Q7

<u>Exercise 8.2.2</u>

(b):

Solution:

f = O(n^3) .
Proof.

Let c = 4 and n0 =1. We will show that for any n ≥ 1, f(n) ≤ 4(n^3)

f(n) = n^3 + 3n^2 + 4 < n^3 + 3n^2

For n >=1, n^3 > n^, so n^3 + 3n^2 <=n^3 + 3n^3

n^3 + 3n^3 = 4(n^3) Therefore for n>=1, f(n) <=4n^3. ■

f = Ω(n ^3) .
Proof.

Let c = 1/2 and n0 =1. We will show that for any n ≥ 1, 2f(n)>= n^3

2f(n) = 2 n^3 + 6n^2 + 8 >= n^3

2n^3 >= n^3 since 6n^2 + 8 is positive

Therefore, for n >= 1, 2n^3 >= n^3

n^3 + 3n^2 + 4 >= (1/2)2n^3 ■

(b):

Solution:

f = O(n).

Proof.

Let c = 3 and n0 =1. We will show that for any $n \geq 1$, f <= 3n since (7 n^2 + 2n − 8) >= 0;

For n >=1, 7n^2 + 2n - 8 <= 7n^2 +2n

So, 7n^2 +2n < =7n^2+2n^2 = 9n^2

We know √(9n^2) = 3n Therefor n >=1, f <= 3n. ■

$f = \Omega(n)$

Proof.

Let c = 7^(1/2) and n0 = 4. We will show that for any $n \geq 1$, f >= n f = √(7n^2 + 2n − 8)

Since 7n^2 + 4n − 8 >= 0, so
For n >=1, 7n^2 + 4n − 8 >= 7n^2 + 4n - 8n = 7n^2 - 4n = n(7n-4) So, √(7n^2 + 2n − 8) >= √n(7n-4) >=√(n*n) >= n

Therefore for n >= 1 f >= n.

Because √(7n^2 + 2n − 8) = O(n) and √(7n^2 + 2n − 8) = Ω(n), √ (7n^2 + 2n − 8) = $\Theta(n)$. ■

<u>Exercise 8.3.5</u>

(a):

Solution:

This program attempts to sort input sequence of numbers. P is the pivot number; switch any number from the left side that is larger than pivot to any number from the right side that is smaller.

For instance, p = 0, the list contains positive and negative values. The list will be sorted when this program is done running. From negative to positive.

(b):

Solution:

Depending on how many integers have already been sorted or skipped. The line will be executed if these values are smaller than the left-hand pivot number or larger than the right-hand pivot number.

For example, if the input list doesn't need to sort, from small to large, the lines are executed n times. If the input list is sorted by large to small, the lines are executed 0 times.

(c):

Solution:

Depends on the how many numbers need to be sorted. In the end, it all boils down to what values the numbers in the sequence have. The maximum number of swaps is n/2 and the minimum number of swaps is 0.

(d):

Solution:

$\Omega(n)$. Executing swap operations exchange two numbers from left side to right and right to left. The maximum number of is n/2. If the list is sorted, swap operation will be executed 0 times.

(e):

Solution:

O(n^2). The upper bound is when the list is decreasing. Total swap times will be n/2.

Q8

Exercise 5.1.1

(b).

Solution:

When the length equals 7: 40^7
When the length equals 8: 40^8
When the length equals 9: 40^9
So the total of is **40^7 + 40^8 + 40^9** strings

(c):

Solution:

When the length equals 7: 40^6 * 14
When the length equals 8: 40^7 * 14
When the length equals 9: 40^8 * 14
Therefore, the total is **40^6 * 14 + 40^7 * 14 + 40^8 * 14** strings

Exercise 5.3.2

(a).

Solution:

2^9 * 3 which is 1536 since they can't appear consecutively therefore the number before has a impact on the options of the next number

Exercise 5.3.3

(b).

Solution:

10 * 9 * 8 * 26 ^ 4 which is 329022720. the options for each digit on the license plate decrease consecutively but for the letters each of them have 26 options (c):

Solution:

10 * 9 * 8 * 26 * 25 * 24 * 23 which is 258336000. Same as above but since the letters can't repeat either therefore each of them have one less option than the precedent one.

Exercise 5.2.3

(a):

Solution:

Define f(s) = {s + [0] if s has an odd number of 1's s + [1] if s has an odd number of 1's}

f-1(s) = {every binary string remove last digit of number}
Because bijection if and only if f has a well defined inverse. f(n) is bijection.

(b):

Solution:

2^9 which is 512. Because they are bijection, |B9|= |E10| =2^9.

Q9

Exercise 5.4.2

(a):

Solution:

2 * 10^4 which is 20000. Because they start with either 824 or 825, There are only two options for the first three digits.

(b):

Solution:

2*10*9*8*7 which is 10080. Only two options for the first three digits and the last four digits can only have 10*9*8*7 .

Exercise 5.5.3

(a):

Solution:

2^10 which is 1024. Since 10 digits and each of them have two options(0,1)

(b):

Solution:

2^7 which is 128. The first there digits have already been decided so we only consider the last four digits and its the same as above

(c):

Solution:

2^7 + 2^8 which is 384. Combine the two possibilities 001(2^7) and 10(2^8).

(d):

Solution:

2 ^ 6 ^ 4 which is 256.

(e):

Solution:

C(10 , 6) which is 210. Its the combination of 6 0’s and other digits

(f):

Solution:

C(9, 6) which is 84. Since the first once has been decided. We only consider the last 9 digits and Its the combination of the 6 0’s

(g):

Solution:

C(5, 1) * C(5 , 3) which is 50. The combination of the 1 with other digits in the first half times the combination of the three 1’s with other digits in the second half

<u>Exercise 5.5.5</u>

(a):

Solution:

(30, 10) ways to select a subset of 10 boys from 30 boys.

(35,10) ways to select a subset of 10 girls from 35 girls.

Based on the rules, there are a total of (35, 10) * (30, 10) options.

Exercise 5.5.8

(c):

Solution:

There are C(26, 5) ways to select.

All five cards must be from Hearts and Diamond As we have total 13 hearts and 13 diamond card in our deck.

(d):

Solution:

There are C(13, 1) * C(48 ,1) ways to select.

We have total 13 ranks in our deck of card. Also each rank contains exactly four card so once we select our rank all four card can be select in a unique way.

(e):

Solution:

There are C(13,2)×C(4,2)×C(4,3) ways to select.

we need 2 card of a rank and another 3 cards of another rank so for this first we need to choose two different rank which can be done in C(13,2) Ways, Now out of these selected rank each rank has four cards but we need to choose 2 from one and 3 from another that can be done in C(4,2) and C(4,3) Ways.

(f):

Solution:

There are C(13,5)×C(4,1)×C(4,1)×C(4,1)×C(4,1)×C(4,1) ways to select.

Firstly select five-different ranks then one cards from each selected rank.

So ranks can be chosen in C(13,5) ways and a card from each rank can be chosen in C(4,1) Ways

<u>Exercise 5.6.6</u>

(a):

Solution:

There are C(44, 5) * C(56, 5) ways.

Choose 5 from each group and times their combination.

(b):

Solution:

There are P(44, 2) * P(56, 2) ways.

Since who is the president or vice president matters so its permutation problem. Choose two from each group and times their permutation.

Q10

<u>Exercise 5.7.2</u>

(a):

Solution:

There are C(52, 5) - C(39, 5) ways to select.

There are C(52, 5) possible 5 card hands. 5 card hands have no club C(39,5). 5 card hands have at least one club C(52, 5) -C(39,5) which is 2023203

(b):

Solution:

There are C(52, 5) - C(13, 5)*4^5 ways to select.

The number of ways to choose 5 different rank is C(13, 5) and then for ever rank there are 4 ways to choose a suit for it. C(13, 5) * 4^5.

The number of 5 card hands that have at least two cards with the same rank C(52, 5) - C(13, 5) * 4^5.

<u>Exercise 5.8.4</u>

(a):

Solution:

5^20.

Since there is no restriction. The 20 books can be given to any of the 5 kids.

(b):

Solution:

$$\frac{20!}{4!4!4!4!4!}$$

Since the books are divided evenly, then each kid got 4 books.

Q11

(a):

Solution:

The number of elements in target is smaller than the number of elements in domain. Therefore its not a one-to-one function.

(b):

Solution:

5! = 120.

(c):

Solution:

P(6, 5) = 720

(d):

Solution:

P(7, 5) = 2520