Metropolis-Hastings

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Apply rejection sampling to Markov Chains

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For k = 1, 2, …

- Sample x' from a wrong $Q(x^k \to x')$

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How to choose A: $\pi(x') = \sum_x \pi(x) T(x \to x')$

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$$= \pi(x') \underbrace{\sum_x T(x' \to x)}_{1}$$

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