Network = 28 * 28 MNIST

filter = 3 * 3

filter_bias = 0

Forward pass

(feature * filter) + filter_bias = Feature Map → ReLU → POOLING → Neural NET → Softmax

Backward pass

Entropy → ReLU' → feature map

NEURAL NETS

Layers - 3

activations - ReLU & Softmax

forward pass

$$\text{feature map}[i,j] = \left(\sum_{i=0}^{x}\sum_{j=0}^{y}\sum_{u=0}^{3}\sum_{v=0}^{3} \text{filter}[u,v] * \text{feature}[i+u, j+v]\right) + \text{filter\_bias}$$

$X = \frac{N-L}{1} + 1 = \frac{28-3}{1} + 1 = 26$

$Y = \frac{N-L}{1} + 1 = \frac{28-3}{1} + 1 = 26$

feature map = ReLU (feature map)

Pooling_Indexes

P = np.zeros(x, y)

x, y = $\frac{\text{f\_length} - \text{pool\_kernel\_length}}{2} + 1$

$= \frac{26-2}{2} + 1 = 13$

indexes = np.zeros((x, y), d_type = tuple)

```
for i in x
  for j in y
    i_start = i * 2 (stride)  if i != 12, i_start = 24
    j_start = j * 2 (stride)  if j != 12, j_start = 24

    window = feature_map[i_start : i_start+2, j_start : j_start+2]

    value = max(window)
    index = argmax(window)
    i_dash = index / 2 (stride)
    j_dash = index % 2
    P[i,j] = value
    indexes[i_start, j_start] = (i_start + i_dash, j_start + j_dash)
```

Neural Network

Layers = 169, 40, 10

weights = [(40, 169), (10, 40)]

biases = [(40, 1), (10, 1)]

$L_1 = A_0 =$ 13 * 13 = Pooled Array = (169, 1)

$L_2 = A_1 = \sum_{i=0}^{169}\sum_{j=0}^{40} \text{ReLU}(w_2^{ji} \cdot A_0^i + b_1) = \text{ReLU}(z^1)$ ∴ shape (40, 1)

$L_3 = A_2 = \sum_{i=0}^{40}\sum_{k=0}^{10} \text{SoftMax}(w_2^{kj} \cdot A_1^j + b_2) = \text{Softmax}(z^2)$ ∴ shape (10, 1)

Cross Entropy = $A_2 - Y$

BACK PROPAGATION

$\frac{d\,\text{Cross Entropy}}{dz^2} = \delta^2 = a^2 - 1$ ∴ shape (10, 1)

$\frac{dC}{dw^2} = \frac{dC}{dz^2} \cdot \frac{dz^2}{dw^2}$

$= \delta^2 \cdot a^{2T}$ ∴ shape = (10, 40) ∴ $w^2 = w^2 - \eta \cdot \frac{dL}{dw^2}$

$\frac{dC}{db^2} = \delta^2$ ∴ shape (10, 1) ∴ $b^2 = b^2 - \eta \cdot \frac{dL}{db^2}$

$\frac{dC}{da^1} = \frac{dC}{dz^2} \cdot \frac{dz^2}{da^1}$

$= \delta^2 \cdot w^2$

$\frac{dC}{dz^1} = \frac{dC}{da^1} \cdot \frac{da^1}{dz^1}$

$= \delta^2 \cdot w^2 \cdot \frac{d}{dz^1} \text{ReLU}(z^1)$

$\delta^1 = \delta^2 \cdot w^2 * \text{ReLU}'(z^1)$ ∴ shape $[(40, 10) \cdot (10, 1)] * (40, 1)] = (40, 1)$ ∴ $b_1 = b_1 - \eta \cdot \frac{dC}{db_1}$

(w^{2T}, δ^2)

$\frac{dC}{dw^1} = \frac{dC}{dz^1} \cdot \frac{dz^1}{dw^1}$

$= \delta^1 \cdot \frac{d}{dw^1}(w^1 \cdot a^0 + b^1)$

$= \delta^1 \cdot a^{0T}$ ∴ shape $[(40, 1) \cdot (1, 169)]$ = (40, 169) ∴ $w_1 = w_1 - \eta \cdot \frac{dC}{dw^1}$

$\frac{dC}{db^1} = \delta^1$

$\frac{dC}{da^0} = \frac{dC}{dz^1} \cdot \frac{dz^1}{da^0}$

$\delta^0 = \delta^1 \cdot w^{1T}$ ∴ shape $[(169, 40) \cdot (40, 1)]$ = (169, 1)

(w^{1T}, δ^1)

```
new_map = np.zeros((26, 26))

Back fill_map()
    for index in δ^0
        new_map[indexes[i]] = δ^0[i]

New_map_ReLU_Prime = new_map * ReLU'(feature_map)

// forward propagation = ReLU(feature_map)
// derivative = ReLU'(feature_map) * derivative_feature_map(new_map)

dw = filter derivative
dw = np.zeros((3 * 3))

fill_dw:
    for u in 0,3
      for v in 0,3
        for i in 0,26
          for j in 0,26
            dw[u,v] += new_map[i,j] * feature[i+u, j+v]

filter = filter - η*dw

d_filter_bias = sum(dw)

filter_bias = filter_bias - η*d_filter_bias
```