Aircraft dynamic model identification on the basis of flight data recorder registers

Maciej Lasek, Piotr Lichota

The Institute of Aeronautics and Applied Mechanics, Warsaw University of Technology, Warsaw, Poland,
e-mail: plichota@meil.pw.edu.pl

We investigate the problem of an aircraft dynamic model parametric identification using dimensional derivatives as an example. Identification is done in offline mode, in the time domain. Flight parameters used for identification are obtained from Flight Data Recorder, that register them during each scheduled flight.
We investigate the possibility of application of Maximum Likelihood Estimation that belongs to the Output Error Methods class. The likelihood function is defined for n-dimensional multivariate normal distribution. Unknown covariance matrix is estimated with the use of measured data and output equation. Output equation is calculated with Runge–Kutta fourth order method. In order to find the cost function minimum we consider using Levenberg-Marquardt Algorithm, where derivatives are calculated with central difference formulas and small perturbations theory.
Mathematical model of an aircraft is obtained through flight dynamics classical approach. Rigid body model of an aircraft is assumed. Coordinate Systems Transformations are done using Euler's Rotation Theorem with angle order typical for flight dynamics. Equations of motion are obtained from Newtons Second Law of Motion in body fixed coordinate system Oxyz, that is located at aircraft's center of gravity. Turbulence is modeled as a bias, and also is an object of identification.
We implement this method in Matlab R2009b environment.

Key words: Dimensional derivatives; Estimation; Flight Data Recorder; Flight Dynamics; Maximum Likelihood Estimation; Levenberg-Marquardt; System Identification

Introduction

In 1911 George Hartley Bryan published „Stability in Aviation", that is actual to this day. Equations and methodology described in his book are used to evaluate modern aircraft, however in some cases something more is needed.

It is not always possible to perform stability test, and even if, it is often associated with high costs. Despite that, obtained results are adequate for particular object or rather object and its condition (that can change in time). Stability tests of airliners are a good example of presented problems – they generate high costs and can not be done e.g. during scheduled flight (due to regulations), what would lower the expense. Overcoming this problem can be done by combining classical approach and system identification.

The idea is simple – to use the Flight Data Recorder (FDR) data to identify particular aircraft aerodynamic derivatives in its condition, that changes due to e.g. maintenance. The benefits of such an approach can be tremendous. FDR usage during each scheduled flight is mandatory – it do not generate additional costs, in opposite to sensors. Moreover, additional sensors montage would cause the need of certification.

In the case of identification on the basis of FDR recordings aircraft condition can be better monitored, that allows to change maintenance programs and lower airline companies expenses. Flight simulators certification would be cheaper, because the need of performing additional flights would disappear.

However, the most important profit would be gain in safety level – obtained data could be used by aircraft accidents investigation committees as an additional information, that describes system dynamics. Therefore, if is worth to pay special attention to combine flight dynamics classical approach and system identification.

Summarizing, it is great interest to determinate undemensional control and stability derivatives of an aircrft, using data obtained from flight data recorders due to possibilities of lowering cost of airfleet operators, simulators' producents and highering the safety level in civil aviation.

Method

Output Error Methods (OEM) are often used for dynamic systems parametric identification. They minimize the difference between measurements and identified model response. The most common method, that belongs to OEM is Maximum Likelihood Estimation (MLE), that minimizes negative log-likelihood function L for measurements z when identified parameters Θ.

$$\Theta = \arg\{\min_{\Theta} L(z|\Theta)\} \tag{1}$$

The log-likelihood function is obtained from n-dimensional Multivariate Normal Distribution probability density function p:

$$p(\mathbf{z}(t_k)|\Theta) = ((2\pi)^n |\mathbf{R}|)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}[\mathbf{z}(t_k)-\mathbf{y}(t_k)]^T \mathbf{R}^{-1}[\mathbf{z}(t_k)-\mathbf{y}(t_k)]\right) \tag{2}$$

where:

- $y(t_k)$ – output vector
- $z(t_k)$ –measurements vector
- $R(t_k)$ –covariance matrix.

Output equation can be written as a vector-valued-function:

$$\mathbf{y} = g(\mathbf{x}(t), \mathbf{u}(t), \Theta) \tag{3}$$

Measurement equation in particular time point t_k is a sum of two components, i.e. output vector in this time point and measurement noise vector $v(t_k)$.

$$\mathbf{z}(t_k) = \mathbf{y}(t_k) + \mathbf{v}(t_k) \tag{4}$$

Estimating covariance matrix as:

$$\mathbf{R} = -\frac{1}{N}[\mathbf{z}(t_k)-\mathbf{y}(t_k)][\mathbf{z}(t_k)-\mathbf{y}(t_k)]^T \tag{5}$$

and variable independence leads to the following form of negative log-likelihood function:

$$\mathbf{R} = -\frac{1}{N}[\mathbf{z}(t_k)-\mathbf{y}(t_k)][\mathbf{z}(t_k)-\mathbf{y}(t_k)]^T \tag{6}$$

Thus, due to constant values of some terms cost function J can be defined as:

$$J(\Theta) = |\mathbf{R}| \tag{7}$$

The best way to evaluate cost function minimum is to use Levenberg-Marquartd Algorithm (LMA), that interpolates between Gauss-Newton Algorithm (GNA), Steepest Descent Method (SD) and combine their best features: speed (GNA) and robust (SD).

In LMA parameter-update formula in i-th iteration is based on parameter values in previous iteration, cost function gradient vector $\nabla_{\Theta} J$ cost function hessian (also called Fisher Information Matrix) $\nabla^2_{\Theta} J$ step-size parameter λ.

$$J(\Theta) = |\mathbf{R}| \tag{8}$$

Symbol I denotes identity matrix.

After neglecting small terms in gradient vector and Fisher Information Matrix, they are defined as follows:

$$\nabla_{\Theta} J = -\sum_{k=1}^{N} \left[\frac{\partial \mathbf{y}(t_k)}{\partial \Theta}\right]^T \mathbf{R}^{-1}[\mathbf{z}(t_k)-\mathbf{y}(t_k)]$$
$$\nabla^2_{\Theta} J = -\sum_{k=1}^{N} \left[\frac{\partial \mathbf{y}(t_k)}{\partial \Theta}\right]^T \mathbf{R}^{-1}\frac{\partial \mathbf{y}(t_k)}{\partial \Theta} \tag{9}$$

Step-size parameter is used for determining the impact of GNA and SD on LMA in each iteration. If the cost function value that was calculated for step-size parameter divided by reduction factor is smaller than cost function from previous iteration, GNA impact is increased (reducing step-size parameter). If not, but if the cost function value that was calculated for step-size parameter is smaller than cost function from previous iteration step-size parameter is remained unchanged. If neither of both conditions are satisfied SD is dominant (increasing step-size parameter).

Flight dynamics and system identification are related trough the output equation which is derived in Oxyz coordinate system from Newton's second law of motion.

Oxyz is a right handed and orthogonal body-fixed coordinate system. Its' origin O can be located at center of gravity. Ox-axis is parallel to mean aerodynamic chord, lies in the plane of symmetry and is directed forward. Oy-axis is directed towards right wing whereas Oz-axis complements the coordinate system.

Momentum Π and angular momentum K_0 that are used for obtaining output can be written as:

$$\frac{\delta \Pi}{dt} + \Omega \times \Pi = \mathbf{F}$$
$$\frac{\tilde{\delta} \mathbf{K}_0}{dt} + \Omega \times \mathbf{K}_0 = \mathbf{M}_0 \tag{10}$$

In the above equations force vector F components are longitudinal force X, lateral force Y and vertical force Z, whereas moment vector L components are rolling moment M, pitching moment N and yawing moment. Symbol δ denotes local derivative and Ω stands for angular velocity.

Obtained equation must be completed by applying kinematic equations, that link Euler angles with angular velocity:

$$P = \dot{\Phi} - \dot{\Psi}\sin\Theta$$
$$Q = \dot{\Theta}\cos\Phi + \dot{\Psi}\cos\Phi\cos\Theta$$
$$R = -\dot{\Theta}\sin\Phi + \dot{\Psi}\cos\Phi\cos\Theta \tag{11}$$

Those equations are obtained through Euler's rotations theorem. Transformations between different coordinate systems are done in following order: yaw angle, pitch angle, roll angle. The usage of Euler's transformation matrices is inseparable from rotations order (whereas in quaternions algebra it does not matter), but for small angles obtained error can be neglected.

The next step in obtaining output equation is the usage of small disturbances theory in which each: velocity component (U,V,W), angular velocity component (P,Q,R) and Euler angle (ϕ,Θ,ψ) is considered as a sum of a value in equilibrium state and a perturbance about mean motion. In this state there is no translational or rotational acceleration. Moreover products and squares of disturbed parameters are negligible, sinus is approximated to the angle value, cosines to unity. Obtained output equation is linear, however it is still complicated. Therefore e.g.

straight, symmetric flight with wing levels can be considered. For stability axis system only longitudinal velocity perturbations exist i.e:

$$
\begin{aligned}
U &= U_0 + \Delta U \\
V &= V_0 \\
W &= W_0 \\
P &= P_0 \\
Q &= Q_0 \\
R &= R_0 \\
\Phi &= \Phi_0 \\
\Theta &= \Theta_0 \\
\Psi &= \Psi_0
\end{aligned}
\tag{12}
$$

Dimensional derivatives can be calculated with respect to each state vector component and are defined as follows (with respect to j-th parameter):

$$
\begin{aligned}
U &= U_0 + \Delta U \\
V &= V_0 \\
W &= W_0 \\
P &= P_0 \\
Q &= Q_0 \\
R &= R_0 \\
\Phi &= \Phi_0 \\
\Theta &= \Theta_0 \\
\Psi &= \Psi_0
\end{aligned}
\tag{13}
$$

where:

I_x, I_y, I_z – moments of inertia around particular axis

m – mass.

In order to simplify equations of motion higher order terms are neglected and additional transformations are performed. Lateral-directional derivatives are now defined as follows:

$$L'_j = L_j + \frac{I_{xz}}{I_{zz}} N_j$$

$$N'_j = \frac{I_{xz}}{I_{xx}} L_j + N_j$$

In a similar manner longitudinal derivatives are modified:

$$
\begin{aligned}
\tilde{M}_U &= M_U + M_{\dot{W}} Z_U \\
\tilde{M}_W &= M_W + M_{\dot{W}} Z_W \\
\tilde{M}_Q &= M_Q + M_{\dot{W}} (Z_Q + U_0)
\end{aligned}
\tag{14}
$$

Input (control) vector u is defined as elevator H, rudder V and ailerons A deflections increments and thrust force Th increment.

Dimensional derivatives of this vector are defined in analogous way as before. Their modified form is obtained with the use of following equations:

$$
\begin{aligned}
\tilde{M}_{\delta H} &= M_{\delta H} + M_{\dot{W}} Z_{\delta H} \\
\tilde{M}_{\delta Th} &= M_{\delta Th} + M_{\dot{W}} Z_{\delta Th}
\end{aligned}
\tag{15}
$$

Due to assumed trim conditions output equation can now be separated in two groups: longitudinal and lateral-directional equations. Therefore both kinds of motion can be analyzed separately in order to obtain parametric identification results faster. Even if system identification is carried out in offline mode (data recorded during flight are analyzed after) it is worth to identify them separately.

Consideration of full set of equations is also possible, but usually not recommended.

Due to OEM assumptions process noise cannot be considered in output equation. However, when dealing with aircrafts it should be considered – because of e.g. turbulence. One way of solving this problem is using another method instead of OEM – e.g. one of Filter Error Methods or Artificial Neural Networks.

Other possible solution for overcoming this problem is to model biases due to velocity and angular velocity components as unknown system parameters and to identify them in the same way as dimensional derivatives. Moreover, velocity and angular velocity components derivatives (with respect to time) also need to be identified. Therefore process noise vector w can be defined as follows:

$$\mathbf{w} = [w_{\dot{U}} \quad w_U \quad w_{\dot{V}} \quad w_V \quad w_{\dot{W}} \quad w_W \quad w_{\dot{P}} \quad w_P \quad \ldots]^T \tag{16}$$

Application of process noise leads to first order homogenous linear differential vector equation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{w} \tag{17}$$

where:

– State matrix

– Input matrix

Those matrices are defined as follows:

$$
\mathbf{A} = \begin{bmatrix}
X_U & 0 & X_W & 0 & X_Q & 0 & -g & 0 \\
0 & Y_V & 0 & Y_P & 0 & Y_R + U_0 & 0 & -g \\
Z_U & 0 & Z_W & 0 & Z_Q + U_0 & 0 & 0 & 0 \\
0 & L'_V & 0 & L'_P & 0 & L'_R & 0 & 0 \\
\tilde{M}_U & 0 & \tilde{M}_W & 0 & \tilde{M}_Q & 0 & 0 & 0 \\
0 & N'_V & 0 & N'_P & 0 & N'_R & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix}
X_{\delta H} & 0 & Z_{\delta H} & 0 & \tilde{M}_{\delta H} & 0 & 0 & 0 \\
0 & Y_{\delta V} & 0 & L'_{\delta V} & 0 & N'_{\delta V} & 0 & 0 \\
0 & Y_{\delta A} & 0 & L'_{\delta A} & 0 & N'_{\delta A} & 0 & 0 \\
X_{\delta Th} & 0 & Z_{\delta Th} & 0 & \tilde{M}_{\delta Th} & 0 & 0 & 0
\end{bmatrix}^T
\tag{18}
$$

Results

Presented results were obtained using a Intel Core i5 at 2.5GHz with 4GB of RAM in Ubuntu 11.04 operating system and Matlab/Simulink R2009b environment. Due to program objectives no additional libraries and toolboxes were used. Derivatives are calculated with central difference formulas, output equation with Runge–Kutta fourth order method.

High cost of identification experiments, valid regulations, lacks in computional fluid dynamics and many others cause that the only acceptional way of validation is based on regression analysis. Averaged goodness of fit of presented results is 0.97, whereas the worst goodness of fit is obtained in lateral-directional chanel: 0.93.

The data used for lateral-directional derivatives parametric identification was obtained from [3] in order to verify correctness of the algorithm. Measurement vector consist of: roll acceleration P, yaw acceleration R, lateral acceleration V, roll velocity P and yaw velocity R. Output

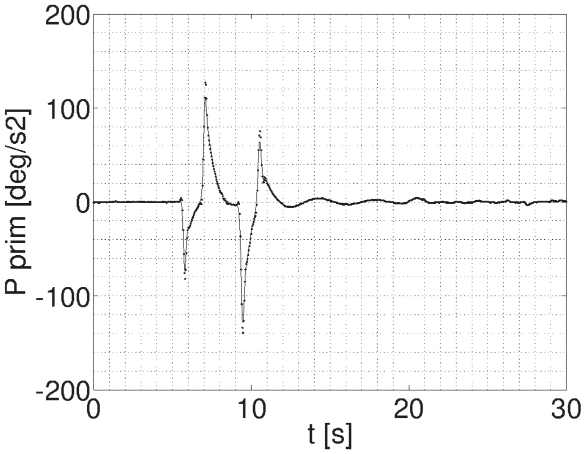

Fig. 1. Roll acceleration.

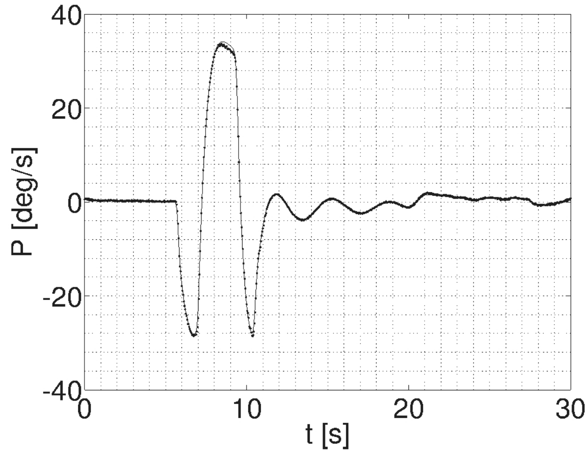

Fig. 4. Roll velocity.

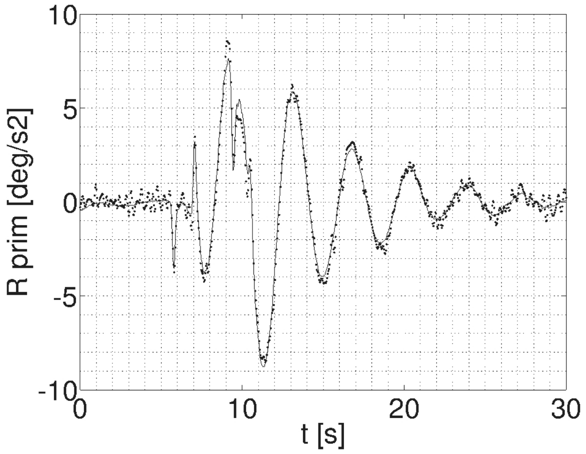

Fig. 2. Yaw acceleration.

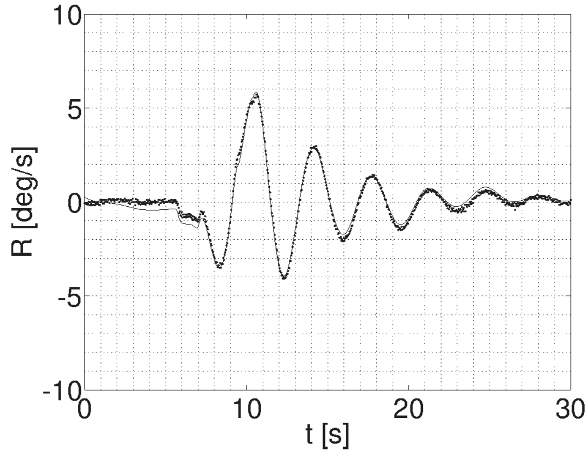

Fig. 5. Yaw velocity.

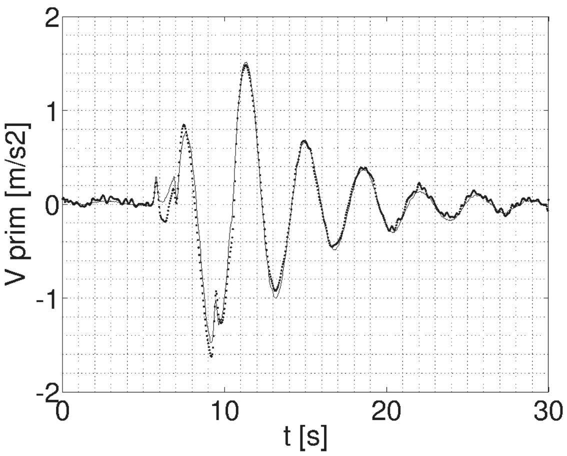

Fig. 3. Lateral acceleration.

vector is composed of the same elenents, however they are calculated on the basis of state equation. In presented plots dots denote measurements. Solid lines stand for identified object.

Conclusions

As it can be seen from presented plots system parameters (dimensional derivatives and biases) were well identified. Therefore MLE with LMA can be used to obtain aircraft dynamics on the basis of FDR. However it is important to remember that identification is much faster and more robust if optimization initial point is well chosen (e.g. inizalization can be done with any of Least Squares Methods).

Despite that time is not a dominant factor it should not be ignored because of the high number of identified parameters and time points that are registered during each scheduled flight. Due to that fact computation time comes even more important than other problems e.g. coordinates transformation order (quaternions algebra as a possible solution for three-dimensional space). For this reason it is necessary to perform additional studies about choosing optimal initial point for LMA algorithm.

References

[1] Etkin B.: Dynamics of Atmospheric Flight, New York, John Wiley & Sons, Inc., 1972

[2] Iliff K. W.: Estimation of Aerodynamic Characteristics from Dynamic Flight Test Data. Stability Parameters, 1978, AGARD-CP-235, Paper 15

[3] Jategaonkar R.: Flight Vehicle System Identification: A Time Domain Methodology, Reston, Progress in Astronautics and Aeronautics, 2006

[4] Raol J.R, Girija G. Singh J.: Modelling and Parameter Estimation of Dynamic Systems, London, The Institution of Electrical Engineers, 2004