

# Introduction to AI

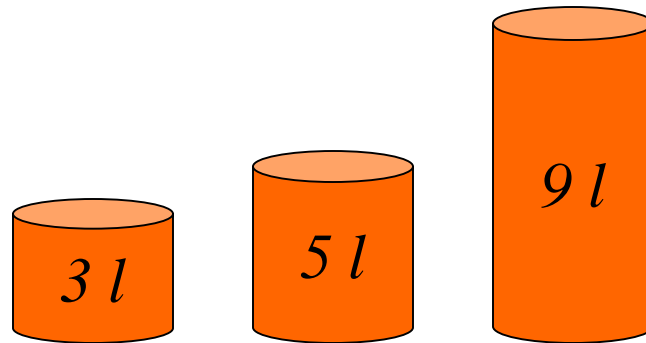
## *Lecture 4: Problem solving and search*

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**LaSalle College**  
Montréal

## Example: Measuring problem!

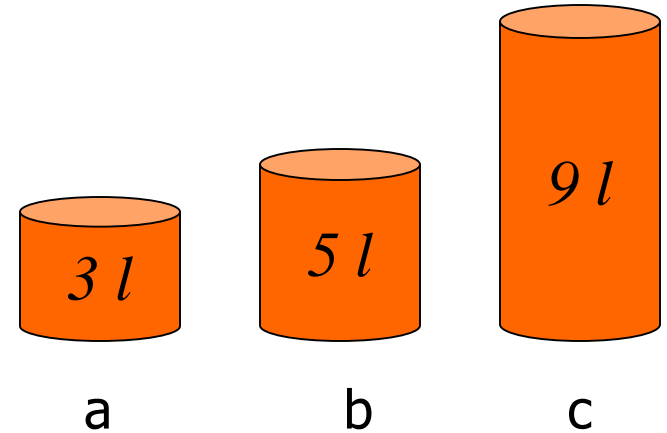


**Problem:** Using these three buckets,  
measure 7 liters of water.

# Example: Measuring problem!

- **(one possible) Solution:**

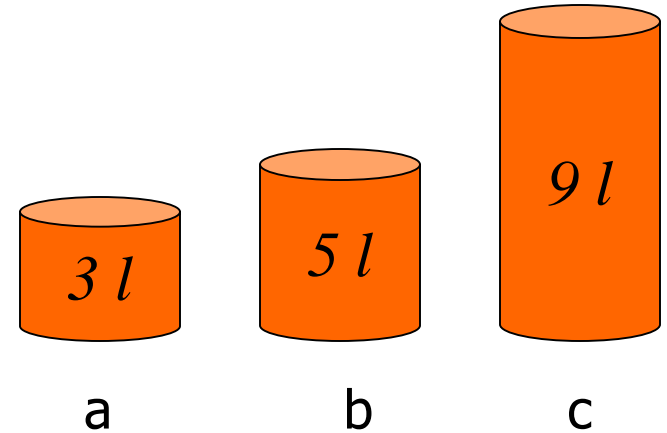
a	b	c	
0	0	0	start



# Example: Measuring problem!

- **(one possible) Solution:**

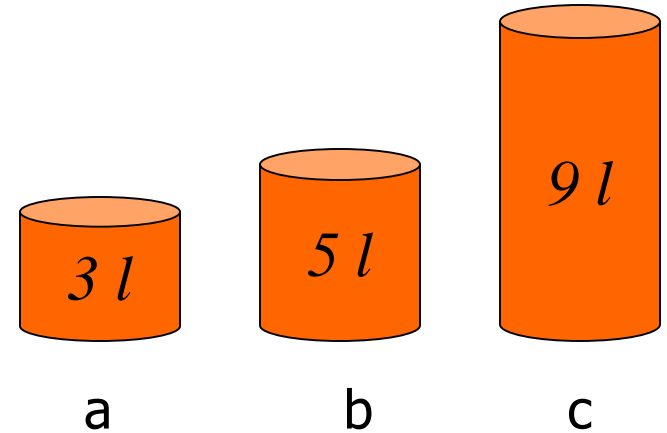
a	b	c	
0	0	0	start
3	0	0	



# Example: Measuring problem!

- (one possible) Solution:**

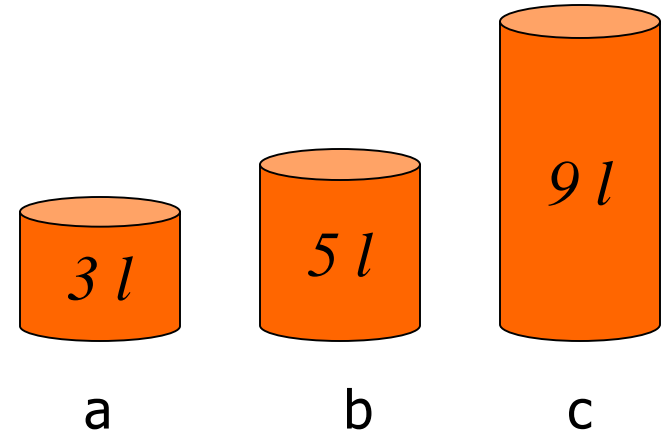
a	b	c	
0	0	0	start
3	0	0	
0	0	3	



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- (one possible) Solution:**

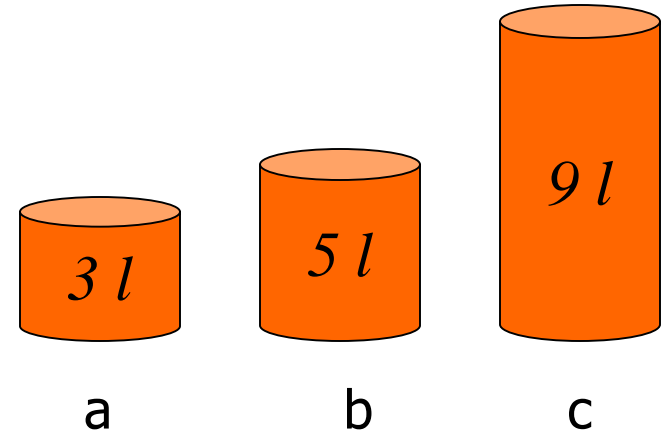
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	



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- (one possible) Solution:**

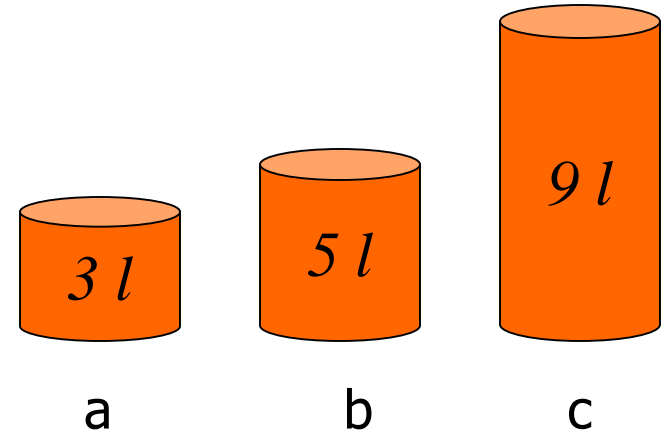
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	



# Example: Measuring problem!

- (one possible) Solution:**

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	

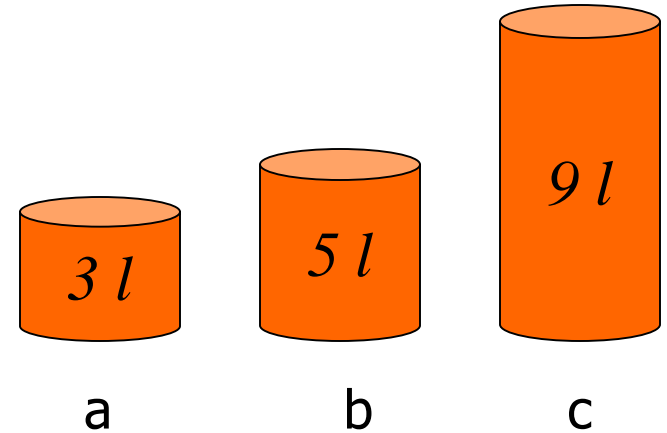




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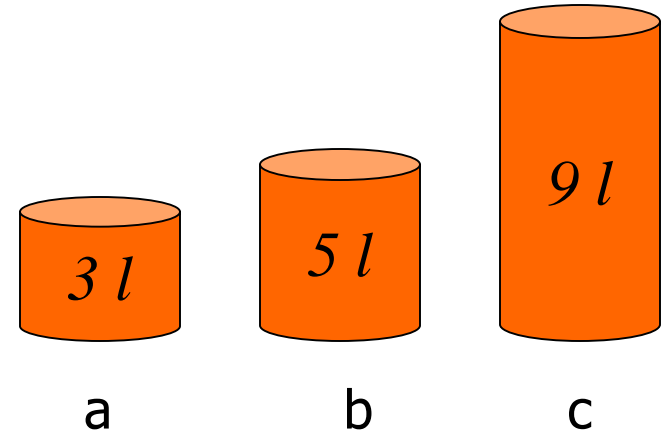
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	



# Example: Measuring problem!

- (one possible) Solution:**

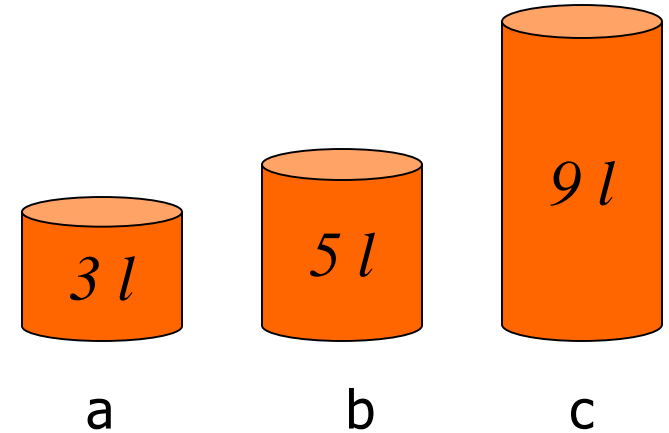
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	



# Example: Measuring problem!

- (one possible) Solution:**

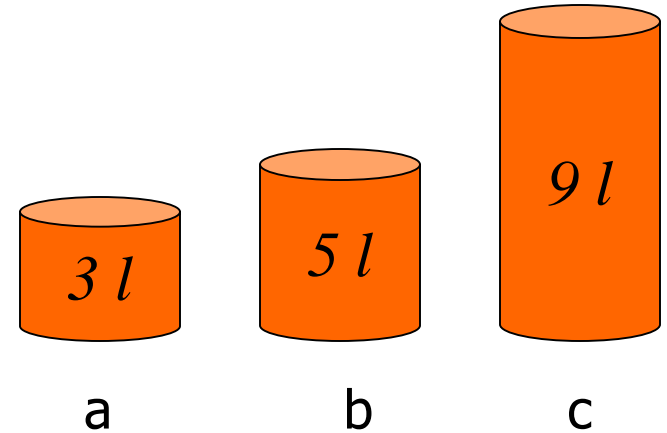
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	



# Example: Measuring problem!

- (one possible) Solution:**

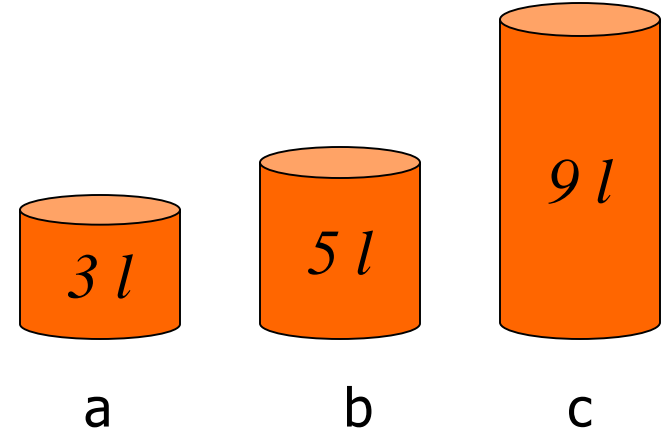
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	
0	5	7	goal



# Example: Measuring problem!

- **Another Solution:**

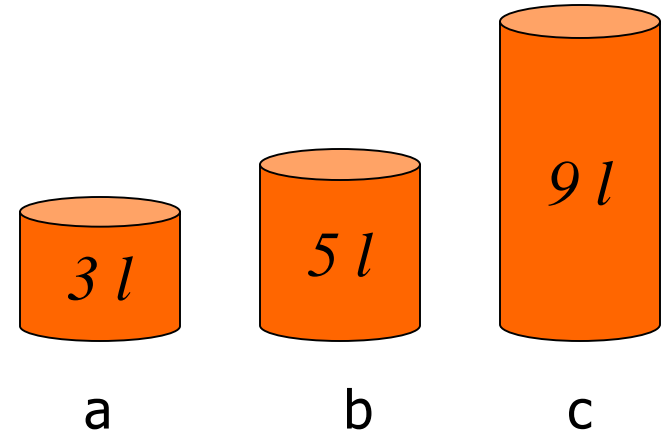
a	b	c	
0	0	0	start
0	5	0	



# Example: Measuring problem!

- **Another Solution:**

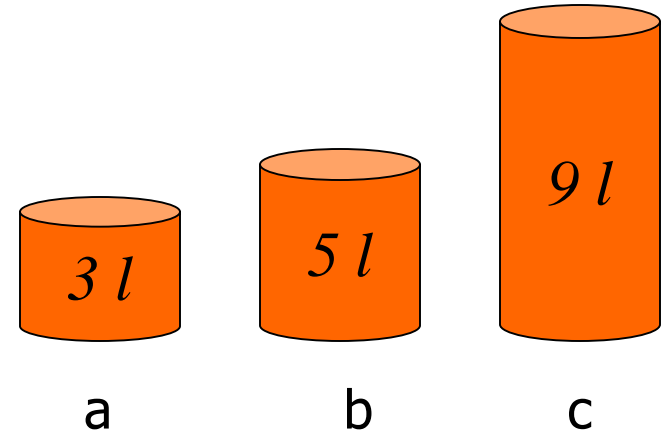
a	b	c	
0	0	0	start
0	5	0	
3	2	0	



# Example: Measuring problem!

- **Another Solution:**

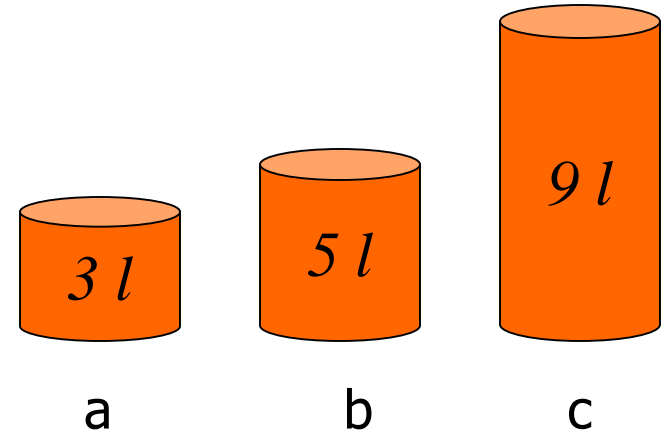
a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	



# Example: Measuring problem!

- **Another Solution:**

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
-	-	-	

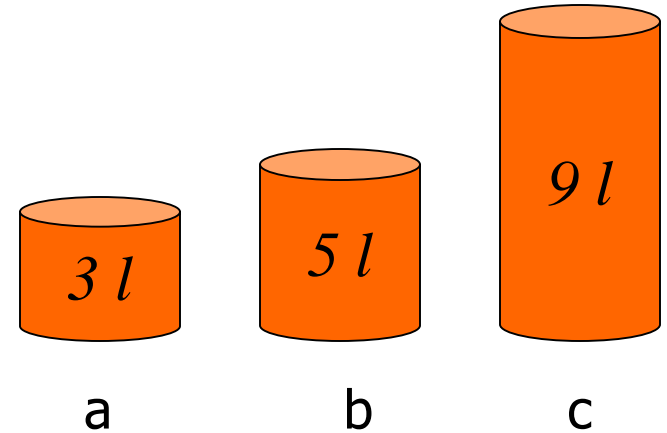




# Example: Measuring problem!

- Another Solution:

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
<b>3</b>	<b>0</b>	<b>7</b>	goal



# Which solution do we prefer?

- **Solution 1:**

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	
0	5	7	goal

- **Solution 2:**

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
3	0	7	goal

# Problem Solving Agent



Measure 7 liters of water using a 3-liter, a 5-liter, and a 9-liter buckets.

- **Formulate goal:** Have 7 liters of water in 9-liter bucket
- **Formulate problem:**
  - States: amount of water in the buckets
  - Operators: Fill bucket from source, empty bucket
- **Find solution:** sequence of operators that bring you from current state to the goal state

## Remember (lecture 3): Environment types

Environment	Accessible	Deterministic	Episodic	Static	Discrete
Operating System	Yes	Yes	No	No	Yes
Virtual Reality	Yes	Yes	Yes/No	No	Yes/No
Office Environment	No	No	No	No	No
Mars	No	Semi	No	Semi	No

The environment types largely determine the agent design.

# Problem types

- **Single-state problem:** deterministic, accessible  
*Agent knows everything about world, thus can calculate optimal action sequence to reach goal state.*
- **Multiple-state problem:** deterministic, inaccessible  
*Agent must reason about sequences of actions and states assumed while working towards goal state.*
- **Contingency problem:** nondeterministic, inaccessible
  - *Must use sensors during execution*
  - *Solution is a tree or policy*
  - *Often interleave search and execution*
- **Exploration problem:** unknown state space  
*Discover and learn about environment while taking actions.*

# Problem types



- **Single-state problem:** deterministic, accessible
  - Agent knows everything about world (the exact state),
  - Can calculate optimal action sequence to reach goal state.
- E.g., playing chess. Any action will result in an exact state

# Problem types



- **Multiple-state problem:**      deterministic, inaccessible
  - Agent does not know the exact state (could be in any of the possible states)
    - May not have sensor at all
  - Assume states while working towards goal state.
- E.g., walking in a dark room
  - If you are at the door, going straight will lead you to the kitchen
  - If you are at the kitchen, turning left leads you to the bedroom
  - ...

# Problem types



- **Contingency problem:**      nondeterministic, inaccessible
  - Must use sensors during execution
  - Solution is a tree or policy
  - Often interleave search and execution
- E.g., a new skater in an arena
  - Sliding problem.
  - Many skaters around



# Problem types



- **Exploration problem:** unknown state space

*Discover and learn about environment while taking actions.*

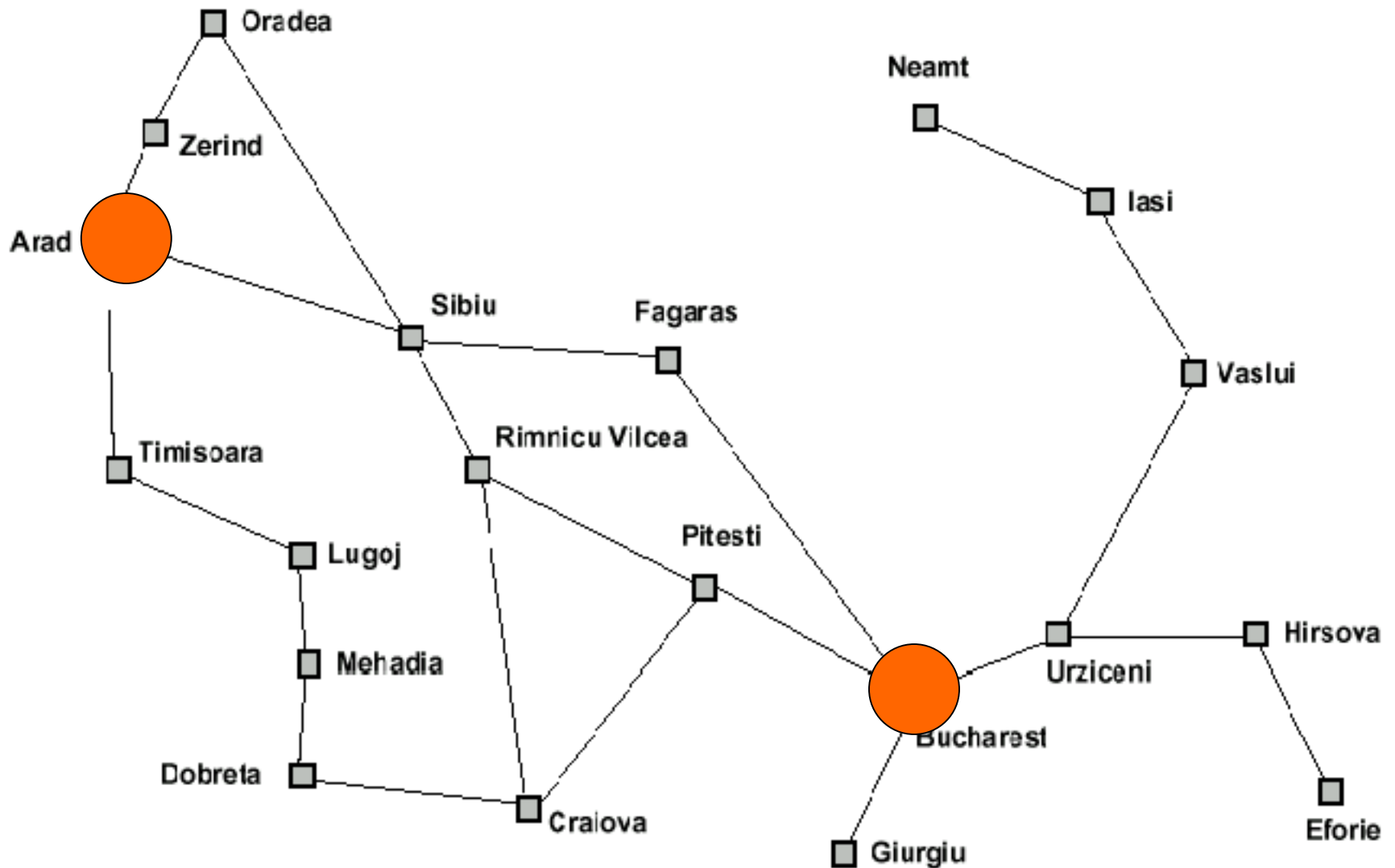
- *E.g., Maze*

## Example: Romania



- In Romania, on vacation. Currently in Arad.
- Flight leaves tomorrow from Bucharest.
- **Formulate goal:**
  - be in Bucharest
- **Formulate problem:**
  - states: various cities
  - operators: drive between cities
- **Find solution:**
  - sequence of cities, such that total driving distance is minimized.

# Example: Traveling from Arad To Bucharest



# Problem formulation

A *problem* is defined by four items:

initial state e.g., "at Arad"

operators (or *successor function*  $S(x)$ )

e.g., Arad  $\rightarrow$  Zerind      Arad  $\rightarrow$  Sibiu      etc.

goal test, can be

*explicit*, e.g.,  $x = \text{"at Bucharest"}$

*implicit*, e.g.,  $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of operators executed, etc.

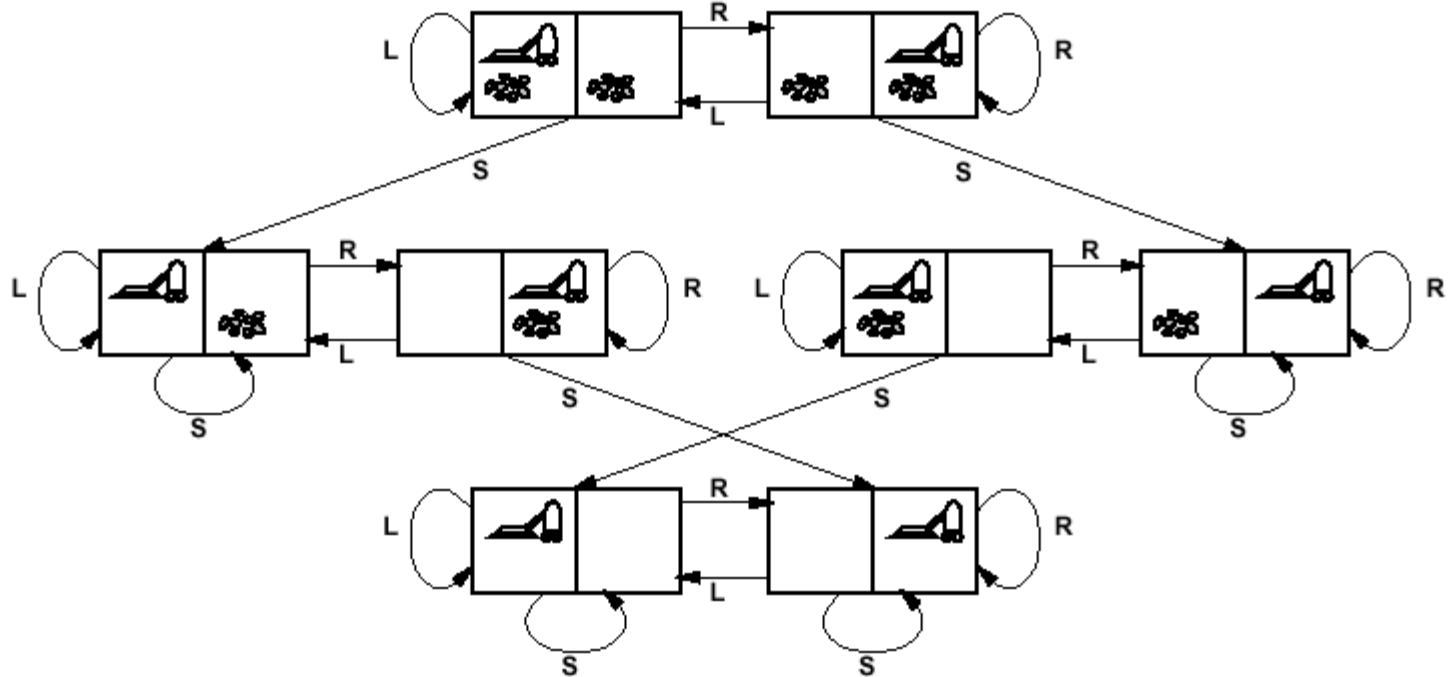
A *solution* is a sequence of operators  
leading from the initial state to a goal state

# Selecting a state space



- Real world is absurdly complex; some abstraction is necessary to allow us to reason on it...
- Selecting the correct abstraction and resulting state space is a difficult problem!
- Abstract states  $\Leftrightarrow$  real-world states
- Abstract operators  $\Leftrightarrow$  sequences or real-world actions  
(e.g., going from city  $i$  to city  $j$  costs  $L_{ij}$   $\Leftrightarrow$  actually drive from city  $i$  to  $j$ )
- Abstract solution  $\Leftrightarrow$  set of real actions to take in the real world such as to solve problem

# Vacuum World



states??

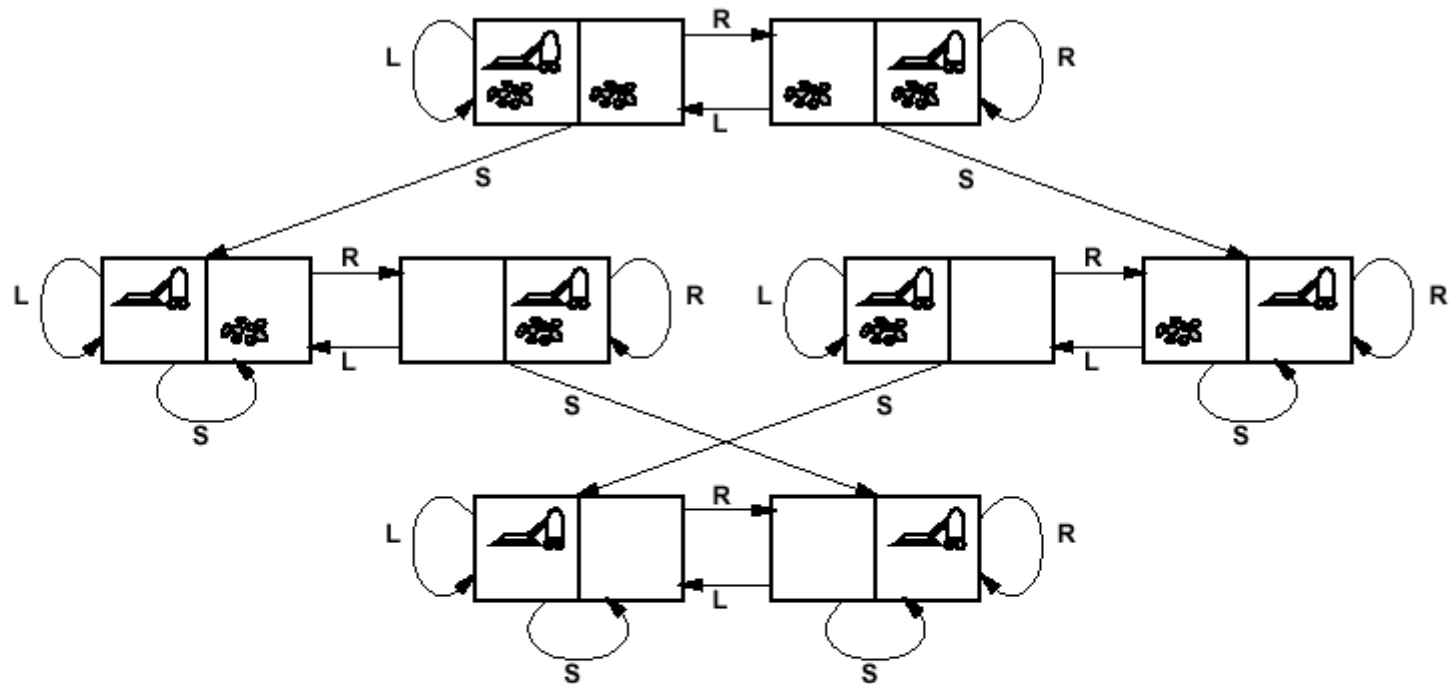
operators??

goal test??

path cost??

*Simplified world: 2 locations, each may or not contain dirt, each may or not contain vacuuming agent.*

# Vacuum World



states?: integer dirt and robot locations (ignore dirt *amounts*)

operators?: *Left, Right, Suck*

goal test?: no dirt

path cost?: 1 per operator

## Example: 8-puzzle

5	4	
6	1	8
7	3	2

start state

1	2	3
8		4
7	6	5

goal state

- State:
- Operators:
- Goal test:
- Path cost:



## Example: 8-puzzle

5	4	
6	1	8
7	3	2

start state

1	2	3
8		4
7	6	5

goal state

- State: integer location of tiles (ignore intermediate locations)
- Operators: moving blank left, right, up, down (ignore jamming)
- Goal test: does state match goal state?
- Path cost: 1 per move

## Example: 8-puzzle

5	4	
6	1	8
7	3	2

start state

1	2	3
8		4
7	6	5

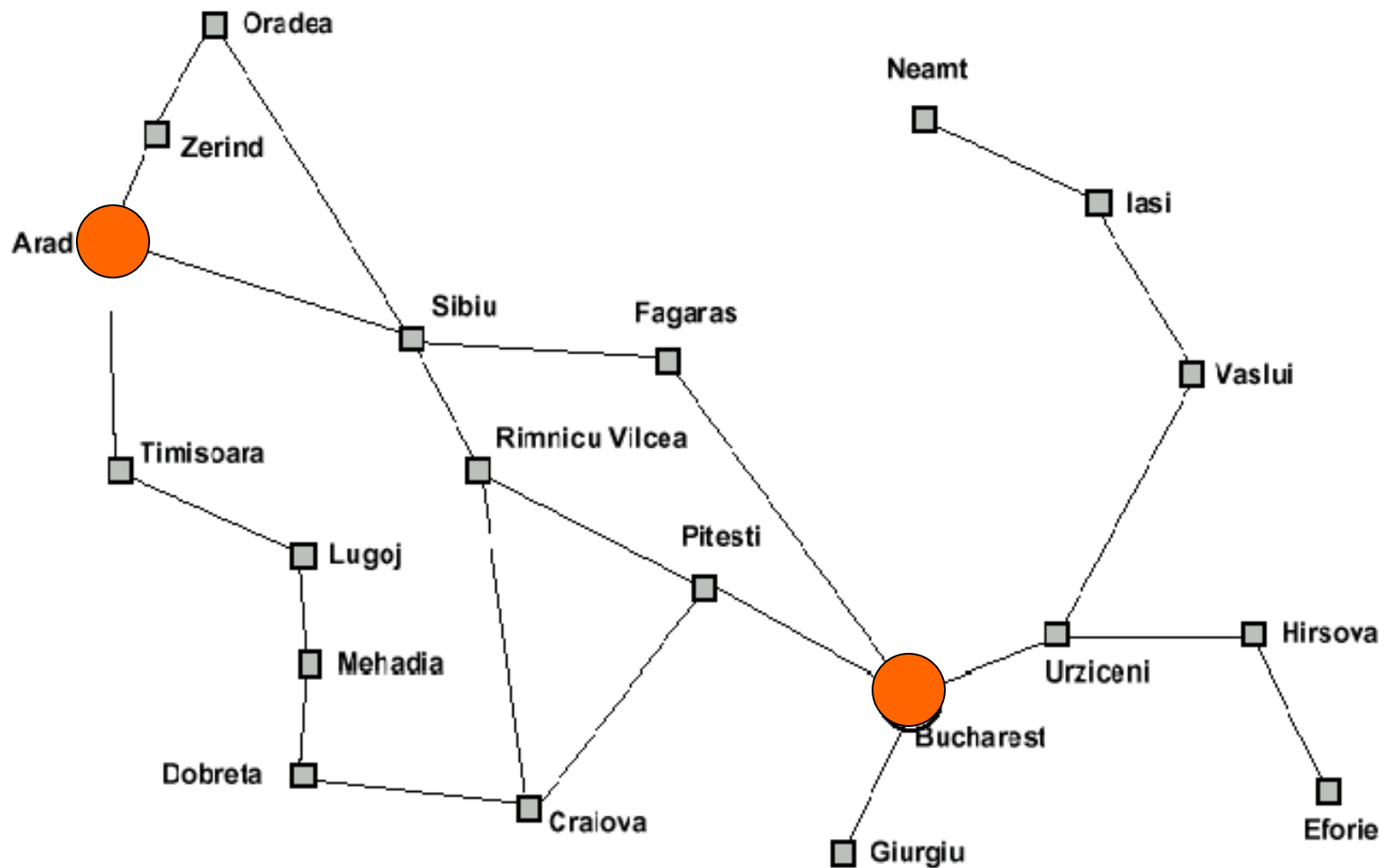
goal state

Why search algorithms?

- 8-puzzle has 362,880 states
- 15-puzzle has  $10^{12}$  states
- 24-puzzle has  $10^{25}$  states

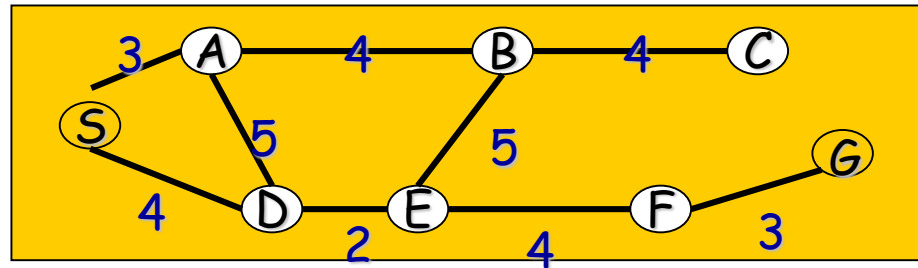
So, we need a principled way to look for a solution in these huge search spaces...

# Example: Traveling from Arad To Bucharest

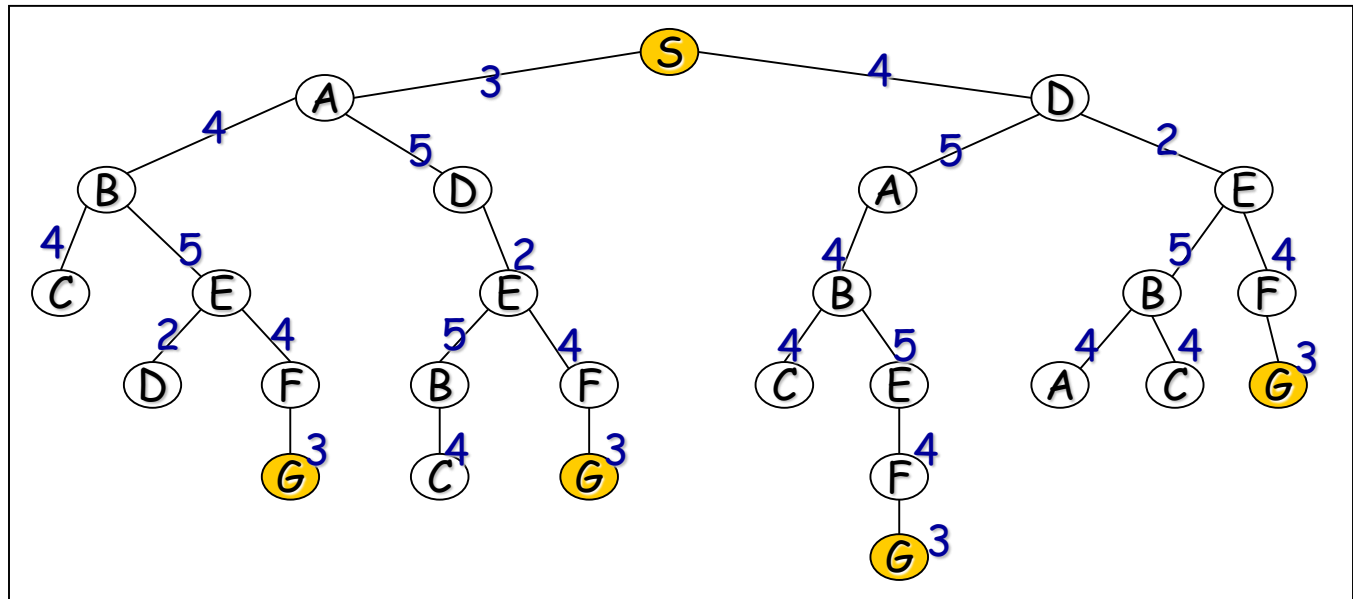


# From problem space to search tree

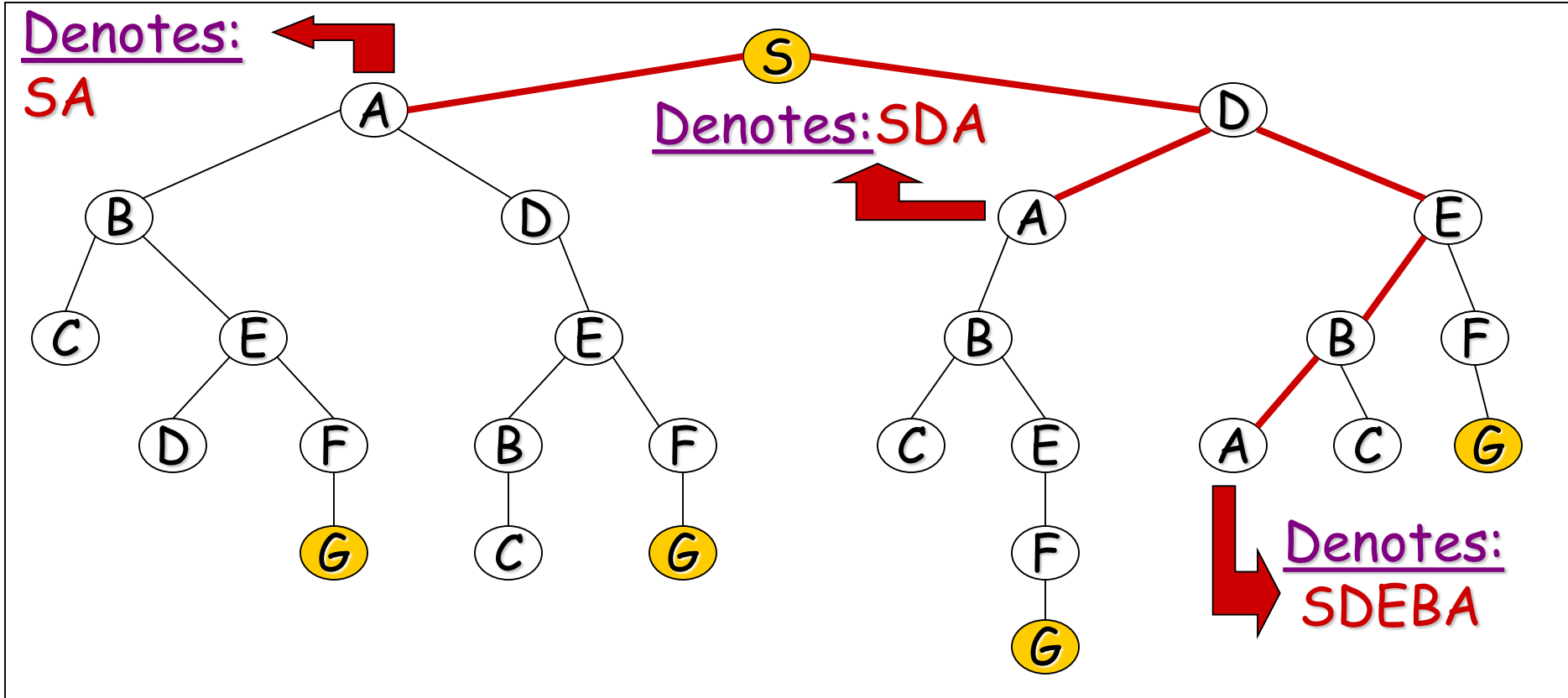
Problem space



Associated  
loop-free  
search tree



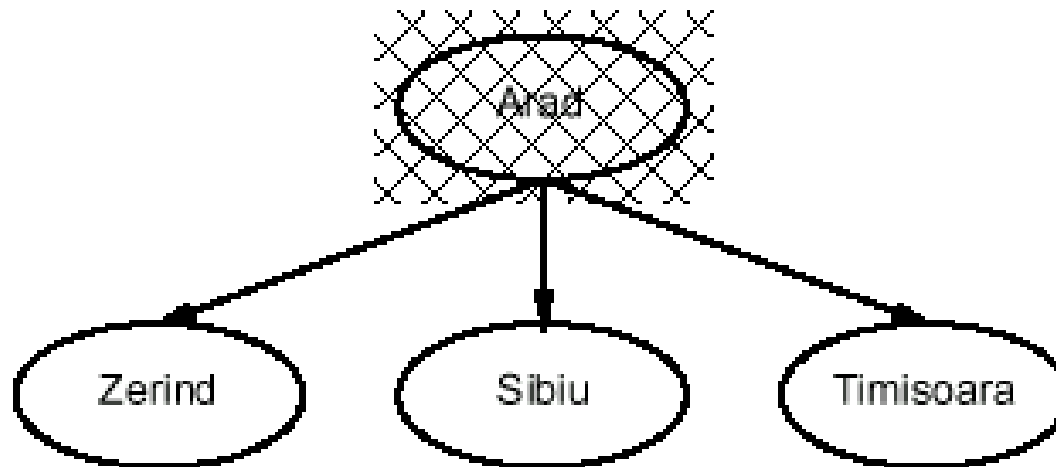
# Paths in search trees



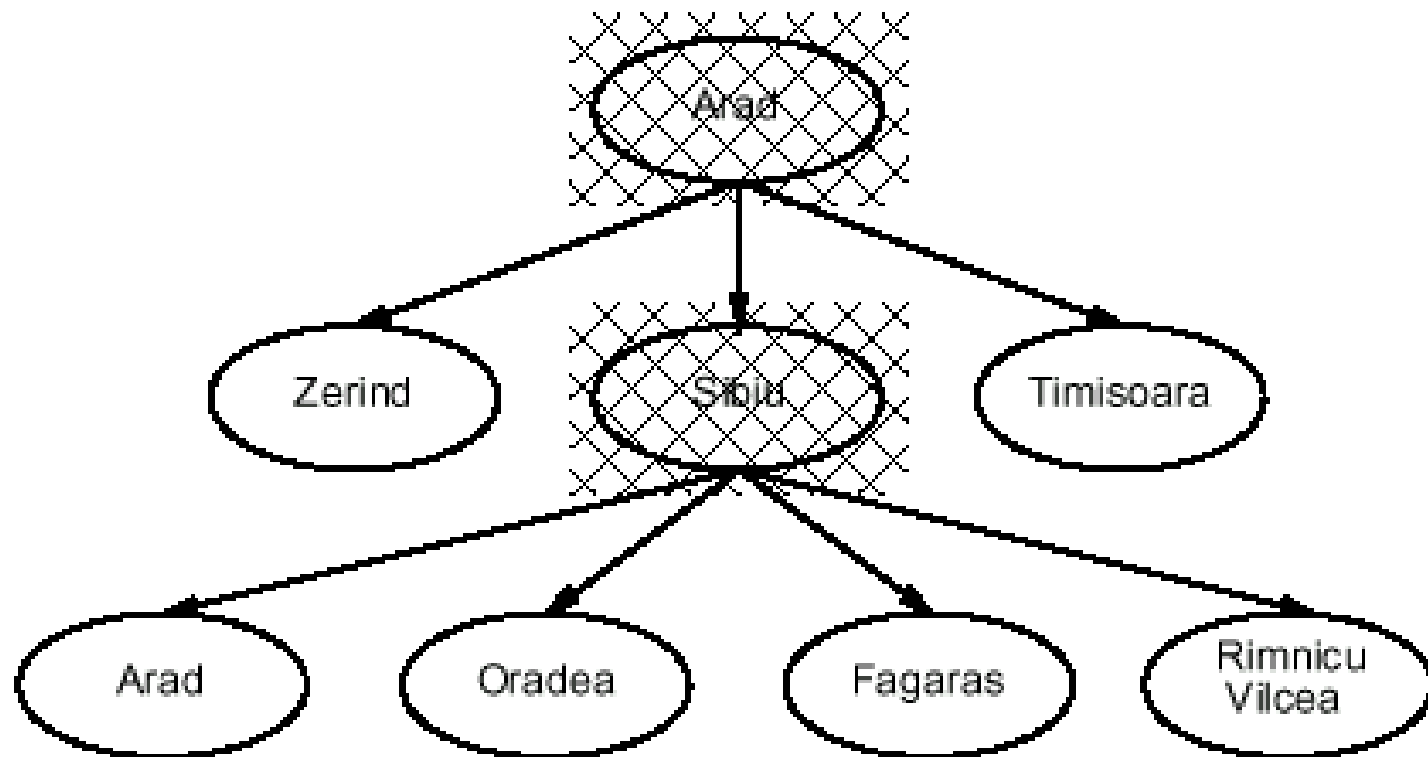
# General search example



## General search example



# General search example





# General search example

