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MODELING AND SIMULATION OF TYMPANIC MEMBRANE VIBRATION

AcoustiMind Team

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ABSTRACT

This work studies the free vibration of the tympanic membrane using analytical ,numerical, PINNs, and interactive simulation methods. The membrane is modeled as a uniformly tensioned sectorial annulus to obtain natural frequencies and mode shapes, with applications in machine-learning-based diagnosis of membrane tension and pathology, and real-time 3D visualization in Unity.

LITERATURE REVIEW

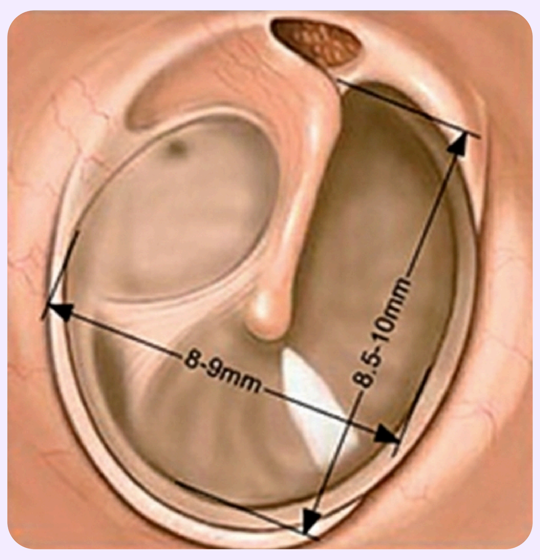
2016 This work studies tympanic membrane free vibration using analytical, numerical, and physics-informed approaches to extract natural frequencies and mode shapes

2023 This study combines transient experiments and finite element modeling to characterize human tympanic membrane vibration with high temporal and spatial accuracy.

2025 Machine-learning models applied to audiometry data can accurately diagnose Meniere's disease and predict endolymphatic hydrops.

MATHEMATICAL MODEL

The tympanic membrane (TM) is modeled as a thin, flexible sectorial annulus membrane under uniform tension, with fixed boundaries at the annular ligament. Bending stiffness and thickness variation are neglected.

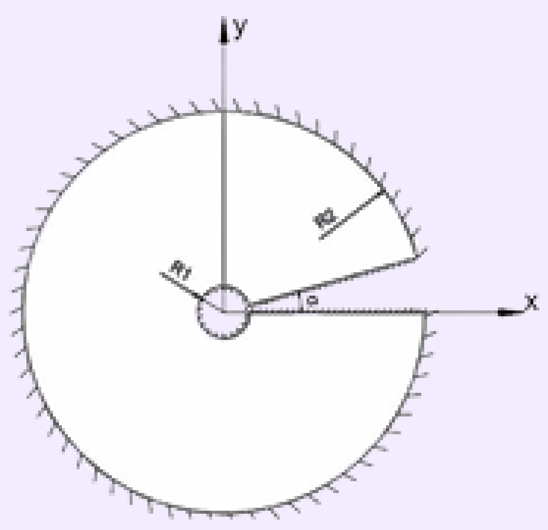


fig(1): The TM Structure diagram

The transverse free vibration of the TM is governed by the 2D membrane wave equation:

eq(1): $$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = \frac{\rho}{T}\frac{\partial^2 w}{\partial t^2}$$

Using separation of variables, the solution reduces to a Bessel eigenvalue problem, from which the natural frequencies and mode shapes are obtained.



fig(2): The Simplified TM Structure

The Free Vibration Model of the Tympanic Membrane:

eq(2): $$w(r,\theta,t) = \sum_{m,n} A_{mn} W_{mn}(r,\theta)\,\sin(\omega_{mn} t + \phi)$$

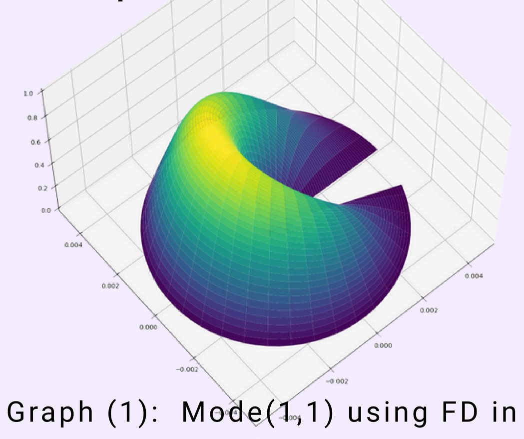
CONCLUSION

This work combines analytical, numerical, physics-informed, and real-time visualization approaches to study tympanic membrane vibration, showing that a simplified sectorial annulus model accurately captures key dynamics and supports diagnostic and interactive applications.

METHODOLOGY

FINITE DIFFERENCE

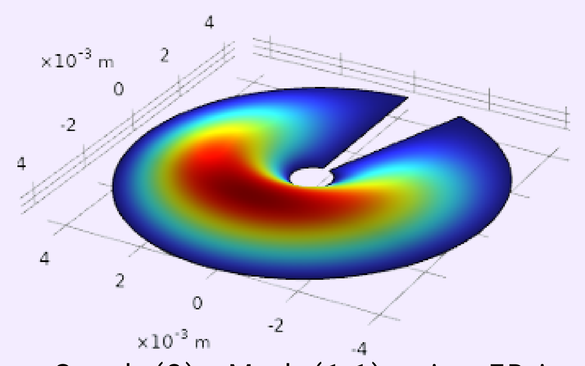
- TM discretized on a polar grid to fit circular geometry.
- Central finite differences applied to the wave equation.
- Eigenvalue problem solved for natural frequencies and mode shapes.



Graph (1): Mode(1,1) using FD in Python

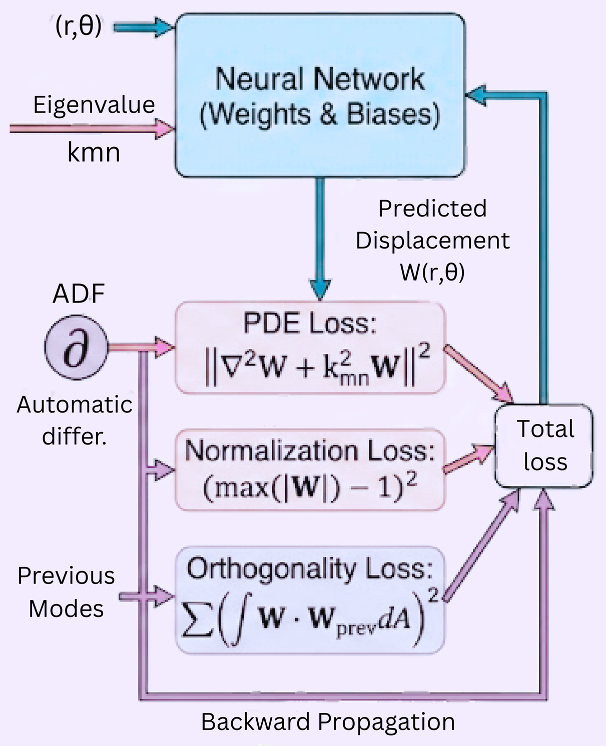
COMSOL SIMULATION

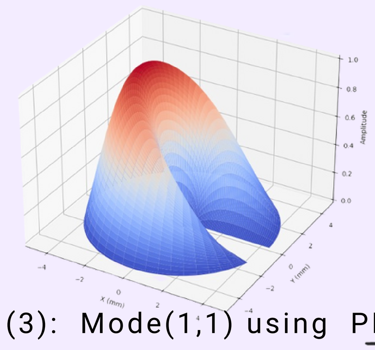
- FEM membrane model with same TM geometry and material.
- Fixed boundaries at annular ligament.
- Eigenfrequency study extracts natural frequencies and visualizes modes.



Graph (2): Mode(1,1) using FD in COMSOL

PINNS





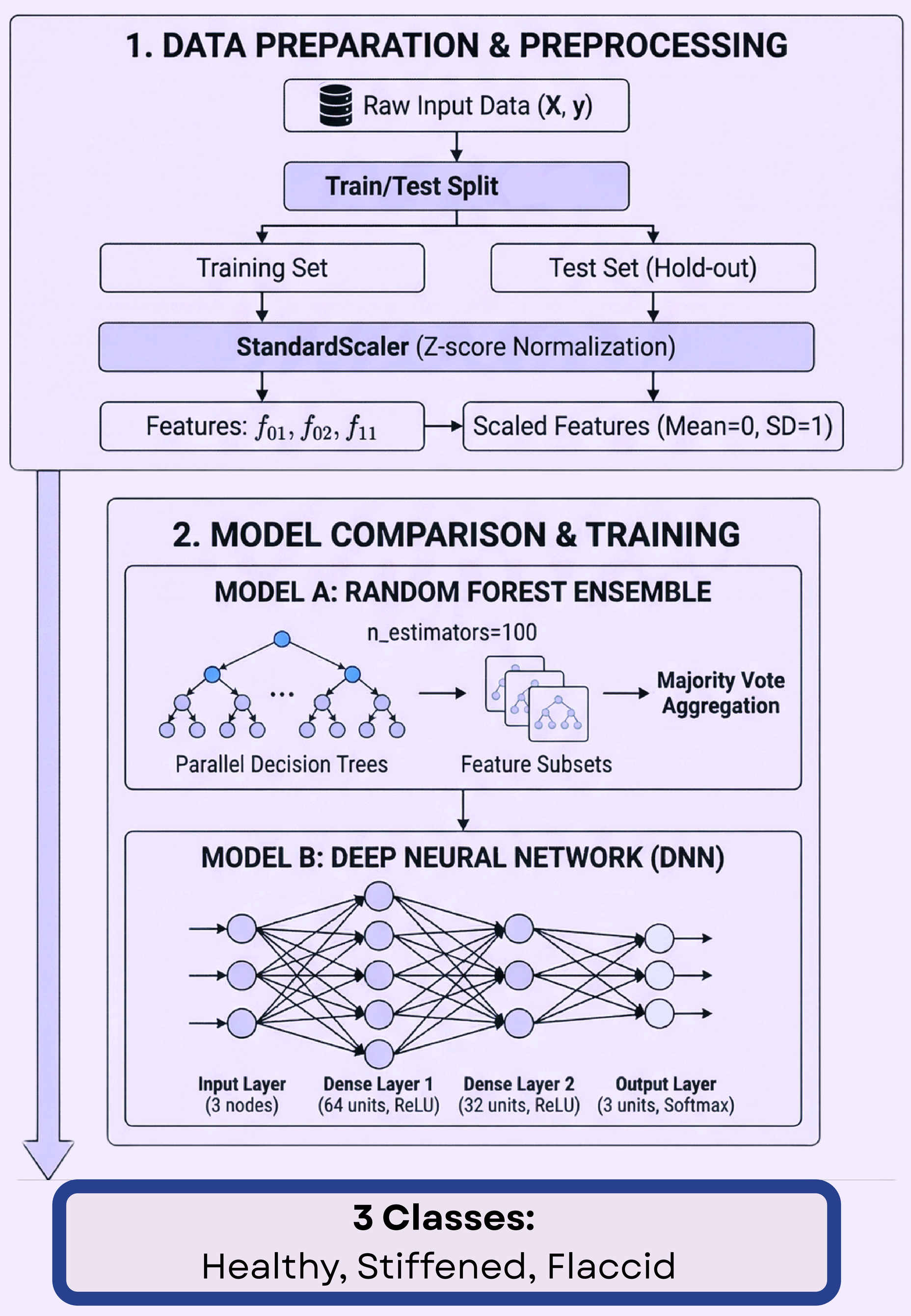
Graph (3): Mode(1,1) using PINNS

RESULTS AND ANALYSIS

1. **Numerical Accuracy (FDM)**
   - High Precision: The FDM solver achieved < 1.5 % average error.
2. **Advanced Modeling (PINN)**
   - Mesh-Free Solver: Successfully reconstructed mode shapes without a mesh with average error < 3% .
   - The PINN accurately reconstructs the continuous displacement field, reproducing sectorial vibration modes consistent with Bessel-function theory.

Analytical (Hz)	Numerical (Hz)	Error of Numerical %	Error of PINNs%
420.46	421.5	0.24	0.25
869.28	883.19	1.6	2.78
1352.2	1346	0.46	0.49

ML-BASED DIAGNOSTIC SYSYTEM

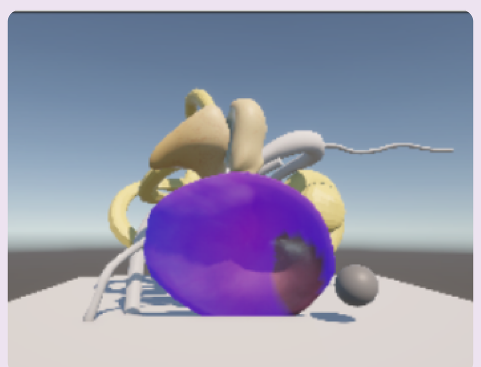


UNITY INTERACTIVE VR SIMULATION

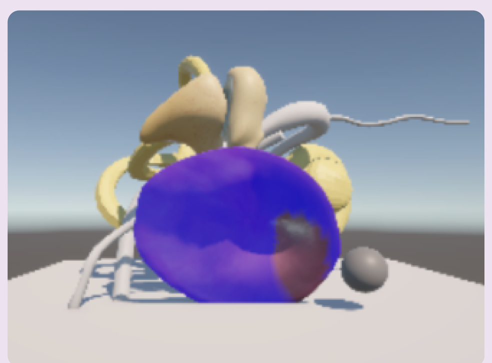
Real-time VR model visualizes TM vibration, modeled based on the PDE solution of the tympanic membrane:

eq(3): $$u(r,t) = A \cdot \sin(2\pi f t) \cdot \sin\left(\pi \frac{r}{R}\right) \cdot P(r)$$

- Pathology factor P(r) simulates damage, dynamically altering vibration.
- Users interact via virtual hands; color overlay shows deformation and tears.
- Supports exploration of healthy vs. injured states and surgical training.



Maximum displacement in TM after damage



Damaged TM with no sound input

FUTURE WORK

- Extend the Unity model to immersive VR for surgical training.
- Enable patient-specific modeling using CT or OCT imaging.
- Integrate augmented reality for real-time tympanic membrane visualization to enhance training, diagnosis, and surgical planning.

REFERENCES

[1] C. Wu, Y. Chen, M. S. H. Al-Furjan, J. Ni, and X. Yang, "Free vibration model and theoretical solution of the tympanic membrane," Computer Assisted Surgery, vol. 21, pp. 62–69, 2016, doi: 10.1080/24699322.2016.1240315.

[2] J. Shan, H. Yamazaki, J. Li, and S. Kawano, "Theoretical and experimental study on traveling wave propagation characteristics of artificial

[3] H. G. Liu, N. Ta, and Z. S. Rao, "Numerical modeling of human middle ear biomechanics using finite element method," J Donghua Univ, 2021;28:115–118.