

# Information Security

## *Lecture 2: Cryptography*

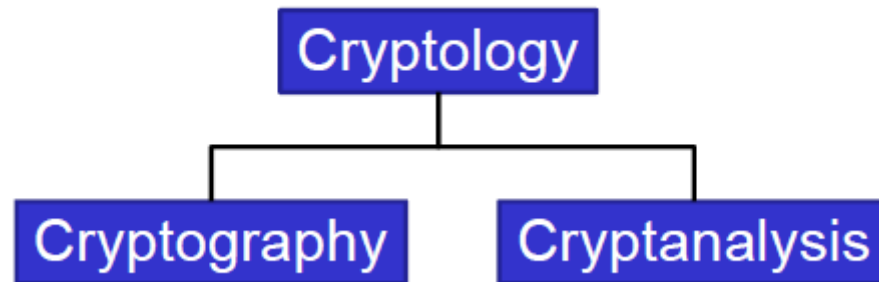
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# Terminology

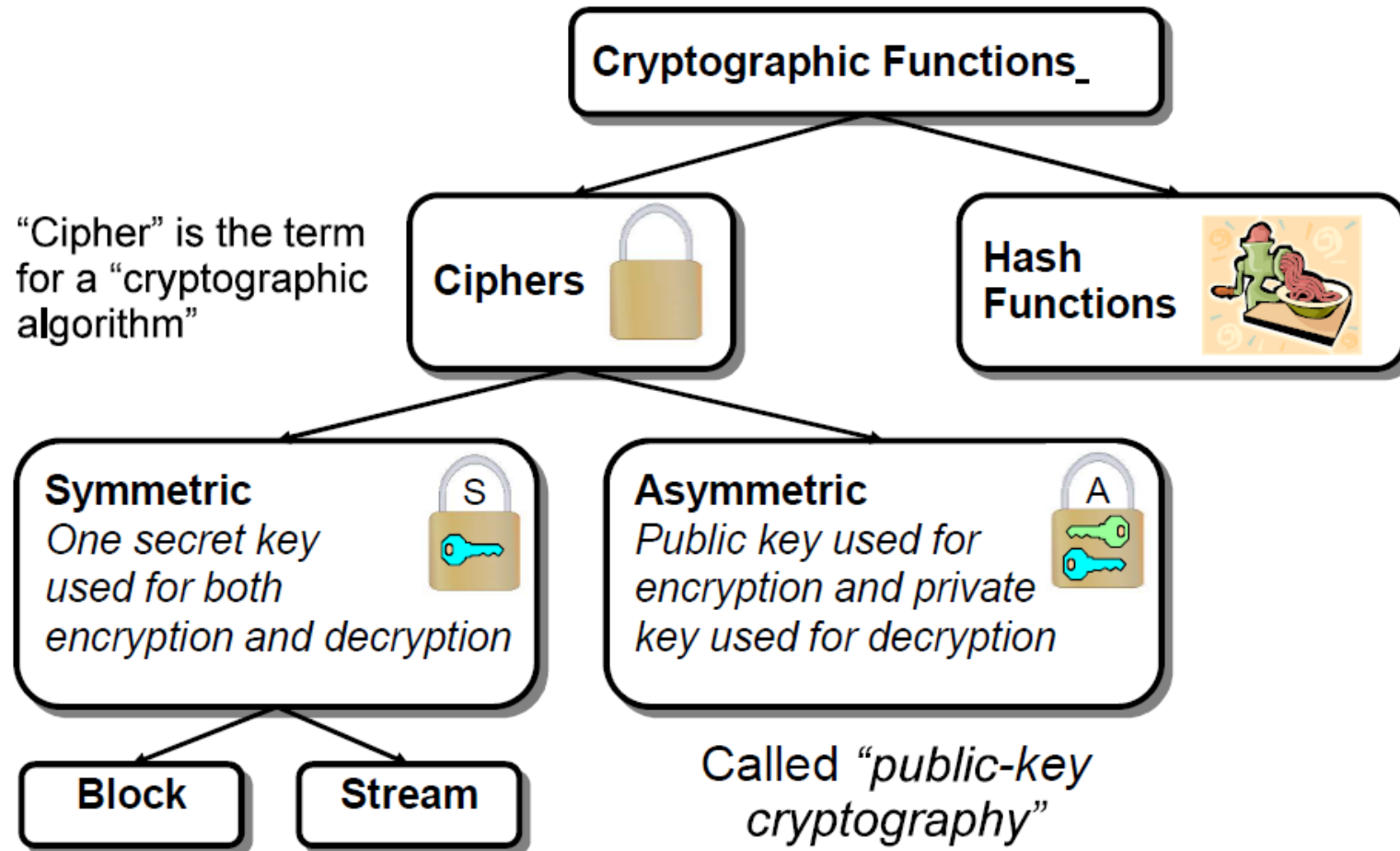
- **Greek Word:** “Krypto” = Hidden / Secret
- Cryptography is the science of secret writing with the goal of hiding the meaning of a message.
- Cryptanalysis is the science of breaking cryptography.
- Cryptology covers both cryptography and cryptanalysis.



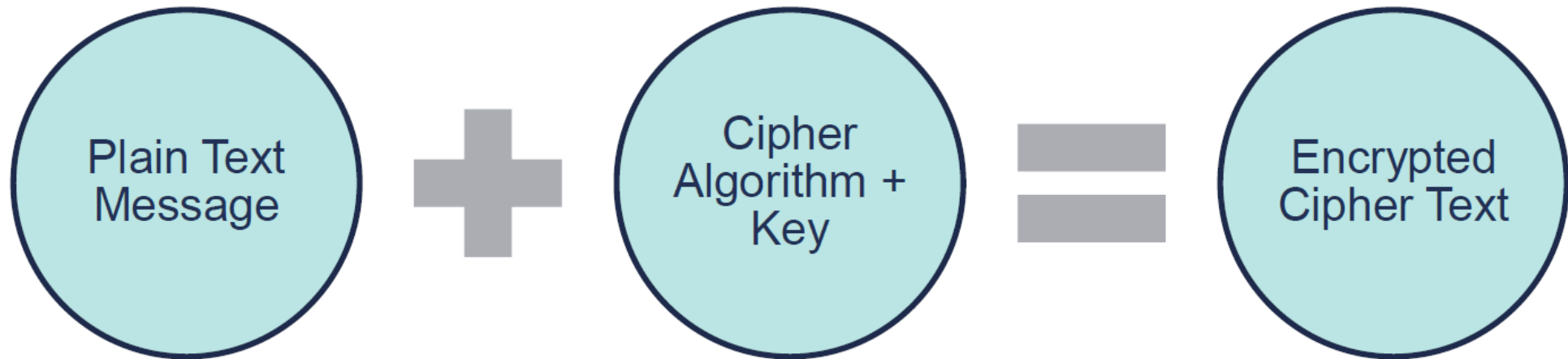
# What can cryptography do?

- Crypto can provide the following security services:
- Confidentiality:
  - Makes data unreadable to entities who do not have the appropriate cryptographic keys, even if they have the data.
- Data Integrity:
  - Entities with the appropriate cryptographic keys can verify that data is correct and has not been altered, either deliberately or accidentally.
- Authentication:
  - Entities who communicate can be assured that the other user/entity or the sender of a message is what it claims to be.
- Digital Signature and PKI (Public-Key Infrastructure):
  - Strong proof of data origin which can be verified by 3rd parties.
  - Scalable (to the whole Internet) distribution of cryptographic keys.

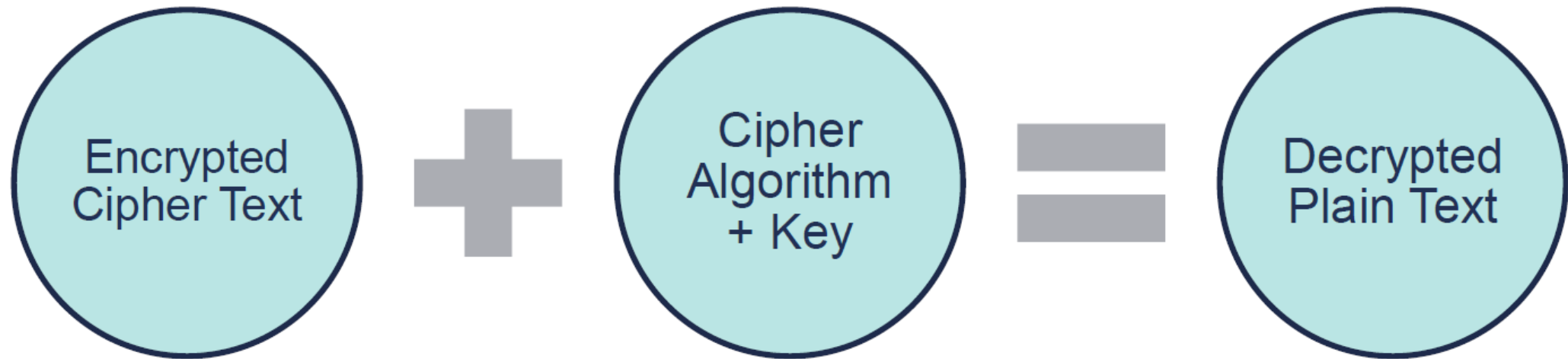
# Taxonomy of cryptographic functions



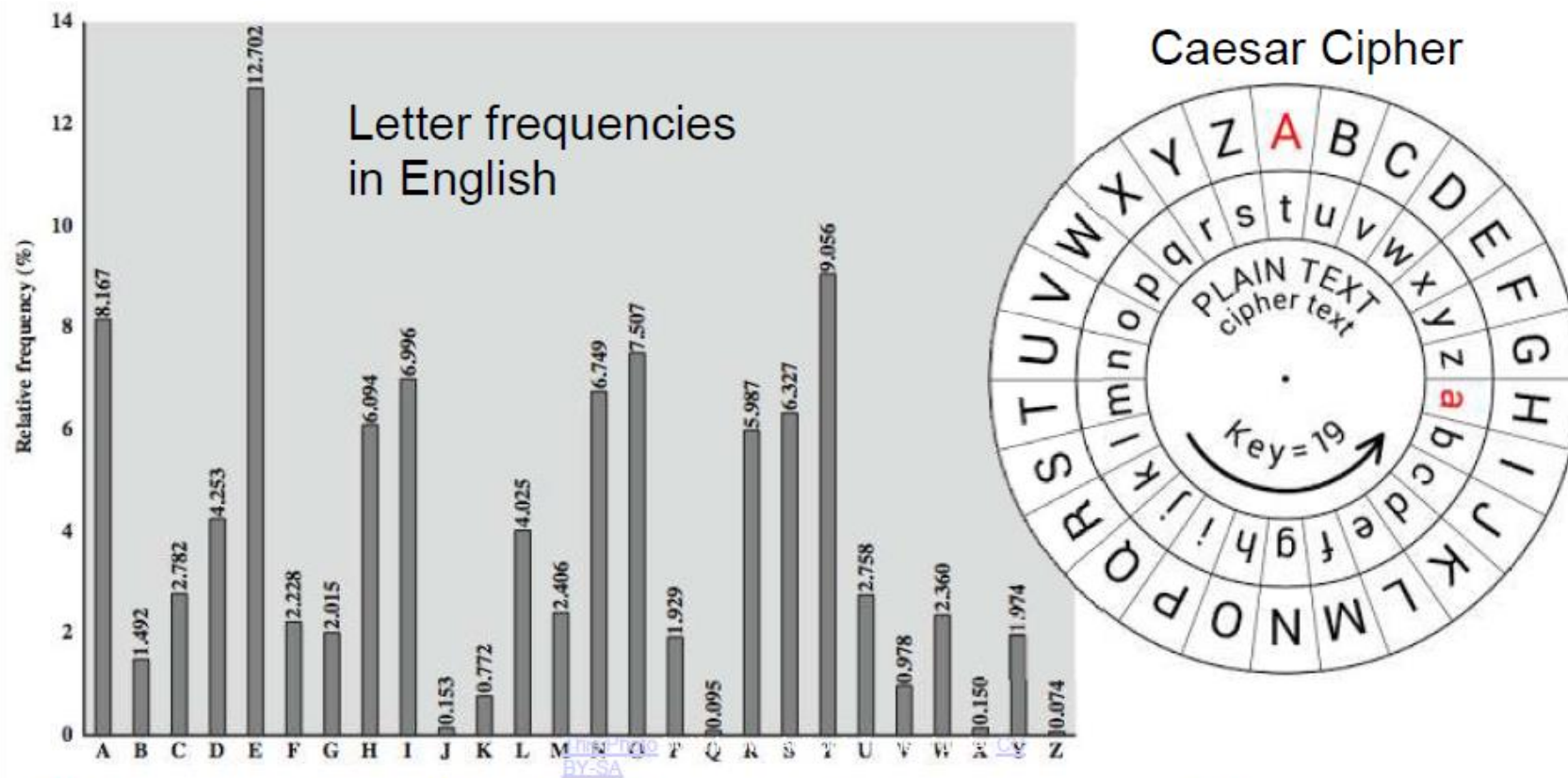
# Encrypting a Message



# Decrypting a Message

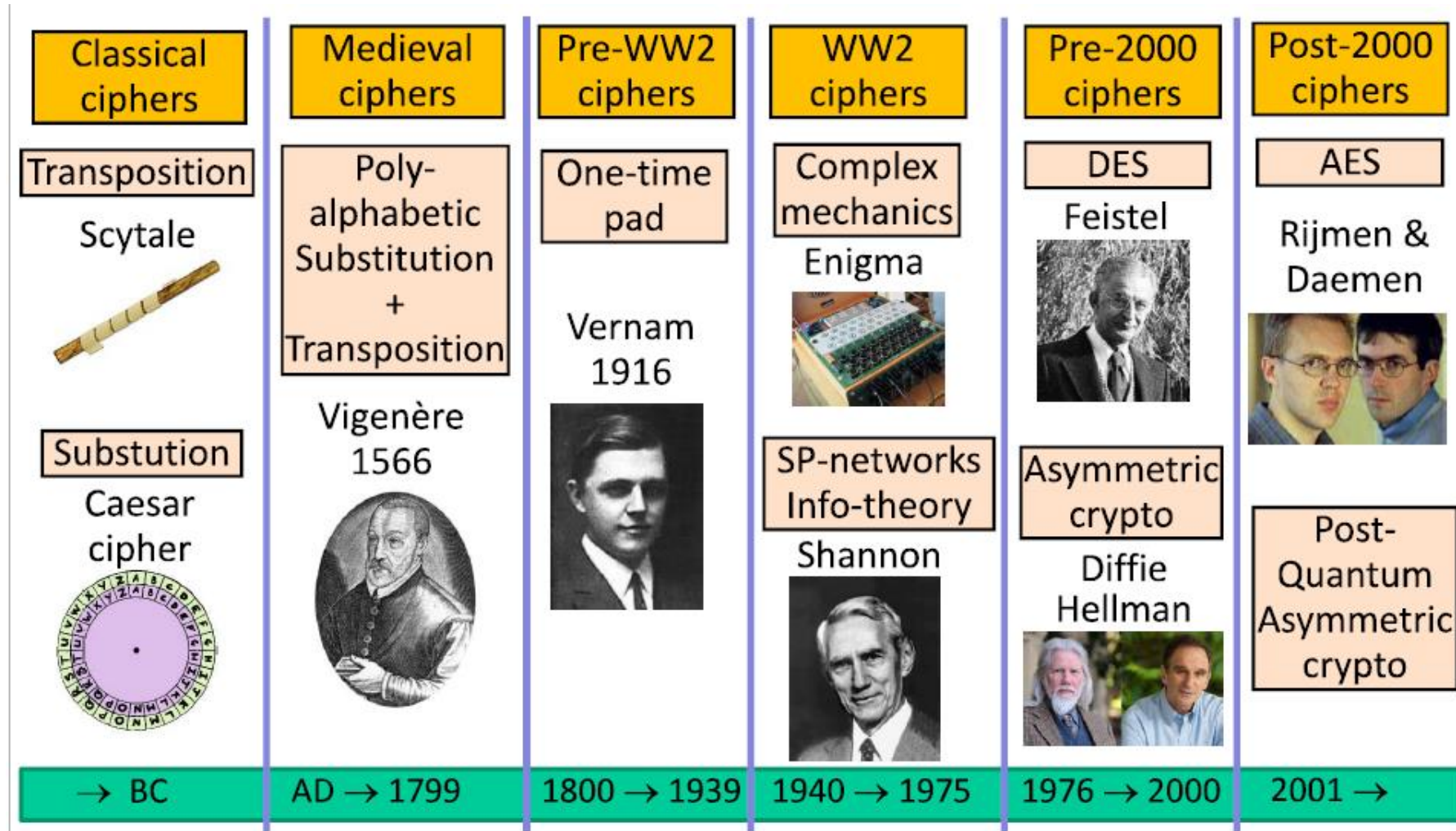


# Letter Frequencies



Historic ciphers, like the Caesar Cipher, are weak because they fail to hide statistical regularities in the ciphertext.

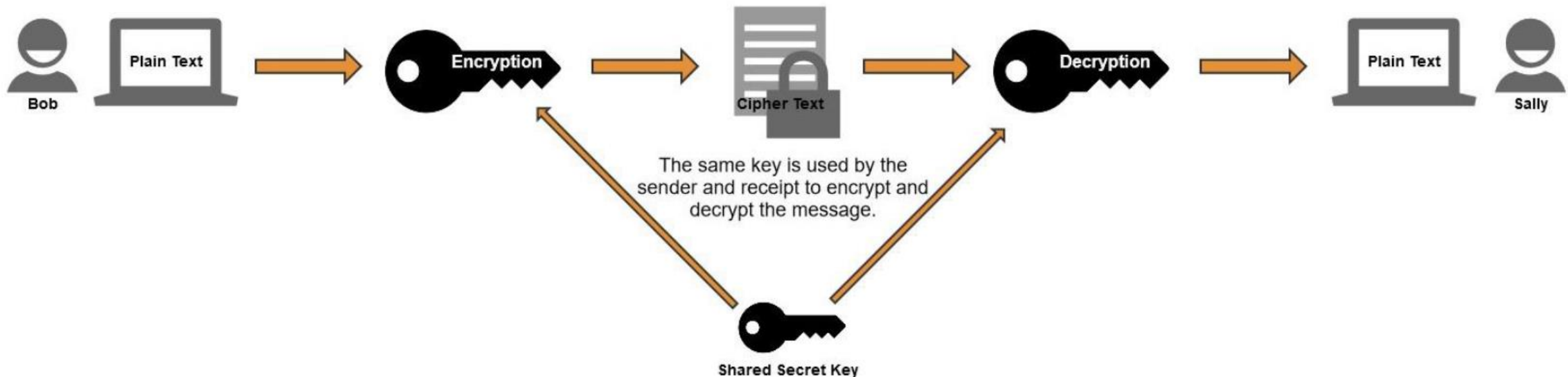
# Evolution of Ciphers





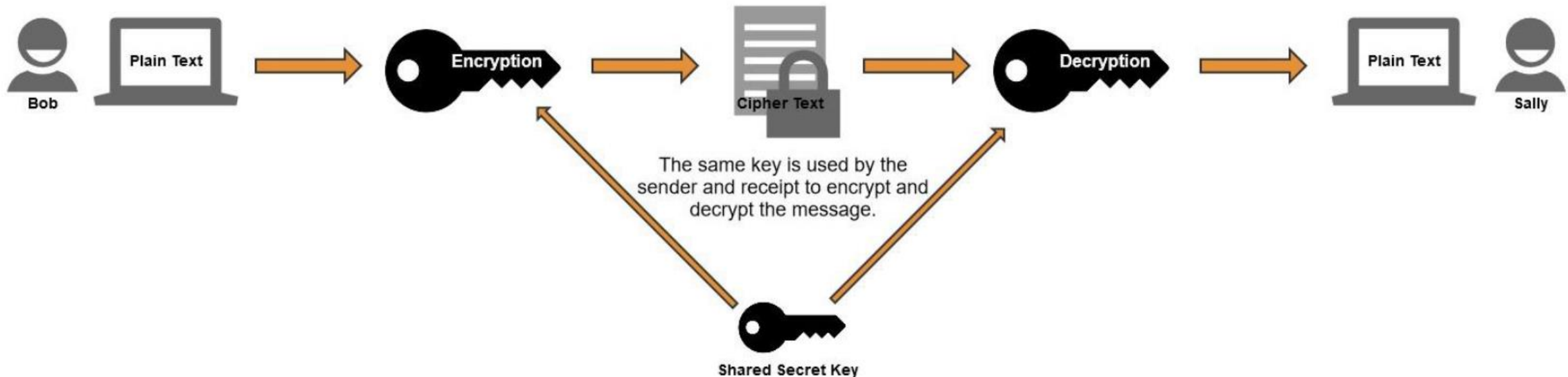
# Symmetric (Private Key) Encryption

- Symmetric encryption uses a **single key** for **encryption** and **decryption**.
- Both the **sender** and **receiver** have the **same key** and use it to encrypt and decrypt all messages.
- It's also known as **secret-key encryption** or **private-key encryption**.



# Symmetric (Private Key) Encryption

- **Symmetric encryption** is much more efficient at encrypting large amounts of data than its counterpart, **asymmetric encryption**.
- The downside of symmetrical encryption is that it makes it hard to initiate communication the first time.



# Symmetric Encryption Algorithm: DES & 3DES

- **Data Encryption Standard (DES)**
- DES is an older algorithm that widely used for a period of time dating back to the 1970's.
- It has been compromised and no longer secure.
- **Triple DES (3DES)**
- 3DES was developed as an improvement over DES.
- It improved the encryption by encrypting the data with DES three times with two, or sometimes three keys.
- While 3DES is a significant improvement over DES, it consumes a lot of processor power and memory resources.
- AES is much less resource-intensive and has replaced 3DES as the current standard.

# Symmetric Encryption Algorithm: AES

- **Advanced Encryption Standard (AES)**
- AES is a very strong encryption algorithm that's commonly used worldwide.
- It's significantly faster than both DES and 3DES and also provides stronger encryption.
  - 128-Bit AES would take billion of years to brute force
- It's also the "official" encryption standard for the U.S. government (since 2002).
- Governments, militaries, banks and corporations rely on it. It's responsible for securing most, if not all of your personal and financial data. There are special CPU instructions for it.

# AES Algorithm (*Rijndael*)

- AES can be performed with the following key sizes: **128 bits, 196 bits and 256 bits**. Generally, increasing the key size also increases the level of security.
- It is also a *block cipher*, meaning that the data is divided into blocks before encryption. AES divides plaintext into blocks of 16 bytes (128 bits).
- we arrange each block of the plaintext into a 4x4 matrix and repeatedly perform a set of operations on it. We call each iteration a *round*, and we perform **10, 12 or 14 rounds** depending on the key length:
  - 10 rounds for a 128-bit key
  - 12 rounds for a 196-bit key
  - 14 rounds for a 256 bit key
- For each round, we generate a *round key* from the main key using the *Rijndael Key Schedule*.

# Asymmetric Encryption

- Asymmetric encryption uses two keys, a public key and a private key created as a matched pair.
  - **Private Key**: Kept secret and never shared.
  - **Public Key**: Shared with others.
- Commonly referred to as:
  - Public Key Encryption
  - Public Key Infrastructure (PKI) Encryption

# How Public Key Encryption Works

- Anything encrypted with the **private key** can only be decrypted with the matched **public key**.
- Anything encrypted with the **public key** can only be decrypted with the matched **private key**.



# Diffie-Hellman

- Diffie-Hellman key exchange is a method of digital encryption that securely exchanges cryptographic keys between two parties over a public channel without their conversation being transmitted over the internet.
- The two parties use symmetric cryptography to encrypt and decrypt their messages.





Private = 5



$(6^5) \text{ MOD } 13$   
 $(7776) \text{ MOD } 13$   
Public = 2



$(9^5) \text{ MOD } 13$   
 $(59049) \text{ MOD } 13$   
Shared Secret = 3



Agree upon two numbers:

**P** Prime Number 13  
**G** Generator of P 6

Randomly generate a Private Key

Calculate Public Key:  
 $(G^{\text{Private}}) \text{ MOD } P$

Exchange Public Keys

Calculate the Shared Secret  
 $(\text{Shared Public}^{\text{Private}}) \text{ MOD } P$



Private = 4



$(6^4) \text{ MOD } 13$   
 $(1296) \text{ MOD } 13$   
Public = 9



$(2^4) \text{ MOD } 13$   
 $(16) \text{ MOD } 13$   
Shared Secret = 3



# THE RIVEST-SHAMIR-ADLEMAN (RSA) ALGORITHM FOR PUBLIC-KEY CRYPTOGRAPHY — THE BASIC IDEA

- It is named after its inventors Ron Rivest, Adi Shamir and Len Adleman.
- Published in 1978
- It is the most widely used public-key encryption algorithm today.
- It provides confidentiality and digital signatures.
- Its security is based on the difficulty of integer factorization

# Basic math concepts

Factor	Prime	Semi-Prime	Modulo
Numbers you can multiply to get original number	Number whose factors are only 1 and itself	Numbers whose factors are prime numbers	Remainder Division
Factors of 12: 1 2 3 4 6 12	2 3 5 7 11 13 29 37 61	Factors of Semi Prime 21: 1 3 7 21	13 MOD 5 = 3
Factors of 7: 1 7	Number divisible by only 1 and itself	Product of two Primes is always Semi Prime	21 MOD 5 = 1
			25 MOD 5 = 0

# RSA Example

- Generating Keys:
  - Select two Prime Numbers (**P**, **Q**)
  - Calculate Product (**P**\***Q**)
  - Calculate Totient (**P**-1)\*(**Q**-1)
  - Select Public Key (**E**)
    - Must be Prime
    - Must be less than Totient
    - Must NOT be a factor of the Totient
  - Select a Private Key (**D**)
    - Product of D and E, divided by T must result in a remainder of 1
    - (**D**\***E**) MOD **T** = 1

Prime #s	<b>P</b>	<b>Q</b>	<b>7</b>	<b>19</b>
Product	<b>N</b>		<b>133</b>	
Totient	<b>T</b>		<b>108</b>	
Public Key	<b>E</b>		<b>29</b>	
Private Key	<b>D</b>		<b>41</b>	

# RSA Encryption and Decryption

- Encryption and Decryption:
  - Encryption:
$$\text{Message}^E \text{ MOD } N = \text{Cipher Text}$$
  - Decryption:
$$\text{Cipher}^D \text{ MOD } N = \text{Message}$$
- Encryption and Decryption:
  - Encryption:
$$\text{Message}^D \text{ MOD } N = \text{Cipher Text}$$
  - Decryption:
$$\text{Cipher}^E \text{ MOD } N = \text{Message}$$

# RSA Example

- Encryption and Decryption:

Message  
60

- Encryption:

$$\text{Message}^E \text{ MOD } N = \text{Cipher Text}$$

- Decryption:

$$\text{Cipher}^D \text{ MOD } N = \text{Message}$$

Prime #s	P	Q	7	19
Product	N		133	
Totient	T		108	
Public Key	E		29	
Private Key	D		41	

- **Encrypt** with **Public Key**, **Decrypt** with **Private Key**

$$(60^{29}) \text{ MOD } 133 = 86$$

$$(86^{41}) \text{ MOD } 133 = 60$$

# RSA Example

- Encryption and Decryption:

Message  
60

- Encryption:

$$\text{Message}^D \text{ MOD } N = \text{Cipher Text}$$

- Decryption:

$$\text{Cipher}^E \text{ MOD } N = \text{Message}$$

Prime #s	P	Q	7	19
Product	N		133	
Totient	T		108	
Public Key	E		29	
Private Key	D		41	

- **Encrypt** with **Private Key**, **Decrypt** with **Public Key**

$$(60^{41}) \text{ MOD } 133 = 72$$

$$(72^{29}) \text{ MOD } 133 = 60$$



# RSA

- How secure is RSA?
  - Security lies in difficulty of factoring Semi Prime numbers
    - If given the number 133, could you extract 7 and 19?
      - Really? ... Prove it, what are the factors of 1909?
- In 1991, RSA Laboratory created the RSA Challenge:
  - Released 54 Semi-Primes of various sizes and asked for Factors
    - Competition ended in 2007 – only 12 factors were identified
    - As of 2020, another 11 were identified (no cash was awarded)
      - Biggest number factored: 829 Bits (Feb 2020)
    - In 29 years, the 1024 bit number has never been factored



# RSA

- How secure is RSA?

**1024 bit Semi-Prime number:**

```
1350664108659952233496032162788059699388814756056670
2752448514385152651060485953383394028715057190944179
8207282164471551373680419703964191743046496589274256
2393410208643832021103729587257623585096431105640735
0150818751067659462920556368552947521350085287941637
7328533906109750544334999811150056977236890927563
```

- 1024 bit RSA keys was recommended standard since 2002
- 2048 bit RSA Keys is recommended standard since 2015