Series Solution for Unsteady Gas Equation with MLDM

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Physic:

1. about the modified Laplace decomposition method (MLDM)

The technique is based on the application of Laplace transform to boundary layers in fluid mechanics. The nonlinear terms can be easily handled by the use of Adomian polynomials. The obtained series solution is combined with the diagonal Padé approximants to handle the boundary condition at infinity.

2. where does this equation come from?

In 2006, Agadjanov developed this method for the solution of Duffing equation. The Laplace decomposition method (LDM) was proved to be compatible with the versatile nature of the physical problems and was applied to a wide class of functional equations Recently a reliable modification of the Laplace decomposition, recently a reliable modification of the Laplace decomposition algorithm has been done by Yasir.

3. Modified Laplace Decomposition Method (MLDM) equation general form

in order to elucidate the solution procedure of the modified Laplace decomposition method, we consider the following general form of second-order nonlinear ordinary differential equation with initial conditions is given:

$$f'' + b_1(x)f' + b_2(x)f = g(y)$$

$$f(0) = \alpha, f'(0) = \beta$$

According to the Laplace decomposition method, apply Laplace transform on both sides of Eq(Laplace denoted by L):

$$s_2L[f] - s_1\alpha - \beta + L[b_1(x)f'] + L[b_2(x)f] = L[g(y)]$$

Using the differentiation property of Laplace transform, we have:

$$L[f]=\frac{\alpha}{s}+\frac{\beta}{s^2}+\frac{1}{s^2}L[g(y)]-\frac{1}{s^2}L[b_1(x)f'+b_2(x)f].$$

The Laplace decomposition method admits a solution in the form:

$$f=\sum_{m=0}^{\infty} f_m.$$

The nonlinear term is decomposed as:

$$g(y)=\sum_{m=0}^{\infty} A_m,$$

where Am are Adomian polynomials of g0 1 2 3,g ,g ,g ,...,gn and it can be calculated by the following formula:

$$A_m=\frac{1}{n!}\frac{d^m}{d\lambda^m}\left[N\left(\sum_{m=0}^{\infty}\lambda^i y_i)\right)\right]_{\lambda=0}, \quad m=0,1,2,3.......$$

Using the four previous equations:

$$L\left[\sum_{m=0}^{\infty} f_m\right]=\alpha+\frac{\beta}{s^2}+\frac{1}{s^2}L\left[\sum_{m=0}^{\infty} A_m\right]-\frac{1}{s^2}L\left[b_1(x)\sum_{m=0}^{\infty} f'_m+b_2(x)\sum_{m=0}^{\infty} f_m\right].$$

By matching the two sides of the above equation, we have the following relation:

$$L[f_0]=\alpha+\frac{\beta}{s^2},$$

$$L[f_1]=\frac{1}{s^2}L[A_0]-\frac{1}{s^2}L[b_1(x)f'_0+b_2(x)f_0],$$
$$L[f_2]=\frac{1}{s^2}L[A_1]-\frac{1}{s^2}L[b_1(x)f'_1+b_2(x)f_1].$$

In general, the recursive relation is given by:

$$L\left[f_{m+1}\right]=\frac{1}{s^2}L\left[A_n\right]-\frac{1}{s^2}L\left[b_1(x)f_m'+b_2(x)f_m\right],\quad m\geq 0.$$

By taking the inverse Laplace transform from both sides of the previous equations, one obtained:

$$f_0(x)=H(x),$$

$$f_{m+1}(x)=L^{-1}\left[\frac{1}{s^2}\mathrm{L}\left[A_m\right]-\frac{1}{s^2}L\left[b_1(x)f_m'+b_2(x)f_m\right]\right],\quad m\geq 0.$$

H(x) represents the term arising from the source term and prescribes the initial condition. The modified Laplace decomposition method suggests that the function H(x) defined above be decomposed into two parts, namely H0(x) and H1(x). Such that:

$$H(x)=H_0(x)+H_1(x).$$

The initial solution is important and the choice of Eq. Three equations before as the initial solution always lead to noise oscillation during the iteration procedure. Instead of the iteration procedure, Eqs Previous, we suggest the following modification:

$$f_0(x)=H_0(x),$$
$$f_1(x)=H_1(x)+L^{-1}\left[\frac{1}{s^2}L\left[A_0\right]-\frac{1}{s^2}L\left[b_1(x)f_0'+b_2(x)f_0\right]\right],$$
$$f_{m+1}(x)=L^{-1}\left[\frac{1}{s^2}L\left[A_m\right]-\frac{1}{s^2}L\left[b_1(x)f_m'+b_3(x)f_m\right]\right],\quad m\geq 1.$$

The solution through the modified Laplace decomposition method highly depends upon the choice of H0(x) and H1(x).

Review:

CSRBF method:
Many problems in science and engineering arise in infinite and semi-infinite domains. Different numerical methods have been proposed for solving problems in various domains such as FEM, FDM and Spectral methods, and meshfree method. The use of the RBF is one of the popular meshfree methods for solving differential equations For many years the global radial basis functions such as Gaussian, Multi quadric, Thin plate spline, Inverse multiqudric and etc was used These functions are globally supported and generate a system of equations with the ill-condition full matrix. To convert the ill-condition matrix to a well-condition matrix, CSRBFs can be used instead of global RBFs. CSRBFs can convert

the global scheme into a local one with banded matrices, Which makes the RBF method more feasible for solving a large-scale problem.
The one-dimensional function y(x) to be interpolated or approximated can be represented by a CSRBF as:

$$y(x) \approx y_n(x) = \sum_{i=1}^{N} \xi_i \phi_i(x) = \Phi^T(x)\Xi,$$

Where

$$\phi_i(x) = \phi(\frac{\|x - x_i\|}{r_\omega}),$$
$$\Phi^T(x) = [\phi_1(x), \phi_2(x), \cdots, \phi_N(x)],$$
$$\Xi = [\xi_1, \xi_2, \cdots, \xi_N]^T,$$

x is the input, rω is the local support domain and ξis is the set of coefficients to be determined. By using the local support domain, we mapped the domain of the problem to the CSRBF local domain. by choosing N to interpolate points (x j, j = 1, 2, · · ·, N) in domain:

$$y_j = \sum_{i=1}^{N} \xi_i \phi_i(x_j)(j = 1, 2, \cdots, N).$$

To summarize the discussion on the coefficients matrix, we define:

$$A\Xi = Y,$$

Where:

$$Y = [y_1, y_2, \cdots, y_N]^T,$$
$$A = [\Phi^T(x_1), \Phi^T(x_2), \cdots, \Phi^T(x_N)]^T$$
$$= \begin{pmatrix} \phi_1(x_1) & \phi_2(x_1) & \cdots & \phi_N(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \cdots & \phi_N(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(x_N) & \phi_2(x_N) & \cdots & \phi_N(x_N) \end{pmatrix}.$$

note that φi(x j) = φ(‖xi − x j‖/rω), by solving the system, the unknown coefficients ξi will be achieved.

References:

[1] R. E. Kidder, Unsteady flow of gas through a semi-infinite porous medium, J. Appl. Mech. 24 (1957) 329–332.

[2] R. P. Agrawal, D. O'Regan, Infinite interval problems modeling the flow of a gas through a semi-infinite porous medium, Studies in Applied Mathematics, 108 (2002) 245–257.

[3] R. C. Roberts, Unsteady flow of gas through a semi-infinite porous medium, Proceeding of the First US National Congress of Applied Mechanics, Ann Arbor, MI (1952) 773–776.

[4] A. M. Wazwaz, The modified decomposition method applied to the unsteady flow of gas through a porous medium, Appl. Math. Comput. 118 (2001) 123–132.

[5] A. Shokri, M. Dehghan, 2010. A not-a-knot meshless method using radial basis functions and predictor-corrector scheme to the numerical solution of improved Boussinesq equation. Comput. Phys. Commun. 181, 1990–2000.

[6] K. Rashidi, H. Adibi, J. A. Rad, K. Parand, 2014. Application of meshfree methods for solving the inverse onedimensional Stefan problem. Eng. Anal. Bound. Elem. 40, 1–21.

[7] K. Parand, A. Taghavi, H. Fani, a Lagrangian method for solving the unsteady gas equation, Int. j. Comput. Meth. Sci. 3 (2009) 40–44.

[8] Mohyud-Din, S.T., M.A. Noor and K.I. Noor, 2009. Homotopy perturbation method for the unsteady flow of gas through a porous medium, Int. J. Mod. Phys. B.

[9] Khuri, S.A., 2001. A Laplace decomposition algorithm applied to a class of nonlinear differential equations, J. Appl. Mathematics, 1 (4): 141-155.

[10] Yusufoglu, E., 2006. Numerical solution of Duffingequation by the Laplace decomposition algorithm, Appl. Math. Comput., 177: 572-580.

[11] Yasir Khan and Naeem Faraz, 2010. A new approach to differential-difference equations, J. Adv. Res. Differ. Equ., 2(2): 1-12.

[12]. Islam, S., Y. Khan, N. Faraz and F. Austin, 2010. Numerical Solution of Logistic Differential Equations by using the Laplace Decomposition Method, World Applied Sciences J., 8(9): 1100-1105.

[13] J. A. Rad,S. Kazem,M. Shaban, K. Parand,A. Yildirim, 2014. Numerical solution of fractional differential equations with a tau method based on legendre and bernstein polynomials. Math. Meth. Appl. Sci. 37, 329–342.

[14] S. Kazem, J. A. Rad, K. Parand, M. Shaban, H. Saberi, The numerical study on the unsteady flow of gas in a semi-infinite porous medium using an RBF collocation method, Int. J. Computer Math. 89 (2012) 2240–2258.

[15] M. A. Noor, S. T. Mohyud-Din, Variational iteration method for unsteady flow of gas through a porous medium using hes polynomials and pade approximants, Comput. Math. Appl. 58 (2009) 2182–2189.´

[16] Y. Khan, N. Faraz, A. Yildirim, Series solution for unsteady gas equation via medium-pade technique, World Applied Sciences Journal 9 9 (2010) 1818–4952.

[17] S. Upadhyay, K. N. Rai, Collocation method applied to the unsteady flow of gas through a porous medium, Comput. Math. Appl. Res. 3 (2014) 251–259.

[18] H. Roohani-Ghehsareh, S. H. Bateni, A. Zaghian, A meshfree method based on the radial basis functions for the solution of two-dimensional fractional evolution equation, Eng. Anal. Bound. Elem. 61 (2015) 52–60.

[19] N. S. O'Brien, K. Djidjili, S. J. Cox, Solving an eigenvalue problem on a periodic domain using a radial basis function finite-difference scheme, Eng. Anal. Bound. Elem. 37 (2013) 1594–1601.

[20] C. A. Bustamante, H. Power, Y. H. Sua, W. F. Florez, A global meshless collocation particular solution method (integrated radial basis function) for two-dimensional stokes flow problems, Appl. Math. Model. 37 (2013) 4538–4547.