Q7

Exercise 3.1.1

(a):

solution: True

27 is a multiple of 3, and $27 \in Z$. Therefore $27 \in A$

(b):

solution: False

27 is not a perfect square number

(c):

solution: True

100 is a perfect square of 10/-10, $100 \in Z$. Therefore $100 \in B$.

(d):

solution: False

$E \subseteq C$ is false, and $C \subseteq E$ is false. false or false is false

(e):

solution: True

All the elements of E are multiples of 3 and set A contains all the elements. Therefore $E \subseteq A$

(f):

solution : False

$12 \in A$, but 12 is not an element in E. Hence its false (g):

solution: False

All of elements of A are multiples of 3, and it does not contain E as an element

Exercise 3.1.2

(a):

solution: False

15 is a single element instead of a set , $15 \in A$

(b):

solution: True

A = {..., -9, -6, -3, 0, 3, 6, 9, 12, 15....}

$\{15\} \subseteq A$, and at least one element in A and is not {15}. Hence $\{15\} \subset A$

(c):

solution: True

empty set is a proper subset of every set. Hence it is true

(d):

solution: True

every set is a subset of itself. Therefore its true

(e):

solution: False

$\varnothing$ is an empty set which cannot be an element of B

Exercise 3.1.5

(b):

solution: { $x \in N$: x is a multiple of 3}; Infinite set

(d):

solution: { $x \in N$: x is a multiple of 10 and $0 \le x \le 1000$}; the cardinality is 101

Exercise 3.2.1

(a):

solution: True

(b):

solution: True

(c):

solution: False

$2 \in X$, and $\{2\} \subseteq X$

(d):

solution: False

$\{3\} \in X$ not 3

(e):

solution: True

{1, 2} is an element of X

(f):

solution: True

1, 2 $\in$ X, therefore {1, 2} is a subset of X

(g):

solution: True

2, 4 $\in$ X, therefore {2, 4} $\subseteq$ X

(h):

solution: False

{2, 4} $\subseteq$ X

(i):

solution: False

{2, {3}} $\subseteq$ X

(j):

solution: False

{3} $\in$ X

(k):

solution : False

There are six elements not seven

Q8

<u>Exercise 3.2.4</u>

(b):

solution: {{2}, {1,2}, {2,3}, {1,2,3}}

$P(A) = \{ \varnothing, \{ 1 \}, \{ 2 \}, \{ 3 \}, \{ 1, 2 \}, \{ 1, 3 \}, \{ 2, 3 \}, \{ 1, 2, 3 \} \}$

Since we only need subsets that contain the element 2, therefore:

$\{X \in P(A): 2 \in X\} = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

Q9

Exercise 3.3.1

(c):

solution: {-3, 1, 17}

(d):

solution: {-5, -3, 0, 1, 4,17}

B ∩ C = {-5, 1}

A ∪ (B ∩ C) = {-5, -1, 0, 1, 4, 17}

(e):

solution: {1}

A ∩ B = {1, 4}

A ∩ B ∩ C = {1}

Exercise 3.3.3

(a):

solution: {1}

$\bigcap_{i=2}^{5} A_i$ = A2 ∩ A3 ∩ A4 ∩ A5 = $\{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\}$

= {1, 2, 4} ∩{1, 3, 9} ∩{1, 4, 16} ∩ {1, 5, 25}

={ 1 }

(b):

solution: { 1, 2, 3, 4, 5, 9, 16, 25}

$\bigcup_{i=2}^{5} A_i$ = A2 ∪ A3 ∪ A4 ∪ A5 = $\{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\}$

= {1, 2, 4} ∪ {1, 3, 9} ∪{1, 4, 16} ∪ {1, 5, 25}

= { 1, 2, 3, 4, 5, 9, 16, 25}

(e):

solution: $\{x \in \mathbf{R} : -1/100 \le x \le 1/100\}$

$\bigcap_{i=1}^{100} C_i = \bigcap_{i=1}^{100} \{x \in \mathbf{R} : -1/i \le x \le 1/i\} = \{x \in \mathbf{R} : -1 \le x \le 1\} \cap \{x \in \mathbf{R} : -1/2 \le x \le 1/2\} \cap$
$\{x \in \mathbf{R} : -1/3 \le x \le 1/3\}$....

$\cap \{x \in \mathbf{R} : -1/100 \le x \le 1/100\}$

for all i (1...100), $\{x \in \mathbf{R} : -1/100 \le x \le 1/100\}$ is a subset of all the other sets.

(f):

solution: $\{x \in \mathbf{R} : -1 \le x \le 1\}$

$\bigcup_{i=1}^{100} C_i = \bigcap_{i=1}^{100} \{x \in \mathbf{R} : -1/i \le x \le 1/i\} = \{x \in \mathbf{R} : -1 \le x \le 1\} \cup \{x \in \mathbf{R} : -1/2 \le x \le 1/2\} \cup$
$\{x \in \mathbf{R} : -1/3 \le x \le 1/3\}$...

$\cup \{x \in \mathbf{R} : -1/100 \le x \le 1/100\}$

for all (1...100), $\{x \in \mathbf{R} : -1 \le x \le 1\}$ contains all the elements in all other sets.

Exercise 3.3.4

(b):

solution: $P(A \cup B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\},\{b, c\},\{a, b, c\}\}$

$A \cup B = \{a, b, c\}$

There are 8 (2^3)subsets of $A \cup B$, therefore the power set $P(A \cup B)$ is $\{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\},\{b, c\},\{a, b, c\}\}$

(d):

solution: $P(A) \cup P(B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}]$

$P(A) \cup P(B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Q10

Exercise 3.5.1

(b):

solution: One example: (foam, tall, non-fat)

$B \times A \times C$ contains all ordered triples in which the first entry in the triple is in B, the second entry is in A, and the third entry is in C. Foam $\in$ B, tall $\in$ A, and non-fat $\in$ C so (foam, tall, non-fat) is an element of $B \times A \times C$

(c):

solution: {(foam, non-fat), (foam, whole), (non-foam, non-fat), (non-foam, fat)}

Exercise 3.5.3

(b):

solution: True

$Z^2 = \{ (a, b): a, b \in Z \}$

$Z \subseteq R$

for every $a, b \in Z$, $a, b \subseteq R$

therefore:

$Z^2 \subseteq R^2$

(c):

solution : True

The elements in Z^2 are pairs. The elements in Z^3 are triples. Therefore the two sets have no elements in common.

(e):

solution: True

for any subset $(a, c) \in A \times C$, then $a \in A$ and $c \in C$

$A \in B$, therefore $\forall\ a \in A \Rightarrow a \in B$

$a \in B$ and $c \in C$

therefore: $(a, c) \in B \times C$

(a, c) is an arbitrary subset of $A \times C$. therefore $\forall\ (a, c) \in A \times C \Rightarrow (a, c) \in B \times C$

Hence: $A \times C \subseteq B \times C$

Exercise 3.5.6

(d):

solution: {01, 011, 001, 0011}

$\{0\} \cup \{0\}^2 = \{0\} \cup \{00\} = \{0, 00\}$

$x \in \{0, 00\}$

{1} ∪ {1}2} = {1} ∪ {1, 11} = {1, 11}

$y \in \{1,11\}$

{xy: where x ∈ {0} ∪ {0}2 and y ∈ {1} ∪ {1}2} = {01, 011, 001, 0011}

(e):

solution: { aaa, aaaa, aba, abaa}

y ∈ {a} ∪ {a}2

$y \in \{a, aa\}$

$\{xy: x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\} = \{ aaa, aaaa, aba, abaa\}$

<u>Exercise 3.5.7</u>

(c):

solution: {aa, ab, ac, ad}

$A \times B = \{ab, ac\}$

$A \times C = \{aa, ab, ad\}$

$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

(f):

solution: { ∅, {ab}, {ac}, {abac}}

$A \times B = \{ab, ac\}$

$P(A \times B) = \{ \emptyset, \{ab\}, \{ac\}, \{abac\}\}$

(g):

solution: { (a, b), (a, c), (a, b, c)}

$P(A) = \{\emptyset, \{a\}\}$

$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \times P(B) = \{ (a, b), (a, c), (a, b, c)\}$

Q11

Exercise 3.6.2

(b):

solution:

$(B \cup A) \cap (\bar{B} \cup A)$	
$(A \cup B) \cap (A \cup \bar{B})$	Commutative laws
$A \cup (B \cap \bar{B})$	Distributive laws
$A \cup \varnothing$	Complement laws
A	Identity laws

(c):

solution:

$\overline{A \cap \bar{B}}$	
$\bar{A} \cup \bar{\bar{B}}$	De Morgan's laws
$\bar{A} \cup B$	Double Complement laws

Exercise 3.6.3

(b):

solution: False

if A = {a, b}, and B = {b}, then A - (B ∩ A) = {a}, which is not equal to A (d):

solution: False

if A = {a, b}, and B = {a, b, c}, then (B-A) ∪ A = {a, b, c}, which is not equal to A

Exercise 3.6.4

(b):

solution:

$A \cap (B - A)$	
$A \cap (B \cap \bar{A})$	Set subtraction laws
$(B \cap \bar{A}) \cap A$	Commutative laws
$B \cap (\bar{A} \cap A)$	Associative laws
$B \cap \varnothing$	Complement laws
$\varnothing$	Domination laws

(c):

solution:

$A \cup (B - A)$	
$A \cup (B \cap \bar{A})$	Set subtraction laws
$(A \cup B) \cap (A \cup \bar{A})$	Distributive laws
$(A \cup B) \cap U$	Complement laws
$A \cup B$	Identity laws