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Discrete Mathematics and Its Applications

Sixth Edition

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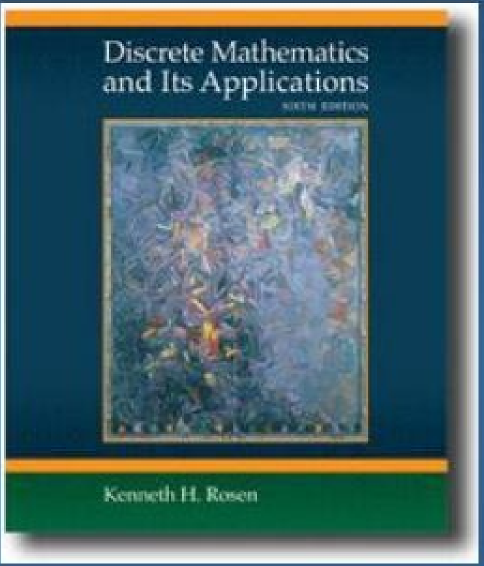

Chapter 1
The Foundations: Logic and Proofs

歐亞書局

- 1.1 Propositional Logic
- 1.2 Propositional Equivalences
- 1.3 Predicates and Quantifiers
- 1.4 Nested Quantifiers
- 1.5 Rules of Inference
- 1.6 Introduction to Proofs
- 1.7 Proof Methods and Strategy

1.1 Propositional Logic

- Logic: to give precise meaning to mathematical statements
- *Proposition*: a declarative sentence that is either true or false, but not both
  - 1+1=2
  - Toronto is the capital of Canada
- Propositional variables: p, q, r, s
- Truth value: true (T) or false (F)

- Compound propositions: news propositions formed from existing propositions using logical operators
- Definition 1: Let p be a proposition. The *negation* of p, denoted by $\neg p$ (or $\bar{p}$), is the statement "It is not the case that p."
  - "not p"

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

- Definition 2: Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q."
- Definition 3: Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q."

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- Definition 4: Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statements

- Definition 5: Let p and q be propositions. The *conditional statement* $p \to q$ is the proposition "if p, then q."
  - p: *hypothesis* (or *antecedent* or *premise*)
  - q: *conclusion* (or *consequence*)
  - Implication
    - "p implies q"
    - Many ways to express this…

TABLE 5 **The Truth Table for the Conditional Statement $p \longrightarrow q$.**

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Converse, Contrapositive, and Inverse

- $p \to q$
- *Converse*: $q \to p$
- *Contrapositive*: $\neg q \to \neg p$
- *Inverse*: $\neg p \to \neg q$
- Two compound propositions are *equivalent* if they always have the same truth value
  - The contrapositive is equivalent to the original statement
  - The converse is equivalent to the inverse

Biconditionals

- Definition 6: Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q."
  - "*bi-implications*"
  - "p is necessary and sufficient for q"
  - "p iff q"

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Implicit Use of Biconditionals

- Biconditionals are not always explicit in natural language
  - Imprecision in natural language
    - "If you finish your meal, then you can have dessert."
    - "You can have dessert if and only if you finish your meal."

TABLE 7 (1.1)

TABLE 7 **The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.**

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

- Negation operator is applied before all other logical operators
- Conjunction operator takes precedence over disjunction operator
- Conditional and biconditional operators have lower precedence
- Parentheses are used whenever necessary

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

Translating English Sentences

- Ex.12: "You can access the Internet from campus only if you are a computer science major or you are not a freshman."
- Ex.13: "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Examples

- Boolean Searches
  - New AND Mexico AND universities
  - (Mexico AND universities) NOT New
- Logic Puzzles
  - Ex. 18:
    - Knights always tell the truth, and knaves always lie
    - A says “B is a knight”
    - B says “The two of us are opposite types”
    - What are A and B?
  - Ex. 19

Logic and Bit Operations

- Bit: binary digit
- Boolean variable: either true or false
  - Can be represented by a bit
- Definition 7: A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

TABLE 9 **Table for the Bit Operators *OR*, *AND*, and *XOR*.**

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

1.2 Propositional Equivalences

- Definition 1:
  - *Tautology*: a compound proposition that is always true
  - *Contradiction*: a compound proposition that is always false
  - *Contingency*: a compound proposition that is neither a tautology nor a contradiction

TABLE 1 (1.2)

TABLE 1 Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Logical Equivalence

- Compound propositions that have the same truth values in all possible cases
- Definition 2: Compound propositions p and q are *logically equivalent* if $p \leftrightarrow q$ is a tautology (denoted by $p \equiv q$ or $p \Leftrightarrow q$)
- De Morgan's Law
  - $\neg (p \wedge q) \equiv \neg p \vee \neg q$
  - $\neg (p \vee q) \equiv \neg p \wedge \neg q$

TABLE 2 De Morgan's Laws.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

TABLE 3 (1.2)

TABLE 3 Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

TABLE 4 **Truth Tables for** $\neg p \vee q$ **and** $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

TABLE 5 (1.2)

TABLE 5 **A Demonstration That $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ Are Logically Equivalent.**

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

TABLE 6 **Logical Equivalences.**

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \vee q \equiv \neg p \rightarrow q$$

$$p \wedge q \equiv \neg (p \rightarrow \neg q)$$

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$

$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$

$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$

$$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$$

TABLE 8 Logical Equivalences Involving Biconditionals.

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Constructing New Logical Equivalence

- How to show logical equivalence
  - Use a truth table (Example 2, 3, 4 in Tables 3, 4, 5)
  - Use logical identities that we already know
    - (Example 6, 7, 8)

1.3 Predicates and Quantifiers

- Predicate logic
- *Predicate*: a property that the subject of the statement can have
  - Ex: $x>3$
    - x: variable
    - >3: predicate
    - P(x): x>3
      - The value of the propositional function P at x
  - $P(x_1, x_2, \ldots, x_n)$: n-place predicate or n-ary predicate

Quantifiers

- Quantification
  - Universal quantification: a predicate is true for every element
  - Existential quantification: there is one or more element for which a predicate is true

The Universal Quantifier

- Domain: domain of discourse (universe of discourse)
- Definition 1: The *universal quantification* of $P(x)$ is the statement "$P(x)$ for all values of x in the domain", denoted by $\forall x\, P(x)$
  - "for all x $P(x)$" or "for every x $P(x)$"
    - Counterexample: an element for which $P(x)$ is false
  - When all elements in the domain can be listed, $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

The Existential Quantifier

- Definition 2: The *existential quantification* of $P(x)$ is the proposition "There exists an element x in the domain such that $P(x)$", denoted by $\exists x\, P(x)$
  - "there is an x such that $P(x)$" or "for some x $P(x)$"
  - When all elements in the domain can be listed, $P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

TABLE 1 (1.3)

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x\, P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Other Quantifiers

- Uniqueness quantifier: $\exists! x\ P(x)$ or $\exists_1 x\ P(x)$
  - There exists a unique x such that $P(x)$ is true
- Quantifiers with restricted domains
  - $\forall x{<}0\ (x^2{>}0)$
    - Conditional: $\forall x(x{<}0 \rightarrow x^2{>}0)$
  - $\exists z{>}0\ (z^2{=}2)$
    - Conjunction: $\exists z(z{>}0 \wedge z^2{=}2)$

- Precedence of quantifiers
  - $\forall$ *and* $\exists$ have higher precedence than all logical operators
  - *Ex:* $\forall x\, P(x) \vee Q(x)$
    - $(\forall x\, P(x)) \vee Q(x)$

Logical Equivalence involving Quantifiers

- Definition 3: statements involving predicates and quantifiers are *logically equivalent* if and only if they have the same truth value no matter which predicates are substituted and which domain is used
  - E.g. $\forall x\,(P(x) \wedge Q(x))$ and $\forall x\,P(x) \wedge \forall x\,Q(x)$

Negating Quantified Expressions

- $\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$
  - Negation of the statement *"Every student in your class has taken a course in Calculus"*
  - *"There is a student in your class who has not taken a course in Calculus"*
- $\neg \exists x\, Q(x) \equiv \forall x\, \neg Q(x)$

TABLE 2 (1.3)

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Translating from English into Logical Expressions

- "Every student in this class has studied calculus"
- "Some student in this class has visited Mexico"
- "Every student in this class has visited either Canada or Mexico"

- Using Quantifiers in system specifications
  - "Every mail message larger than one megabyte will be compressed"
  - "If a user is active, at least one network link will be available"
- Examples from Lewis Carroll
  - "All lions are fierce"
  - "Some lions do not drink coffee"
  - "Some fierce creatures do not drink coffee"

Logic Programming

- Prolog
  - Facts
    - E.g.
      - instructor(chan, math)
      - instructor(patel, os)
      - enrolled(kevin, math)
      - enrolled(kevin, os)
      - enrolled(juana, math)
  - Rules
    - E.g.
      - teaches(P,S) :- instructor(P,C), enrolled(S,C)
  - ?teaches(X, kevin)

1.4 Nested Quantifiers

- Two quantifiers are nested if one is within the scope of the other
  - $\forall x \, \exists y \, (x+y=0)$
  - $\forall x \, \forall y \, ((x>0) \wedge (y<0) \rightarrow (xy<0))$
- Thinking of quantification as loops
  - $\forall x \forall y \, P(x, y)$
  - $\forall x \exists y \, P(x, y)$
  - $\exists x \forall y \, P(x, y)$
  - $\exists x \exists y \, P(x, y)$

- The order of quantifiers is important unless all quantifiers are universal quantifiers or all are existential quantifiers
  - $\forall x \forall y\, P(x, y)$ *vs.* $\forall y \forall x\, P(x, y)$
    - $P(x,y)$: "$x+y=y+x$"
  - $\forall x \exists y\, Q(x, y)$ *vs.* $\exists y \forall x\, Q(x, y)$
    - $Q(x,y)$: "$x+y=0$"

TABLE 1 (1.4)

TABLE 1 Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

Translating mathematical statements into statements involving nested quantifiers

- “The sum of two positive integers in always positive”
- “Every real number except zero has a multiplicative inverse”
  (a multiplicative inverse of a real number x is a real number y such that $xy=1$)

Translating from nested quantifiers into English

- $\forall x\, (C(x) \vee \exists y\, (C(y) \wedge F(x, y))$
  - $C(x)$: "*x has a computer*"
  - $F(x,y)$: "*x and y are friends*"
  - *Domain of x, y*: "*all students in your school*"
- $\exists x\, \forall y\, \forall z\, (F(x, y) \wedge F(x, z) \wedge (y \neq z) \rightarrow \neg F(y, z))$
  - $F(a, b)$: "*a and b are friends*"
  - *Domain of x, y, z*: "*all students in your school*"

- Translating English sentences into logical expressions
  - “If a person is female and is a parent, then this person is someone’s mother”
  - “Everyone has exactly one best friend”
- Negating nested quantifiers
  - Negation of $\forall x \, \exists y \, (xy=1)$
  - “There does not exist a woman who has taken a flight on every airline in the world”

1.5 Rules of Inference

- Proofs: valid arguments that establish the truth of mathematical statements
  - Argument: a sequence of statements that end with a conclusion
  - Valid: the conclusion must follow from the preceding statements (premises) of the argument

Valid Arguments in Propositional Logic

- Ex:
  - “If you have a current password, then you can log onto the network”
  - “You have a current password”
  - Therefore, “You can log onto the network”
- $$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

- Definition 1: *argument*: a sequence of propositions
  - *Premises*
  - *Conclusion:* the final proposition
  - *Argument form*: a sequence of compound propositions involving propositional variables

Rules of Inference fro Propositional Logic

- Rules of inference
  - *Modus ponens* (law of detachment)
    - $(p \wedge (p \rightarrow q)) \rightarrow q$
  - Ex.1

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$[(p) \land (q)] \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution

- Ex.6:
  - “It it not sunny this afternoon and it is colder than yesterday”
  - “We will go swimming only if it is sunny”
  - “If we do not go swimming, then we will take a canoe trip”
  - “If we take a canoe trip, then we will be home by sunset”
  - Conclusion: “We will be home by sunset”

Fallacies

- $((p \to q) \wedge q) \to p$ is not a tautology
  - *Fallacy of affirming the conclusion*
  - Ex.10:
    - *"If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics.*
      Therefore, you did every problem in this book."
- $((p \to q) \wedge \neg p) \to \neg q$ is not a tautology
  - *Fallacy of denying the hypothesis*
  - Ex.11

Rules of Inference for Quantified Statements

- Universal instantiation
  - $\forall x\, P(x),\ .\because P(c)$
- Universal generalization
  - $P(c)$ *for any* $c,\ .\because \forall x\, P(x)$
- Existential instantiation
  - $\exists x\, P(x),\ .\because P(c)$ *for some element* c
- Existential generalization
  - $P(c)$ *for some element* $c,\ .\because \exists x\, P(x)$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\dfrac{\forall x\, P(x)}{\therefore\; P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\; \forall x\, P(x)}$	Universal generalization
$\dfrac{\exists x\, P(x)}{\therefore\; P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\; \exists x\, P(x)}$	Existential generalization

- Combining rules of inference for propositions and quantified statements
  - Universal modus ponens
    - $\forall x\ (P(x) \rightarrow Q(x))$
      $P(a)$, *where a is a particular element in the domain*
      $\therefore Q(a)$
  - Universal modus tollens
    - $\forall x\ (P(x) \rightarrow Q(x))$
      $\neg Q(a)$, *where a is a particular element in the domain*
      $\therefore \neg P(a)$

1.6 Introduction to Proofs

- Some terminology
  - *Theorem*: a statement that can be shown to be true
  - *Axioms*: statements assumed to be true
    - A proof is a valid argument that establishes the truth of a theorem
      - Including axioms, premises of the theorem, and previously proven theorems
  - *Lemma*: less important theorems that is helpful in the proof
  - *Corollary*: a theorem that can be directly established from a theorem that has been proved
  - *Conjecture*: a statement that is being proposed to be true

Direct Proofs

- $p \rightarrow q$
  - Assume that p is true
  - Showing that q must also be true
- Definition 1: The integer n is *even* if there exists an integer k such that $n=2k$, and n is *odd* if there exists an integer k such that $n=2k+1$.
- Ex.1: Prove that "if n is an odd integer, then n^2 is odd."

Proof by Contraposition

- $p \to q$
  - $\neg q \to \neg p$
    - Take $\neg q$ as a hypothesis
    - Then show that $\neg p$ must follow
- Ex.3: prove that if n is an integer and $3n+2$ is odd, then n is odd.
- *Vacuous proof*
  - If we can show that p is false, then we have a vacuous proof of $p \to q$ is true
- *Trivial proof*
  - If we know that the conclusion q is true, $p \to q$ must also be true

A little proof strategy

- First, evaluate whether a direct proof looks promising
- Otherwise, try the same thing with a proof by contraposition

- Definition 2: The real number r is *rational* is there exist integers p and q with $q \neq 0$ such that $r = p/q$. A real number that is not rational is called *irrational*.
- Ex.7: Prove that the sum of two rational numbers is rational.
- Ex.8: Prove that if n is an integer and n^2 is odd, then n is odd.

Proof by Contradiction

- If we can show that $\neg p \rightarrow (r \wedge \neg r)$ is true for some proposition r, we can prove that p is true
- Ex.9: show that at least four of any 22 days must fall on the same day of the week.
- Ex.10: prove that $\sqrt{2}$ is irrational.

- Proof of equivalence
  - To prove that $p \leftrightarrow q$, we show that $p \to q$ and $q \to p$ are both true
- Counterexamples
  - To show that $\forall x\, P(x)$ is false, we need only find a counterexample
- Mistakes in proofs
  - *Fallacy of begging the question*
    - Circular reasoning

1.7 Proof Methods and Strategy

- Proof by cases
  - $((p_1 \vee p_2 \vee \ldots \vee p_n) \rightarrow q) \leftrightarrow (p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \ldots \wedge (p_n \rightarrow q)$
- Exhaustive proof
  - A special type of proof by cases that exhaust all possibilities (if there are relatively small number of examples)

- Without loss of generality (WLOG)
  - By proving one case of a theorem, no additional argument is required to prove other specified cases

Existence Proofs

- $\exists x\ P(x)$
  - Constructive: find an element a such that $P(a)$ is true
  - Nonconstructive
    - E.g.: proof by contradiction

FIGURE 1 (1.7)

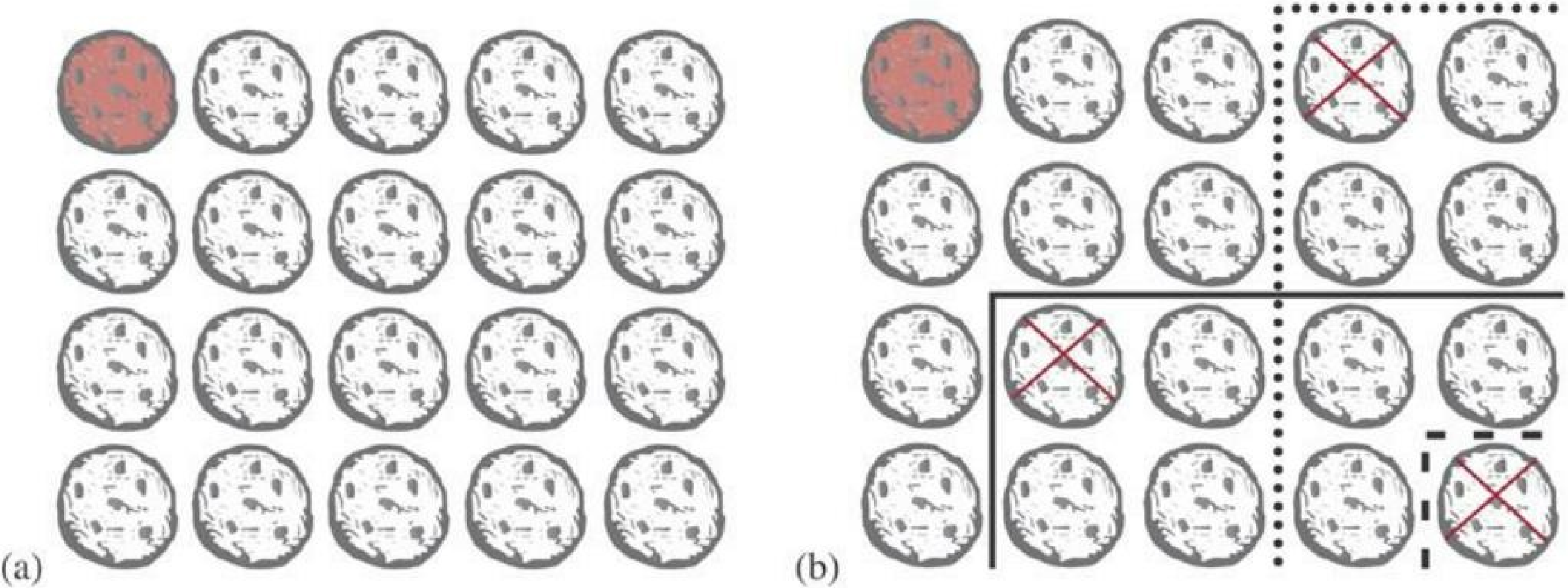

FIGURE 1 (a) Chomp, the Top Left Cookie is Poison (b) Three Possible Moves.

Uniqueness Proofs

- Existence: we show that an element x with the desired property exists
- Uniqueness: we show that if $y \neq x$, then y does not have the desired property

Proof strategies

- Forward and backward reasoning
- Adapting existing proofs
- Looking for counterexamples

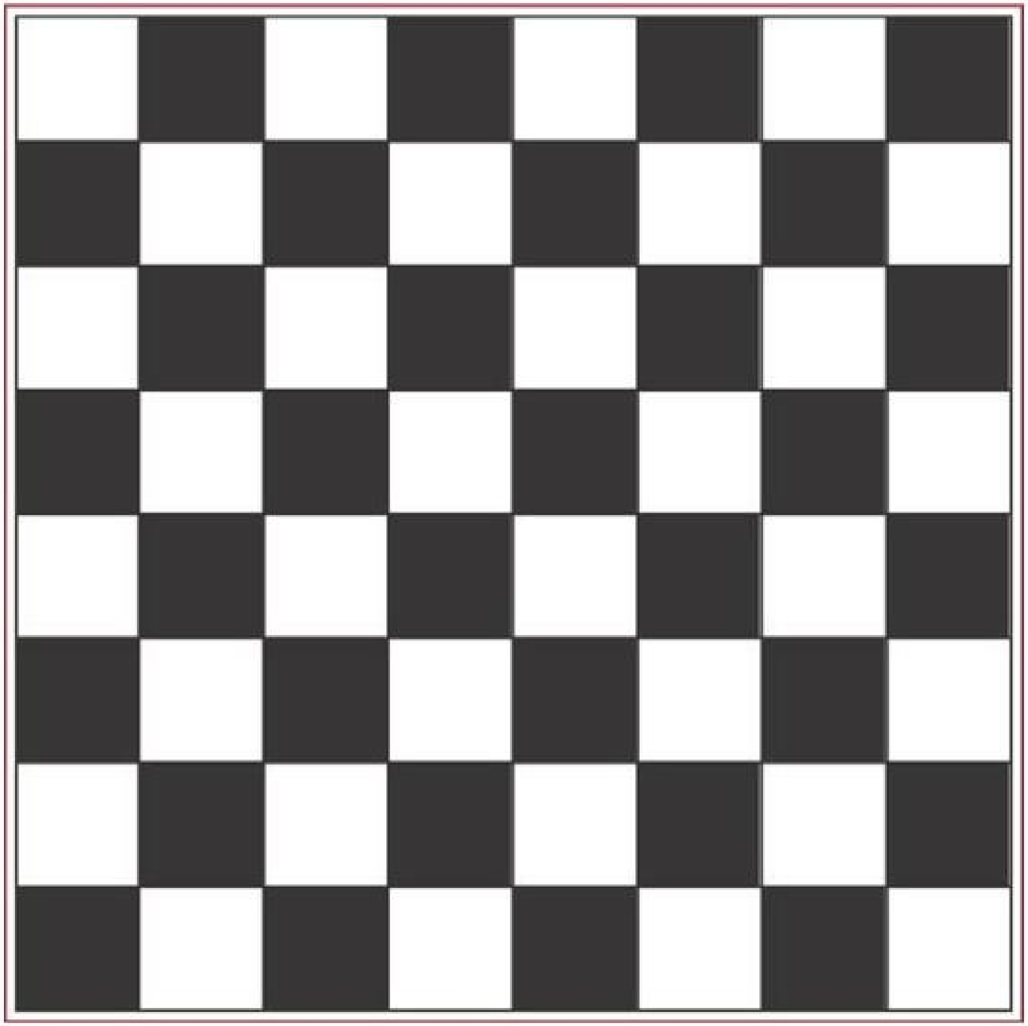

FIGURE 2 The Standard Checkerboard.

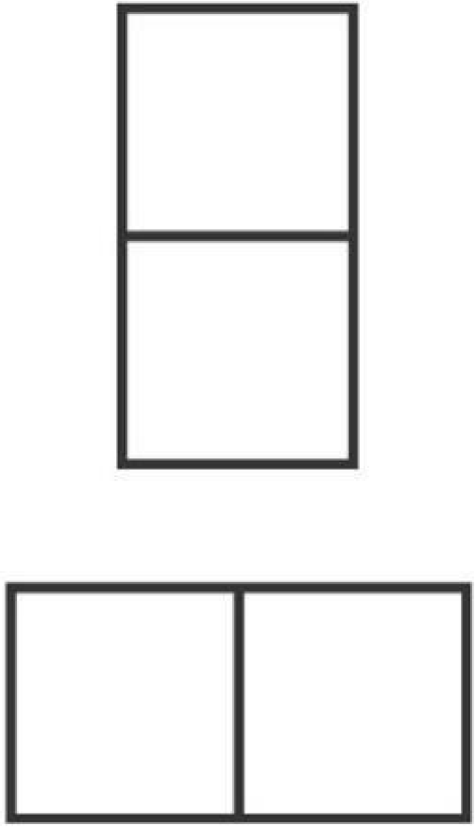

FIGURE 3 Two Dominoes.

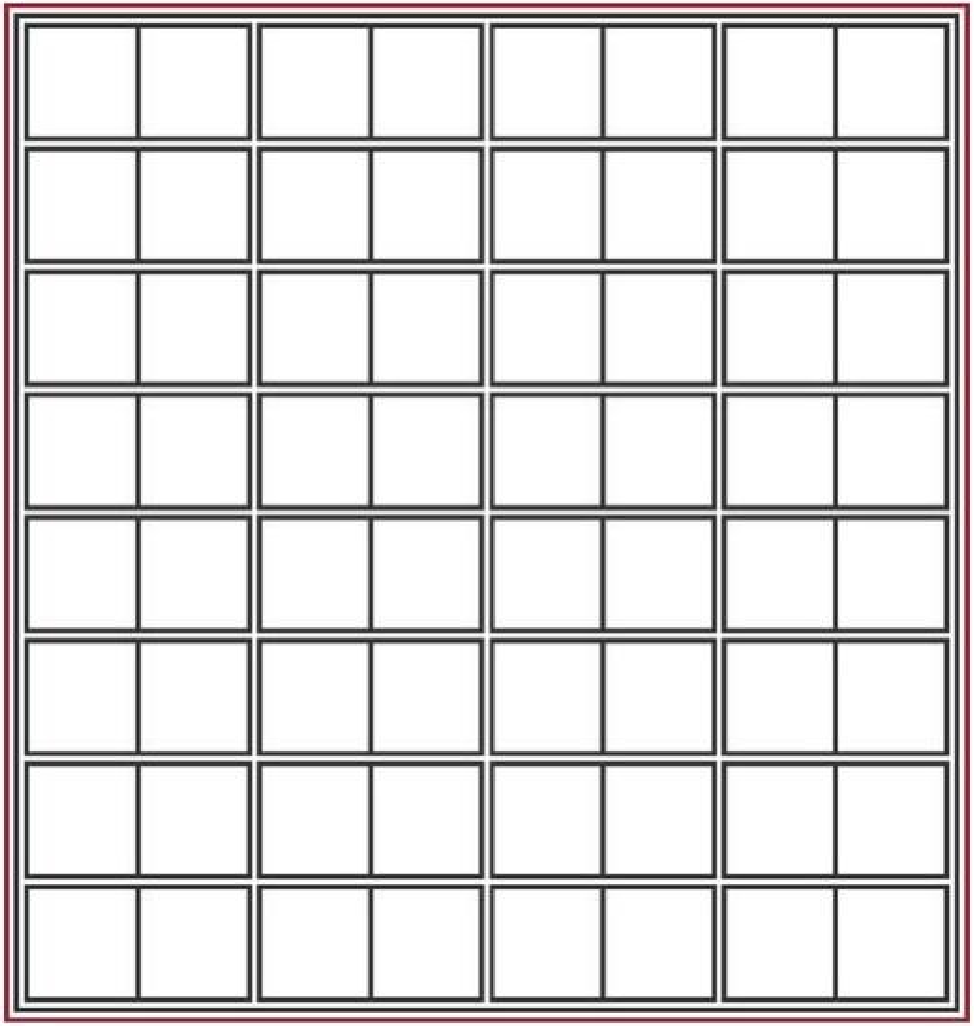

FIGURE 4 Tiling the Standard Checkerboard.

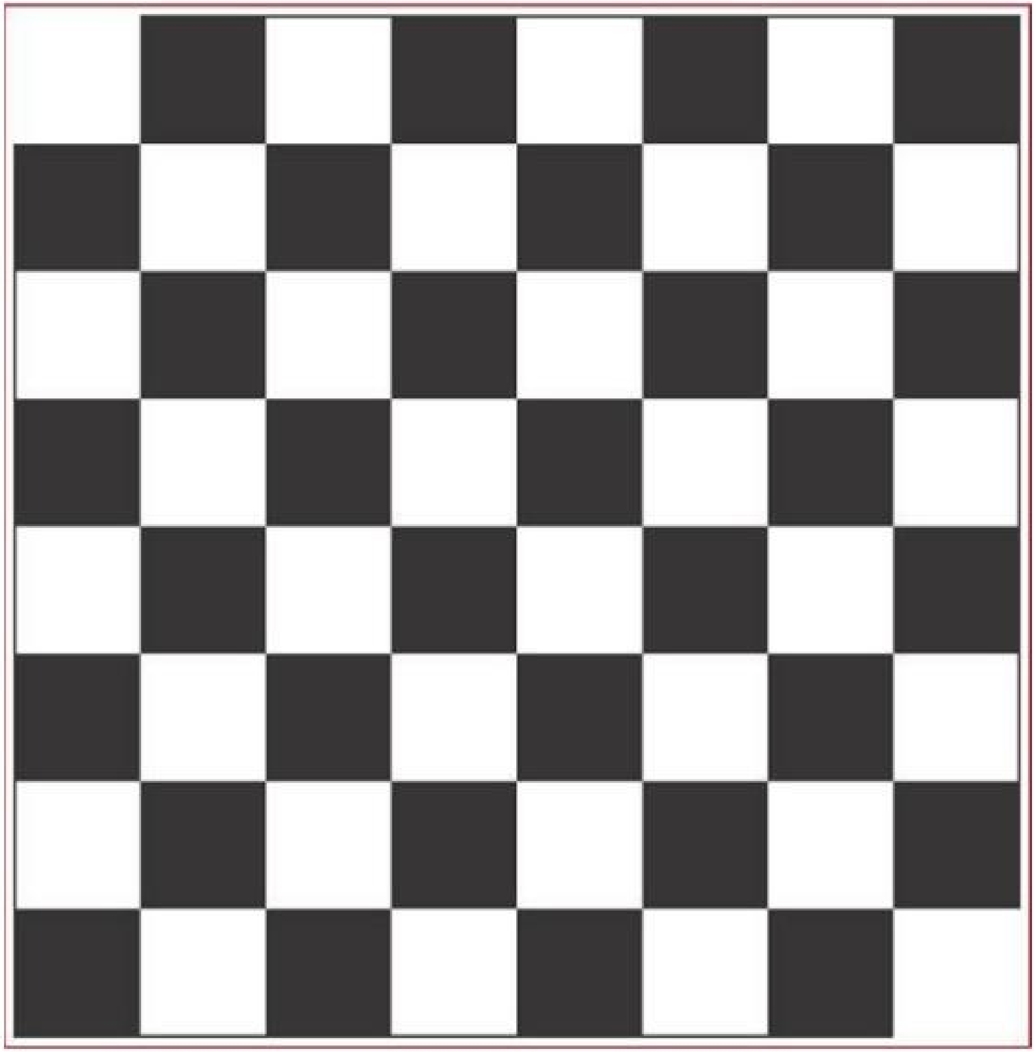

FIGURE 5 The Standard Checkerboard with the Upper Left and Lower Right Corners Removed.

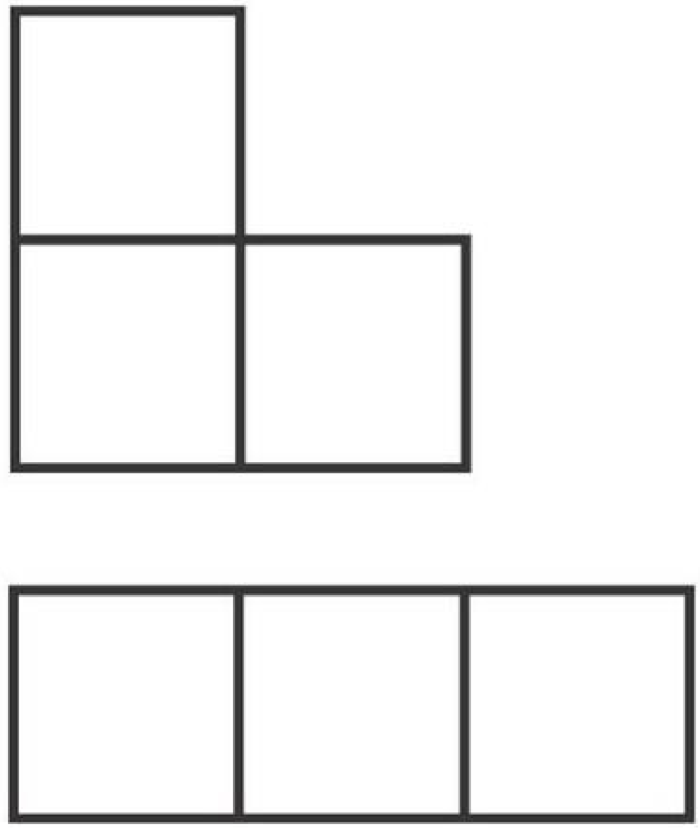

FIGURE 6 A Right Triomino and a Straight Triomino.

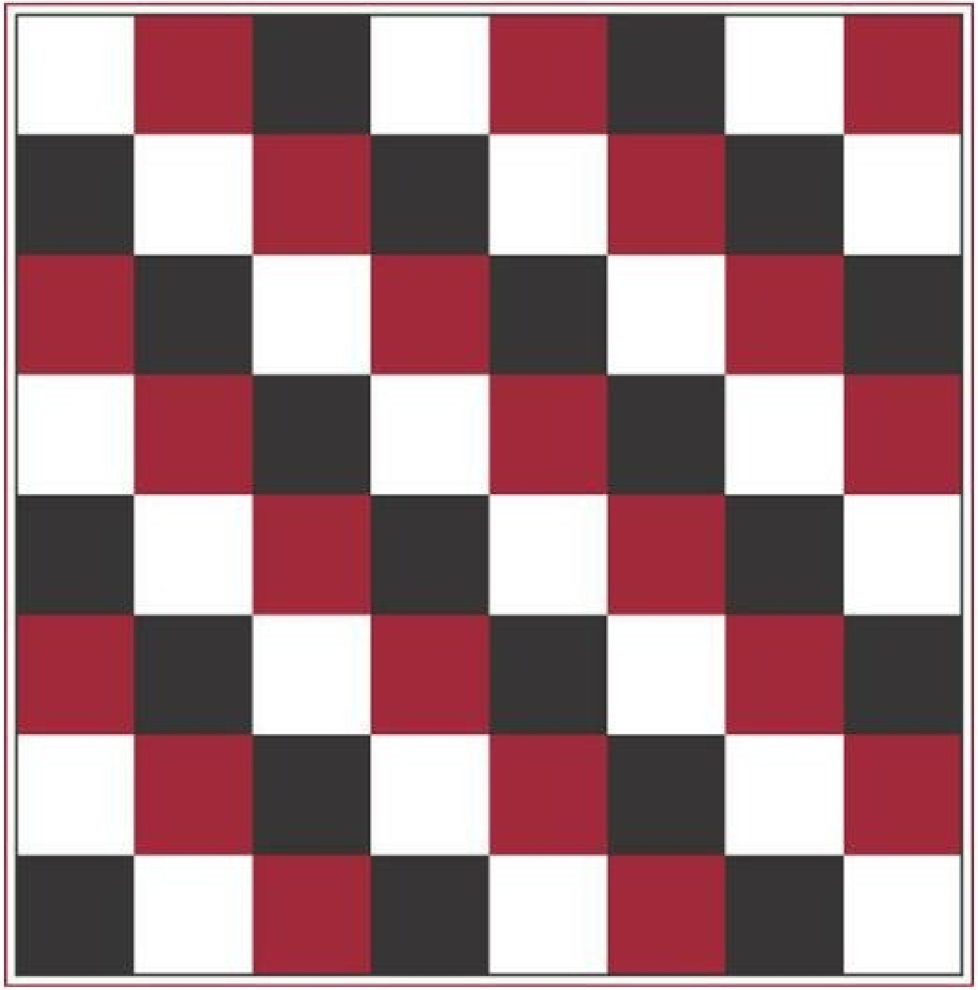

FIGURE 7 Coloring the Squares of the Standard Checkerboard with Opposite Corners Removed with Three Colors.

Thanks for Your Attention!