Very unrigorous maths

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Let: $r = critrate$, $d = critdamage$, $x = r/d$, $v = 2r + d$. We want to approximate $1 + rd$ for values of x close to $1/2$.

Clearly $r = \frac{(v-d)}{2} = xd$, so $d = \frac{v}{2(x+\frac{1}{2})}$
so

$$1 + rd = 1 + xd^2 = 1 + \frac{xv^2}{4(x + \frac{1}{2})^2}$$

Now suppose $x = \frac{1}{2} + \epsilon$, then:

$$1 + \frac{xv^2}{4(x + \frac{1}{2})^2} = 1 + \frac{(\frac{1}{2} + \epsilon)v^2}{4(1 + \epsilon)^2} = 1 + \frac{(\frac{1}{2} + \epsilon)v^2}{4(1 + 2\epsilon + \epsilon^2)}$$

For small ϵ,

$$\frac{\frac{1}{2} + \epsilon}{1 + 2\epsilon + \epsilon^2} \approx \frac{1}{2}$$

So we have the approximation

$$1 + rd \approx 1 + \frac{v^2}{8} \tag{1}$$

for $x = \frac{r}{d} \approx \frac{1}{2}$.

As for when $0.7v$ is a good approximation, notice that most players probably have between 2 and 4 crit value (0.5 crit rate, 1 crit damage to 1 crit rate, 2 crit damage), so we can try Taylor expanding (1) around $v = a$ where a is somewhere between 2 and 4, then we have

$$1 + \frac{(a + v - a)^2}{8} \approx 1 + \frac{a^2 + 2(v - a)a}{8} = 1 + \frac{-a^2}{8} + \frac{av}{4}$$

We want the constant term to cancel out so $a = 2\sqrt{2}$, which gives $\frac{a}{4} = \frac{\sqrt{2}}{2}$, which is close to 0.7. So $0.7v$ is (probably) a reasonable approximation when crit value is around $2\sqrt{2}$ and r/d is close to $1/2$.