MTHG202 Special Functions and Partial Differential Equations

MODELING AND SIMULATION OF TYMPANIC MEMBRANE VIBRATION

AcousticMind

Overview

- Introduction
- Literature Review
- Objectives
- Problem Definition
- Methodology
- Results
- Implemented Application
- Conclusion

Introduction

- Myringoplasty is a routine surgical procedure used to repair perforations in the tympanic membrane (TM).
- The middle ear anatomy cannot be fully reproduced during surgery, the procedure largely depends on the surgeon's experience.
- Accurate mechanical modeling of the TM is therefore essential to improve **understanding**, **diagnosis,** and **training**.

Fig(1): Ear Surgery Illustration

Literature Review

[1] Analytical Free Vibration Modeling of the Tympanic Membrane

- Wu et al. modeled the tympanic membrane (TM) as a sectorial annulus under uniform tension with fixed boundaries.
- The 2D wave equation in polar coordinates was solved via separation of variables, reducing it to a Bessel equation to obtain natural frequencies and mode shapes.
- Analytical results agreed well with finite element simulations and experiments, validating the simplified model.

[2] Finite Element Modeling of Middle Ear Sound Transmission

- Chen et al. developed a 3D finite element model of the middle ear based on high-resolution CT images, modeling the tympanic membrane as a three-layer structure.
- The model achieved accurate acoustic-structural coupling and was validated using experimental umbo displacement data.
- Results showed that increased tympanic membrane stiffness and thickness reduce sound transmission, matching clinical findings.

Objectives

- To investigate the free vibration dynamics of the TM using three complementary approaches:

1. **Analytical modeling** based on physical membrane theory.
2. **Numerical simulation** to validate and visualize vibration modes.
3. **Data-driven learning** to enable automated pattern recognition and diagnosis.

- Beyond theoretical analysis, we demonstrate two practical applications:

1. **An automated diagnosis system** based on vibration characteristics.
2. **A real-time interactive simulation in Unity** for intuitive visualization and surgical training.

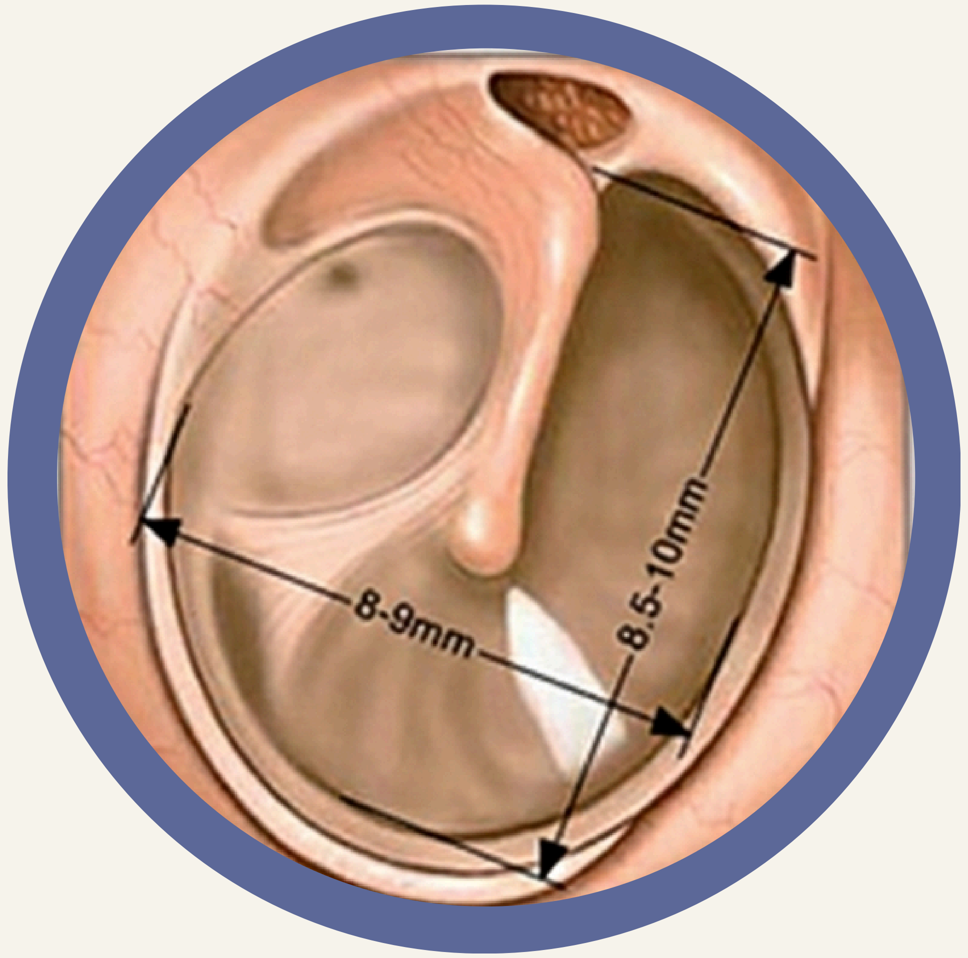
PROBLEM DEFINITION

- In this project, we study and describe the free vibration behavior of the human tympanic membrane using mathematical modeling.
- The tympanic membrane is modeled using partial differential equations, and its vibration characteristics are analyzed and compared using analytical, numerical, simulation, and machine learning approaches.

METHODOLOGY

Analytical Solution

- The tympanic membrane is modeled as a thin, tension-dominated membrane.
- Its motion is governed by a 2D time-dependent wave equation in polar coordinates.



Fig(2): TM structure

Derived analytically using **separation of variables** and **Bessel functions.**

This allows analytical extraction of natural frequencies and mode shapes.

This splits the problem into:

- A spatial eigenvalue problem
- A temporal ordinary differential equation

$$w(r,\theta,t) = \sum_{m=1}^{\infty}\sum_{k=1}^{\infty} A_{mn} W_{mn}(r,\theta)\sin(\omega_{mn}t+\varphi)$$

METHODOLOGY

Role of Time in the Model

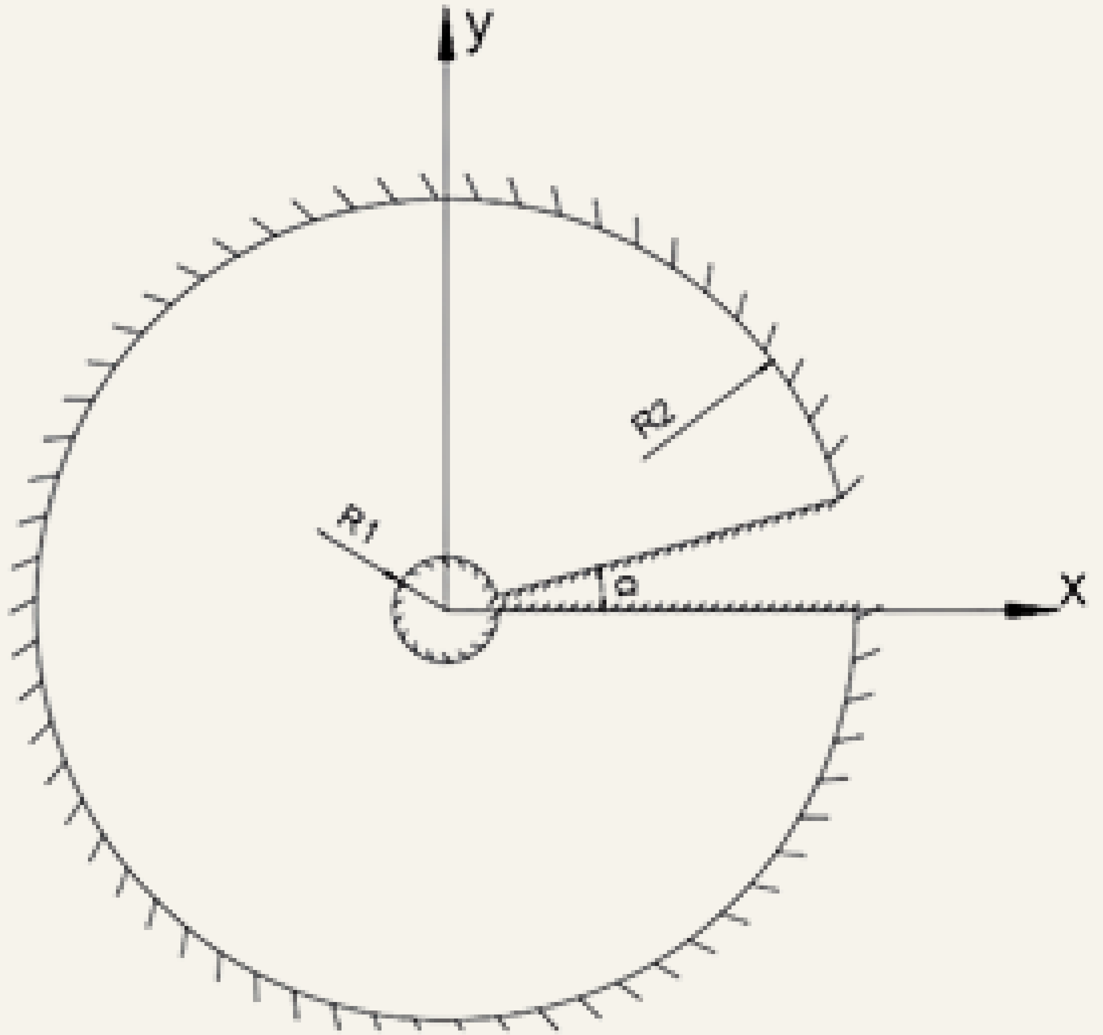
Although the governing equation is time-dependent, time is analytically separated from space ,from separation of variables,

$$\ddot{q}(t) + \omega^2 q(t) = 0$$

physical meaning :

- the solution is pure harmonic oscillation
- Time controls oscillation, not deformation shape

So, The system is time-dependent , but modal analysis separates time analytically , allowing numerical and simulations to focus only on eigenmodes computing Mode shapes [$W_{mn}(r, \theta)$] and Natural frequencies ω_{mn}



Fig(3): Simplified TM structure

METHODOLOGY

Finite Difference

(Numerical Solution)

Objectives

To validate analytical solution by extracting natural frequencies and vibration mode shapes.

Central Finite difference

$$\frac{\partial^2 w}{\partial \theta^{-2}} \approx \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{(\Delta\theta)^{2}}$$

1. Using Python(Scipy)

- Discretization
- Grid Structure
- Matrix Formulation
- Eigenvalue Calculation (Sparse eigenvalue solver)

2. using COMSOL

- 2D Modeled stimulation
- Boundary Conditions: Fixed at 0
- Study Type: Eigenfrequency study
- Visualization: Out-of-plane displacement scaled for visibility(3D)

METHODOLOGY

Finite Difference

(Numerical Solution)

Why Use the Finite Difference Method?

Speed & Efficiency

It transforms the differential equation into a system of **linear equations** ($Ax = b$), allowing the 2D problem to be solved on a standard CPU within milliseconds.

Deterministic Results

Repeated execution of the code (e.g., 100 runs) consistently produces identical results.

METHODOLOGY

Why Use PINNs?

Key Advantages

flexibility , future potential and Resolution

Mesh-Free Solution (No Grid Required)

It handles complex, curved, or changing geometries effortlessly without needing a human to design a grid

Infinite Resolution (Continuous Function)

The output is a continuous mathematical function. You can zoom in infinitely on any part of the eardrum and get an exact value without losing quality

METHODOLOGY

Why Use PINNs?

Handling Noisy Data

PINN Can combine the Physics (Helmholtz equation) with the Data (measurements) to find a clean, physically correct solution that ignores the noise

The "Inverse" Problem (Medical Diagnosis)

- The model can watch a vibrating membrane and reverse-engineer its physical properties.
- Bidirectional Solving also code treats physical parameters as trainable variables, just like neural network weights.

METHODOLOGY

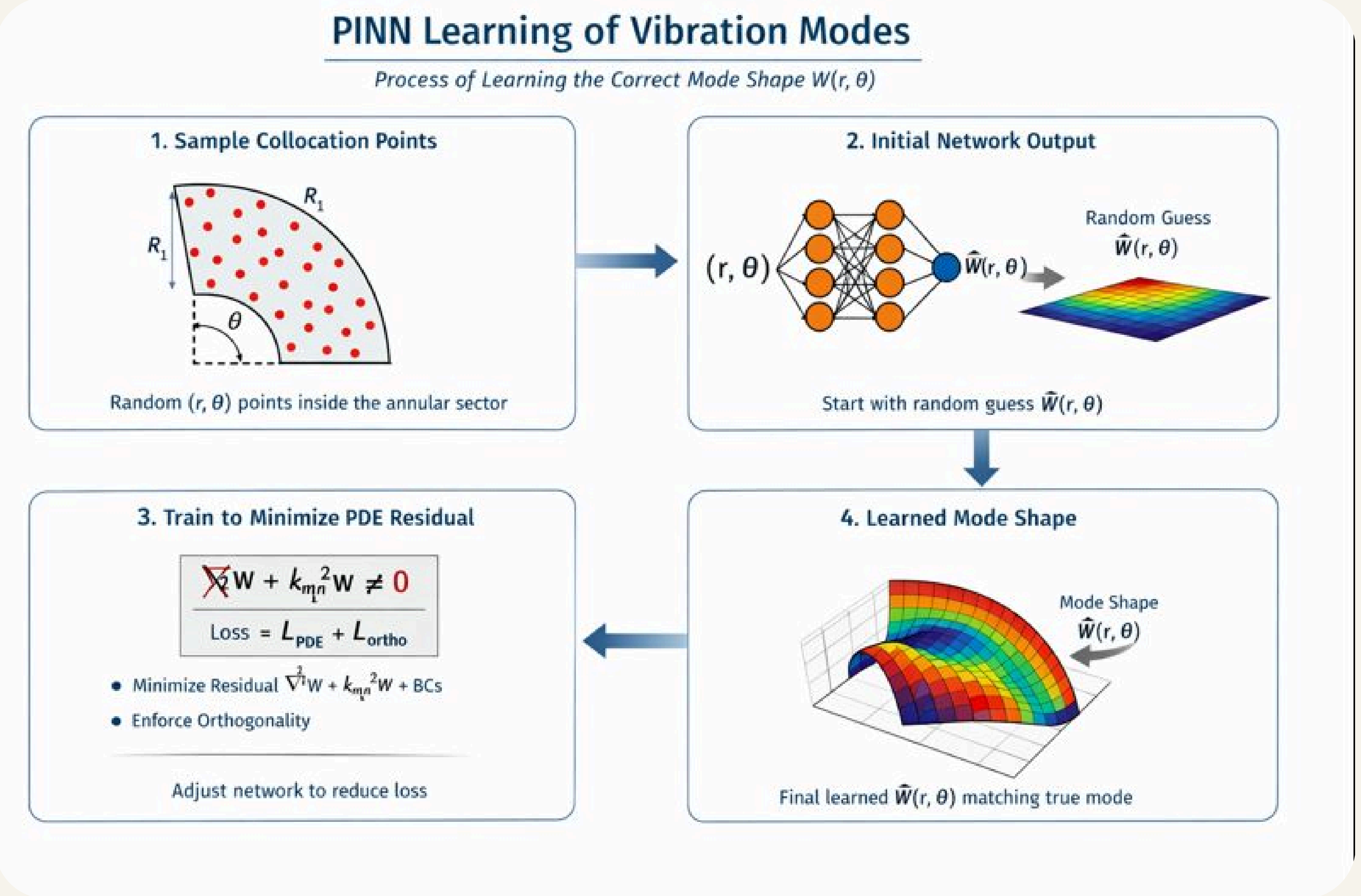
PINNS

PINNs use a neural network to approximate the vibration modes while enforcing the governing physics directly, eliminating the need for traditional discretization.

PINN Architecture:

(Neural Network Structure)

- **Input layer:** (r, θ)
- **Hidden layers:**
    - 3 layers
    - 128 neurons per layer



Fig(4): PINN model workflow

METHODOLOGY
PINNS

- **Eigenvalues (Frequency):** Solved via Classical Math (Special Functions).
- **Why:** Eliminates training instability. We do not ask the PINNS to "guess" the frequency, we provide it as a ground truth constant.
- Eigenfunctions (Mode Shapes) corresponding to those frequencies: Learned by the Neural Network.
- **Method:** PINN minimizing the Helmholtz(wave equation) residual.
- **Why:** Acts as a powerful mesh-free interpolator that handles the complex spatial derivatives.

Results

Model Parameters & Validation Setup

Parameters were selected based on physiological values from existing literature to ensure biological relevance.

1. Geometry:

- Inner Radius (R_1): 0.5 mm
- Outer Radius (R_2): 4.5 mm
- Thickness (d): 0.1 mm

2. Material Properties:

- Young's Modulus (E): 33.4 N/mm^2
- Density (rho): 0.00012 g/mm^2
- Tension (T): 1.35 N/mm^2

Result

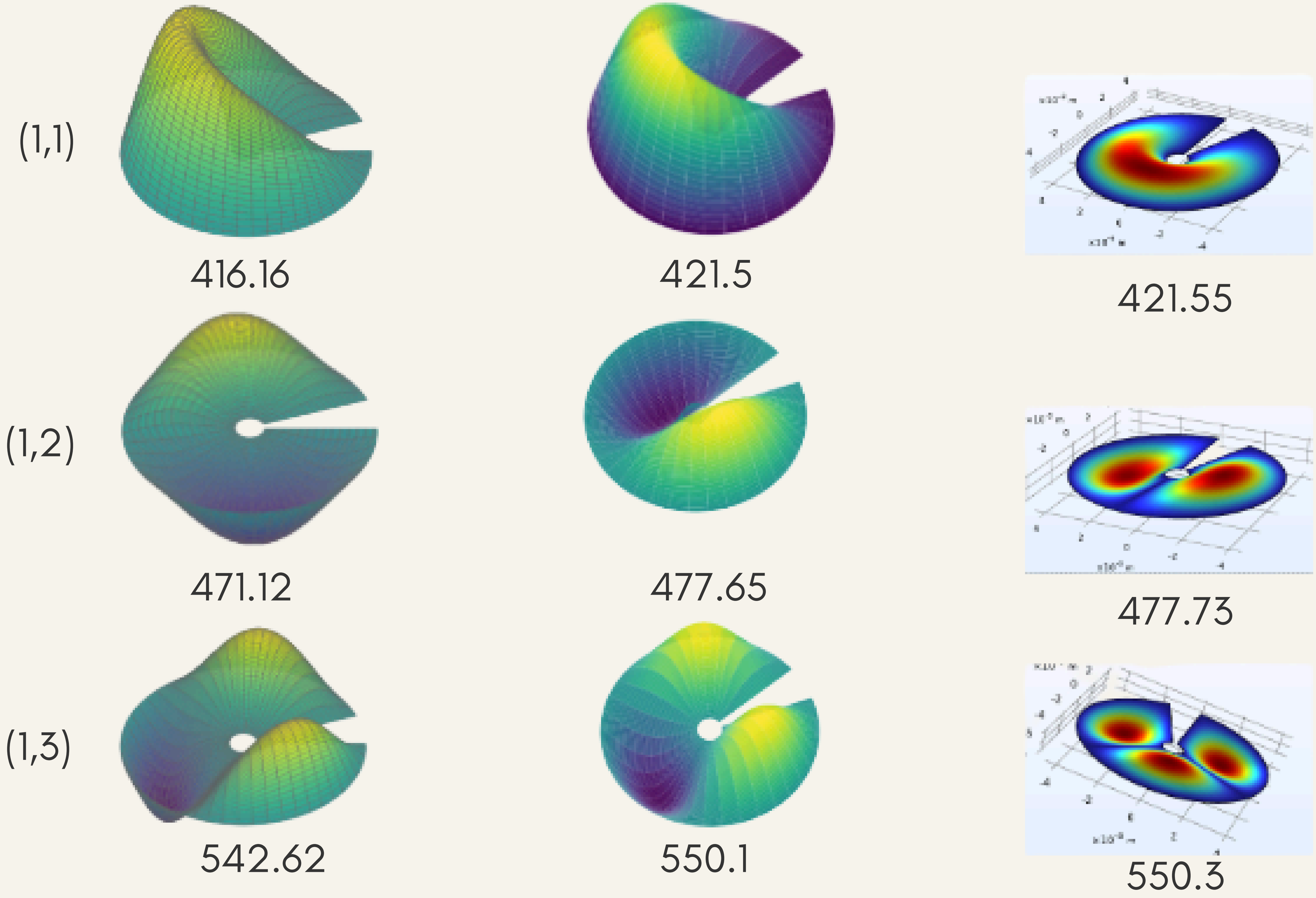
Mode	Analytical (Hz)	Numerical using python (Hz)	Error (%)	Numerical using Comsol (Hz)	Error (%)
(1,1)	416.46	421.5	1.21	421.55	1.22
(1,2)	471.55	477.73	1.31	477.65	1.29
(1,3)	542.62	550.3	1.42	550.1	1.38
(2,2)	869.28	883.19	1.60	882.6	1.53
(2,3)	951.69	947.03	0.49	946.3	0.57
(3,3)	1352.29	1346	0.47	1343.8	0.63

Table(1): Comparison of natural frequencies and error analysis

Result

Visualizing Vibration Mode Shapes

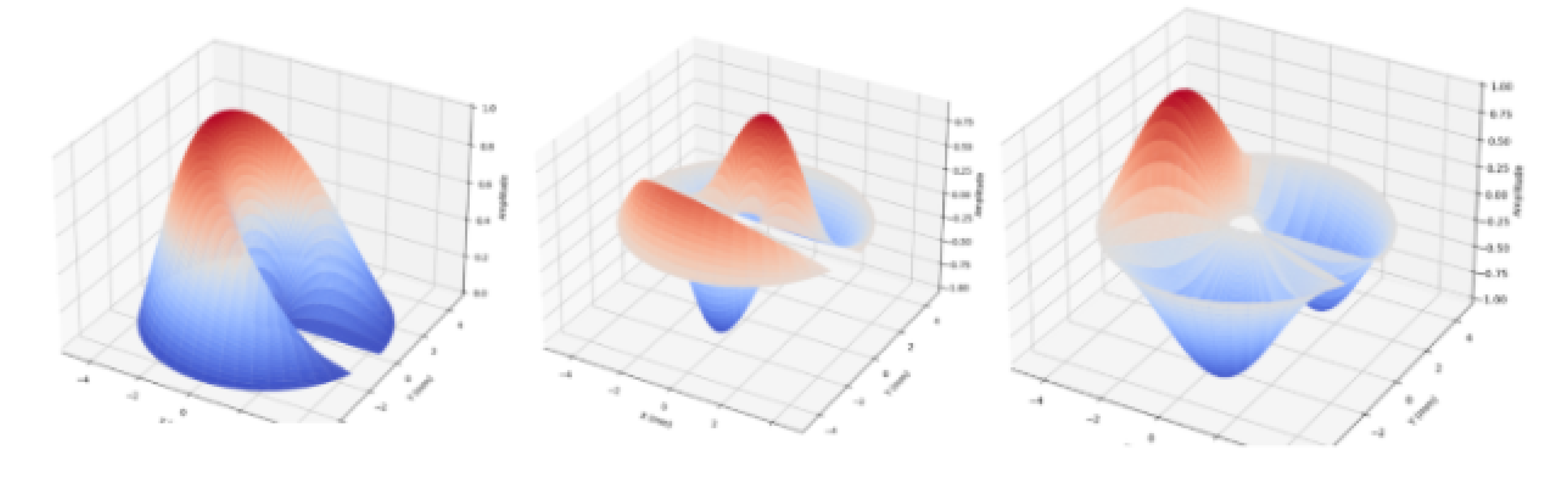
- The physical behavior (shape deformation) is consistent across all three platforms.
- The "cutout" geometry of the membrane is accurately captured in the Numerical and Simulation models.



Visual comparison of modes (1,1) through (1,3).

Result

PINN results



Visualization of mode(1,1) ,(2,2),and (3,1) using PINNs

Result

Summary of Findings

- **Consistency:** The order of natural frequencies is identical across all methods.
- **Validity:** The physical behavior of the tympanic membrane is correctly captured.
- **Versatility:** Both traditional numerical methods and modern ML (PINN) approaches are viable for this model.

IMPLEMENTED APPLICATIONS

1. Application for diagnosis

Physical Insight

- Tympanic membrane behaves as a **tensioned elastic resonator**
- Pathological changes **alter modal resonance frequencies**. which depend on Shape, Mass, Tention...
- **LDV** provides resonance frequencies, but clinical classification is still **experience-dependent** and requires some tests & confirmation.

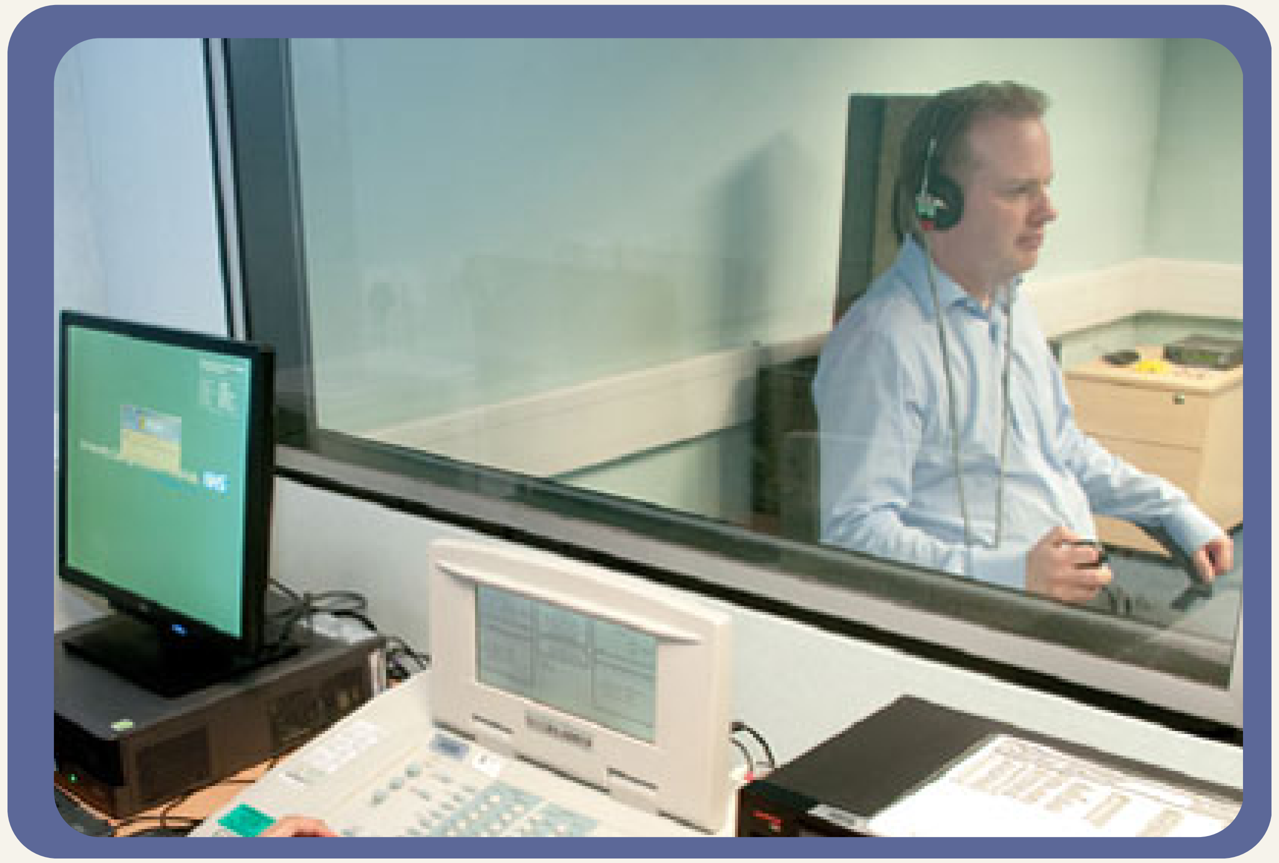
Fig(4): Physical Structure of TM

IMPLEMENTED APPLICATIONS

1.Application for diagnosis

Moving beyond pure-tone testing using a **physics-based eardrum vibration model**

- The issue was that we didn't have enough dataset.
- A **PDE-based biomechanical model** generates resonance frequency responses under varying mechanical conditions
- These resonance features are used to train **machine learning classifiers** for eardrum condition diagnosis



Fig(5): Pure-Tone Testing

Models Used:

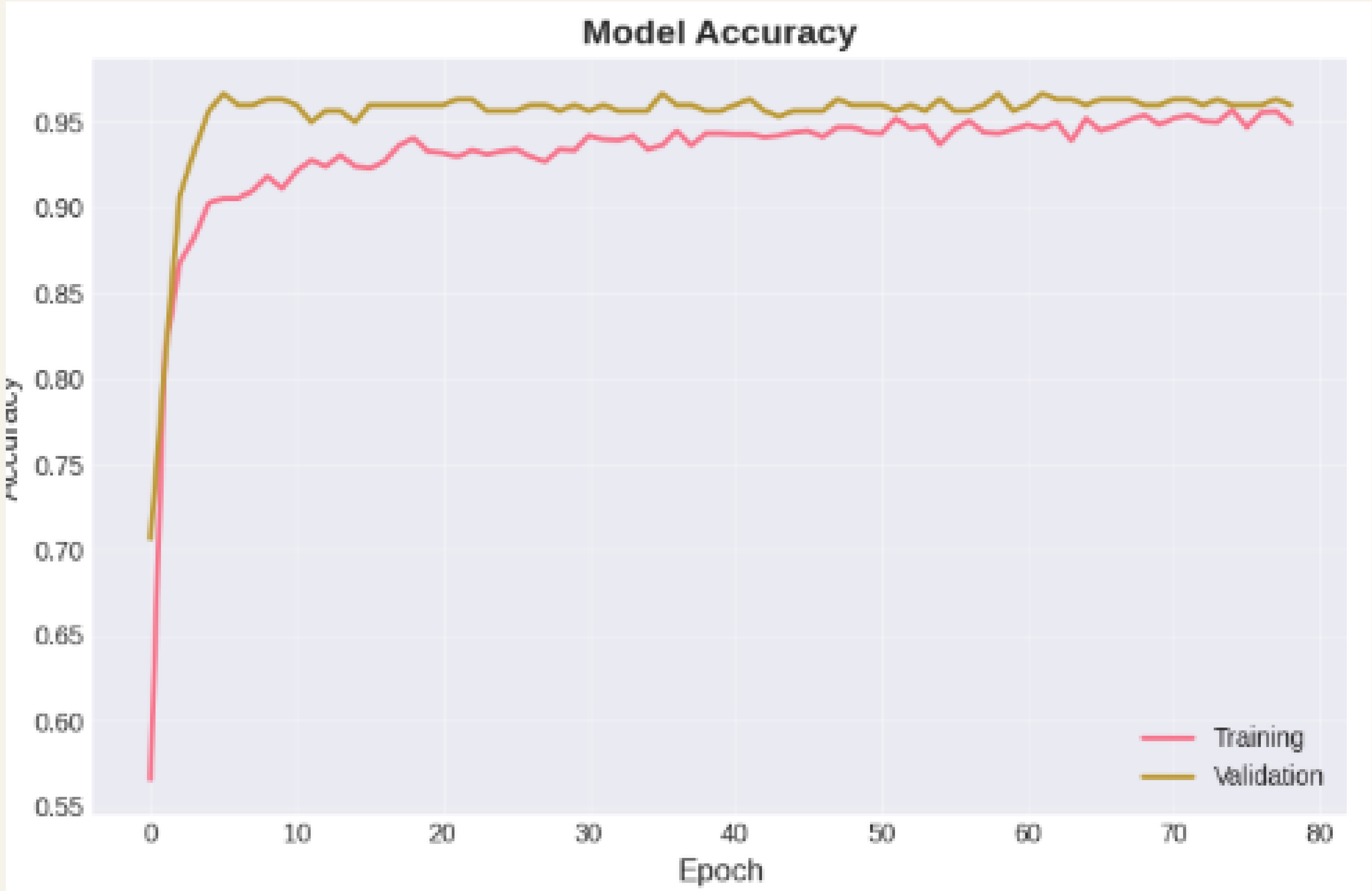
- Random Forest Classifier
- Deep Neural Network (DNN)

Classification Targets:

- Healthy
- Stiffened
- Flaccid

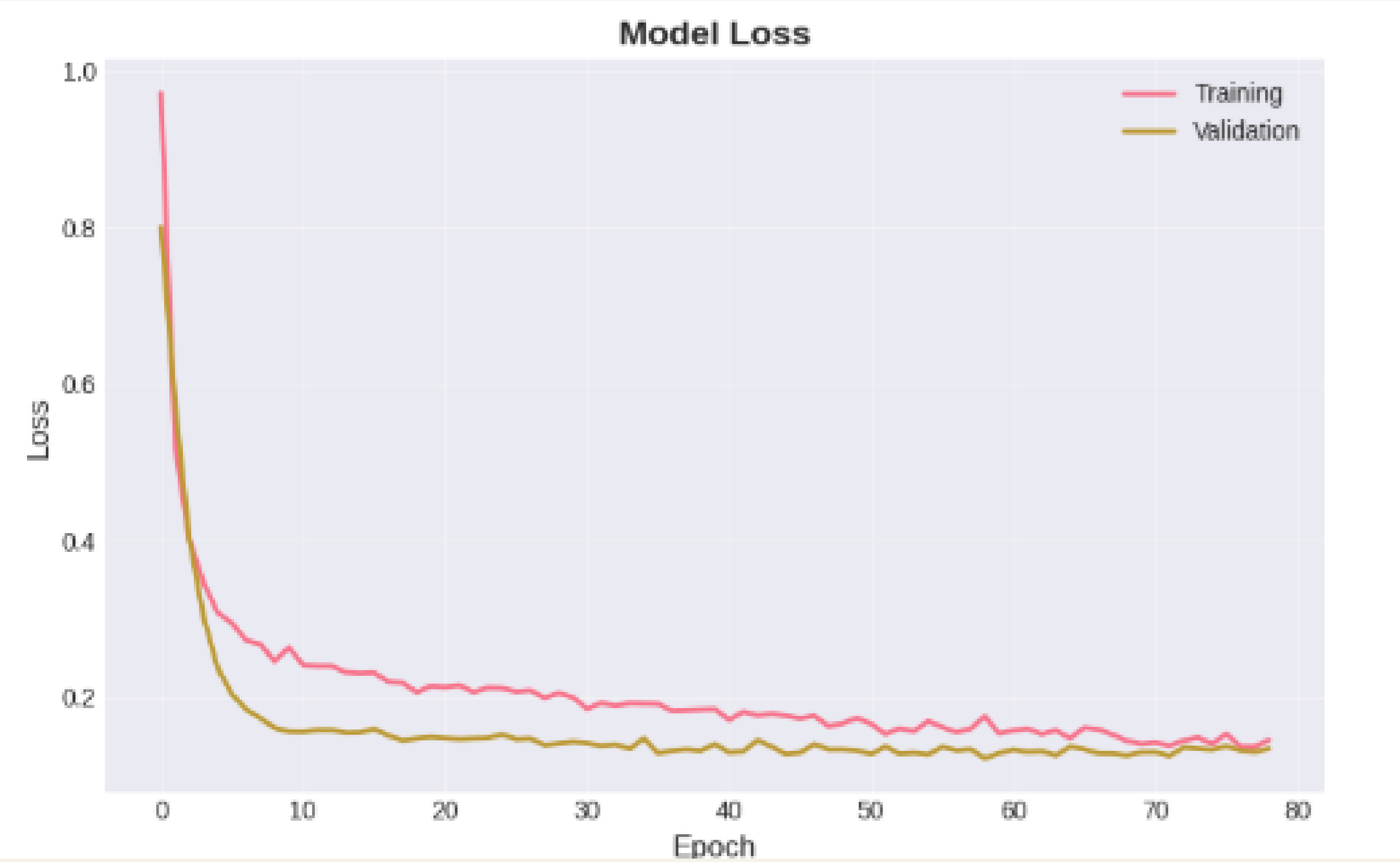
Learning Curves

1. Accuracy



Neural network training and validation accuracy over 80 epochs

2. Loss

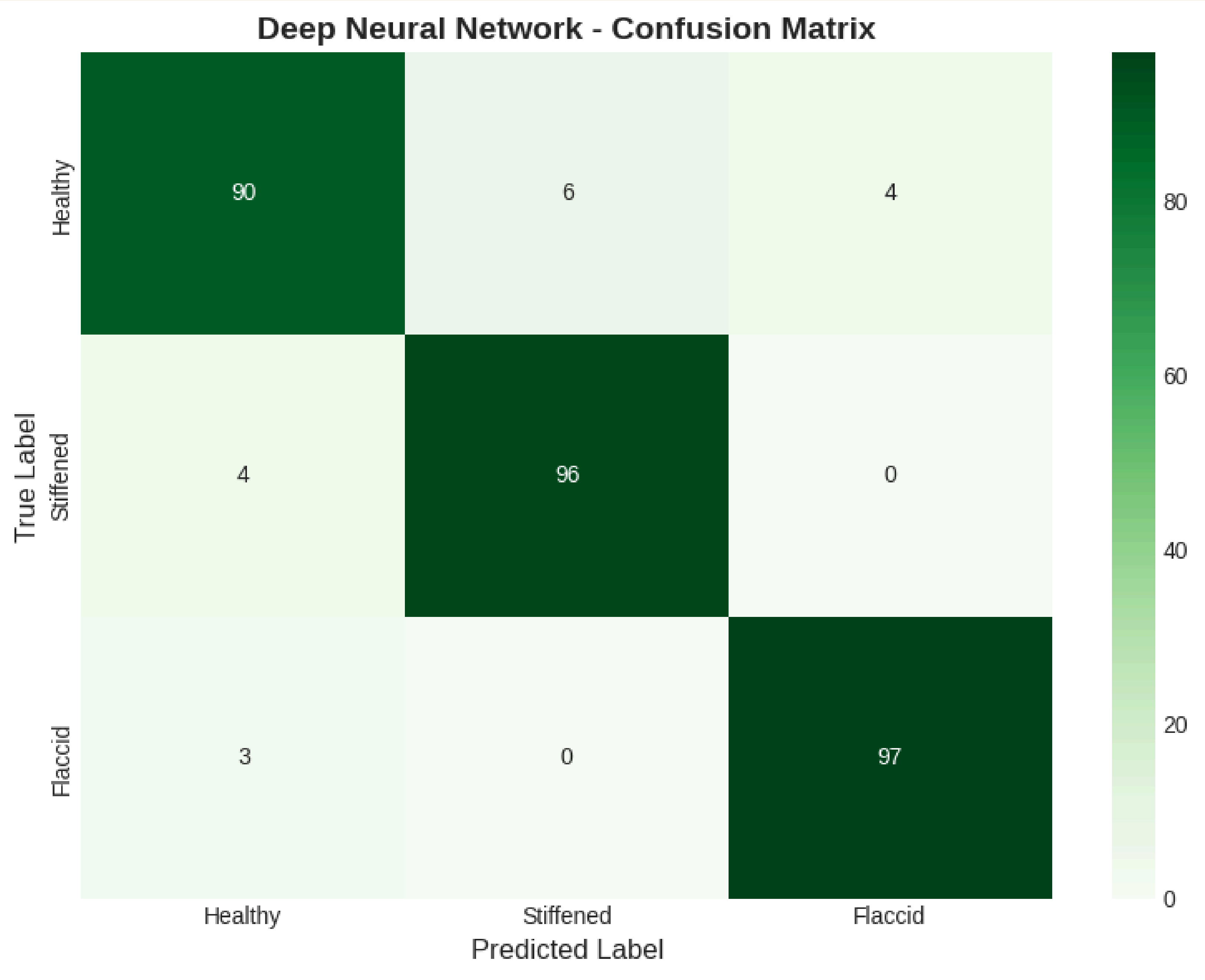


Neural network training and validation loss over 80 epochs.

Performence

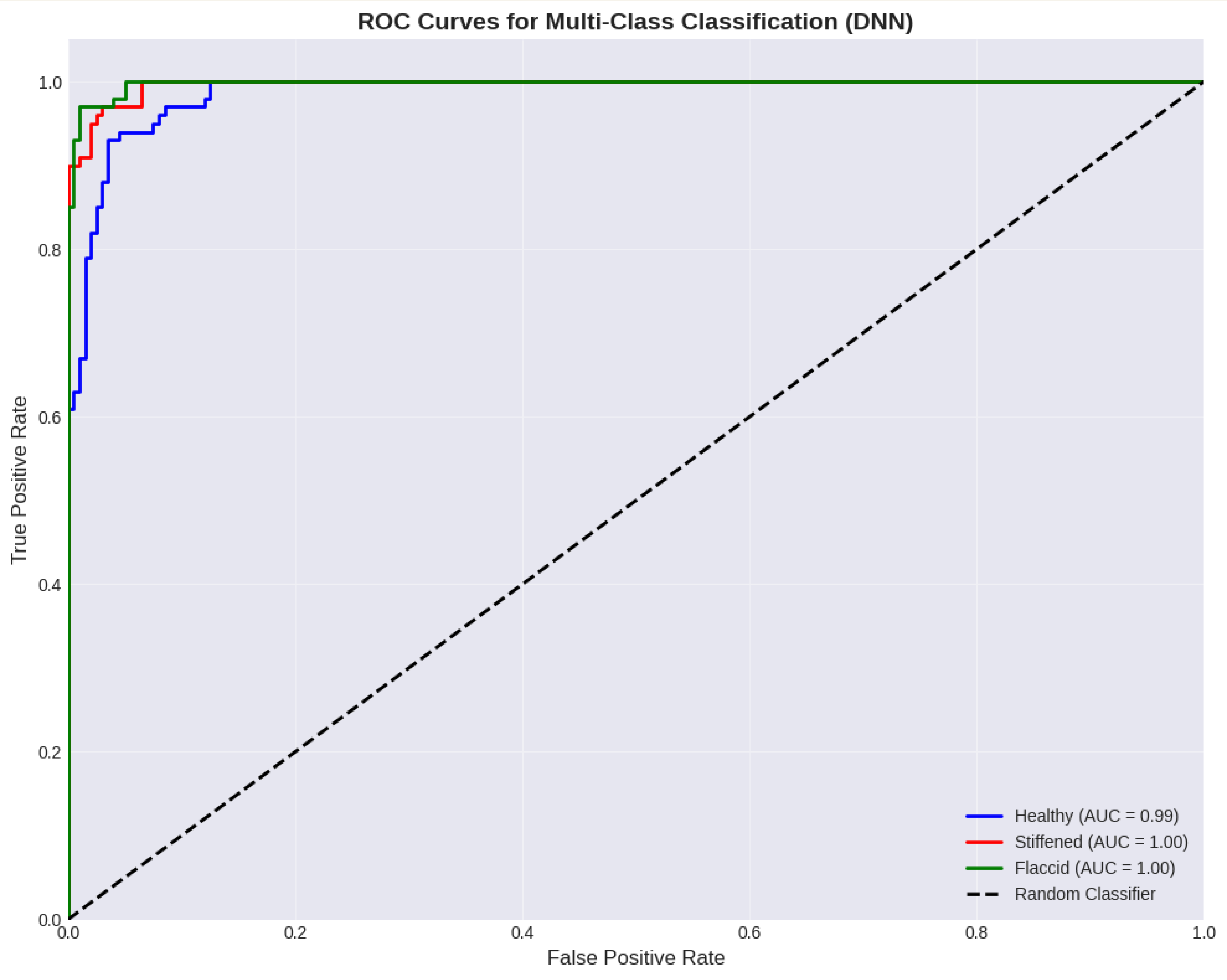
Achieved >93% accuracy !

1. Confusion Matrix



Deep learning Confusion Matrix

2. ROC Curves



ROC CURVES FOR MULTI-CLASS CLASSIFICATION

IMPLEMENTED APPLICATIONS

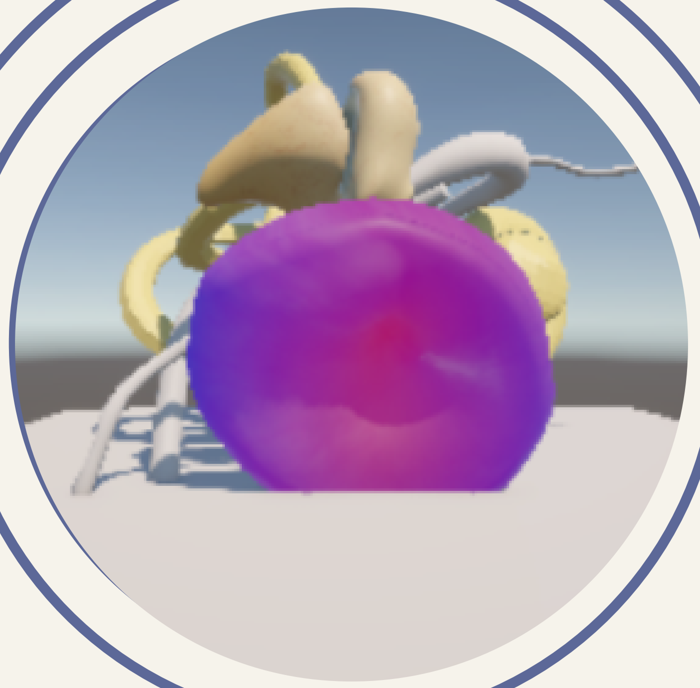
2.Unity Interactive Application

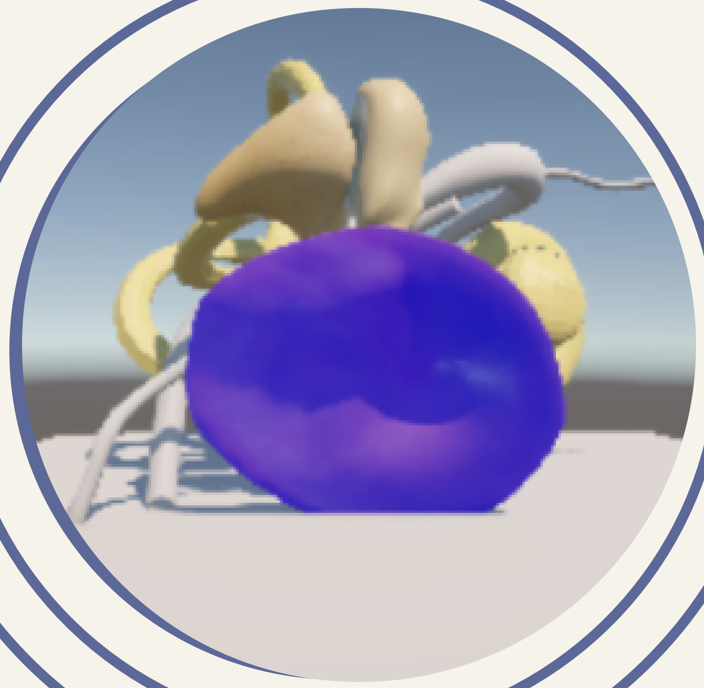
Objective: Intuitive exploration of auditory biomechanics in a VR environment.

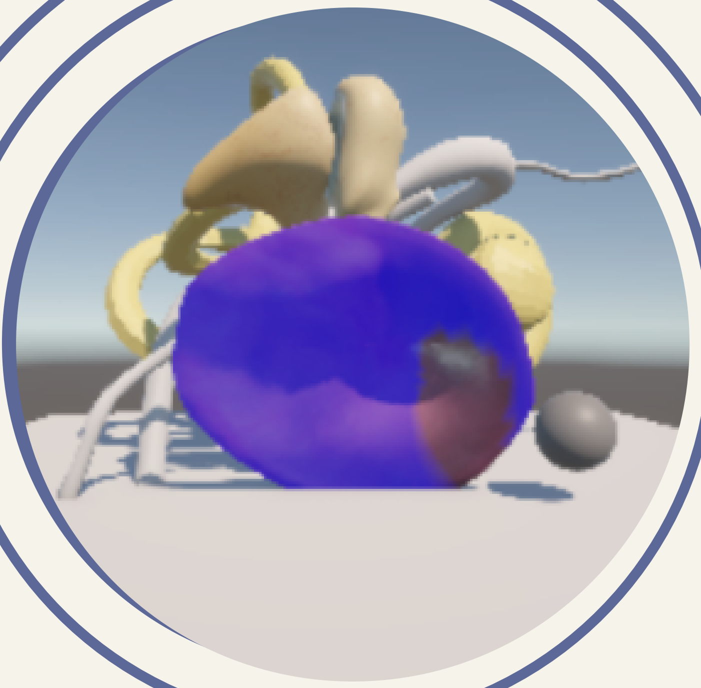
- **Analytical Core:** Vibration is driven by a radial-dependent sinusoidal model to ensure computational efficiency for real-time rendering.
- **Interactive Pathology:**
  - Factor P(r): Represents tissue health (1 = healthy, <1 = damaged).
  - Tearing Mechanism: Exceeding force thresholds creates visual holes and alters local vibration response.
- **Visual Feedback:** Dynamic color overlays (Red = Max Displacement, Transparent = Tear) provide immediate user feedback.
- **Clinical Potential:** Serves as a foundational tool for surgical training and pre-operative planning.

unity

Unity Interactive Application

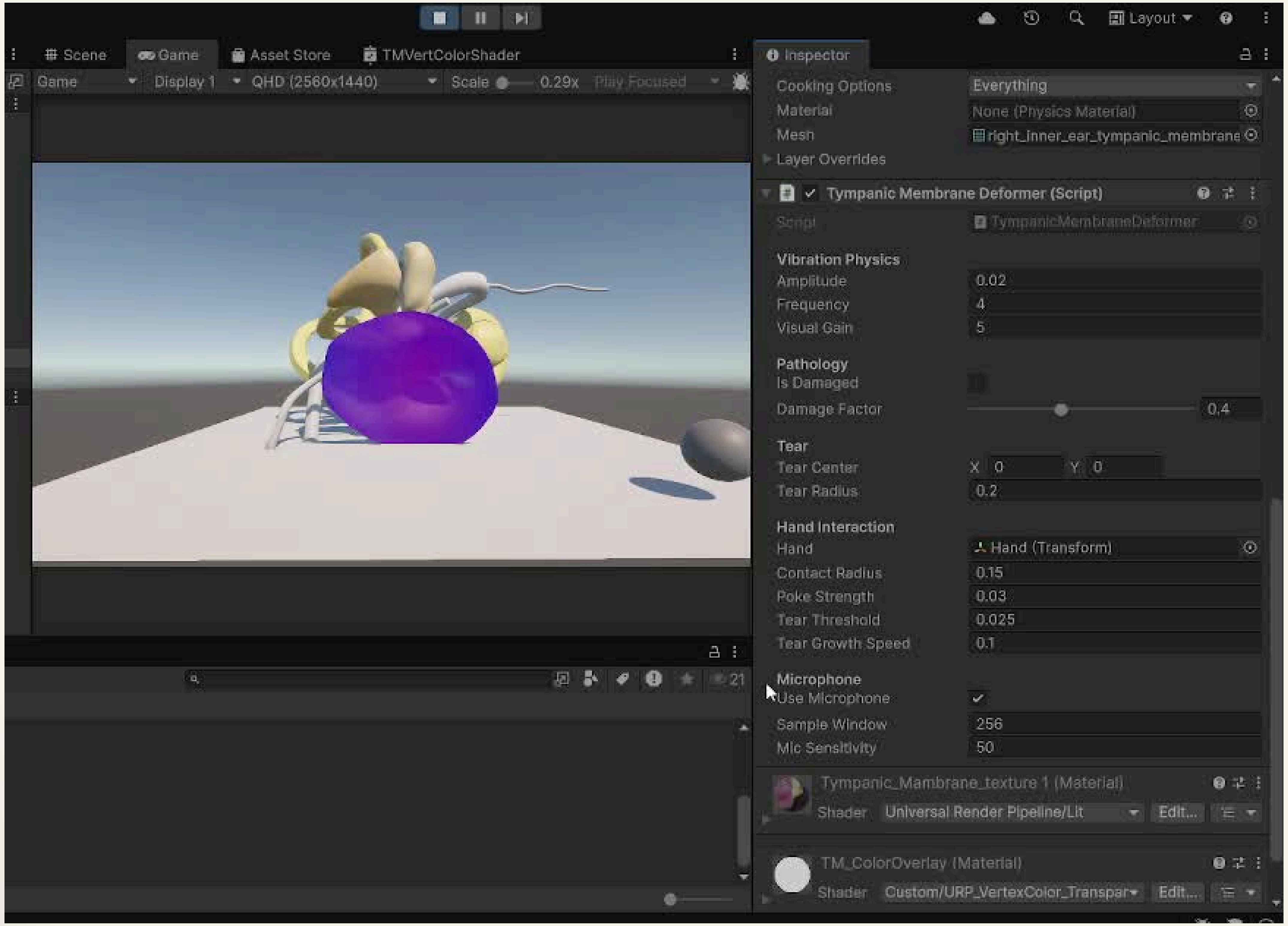




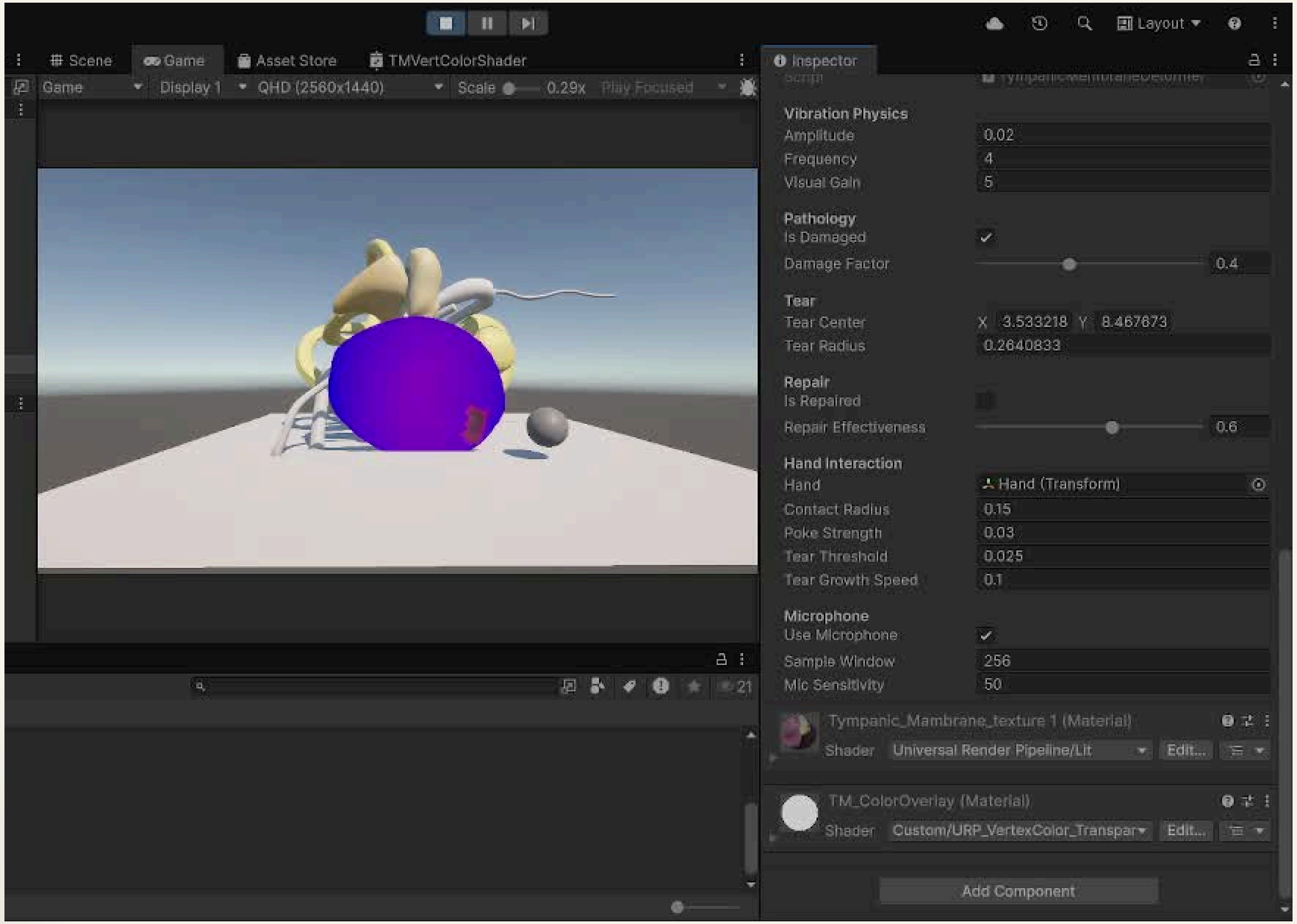


unity

Video Demonstration



Video Demonstration



Conclusion

This study establishes a unified multimodal framework for analyzing tympanic membrane dynamics by integrating analytical modeling, numerical simulation, and machine learning. By validating a sectorial annulus model against **Finite Difference** and **COMSOL** methods, we confirmed that simplified mathematical models can accurately capture dominant vibration behaviors.

Beyond theory, we demonstrated practical applications through a **physics-informed neural network** for automated pathology diagnosis and a real-time **Unity simulation** for intuitive visualization. Together, these approaches provide a robust foundation for future advancements in **patient-specific modeling**, **clinical diagnostics,** and **immersive surgical training tools**.

Reference

[1] C. Wu, Y. Chen, M. S. H. Al-Furjan, J. Ni, and X. Yang,“Free vibration model and theoretical solution of the tympanic membrane,” Computer Assisted Surgery, vol. 21, pp. 62-69, 2016,doi: 10.1080/24699322.2016.1240315.

[2] J. Garcia-Manrique, C. Furlong, A. Gonzalez-Herrera, and J. T. Cheng, “Numerical model characterization of the sound transmission mechanism in the tympanic membrane from a high-speed digital holographic experiment in transient regime,”Acta Biomaterialia, vol. 159, pp. 63-73, Mar. 2023, doi:10.1016/j.actbio.2023.01.048.

[3] P. J. Prendergast, P. Ferris, H. J. Rice, and A. W. Blayney, “3-4; ProQuest Nursing & Allied Health Source pg.”

[4] J. Shan, H. Yamazaki, J. Li, and S. Kawano, ”Theoretical and experimental study on traveling wave propagation characteristics ofartificial basilar membrane,” Scientific Reports, vol. 15, no. 24041,2025.

THANK YOU

For your attention