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Direct adaptive CMAC PI control for uncertain nonlinear systems with measurable output feedback

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Abstract. A stable direct adaptive CMAC PI controller for a class of uncertain nonlinear systems is investigated under the constrain that only the system output is available. First, a state observer is used to estimate unmeasured states of the systems. Then, the PI control structure is used for improving robustness in the closed-loop system and avoiding affection of uncertainties and external disturbances. The global asymptotic stability of the closed-loop system is guaranteed according to the Lyapunov stability criterion. Simulation results indicate that the proposed approach is capable of achieving a good trajectory following performance without the knowledge of plant parameters.

Introduction

The control of uncertain nonlinear systems has been an investigated problem because of its wide applications in practical systems. There has been considerable attention over the years on researches using neural networks (NNs) based on human heuristic and learning algorithms [1,2]. Especially, the cerebellar model articulation controller (CMAC) was first developed by Albus in the 1970s [3,4], and is regarded as a nonfully connected perceptron-like associative memory network with overlapping receptive-fields. The contents of these memory locations are referred as weights, and the output of this network is a linear combination of these weights in the memory addressed by the activated inputs [5]. In the meantime, sliding mode control (SMC) has successfully been developed as a popular robust strategy to a wide variety of systems having uncertainties and invariance to unknown disturbance [6]. Integrating CMAC and SMC into controller design have acquired superior performance [7]. In extending that research, we introduce a PI-type adaptive CMAC control structure to cope with the bounded disturbances, which also provides robustness in the presence of large disturbances.

However, these control methods are based on the assumption that the system states are known or available for feedback. If system states are not available, an observer needs to estimate the unmeasured states and the SPR-Lyapunov design approach has been used to derive the adaptive and control laws [8], and the stability of the closed-loop system can be guaranteed..

Problem Formulation

Consider the nth-order nonlinear system described in the following form:

$$\begin{aligned} x^{(n)} &= f(\boldsymbol{x}) + bu + d \\ y &= x \end{aligned} \tag{1}$$

where $f(\boldsymbol{x})$ is unknown function, u and y are the control input and output, respectively, b is an unknown positive constant, and $\boldsymbol{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T = [x_1, x_2, \cdots, x_n]^T$ is the state vector, d is the disturbance and it is assumed to have an upper bound D_N . Rewriting (1) in the following form

$$\begin{aligned} \dot{\boldsymbol{x}} &= A\boldsymbol{x} + B(f(\boldsymbol{x}) + bu + d) \\ y &= C^T \boldsymbol{x} \end{aligned} \tag{2}$$

where $A = \begin{bmatrix} \boldsymbol{0}_{(n-1)\times 1} & \boldsymbol{I}_{n-1} \\ 0 & \boldsymbol{0}_{1\times(n-1)} \end{bmatrix}$, $B = [0, 0, \cdots, 0, 1]^T$, $C = [1, 0, \cdots, 0]^T$ and $\dot{\boldsymbol{x}} = [\dot{x}, \cdots, x^{(n-1)}, x^{(n)}]^T$.

Give the desired trajectory y_d and define the tracking error as $e = y_d - y$, desired trajectory vector $\boldsymbol{y}_d = [y_d, \dot{y}_d, \cdots, y_d^{(n-1)}]^T$, output tracking error vector $\boldsymbol{e} = \boldsymbol{y}_d - \boldsymbol{x}$. Set the sliding surface as

$$s = e^{(n-1)}(t) - e^{(n-1)}(0) + \int_0^t K_c^T \boldsymbol{e}(\tau)\, \mathrm{d}\tau \tag{3}$$

The tracking control problem can be formulated by keeping the error vector $\boldsymbol{e}$ on the sliding surface

$$\dot{s} = y_d^{(n)} + K_c^T \boldsymbol{e} - f(x) - bu - d \tag{4}$$

where $K_c = [k_1^c, k_2^c, \cdots, k_n^c]^T$ is the feedback gain vector. If K_c is chosen such that the characteristic polynomial $h(p) = p^n + k_n^c p^{(n-1)} + \cdots + k_2^c p + k_1^c$ is Hurwitz, then it implies that the tracking error trajectory will converge to zero when time tends to infinity, i.e. $\lim_{t\to\infty} e(t) = 0$. There exists a positive-definite symmetric matrix P_1 which satisfies

$$(A - BK_c^T)^T P_1 + P_1(A - BK_c^T) = -Q_1 \qquad (Q_1 \text{ is a positive-definite matrix}) \tag{5}$$

Description of the CMAC system

The architecture of the CMAC system shown in Fig. 1, includes input space $\boldsymbol{I} = [I_1, I_2, \cdots, I_n]^T$, association memory space, receptive-field space, weight memory space and output space. The Gaussian function is chosen as the receptive-field basis function which can be expressed as

$$\phi_{ik}(I_i) = \exp[-(I_i - m_{ik})^2 / \sigma_{ik}^2] \quad \text{for } k = 1,2,\cdots,N_b. \tag{6}$$

where $\phi_{ik}(I_i)$ represents the kth block receptive-field basis function of the input I_i with the mean m_{ik} and the variance σ_{ik}, N_b is the number of block for each input dimension. The multidimensional receptive-field function $b_p(\boldsymbol{I})$ is associated with pth receptive-field and can be expressed as $b_p(\boldsymbol{I}) = \prod_{i=1}^{n} \phi_{ik}(I_i)$ for $p = 1,2,\cdots,N_h$ where N_h is the number of memory.

Define N as the number of sample points in each input space and L as the number of weight of CMAC that is activated by the corresponding input vector. Fig. 2 depicts the structure of a two-dimensional CMAC with 7 elements (N=7) and 3 elements is accumulated as a block. CMAC uses a set of indices as an address in accordance with the current state to extract the stored data. The associated index $\boldsymbol{a}_{25}$ corresponding to the input vector (the state $(I_1, I_2) = (3,3)$) is expressed as:

$$\boldsymbol{a}_{25}^T = \begin{array}{cccccccccccccccc} Aa & Ab & Ac & Ba & Bb & Bc & \cdots & Ed & Ee & Ef & \cdots & Hg & Hh & Hi & \cdots & Ii \\ [0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0] \end{array}$$

By considering the input vector v_k, the output y_{v_k} of CMAC can be mathematically expressed as

$$y_{v_k} = \boldsymbol{a}_k^T Diag(b_1, b_2, \cdots, b_{N_h})\boldsymbol{w} = \xi_{v_k}^T \boldsymbol{w} = \boldsymbol{w}^T \xi_{v_k} \tag{7}$$

where $\boldsymbol{\zeta}_{v_k}^T = \boldsymbol{a}_k^T Diag(b_1, b_2, \cdots, b_{N_h})$ and the weight vector $\boldsymbol{w}$ of CMAC is $\boldsymbol{w} = [w_1, w_2, \cdots, w_{N_h}]^T$.

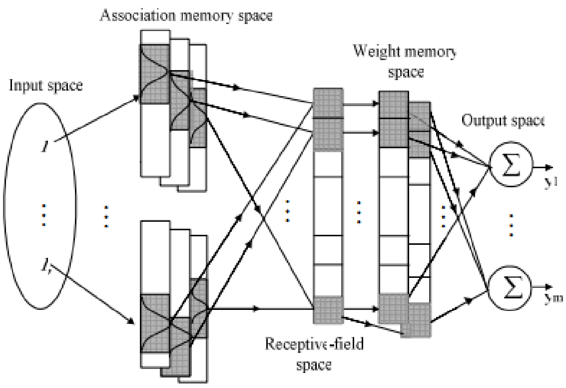

Fig. 1 Network architecture of a CMAC

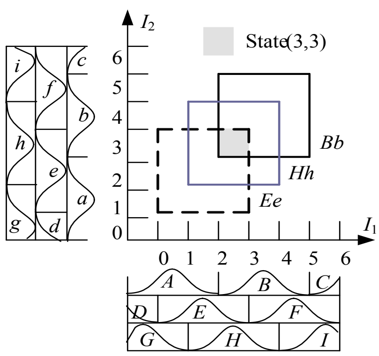

Fig. 2 Block division of a 2-D CMAC

Direct adaptive CMAC PI control with observer

In the previous section, due to assumption that the state vector $\boldsymbol{x}$ is immeasurable and only the system output is available, by replacing $f(\boldsymbol{x})$, $\boldsymbol{x}$, $\boldsymbol{e}$ and s with their estimates $\hat{f}(\hat{\boldsymbol{x}})$, $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{e}}$ and $\hat{s}(\hat{\boldsymbol{e}})$, where $\hat{\boldsymbol{e}} = \boldsymbol{y}_d - \hat{\boldsymbol{x}}$. The CMAC controller is chosen as (7), i.e.

$$u_D(\hat{\boldsymbol{x}}) = u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{w}) = \boldsymbol{w}^T \boldsymbol{\zeta}_k(\hat{\boldsymbol{x}}) \tag{8}$$

where $\boldsymbol{\zeta}_k^T = \boldsymbol{a}_k^T Diag(b_1, b_2, \cdots, b_{N_h})$, $\boldsymbol{w} = [w_1, w_2, \cdots, w_{N_h}]^T$. Thus, the resulting control law is

$$u = u_D + u_a + u_s \tag{9}$$

where u_D is the direct adaptive CMAC controller as (8), u_a and u_s are employed to compensate the external disturbance and modeling error. For increasing robustness, an optimal control law can be

$$u^* = b^{-1}[-f(\boldsymbol{x}) - \Psi(\hat{s} \mid \boldsymbol{\theta}) + y_d^{(n)} + K_c^T \hat{\boldsymbol{e}}] \tag{10}$$

where $\Psi(\hat{s} \mid \boldsymbol{\theta})$ is a Proportional-Integral (PI) type error and described by the following equation

$$\Psi(\hat{s} \mid \boldsymbol{\theta}) = \begin{cases} \boldsymbol{\theta}^T \psi(\hat{s}) = k_p \hat{s} + k_I \int \hat{s} dt, & |\hat{s}| \le \Phi \\ \hat{I}_s \operatorname{sgn}(\hat{s}), & |\hat{s}| > \Phi \end{cases} \tag{11}$$

where $\boldsymbol{\theta} = [k_p, k_I]^T$, $\psi(\hat{s}) = [\hat{s}, \int \hat{s} dt]^T$, $\hat{I}_s = \hat{\Omega} + \hat{D}$ is the estimate of $I_s = \Omega + D$, Ω and D are bound of minimum approximation error and external disturbance, respectively. Applying (9) and (10) into (2) and after some manipulation, the error dynamic equation

$$\begin{aligned} \dot{\boldsymbol{e}} &= A\boldsymbol{e} - BK_c^T \hat{\boldsymbol{e}} + B[b(u^* - u_D)] + B[\Psi(\hat{s} \mid \boldsymbol{\theta}) - b(u_a + u_s) - d] \\ e_1 &= \boldsymbol{C}^T e \end{aligned} \tag{12}$$

Define the optimal parameters of neural systems as $\boldsymbol{w}^* = \arg\min_{w \in \Omega_1} \left[\sup_{\boldsymbol{x} \in U_1, \hat{\boldsymbol{x}} \in U_2} \left| u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{w}) - u^*(\boldsymbol{x}) \right| \right]$ and $\boldsymbol{\theta}^* = \arg\min_{\theta \in \Omega_2} \left[\sup_{\boldsymbol{x} \in U_1, \hat{\boldsymbol{x}} \in U_2} \left| \Psi(\hat{s} \mid \boldsymbol{\theta}) - \Psi(\hat{s} \mid \boldsymbol{\theta}^*) \right| \right]$, where Ω_1 and Ω_2 are constraint sets for $\boldsymbol{w}$ and $\boldsymbol{\theta}$, Let the minimum approximation error be defined as $\varpi = [u^* - \hat{u}_D(\hat{\boldsymbol{x}} \mid \boldsymbol{w}^*)] + \Psi(\hat{s} \mid \boldsymbol{\theta}^*)$.

Assumption 1: CMAC approximation error ϖ is bounded, i.e., ω satisfies $\|\varpi\| \le M$.

Design the following observer to estimate the error vector $\boldsymbol{e}$ in (12)

$$\begin{aligned} \dot{\hat{\boldsymbol{e}}} &= A\hat{\boldsymbol{e}} - BK_c^T \hat{\boldsymbol{e}} + K_o(e_1 - \hat{e}_1) \\ \hat{e}_1 &= C^T \hat{\boldsymbol{e}} \end{aligned} \tag{13}$$

where $K_o^T = [k_n^o, k_{n-1}^o, \cdots, k_1^o]$ is the observer gain vector, which is selected such that the characteristic polynomial of $A - K_o C^T$ is strictly Hurwitz because (C, A) is observable. Define the observation errors $\tilde{\boldsymbol{e}} = \boldsymbol{e} - \hat{\boldsymbol{e}}$ and $\tilde{e}_1 = e_1 - \hat{e}_1$. Subtracting (13) from (12) yields

$$\begin{aligned} \dot{\tilde{\boldsymbol{e}}} &= (A - K_o C^T)\tilde{\boldsymbol{e}} + Bb(u^* - u_D) + B[\Psi(\hat{s} \mid \boldsymbol{\theta}) - b(u_a + u_s) - d] \\ \tilde{e}_1 &= C^T \tilde{\boldsymbol{e}} \end{aligned} \tag{14}$$

The output error dynamics of (14) can be given as

$$\tilde{e}_1 = H(p)[b(u^* - u_D) + \Psi(\hat{s} \mid \boldsymbol{\theta}) - b(u_a + u_s) - d] \tag{15}$$

where $H(p) = C^T(pI - (A - K_o C^T))^{-1} B$ is the transfer function of (14), p denotes the Laplace operator d/dt. The SPR-Lyapunov design approach [8] is used to analyze the stability of error dynamics (15). Equation (15) can be written as

$$\tilde{e}_1 = H(p)L(p)\{[L^{-1}(p)b(u^* - u_D) + L^{-1}(p)[\Psi(\hat{s} \mid \boldsymbol{\theta}) - b(u_a + u_s) - d]\} \tag{16}$$

where $L(p) = p^m + b_1 p^{m-1} + \cdots + b_m$, $m = n - 1$, is chosen such that $H(p)L(p)$ is a proper SPR transfer function. Then the state-space realization of (16) can be expressed as

$$\dot{\tilde{e}} = A_c\tilde{e} + B_c[-b\varpi_1 + b\zeta_1(\hat{x})\tilde{w} + \tilde{\theta}^T\psi_1(\hat{s}) - b(u_{a_1} + u_{s1}) - d_1]$$
$$e_1 = C_c^T\tilde{e} \tag{17}$$

where $\tilde{w} = w^* - w$, $A_c = A - K_oC^T \in \Re^{nxn}$, $B_c = [1, b_1, b_2, \cdots, b_m]^T \in \Re^n$, $C_c = [1,0,0,\cdots,0]^T \in \Re^n$ and $[u_1^*, u_{D1}, \varpi_1, \zeta_1, \Psi_1, \psi_1, u_{a_1}, u_{s1}, d_1] = L^{-1}(s)[u^*, u_D, \varpi, \zeta, \Psi, \psi, u_a, u_s, d]$. Because $H(p)L(p)$ is a SPR dynamic system, there exists $P_2 = P_2^T > 0$ such that

$$A_c^TP_2 + P_2A_c = -Q_2$$
$$P_2B_c = C_c \tag{18}$$

where $Q_2 = Q_2^T > 0$ is the given positive definite matrix. It is noted that $\tilde{e}^TP_2B_c = \tilde{e}^TC_c = \tilde{e}_1$.

Theorem 1: Consider the nonlinear system (1) and assume that only output variable is measurable. Construct the adaptive CMAC controller as (9), and $u_D(\hat{x} \mid w)$ is given by (10), and the auxiliary compensation control and the adaptation laws are chosen as

$$u_a = L(p)K_o^TP_1\hat{e}b^{-1} \tag{19}$$

$$u_s = kL(p)\operatorname{sgn}(\tilde{e}^TP_2B_c)b^{-1} \qquad k \geq M \tag{20}$$

$$\dot{w} = -\gamma_D\tilde{e}^TP_2B_cb\xi_1(\hat{x}) \tag{21}$$

$$\dot{\theta} = -\gamma_s\tilde{e}^TP_2B_c\psi_1(\hat{s}) \tag{22}$$

where $\gamma_D > 0$ and $\gamma_s > 0$ are positive gain constants to be designed, $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$ are the solutions of the equation (5) and (18), respectively. Then the proposed control scheme can ensures that all the closed-loop signals are bounded, and the tracking errors converge to zero.

Proof: Choose the Lyapunov function as $V = \frac{1}{2}\hat{e}^TP_1\hat{e} + \frac{1}{2}\tilde{e}^TP_2\tilde{e} + \frac{1}{2\gamma_\theta}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\gamma_D}\tilde{w}^T\tilde{w}$ with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$. Taking the time derivative of V yields

$$\dot{V} = \tfrac{1}{2}\hat{e}^T[(A - BK_c)^TP_1 + P_1(A - BK_c)]\hat{e} + \hat{e}^TP_1K_oC^T\tilde{e} - \tilde{e}^TP_2B_cbu_{a1} - \tilde{e}^TP_2B_cbu_{s1} \tag{23}$$
$$+ \tfrac{1}{2}\tilde{e}^T(A_c^TP_2 + P_2A_c)\tilde{e} + \tilde{e}^TP_2B_c[-b\varpi_1 + b\zeta_1(\hat{x})\tilde{w} + \tilde{\theta}^T\psi_1(\hat{s}) - d_1] + (1/\gamma_\theta)\tilde{\theta}^T\dot{\tilde{\theta}} + (1/\gamma_D)\tilde{w}^T\dot{\tilde{w}}$$

Substituting (6) and (19) into (23) and the fact $\dot{\tilde{w}} = -\dot{w}$, $\dot{\tilde{\theta}} = -\dot{\theta}$, the above equation becomes

$$\dot{V} = -\tfrac{1}{2}\hat{e}^TQ_1\hat{e} - \tfrac{1}{2}\tilde{e}^TQ_2\tilde{e} + \tilde{e}^TPB_c(\varpi_1 - bu_{s1})$$
$$\leq -\tfrac{1}{2}\hat{e}^TQ_1\hat{e} - \tfrac{1}{2}\tilde{e}^TQ_2\tilde{e} + |\tilde{e}^TPB_c|[(M - k)] \tag{24}$$

According to $k \geq M$, then (24) becomes $\dot{V} \leq -\frac{1}{2}\hat{e}^TQ_1\hat{e} - \frac{1}{2}\tilde{e}^TQ_2\tilde{e}$. Denoting $Q = diag[Q_1, Q_2]$ and $E^T = [\hat{e}^T, \tilde{e}^T]$, $\dot{V}$ becomes $\dot{V} \leq -\frac{1}{2}E^TQE < 0$.Therefore, it imply that $x, \hat{x}, e, \hat{e} \in L_\infty$. Integrating $\dot{V}$ from $t = 0$ to $t = T$ and after some manipulation yields $\int_0^T \frac{1}{2}E^TQEdt \leq V(0) - V(T) < \infty$. Since $\frac{1}{2}E^TQE$ is bounded, using Barbalat's lemma, we have $\lim_{t\to\infty} E(t) = 0$

Simulation results

Consider a Duffing forced oscillation system as follows

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u(t) + d \tag{25}$$
$$y = x_1.$$

where u is the control input and d is the bounded external disturbance. The control objective is to maintain the output x_1 of the system to track the desired trajectory $y_d = 0.1\sin(t)$. The external disturbance d is assumed to be a square wave with amplitude ± 0.1 and period 2π. The Gaussian basis functions in each input space are chosen as $\phi_{ik}(I_i) = \exp[-(I_i - m_{ik})^2 / \sigma_{ik}^2]$ for $i = 1,2$ and $k = 1, \cdots, 8$. The parameters of Gaussian basis function are chosen as $m_{i1} = -4$, $m_{i2} = -3$, $m_{i3} = -2$,

$m_{i4} = -1$, $m_{i5} = 1$, $m_{i6} = 2$, $m_{i7} = 3$, $m_{i8} = 4$, and $\sigma_{ik} = 0.5$. Select filter $L^{-1}(s) = 1/(s+2)$. Select the positive matrices $Q = Diag[10\ 10]$ and choose the feedback and observer gain vector as $K_c^T = [144, 24]$ and $K_o^T = [60, 900]$. The initial states are chosen as $\boldsymbol{x}(0) = [0.2, 0.2]^T$, $\hat{\boldsymbol{x}}(0) = [1.5, 1.5]^T$ and let adaptation gains as $r_D = r_s = 100$. In Fig. 3, the system output catches up to the desired trajectory well. Fig. 4 shows that the tracking error has been attenuated efficiently.

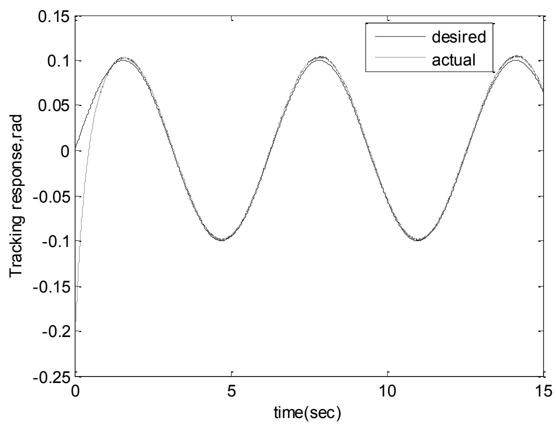

Fig. 3. Trajectories of the output y (dash)and desired y_d (solid).

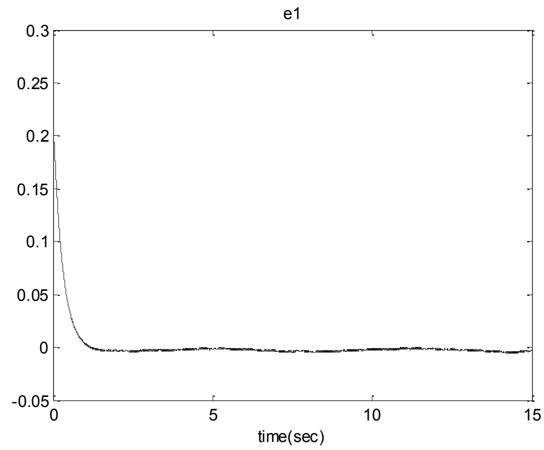

Fig. 4. The tracking error

6. Conclusion

This paper demonstrates the implementation and design of a direct adaptive CMAC PI controller with observer for the tracking of a nonlinear system. This design obtains robustness in the sense that the self-tuning mechanism can automatically adapt the CMAC PI controller by using a learning algorithm and the global asymptotic stability of the algorithm is established via Lyapunov stability criterion. Finally, the proposed control scheme is applied to the Duffing forced oscillation system. Simulation results indicate that the proposed approach is capable of achieving a good trajectory following performance without the knowledge of plant parameters.

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