

# Introduction to AI

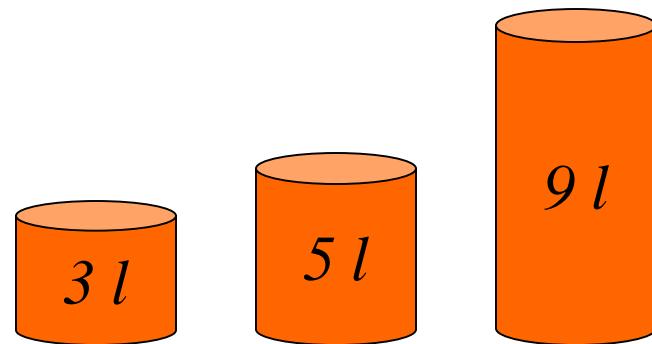
*Lecture 4: Problem solving and search*

*Mona Taghavi*



**LaSalle College**  
Montréal

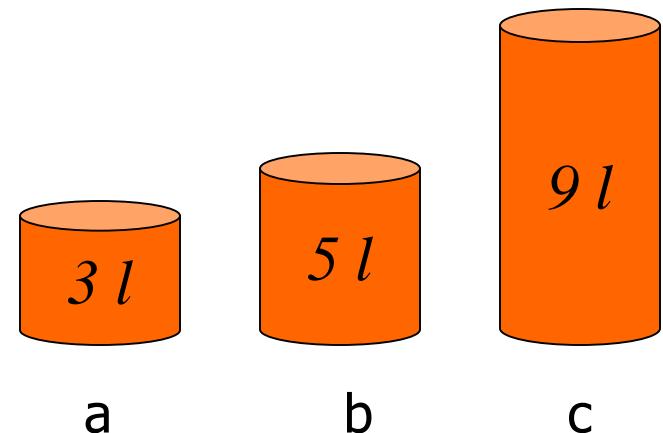
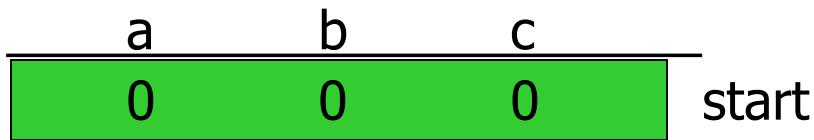
# Example: Measuring problem!



**Problem:** Using these three buckets,  
measure 7 liters of water.

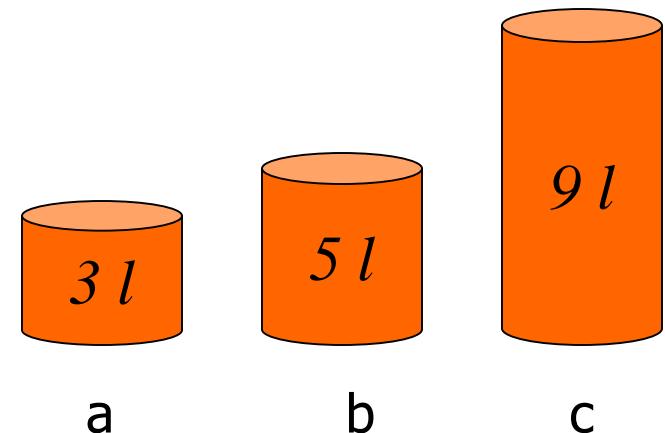
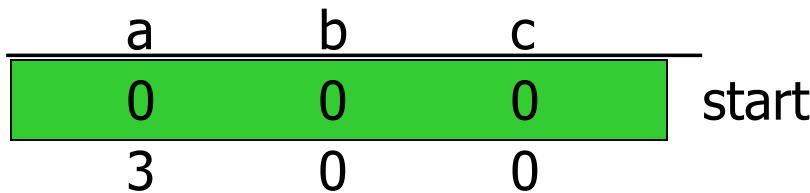
# Example: Measuring problem!

- **(one possible) Solution:**



# Example: Measuring problem!

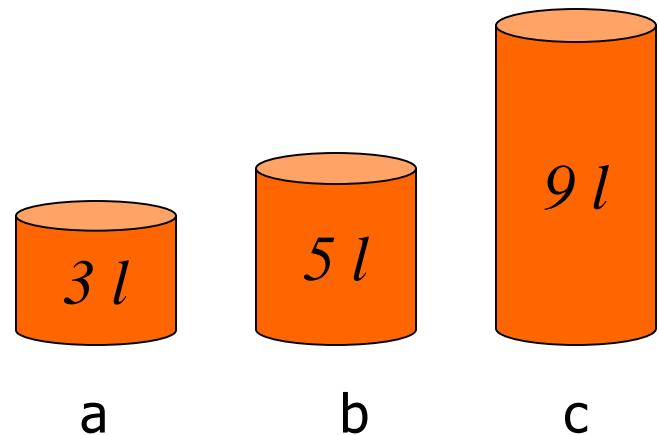
- **(one possible) Solution:**



# Example: Measuring problem!

- **(one possible) Solution:**

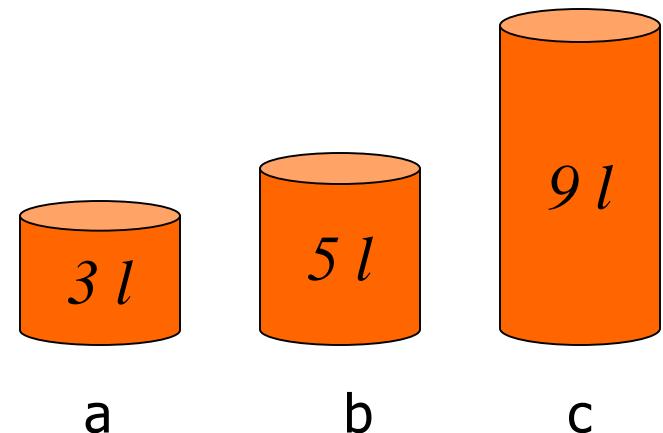
a	b	c	
0	0	0	start
3	0	0	
0	0	3	



# Example: Measuring problem!

- **(one possible) Solution:**

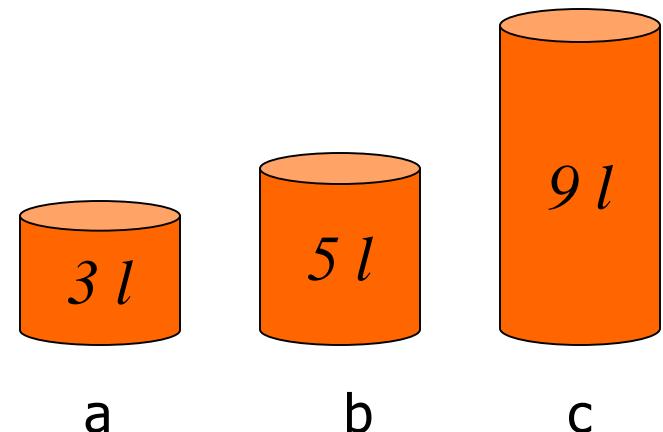
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	



# Example: Measuring problem!

- **(one possible) Solution:**

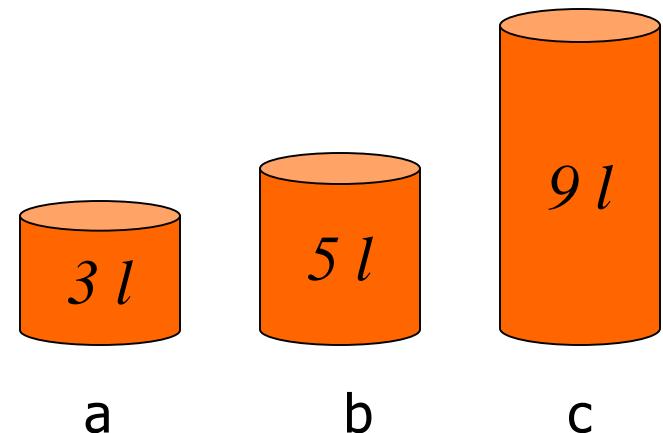
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	



# Example: Measuring problem!

- **(one possible) Solution:**

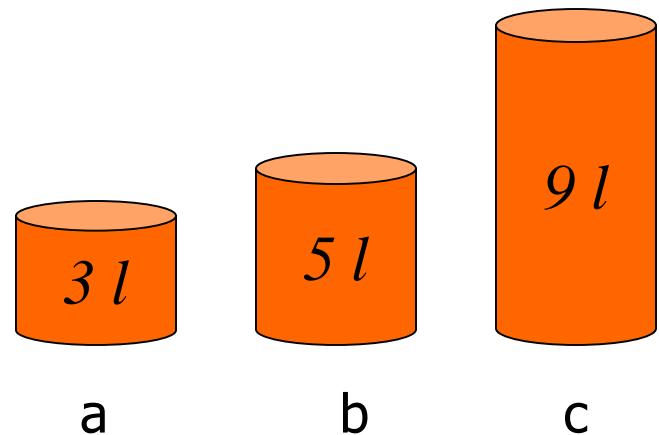
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	



# Example: Measuring problem!

- **(one possible) Solution:**

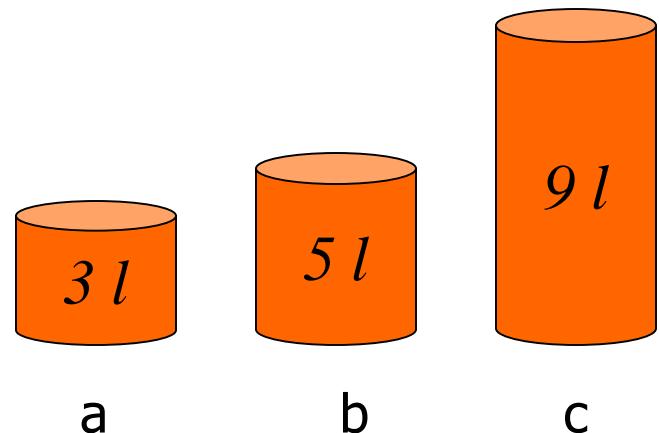
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	



# Example: Measuring problem!

- **(one possible) Solution:**

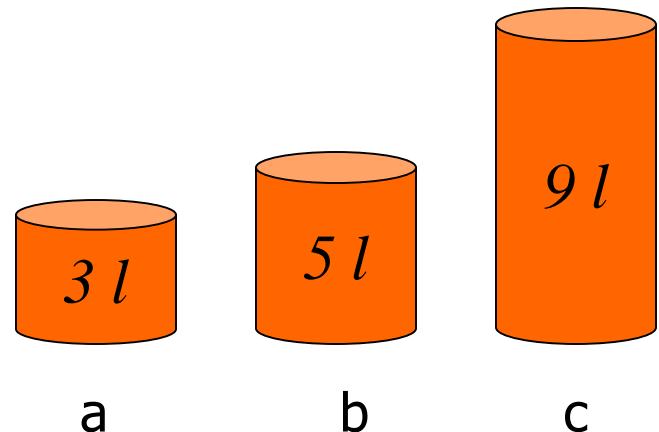
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	



# Example: Measuring problem!

- **(one possible) Solution:**

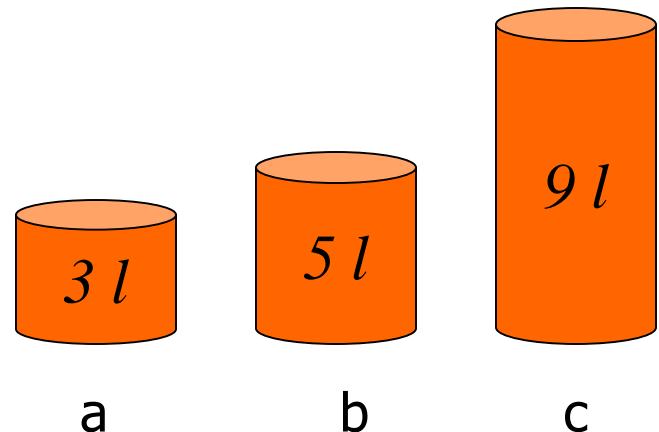
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	



# Example: Measuring problem!

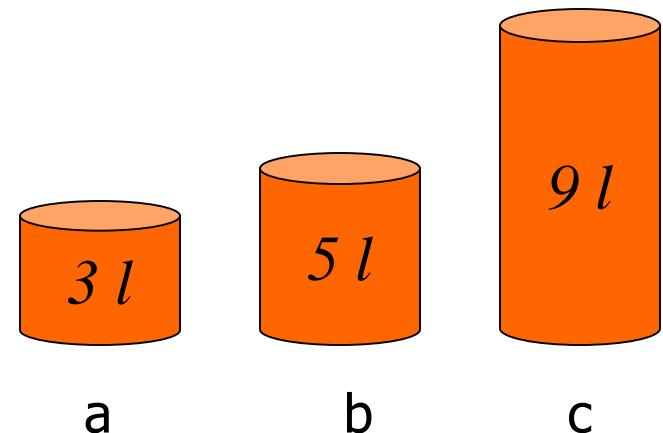
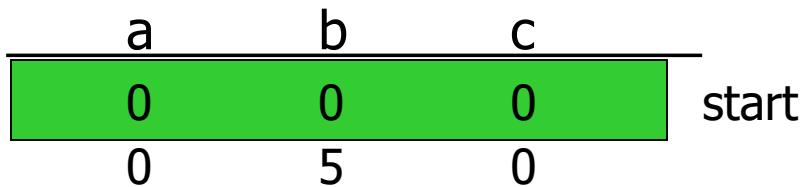
- **(one possible) Solution:**

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	
0	5	7	goal



# Example: Measuring problem!

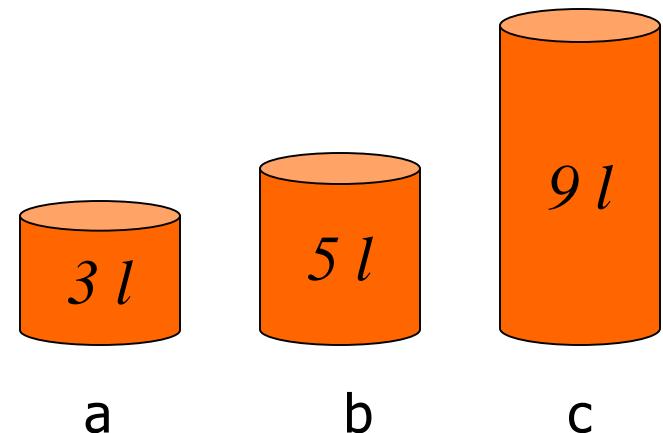
- **Another Solution:**



# Example: Measuring problem!

- Another Solution:

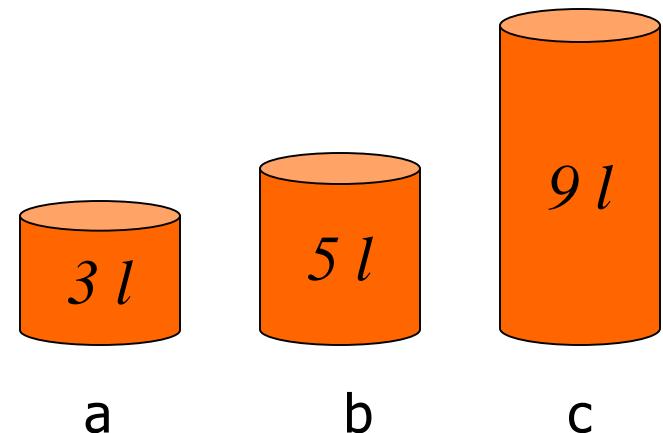
a	b	c	
0	0	0	start
0	5	0	
3	2	0	



# Example: Measuring problem!

- Another Solution:

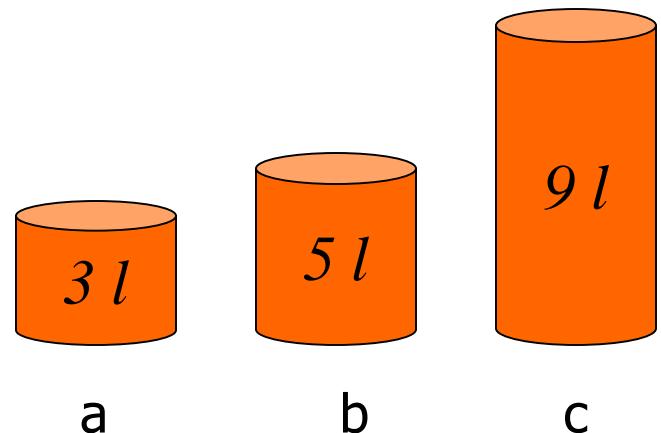
a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	



# Example: Measuring problem!

- Another Solution:

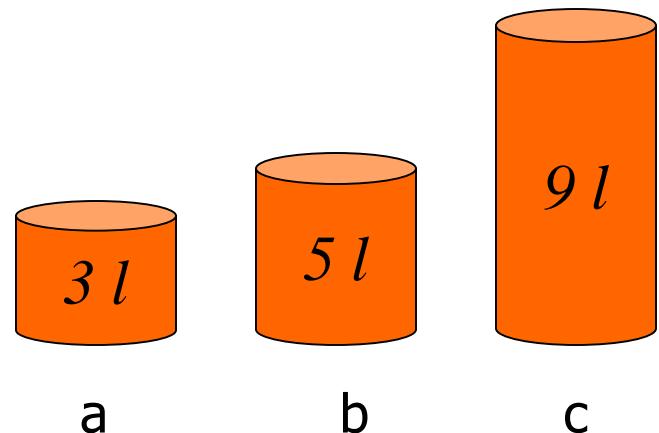
a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
-	-	-	



# Example: Measuring problem!

- Another Solution:

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
<b>3</b>	<b>0</b>	<b>7</b>	goal



# Which solution do we prefer?

- **Solution 1:**

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	
0	5	7	goal

- **Solution 2:**

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
3	0	7	goal

# Problem Solving Agent

Measure 7 liters of water using a 3-liter, a 5-liter, and a 9-liter buckets.

- **Formulate goal:** Have 7 liters of water in 9-liter bucket
- **Formulate problem:**
  - States: amount of water in the buckets
  - Operators: Fill bucket from source, empty bucket
- **Find solution:** sequence of operators that bring you from current state to the goal state

# Remember (lecture 3): Environment types

Environment	Accessible	Deterministic	Episodic	Static	Discrete
Operating System	Yes	Yes	No	No	Yes
Virtual Reality	Yes	Yes	Yes/No	No	Yes/No
Office Environment	No	No	No	No	No
Mars	No	Semi	No	Semi	No

The environment types largely determine the agent design.

# Problem types

- **Single-state problem:** deterministic, accessible  
*Agent knows everything about world, thus can calculate optimal action sequence to reach goal state.*
- **Multiple-state problem:** deterministic, inaccessible  
*Agent must reason about sequences of actions and states assumed while working towards goal state.*
- **Contingency problem:** nondeterministic, inaccessible
  - *Must use sensors during execution*
  - *Solution is a tree or policy*
  - *Often interleave search and execution*
- **Exploration problem:** unknown state space  
*Discover and learn about environment while taking actions.*

# Problem types



- **Single-state problem:** deterministic, accessible
  - Agent knows everything about world (the exact state),
  - Can calculate optimal action sequence to reach goal state.
- E.g., playing chess. Any action will result in an exact state

# Problem types



- **Multiple-state problem:** deterministic, inaccessible
  - Agent does not know the exact state (could be in any of the possible states)
    - May not have sensor at all
  - Assume states while working towards goal state.
  - E.g., walking in a dark room
    - If you are at the door, going straight will lead you to the kitchen
    - If you are at the kitchen, turning left leads you to the bedroom
    - ...

# Problem types



- **Contingency problem:** nondeterministic, inaccessible
  - Must use sensors during execution
  - Solution is a tree or policy
  - Often interleave search and execution
- E.g., a new skater in an arena
  - Sliding problem.
  - Many skaters around

# Problem types



- **Exploration problem:** unknown state space

*Discover and learn about environment while taking actions.*

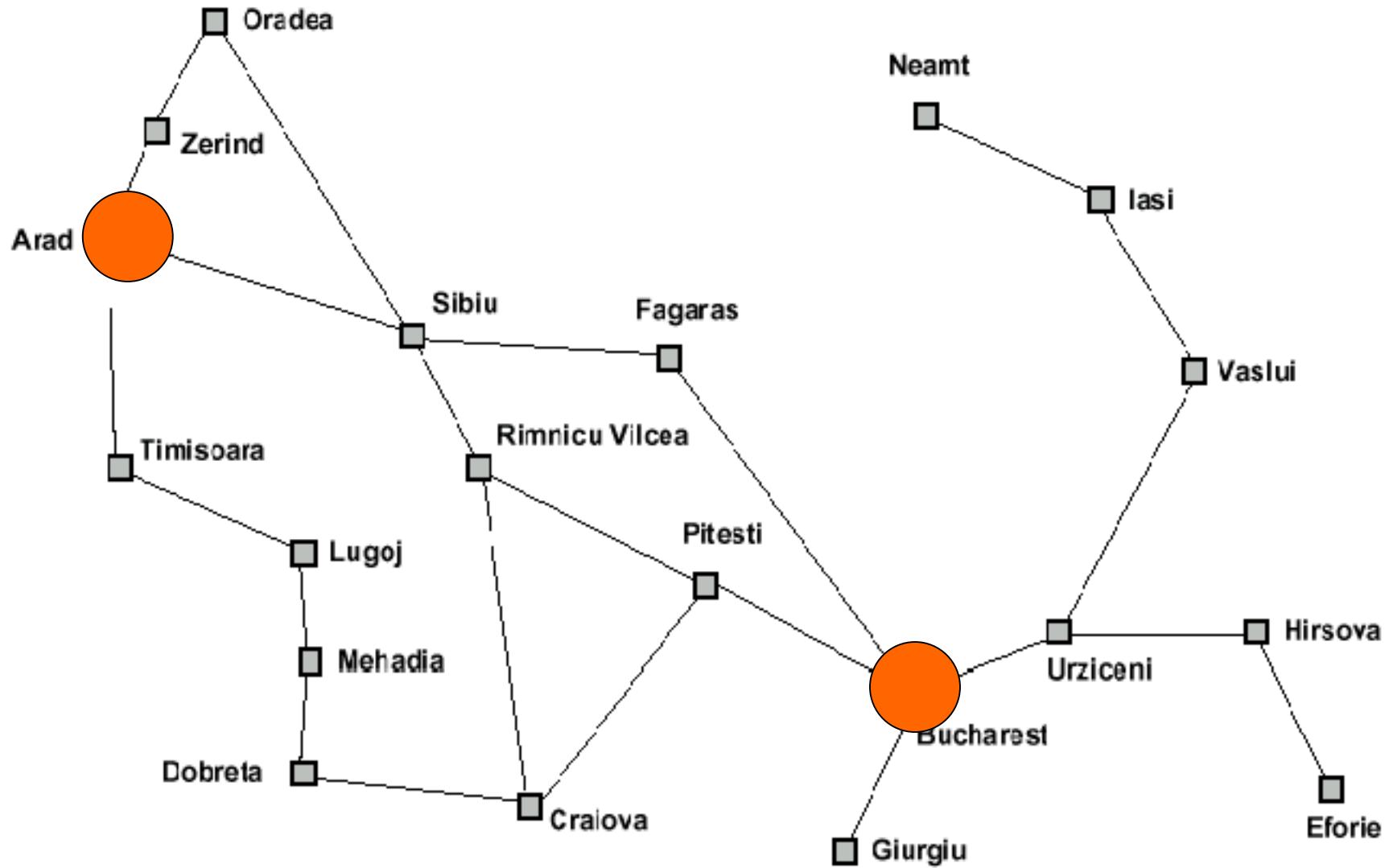
- *E.g., Maze*

## Example: Romania



- In Romania, on vacation. Currently in Arad.
- Flight leaves tomorrow from Bucharest.
- **Formulate goal:**
  - be in Bucharest
- **Formulate problem:**
  - states: various cities
  - operators: drive between cities
- **Find solution:**
  - sequence of cities, such that total driving distance is minimized.

# Example: Traveling from Arad To Bucharest



# Problem formulation

A *problem* is defined by four items:

initial state e.g., "at Arad"

operators (or successor function  $S(x)$ )

e.g., Arad → Zerind      Arad → Sibiu      etc.

goal test, can be

explicit, e.g.,  $x = \text{"at Bucharest"}$

implicit, e.g.,  $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of operators executed, etc.

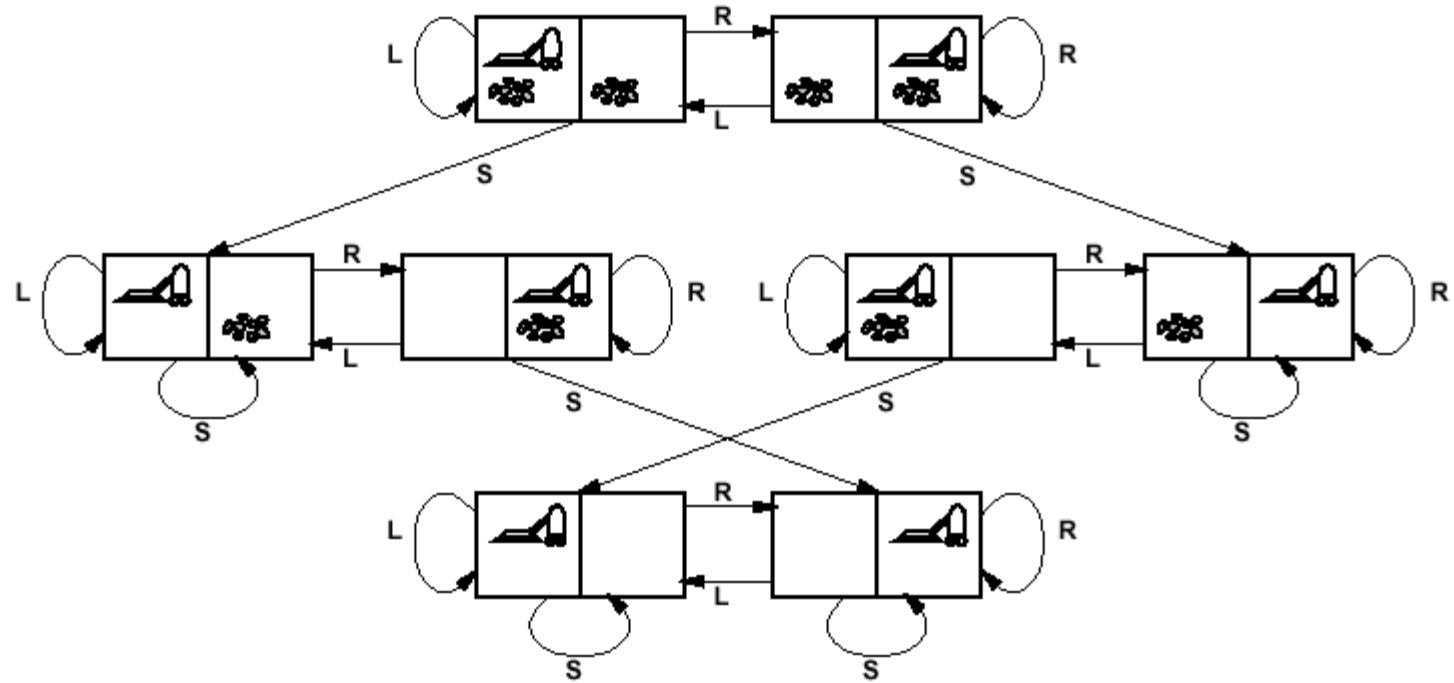
A *solution* is a sequence of operators

leading from the initial state to a goal state

# Selecting a state space

- Real world is absurdly complex; some abstraction is necessary to allow us to reason on it...
- Selecting the correct abstraction and resulting state space is a difficult problem!
- Abstract states       $\Leftrightarrow$       real-world states
- Abstract operators     $\Leftrightarrow$       sequences or real-world actions  
(e.g., going from city i to city j costs  $L_{ij}$   $\Leftrightarrow$  actually drive from city i to j)
- Abstract solution      $\Leftrightarrow$       set of real actions to take in the real world such as to solve problem

# Vacuum World



states??

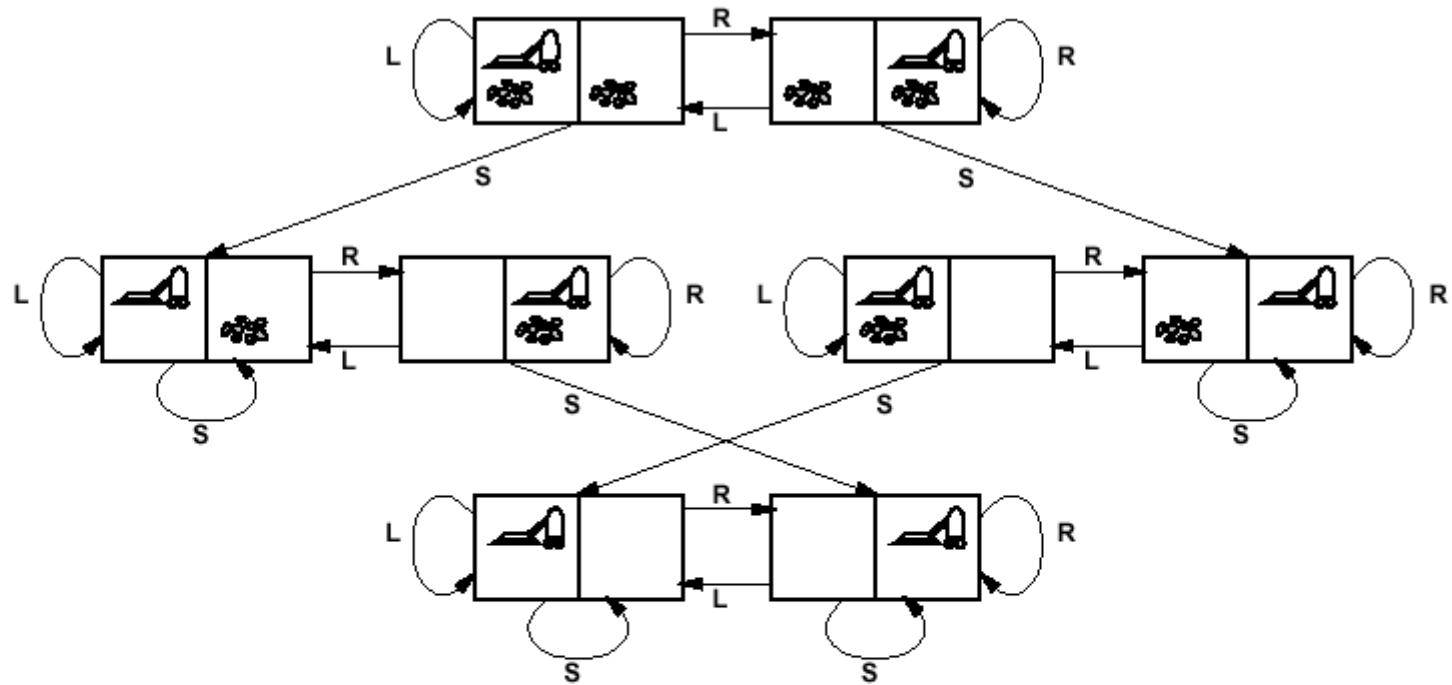
operators??

goal test??

path cost??

*Simplified world: 2 locations, each may or not contain dirt, each may or not contain vacuuming agent.*

# Vacuum World



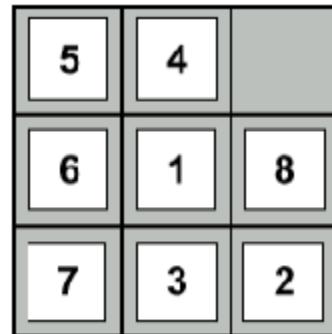
states??: integer dirt and robot locations (ignore dirt *amounts*)

operators??: *Left*, *Right*, *Suck*

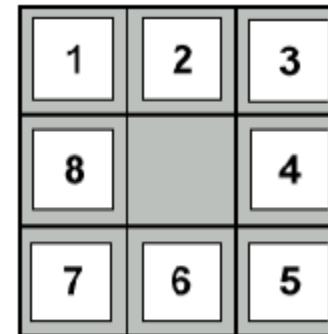
goal test??: no dirt

path cost??: 1 per operator

# Example: 8-puzzle



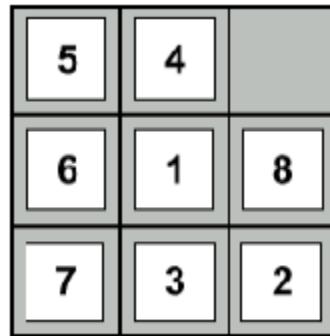
start state



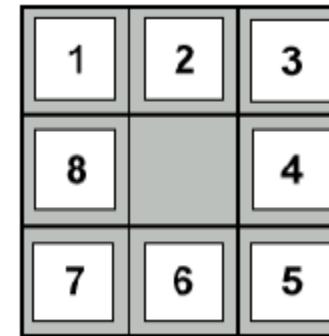
goal state

- State:
- Operators:
- Goal test:
- Path cost:

## Example: 8-puzzle



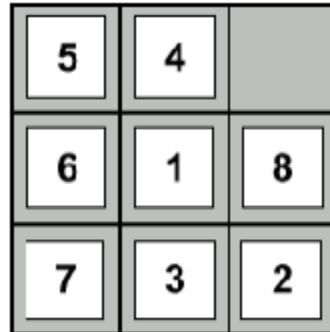
start state



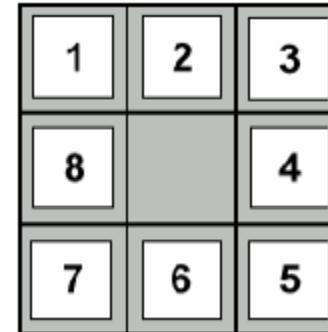
goal state

- State: integer location of tiles (ignore intermediate locations)
- Operators: moving blank left, right, up, down (ignore jamming)
- Goal test: does state match goal state?
- Path cost: 1 per move

## Example: 8-puzzle



start state



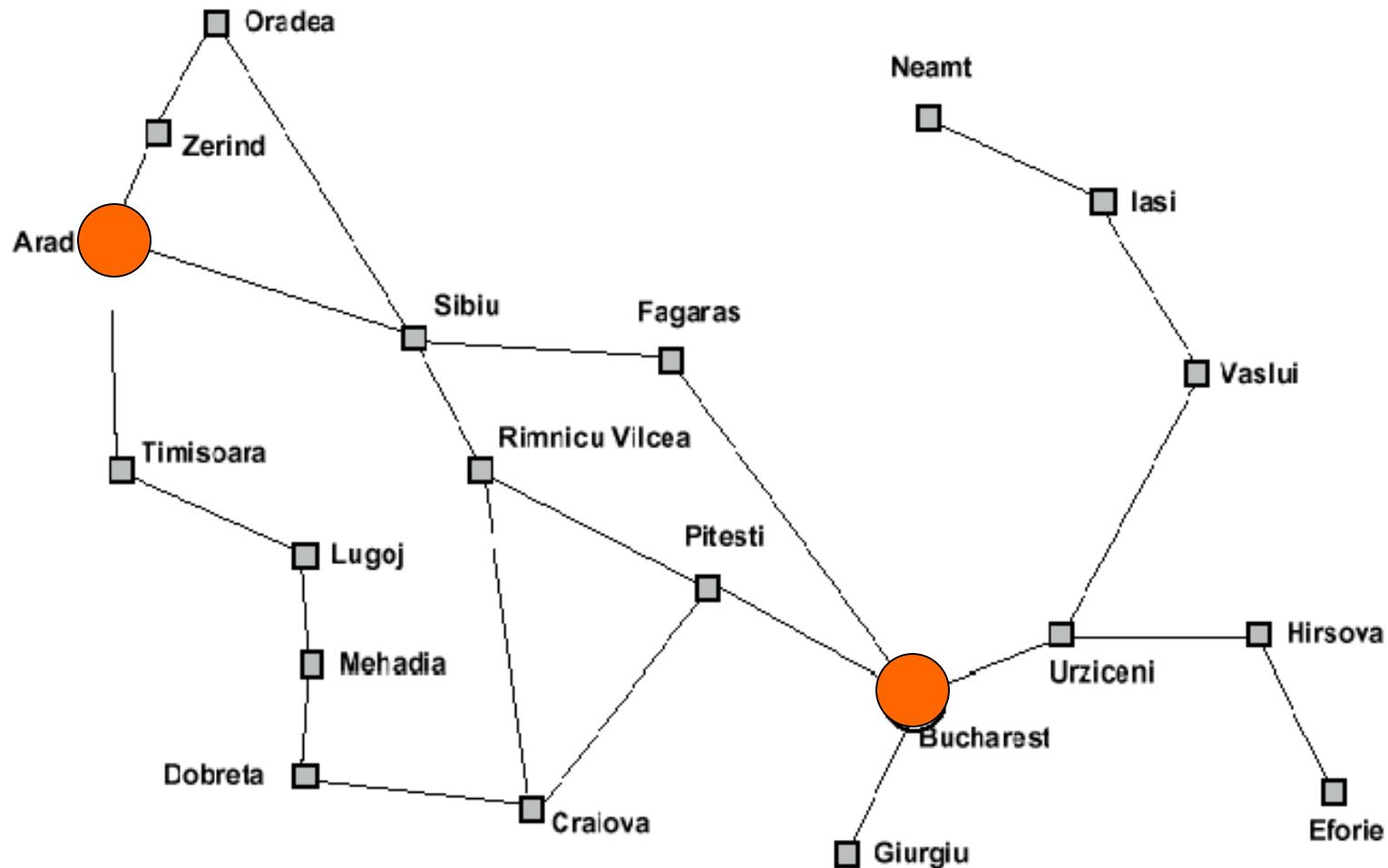
goal state

### Why search algorithms?

- 8-puzzle has 362,800 states
- 15-puzzle has  $10^{12}$  states
- 24-puzzle has  $10^{25}$  states

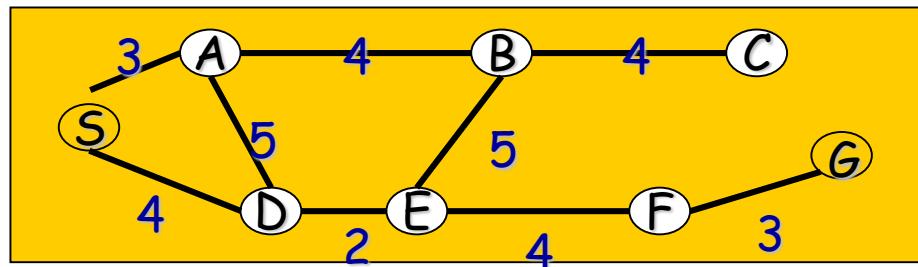
So, we need a principled way to look for a solution in these huge search spaces...

# Example: Traveling from Arad To Bucharest

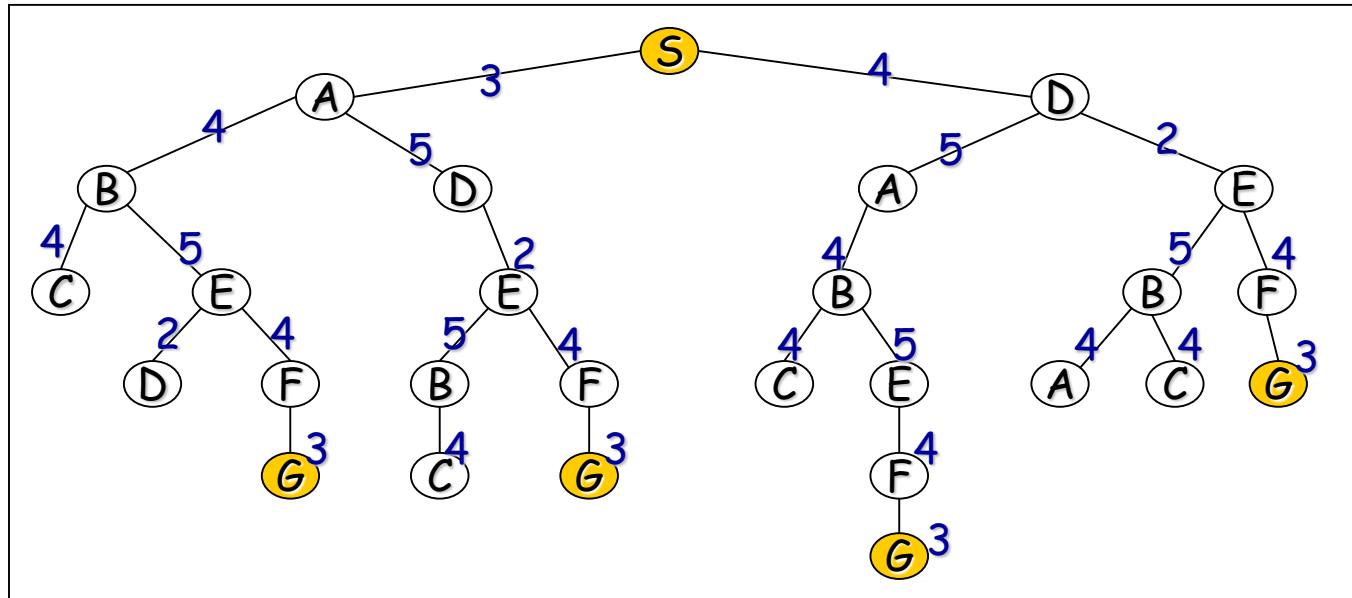


# From problem space to search tree

Problem space



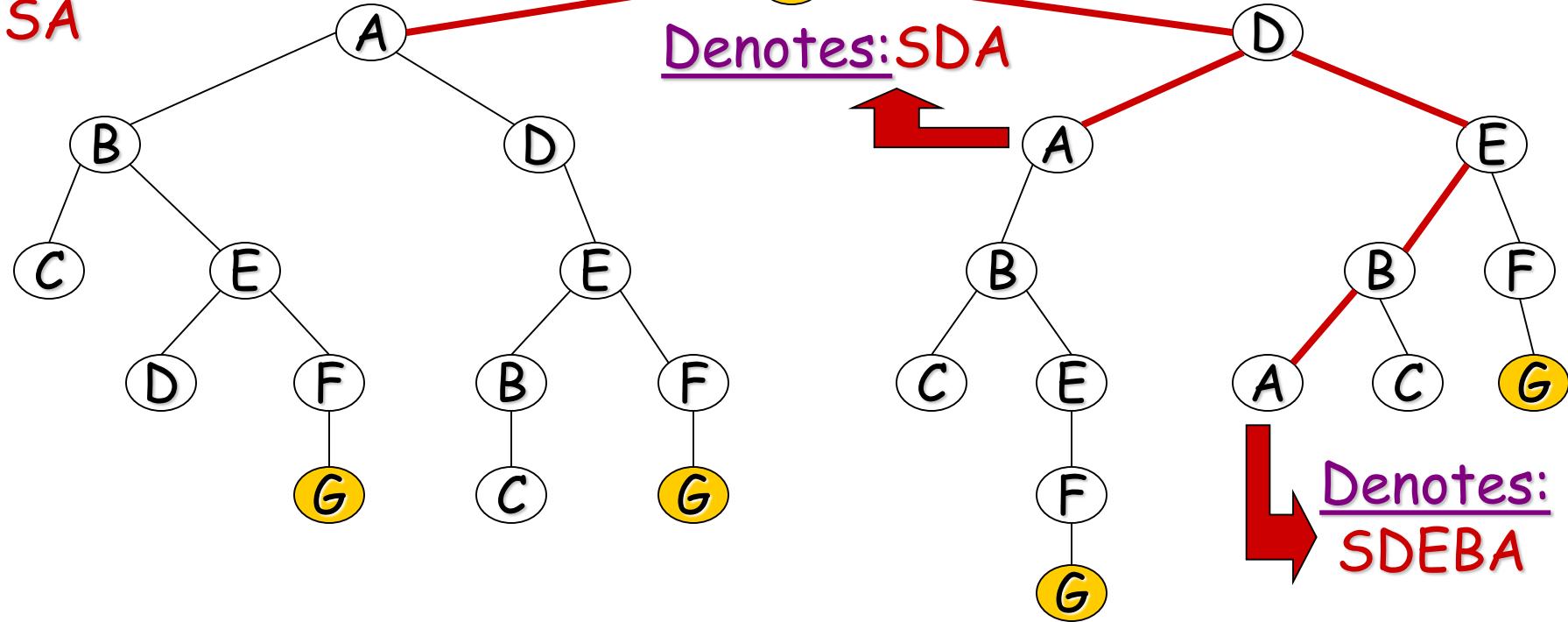
Associated  
loop-free  
search tree



# Paths in search trees

Denotes:

SA

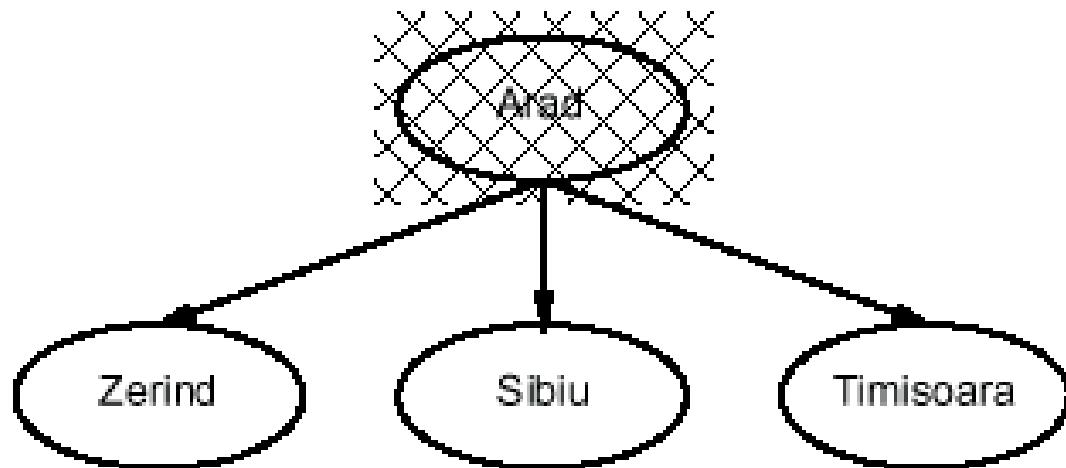


# General search example

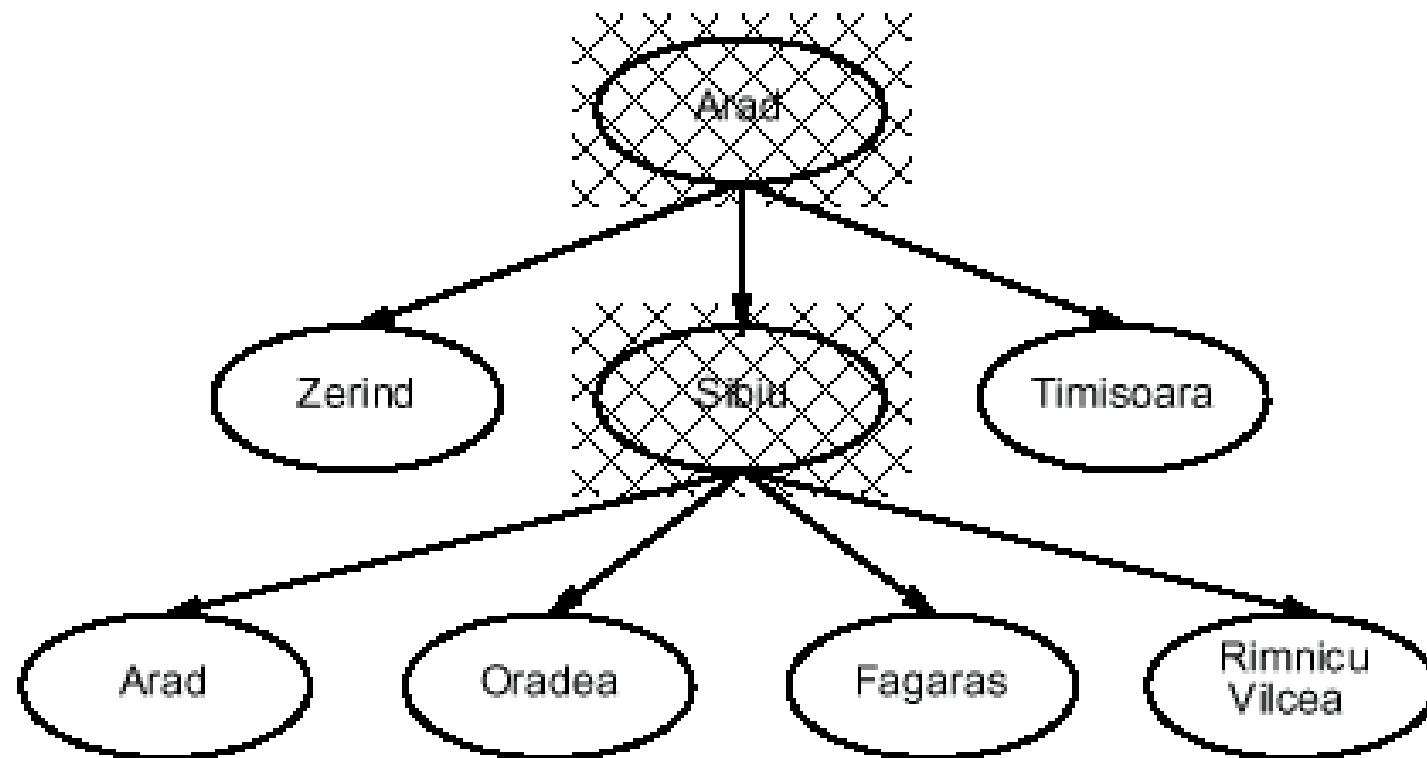


Arad

# General search example



# General search example



# General search example

