# Final Exam

Age of the Universe using Hubble Ultra Deep field Images

**Mohammad Mahmoud Ibrahim** **202200438**

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# Contents

# 1 Introduction

The Hubble Ultra Deep Field (HUDF) is an image of a small region of space in the constellation Fornax, taken by the Hubble Space Telescope. It was the deepest image of the universe, prior to James Webb Space Telescope (JWST) revealing around 10 thousand galaxies at various stages of evolution. We will utilize the HUDF to gather redshift and distance data of 23 galaxies to then repeat Hubble's procedure and look for a linear relationship between redshift and distance to estimate Hubble's constant, $H_0$. We then extrapolate from this relationship to estimate the age of the universe.

# 2 HUDF Data

We will use the "Appreciating Hubble at Hyper-Speed" software, copyrighted by Matt Mechtely and Roger Windhorst, to gather data from the HUDF. The following table contains the redshift and distance data in Mpc of 25 galaxies from the HUDF, along with their ID.

| # | Galaxy ID | Distance (Mpc) | Redshift (z) |
|---|---|---|---|
| 1 | 660985 | 594.83 | 0.15 |
| 2 | 731 | 632.90 | 0.16 |
| 3 | 9455 | 670.98 | 0.17 |
| 4 | 3088 | 747.13 | 0.19 |
| 5 | 3822 | 785.21 | 0.20 |
| 6 | 3484 | 859.72 | 0.22 |
| 7 | 3268 | 869.97 | 0.23 |
| 8 | 5620 | 1224.74 | 0.32 |
| 9 | 663290 | 1299.45 | 0.34 |
| 10 | 6188 | 1331.47 | 0.35 |
| 11 | 355 | 1470.40 | 0.39 |
| 12 | 423 | 1606.79 | 0.43 |
| 13 | 4929 | 1674.57 | 0.45 |
| 14 | 8049 | 1707.61 | 0.46 |
| 15 | 3492 | 1740.66 | 0.47 |
| 16 | 8257 | 1839.79 | 0.50 |
| 17 | 7556 | 1872.00 | 0.51 |
| 18 | 2753 | 2032.23 | 0.56 |
| 19 | 9253 | 2218.92 | 0.62 |
| 20 | 8275 | 2249.49 | 0.63 |
| 21 | 2322 | 2280.07 | 0.64 |
| 22 | 4587 | 2370.19 | 0.67 |
| 23 | 4527 | 2517.49 | 0.72 |

Table 1: Redshift and Distance Data of 25 Galaxies from HUDF

Furthermore, a plot was made of velocity in km/s, (calculated using v=zc, where c is the speed of light) against distance in Mpc on OriginLab. The plot is shown in Figure 1.

A linear fit was made to the data, and the residuals of the regression were plotted. The resulting plot is shown in Figure 2.

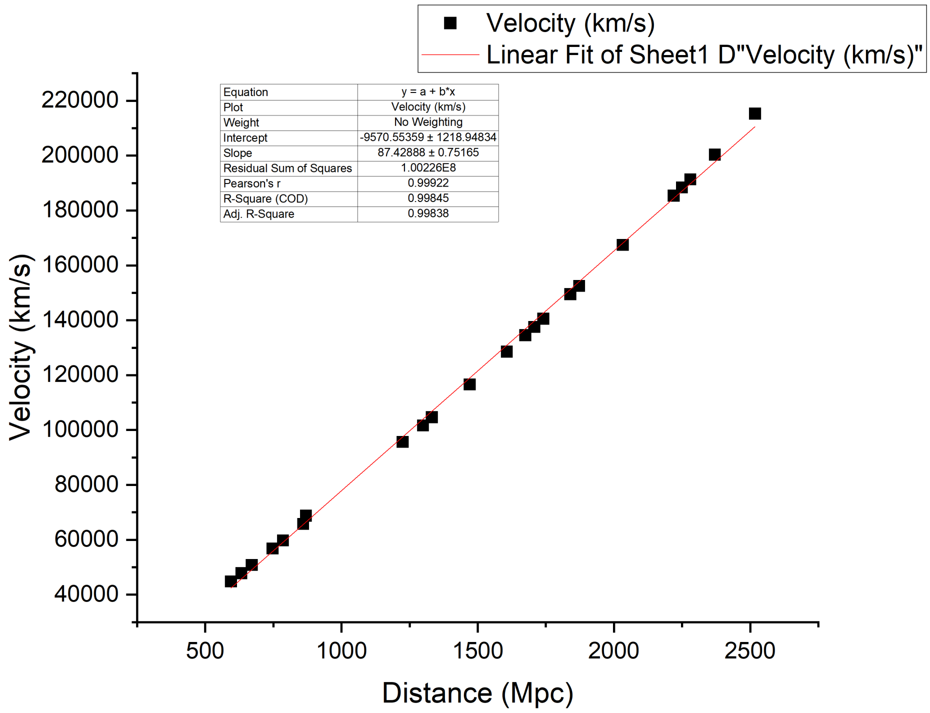


Figure 1: Hubble's Law Plot of Velocity vs Distance

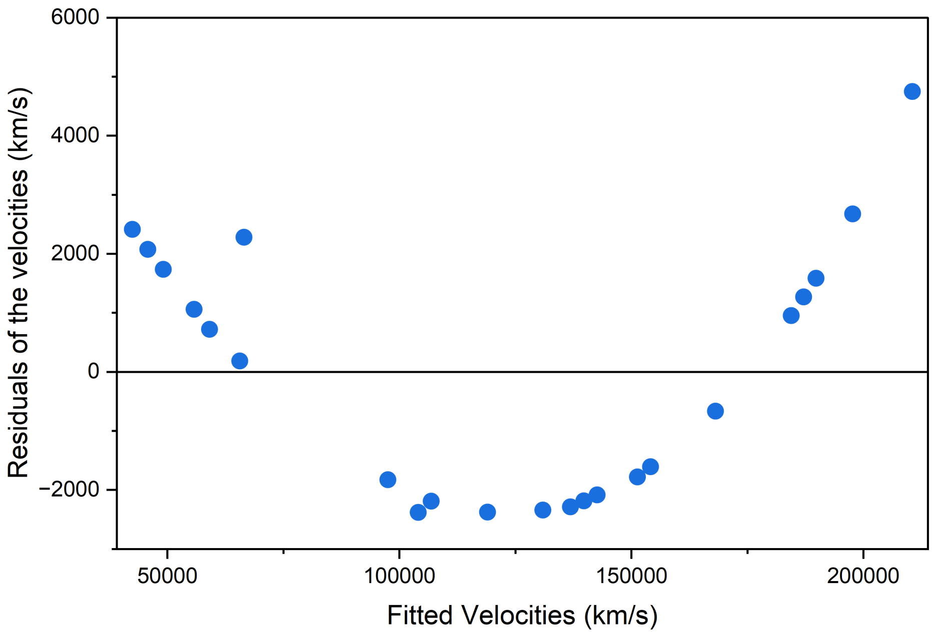


Figure 2: Residuals of the Linear Fit

## 3 Report Questions

Q(I) Evaluate the goodness of the fit and provide commentary on it. Additionally, provide an explanation of the resulting residual plot.
Answer: A linear fit was made to the data, with a goodness of fit of $R^2 = 0.998$. Which is very significant, indicating a strong linear relationship between velocity and distance. This gives good validity for Hubble's law. The residuals plot is very uniform, showing an almost parabolic distribution around zero, indicating that the linear model may only be a good approimation for an undelying non-linear relationship.

Q(II) Hubble discovered such a correlation and formulated the well known hubble's law ($v = H_0 d$), compute your estimated value for the Hubble constant specifying the units used in the calculation.
Answer: The slope of the linear fit is $H_0 = 87.4 \pm 0.8$ km/s/Mpc, which is our approximation of the Hubble constant using the given data set.

Q(III) Discuss the movement of galaxies as follows:

a. On your graph, as you move to larger and larger distances what happens to the velocity of the galaxies?
Answer: The velocity increases almost linearly with distance, which is consistent with Hubble's law. However, as I initially accidentally collected data for galaxies till z¿3 and plotted them, I noticed that the redshift (saying velocities seems incorrect as z¿1 implies v¿c) increases exponentially with distance for galaxies with z¿1, which is not consistent with Hubble's law.

b. Are these galaxies moving away from us or towards us?
Answer: A positive redshift indicates that the galaxies are moving away from us.

c. Do you always see galaxies moving this way, regardless of direction? That is are all galaxies moving in the same way, regardless of which direction in the sky we look?
Answer: On average, yes. If we calculate the average velocities of the galaxies in any solid angle of the sky, we will find that they are all moving away from us with the nearly linear relationship of Hubble. However, since the galaxies themselves have their own dynamics independent of the expansion of the universe, we may find some galaxies moving towards us such as Andromeda, which is on a collision course with the Milky Way.

Q(IV) There are two reasons why we might see the results discovered by Hubble: (1) we are the center of the universe, and all galaxies move from this point in space or (2) space itself is expanding, carrying the galaxies with it. Science has shown that the Earth is not the center of the solar system, and similarly, it seems unlikely that our galacy is the center of the universe. Explain how the expansion of space itself could explain the movement of the galaxies.
Answer: If space itself is expanding, it means the distance between any two points in space is increasing over time, which would mean that the distance between the galaxies is seen to be increasing over time, which would be observed as the galaxies moving away from us. As if a balloon is being inflated, and the points on the surface are getting farther apart. This also makes sense with farther points moving away faster, as there is more space between them and hence more expansion.

Q(V) If this is purely new to you, from a scientific point of view, what can we infer knowing that the space is expanding? What questions can this interesting result lead to?
Answer: If space is expanding, if we extrapolate this back in time, we can infer that the universe used to be all contained in a singularity, which is the Big Bang hypothesis. Other questions to think of would be if the expansion is accelerating or decelerating, and we could figure this out by looking at the redshift of much older galaxies.

Q(VI) If the accepted value of Hubble's constant is $H_0 = 73.8$ km/s/Mpc, what is the percent error in your measurement?

Answer: The percent error is:

$$\text{Percent Error} = \left|\frac{H_0 - H_{0,accepted}}{H_{0,accepted}}\right| \times 100\% = \left|\frac{87.4 - 73.8}{73.8}\right| \times 100\% \approx 18.4\%$$

Which is quite large, however we will discuss the possible sources of error at the end.

Q(VII) If the universe has been expanding at the Hubble rate since its beginning we can caluclate the age of the universe

a. Notice the units of the Hubble constant. Both km and Mpc are units of distance. With a little work, we see that the Hubble's constant is a rate witrh units 1/s. Convert Mpc to km, cancel out these distance units, and put the constant in units of 1/s units.
Answer: 1 Mpc = $3.08 \times 10^{19}$ km, so:

$$H_0 = 87.4 \text{ km/s/Mpc} = 87.4 \text{ km/s} \times \frac{1}{3.08 \times 10^{19} \text{ km}} = 2.84 \times 10^{-18} \text{ s}^{-1}$$

b. Multiply your answer by the number of seconds in one year ($3.15 \times 10^7$s/yr) and invert your answer to get the age of the universe in years
Answer: The age of the universe is:

$$t = \frac{1}{H_0} = \frac{1}{2.84 \times 10^{-18} \text{ s}^{-1}} \approx 3.52 \times 10^{17} \text{ s} \approx 1.12 \times 10^{10} \text{ years} \approx 11.2 \text{ billion years}$$

Q(VIII) Explain the possible sources of error in the measurment of the Hubble constant.
Answer: There are a lot of possible sources of error:

- The biggest source of error might be the use of galaxies with high redshift, as has been apparent throughout the report, as z increases, Hubble's law begins to break down.
- There may be errors in the redshift and distance measurements of the galaxies.
- The galaxies themselves have their own dynamics, which we have completely ignored, and may cause them to move towards or away from us, which would skew the results.
- The number of galaxies used is quite small, and the sample may not be representative of the universe as a whole.

## 4 Conclusion

In this report, we have used the Hubble Ultra Deep Field images to gather data on the redshift and distances of 23 galaxies. We then plotted the velocity against distance and found a strong linear relationship, consistent with Hubble's law. We estimated the Hubble constant to be $H_0 = 87.4 \pm 0.8$ km/s/Mpc, which gives us an age of the universe of approximately 11.2 billion years. However, we also discussed the possible sources of error in our measurement, including the use of high redshift galaxies and the dynamics of galaxies themselves.