Northeastern University, Khoury College of Computer Science

CS 6220 Data Mining | Assignment 3

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Question 2

The likelihood function:

$$L(\lambda) = \prod_{i=1}^{n} P(x_1|\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$= \frac{\prod_{i=1}^{n} e^{-\lambda} \quad \prod_{i=1}^{n} \lambda^{x_i}}{\prod_{i=1}^{n} x_i!}$$

$$= \frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}$$

The log-likelihood function:

$$\ell(\lambda) = \log L(\lambda) = \log\left(\frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}\right)$$

$$= \log(e^{-n\lambda}) + \log(\lambda^{\sum_{i=1}^{n} x_i}) - \log\left(\prod_{i=1}^{n} x_i!\right)$$

$$= -n\lambda + \sum_{i=1}^{n} x_i \log\lambda - \log\left(\prod_{i=1}^{n} x_i!\right)$$

The derivatives with respect to λ:

$$\frac{\partial \ell(\lambda)}{\partial \lambda} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i$$

$$\frac{\partial^2 \ell(\lambda)}{\partial^2 \lambda} = -\frac{1}{\lambda^2}\sum_{i=1}^{n} x_i$$

$$\left.\frac{\partial \ell(\lambda)}{\partial \lambda}\right|_{\lambda=\hat{\lambda}} = 0 \iff -n + \frac{1}{\hat{\lambda}}\sum_{i=1}^{n} x_i = 0$$

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Because: $\frac{\partial^2 \ell(\lambda)}{\partial^2 \lambda} = -\frac{1}{\lambda^2}\sum_{i=1}^{n} x_i < 0 \quad \forall \lambda \in (0, \infty)$

$L(\lambda)$ is maximised at $\lambda = \hat{\lambda}$

The MLE is $\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i$.