

Week 3 Questions

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Introduction

We are going to use the same data as for the last two weeks of the ‘Weekly Questions’ to primarily look at polynomials and B -splines.

id		year		assets		capex	
Min.	: 1050	Min.	:1990	Min.	: 0.72	Min.	:-401.609
1st Qu.:	10391	1st Qu.:	1999	1st Qu.:	59.32	1st Qu.:	1.572
Median :	25390	Median :	2004	Median :	216.73	Median :	8.203
Mean :	54440	Mean :	2004	Mean :	1434.82	Mean :	74.055
3rd Qu.:	104598	3rd Qu.:	2009	3rd Qu.:	914.16	3rd Qu.:	40.154
Max.	:270705	Max.	:2014	Max.	:80033.55	Max.	:7150.000
ltd		ebitda		ppe			
Min.	: 0.000	Min.	:-1134.000	Min.	: 0.00		
1st Qu.:	0.000	1st Qu.:	2.293	1st Qu.:	15.13		
Median :	3.129	Median :	21.395	Median :	67.45		
Mean :	293.976	Mean :	199.175	Mean :	768.24		
3rd Qu.:	100.000	3rd Qu.:	123.826	3rd Qu.:	341.80		
Max.	:24380.700	Max.	: 7723.890	Max.	:59762.83		
sales		ads		rd			
Min.	: 0.05	Min.	: 0.000	Min.	: 0.000		
1st Qu.:	58.35	1st Qu.:	0.537	1st Qu.:	0.000		
Median :	229.86	Median :	3.055	Median :	3.996		
Mean :	1453.51	Mean :	40.312	Mean :	43.057		
3rd Qu.:	992.47	3rd Qu.:	18.500	3rd Qu.:	23.247		
Max.	:108465.00	Max.	:2840.000	Max.	:3146.829		
bookval		mv		indclass		tobinsQ	
Min.	: 0.04	Min.	: 0.10	Min.	: 1.00	Min.	: 0.000004
1st Qu.:	34.52	1st Qu.:	60.49	1st Qu.:	15.00	1st Qu.:	1.23689
Median :	128.56	Median :	275.95	Median :	36.00	Median :	2.10815
Mean :	654.89	Mean :	1759.15	Mean :	29.35	Mean :	3.21056
3rd Qu.:	482.49	3rd Qu.:	1231.72	3rd Qu.:	38.00	3rd Qu.:	3.58202
Max.	:41466.76	Max.	:50174.89	Max.	:49.00	Max.	:193.71598
ltdratio		capexratio		rdratio		adsratio	
Min.	:0.00000	Min.	:-0.25973	Min.	:0.0000	Min.	:0.000000
1st Qu.:	0.00000	1st Qu.:	0.01781	1st Qu.:	0.0000	1st Qu.:	0.005603
Median :	0.03931	Median :	0.03282	Median :	0.0290	Median :	0.014364
Mean :	0.11473	Mean :	0.05643	Mean :	0.0908	Mean :	0.032936
3rd Qu.:	0.19905	3rd Qu.:	0.05886	3rd Qu.:	0.1159	3rd Qu.:	0.035927
Max.	:0.85654	Max.	: 5.04205	Max.	:7.0207	Max.	:1.757624
pperatio		ebitdaratio					
Min.	:0.0000	Min.	:-4.06708				
1st Qu.:	0.1830	1st Qu.:	0.04678				
Median :	0.3484	Median :	0.11329				
Mean :	0.4367	Mean :	0.08921				
3rd Qu.:	0.6103	3rd Qu.:	0.17485				
Max.	:3.7330	Max.	: 1.50841				

Polynomial-based models

```
deg <- 2

polyfit2 <- lm(tobinsQ ~ poly(ltldratio, degree = deg) + poly(capexratio,
  degree = deg) + poly(rdratio, degree = deg) + poly(adsratio,
  degree = deg) + poly(pperatio, degree = deg) + poly(ebitdaratio,
  degree = deg) + poly(year, degree = deg) + poly(assets, degree = deg) +
  poly(capex, degree = deg) + poly(ltd, degree = deg) + poly(ebitda,
  degree = deg) + poly(ppe, degree = deg) + poly(sales, degree = deg) +
  poly(ads, degree = deg) + poly(rd, degree = deg) + poly(bookval,
  degree = deg) + poly(mv, degree = deg) + as.factor(indclass),
  data = newdat)
stepfit2 <- step(polyfit2, trace = 0)
pander(Anova(polyfit2))
```

Table 1: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
poly(ltldratio, degree = deg)	4358	2	95.9	4.43e-42
poly(capexratio, degree = deg)	69.1	2	1.52	0.2187
poly(rdratio, degree = deg)	469.3	2	10.33	3.306e-05
poly(adsratio, degree = deg)	155.9	2	3.43	0.0324
poly(pperatio, degree = deg)	203.9	2	4.486	0.01128
poly(ebitdaratio, degree = deg)	2418	2	53.2	9.698e-24
poly(year, degree = deg)	918.4	2	20.21	1.724e-09
poly(assets, degree = deg)	700.5	2	15.41	2.06e-07
poly(capex, degree = deg)	25.59	2	0.5631	0.5695
poly(ltd, degree = deg)	87.51	2	1.926	0.1458
poly(ebitda, degree = deg)	1509	2	33.21	4.113e-15
poly(ppe, degree = deg)	564.2	2	12.41	4.107e-06
poly(sales, degree = deg)	410.2	2	9.027	0.0001209
poly(ads, degree = deg)	225.4	2	4.959	0.00703
poly(rd, degree = deg)	1583	2	34.82	8.223e-16
poly(bookval, degree = deg)	7988	2	175.8	4.411e-76
poly(mv, degree = deg)	33044	2	727.1	1.438e-300
as.factor(indclass)	5579	40	6.138	5.158e-31
Residuals	305631	13450	NA	NA

```
pander(Anova(stepfit2))
```

Table 2: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
poly(ltldratio, degree = deg)	4376	2	96.27	3.057e-42
poly(rdratio, degree = deg)	533.4	2	11.74	8.083e-06
poly(adsratio, degree = deg)	192.1	2	4.226	0.01463
poly(pperatio, degree = deg)	175.5	2	3.861	0.02107
poly(ebitdaratio, degree = deg)	2420	2	53.24	9.318e-24
poly(year, degree = deg)	1019	2	22.42	1.901e-10
poly(assets, degree = deg)	902	2	19.84	2.479e-09
poly(ebitda, degree = deg)	1650	2	36.31	1.877e-16

	Sum Sq	Df	F value	Pr(>F)
poly(ppe, degree = deg)	622.8	2	13.7	1.136e-06
poly(sales, degree = deg)	396.9	2	8.732	0.0001622
poly(ads, degree = deg)	255.4	2	5.619	0.003637
poly(rd, degree = deg)	1572	2	34.58	1.049e-15
poly(bookval, degree = deg)	10420	2	229.3	1.245e-98
poly(mv, degree = deg)	33920	2	746.3	4.36e-308
as.factor(indclass)	5615	40	6.177	2.673e-31
Residuals	305797	13456	NA	NA

```
pander(vif(polyfit2))
```

	GVIF	Df	GVIF^(1/(2*Df))
poly(ltdratio, degree = deg)	1.941	2	1.18
poly(capexratio, degree = deg)	1.587	2	1.122
poly(rdratio, degree = deg)	2.119	2	1.207
poly(adsratio, degree = deg)	1.513	2	1.109
poly(pperatio, degree = deg)	2.605	2	1.27
poly(ebitdaratio, degree = deg)	1.717	2	1.145
poly(year, degree = deg)	1.211	2	1.049
poly(assets, degree = deg)	918	2	5.504
poly(capex, degree = deg)	22.95	2	2.189
poly(ltd, degree = deg)	39.68	2	2.51
poly(ebitda, degree = deg)	36.05	2	2.45
poly(ppe, degree = deg)	39.79	2	2.512
poly(sales, degree = deg)	14.59	2	1.954
poly(ads, degree = deg)	4.043	2	1.418
poly(rd, degree = deg)	3.696	2	1.387
poly(bookval, degree = deg)	144	2	3.464
poly(mv, degree = deg)	8.955	2	1.73
as.factor(indclass)	12.79	40	1.032

```
pander(vif(stepfit2))
```

	GVIF	Df	GVIF^(1/(2*Df))
poly(ltdratio, degree = deg)	1.68	2	1.139
poly(rdratio, degree = deg)	1.855	2	1.167
poly(adsratio, degree = deg)	1.466	2	1.1
poly(pperatio, degree = deg)	2.518	2	1.26
poly(ebitdaratio, degree = deg)	1.714	2	1.144
poly(year, degree = deg)	1.178	2	1.042
poly(assets, degree = deg)	130.3	2	3.378
poly(ebitda, degree = deg)	34.1	2	2.416
poly(ppe, degree = deg)	21.54	2	2.154
poly(sales, degree = deg)	12.3	2	1.873
poly(ads, degree = deg)	3.781	2	1.394
poly(rd, degree = deg)	3.14	2	1.331
poly(bookval, degree = deg)	63.64	2	2.824
poly(mv, degree = deg)	8.568	2	1.711
as.factor(indclass)	10.37	40	1.03

```
require(pander)
pander(AIC(polyfit2, stepfit2))
```

	df	AIC
polyfit2	76	80703
stepfit2	70	80698

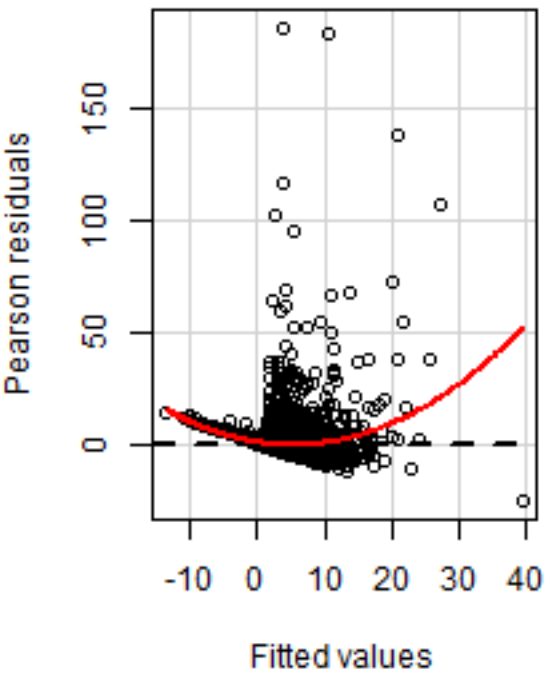
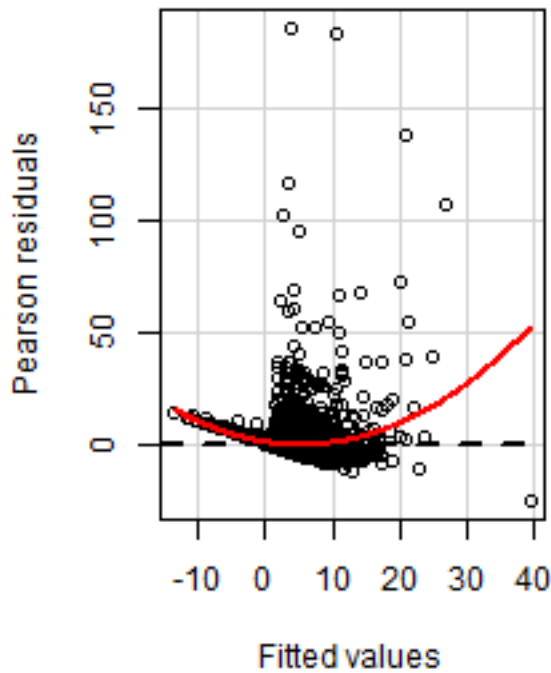
```
pander(BIC(polyfit2, stepfit2))
```

	df	BIC
polyfit2	76	81274
stepfit2	70	81224

```
par(mfrow = c(1, 2))
residualPlots(polyfit2, terms = ~1)
```

Test stat Pr(>|t|)
Tukey test 27.454 0

```
residualPlots(stepfit2, terms = ~1)
```

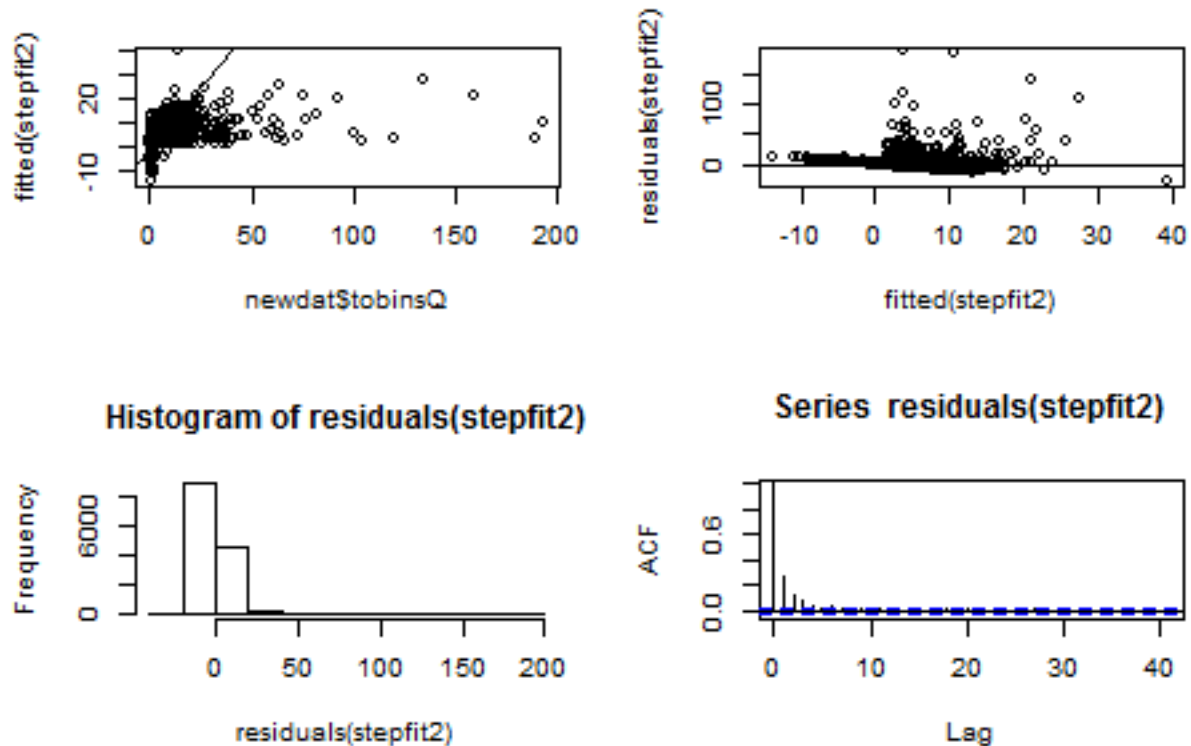


Test stat Pr(>|t|)
Tukey test 27.261 0

```

par(mfrow = c(2, 2))
plot(newdat$tobinsQ, fitted(stepfit2))
abline(0, 1)
plot(fitted(stepfit2), residuals(stepfit2))
abline(h = 0)
hist(residuals(stepfit2))
acf(residuals(stepfit2))

```



```
ncvTest(stepfit2)
```

Non-constant Variance Score Test
 Variance formula: ~ fitted.values
 Chisquare = 31700.44 Df = 1 p = 0

```
runs.test(residuals(stepfit2))
```

Runs Test - Two sided

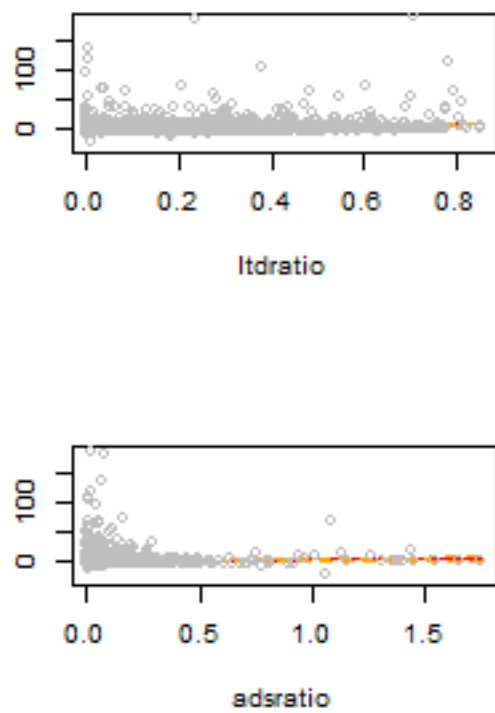
data: residuals(stepfit2)
 Standardized Runs Statistic = -62.334, p-value < 2.2e-16

```

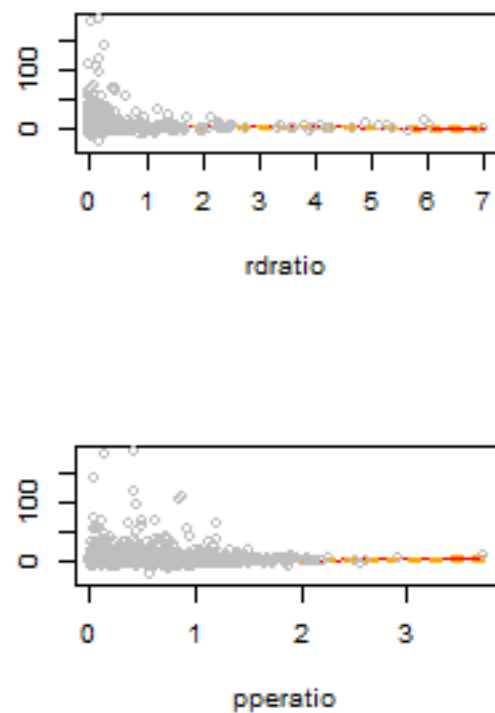
par(mfrow = c(2, 2))
termplot(stepfit2, se = T, partial.resid = TRUE)

```

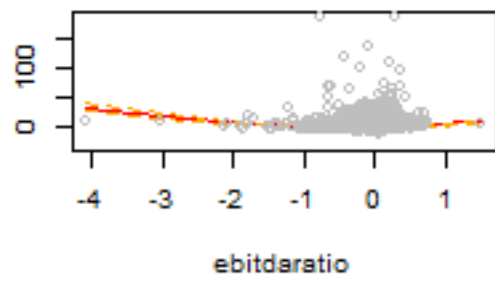
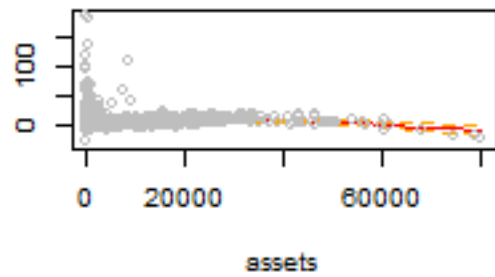
Partial for poly(adsratio, degree = d) Partial for poly(ltdratio, degree = de



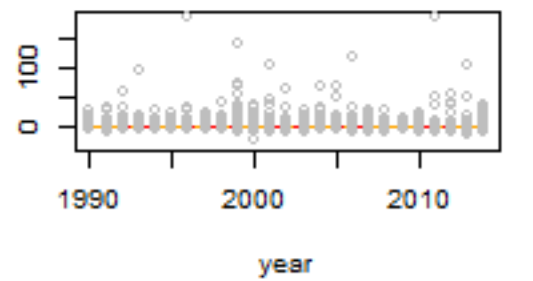
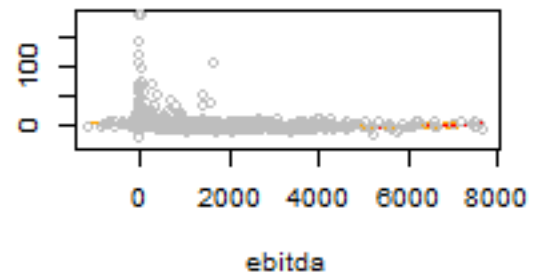
Partial for poly(rdratio, degree = d) Partial for poly(pperatio, degree = de



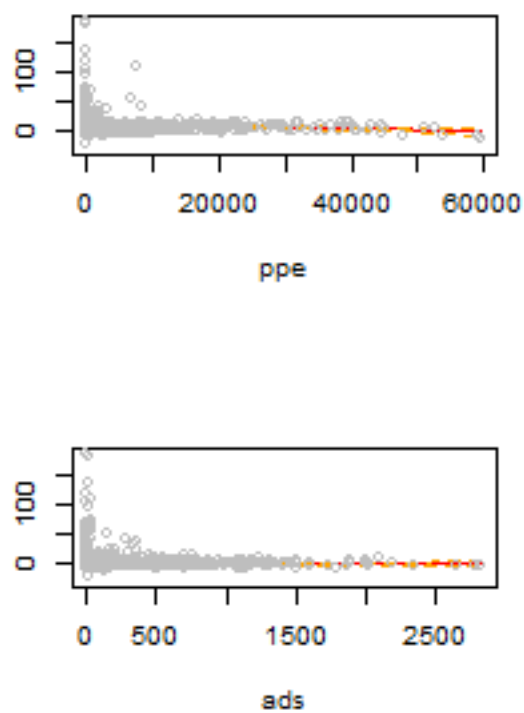
Partial for poly(assets, degree = de



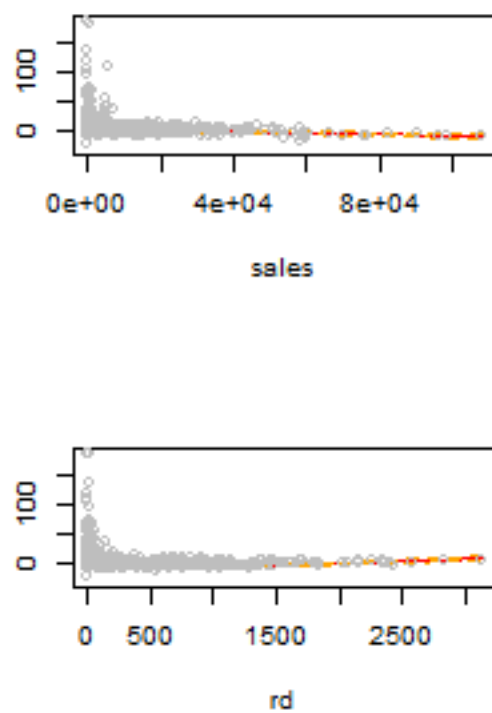
Partial for poly(ebitda, degree = de

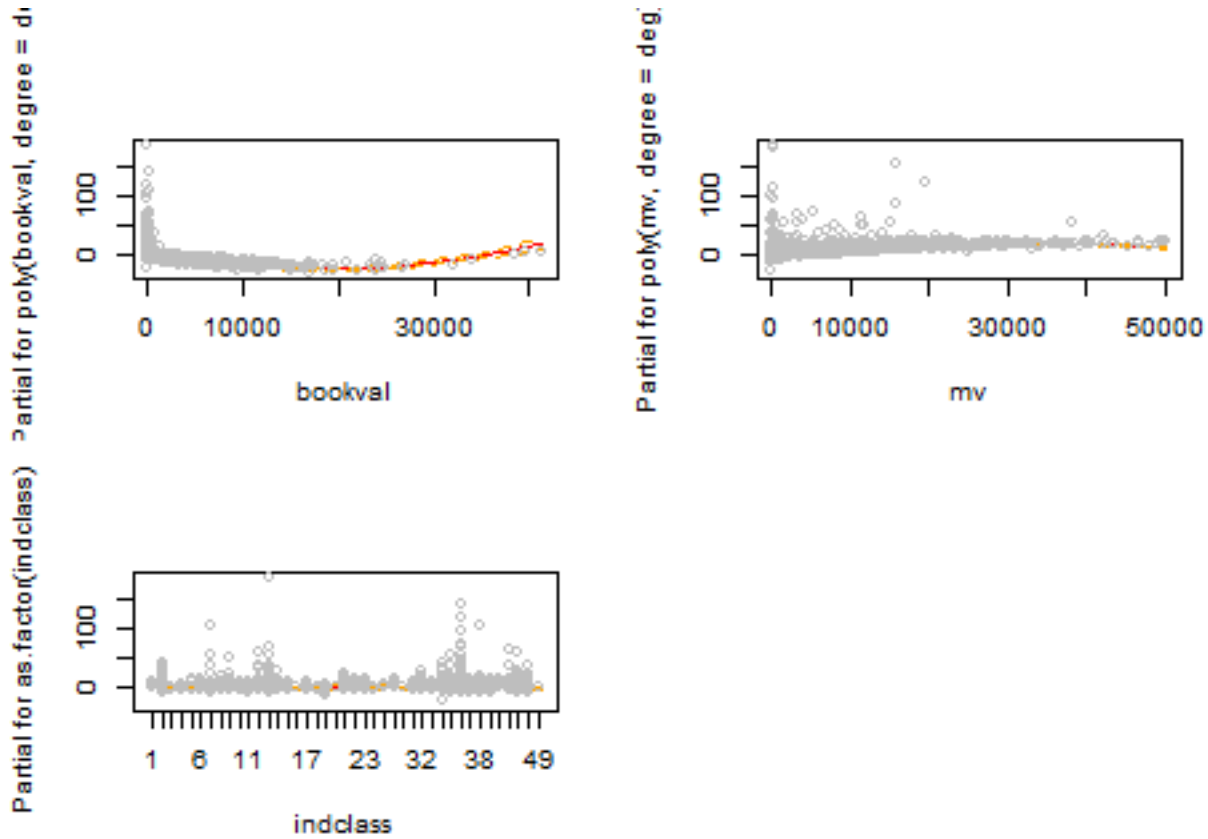


Partial for poly(ads, degree = deg Partial for poly(ppe, degree = deg



Partial for poly(sales, degree = deg Partial for poly(rd, degree = deg





```
deg <- 3
polyfit3 <- lm(tobinsQ ~ poly(ltdratio, degree = deg) + poly(capexratio,
  degree = deg) + poly(rdratio, degree = deg) + poly(adsratio,
  degree = deg) + poly(pperatio, degree = deg) + poly(ebitdaratio,
  degree = deg) + poly(year, degree = deg) + poly(assets, degree = deg) +
  poly(capex, degree = deg) + poly(ltd, degree = deg) + poly(ebitda,
  degree = deg) + poly(ppe, degree = deg) + poly(sales, degree = deg) +
  poly(ads, degree = deg) + poly(rd, degree = deg) + poly(bookval,
  degree = deg) + poly(mv, degree = deg) + as.factor(indclass),
  data = newdat)

stepfit3 <- step(polyfit3, trace = 0)
pander(Anova(polyfit3))
```

Table 7: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
poly(ltdratio, degree = deg)	4985	3	77.9	5.973e-50
poly(capexratio, degree = deg)	133.9	3	2.093	0.09886
poly(rdratio, degree = deg)	481.7	3	7.528	4.976e-05
poly(adsratio, degree = deg)	168.1	3	2.627	0.04859
poly(pperatio, degree = deg)	291.9	3	4.562	0.003377
poly(ebitdaratio, degree = deg)	5299	3	82.8	4.497e-53
poly(year, degree = deg)	520	3	8.126	2.105e-05
poly(assets, degree = deg)	982.5	3	15.35	5.721e-10
poly(capex, degree = deg)	223.1	3	3.486	0.01508

	Sum Sq	Df	F value	Pr(>F)
poly(ltd, degree = deg)	304.8	3	4.762	0.002548
poly(ebitda, degree = deg)	2608	3	40.76	3.275e-26
poly(ppe, degree = deg)	1210	3	18.92	3.084e-12
poly(sales, degree = deg)	130.6	3	2.041	0.1058
poly(ads, degree = deg)	184.3	3	2.88	0.03451
poly(rd, degree = deg)	1029	3	16.08	1.975e-10
poly(bookval, degree = deg)	12470	3	194.9	1.053e-123
poly(mv, degree = deg)	41184	3	643.6	0
as.factor(indclass)	4140	40	4.852	6.527e-22
Residuals	286549	13433	NA	NA

```
pander(Anova(stepfit3))
```

Table 8: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
poly(ltdratio, degree = deg)	4985	3	77.9	5.973e-50
poly(capexratio, degree = deg)	133.9	3	2.093	0.09886
poly(rdratio, degree = deg)	481.7	3	7.528	4.976e-05
poly(adsratio, degree = deg)	168.1	3	2.627	0.04859
poly(pperatio, degree = deg)	291.9	3	4.562	0.003377
poly(ebitdaratio, degree = deg)	5299	3	82.8	4.497e-53
poly(year, degree = deg)	520	3	8.126	2.105e-05
poly(assets, degree = deg)	982.5	3	15.35	5.721e-10
poly(capex, degree = deg)	223.1	3	3.486	0.01508
poly(ltd, degree = deg)	304.8	3	4.762	0.002548
poly(ebitda, degree = deg)	2608	3	40.76	3.275e-26
poly(ppe, degree = deg)	1210	3	18.92	3.084e-12
poly(sales, degree = deg)	130.6	3	2.041	0.1058
poly(ads, degree = deg)	184.3	3	2.88	0.03451
poly(rd, degree = deg)	1029	3	16.08	1.975e-10
poly(bookval, degree = deg)	12470	3	194.9	1.053e-123
poly(mv, degree = deg)	41184	3	643.6	0
as.factor(indclass)	4140	40	4.852	6.527e-22
Residuals	286549	13433	NA	NA

```
pander(AIC(polyfit3, stepfit3))
```

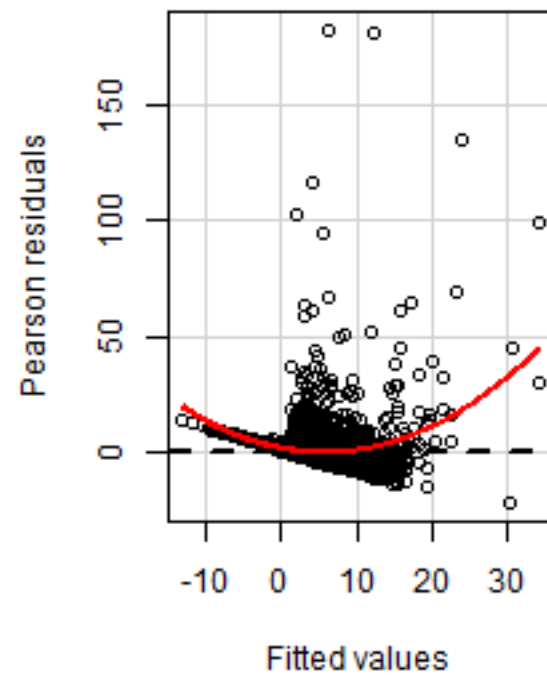
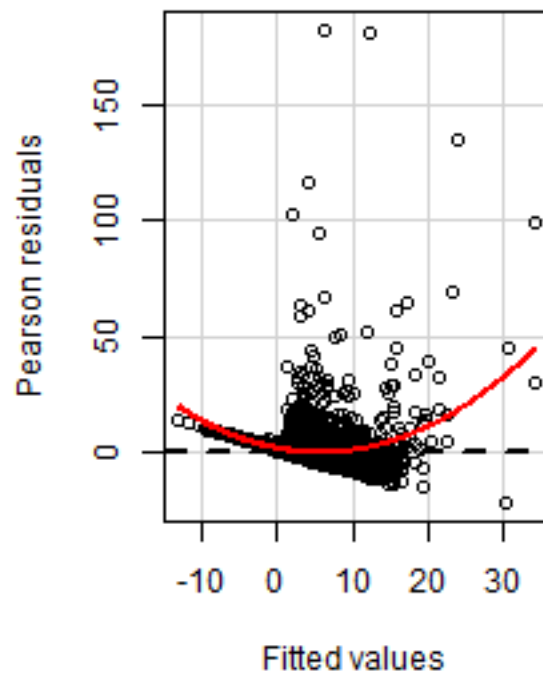
	df	AIC
polyfit3	93	79865
stepfit3	93	79865

```
pander(BIC(polyfit3, stepfit3))
```

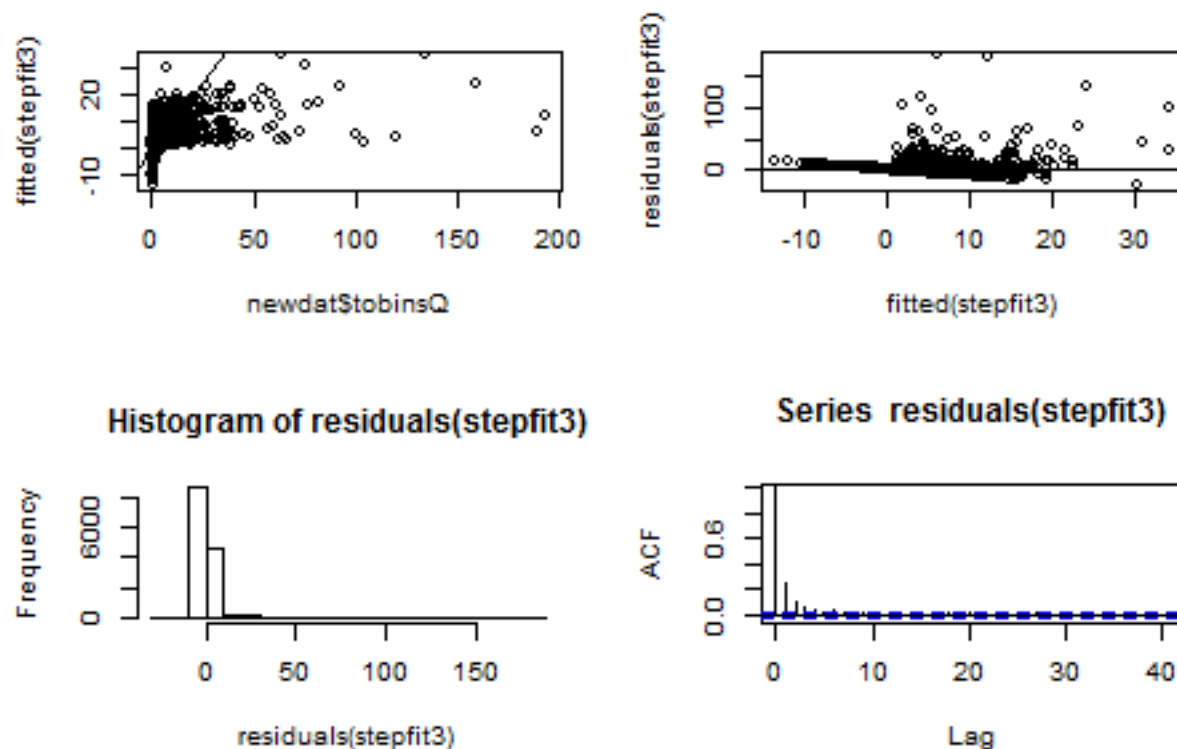
	df	BIC
polyfit3	93	80564
stepfit3	93	80564

```
par(mfrow = c(1, 2))
residualPlots(polyfit3, terms = ~1)
```

```
Test stat Pr(>|t|)
Tukey test    35.332      0
residualPlots(stepfit3, terms = ~1)
```



```
Test stat Pr(>|t|)
Tukey test    35.332      0
par(mfrow = c(2, 2))
plot(newdat$tobinsQ, fitted(stepfit3))
abline(0, 1)
plot(fitted(stepfit3), residuals(stepfit3))
abline(h = 0)
hist(residuals(stepfit3))
acf(residuals(stepfit3))
```



```
ncvTest(stepfit3)
```

```
Non-constant Variance Score Test
Variance formula: ~ fitted.values
Chisquare = 41794.4    Df = 1    p = 0
```

```
runs.test(residuals(stepfit3))
```

```
Runs Test - Two sided
```

```
data: residuals(stepfit3)
Standardized Runs Statistic = -61.853, p-value < 2.2e-16
```

B-spline based models

```
deg <- 2
bsfit3 <- lm(tobinsQ ~ bs(ltdratio, degree = deg, knots = mean(ltdratio)) +
  bs(capexratio, degree = deg, knots = mean(capexratio)) +
  bs(rdratio, degree = deg, knots = mean(rdratio)) + bs(adsratio,
  degree = deg, knots = mean(adsratio)) + bs(pperatio, degree = deg,
  knots = mean(pperatio)) + bs(ebitdaratio, degree = deg, knots = mean(ebitdaratio)) +
  bs(year, degree = deg, knots = mean(year)) + bs(assets, degree = deg,
  knots = mean(assets)) + bs(capex, degree = deg, knots = mean(capex)) +
  bs(ltd, degree = deg, knots = mean(ltd)) + bs(ebitda, degree = deg,
  knots = mean(ebitda)) + bs(ppe, degree = deg, knots = mean(ppe)) +
```

```

bs(sales, degree = deg, knots = mean(sales)) + bs(ads, degree = deg,
knots = mean(ads)) + bs(rd, degree = deg, knots = mean(rd)) +
bs(bookval, degree = deg, knots = mean(bookval)) + bs(mv,
degree = deg, knots = mean(mv)) + as.factor(indclass), data = newdat)

stepfitbs3 <- step(bsfit3, trace = 0)
pander(Anova(bsfit3))

```

Table 11: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
bs(ltdratio, degree = deg, knots = mean(ltdratio))	1811	3	31.4	3.202e-20
bs(capexratio, degree = deg, knots = mean(capexratio))	185.5	3	3.217	0.02182
bs(rdratio, degree = deg, knots = mean(rdratio))	101.7	3	1.765	0.1516
bs(adsratio, degree = deg, knots = mean(adsratio))	89.28	3	1.548	0.1997
bs(pperatio, degree = deg, knots = mean(pperatio))	41.93	3	0.7272	0.5356
bs(ebitdaratio, degree = deg, knots = mean(ebitdaratio))	2880	3	49.94	4.317e-32
bs(year, degree = deg, knots = mean(year))	72.86	3	1.264	0.285
bs(assets, degree = deg, knots = mean(assets))	838.2	3	14.54	1.884e-09
bs(capex, degree = deg, knots = mean(capex))	80.34	3	1.393	0.2427
bs(ltd, degree = deg, knots = mean(ltd))	366.5	3	6.356	0.0002662
bs(ebitda, degree = deg, knots = mean(ebitda))	1012	3	17.55	2.294e-11
bs(ppe, degree = deg, knots = mean(ppe))	503.3	3	8.73	8.806e-06
bs(sales, degree = deg, knots = mean(sales))	417.1	3	7.235	7.576e-05
bs(ads, degree = deg, knots = mean(ads))	119.9	3	2.079	0.1006
bs(rd, degree = deg, knots = mean(rd))	606.5	3	10.52	6.601e-07
bs(bookval, degree = deg, knots = mean(bookval))	23693	3	410.9	2.056e-255
bs(mv, degree = deg, knots = mean(mv))	66171	3	1148	0
as.factor(indclass)	1641	40	2.135	4.093e-05
Residuals	258170	13433	NA	NA

```

pander(Anova(stepfitbs3))

```

Table 12: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
bs(ltdratio, degree = deg, knots = mean(ltdratio))	1837	3	31.84	1.686e-20
bs(capexratio, degree = deg, knots = mean(capexratio))	149	3	2.584	0.0515
bs(ebitdaratio, degree = deg, knots = mean(ebitdaratio))	3514	3	60.92	4.138e-39
bs(assets, degree = deg, knots = mean(assets))	988.1	3	17.13	4.235e-11
bs(ltd, degree = deg, knots = mean(ltd))	330.9	3	5.737	0.000643
bs(ebitda, degree = deg, knots = mean(ebitda))	1040	3	18.03	1.125e-11
bs(ppe, degree = deg, knots = mean(ppe))	716.6	3	12.42	4.137e-08
bs(sales, degree = deg, knots = mean(sales))	445.5	3	7.724	3.752e-05
bs(rd, degree = deg, knots = mean(rd))	554.2	3	9.608	2.474e-06
bs(bookval, degree = deg, knots = mean(bookval))	24054	3	417	4.768e-259
bs(mv, degree = deg, knots = mean(mv))	68757	3	1192	0
as.factor(indclass)	1940	40	2.523	3.867e-07
Residuals	258628	13451	NA	NA

```
pander(AIC(bsfit3, stepfitbs3))
```

	df	AIC
bsfit3	93	78455
stepfitbs3	75	78443

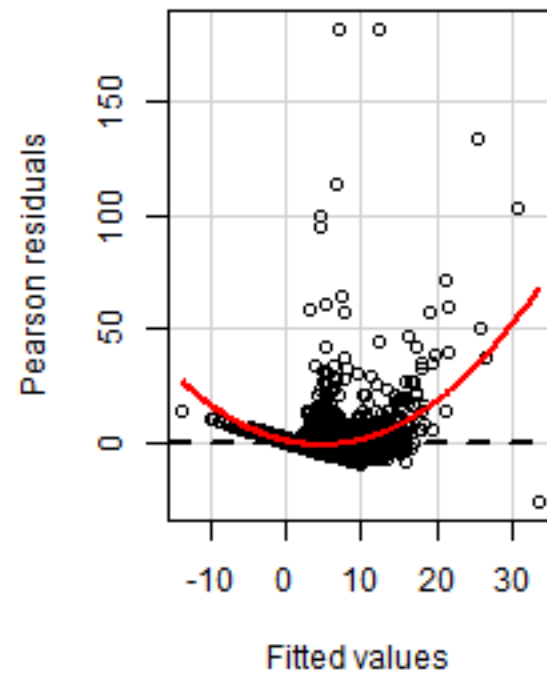
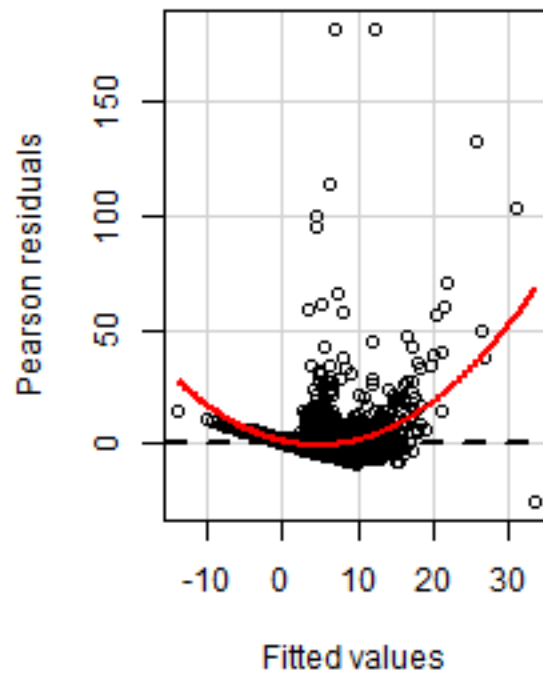
```
pander(BIC(bsfit3, stepfitbs3))
```

	df	BIC
bsfit3	93	79153
stepfitbs3	75	79006

```
par(mfrow = c(1, 2))
residualPlots(bsfit3, terms = ~1)
```

	Test stat	Pr(> t)
Tukey test	59.46	0

```
residualPlots(stepfitbs3, terms = ~1)
```

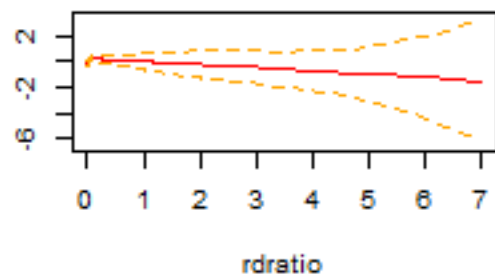
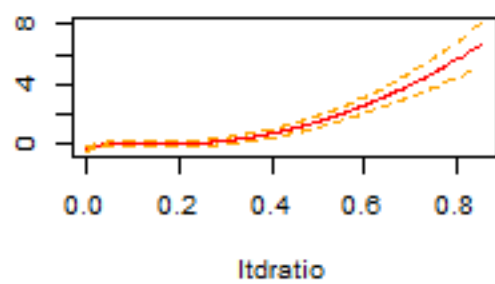


```

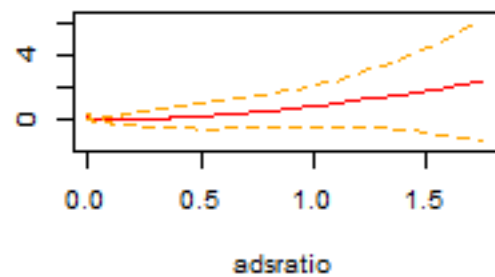
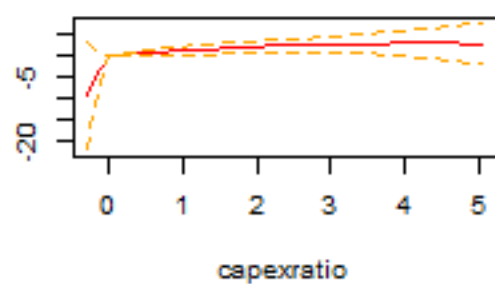
      Test stat Pr(>|t|)
Tukey test   58.571      0
par(mfrow = c(2, 2))
termplot(bsfit3, se = T, ylim = "free")

```

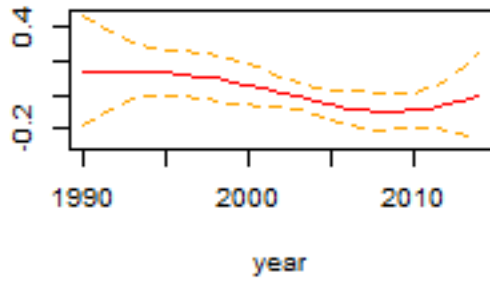
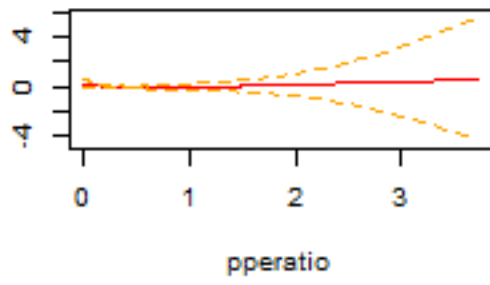
· bs(rdratio, degree = deg, knots = mbs(ltdratio, degree = deg, knots = n



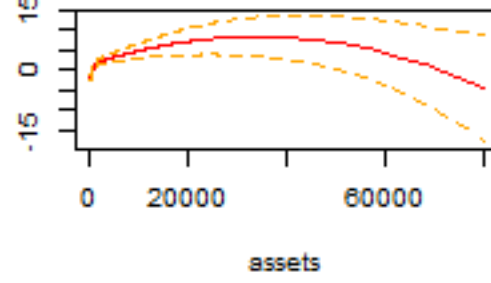
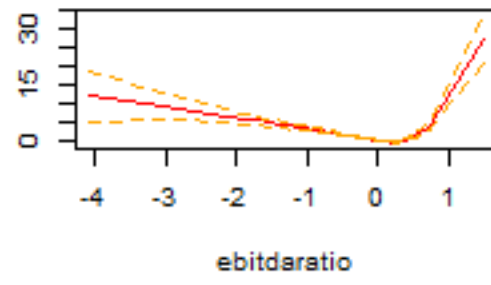
s(adsratio, degree = deg, knots = mapexratio, degree = deg, knots = n



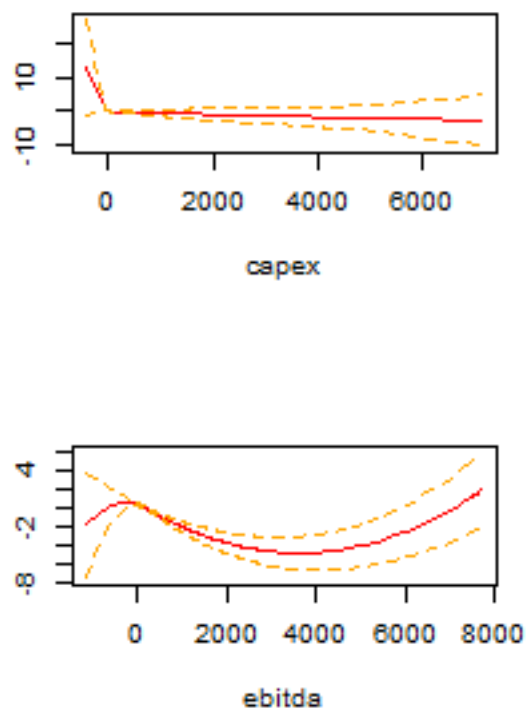
or bs(year, degree = deg, knots = m(ppratio, degree = deg, knots = n



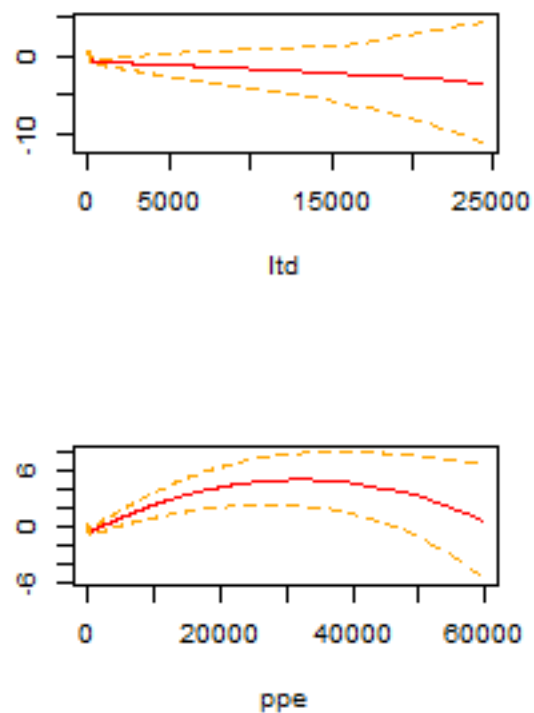
bs(assets, degree = deg, knots = m(ebitdaratio, degree = deg, knots = n



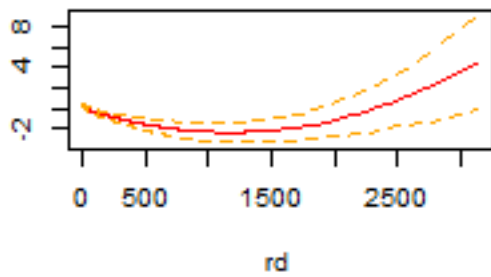
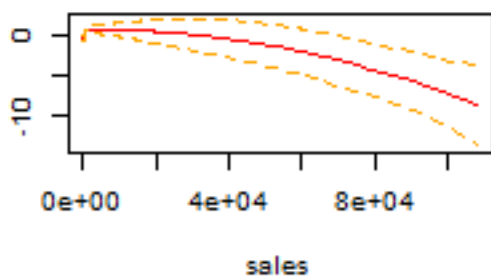
bs(ebitda, degree = deg, knots = n) or bs(capex, degree = deg, knots = n



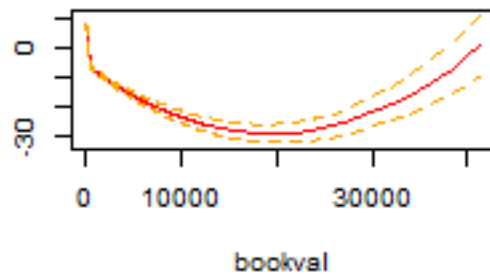
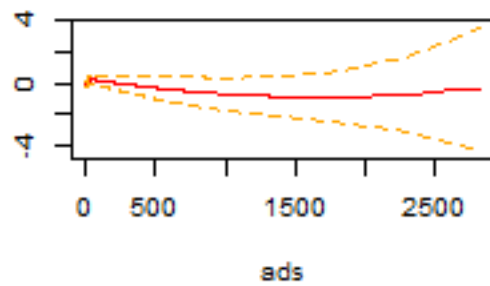
or bs(ppe, degree = deg, knots = n) for bs(ltd, degree = deg, knots = n



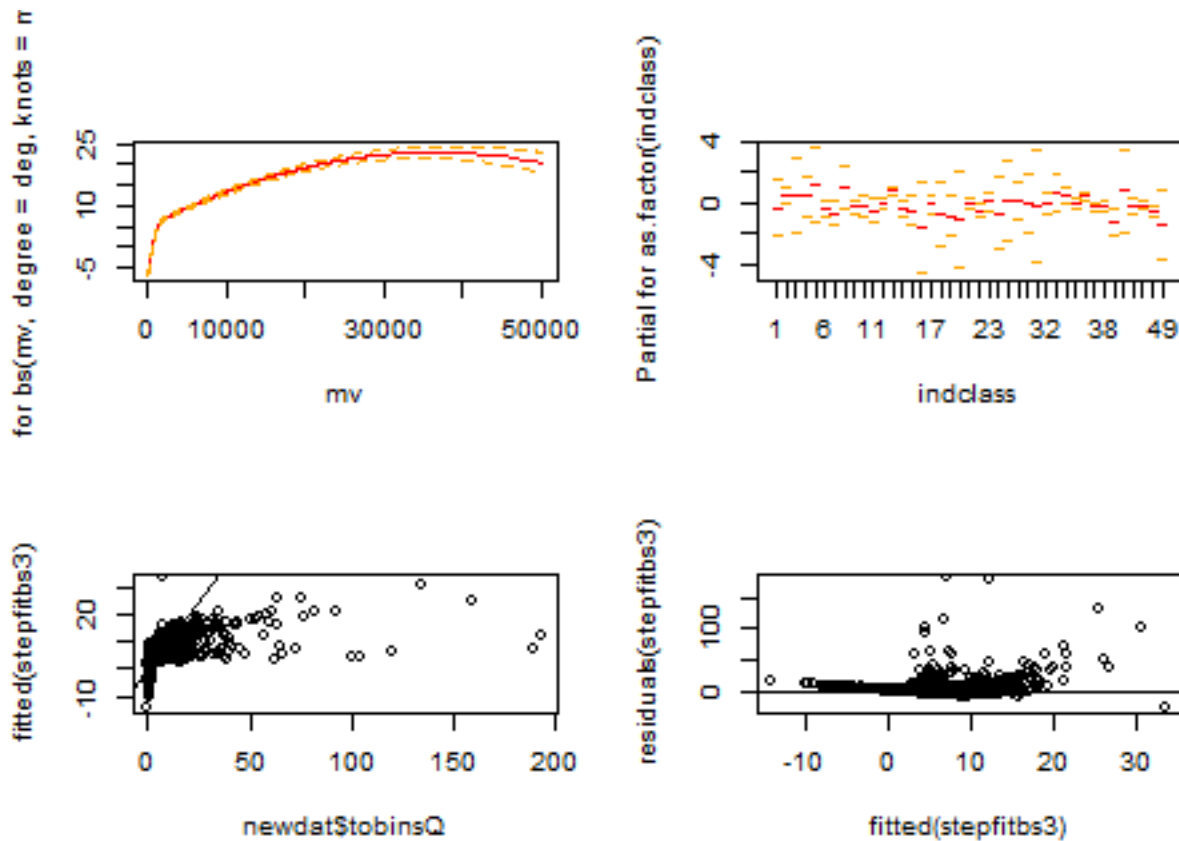
l for bs(rd, degree = deg, knots = m) bs(sales, degree = deg, knots = n



s(bookval, degree = deg, knots = m) or bs(ads, degree = deg, knots = n



```
plot(newdat$tobinsQ, fitted(stepfitbs3))
abline(0, 1)
plot(fitted(stepfitbs3), residuals(stepfitbs3))
abline(h = 0)
```



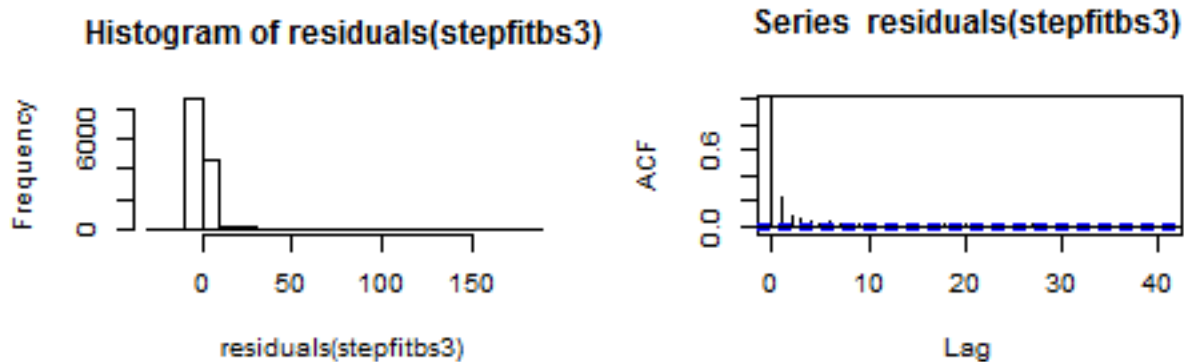
```
hist(residuals(stepfitbs3))
acf(residuals(stepfitbs3))
ncvTest(stepfitbs3)
```

Non-constant Variance Score Test
 Variance formula: ~ fitted.values
 Chisquare = 44797.37 Df = 1 p = 0

```
runs.test(residuals(stepfitbs3))
```

Runs Test - Two sided

data: residuals(stepfitbs3)
 Standardized Runs Statistic = -55.885, p-value < 2.2e-16

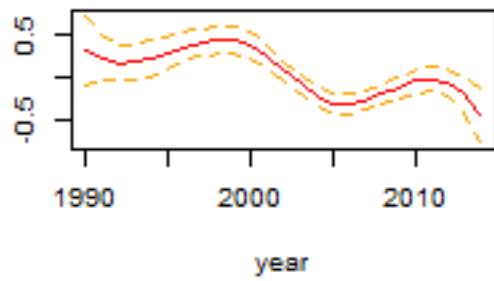
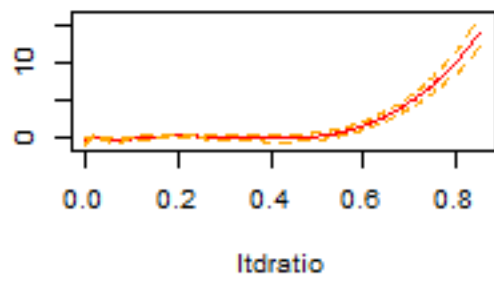


```
deg <- 3
bsfit4 <- lm(tobinsQ ~ bs(ltdratio, degree = deg, knots = quantile(ltdratio[ltdratio >
  min(ltdratio)])[2:3]) + bs(capexratio, degree = deg, knots = quantile(capexratio[capexratio >
  min(capexratio)])[2:3]) + bs(rdratio, degree = deg, knots = quantile(rdratio[rdratio >
  min(rdratio)])[2:3]) + bs(adsratio, degree = deg, knots = quantile(adsratio[adsratio >
  min(adsratio)])[2:3]) + bs(pperatio, degree = deg, knots = quantile(pperatio[pperatio >
  min(pperatio)])[2:3]) + bs(ebitdaratio, degree = deg, knots = quantile(ebitdaratio[ebitdaratio >
  min(ebitdaratio)])[2:3]) + bs(year, degree = deg, knots = quantile(year[year >
  min(year)])[2:3]) + bs(assets, degree = deg, knots = quantile(assets[assets >
  min(assets)])[2:3]) + bs(capex, degree = deg, knots = quantile(capex[capex >
  min(capex)])[2:3]) + bs(ltd, degree = deg, knots = quantile(ltd[ltd >
  min(ltd)])[2:3]) + bs(ebitda, degree = deg, knots = quantile(ebitda[ebitda >
  min(ebitda)])[2:3]) + bs(ppe, degree = deg, knots = quantile(ppe[ppe >
  min(ppe)])[2:3]) + bs(sales, degree = deg, knots = quantile(sales[sales >
  min(sales)])[2:3]) + bs(ads, degree = deg, knots = quantile(ads[ads >
  min(ads)])[2:3]) + bs(rd, degree = deg, knots = quantile(rd[rd >
  min(rd)])[2:3]) + bs(bookval, degree = deg, knots = quantile(bookval[bookval >
  min(bookval)])[2:3]) + bs(mv, degree = deg, knots = quantile(mv[mv >
  min(mv)])[2:3]) + as.factor(indclass), data = newdat)

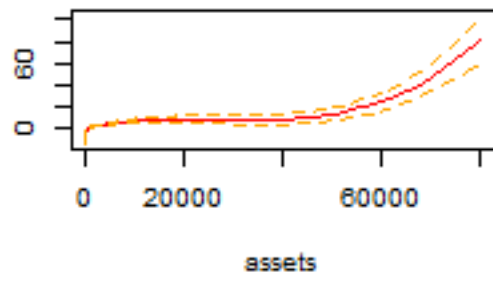
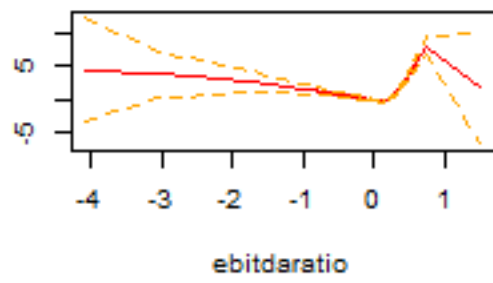
stepfitbs4 <- step(bsfit4, trace = 0)

par(mfrow = c(2, 2))
termplot(stepfitbs4, se = T, ylim = "free")
```

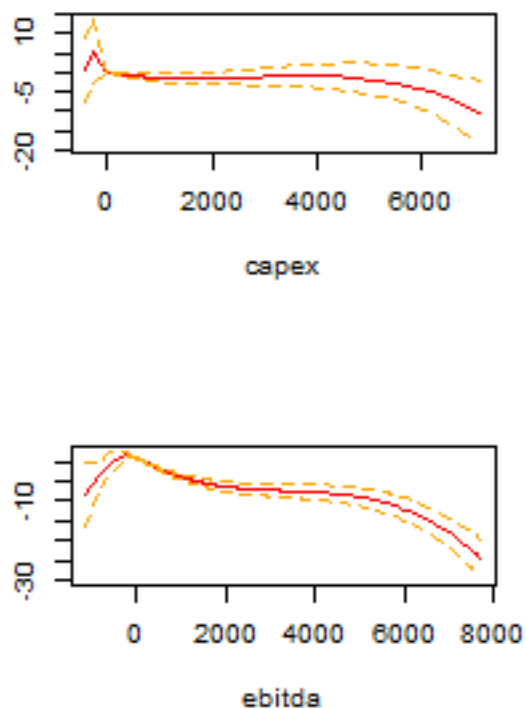
egree = deg, knots = quantile(year[ree = deg, knots = quantile(ltdratio)



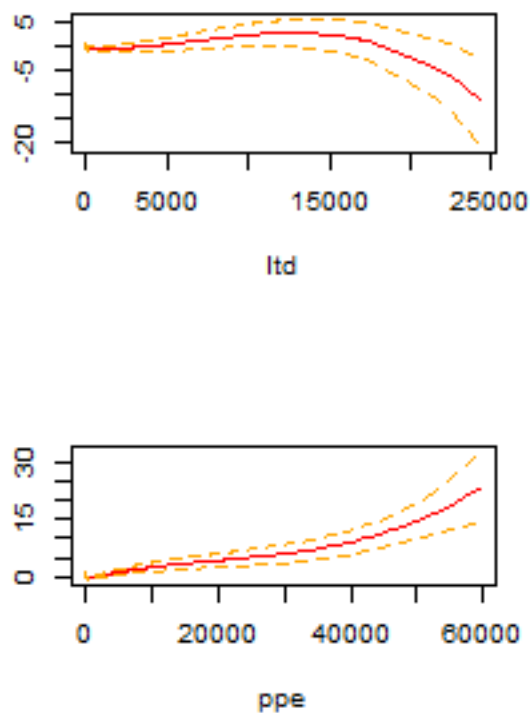
iree = deg, knots = quantile(assets[ree = deg, knots = quantile(ebitdaratio)



```
jree = deg, knots = quantile(ebitda|jree = deg, knots = quantile(capex|
```



```
degree = deg, knots = quantile(ppe|degree = deg, knots = quantile(ltd|
```



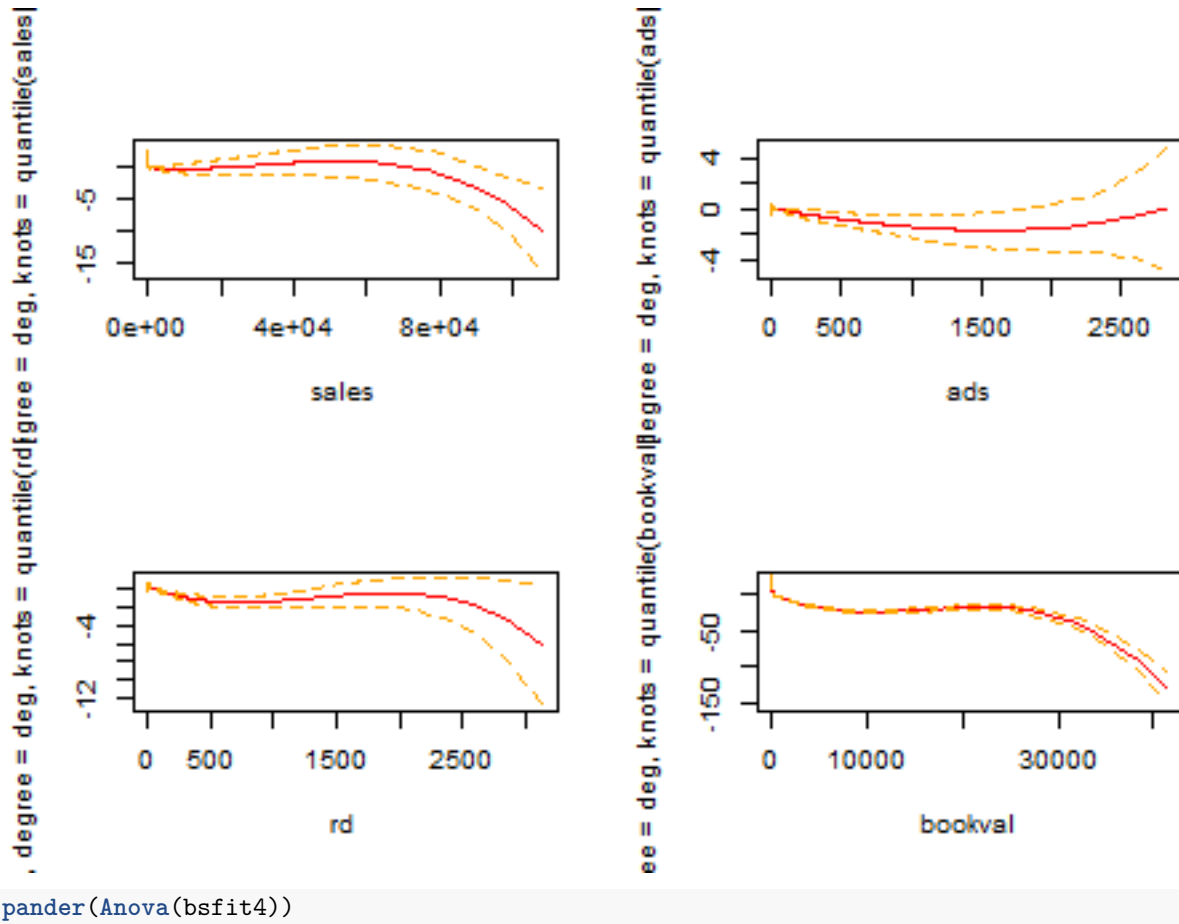


Table 15: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
bs(ltdratio, degree = deg, knots = quantile(ltdratio[ltdratio > min(ltdratio)])[2:3])	3429	5	40.77	8.888e-42
bs(capexratio, degree = deg, knots = quantile(capexratio[capexratio > min(capexratio)])[2:3])	54.21	5	0.6447	0.6656
bs(rdratio, degree = deg, knots = quantile(rdratio[rdratio > min(rdratio)])[2:3])	132.1	5	1.571	0.1644
bs(adsratio, degree = deg, knots = quantile(adsratio[adsratio > min(adsratio)])[2:3])	91.92	5	1.093	0.3618
bs(pperatio, degree = deg, knots = quantile(pperatio[pperatio > min(pperatio)])[2:3])	57.45	5	0.6832	0.6362
bs(ebitdaratio, degree = deg, knots = quantile(ebitdaratio[ebitdaratio > min(ebitdaratio)])[2:3])	3039	5	36.14	6.783e-37
bs(year, degree = deg, knots = quantile(year[year > min(year)])[2:3])	611.6	5	7.273	8.193e-07

	Sum Sq	Df	F value	Pr(>F)
bs(assets, degree = deg, knots = quantile(assets[assets > min(assets)]))[2:3])	5245	5	62.37	1.625e-64
bs(capex, degree = deg, knots = quantile(capex[capex > min(capex)]))[2:3])	259.8	5	3.089	0.008645
bs(ltd, degree = deg, knots = quantile(ltd[ltd > min(ltd)]))[2:3])	401.5	5	4.775	0.0002314
bs(ebitda, degree = deg, knots = quantile(ebitda[ebitda > min(ebitda)]))[2:3])	2650	5	31.51	5.083e-32
bs(ppe, degree = deg, knots = quantile(ppe[ppe > min(ppe)]))[2:3])	1723	5	20.5	1.909e-20
bs(sales, degree = deg, knots = quantile(sales[sales > min(sales)]))[2:3])	262.2	5	3.118	0.008137
bs(ads, degree = deg, knots = quantile(ads[ads > min(ads)]))[2:3])	232.7	5	2.768	0.01671
bs(rd, degree = deg, knots = quantile(rd[rd > min(rd)]))[2:3])	486.8	5	5.789	2.405e-05
bs(bookval, degree = deg, knots = quantile(bookval[bookval > min(bookval)]))[2:3])	44053	5	523.9	0
bs(mv, degree = deg, knots = quantile(mv[mv > min(mv)]))[2:3])	68811	5	818.3	0
as.factor(indclass)	1298	40	1.929	0.0003886
Residuals	225348	13399	NA	NA

`pander(Anova(stepfitbs4))`

Table 16: Anova Table (Type II tests)

	Sum Sq	Df	F value	Pr(>F)
bs(ltdratio, degree = deg, knots = quantile(ltdratio[ltdratio > min(ltdratio)]))[2:3])	3374	5	40.02	5.567e-41
bs(ebitdaratio, degree = deg, knots = quantile(ebitdaratio[ebitdaratio > min(ebitdaratio)]))[2:3])	3158	5	37.45	2.822e-38
bs(year, degree = deg, knots = quantile(year[year > min(year)]))[2:3])	763.6	5	9.056	1.315e-08
bs(assets, degree = deg, knots = quantile(assets[assets > min(assets)]))[2:3])	6174	5	73.22	6.583e-76
bs(capex, degree = deg, knots = quantile(capex[capex > min(capex)]))[2:3])	225.7	5	2.677	0.02007
bs(ltd, degree = deg, knots = quantile(ltd[ltd > min(ltd)]))[2:3])	382.9	5	4.54	0.0003876
bs(ebitda, degree = deg, knots = quantile(ebitda[ebitda > min(ebitda)]))[2:3])	2895	5	34.34	5.375e-35

	Sum Sq	Df	F value	Pr(>F)
bs(ppe, degree = deg, knots = quantile(ppe[ppe > min(ppe)])[2:3])	1633	5	19.37	2.889e-19
bs(sales, degree = deg, knots = quantile(sales[sales > min(sales)])[2:3])	448.2	5	5.316	6.963e-05
bs(ads, degree = deg, knots = quantile(ads[ads > min(ads)])[2:3])	209.8	5	2.488	0.02928
bs(rd, degree = deg, knots = quantile(rd[rd > min(rd)])[2:3])	786.8	5	9.331	6.914e-09
bs(bookval, degree = deg, knots = quantile(bookval[bookval > min(bookval)])[2:3])	45926	5	544.7	0
bs(mv, degree = deg, knots = quantile(mv[mv > min(mv)])[2:3])	76835	5	911.2	0
Residuals	226974	13459	NA	NA

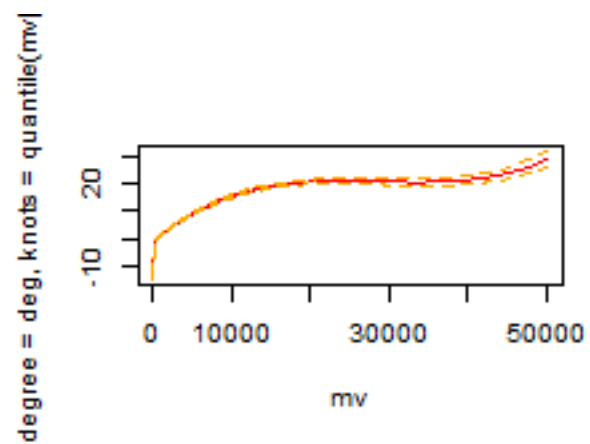
```
pander(AIC(bsfit4, stepfitbs4))
```

	df	AIC
bsfit4	127	76684
stepfitbs4	67	76661

```
pander(BIC(bsfit4, stepfitbs4))
```

	df	BIC
bsfit4	127	77638
stepfitbs4	67	77164

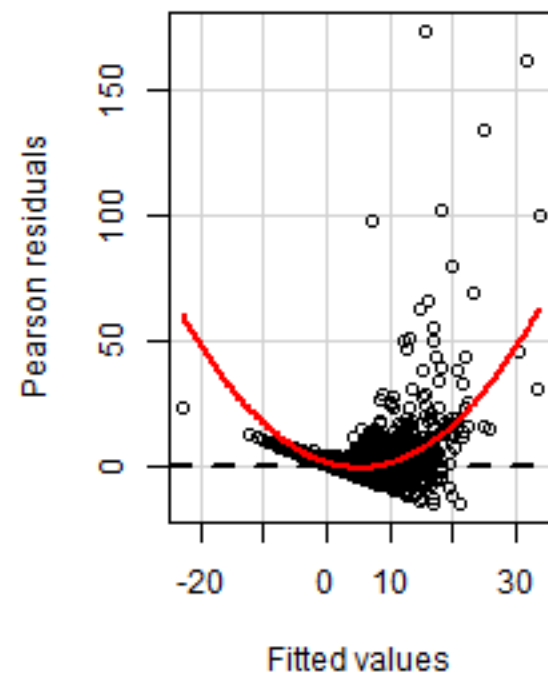
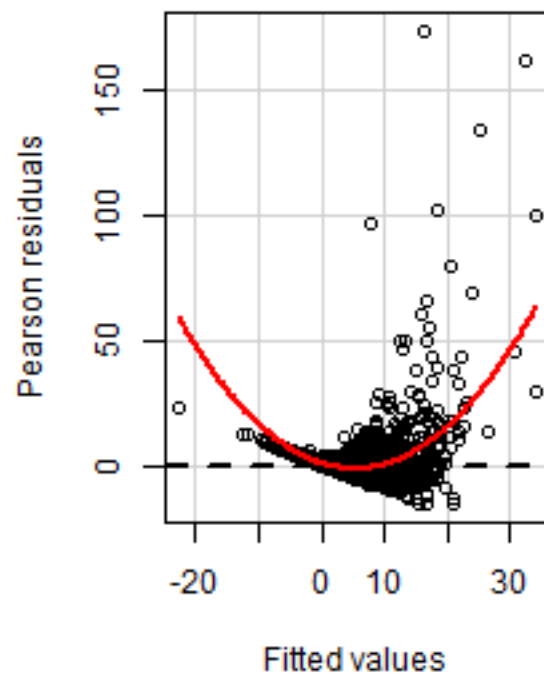
```
par(mfrow = c(1, 2))
```



```
residualPlots(bsfit4, terms = ~1)
```

	Test stat	Pr(> t)
Tukey test	85.44	0

```
residualPlots(stepfitbs4, terms = ~1)
```



```

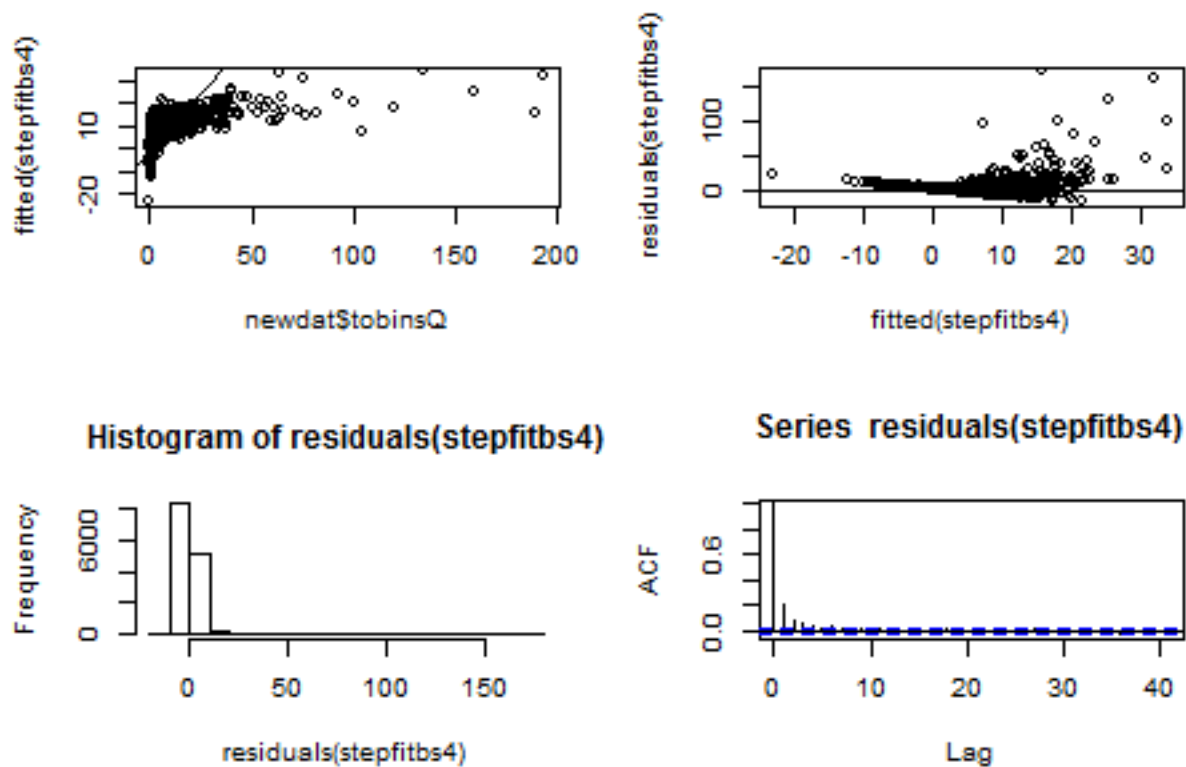
          Test stat Pr(>|t|)
Tukey test  83.616      0

```

```

par(mfrow = c(2, 2))
plot(newdat$tobinsQ, fitted(stepfitbs4))
abline(0, 1)
plot(fitted(stepfitbs4), residuals(stepfitbs4))
abline(h = 0)
hist(residuals(stepfitbs4))
acf(residuals(stepfitbs4))

```



```
ncvTest(stepfitbs4)
```

```
Non-constant Variance Score Test
Variance formula: ~ fitted.values
Chisquare = 110418.6    Df = 1    p = 0    P 小 , non-constant
```

```
runs.test(residuals(stepfitbs4))
```

Runs Test - Two sided

```
data: residuals(stepfitbs4)
Standardized Runs Statistic = -50.278, p-value < 2.2e-16
```

Questions

1. Which of the following is FALSE?

符合

- The models fitted here are inappropriate in that they do not return fitted values which conform to the natural (hard) boundary of the response range. One possible remedy to this is to use a link function which prevents negative predictions being returned.
- Interestingly, while the covariates that were dropped as part of the stepwise selection procedure which resulted in `polyfit2` did not have concerning $\text{GVIF}^{(1/(2 \cdot \text{Df}))}$ values (e.g. greater than 5), the omission of those variables resulted in a reduction of these $\text{GVIF}^{(1/(2 \cdot \text{Df}))}$ values for some of the other covariates.
- Despite the use of quadratic polynomials, the `stepfit2` model still shows evidence of ill-fitting behaviour; the Tukey's test for non-additivity for the `stepfit2` model has a small test statistic and a small p -value.
- The differences in the `stepfit2` model compared with the `polyfit2` model are not surprising when we consider the p -values exhibited by the terms in the `polyfit2` model.

2. TRUE or FALSE? Both the AIC and BIC indicate that the `stepfit2` model is a better model than the `polyfit2` model. While the difference in the number of parameters between these two models is 6, the cost of including these parameters in the model (as quantified using the penalty for the AIC/BIC) outweigh the fit benefits of including these parameters.

3. Which of the following is FALSE?

- In keeping with all models fitted to date for these data, the largest response values are underpredicted.
- For all models fitted with graphical diagnostics shown in this document, there is evidence of non-constant error variance both visually and quantified numerically using the Breusch-Pagan test.
- For all models fitted with graphical diagnostics shown in this document, there is evidence of non-normality in the errors.
- For all models fitted with graphical diagnostics shown in this document, there is no evidence of correlation in the errors both visually and quantified numerically using the runs test.

4. Which of the following is FALSE?

- Due to overplotting, the partial relationships for the `stepfit2` model are largely obscured.
- Despite the `stepfit3` model having 23 additional parameters compared with the `stepfit2` model, both the AIC and BIC scores favour the `stepfit3` model (even though the BIC has a larger penalty than the AIC in this case).
- While the stepwise model selection results based on the AIC for the quadratic polynomial-based model resulted in the loss of three covariates (compared to the 'full' model), these covariates were retained in the model when cubic polynomials were used instead. This reminds us that model selection does not depend on how each covariate (i.e. in what form) enters the model.
- The model selection results for the `stepfit3` model and the associated p -values for each term are not surprising since the stepwise results are based on the AIC and it is entirely possible to have terms retained under the AIC (or BIC) when the p -values are in excess of 5%.

5. Which of the following is FALSE?

- The model fitted using quadratic B -splines fitted with one internal knot at the mean for each covariate returned a better model fit than the corresponding model (with the same covariates) fitted using cubic polynomials instead. This is true even though both models estimated the same number of parameters which indicates the gains that can be made when parameters are used more wisely.
- While some covariates in the `bsfit3` model were omitted when subjected to stepwise selection based on the AIC, the `stepfitbs3` model retained at least one covariate with a p -value $> 5\%$.
- The stepwise results for the model fitted using quadratic B -splines fitted with one internal knot at the mean resulted in the loss of 5 covariates from the model and still the AIC and BIC scores

suggest the model is favoured when these are omitted.

- There are some differences apparent in the fitted functions using B-splines with knots placed at the mean compared with those with knots placed at quantiles.
6. TRUE or FALSE? While knots placed at quantiles and using degree 3 B-splines ‘cost’ an additional 2 parameters per covariate in this case (compared with a model with degree 2 B-splines and one internal knot placed at the mean of each covariate), the stepwise selected model for the model fitted with splines with knots of this sort, still indicated this model was preferred over the model selected when a knot was placed at the mean for each covariate.
 7. TRUE or FALSE? There was a substantial loss of model covariates when B-splines were fitted with knots placed at quantiles compared to the loss in the number of covariates seen with other less complicated models when subjected to stepwise selection. Despite this loss, all terms which remained were statistically significant at the 5% level.