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(see week 2)

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But if x is image, why not

$$\mu(t) = \mathrm{CNN}_1(t)$$

$$\Sigma(t) = \mathrm{CNN}_2(t)$$

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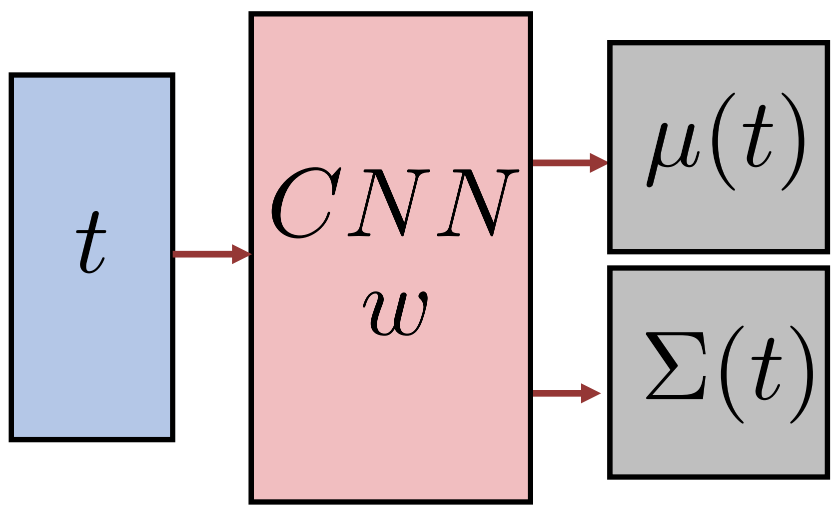

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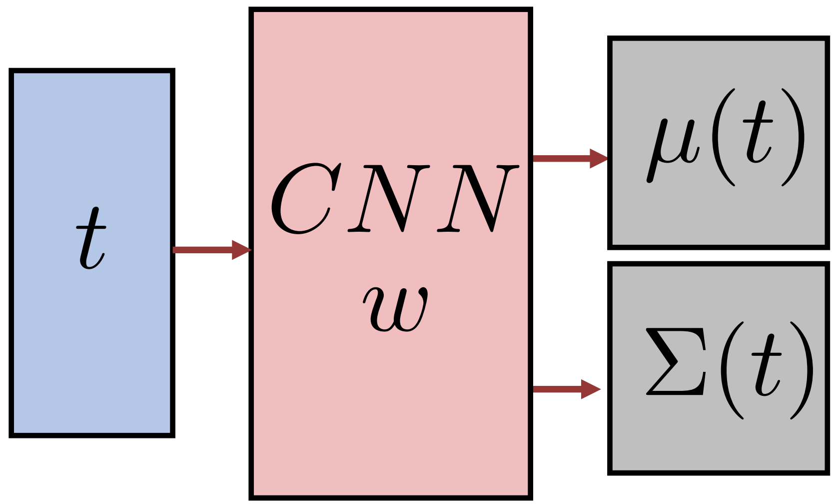

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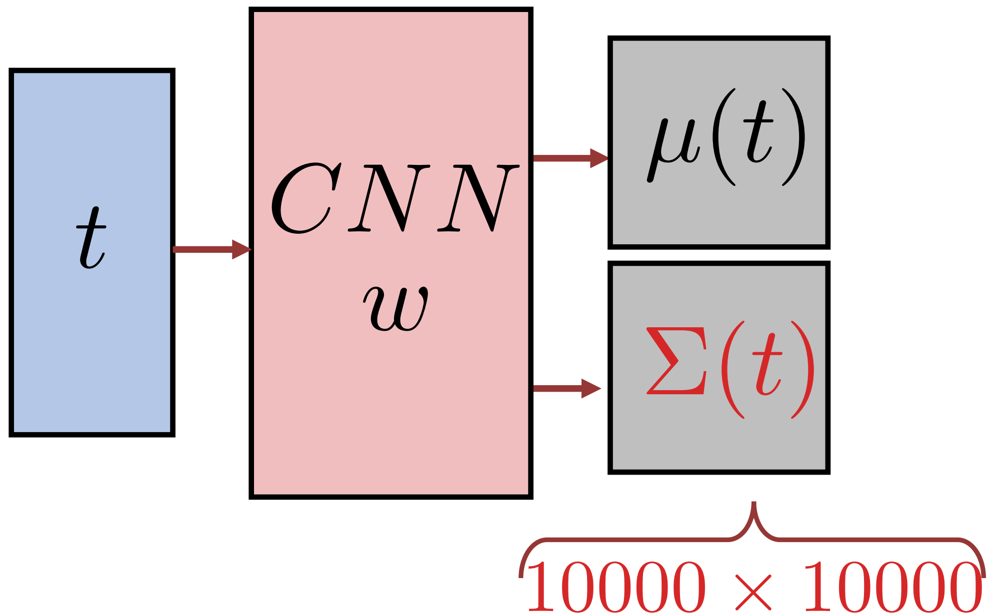

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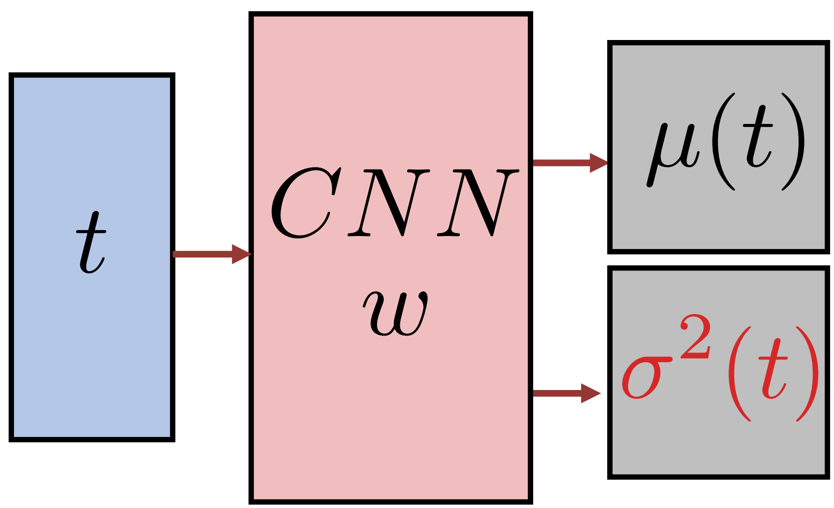

Scaling up Expectation Maximization

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Latent Variable model — use EM! But E-step is intractable

Need to compute $p(T \mid X, w)$

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$$T_s \sim q(T)$$

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