Q1

A. Base b → decimal

$(a_n a_{n-1} \cdots a_2 a_1 a_0)_b = a_0 \times b^0 + a_1 \times b^1 + \cdots + a_n \times b^n$

1. $(10011011)_2 = 1\times2^0 + 1\times2^1 + 0\times2^2 + 1\times2^3 + 1\times2^4 + 0\times2^5 + 0\times2^6 + 1\times2^7 = (155)_{10}$

2. $(456)_7 = (6\times7^0 + 5\times7^1 + 4\times7^2) = (237)_{10}$

3. $(38A)_{16} = (10\times16^0 + 8\times16^1 + 3\times16^2) = (906)_{10}$

4. $(2214)_5 = 4\times5^0 + 1\times5^1 + 2\times5^2 + 2\times5^3 = (309)_{10}$

B. decimal → binary / hex → binary

1. $\boxed{(69)_{10} = (1000101)_2}$

Steps:
$$69 = 1\times2^6 + 5$$
$$5 = 0\times2^5 + 5$$
$$5 = 0\times2^4 + 5$$
$$5 = 0\times2^3 + 5$$
$$5 = 1\times2^2 + 1$$
$$1 = 0\times2^1 + 1$$
$$1 = 1\times2^0 + 0$$

by (1), (2), (3), (4), (5), (6), (7)

results: 1000101

2. $\boxed{(485)_{10} = (111100101)_2}$

Steps:
$$485 = 1\times2^8 + 229$$
$$229 = 1\times2^7 + 101$$
$$101 = 1\times2^6 + 37$$
$$37 = 1\times2^5 + 5$$
$$5 = 0\times2^4 + 5$$

1, 2, 3, 4, 5, 6, 7, 8, 9

results: 111100101

3. $\boxed{(6D1A)_{16} = (110110100011010)_2}$

steps: 1. hex → decimal

$$(6D1A)_{16} = 10\times16^0 + 1\times16^1 + 13\times16^2 + 6\times16^3 = (27930)_{10}$$

2: decimal → binary
$$27930 = 1\times2^{14} + 11546$$
$$11546 = 1\times2^{13} + 3354$$
$$3354 = 1\times2^{11} + 1306$$
$$1306 = 1\times2^{10} + 282$$
$$282 = 0\times2^9 + 282$$
$$282 = 1\times2^8 + 26$$
$$26 = 0\times2^7 + 26$$
$$26 = 0\times2^6 + 26$$
$$26 = 0\times2^5 + 26$$
$$26 = 1\times2^4 + 10$$
$$10 = 1\times2^3 + 2$$
$$2 = 0\times2^2 + 2$$
$$2 = 1\times2^1 + 0$$
$$0 = 0\times2^0 + 0$$

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

results: 110110100011010

C. binary → hex / decimal → hex

1. $\boxed{(1101011)_2 = (6B)_{16}}$ $*(6b)_{16}$

Step 1. $1101011_2 = 1\times2^0 + 1\times2^1 + 0\times2^2 + 1\times2^3 + 0\times2^4 + 1\times2^5 + 1\times2^6 = (107)_{10}$

Step 2:
$$107 = 6\times16^1 + 11$$
$$11 = 11\times16^0 + 0$$

1.2 results = 6, 11 → 6B.

2. $\boxed{(895)_{10} = (37F)_{16}}$

Steps:
$$895 = 3\times16^2 + 127$$
$$127 = 7\times16^1 + 15$$
$$15 = 15\times16^0 + 0$$

1.2.3 results = 3, 7, 15 → 37F

Q2:

Q2 :

$$\boxed{1.\ (7566)_8 + (4515)_8 = (14303)_8}$$

steps = convert octal → decimal

$$(7566)_8 = 6\times 8^0 + 6\times 8^1 + 5\times 8^2 + 7\times 8^3 = (3958)_{10}$$

$$(4515)_8 = 5\times 8^0 + 1\times 8^1 + 5\times 8^2 + 4\times 8^3 = (2381)_{10}$$

$$(3958 + 2381)_{10} \longrightarrow (14303)_8$$

$$\boxed{2.\ (10110011)_2 + (1101)_2 = (11000000)_2}$$

steps = binary → decimal

$$(10110011)_2 = 1\times 2^0 + 1\times 2^1 + 0\times 2^2 + 0\times 2^3 + 1\times 2^4 + 1\times 2^5 + 0\times 2^6 + 1\times 2^7 = (179)_{10}$$

$$(1101)_2 = (13)_{10}$$

$$(179 + 13)_{10} = (11000000)_2$$

$$\boxed{3.\ (7A66)_{16} + (45C5)_{16} = (C02B)_{16}}$$

steps = hex → decimal

$$(7A66)_{16} = (31334)_{10}$$

$$(45C5)_{16} = (17861)_{10}$$

$$(31334 + 17861)_{10} = (C02B)_{16}$$

$$\boxed{4.\ (3022)_5 - (2433)_5 = (34)_5}$$

steps = $(3022)_5 = (387)_{10}$

$$(2433)_5 = (368)_{10}$$

$$(387 - 368)_{10} = (34)_5$$

Q3:

Q3 =

A. 1. $(124)_{10} = (01111100)_2$

Steps = ① It's a positive number
② decimal → binary
③ pad to 8 bits

2. $(-124)_{10} = (10000100)_2$

Steps = $(124)_{10} = (1111100)_2$

```
  01111100
¬ 10000011
+        1
----------
  10000100
```

3. $(109)_{10} = (01101101)_2$

steps = convert to binary / pad to 8 bits

4. $(-79)_{10} = (10110001)_2$

steps = $(79)_{10} = (1001111)_2$

```
  01001111
¬ 10110000
+        1
----------
  10110001
```

B = 1. 0011110 8bit 2's compl = 30

steps ① positive number
② binary → decimal

2. 11100110 (8bit 2's comp) = −26

steps = ① negative number
② 11100110

```
¬ 00011001
+        1
----------
  00011010
```

③ $(00011010)_2 = 26$

3. 00101101 8bit 2's comp = 45

Steps = ① positive
② $1\times2^0+0\times2^1+1\times2^2+1\times2^3+0\times2^4+1\times2^5+0\times2^6+0\times2^7 = 45$

4. 10011110 (8bit 2's comp) = −98

steps = ① negative number

```
  10011110
¬ 01100001
+        1
----------
  01100010
```

③ $(01100010)_2 = (98)_{10}$

Q4

1. Exercise 1.2.4

(b)

P	q	¬(p ∨ q)
T	T	F
T	F	F
F	T	F
F	F	T

(c)

P	q	r	p ∧ ¬q	r ∨ (p ∧¬q)
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

2. Exercise 1.3.4

(b)

P	q	p—>q	q—>p	(p → q) → (q → p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d)

P	q	¬q	(p ↔ q)	p ↔ ¬q	(p ↔ q) ⊕ (p ↔ ¬q)
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Q5

1. Exercise 1.2.7

(b)

$(B\wedge D) \vee (M \wedge D) \vee (B \wedge M)$

(c)

$B\vee(D\wedge M)$

2. Exercise 1.3.7

(b)

$(s \vee y) \rightarrow p$

(c)

$p \rightarrow y$

(d)

$p \leftrightarrow (s \wedge y)$

(e)

$p \rightarrow (s \vee y)$

3. Exercise 1.3.9

(c)

$(y \vee p) \rightarrow c$

(d)

$c \rightarrow p$

Q6

1. Exercise 1.3.6

(b)
If Joe is eligible for the honors program, then he maintained a B average

(c)
If Rajiv go on the roller coaster, then he is at least four feet tall

(d)
If Rajiv is at least four feet tall, then he can go on the roller coaster

2. Exercise 1.3.10

(c)
False
$p \vee r$ is true, $q \wedge r$ is false, the. Then the biconditional statement is false

(d)
Unknown
If r is true, $p \wedge r$ is true, $q \wedge r$ is false, then the expression is false;
If r is false, $p \wedge r)$ is false, $q \wedge r$ is false, then the expression is true.

(e)
Unknown
If r is true, $r \vee q$ is true, then the expression is true;
If r is false, $r \vee q$ is false, then the expression is false

(f)
True
$p \wedge q$ is false
If the hypothesis is false, the statement is true no matter if the conclusion is true or false

Q7

Exercise 1.4.5

(b)
¬j → (l ∨ ¬r)
(r ∧ ¬l) → j

j	**l**	**r**	**¬j → (l ∨ ¬r)**	(r ∧ ¬l) → j
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

(c)
j → ¬l
¬j → l

j	**i**	j → ¬l	**¬j → l**
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

(d)
(r ∨ ¬l) → j
j → (r ∧ ¬l)

j	**l**	**r**	(r ∨ ¬l) → j	j → (r ∧ ¬l)
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Q8

1. Exercise 1.5.2

(c) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$p \to q$	
$\neg p \vee q$	Conditional identities
$p \to r$	
$\neg p \vee r$	Conditional identities
$(\neg p \vee q) \wedge (\neg p \vee r)$	
$\neg p \vee (q \wedge r)$	Distributive laws
$p \to (q \wedge r)$	Conditional identities

(f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg p \wedge \neg (\neg p \wedge q)$	De Morgan's laws
$\neg (\neg p \wedge q)$	
$\neg \neg p \vee \neg q$	De Morgan's laws
$\neg \neg p$	
p	Double negation law
$p \vee \neg q$	
$\neg p \wedge (p \vee \neg q)$	
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	Distributive laws
$\neg p \wedge p$	
F	Complement laws
F v $(\neg p \wedge \neg q)$	
$(\neg p \wedge \neg q)$	Identity laws

(i) $(p \wedge q) \to r \equiv (p \wedge \neg r) \to \neg q$

$(p \wedge q) \to r$	
$\neg(p \wedge q) \vee r$	Conditional identities
$\neg(p \vee r) \wedge (r \vee q)$	Distributive laws
$\neg(p \vee r)$	
$\neg p \wedge \neg r$	De Morgan's laws
$\neg p \wedge \neg r \wedge (r \vee q)$	
$(\neg p \wedge \neg r \wedge r) \vee (q \wedge \neg p \wedge \neg r)$	Distributive laws
$\neg r \wedge r$	
F	Complement laws
F v $(q \wedge \neg p \wedge \neg r)$	Domination laws
$(q \wedge \neg p \wedge \neg r)$	Identity laws
$q \wedge \neg p$	
$\neg(p \vee \neg q)$	
$\neg(p \vee \neg q) \wedge \neg r$	
$\neg\neg((p \vee \neg q) \vee r)$	De Morgan's laws
$((p \vee \neg q) \vee r)$	Double negation laws
$p \vee r \vee \neg q$	Communication laws
p	
$\neg\neg p$	Double negation laws
$p \vee r \vee \neg q$	
$(p \wedge \neg r) \to \neg q$	
$\neg(p \wedge \neg r) \vee \neg q$	Conditional identities
$\neg(p \wedge \neg r)$	
$\neg p \vee \neg\neg r$	De Morgan's laws
$\neg p \vee \neg\neg r \vee \neg q$	
$\neg p \vee r \vee \neg q$	Double negation laws

Q9

1. Exercise 1.6.3

(c)

$\exists x(x = x^2)$

(d)

$\forall x(x <= x^2)$

2. Exercise 1.7.4

(b)

$\forall x(\neg S(x) \wedge W(x))$

(c)

$\forall x(S(x) \rightarrow \neg W(x))$

(d)

$\exists x(S(x) \wedge W(x))$

Q10

1. Exercise 1.7.9

(c) $\exists x((x = c) \rightarrow P(x))$
False, P(c) is false

(d)$\exists x(Q(x) \wedge R(x))$

True, at least one $Q(x) \wedge R(x)$ is true

(e)$Q(a) \wedge P(d)$
True, Q(a) is true and P(d) is true

(f) $\forall x ((x \neq b) \rightarrow Q(x))$
True, all Q(x) when $x \neq b$ is true

(g) $\forall x (P(x) \vee R(x))$

True, $P(x) \vee R(x)$ is always true for any x

(h) $\forall x (R(x) \rightarrow P(x))$
True, all $R(x) \rightarrow P(x)$ is true

(i) $\exists x(Q(x) \vee R(x))$
True, there is at least one x such statement is true

2. Exercise 1.9.2

(b) $\exists x \forall y Q(x, y)$
True, $\forall y Q(1, y)$ are all true

(c) $\exists y \forall x P(x, y)$
True, $\forall x P(x, 1)$ are all true

(d)$\exists x \exists y S(x, y)$
False, there is no such x, y that S(x, y) is true

(e) $\forall x \exists y Q(x, y)$
False, there is no y such that all $\forall x$ Q(x, y) all are true

(f) $\forall x \exists y P(x, y)$
True, y = 1 such that $\forall x$ P(x, y) is true

(g) $\forall x \forall y P(x, y)$
False, Counter-example: P(1,2), P(2,2), P(3,3)

(h) $\exists x \exists y Q(x, y)$
True, there is at least one Q(x, y) is true

(i) $\forall x \forall y \neg S(x, y)$
True, all S(x, y) is false, then $\neg S(x, y)$ all are true

Q11

1. Exercise 1.10.4

(c) There are two numbers whose sum is equal to their product.

$\exists x \exists y (x + y = x * y)$

(d) The ratio of every two positive numbers is also positive.

$\forall x \forall y((x > 0 \wedge y >0) \rightarrow (x/y) > 0)$

(e) The reciprocal of every positive number less than one is greater than one.

$\forall x ((x >0 \wedge x < 1) \rightarrow (1/x) > 1)$

(f) There is no smallest number.

$\neg \exists x \forall y(x <= y)$

(g) Every number besides 0 has a multiplicative inverse.

$\forall x \exists y((x \neq 0) \rightarrow (xy=1))$

2. Exercise 1.10.7

(c) There is at least one new employee who missed the deadline

$\exists x(N(x) \wedge D(x))$

(d) Sam knows the phone number of everyone who missed the deadline

$\exists y (P(Sam, y) \wedge D(y))$

(e) There is a new employee who knows everyone's phone number.

$\exists x \forall y (N(x) \wedge P(x, y))$

(f) Exactly one new employee missed the deadline.

$\exists! x (N(x) \wedge D(x))$

3. Exercise 1.10.10

(c) Every student has taken at least one class besides Math 101

$\forall x \exists y((y \neq \text{Math } 101) \rightarrow T(x, y))$

(d) There is a student who has taken every math class besides Math 101

$\exists x \forall y ((y \neq \text{Math } 101) \rightarrow T(x, y))$

(e) Everyone besides Sam has taken at least two different math classes

$\forall x \forall y1 \forall y2((y1 \neq y2) \wedge (x \neq Sam) \wedge T(x, y1) \wedge T(x, y2))$

(f) Sam has taken exactly two math classes

$\exists y \exists z \forall w ((z \neq y) \wedge T(Sam, y) \wedge T(Sam, z) \wedge ((w \neq y \wedge w \neq z) \rightarrow \neg T(Sam, w)))$

Q12

1. Exercise 1.8.2

(b) Every patient was given the medication or the placebo or both.

- Logical expression: $\forall x\, (D(x) \vee P(x) \vee (D(x) \wedge P(x)))$
- Negation: $\neg\forall x\, (D(x). \vee P(x) \vee (D(x) \wedge P(x)))$
- Applying De Morgan's law: $\exists x\, (\neg D(x) \wedge \neg P(x) \wedge (\neg D(x) \vee \neg P(x)))$
- English: Some patient was not given the placebo and not given the medication and not either of them

(c) There is a patient who took the medication and had migraines

- Logical expression: $\exists x\, (D(x) \wedge M(x))$
- Negation: $\neg\exists x(D(x) \wedge M(x))$
- Applying De Morgan's law: $\forall x(D(x) \vee \neg M(x))$
- English: every patient was either not given the medication or not had migraines

(d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \vee q$.)

- Logical expression: $\forall x(P(x) \to M(x)) = \forall x\, (\neg P(x) \vee M(x))$
- Negation: $\neg\forall x\, (\neg P(x) \vee M(x))$
- Applying De Morgan's law: $\exists x(P(x) \wedge M(x))$
- English: some patient was given the placebo and did not have migraines
-

(e)There is a patient who had migraines and was given the placebo.

- Logical expression: $\exists x\, (M(x) \wedge P(x))$
- Negation: $\neg\exists x\, (M(x) \wedge P(x))$
- Applying De Morgan's law: $\forall x\, (\neg M(x) \vee \neg P(x))$
- English: every patient was not given the placebo or did not have migraines

2. Exercise 1.9.4
(c) $\exists x\, \forall y\, (P(x, y) \to Q(x, y))$

$\forall x\, \exists y\, (P(x, y) \wedge \neg Q(x, y))$

(d) $\exists x\, \forall y\, (P(x, y) \leftrightarrow P(y, x))$

$\forall x\, \exists y\, (\neg(P(x, y) \to P(y, x)) \vee \neg\, (P(y, x) \to P(x, y)))$

(e) $\exists x\, \exists y\, P(x, y) \wedge \forall x\, \forall y\, Q(x, y)$

$\forall x\, \forall y\, \neg P(x, y) \vee \neg\forall x\, \forall y\, Q(x, y)$