Q9

Exercise 4.1.3

(b):
Solution: Not a function
When x = 2 or -2 the value of the denominator is 0. Then f(x) remains undefined

(c):
Solution: Function
It is a function. R to R The range is N

Exercise 4.1.5

(b):
Solution: {4, 9, 16, 25}
f(2) = 2*2 = 4
f(3) = 3*3 = 9
f(4) = 4*4 = 16
f(5) = 5*5 = 25

(d):
Solution: {0,1,2,3,4,5}
f(00000) = 0
f(00001) = 1
f(00010) = 1
f(00011) = 2
f(00100) = 1
f(00101) = 2
f(00110) = 2
f(00111) = 3

f(01000) = 1
f(01001) = 2
f(01010) = 2

f(01011) = 3
f(01100) = 4
f(01101) = 3
f(01110) = 3
f(01111) = 4

f(10000) = 1
f(10001) = 2
f(10010) = 2
f(10011) = 3
f(10100) = 2
f(10101) = 3
f(10110) = 4
f(10111) = 4

f(11000) = 2
f(11001) = 3
f(11010) = 3
f(11011) = 4
f(11100) = 3
f(11101) = 4
f(11110) = 4
f(11111) = 5

(h) :

Solution: {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

A ×A = {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

f: A × A →Z×Z, where f(x, y) = (y, x)

The range is: {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

(i):

Solution: {(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)}

A ×A = {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

$f: A \times A \rightarrow Z \times Z$, where $f(x, y) = (x, y+1)$

The range is: $\{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

(l):

Solution: {{}, {2}, {3}, {2, 3}}

$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

$f: P(A) \rightarrow P(A)$. For $X \subseteq A$, $f(X) = X - \{1\}$

The range is $\{\{\}, \{2\}, \{3\}, \{2, 3\}\}$

Q10

<u>Exercise 4.2.2</u>

(c):

Solution: one-to-one but not onto

For example, there is no $x \in Z$ such that $h(x) = 3$.

(g):

Solution: one-to-one but not onto

For example there is no $x \in Z$ and $y \in Z$ such that $h(x, y) = (1, 3)$ Because 2y must be an even integer if $y \in Z$

<u>Exercise 4.2.4</u>

(b):

Solution: Neither one-to-one nor onto

For example, $f(000) = 100 = f(100)$ and there is no function $f(\{0, 1\}^3) = 000$

(c):

Solution: one-to-one and onto

$f: \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example $f(011) = 110$.

(d):

Solution: one-to-one but not onto

For example, there is no function $f(\{0, 1\}^3) = 0001$

(g):

Solution: Neither one-to-one nor onto

For example. $f(\{1, 2, 3\}) = \{2, 3\} = f(\{2, 3\})$ and there is no $f(P(A)) = \{1\}$

Exercise 4.2.5

one-to-one, but not onto $f: Z \longrightarrow Z+$. $f(x) = 3x + 3$

because there is no $f(x) = 2$

onto, but not one-to-one $f: Z \longrightarrow Z+$. $f(x) = |x|$

because $f(-1) = f(1) = 1$ and any positive integer y in the target, then $x = +- y$ is a real number

one-to-one and onto $f: Z \longrightarrow Z+$. $f(x) = 2|x|+1$ if $x <= 0$

$f(x) = 2|x|$ if $x > 0$

neither one-to-one nor onto $f: Z \longrightarrow Z+$. $f(x) = 3|x| + 3$

Because $f(-1) = f(1) = 6$ and there is no $f(x) = 2$

Q11

Exercise 4.3.2

(c):
Solution: f^{-1}: R → R. $f^{-1}(x) = x/2 - 3/2$

(d):
Solution: The function is not one-to-one
f({1, 2}) = f({2, 3}) = 2, so f^{-1} is not well-defined.

(g):
Solution: f^{-1}: {0, 1}^3→{0, 1}^3
The output of f^-1 is obtained by taking the input string and reversing the bits.
$f^{-1}(\{0, 1\}^3)$:

domain	target
000	000
100	001
010	010
110	011
001	100
101	101
011	110
111	111

(i):
Solution: $f^{-1}(x, y) = (x-5, y+2)$
For example: f(3, -1) = (8, -3), then $f^{-1}(8, -3) = (3, -1)$

Exercise 4.4.8

(c):

Solution: $2x^2 + 5$

f o h = $f(h(x)) = f(x^2 + 1) = 2(x^2 + 1) + 3 = 2x^2 + 5$

(d):

Solution: $4x^2 + 12x + 10$

h o f = $h(f(x)) = h(2x + 3) = (2x+3)^2 + 1 = 4x^2 + 12x + 10$

Exercise 4.4.2

(b):

Solution: 121

f o h(52) = f(h(52)) = f(11) = 121

(c):

Solution: 16

g o h o f(4) = g(h(f(4))) = g(h(16)) = g(4) = 16

(d):

Solution: $\lceil x^2/5 \rceil$

h o f = $h(f(x)) = \lceil x^2/5 \rceil$

Exercise 4.4.6

(c):

Solution: 111

h(f(010)) = h(110) = 111

(d):

Solution: {101, 111}

(h o f)(x) = h(f(x)) = h({100, 101, 111, 110}) = {101, 111}

(e):

Solution: {001, 101, 111, 011}

(g o f)(x) = g(f(x)) = g({100, 101, 111, 110}) = {001, 101, 111, 011})

Extra Credit: Exercise 4.4.4

(c):

Solution: No

We will show that if g o f is one-to-one, then f must be one-to-one. Assume f is not one-to-one, then there are some a!=b such that f(a) = f(b) for some a, b $\in$X We know g(f(a)) = g(f(b))=> (g o f)(a) = (g o f)(b) since g o f is one-to-one. Therefore, a=b. It is a contradiction to a!=b. So if g o f is one-to-one, then f must be one-to-one.

(d):

Solution: Yes

The diagram below illustrates an example: