDf = pd.DataFrame(data=svd_components, columns=['singular value ' + str(i)␣
↪for i in range(1, standardized_returns_cov.shape[1])],␣
↪index=standardized_returns_cov.index)

plt.figure(figsize=(10, 6))
plt.plot(svd.singular_values_)
plt.xlabel('Singular Value Index')
plt.ylabel('Singular Value')
plt.title('Singular Values from SVD')
plt.show()
```