Origami Numbers
A Short Overview of the Lemmata and Theorems

Nora Depenheuer Joachim Roscher

February 8, 2025

1 First some basic line stuff

- vec_well_defined: no 0 in a fraction denominator of a vector
- vec_ne_zero: $l.vec \neq 0 \forall l$ line
- vec_abs_one: $|l.vec| = 1 \forall l$ line
- diff_ne_zero: $l.z_2 - l.z_1 \neq 0 \forall l$ line
- diff_ne_zero': $l.z_1 - l.z_2 \neq 0 \forall l$ line
- z_1_on_l: $l.z_1 \in l.points \forall l$ line
- z_2_on_l: $l.z_2 \in l.points \forall l$ line
- line_eq_symm: $l_1.eql_2 \iff l_2.eql_1 \forall l_1, l_2$ lines
- line_eq_symm': $\neg l_1.eql_2 \iff \neg l_2.eql_1 \forall l_1, l_2$ lines
- line_eq_is_equivalence_relation: exactly the title
- line_eq_self: $l.eql \forall l$ line
- line_eq_if_switched_points: $l.eq < l.z_2, l.z_1 > foralll$ line
- line_eq_iff_both_points_lie_in_the_other: $l_1.eql_2 \iff l_1.z_1 \in l_2.points \wedge l_1.z_2 \in l_2.points \forall l_1, l_2$ lines
- line_eq_iff_both_points_lie_in_the_other': $l_1.eql_2 \iff l_2.z_1 \in l_1.points \wedge l_2.z_2 \in l_1.points \forall l_1, l_2$ lines
- line_eq_if_add_vec: $l.eq < l.z_1, k \cdot l.vec > \forall k \neq 0, l$ line
- line_ne_iff: $\exists x \in l_1.points \wedge x \notin l_2.points \iff \neg l_1.eql_2 \forall l_1, l_2$ lines
- line_ne_iff': $\exists x \in l_2.points \wedge x \notin l_1.points \iff \neg l_1.eql_2 \forall l_1, l_2$ lines
- Parallel_symm: $l_1 \parallel l_2 \iff l_2 \parallel l_1 \forall l_1, l_2$ lines
- Not_parallel_if_parallel: $\neg l_1 \parallel l_2 \Rightarrow l_2 \parallel l_3 \Rightarrow \neg l_1 \parallel l_3$

2 Now some orgiami stuff

- $\mathbb{O}_n.points$_inc: $\mathbb{O}_n.points\ n \subseteq \mathbb{O}_n.points\ m\ \forall n \leq m$
- $\mathbb{O}_n.lines$_inc: $\mathbb{O}_n.lines\ n \subseteq \mathbb{O}_n.lines\ m\ \forall n \leq m$
- O4_not_parallel: $l \nparallel$ (O4 z l)
- O4_perpendicular: $l.vec \cdot \overline{\text{(O4 z l)}.vec}.re = 0$

- in_$\mathbb{O}$_if_eq: $z \in \mathbb{O} \wedge z' = z \implies z' \in \mathbb{O}$
- in_$\mathbb{O}$_lines_if_eq: $l \in \mathbb{O}.lines \wedge l'.eq\ l \implies l' \in \mathbb{O}.lines$
- in_$\mathbb{O}$_lines_if_eqq: $l \in \mathbb{O}.lines \wedge l' = l \implies l' \in \mathbb{O}.lines$
- O1_in_$\mathbb{O}$: O1 $z_1\ z_2 \in \mathbb{O}.lines$
- O2_in_$\mathbb{O}$: O2 $z_1\ z_2 \in \mathbb{O}.lines$
- O3_in_$\mathbb{O}$: O3 $l_1\ l_2 \in \mathbb{O}.lines$
- O3'_in_$\mathbb{O}$: O3' $l_1\ l_2 \in \mathbb{O}.lines$
- O4_in_$\mathbb{O}$: O4 $z\ l \in \mathbb{O}.lines$
- O5_in_$\mathbb{O}$: O5 $z_1\ z_2\ l \in \mathbb{O}.lines$
- O6_in_$\mathbb{O}$: O6 $z_1\ z_2\ l_1\ l_2 \in \mathbb{O}.lines$
- Isect_in_$\mathbb{O}$: $l_1 \cap l_2 \in \mathbb{O}$
- $\mathbb{O}$_E1: E1 $z\ l\ hz\ hl \in \mathbb{O}.lines$
- $\mathbb{O}'$_E1': $\exists l' \in \mathbb{O}, l'.z_1 = z \wedge l'.z_2 = z - l.vec \forall z \in \mathbb{O}, l \in \mathbb{O}.lines$
- E1_parallel_l: (E1 $z\ l) \parallel l$
- O4_not_parallel_to_E1: $\neg$ (O4 $z\ l) \parallel$ (E1 $z\ l\ hz\ hl$)
- O3_on_O4_and_E1: (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$))$.z_1 = z \wedge$ (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$))$.z_2 = z + i \cdot l.vec - l.vec \wedge$ (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$))$.vec = (i-1) \cdot l.vec/|i-1|$
- l_not_parallel_to_O3_on_O4_and_E1: $\neg l \parallel$ (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$))
- O4_not_parallel_to_O4_on_O3_on_O4_and_E1: $\neg$(O4 $z\ l) \parallel$ (O4 ($l \cap$ (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$)) (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$)))
- O4_on_z_1_and_l_4: (O4 ($l \cap$ (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$)) (O3 (O4 $z\ l$) (E1 $z\ l\ hz\ hl$)))$.vec = -(i+1) \cdot l.vec/|i-1|$
- $\mathbb{O}$_E2: E2 $z\ l\ hz\ hl \in \mathbb{O}$
- E2_ne_z: E2 $z\ l\ hz\ hl \neq z$
- zero_in_$\mathbb{O}$: $0 \in \mathbb{O}$
- one_in_$\mathbb{O}$: $1 \in \mathbb{O}$
- $\mathbb{O}$_reAxis: reAxis $\in \mathbb{O}.lines$
- $\mathbb{O}$_imAxis: imAxis $\in \mathbb{O}.lines$
- $\mathbb{O}$_i: $i \in \mathbb{O}$

3 Now the actual project goal

- $\mathbb{O}$_neg: $-z \in \mathbb{O} \forall z \in \mathbb{O}$
- $\mathbb{O}$_double: $2 \cdot z \in \mathbb{O}$
- $\mathbb{O}$_add_multiples: $z_1 + z_2 \in \mathbb{O} \forall z_1, z_2 \in \mathbb{O}, z_1 = k \cdot z_2$
- $\mathbb{O}$_add: $z_1 + z_2 \in \mathbb{O} \forall z_1, z_2 \in \mathbb{O}$
- $\mathbb{O}$_re: $z.re \in \mathbb{O} \forall z \in \mathbb{O}$

- $\mathbb{O}$_i_mul: $i \cdot z \in \mathbb{O} \forall z \in \mathbb{O}$
- $\mathbb{O}$_real_mul_real: $a \cdot b \in \mathbb{O} \forall a, b \in \mathbb{O} \cap \mathbb{R}$
- $\mathbb{O}$_real_mul_cmpl: $a \cdot z \in \mathbb{O} \forall a \in \mathbb{O} \cap \mathbb{R}, z \in \mathbb{O} \setminus \mathbb{R}$
- $\mathbb{O}$_im: $z.im \in \mathbb{O} \forall z \in \mathbb{O}$
- $\mathbb{O}$_mul: $z_1 \cdot z_2 \in \mathbb{O} \forall z_1, z_2 \in \mathbb{O}$
- $\mathbb{O}$_pow_nat: $z^n \in \mathbb{O} \ \forall z \in \mathbb{O}, \ n \in \mathbb{N}$
- $\mathbb{O}$_real_inv_cmpl: $z/a \in bO \forall z \in \mathbb{O} \setminus \mathbb{R}, a \in \mathbb{O} \cap \mathbb{R}$
- $\mathbb{O}$_real_inv_real: $a/b \in \mathbb{O} \forall a, b \in \mathbb{O} \cap \mathbb{R}$
- $\mathbb{O}$_inv: $\exists z' \in \mathbb{O} : z \cdot z' = 1 \forall z \in \mathbb{O}$
- $\mathbb{O}$_div: $z_1/z_2 \in \mathbb{O} \forall z_1, z_2 \in \mathbb{O}$
- $\mathbb{O}$_pow_int: $z^n \in \mathbb{O} \ \forall z \in \mathbb{O}, \ n \in \mathbb{Z}$
- $\mathbb{O}$_isField: $\mathbb{O}Field$ is a field with $+, \cdot$ from $\mathbb{C}$
- $\mathbb{O}$_square_roots_nonneg_real: $\sqrt{z} \in \mathbb{O} \ \forall z \in \mathbb{O}, z \in \mathbb{R}_{\geq 0}$
- $\mathbb{O}$_abs: Complex.abs (z) $\in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_vec: l.vec $\in \mathbb{O} \ \forall l \in \mathbb{O}$.lines
- $\mathbb{O}$_sin_arg: Complex.sin (z.arg) $\in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_cos_arg: Complex.cos (z.arg) $\in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_sin_arg_div_two: Complex.sin (z.arg / 2) $\in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_sin_arg_div_three: Complex.sin (z.arg / 3) $\in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_half_angle: $e^{i \cdot z.arg/2} \in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_square_root: $\exists z' \in \mathbb{O} : z' \cdot z' = z \forall z \in \mathbb{O}$
- $\mathbb{O}$_square_roots: $z^2 \in \mathbb{O} \implies z \in \mathbb{O} \ \forall z$
- $\mathbb{O}$_square_roots': Polynomial.nthRoots 2 $z \ \subseteq \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_slope: l.vec.im/l.vec.re $\in \mathbb{O} \ \forall l \in \mathbb{O}$.lines
- $\mathbb{O}$_cubics: The roots of a cubic with coefficients in $\mathbb{O} \cap \mathbb{R}$ lie in $\mathbb{O}$
- $\mathbb{O}$_trisect_angle: $e^{i \cdot z.arg/3} \in \mathbb{O} \ \forall z \in \mathbb{O}$
- $\mathbb{O}$_cube_roots_real: $z^3 \in \mathbb{O} \implies z \in \mathbb{O} \ \forall z \in \mathbb{R}$
- $\mathbb{O}$_cube_root: $\exists z' \in \mathbb{O} : z' \cdot z' \cdot z' \cdot z' = z \forall z \in \mathbb{O}$
- $\mathbb{O}$_cube_roots_of_unity: $1, e^{i2\pi/3}, e^{i4\pi/3} \in \mathbb{O}$
- $\mathbb{O}$_cube_roots: $z^3 \in \mathbb{O} \implies z \in \mathbb{O} \ \forall z$
- $\mathbb{O}$_cube_roots': Polynomial.nthRoots 3 $z \ \subseteq \mathbb{O} \ \forall z \in \mathbb{O}$