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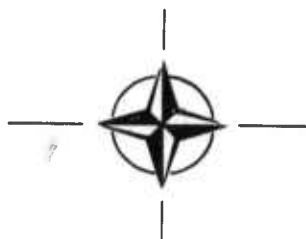
**MEASUREMENT OF AIRCRAFT  
MOMENTS OF INERTIA**

by

N. LEONARD WENER

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NORTH ATLANTIC TREATY ORGANIZATION  
ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

MEASUREMENT OF AIRCRAFT MOMENTS OF INERTIA

by

N. Leonard Wener

This Report was presented at the Fifteenth Meeting of the Flight Test Techniques and Instrumentation Panel, held from 21st to 23rd September, 1959, in Aachen, Germany.

## SUMMARY

In support of a program to extract stability derivatives from flight test data, the moments and product of inertia were measured on an F-106A airplane. The spring oscillation method was used, giving some valid results for pitching and rolling moments of inertia and for inclination of the principal axes. Yawing moments of inertia, as measured, were not satisfactory. Explanation is offered for some difficulties with the results. Also described is the combination of experimental results with analytical methods, used to produce a final set of moments and product of inertia.

The magnitude of such experiments is indicated by estimates of the manpower and time required to conduct them. This, combined with the quality of the results which might be expected, permits recommendations to be made regarding future tests of this type. Finally, a new approach to the experimental problem is described.

## SOMMAIRE

A l'appui d'un programme de travaux visant la détermination des dérivées de stabilité à partir des résultats d'essais en vol, des mesures des moments et des produits d'inertie ont été effectuées sur un avion F-106A. La technique employée a été celle qui fait subir à un avion muni de ressorts hélicoides des oscillations; des résultats valables ont été obtenus en ce qui concerne la mesure des moments en tangage et en roulis, ainsi que l'inclinaison des axes principales. La mesure des moments d'inertie en lacet n'a pas donné de résultats satisfaisants. On cherche à expliquer l'origine de certaines difficultés auxquelles ont donné lieu les résultats obtenus, et décrit comment une combinaison des résultats expérimentaux avec les valeurs théoriques a permis d'établir en définitive les valeurs des moments et du produit d'inertie.

L'importance de tels essais est indiquée par une évaluation de la main d'oeuvre et du temps nécessaire à leur réalisation. Cette considération, avec celle portant sur la qualité des résultats à laquelle on pourrait s'attendre, permet de formuler des recommandations quant à l'exécution à l'avenir d'essais de ce genre.

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# NOTATION

$I_y$	pitching moment of inertia about fulcrum
$C_x, C_y, C_z$	spring constants (lb/ft)
$L_x, L_y, L_z$	moment arms of springs (ft)
$\theta$	angle of rotation of airplane about fulcrum (rad.)
$t$	time
$\omega$	oscillation frequency (rad/sec)
$I_{xx}$	rolling moment of inertia (slug-ft <sup>2</sup> )
$I_{yy}$	pitching moment of inertia (slug-ft <sup>2</sup> )
$I_{zz}$	yawing moment of inertia (slug-ft <sup>2</sup> )
$I_{xz}$	product of inertia
$W$	weight of airplane (lb)
$h_x, h_y$	vertical distances from oscillation axis to c.g., positive upward (ft)
$l_x, l_y$	distances from c.g. to oscillation axis (ft)
$V$	volume of entrapped air (ft <sup>3</sup> )
$\rho$	air density (slugs/ft <sup>3</sup> )
$g$	gravitational acceleration (ft/sec <sup>2</sup> )
$I_{x(\text{add. mass})}$	virtual rolling moment of inertia increment due to surrounding air mass
$I_{y(\text{add. mass})}$	virtual pitching moment of inertia increment due to surrounding air mass
$I_{z(\text{add. mass})}$	virtual yawing moment of inertia increment due to surrounding air mass
$I_{x(\text{gear})}$	rolling moment of inertia, about oscillation axis, of equipment attached to airplane only for test
$I_{y(\text{gear})}$	pitching moment of inertia, about oscillation axis, of equipment attached to airplane only for test
$I_{z(\text{gear})}$	yawing moment of inertia, about oscillation axis, of equipment attached to airplane only for test



## MEASUREMENT OF AIRCRAFT MOMENTS OF INERTIA

N. Leonard Wener\*

### 1. INTRODUCTION

There can be no question as to the need for knowledge of the mass and inertia characteristics of aircraft. That these must be obtained experimentally is not at all obvious. We will see why such an experiment, although expensive, might be justified, as indeed it was justified for the Convair F-106A Interceptor.

A program was initiated to obtain the stability and control derivatives of the F-106A airplane from flight test data. To solve the equations of motion, inversely, for the dimensionless stability derivatives, it is necessary to provide the moments and product of inertia. Because the flight measurements are referenced to airframe axes, the inertias to be determined are similarly oriented.

As for all aircraft, the moments and product of inertia were computed for the F-106A piece by piece, but these computations had not been confirmed by experimental means. To use such data in determining the stability derivatives would leave an element of doubt in the results. Therefore, it was decided that the measurement of the moments of inertia was justified.

### 2. TEST PROCEDURES

The technique selected was that of oscillating the airplane about a fixed fulcrum with the restoring force provided by a helical spring. An example is shown in Figure 1. Damping is primarily due to pivot friction and internal losses in the spring. These can be made small enough to be neglected in the equation of motion. In the simplest case, where the axis of rotation is parallel to the principal lateral axis of the airplane and in the same X-Y plane with it, the pitching motion is expressed by

$$I_y \ddot{\theta} + C_y L_y^2 \tan \theta = 0 \quad (1)$$

or, for small angles,

$$I_y \ddot{\theta} + C_y L_y^2 \theta = 0 \quad (2)$$

where  $I_y$  = pitching moment of inertia about fulcrum

$C_y$  = spring stiffness (lb/ft)

$L_y$  = moment arm of spring (ft)

$\theta$  = angle of rotation of airplane about fulcrum (rad).

The solution of (2) shows  $\theta(t)$  to be a sinusoidal function, for which the following relationship holds:

$$\ddot{\theta}(t) = -\omega^2 \theta(t) = -\omega^2 |\bar{\theta}| \sin \omega t \quad (3)$$

---

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where  $\omega$  = oscillation frequency, rad/sec.

Substituting (3) in (2) gives

$$-I_y \omega^2 \theta(t) + C_y L_y^2 \theta(t) = 0 \quad (4)$$

Solving for  $I_y$ ,

$$I_y = \frac{C_y L_y^2}{\omega^2} \quad (5)$$

Thus, knowing the spring stiffness and its moment arm, it is necessary only to measure the oscillation frequency in order to determine the moment of inertia. The technique of performing this experiment is described in precise detail in Reference 1, which was used as a guide in measuring pitching and rolling moments of inertia for the F-106A. In the case of the yawing moment of inertia and the product of inertia, Reference 1 presents the skewed axis method. The airplane is oscillated about axes, each of which is inclined to the pitch and yaw axes. The two skewed axes should be inclined at large angles to these axes, and to each other, to permit accurate calculation of  $I_{zz}$  and  $I_{xz}$ . The results of such a test, presented in Reference 1, were very poor. As a consequence, a different approach was used to measure  $I_{zz}$  and  $I_{xz}$  on the F-106A.

## 2.1 Pitching Moment of Inertia

The airplane was supported at the two wing jacking points by knife-edge fulcrums (Figs. 2 and 3). The nose-jack fitting was supported on a set of coil springs (Fig. 4). Specially designed safety supports were placed under the wings and fuselage to catch the airplane in an emergency (Figs. 2, 3 and 5). A word about these safety supports is in order. There is no reason why an airplane should be dropped during such experiments. They are not hazardous, and every step can be planned well in advance. Nevertheless, failure of a part or a person must always be counted as a possibility. The safety supports were the 'accident insurance policy' we bought to protect an expensive airplane. Fortunately, no emergencies arose and the supports were never needed.

When the airplane was placed in the test condition, it was excited to oscillate in pitch. A few cycles after the input was removed, a recording of the motion was made on an oscillograph. The sensor was an inductive transducer mounted so as to detect compression of the spring cage (Fig. 4).

Equation (5) gives  $I_y$  for a simplified case. It remains for us to consider a more general case and to obtain  $I_{yy}$  by parallel-axis transfer to the reference axis through the center of gravity. Where the center of gravity is displaced vertically from the fulcrum, a torque is exerted, having a magnitude of  $-Wh_y$ , where  $W$  = weight of aircraft in lb, and  $h_y$  = vertical displacement of the center of gravity from the fulcrum (positive for c.g. above the fulcrum). Another correction is for the 'additional mass' effect of oscillating the airplane in an air mass rather than in a vacuum. This was determined by the method described in Reference 2 and is small, although not negligible. The inertia of the entrapped air was calculated but is insignificant. The final correction is for the moment of inertia of equipment attached to the airplane only for

the test; e.g., the spring cage upper plate. The equation for pitching moment of inertia about the lateral axis through the c.g. is

$$I_{yy} = \frac{C_y L_y^2 - W h_y}{\omega^2} - \frac{W l_y^2}{g} - V \rho l_y^2 - I_{y(\text{add. mass})} - I_{y(\text{gear})} \quad (6)$$

- where  $W$  = weight of airplane (lb)
- $h_y$  = vertical distance from oscillation axis to c.g., positive upward (ft)
- $l_y$  = distance from c.g. to oscillation axis (ft)
- $g$  = gravitational acceleration (ft/sec<sup>2</sup>)
- $V$  = volume of entrapped air (ft<sup>3</sup>)
- $\rho$  = air density (slugs/ft<sup>3</sup>)
- $I_{y(\text{add. mass})}$  = virtual pitching moment of inertia increment due to surrounding air mass
- $I_{y(\text{gear})}$  = pitching moment of inertia, about oscillation axis, of equipment attached to airplane only for test

## 2.2 Rolling Moment of Inertia

Whereas the pitching experiment utilized the standard jacking points of the airplane, the roll inertia test posed the problem that only the nose jack point was available along the fuselage (X-axis). It was necessary to construct a cradle to support the aft section of the fuselage and which could serve as one end of the roll oscillation axis. Figure 6 shows the construction and installation of this cradle. The restraining springs, placed at the wing jack points, were preloaded to bear approximately half of the airplane weight. This assured that the dynamic load would not go to zero during the oscillation. Also, it reduced the fulcrum loads, thereby reducing the friction damping. The test setup is shown in Figure 7.

The computation of rolling moment of inertia is very similar to the pitch case, and is as follows:

$$I_{xx} = \frac{C_x L_x^2 - W h_x}{\omega^2} - \frac{W l_x^2}{g} - V \rho l_x^2 - I_{x(\text{add. mass})} - I_{x(\text{gear})} \quad (7)$$

where  $C_x$  = spring constant (lb/ft)

$L_x$  = moment arm of springs (ft)

$h_x$  = vertical distance from oscillation axis to c.g., positive upward (ft)

$l_x$  = distance from c.g. to oscillation axis (ft).

### 2.3 Yawing Moment of Inertia

The airplane was hoisted by a mobile crane, using the standard F-106 lifting sling, as shown in Figure 8. The restraining springs were attached to the lower wing surface through fittings provided for external fuel tanks. An adapter produced the adjustment necessary to level the springs, whatever the pitch attitude of the airplane. The forward ends of the springs were attached to special abutments on the hangar floor. Figure 9 provides details of the spring installation. An exploded view of one spring set and wing attachment adapter is given in Figure 10.

Although the lifting sling provided adjustment of the airplane's pitch attitude, the resolution was limited. It was not possible to orient the reference Z-axis closer than about one degree from the vertical; however, correction for this is possible. However, more serious problems than this were present. Because the oscillation axis is not the principal Z-axis, any yawing motion will induce a rolling motion, the relative magnitudes being determined by the products of inertia for the oscillation axis. The induced rolling produces bending of the springs and binding of their guide rods. Further difficulties were friction at the crane hook swivel; 'spring' in the lifting sling; and motion at the abutment ends of the restraining springs. Generally, these errors increase the oscillation period, leading to high values of  $I_{zz}$ .

The equation for the yawing moment of inertia is

$$I_{zz} \approx \frac{C_z L_z^2}{\omega^2} - I_{z(\text{add. mass})} - I_{z(\text{gear})} \quad (8)$$

where  $C_z$  = spring constant (lb/ft)

$L_z$  = moment arm of springs (ft).

Equation (8) does not include the correction for axis misalignment because, as will be seen later, this effect is negligible in comparison with other errors.

### 2.4 Product of Inertia

The determination of  $I_{xz}$  was made by the 'null' method described in Reference 3. The airplane was oscillated in yaw at five pitch attitudes, at increments of about two degrees (Figs. 11 and 12). Rolling motion was measured, together with the yaw. The ratio of roll-to-yaw versus pitch attitude is shown in Figure 13. At a roll-to-yaw ratio of zero, the product of inertia is zero; therefore, the axis of oscillation must be a principal axis. The pitch angle at which the roll-to-yaw ratio vanishes is the inclination of the principal axis.

## 3. RESULTS

The tests described in the preceding section were performed on the F-106A airplane in two configurations:

- (a) Zero fuel
- (b) All but one fuel tank filled, one tank empty.

The experimental results are given in Table 1 together with the predicted moments and product of inertia. The predicted values were computed for an F-106A having slightly different mass distribution than the test airplane\*. Although exact comparisons are not valid (the differences in weight are only 0.5-1.5%, and the c.g. differences are 2.1 to 2.4% MAC), general comparisons are useful.

The numbers in Table 1 represent average values wherever more than one valid reading was obtained for any variable. For example, two calibrations were made for all the spring sets used in the experiment. The average was used in computing the moments of inertia listed in the table. The tolerances given were determined by using in the computations those values of the variables which produced the most extreme results. They must be considered more as order-of-magnitude variations than as probability boundaries. It was not possible to obtain repetitive measurements sufficient to provide statistical significance to the variations.

Several things are apparent from Table 1. Comparison between test and prediction yields varying degrees of agreement (or disagreement). The test results for  $I_{zz}$  are not only much higher than predictions, but exceed the sum of  $I_{xx}$  and  $I_{yy}$ . Since this is not physically possible, one or more of the moments of inertia must be in error. The nature of the yawing inertia test was such that it was far more subject to error than were the tests for pitching or rolling moments of inertia. The latter had firmly fixed axes, whereas the yaw axis was maintained only by the weight of the airplane on the suspension cable. Also, the roll coupling produced distortion of the springs, leading to errors in the spring stiffness constant and distorting the oscillatory motion from a sinusoid. For these reasons, the experimental data for  $I_{zz}$  were rejected.

For the rolling moment of inertia, zero fuel, it was found that the springs were nonlinear at compression in excess of three inches. The static loading compressed the springs 3.25 in.; thus, the oscillation took place with a nonlinear restoring force. This resulted in a greater uncertainty ( $\pm 2300$  slug ft<sup>2</sup>) than for the rolling inertia with full fuel ( $\pm 700$  slug ft<sup>2</sup>). The difference between the two configurations, the rolling inertia of the fuel, was much less than predicted. It is unlikely that the predicted inertia of the fuel could be this much in error. Allowing for the tolerance band, the increment approaches  $(21,800 - 13,100) = 8700$  slug ft<sup>2</sup> as compared with the predicted value of 10,200 slug ft<sup>2</sup>.

The measured pitching moment of inertia shows excellent agreement with predictions for zero fuel. With full fuel, the measurement exceeds the prediction by about four percent. The increment due to the fuel shows a larger pitching inertia than predicted, which is the reverse of the situation for rolling. Again, the error boundaries can account for the majority of this difference.

The inclination,  $\epsilon$ , of the principal axes shows good agreement between test and prediction for the full fuel case, not so good for the empty airplane. Despite the poor results for yawing moment of inertia, it is believed that the measured  $\epsilon$  is valid. Although it is probable that the roll-to-yaw ratios of Figure 13 contain errors, the null point (zero roll) should remain little affected. The slope of the curve at the abscissa permits reasonably accurate determination of  $\epsilon$ . This could have been improved,

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\*The actual differences, due to test instrumentation, are less than the difference due to inability to predict gross weight and c.g. precisely.

however, if finer adjustment of the lifting sling had been available.

The product of inertia is given by

$$I_{xz} = \frac{1}{2} \tan 2\epsilon (I_{zz} - I_{xx}) \quad (9)$$

For small  $\epsilon$ ,

$$I_{xz} \approx \epsilon (I_{zz} - I_{xx}) \quad (10)$$

where  $I_{zz}$  is the 'derived', not the measured, value. Because  $I_{zz}$  was derived from  $(I_{xx} + I_{yy} - \text{Constant})$ , the tolerance on  $(I_{zz} - I_{xx})$  is taken to be the same as for  $I_{yy}$ . This, combined with the uncertainty in  $\epsilon$ , determined the listed error boundaries for  $I_{xz}$ .

The derived yawing moment of inertia, given in Table 1, was obtained from the sum of the measured pitching and rolling moments of inertia less the predicted difference between  $I_{zz}$  and  $(I_{xx} + I_{yy})$ . This can be expressed by

$$I_{zz(\text{derived})} = (I_{xx} + I_{yy})_{(\text{measured})} + (I_{zz} - I_{xx} - I_{yy})_{(\text{predicted})} \quad (11)$$

This procedure is defended on the ground that the measured pitching and rolling inertias agree reasonably well with the predictions. While the result does not constitute a purely experimental value of  $I_{zz}$ , it is certainly a more reasonable solution than the test measurements provide.

To summarize the results, the predicted moments and product of inertia are sufficiently close to the measured values (where the measurements are considered to be valid) to satisfy most requirements for dynamic analyses.

#### 4. MANPOWER AND TIME REQUIREMENTS

Any decision to measure the moments of inertia of an aircraft must take into account the probable cost. It will not be attempted to put a price on the F-106 inertia tests. These could not readily be translated into probable costs for a similar test in areas having differing labor and materials prices. However, it should be of value to present the approximate manpower requirements of the tests previously described. Also included, although not previously mentioned in this report, is the measurement of center of gravity location (both X and Z coordinates) by weighing the aircraft at several attitudes.

Planning the inertia tests must begin some months in advance. This is necessary to design the test equipment required: knife edges and V-blocks, jack adapters, springs and cages, safety supports, etc. In the case of the F-106A, the springs, spring cages, and jack adapters were available at the U.S. Air Force Flight Test Center, Edwards Air Force Base, California, where the tests were conducted. This equipment was designed and built by the Cornell Aeronautical Laboratory, Buffalo, N.Y., and is described in Reference 1. Step-by-step plans for conducting the tests are described in Reference 1, together with sample forms for carrying out all calculations. This guide is highly recommended.

The special test equipment required about 600 man-hours for design and 1400 man-hours for construction. Planning and supervision of the test program and calculation of the results required approximately 3000 man-hours. The actual testing took six weeks (30 working days) and required a crew to set up the necessary test configurations and to maintain the airplane. This totalled about 2400 man-hours. Although the maintenance crew would have been needed, whether or not the inertia tests were performed, their labor is properly charged to the tests. Summarizing the manpower expenditures:

Engineering and design:	3600 man hours
Shop construction, setup and maintenance:	3800 man hours

As with most endeavors, the experience gained during the F-106A test program would permit it to be repeated with considerable savings. Careful planning of the tests, especially in scheduling the operations to tie up the airplane for a minimum time, will hold down the cost. On the other hand, exceedingly great care and precision are required in making measurements, which tends to increase the time required. The need for accuracy and speed are not necessarily contradictory; however, where a choice must be made, the dictates of precision must be met.

## 5. A NEW APPROACH

The spring oscillation method of determining moments of inertia has been described in the preceding sections of this report and in References 1 and 3. This method (or any others which have been tried) is expensive and time consuming, particularly because of the special fixtures which must be designed and built to suit each different airplane. Furthermore, the results of recent yawing inertia tests have been poor.

Cornell Aeronautical Laboratory, Inc., under contract to the U.S. Air Force, (Flight Control Laboratory, Wright Air Development Center), devised a method for measuring all moments and products of inertia of aircraft up to 300,000 lb gross weight. The technique employs forced oscillation of a platform on which the airplane is rigidly mounted. The forces and moments acting on the airplane are measured at the mounting points. These are resolved and summed for orthogonal coordinates during oscillation about each of three fixed axes. These forces and moments and the time history of the motion are used to compute the moments and products of inertia.

The platform is supported on four hydrostatically-floated bearings which are segments of a single sphere. Guide rollers can be engaged which restrict the oscillation to a single axis. Figure 14 shows a model of the platform set up to oscillate in roll. Three-element force transducers were designed to measure the forces applied to the airplane at each support point. One noteworthy advantage of this system is that the airplane can be supported on any number of jacks, since the oscillation axis is provided by the platform. This multipoint support can reduce the errors caused by structural flexing of the airplane during the oscillation.

Construction of this test facility will begin this year in the Weight and Balance Hangar at the Air Force Flight Test Center, Edwards Air Force Base, California. It offers a fresh approach to the problem of measuring aircraft moments of inertia, which simplifies the test setup problem. When the facility is completed and experience has been obtained

in its use, the results should provide an interesting paper for a future meeting of the AGARD Flight Test Panel.

## 6. CONCLUSIONS AND RECOMMENDATIONS

The results of the inertia tests on the F-106A suggest the following conclusions:

- (1) For most dynamic analyses, carefully predicted moments of inertia should suffice;
- (2) Where more accurate moments of inertia are needed (e.g., zero-length launch of airplanes), the spring oscillation method is capable of yielding good results, *provided that* (a) extreme care is exercised in conducting the test, (b) flexibility of the airframe or supporting fixtures is negligible, and (c) fuel sloshing is avoided.
- (3) Test measurements should be made by more than one person and repeated as many times as practicable. Two surveyor's levels (or transits) should be used to double check measurements of the test setup.
- (4) The results obtained with the inertia measuring platform to be installed at Edwards Air Force Base, California, should be studied with a view to future planning.



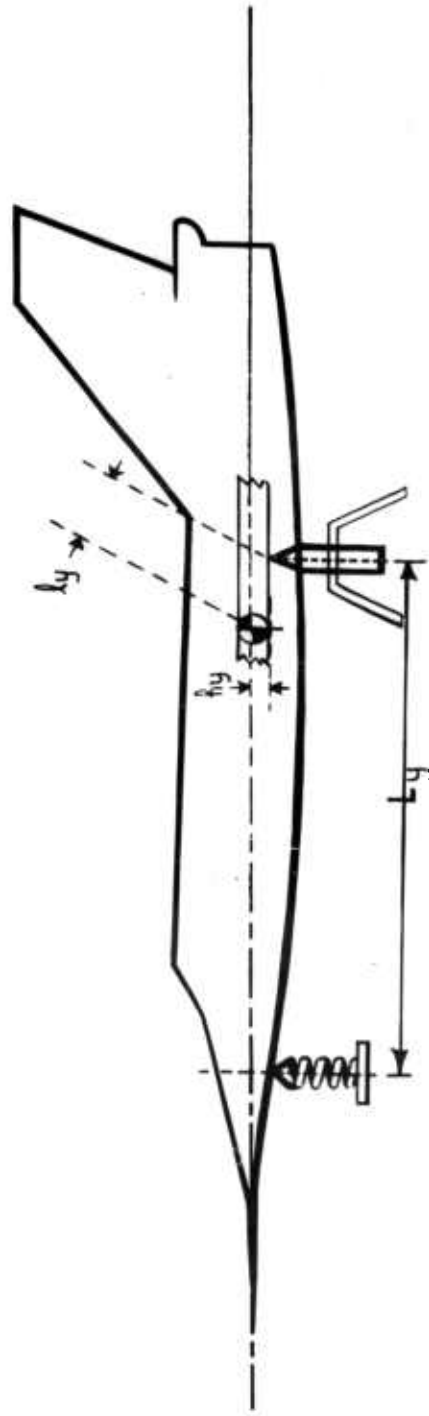
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TABLE 1

Measured and Predicted Moments and Product of  
Inertia for the F-106A Airplane

	ZERO FUEL		FULL FUEL	
	Measured	Predicted	Measured	Predicted
Gross Weight	X	1.016 X	Y	1.004 Y
c.g., % MAC	25.76	28.2	27.43	29.5
$I_{xx}$ (slug-ft <sup>2</sup> )	15,400 ( $\pm 2300$ )	13,300	21,100 ( $\pm 800$ )	23,500
$I_{yy}$ (slug-ft <sup>2</sup> )	160,000 ( $\pm 3500$ )	162,000	191,000 ( $\pm 1500$ )	184,000
$I_{zz}$ (slug-ft <sup>2</sup> )	209,000 ( $\pm 3500$ )	172,000	248,000 ( $\pm 3000$ )	202,000
$\epsilon$ (degrees)	1.8 ( $\pm 0.1$ )	2.24	1.5 ( $\pm 0.1$ )	1.57
	Derived		Derived	
$I_{zz}$ (slug-ft <sup>2</sup> )	172,100 ( $\pm 4200$ )	172,000	208,500 ( $\pm 1700$ )	202,000
$I_{xz}$ (slug-ft <sup>2</sup> )	4,920 ( $\pm 250$ )	6,200	4,900 ( $\pm 200$ )	4,900



**Fig. 1 Test setup for pitching moment of inertia**

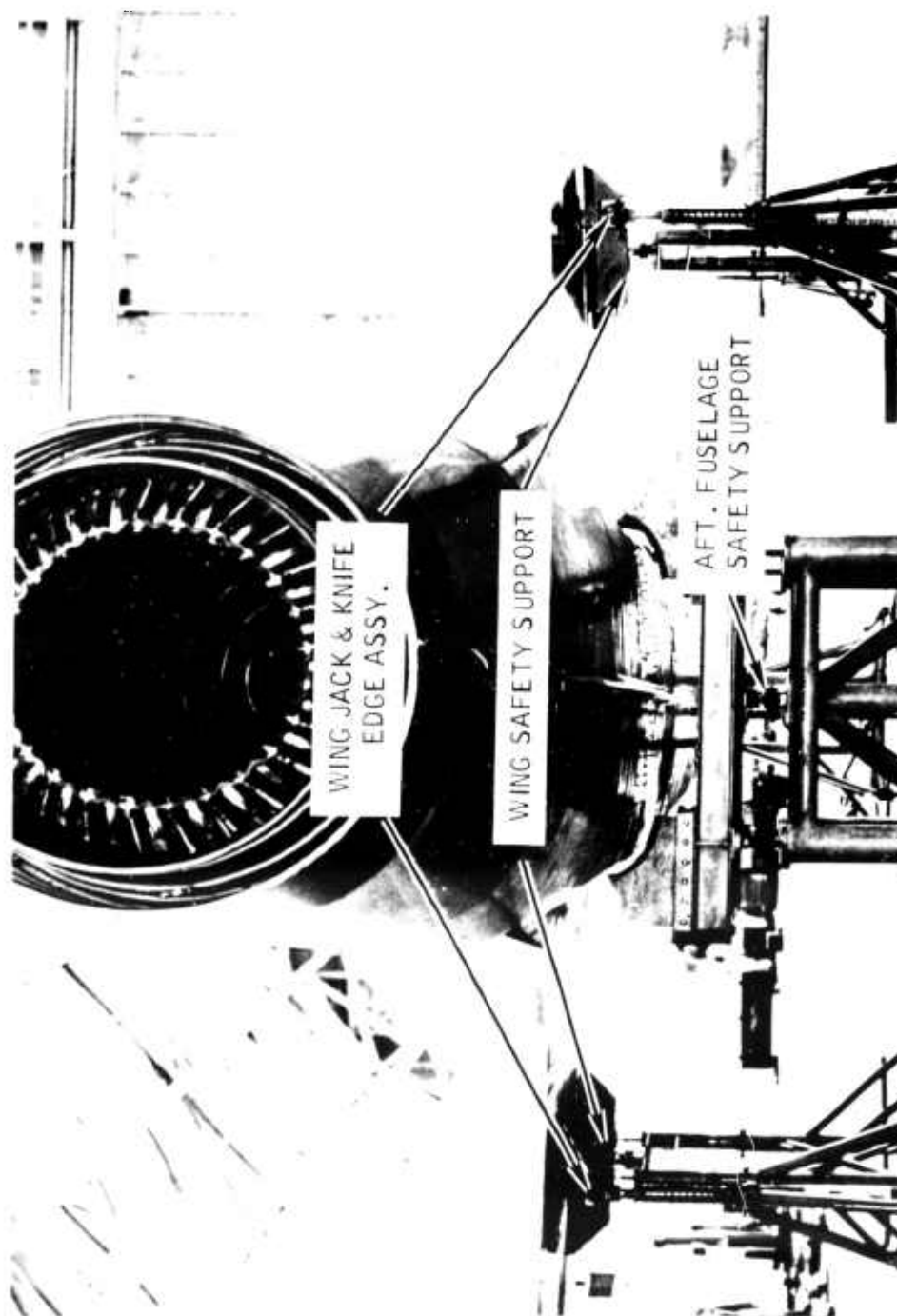


Fig.2 Pitch condition: aft view of test setup



Fig. 3 Pitch condition knife edge and wing safety support details

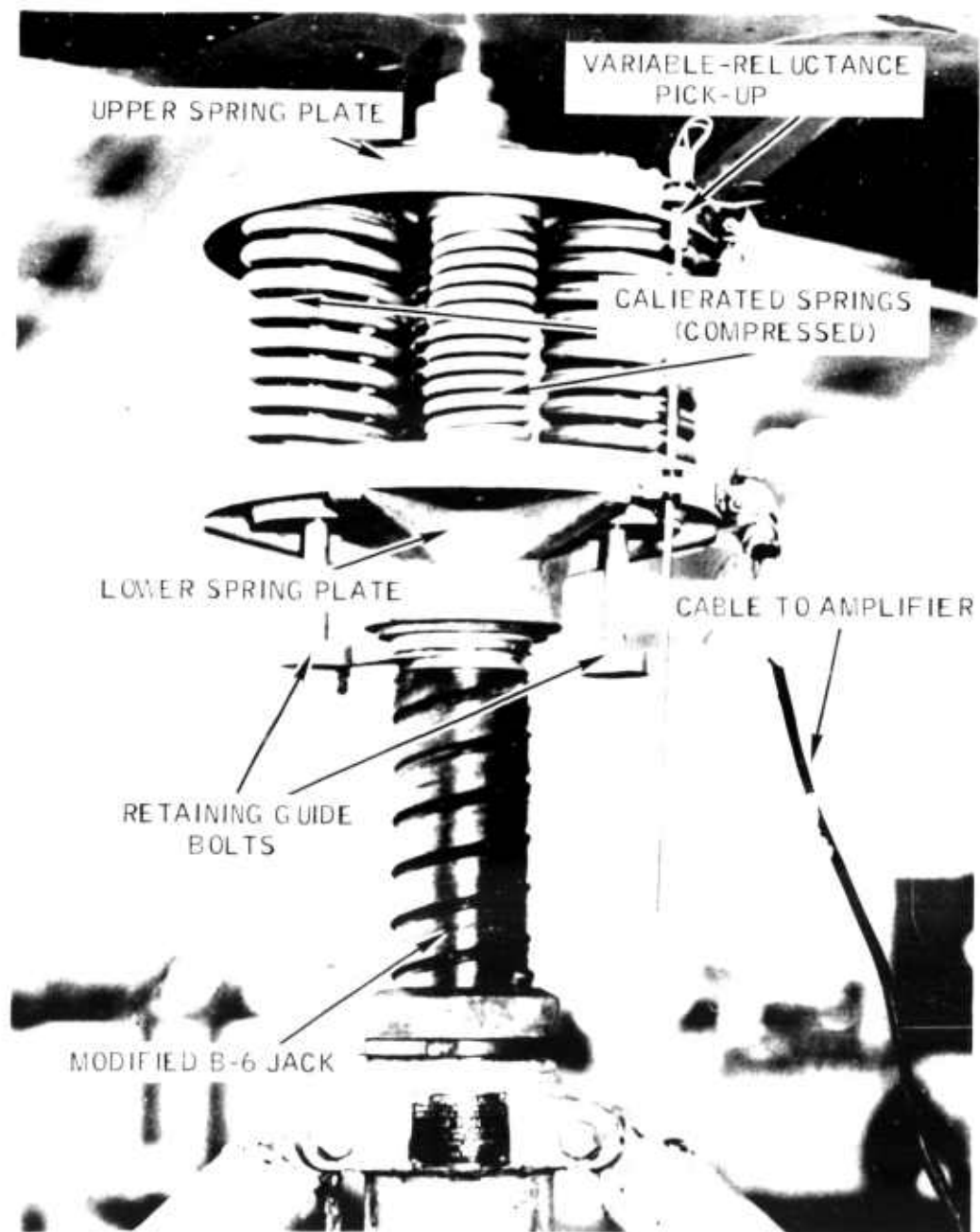


Fig.4 Pitch condition spring cage assembly



Fig. 5 Pitch condition: overall test setup

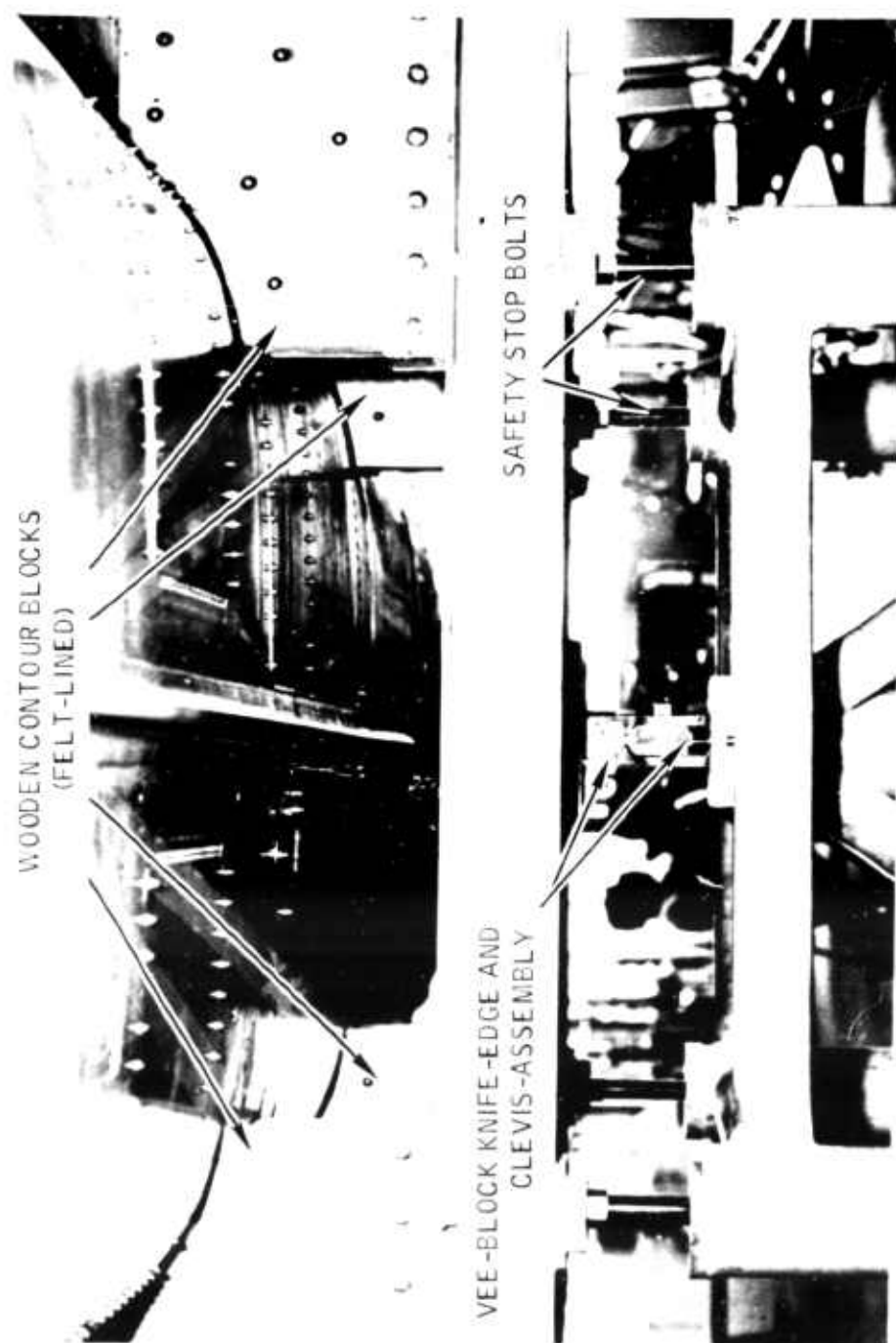


Fig. 6 Roll condition: aft fuselage knife-edge. Assembly and safety support





Fig. 7 Roll condition overall test setup



Fig. 8 Yaw condition: overall test setup

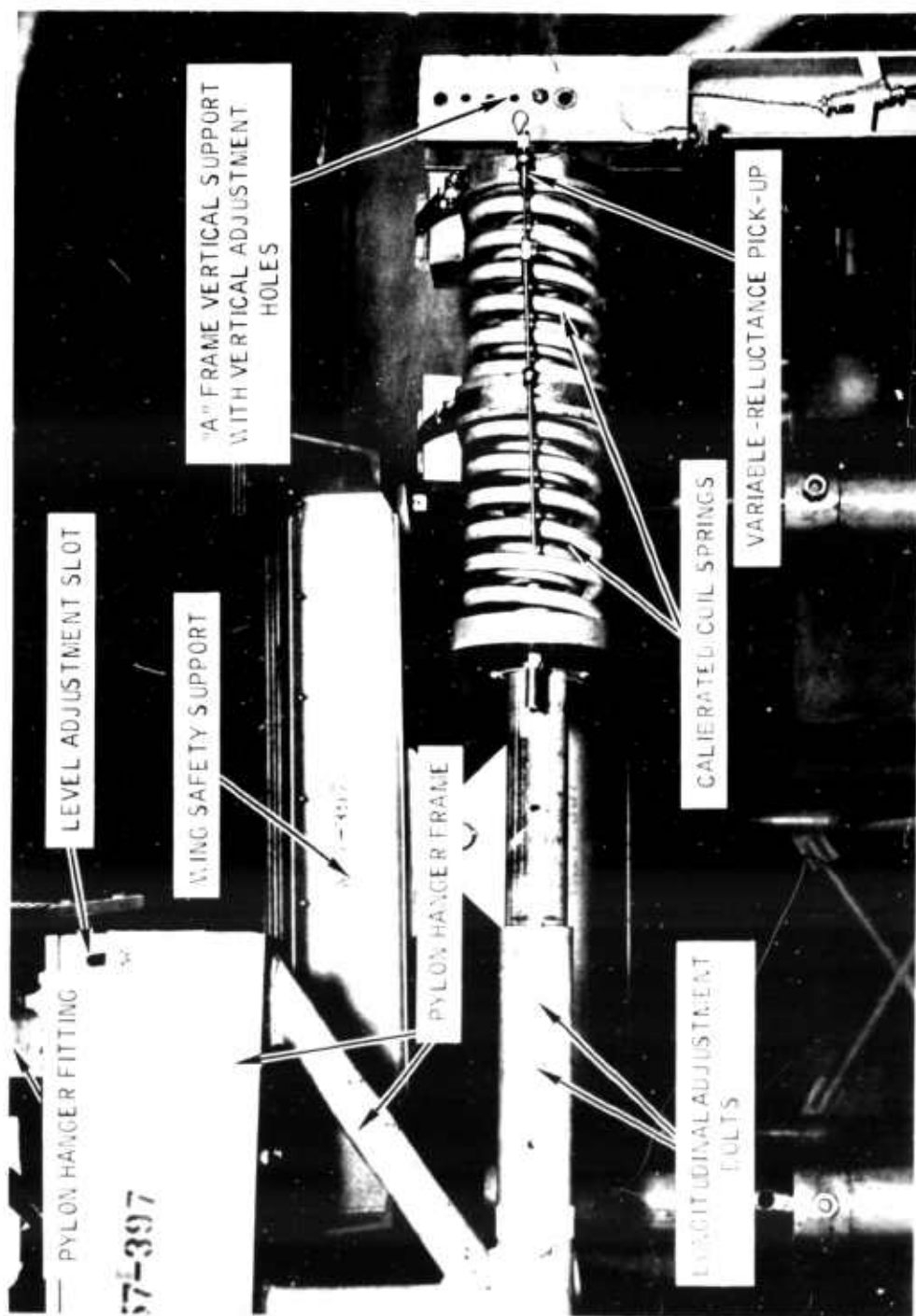


FIG. 9 Yaw condition: spring-cage assembly mounting details

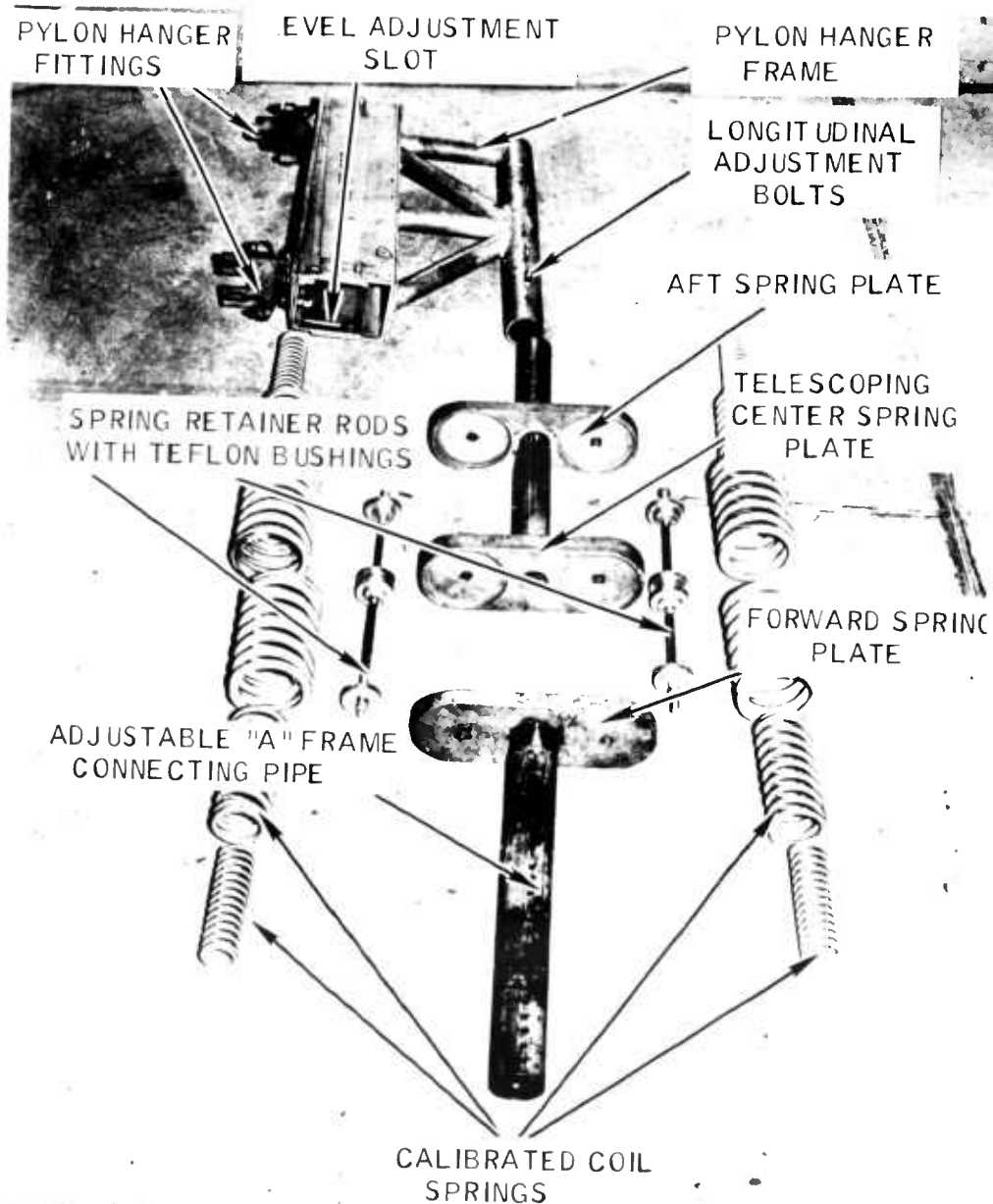


Fig.10 Yaw condition: spring cage assembly

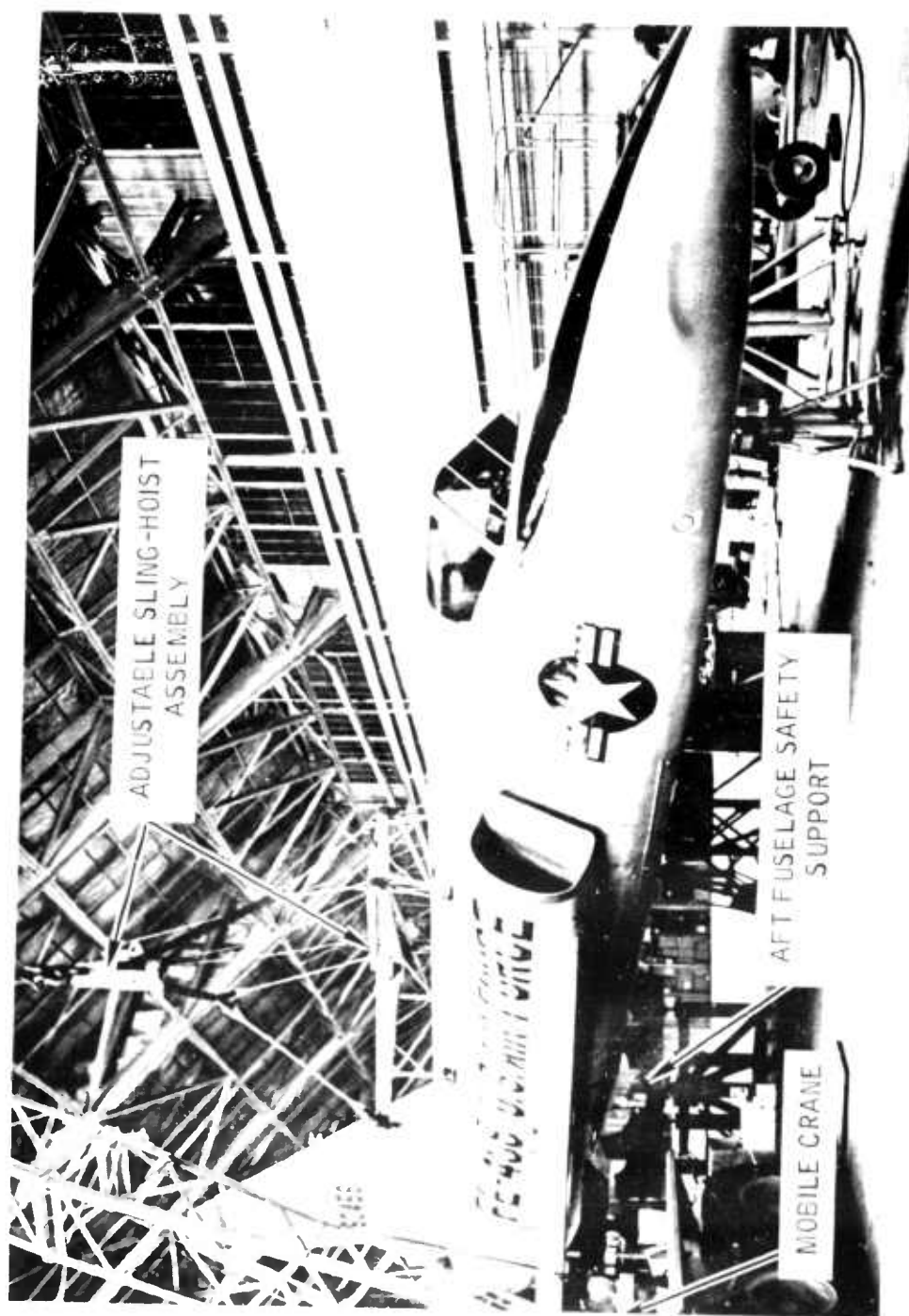


FIG. 11 Determination of maximum roll-to-yaw ratio: nose-down attitude



Fig.12 Determination of maximum roll-to-yaw ratio: nose-up attitude

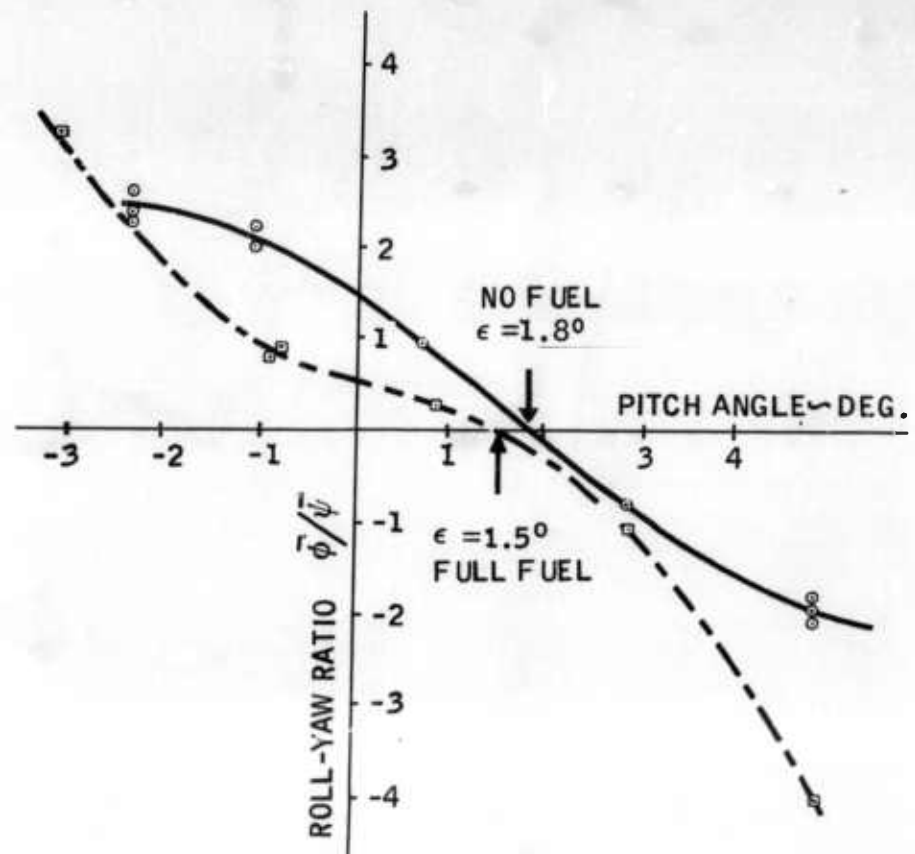


Fig.13 Inclination of principal axes of F-106A airplane

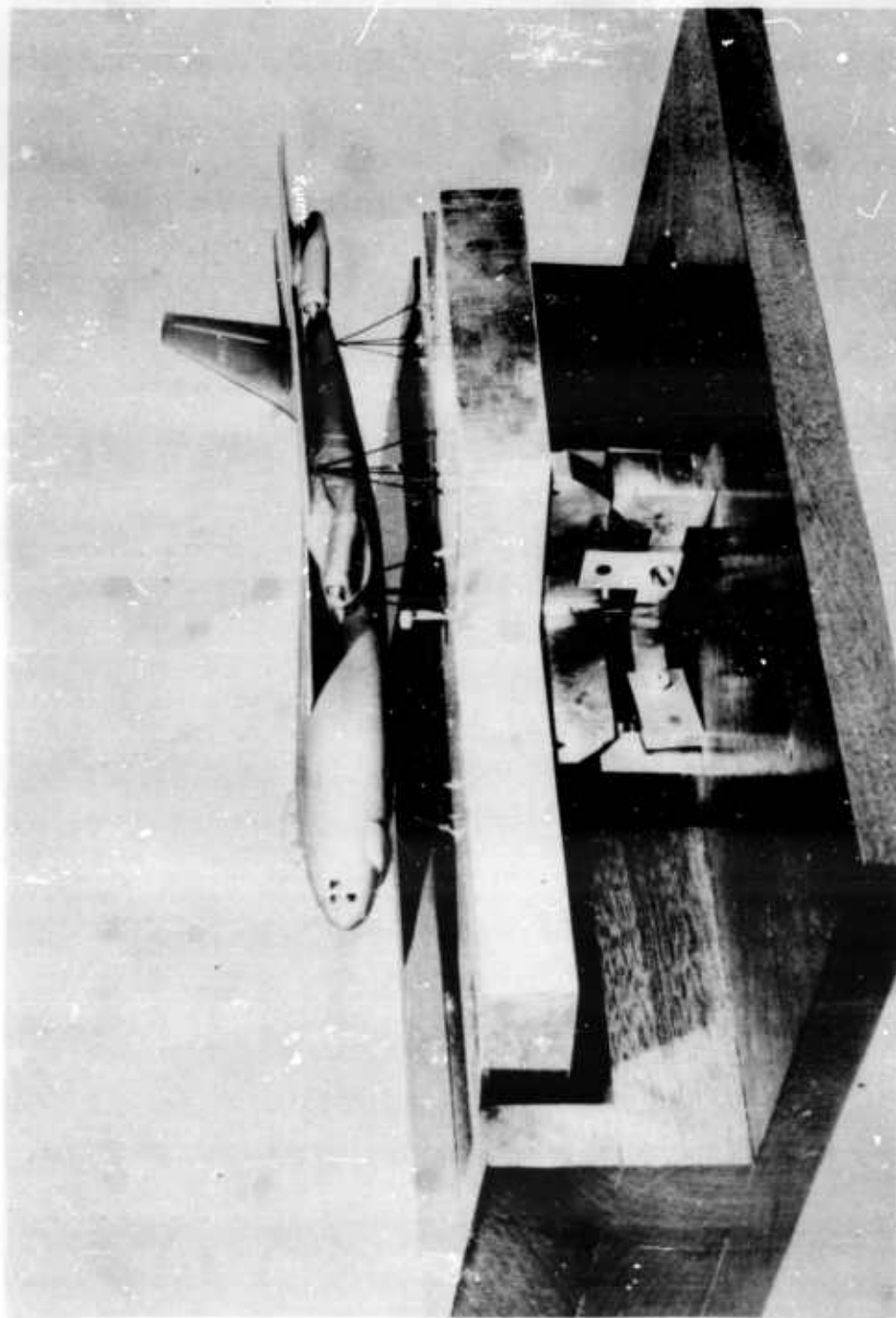


Fig.14 Inertia platform model: roll mode



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