Q5

1. Exercise 1.12.2 section b, e:

section (b):

1	$p \to (q \land r)$	Hypothesis
2	$\neg q$	Hypothesis
3	$\neg q \lor \neg r$	Addition, 2
4	$\neg (q \land r)$	De Morgans, 3
5	$\neg p$	Modus tollens, 1, 4

section (e):

1	$p \lor q$	Hypothesis
2	$q \lor p$	Commutation, 1
3	$\neg q \to p$	Conditional , 2
4	$\neg p \lor r$	Hypothesis
5	$p \to r$	Conditional, 4
6	$\neg q \to r$	Hypothetical Syllogism, 3, 5
7	$\neg q$	Hypothesis
8	r	Modus Ponens, 6, 7

2. Exercise 1.12.3 section c:

section (c):

1	$p \lor q$	Hypothesis
2	$\neg p$	Hypothesis
3	$\neg p \to q$	Conditional, 1
4	q	Modus Ponens, 3, 2

3. Exercise 1.12.5 section c, d

section(c):

The form of the argument is:

$$(c \land h) \to j$$
$$\neg j$$
$$\therefore \neg c$$

Not valid:

1	$\neg j$	Hypothesis
2	$(c \land h) \to j$	Hypothesis
3	$\neg (c \land h)$	Modus tollens, 1,2
4	$\neg c \lor \neg h$	De Morgan, 3

Not valid = when using truth tables, when c =T and h=j=F, the hypotheses of $(c \land h) \to j$ and $\neg j$ are true while the conclusion $\neg c$ is false.

section(d):

The form of the argument is:

$$(c \land h) \to j$$
$$\neg j$$
$$h$$
$$\therefore \neg c$$

Valid:

1	$\neg j$	Hypothesis
2	$(c \land h) \to j$	Hypothesis
3	$\neg (c \land h)$	Modus tollens, 1,2
4	$\neg c \lor \neg h$	DeMorgans, 3
5	$\neg h \lor \neg c$	Commutation, 4
6	$h \to \neg c$	Conditional, 5

7	h	Hypothesis
8	$\neg c$	Modus Ponens, 6, 7

b) Exercise 1.13.3 section b:

$\exists x\ (P(x) \vee Q(x))$
$\exists x\ \neg Q(x)$
$\therefore\ \exists x\ P(x)$

	P	Q
a	F	T
b	F	F

$\exists x\ (P(x) \vee Q(x))$ is true because P(a) is false and Q(b) is true, the statement is true. $\exists x\ \neg Q(x)$ is true because Q(b) is false therefore $\neg Q(b)$ is true
However, since P(a) = P(b) is false, the statement $\exists x\ P(x)$ is false. Both hypothesis is true but the conclusion is false

2. Exercise 1.13.5
section(d):

M(x): x missed class
D(x): x got a detention

$\forall x(M(x) \rightarrow D(x))$
Penelope, a student in class
$\neg M(\text{Penelope})$
$\therefore\ \neg D(\text{Penelope})$

Invalid
The argument is not valid. M(Penelope) $\rightarrow$ D(Penelope) is true based on t=universal instantiation. However, when D(Penelope) = $\neg$M(Penelope) = T, the hypotheses are true and the conclusion is false. When Penelope did not miss the class and got a detention

section (e):

$\forall x\ (M(x) \vee D(x)\ \rightarrow \neg A(x))$
Penelope, a student in class
A(Penelope)
$\therefore\ \neg D(\text{Penelope})$

Valid

1	$\forall x\ (M(x) \vee D(x) \rightarrow \neg A(x))$	Hypothesis
2	Penelope, a student in class	Hypothesis
3	$(M(\text{Penelope}) \vee D(\text{Penelope}) \rightarrow \neg A(\text{Penelope})$	Universal instantiation,1,2
4	A(Penelope)	Hypothesis
5	$\neg\neg A(\text{Penelope})$	Double negation, 4
6	$\neg(M(\text{Penelope}) \vee D(\text{Penelope}))$	Modus Tollens, 3,5
7	$\neg M(\text{Penelope}) \wedge \neg D(\text{Penelope})$	DeMorgans, 6
8	$\neg D(\text{Penelope}) \wedge \neg M(\text{Penelope})$	Commutation, 7
9	$\neg D(\text{Penelope})$	Simplification, 8

Q6

<u>Exercise 2.2.1</u>
section(d):

Proof.
Direct proof. Assume that x and y are odd integers. We will show that the product of x * y is also an odd integer.
Since x , y are odd numbers, x = 2m + 1 for some integer m. y = 2n + 1 for some integer n. Plugging the two expressions for x, y into x * y.
(2m + 1) * (2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1
Since m, n are integers, then 2mn + m + n is also an integer, Since x * y = 2k + 1, where k = 2mn + m + n is an integer, x * y is equal to 2 times an integer plus 1, then x * y is also an odd integer. ■

section(c):

Proof.
Direct proof. Assume that x is a real number and $x \leq 3$. We will show that $12 - 7x + x2 \geq 0$.
Subtract x from both sides of the equality $3 \geq x$ to get $3 - x \geq 0$, Since $4 - x \geq 3 - x$, then
$4 - x \geq 3 - x \geq 0$
Since $4 - x \geq 0$ and $3 - x \geq 0$, then the product of 4 - x and 3 - x is at least 0:
$(4 - x)(3 - x) \geq 0$, then $12 -7x + x2 \geq 0$. ■

Q7

<u>Exercise 2.3.1</u>
section(d):

Proof.
Proof by contrapositive. We assume that n is an even integer and show that $n^2 - 2n + 7$ is an odd number.
If n is an even number, then n = 2k for some integer k. Plugging in the expression 2k for n into $n^2 - 2n + 7$ gives
$(2k)^2 - 2(2k) + 7 = 4k^2 - 4k + 7 = 2(2k^2 - 2k + 3) + 1$
Since k is an integer, $k^2 - 2k + 3$ is an integer. $n^2 - 2n + 7$ is equal to 2 times an integer plus 1. Therefore $n^2 - 2n + 7$ is an odd number. ■

section(f):

Proof:
Proof by contrapositive. We assume that for non-zero number x that 1/x is not irrational, and prove that x must be rational.
Since all real numbers are either rational or irrational, 1/x is not an irrational non-zero number, then 1/x is a rational non-zero number. if 1/x = a/b for a, b $\in$ R, then x = b/a where a, b $\in$ R. Therefore x is a non-zero rational number. ■

section(g):

Proof:
Proof by contrapositive. We assume for real numbers x and y that x > y, and prove that $x^3 + xy^2 > x^2y + y^3$.
Since x > y, then x, y can't be both zeros. Since $x^2 + y^2 > 0$ for every pair of real number x, y (x > y)
For the equality x > y, both sides time a positive number $x^2 + y^2 > 0$ does not change the equality.
$x*(x^2 + y^2) > y *(x^2 + y)$, then $x^3 + xy^2 > x^2y + y^3$. ■

section(l):

Proof:
Proof by contrapositive. We assume for real numbers x, y that it is not true that x > 10 or y > 10, and prove x + y ≤ 20.
Since it is not true that x > 10 or y < 10, then neither x or y is greater than 10 and therefore they are both less or equal to 10
x ≤ 10 and y ≤ 10, then x + y ≤ 20. ■

Q8

Exercise 2.4.1, section c, e

section(c):

Proof.

Proof by contradiction. Suppose that the average of three real numbers x, y, z is less than all three numbers.

$(x + y + z) /3 < x$, $(x +y +z) / 3< y$ and $(x + y + z) / 3< z$, then

$(x + y + z) / + (x + y + z) /3 + (x + y +z) /3 < x + y + z$

$x + y + z < x + y + z$, which contradicts the fact that $x + y + z = x + y + z$

section(e):

Proof.

Proof by contradiction. Suppose that there is a smallest integer k. k - 1 is an integer and k - 1 < k, which contradicts that k is the smallest integer. ■

Q9

Exercise 2.5.1 section c

Proof.
We consider two cases: both x, y are even and both x, y are odd.

Case 1: x is even and y is even, then $x = 2m$ for some integer m and $y = 2n$ for some integer n.
$x + y = 2m + 2n = 2(m + n)$
Since m, n are integers, $m + n$ is also an integer. Therefore $x + y$ is equal to 2 times an integer and $x + y$ is even.

Case 2: x is odd and y is odd, then $x = 2m + 1$ for some integer m and $y = 2n + 1$ for some integer n.

$x + y = (2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$

Since m, n are integers, $m + n + 1$ is also an integer. Therefore $x = y$ is equal to 2 times an integer and $x + y$ is even. ■