#

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

Solution

We can use the Nyquist formula as shown:

$$265{,}000 = 2 \times 20{,}000 \times \log_2 L$$

$$\log_2 L = 6.625 \qquad L = 2^{6.625} = 98.7 \text{ levels}$$

Since this result is neither an integer nor a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

Note: Number of levels here can be 98 or 99 also, but a power of 2 is more acceptable.

#

Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity C is calculated as

$$C = B \log_2 (1 + \text{SNR}) = B \log_2 (1 + 0) = B \log_2 1 = B \times 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

#

We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000. The signal-to-noise ratio is usually 3162. For this channel the capacity is calculated as

$$C = B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163$$
$$= 3000 \times 11.62 = 34{,}860 \text{ bps}$$

This means that the highest bit rate for a telephone line is 34.860 kbps.

If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

#

The signal-to-noise ratio is often given in decibels. Assume that SNR_{dB} = 36 and the channel bandwidth is 2 MHz. The theoretical channel capacity can be calculated as:

$$SNR_{dB} = 10 \log_{10} SNR \;\Rightarrow\; SNR = 10^{SNR_{dB}/10} \;\Rightarrow\; SNR = 10^{3.6} = 3981$$

$$C = B \log_2 (1 + SNR) = 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}$$

Example #1

The bandwidth of a subscriber line is 4 kHz for voice or data. The bandwidth of this line for data transmission can be up to 56,000 bps using a sophisticated modem to change the digital signal to analog.

Example #2

If the telephone company improves the quality of the line and increases the bandwidth to 8 kHz, we can send 112,000 bps by using the same technology as mentioned in Example 1.

#

A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

Solution

We can calculate the throughput as

$$\boxed{\text{Throughput} = \frac{12{,}000 \times 10{,}000}{60} = 2 \text{ Mbps}}$$

The throughput is almost one-fifth of the bandwidth in this case.

#

What is the propagation time if the distance between two points is 12,000 km? Assume the propagation speed to be 2.4×10^8 m/s in cable.

Solution

We can calculate the propagation time as

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

#

What are the propagation time and the transmission time for a 2.5-kbyte message (an e-mail) if the bandwidth of the network is 1 Gbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4×10^8 m/s.

Solution

We can calculate the propagation and transmission time as shown on the next slide:

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{2500 \times 8}{10^9} = 0.020 \text{ ms}$$

Note that in this case, because the message is short and the bandwidth is high, the dominant factor is the propagation time, not the transmission time. The transmission time can be ignored.

#

What are the propagation time and the transmission time for a 5-Mbyte message (an image) if the bandwidth of the network is 1 Mbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4×10^8 m/s.

Solution

We can calculate the propagation and transmission times as shown on the next slide.

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{5{,}000{,}000 \times 8}{10^6} = 40 \text{ s}$$

Note that in this case, because the message is very long and the bandwidth is not very high, the dominant factor is the transmission time, not the propagation time. The propagation time can be ignored.

A digitized voice channel is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). We assume that each sample requires 8 bits. What is the required bit rate

Solution

The bit rate can be calculated as

$$\boxed{2 \times 4000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}}$$

What is the bit rate for full high-definition TV (FHD TV)?
(Let, 24 bits are needed for representing each pixel)

Solution

FHD TV uses digital signals to broadcast high quality video signals. The FHD TV screen is normally a ratio of 16 : 9. There are 1920 by 1080 pixels per screen, and the screen is renewed 30 times per second. Twenty-four bits represent one color pixel.

$$\boxed{1920 \times 1080 \times 30 \times 24 = 1{,}492{,}992{,}000 \text{ or } 1.5 \text{ Gbps}}$$

The TV stations reduce this rate to 20 to 40 Mbps through compression.

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.

Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$\boxed{B = f_h - f_l = 900 - 100 = 800 \text{ Hz}}$$

The spectrum has only five spikes, at 100, 300, 500, 700, and 900 Hz (see Figure 2).

A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.

Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$\boxed{B = f_h - f_l \quad \Rightarrow \quad 20 = 60 - f_l \quad \Rightarrow \quad f_l = 60 - 20 = 40 \text{ Hz}}$$

The spectrum contains all integer frequencies. We show this by a series of spikes (see Figure 3).

A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

Solution

The lowest frequency must be at 40 kHz and the highest at 240 kHz. *Figure 4* shows the frequency domain and the bandwidth.

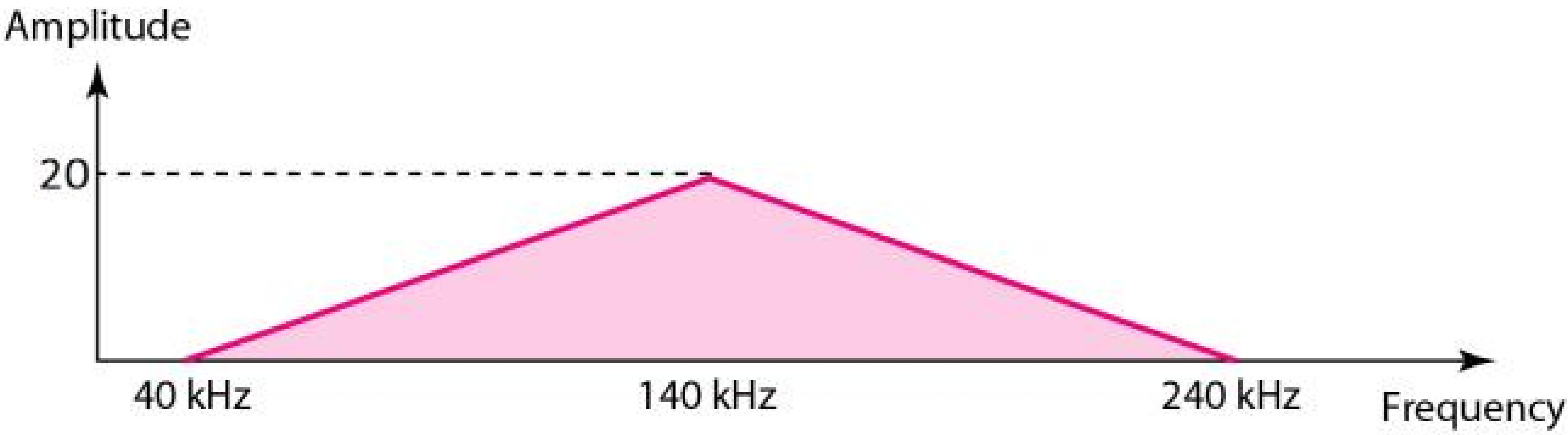

Figure 4 ***The bandwidth for previous example***

Some Examples of Non-periodic Composite Signal

- An example of a nonperiodic composite signal is the signal propagated by an AM radio station. In the United States, each AM radio station is assigned a 10-kHz bandwidth. The total bandwidth dedicated to AM radio ranges from 530 to 1700 kHz. We will show the rationale behind this 10-kHz bandwidth in Chapter 5.
- Another example of a nonperiodic composite signal is the signal propagated by an FM radio station. In the United States, each FM radio station is assigned a 200-kHz bandwidth. The total bandwidth dedicated to FM radio ranges from 88 to 108 MHz. We will show the rationale behind this 200-kHz bandwidth in Chapter 5.
- Another example of a nonperiodic composite signal is the signal received by an old-fashioned analog black-and-white TV. A TV screen is made up of pixels. If we assume a resolution of 525 × 700, we have 367,500 pixels per screen. If we scan the screen 30 times per second, this is 367,500 × 30 = 11,025,000 pixels per second. The worst-case scenario is alternating black and white pixels. We can send 2 pixels per cycle. Therefore, we need 11,025,000 / 2 = 5,512,500 cycles per second, or Hz. The bandwidth needed is 5.5125 MHz

Some Examples

#

A digital signal has eight levels. How many bits are needed per level? We calculate the number of bits from the formula

$$\boxed{\text{Number of bits per level} = \log_2 8 = 3}$$

Each signal level is represented by 3 bits.

#

A digital signal has nine levels. How many bits are needed per level? We calculate the number of bits by using the formula. Each signal level is represented by 3.17 bits. However, this answer is not realistic. The number of bits sent per level needs to be an integer. For this example, 4 bits can represent one level.

#

Assume we need to download text documents at the rate of 100 pages per minute. What is the required bit rate of the channel? (A page is an average of 24 lines with 80 characters in each line)

Solution

A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits, the bit rate is

$$\boxed{100 \times 24 \times 80 \times 8 = 1{,}636{,}000 \text{ bps} = 1.636 \text{ Mbps}}$$

The power we use at home has a frequency of 60 Hz. The period of this sine wave can be determined as follows:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0166 \text{ s} = 0.0166 \times 10^3 \text{ ms} = 16.6 \text{ ms}$$

Express a period of 100 ms in microseconds.

Solution

From Table 3.1 we find the equivalents of 1 ms (1 ms is 10^{-3} s) and 1 s (1 s is 10^{6} μs). We make the following substitutions:.

$$\boxed{100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 100 \times 10^{-3} \times 10^{6}\ \mu\text{s} = 10^{2} \times 10^{-3} \times 10^{6}\ \mu\text{s} = 10^{5}\ \mu\text{s}}$$

The period of a signal is 100 ms. What is its frequency in kilohertz?

Solution

First we change 100 ms to seconds, and then we calculate the frequency from the period (1 Hz = 10^{-3} kHz).

$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 10^{-1} \text{ s}$$

$$f = \frac{1}{T} = \frac{1}{10^{-1}} \text{ Hz} = 10 \text{ Hz} = 10 \times 10^{-3} \text{ kHz} = 10^{-2} \text{ kHz}$$

A sine wave is offset 1/6 cycle with respect to time 0. What is its phase in degrees and radians?

Solution

We know that 1 complete cycle is 360°. Therefore, 1/6 cycle is

$$\frac{1}{6} \times 360 = 60^\circ = 60 \times \frac{2\pi}{360} \text{ rad} = \frac{\pi}{3} \text{ rad} = 1.046 \text{ rad}$$

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.

Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$\boxed{B = f_h - f_l = 900 - 100 = 800 \text{ Hz}}$$

The spectrum has only five spikes, at 100, 300, 500, 700, and 900 Hz (see Figure 2).

A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.

Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$\boxed{B = f_h - f_l \;\Rightarrow\; 20 = 60 - f_l \;\Rightarrow\; f_l = 60 - 20 = 40 \text{ Hz}}$$

The spectrum contains all integer frequencies. We show this by a series of spikes (see Figure 3).

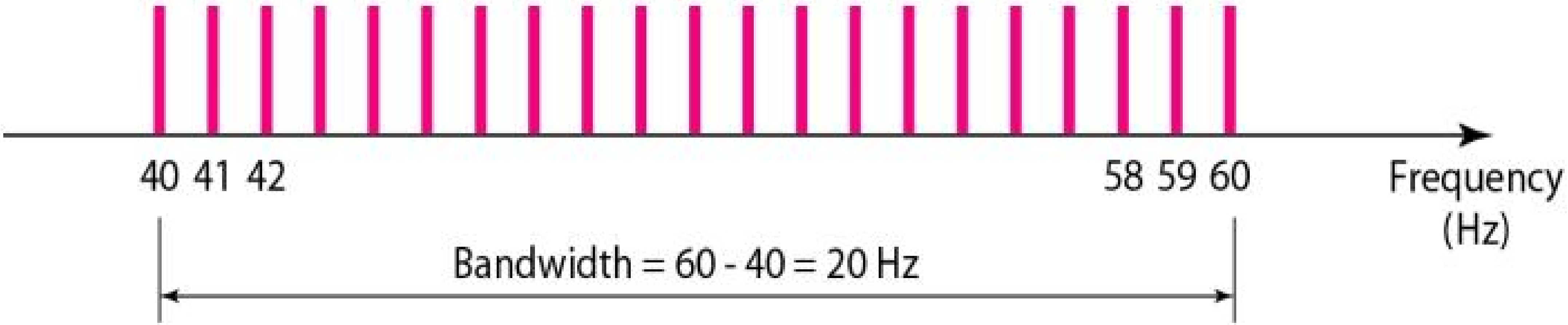

Figure 3: ***The bandwidth for previous example***

A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

Solution

The lowest frequency must be at 40 kHz and the highest at 240 kHz. *Figure 4* shows the frequency domain and the bandwidth.

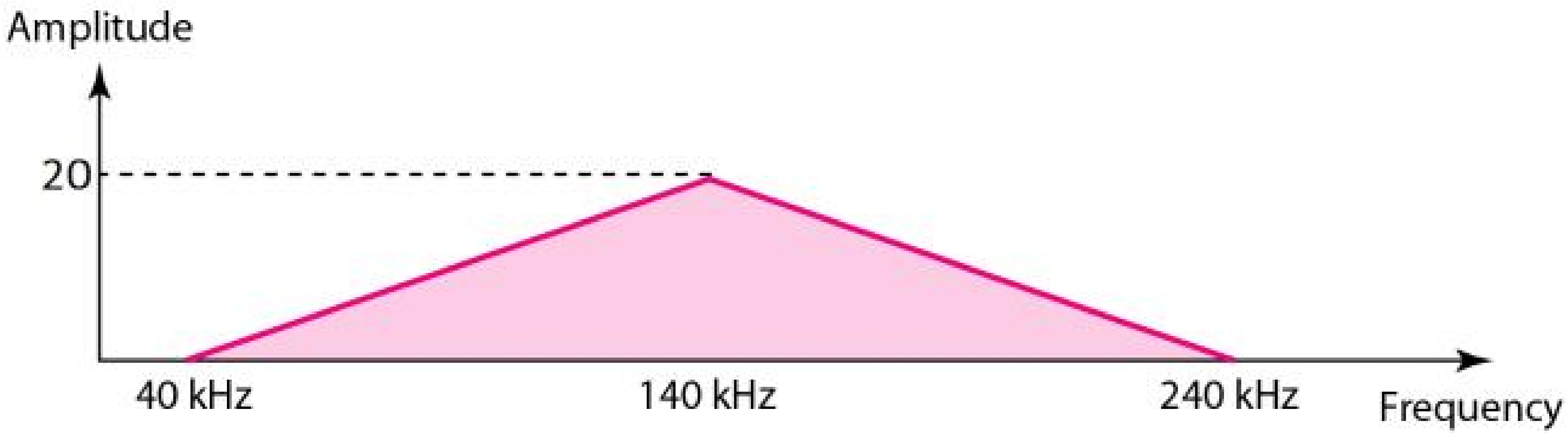

Figure 4 ***The bandwidth for previous example***

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Solution

The bit rate can be calculated as

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$$\boxed{1920 \times 1080 \times 30 \times 24 = 1{,}492{,}992{,}000 \text{ or } 1.5 \text{ Gbps}}$$

The TV stations reduce this rate to 20 to 40 Mbps through compression.

Suppose a signal travels through a transmission medium and its power is reduced to one-half. This means that P2 is (1/2)P1. In this case, the attenuation (loss of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{0.5 P_1}{P_1} = 10 \log_{10} 0.5 = 10(-0.3) = -3 \text{ dB}$$

A loss of 3 dB (–3 dB) is equivalent to losing one-half the power.

A signal travels through an amplifier, and its power is increased 10 times. This means that $P_2 = 10P_1$. In this case, the amplification (gain of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{10 P_1}{P_1}$$

$$= 10 \log_{10} 10 = 10(1) = 10 \text{ dB}$$

One reason that engineers use the decibel to measure the changes in the strength of a signal is that decibel numbers can be added (or subtracted) when we are measuring several points (cascading) instead of just two. In *Figure 3* a signal travels from point 1 to point 4. In this case, the decibel value can be calculated as

$$\boxed{dB = -3 + 7 - 3 = +1}$$

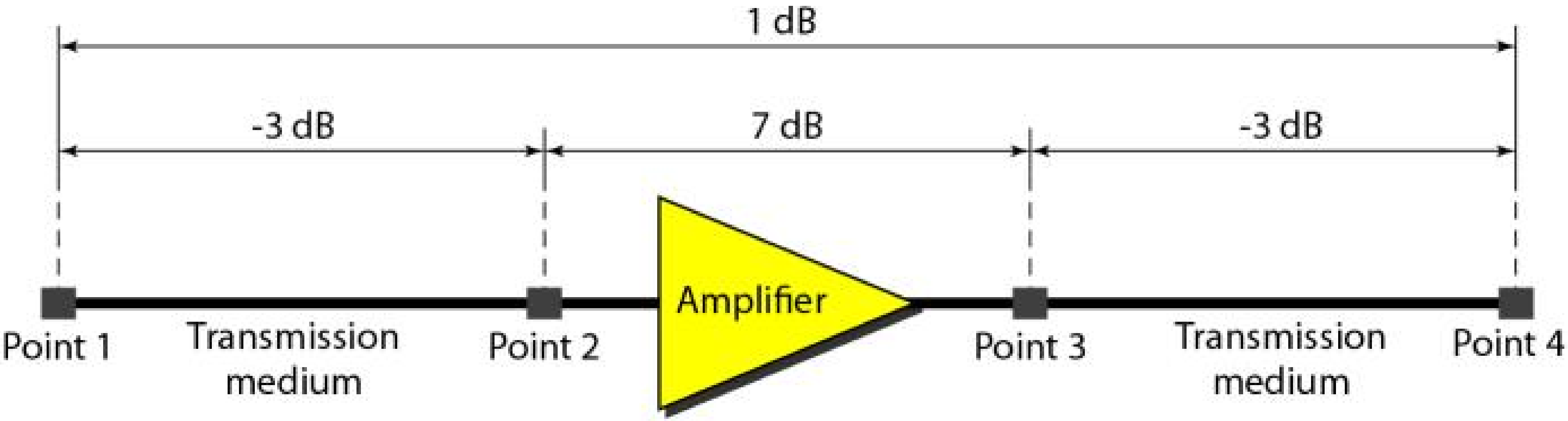

Figure 3: Decibels for the previous example

Sometimes the decibel is used to measure signal power in milliwatts. In this case, it is referred to as dB_m and is calculated as $dB_m = 10 \log10 P_m$, where P_m is the power in milliwatts. Calculate the power of a signal with $dB_m = -30$.

Solution

We can calculate the power in the signal as

$$\boxed{\begin{array}{c} \mathrm{dB_m} = 10 \log_{10} P_m = -30 \\ \log_{10} P_m = -3 \qquad P_m = 10^{-3}\ \mathrm{mW} \end{array}}$$

The loss in a cable is usually defined in decibels per kilometer (dB/km). If the signal at the beginning of a cable with −0.3 dB/km has a power of 2 mW, what is the power of the signal at 5 km?

Solution

The loss in the cable in decibels is 5 × (−0.3) = −1.5 dB. We can calculate the power as

$$\mathrm{dB} = 10 \log_{10} \frac{P_2}{P_1} = -1.5$$

$$\frac{P_2}{P_1} = 10^{-0.15} = 0.71$$

$$P_2 = 0.71 P_1 = 0.7 \times 2 = 1.4 \text{ mW}$$

The power of a signal is 10 mW and the power of the noise is 1 μW; what are the values of SNR and SNR_{dB} ?

Solution

The values of SNR and SNR_{dB} can be calculated as follows:

$$SNR = \frac{10{,}000\ \mu W}{1\ mW} = 10{,}000$$
$$SNR_{dB} = 10 \log_{10} 10{,}000 = 10 \log_{10} 10^4 = 40$$

The values of SNR and SNR_{dB} for a noiseless channel are

$$SNR = \frac{\text{signal power}}{0} = \infty$$
$$SNR_{dB} = 10 \log_{10} \infty = \infty$$

We can never achieve this ratio in real life; it is an ideal.

# # #

A signal is carrying data in which one data element is encoded as one signal element (r = 1). If the bit rate is 100 kbps, what is the average value of the baud rate if case factor 'c' is between 0 and 1?

Solution

We assume that the average value of c is 1/2 . The baud rate is then

$$\boxed{S = c \times N \times \frac{1}{r} = \frac{1}{2} \times 100{,}000 \times \frac{1}{1} = 50{,}000 = 50 \text{ kbaud}}$$

In a digital transmission, the receiver clock is 0.1 percent faster than the sender clock. How many extra bits per second does the receiver receive if the data rate is 1 kbps? How many if the data rate is 1 Mbps?

Solution

At 1 kbps, the receiver receives 1001 bps instead of 1000 bps.

1000 bits sent	1001 bits received	1 extra bps

At 1 Mbps, the receiver receives 1,001,000 bps instead of 1,000,000 bps.

1,000,000 bits sent	1,001,000 bits received	1000 extra bps

# # #

A system is using NRZ-I to transfer 10-Mbps data. What are the average signal rate and minimum bandwidth?

Solution

The average signal rate is $S = N/2 = 500$ kbaud. The minimum bandwidth for this average baud rate is;

$B_{min} = S = 500$ kHz.

- Data element = 8 ; data patterns = 2^8 = 256 and
- Signal element = 6 ; signal patterns = 3^6 = 729.
- There are 729 – 256 = 473 redundant signal elements that provide synchronization and error detection.

# # #

We need to send data at a 1-Mbps rate. What is the minimum required bandwidth, using a combination of 4B/5B and NRZ-I or Manchester coding?

Solution

First 4B/5B block coding increases the bit rate to 1.25 Mbps. The minimum bandwidth using NRZ-I is N/2 or 625 kHz. The Manchester scheme needs a minimum bandwidth of 1.25 MHz. The first choice needs a lower bandwidth, but has a DC component problem; the second choice needs a higher bandwidth, but does not have a DC component problem.

###

A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

The bandwidth of a low-pass signal is between 0 and f, where f is the maximum frequency in the signal. Therefore, we can sample this signal at 2 times the highest frequency (200 kHz). The sampling rate is therefore 400,000 samples per second.

What is the SNR_{dB} in the example of Figure 1?

Solution

We can use the formula ($SNR_{dB} = 6.02\, n_b + 1.76$) to find the quantization. We have eight levels and 3 bits per sample, so

$$SNR_{dB} = 6.02(3) + 1.76 = 19.82\ dB$$

Increasing the number of levels increases the SNR.

###

A telephone subscriber line must have an SNR_{dB} above 40. What is the minimum number of bits per sample?

Solution

We can calculate the number of bits as

$$\boxed{\mathrm{SNR_{dB}} = 6.02n_b + 1.76 = 40 \longrightarrow n = 6.35}$$

Telephone companies usually assign 7 or 8 bits per sample.

\# # #

We want to digitize the human voice. What is the bit rate, assuming 8 bits per sample?

Solution

The human voice normally contains frequencies from 0 to 4000 Hz. So the sampling rate and bit rate are calculated as follows:

$$\text{Sampling rate} = 4000 \times 2 = 8000 \text{ samples/s}$$
$$\text{Bit rate} = 8000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}$$

[**Formula:** Bit rate = Sampling rate (F_s) X No. of bits per sample (n_b)]

#

An analog signal carries 4 bits per signal element. If 1000 signal elements are sent per second, find the bit rate.

Solution

In this case, $r = 4$, $S = 1000$, and N is unknown. We can find the value of N from

$$\boxed{S = N \times \frac{1}{r} \qquad \text{or} \qquad N = S \times r = 1000 \times 4 = 4000 \text{ bps}}$$

#

An analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

Solution

In this example, $S = 1000$, $N = 8000$, and r and L are unknown. We find first the value of r and then the value of L.

$$S = N \times \frac{1}{r} \quad \Rightarrow \quad r = \frac{N}{S} = \frac{8000}{1000} = 8 \text{ bits/baud}$$

$$r = \log_2 L \quad \Rightarrow \quad L = 2^r = 2^8 = 256$$

#

We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What are the carrier frequency and the bit rate if we modulated our data by using ASK with d = 1?

Solution

The middle of the bandwidth is located at 250 kHz. This means that our carrier frequency can be at f_c = 250 kHz. We can use the formula for bandwidth to find the bit rate (with d = 1 and r = 1).

$$B = (1 + d) \times S = 2 \times N \times \frac{1}{r} = 2 \times N = 100 \text{ kHz} \Rightarrow N = 50 \text{ kbps}$$

#

We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What should be the carrier frequency and the bit rate if we modulated our data by using FSK with d = 1?

Solution

This problem is similar to the previous example on ASK , but we are modulating by using FSK. The midpoint of the band is at 250 kHz. We choose 2Δf to be 50 kHz; this means

$$B = (1 + d) \times S + 2\Delta f = 100 \quad \Rightarrow \quad 2S = 50 \text{ kHz} \quad S = 25 \text{ kbaud} \quad N = 25 \text{ kbps}$$

#

Find the bandwidth for a signal transmitting at 12 Mbps for QPSK. The value of d = 0.

Solution

For QPSK, 2 bits is carried by one signal element. This means that r = 2. So the signal rate (baud rate) is S = N × (1/r) = 6 Mbaud. With a value of d = 0, we have B = S = 6 MHz.

#

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The Data Rate of the noiseless channel (Nyquist bit rate) can be calculated as

$$\boxed{\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}}$$

#

Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The Nyquist bit rate can be calculated as

$$\boxed{\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12{,}000 \text{ bps}}$$