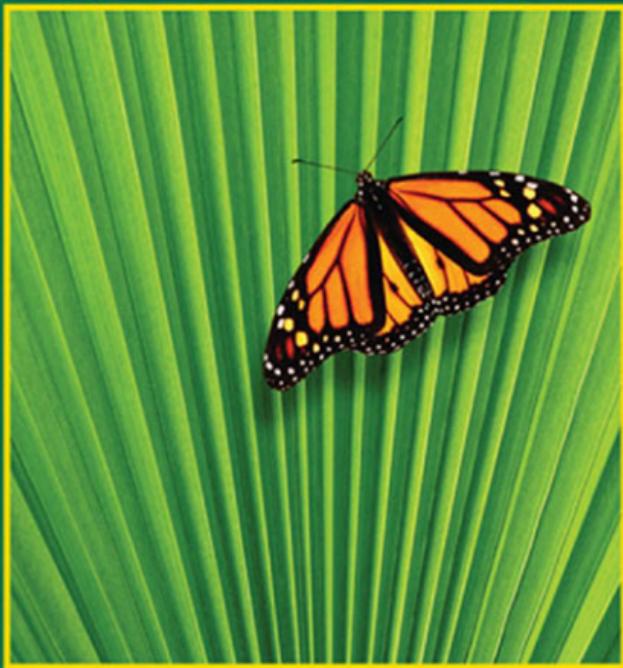


NINTH EDITION

FUNDAMENTALS OF

MATHEMATICS

Van Dyke ■ Rogers ■ Adams



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EDITION

FUNDAMENTALS OF **MATHEMATICS**

**James Van Dyke
James Rogers
Hollis Adams**

Portland Community College

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Fundamentals of Mathematics, Ninth Edition

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To

Carol Van Dyke ♦ Elinore Rogers ♦ Jessica Adams ♦ Ben Adams

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TO THE STUDENT

“It looks so easy when you do it, but when I get home . . .” is a popular lament of many students studying mathematics.

The process of learning mathematics evolves in stages. For most students, the first stage is listening to and watching others. In the middle stage, students experiment, discover, and practice. In the final stage, students analyze and summarize what they have learned. Many students try to do only the middle stage because they do not realize how important the entire process is.

Here are some steps that will help you to work through all the learning stages:

1. Go to class every day. Be prepared, take notes, and most of all, think actively about what is happening. Ask questions and keep yourself focused. This is prime study time.
2. Begin your homework as soon after class as possible. Start by reviewing your class notes and then read the text. Each section is organized in the same manner to help you find information easily. The objectives tell you what concepts will be covered, and the vocabulary lists all the new technical words. There is a **How & Why** section for each objective that explains the basic concept, followed by worked sample problems. As you read each example, make sure you understand every step. Then work the corresponding **Warm-Up** problem to reinforce what you have learned. You can check your answer at the bottom of the page. Continue through the whole section in this manner.
3. Now work the exercises at the end of the section. The A group of exercises can usually be done in your head. The B group is harder and will probably require pencil and paper. The C group problems are more difficult, and the objectives are mixed to give you practice at distinguishing the different solving strategies. As a general rule, do not spend more than 15 minutes on any one problem. If you cannot do a problem, mark it and ask someone (your teacher, a tutor, or a study buddy) to help you with it later. Do not skip the **Maintain Your Skills** problems. They are for review and will help you practice earlier procedures so you do not become “rusty.” The answers to the odd exercises are in the back of the text so you can check your progress.
4. In this text, you will find **State Your Understanding** exercises in every section. Taken as a whole, these exercises cover *all* the basic concepts in the text. You may do these orally or in writing. Their purpose is to encourage you to analyze or summarize a skill and put it into words. We suggest that you do these in writing and keep them all together in a journal. Then they are readily available as a review for chapter tests and exams.
5. When preparing for a test, work the material at the end of the chapter. The **True/False Concept Review** and the **Chapter Test** give you a chance to review the concepts you have learned. You may want to use the chapter test as a practice test.

If you have never had to write in a math class, the idea can be intimidating. Write as if you are explaining to a classmate who was absent the day the concept was discussed. Use your own words—*do not copy out of the text*. The goal is that you understand the concept, not that you can quote what the authors have said. Always use complete sentences, correct spelling, and proper punctuation. Like everything else, writing about math is a learned skill. Be patient with yourself and you will catch on.

Since we have many students who do not have a happy history with math, we have included **Good Advice for Studying**—a series of eight essays that address various problems that are common for students. They include advice on time organization, test taking, and reducing math anxiety. We talk about these things with our own students, and hope that you will find some useful tips.

We really want you to succeed in this course. If you go through each stage of learning and follow all the steps, you will have an excellent chance for success. But remember, you are in control of your learning. The effort that you put into this course is the single biggest factor in determining the outcome. Good luck!

*James Van Dyke
James Rogers
Hollis Adams*

CLAST SKILLS AND THEIR LOCATIONS IN THE BOOK

Arithmetic Skills	Location in Book
Add, subtract, multiply, and divide rational numbers in fractional form	Sections 3.3, 3.4, 3.6–3.10
Add, subtract, multiply, and divide rational numbers in decimal form	Sections 4.3–4.6, 4.8
Calculate percent increase and percent decrease	Section 6.6
Solve the sentence “ $a\%$ of b is c ,” where two of the values of the variables are given	Section 6.5
Recognize the meaning of exponents	Sections 1.5, 2.5, 2.6, 4.5
Recognize the role of the base number in determining place value in the base 10 numeration system	Sections 1.1, 4.1
Identify equivalent forms of decimals, percents, and fractions	Sections 4.2, 4.7, 6.2–6.4
Determine the order relation between real numbers	Sections 1.1, 3.5, 4.2
Identify a reasonable estimate of a sum, average, or product	Sections 1.2, 1.3, 4.8
Infer relations between numbers in general by examining particular number pairs	Sections 1.1, 2.2, 2.3
Solve real-world problems that do not involve the use of percent	Chapters 1–5, 7, 8
Solve real-world problems that involve the use of percent	Sections 6.6–6.8
Solve problems that involve the structure and logic of arithmetic	Throughout
Geometry and Measurement Skills	
Round measurements	Sections 1.1, 4.1, Chapter 7
Calculate distance, area, and volume	Sections 1.2, 1.3, 7.3–7.5
Classify simple plane figures by recognizing their properties	Sections 1.2, 1.3, 7.3, 7.4
Identify units of measurement for geometric objects	Sections 1.2, 1.3, 7.3–7.5
Infer formulas for measuring geometric figures	Sections 1.2, 1.3, Chapter 7
Select applicable formulas for computing measures of geometric figures	Chapter 7
Solve real-world problems involving perimeters, areas, and volumes of geometric figures	Sections 1.2, 1.3, 7.3–7.5
Solve real-world problems involving the Pythagorean theorem	Section 7.6
Algebra Skills	
Add, subtract, multiply, and divide real numbers	Sections 8.2–8.5
Apply the order-of-operations agreement	Sections 1.6, 3.10, 4.8, 8.6
Use scientific notation	Section 4.5
Solve linear equations and inequalities	Sections 1.2, 1.4, 1.6, 3.4, 3.9, 4.3, 4.6, 4.8, 8.7
Use formulas to compute results	Throughout
Recognize statements and conditions of proportionality and variation	Chapter 5
Solve real-world problems involving the use of variables	Sections 1.2, 1.4, 1.6, 3.4, 3.9, 4.3, 4.6, 4.8, 8.7
Statistics Skills, Including Probability	
Identify information contained in graphs	Sections 1.8, 6.6
Determine the mean, median, and mode	Sections 1.7, 3.10, 4.6
Recognize properties and interrelationships among the mean, median, and mode	Sections 1.7, 3.10, 4.6

ELM MATHEMATICAL SKILLS

The following table lists the California ELM Mathematical Skills and where coverage of these skills can be found in the text. Locations of the skills are indicated by chapter section or chapter.

Numbers and Data Skills	Location in Book
Carry out basic arithmetic calculations	Chapters 1, 3–4
Understand and use percent in context	Chapter 7
Compare and order rational numbers expressed as fractions and/or decimals	Sections 3.5, 4.2
Solve problems involving fractions and/or decimals in context	Chapters 3, 4
Interpret and use ratio and proportion in context	Chapter 5
Use estimation appropriately	Sections 1.2, 1.3, 4.8
Evaluate reasonableness of a solution to a problem	Sections 1.2, 1.3, 4.6
Evaluate and estimate square roots	Section 7.6
Represent and understand data presented graphically (including pie charts, bar and line graphs, histograms, and other formats for visually presenting data used in print and electronic media)	Sections 1.8, 6.6
Calculate and understand the arithmetic mean	Sections 1.7, 3.10, 4.6
Calculate and understand the median	Sections 1.7, 4.6

Algebra Skills

Use properties of exponents	Sections 1.5, 2.5, 2.6, 4.5
Solve linear equations (with both numerical and literal coefficients)	Sections 1.2, 1.4, 1.6, 3.4, 3.9, 4.3, 4.6, 4.8, 8.7

Geometry Skills

Find the perimeter, area, or volume of geometric figures (including triangles, quadrilaterals, rectangular parallelepipeds, circles, cylinders, and combinations of these figures)	Sections 1.2, 1.3, 7.3, 7.4, 7.5
Use the Pythagorean theorem	Section 7.6
Solve geometric problems using the properties of basic geometric figures (including triangles, quadrilaterals, polygons, and circles)	Sections 1.2, 1.3, 7.3, 7.4, 7.5, and in problem sets throughout

TASP SKILLS AND THEIR LOCATIONS IN THE BOOK

Fundamental Skills of Mathematics	Location in Book
Solve word problems involving integers, fractions, decimals, and units of measurement	Chapters 3–8
Solve problems involving data interpretation and analysis	Sections 1.2, 1.8, 6.6, and in problem sets throughout

Algebra Skills

Solve one- and two- variable equations	Sections 1.2, 1.4, 1.6, 3.4, 3.9, 4.3, 4.6, 4.8, 8.7
Solve word problems involving one and two variables	Sections 1.2, 1.4, 1.6, 3.4, 3.9, 4.3, 4.6, 4.8, 8.7

Geometry Skills

Solve problems involving geometric figures	Sections 1.2, 1.3, 7.3, 7.4, 7.5, and in problem sets throughout
--	--

Problem-Solving Skills

Solve applied problems involving a combination of mathematical skills	Sections 1.6, 3.10, 4.8, 5.3, 6.6–6.8
---	---------------------------------------

TO THE INSTRUCTOR

Fundamentals of Mathematics, Ninth Edition, is a work text for college students who need to review the basic skills and concepts of arithmetic in order to pass competency or placement exams, or to prepare for courses such as business mathematics or elementary algebra. The text is accompanied by a complete system of ancillaries in a variety of media, affording great flexibility for individual instructors and students.

A Textbook for Adult Students

Though the mathematical content of *Fundamentals of Mathematics* is elementary, students using the text are most often mature adults, bringing with them adult attitudes and experiences and a broad range of abilities. Teaching elementary content to these students, therefore, is effective when it accounts for their distinct and diverse adult needs. As you read about and examine the features of *Fundamentals of Mathematics* and its ancillaries, you will see how they especially meet three needs of your students:

- Students must establish good study habits and overcome math anxiety.
- Students must see connections between mathematics and the modern, day-to-day world of adult activities.
- Students must be paced and challenged according to their individual level of understanding.

A Textbook of Many Course Formats

Fundamentals of Mathematics is suitable for individual study or for a variety of course formats: lab, both supervised and self-paced; lecture; group; or combined formats. For a lecture-based course, for example, each section is designed to be covered in a standard 50-minute class. The lecture can be interrupted periodically so that students individually can work the **Warm-Up** exercises or work in small groups on the group work. In a self-paced lab course, **Warm-Up** exercises give students a chance to practice while they learn, and get immediate feedback since warm-up answers are printed on the same page. Using the text's ancillaries, instructors and students have even more options available to them. Computer users, for example, can take advantage of complete electronic tutorial and testing systems that are fully coordinated with the text.

Teaching Methodology

As you examine the Ninth Edition of *Fundamentals of Mathematics*, you will see distinctive format and pedagogy that reflect these aspects of teaching methodology:

Teaching by Objective Each section focuses on a short list of objectives, stated at the beginning of the section. The objectives correspond to the sequence of exposition and tie together other pedagogy, including the highlighted content, the examples, and the exercises.

1.1 Whole Numbers and Tables: Writing, Rounding, and Inequalities

OBJECTIVES

1. Write word names from place value names and place value names from word names.
2. Write an inequality statement about two numbers.
3. Round a given whole number.
4. Read tables.

VOCABULARY

The **digits** are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The **natural numbers (counting numbers)** are 1, 2, 3, 4, 5, and so on. The **whole numbers** are 0, 1, 2, 3, 4, 5, and so on. Numbers larger than 9 are written in **place value name** by writing the digits in positions having standard **place value**. **Word names** are written words that represent numerals. The word name of 213 is two hundred thirteen. The symbols **less than**, $<$, and **greater than**, $>$, are used to compare two whole numbers that are not equal. So, $11 < 15$, and $21 > 5$. To **round** a whole number means to give an approximate value. The symbol “ \approx ” means “approximately equal to.”

Teaching by Application Each chapter leads off with an application that uses the content of the chapter. Exercise sets have applications that use this material or that are closely related to it. Applications are included in the examples for most objectives. Other applications appear in exercise sets. These cover a diverse range of fields, demonstrating the utility of the content in business, environment, personal health, sports, and daily life.

WHOLE NUMBERS



Whole Numbers
and Tables:
Writing, Rounding,
and Inequalities

1.2 Adding and Subtracting Whole Numbers

APPLICATION

The top ten grossing movies in the United States for 2004 are given in Table 1.1.

Table 1.1 Top Grossing Movies for 2004

<i>Shrek 2</i>	\$436,471,036
<i>Spider-Man 2</i>	\$373,377,893
<i>The Passion of the Christ</i>	\$370,274,604
<i>Meet the Fockers</i>	\$273,488,020
<i>The Incredibles</i>	\$258,938,368
<i>Harry Potter and the Prisoner of Azkaban</i>	\$249,358,727
<i>The Day After Tomorrow</i>	\$186,739,919
<i>The Bourne Supremacy</i>	\$176,049,130
<i>National Treasure</i>	\$169,378,371
<i>The Polar Express</i>	\$162,458,888

Source: Internet Movie Database.

Group Discussion

1. How many of the top grossing movies for 2004 were animated? How many were suitable for children 12 and under?
2. Which movies were comedies? Which were action-adventure?

Emphasis on Language New words of each section are explained in the vocabulary segment that precedes the exposition. Exercise sets include questions requiring responses written in the students' own words.

1.7 Average, Median, and Mode

VOCABULARY

The **average**, or **mean**, of a set of numbers is the sum of the set of numbers divided by the total number of numbers in the set.

The **median** of a set of numbers, ordered from smallest to largest, is either the middle number of the set or the average of the two middle numbers in the set.

The **mode** of a set of numbers is the number or numbers that appear the most often in the set.

OBJECTIVES

- Find the average of a set of whole numbers.
- Find the median of a set of whole numbers.
- Find the mode of a set of whole numbers.

Emphasis on Skill, Concept, and Problem Solving Each section covers concepts and skills that are fully explained and demonstrated in the exposition for each objective.

How & Why

OBJECTIVE 1

Find the average of a set of whole numbers.

The *average* or *mean* of a set of numbers is used in statistics. It is one of the ways to find the middle of a set of numbers (like the average of a set of test grades). Mathematicians call the average or mean a "measure of central tendency." The average of a set of numbers is found by adding the numbers in the set and then dividing that sum by the number of numbers in the set. For example, to find the average of 11, 21, and 28:

$$11 + 21 + 28 = 60 \quad \text{Find the sum of the numbers in the set.}$$
$$60 \div 3 = 20 \quad \text{Divide the sum by the number of numbers, 3.}$$

The average is 20.

The "central" number or average does not need to be one of the members of the set. The average, 20, is not a member of the set.

To find the average of a set of whole numbers

- Add the numbers.
- Divide the sum by the number of numbers in the set.

Carefully constructed examples for each objective are connected by a common strategy that reinforces both the skill and the underlying concepts. Skills are not treated as isolated feats of memorization but as the practical result of conceptual understanding: Skills are strategies for solving related problems. Students see the connections between problems that require similar strategies.

Examples A–E



BASIC Mathematics Now™

DIRECTIONS: Find the average.

STRATEGY: Add the numbers in the set. Divide the sum by the number of numbers in the set.

- A. Find the average of 212, 189, and 253.

$$212 + 189 + 253 = 654 \quad \text{Add the numbers in the group.}$$
$$654 \div 3 = 218 \quad \text{Divide the sum by the number of numbers.}$$

The average is 218.

- B. Find the average of 23, 57, 352, and 224.

$$23 + 57 + 352 + 224 = 656 \quad \text{Add the numbers in the group.}$$
$$656 \div 4 = 164 \quad \text{Divide the sum by the number of numbers.}$$

The average is 164.

Warm-Ups A–E

- A. Find the average of 251, 92, and 449.

- B. Find the average of 12, 61, 49, 82, and 91.

Answers to Warm-Ups

- A. 264 B. 59

Emphasis on Success and Preparation

Integrated throughout the text, the following features focus on study skills, math anxiety, calculators, and simple algebraic equations.

Good Advice for Studying is continued from the previous editions. Originally written by the team of Dorette Long and Sylvia Thomas of Rogue Community College, these essays address the unique study problems that students of *Fundamentals of Mathematics* experience. Students learn general study skills and study skills specific to mathematics and to the pedagogy and ancillaries of *Fundamentals of Mathematics*. Special techniques are described to overcome the pervasive problems of math anxiety. Though an essay begins each chapter, students may profit by reading all the essays at once and then returning to them as the need arises.

GOOD ADVICE FOR STUDYING

Strategies for Success

Are you afraid of math? Do you panic on tests or “blank out” and forget what you have studied, only to recall the material after the test? Then you are just like many other students. In fact, research studies estimate that as many as 50% of you have some degree of math anxiety.

What is math anxiety? It is a learned fear response to math that causes disruptive, debilitating reactions to tests. It can be so encompassing that it becomes a dread of doing **anything** that involves numbers. Although some anxiety at test time is beneficial—it can motivate and energize you, for example—numerous studies show that too much anxiety results in poorer test scores. Besides performing poorly on tests, you may be distracted by worrisome thoughts and be unable to concentrate and recall what you’ve learned. You may also set unrealistic performance standards for yourself and imagine catastrophic consequences for your failure to be successful in math. Your physical signs could be muscle tightness, stomach upset, sweating, headache, shortness of breath, shaking, or rapid heartbeat.

The good news is that anxiety is a learned behavior and therefore can be unlearned. If you want to stop feeling anxious, the choice is up to you. You can choose to learn behaviors that are more useful to achieve success in math. You can learn and choose the ways that work best for you.

To achieve success, you can focus on two broad strategies. First, you can study math in ways **proven** to be effective in learning mathematics and taking tests. Second, you can learn to physically and mentally **relax**, to manage your anxious feelings, and to think rationally and positively. Make a time commitment to practice relaxation techniques, study math, and record your thought patterns. A commitment of 1 or 2 hours a day may be necessary in the beginning. Remember, it took time to learn your present study habits and to be anxious. It will take time to unlearn these behaviors. After you become proficient with these methods, you can devote less time to them.

Begin now to learn your strategies for success. Be sure you have read *To the Student* at the beginning of this book. The purpose of this section is to introduce you to the authors’ plan for this text. *To the Student* will help you to understand the authors’ organization or “game plan” for your math experience in this course.

At the beginning of each chapter, you will find more Good Advice for Studying sections, which will help you study and take tests more effectively, as well as help you manage your anxiety. You may want to read ahead so that you can improve even more quickly. Good luck!



Calculator examples, marked by the symbol , demonstrate how a calculator may be used, though the use of a calculator is left to the discretion of the instructor. Nowhere is the use of a calculator required. Appendix A reviews the basics of operating a scientific calculator.

C. Find the average of 777, 888, 914, and 505.

D. The average of 42, 63, 21, 39, and ? is 50. Find the missing number.

E. The Alpenrose Dairy ships the following number of gallons of milk to local groceries: Monday, 1045; Tuesday, 1325; Wednesday, 2005; Thursday, 1810; and Friday, 2165. What is the average number of gallons shipped each day?

CALCULATOR EXAMPLE

C. Find the average of 673, 821, 415, and 763.

STRATEGY: Enter the sum, in parentheses, and divide by 4.

$$(673 + 821 + 415 + 763) \div 4$$

The average is 668.

D. The average of 38, 26, 12, and ? is 28. Find the missing number.

STRATEGY: Because the average of the four numbers is 28, we know that the sum of the four numbers is 4(28) or 112. To find the missing number, subtract the sum of the three given numbers from 112.

$$112 - (38 + 26 + 12) = 112 - (76) \\ = 36$$

So the missing number is 36.

E. In order to help Pete lose weight the dietician has him record his caloric intake for a week. He records the following: Monday, 3120; Tuesday, 1885; Wednesday, 1600; Thursday, 2466; Friday, 1434; Saturday, 1955; and Sunday, 2016. What is Pete's average caloric intake per day?

STRATEGY: Add the calories for each day and then divide by 7, the number of days.

$$\begin{array}{r} 3120 & 2068 \\ 1885 & 7 \overline{)14476} \\ 1600 & \underline{14} \\ 2466 & \quad 4 \\ 1434 & \quad 0 \\ 1955 & \quad 47 \\ 2016 & \quad 42 \\ \hline 14476 & \quad 56 \\ & \quad 56 \\ & \quad 0 \end{array}$$

Pete's average caloric intake is 2068 calories per day.

Getting Ready for Algebra segments follow Sections 1.2, 1.4, 1.6, 3.4, 3.9, 4.3, 4.6, and 4.8. The operations from these sections lend themselves to solving simple algebraic equations. Though entirely optional, each of these segments includes its own exposition, examples with warm-ups, and exercises. Instructors may cover these segments as part of the normal curriculum or assign them to individual students.

Getting Ready for Algebra

How & Why

In Section 1.2, the equations involved the inverse operations addition and subtraction. Multiplication and division are also inverse operations. We can use this idea to solve equations containing those operations. For example, if 4 is multiplied by 2, $4 \cdot 2 = 8$, the product is 8. If the product is divided by 2, $8 \div 2$, the result is 4, the original number. In the same manner, if 12 is divided by 3, $12 \div 3 = 4$, the quotient is 4. If the quotient is multiplied by 3, $4 \cdot 3 = 12$, the original number. We use this idea to solve equations in which the variable is either multiplied or divided by a number.

When a variable is multiplied or divided by a number, the multiplication symbols (\cdot or \times) and the division symbol (\div) normally are not written. We write $3x$ for 3 times x and $\frac{x}{3}$ for x divided by 3.

Consider the following:

$$\begin{aligned} 5x &= 30 \\ \frac{5x}{5} &= \frac{30}{5} && \text{Division will eliminate multiplication.} \\ x &= 6 \end{aligned}$$

OBJECTIVE

Solve an equation of the form $ax = b$ or $\frac{x}{a} = b$, where x , a , and b are whole numbers.

Pedagogy

The pedagogical system of *Fundamentals of Mathematics* meets two important criteria: coordinated purpose and consistency of presentation.

Each section begins with numbered **Objectives**, followed by definitions of new **Vocabulary** to be encountered in the section. Following the vocabulary, **How & Why** segments, numbered to correspond to the objectives, explain and demonstrate concepts

and skills. Throughout the **How & Why** segments, **skill boxes** clearly summarize and outline the skills in step-by-step form. Also throughout the segments, **concept boxes** highlight appropriate properties, formulas, and theoretical facts underlying the skills. Following each **How & Why** segment are **Examples** and **Warm-Ups**. Each example of an objective is paired with a warm-up, with workspace provided. Solutions to the warm-ups are given at the bottom of the page, affording immediate feedback. The examples also include, where suitable, a relevant application of the objective. Examples similar to each other are linked by common **Directions** and a common **Strategy** for solution. Directions and strategies are closely related to the skill boxes. Connecting examples by a common solution method helps students recognize the similarity of problems and their solutions, despite their specific differences. In this way, students may improve their problem-solving skills. In both **How & Why** segments and in the **Examples**, **Caution** remarks help to forestall common mistakes.

Exercises, Reviews, Tests

Thorough, varied, properly paced, and well-chosen exercises are a hallmark of *Fundamentals of Mathematics*. Exercise sets are provided at the end of each section and a review set at the end of each chapter. Workspace is provided for all exercises and each exercise set can be torn out and handed in without disturbing other parts of the book.

Section exercises are paired so that virtually each odd-numbered exercise, in Sections A and B, is paired with an even-numbered exercise that is equivalent in type and difficulty. Since answers for odd-numbered exercises are in the back of the book, students can be assigned odd-numbered exercises for practice and even-numbered exercises for homework.

Section exercises are categorized to satisfy teaching and learning aims. Exercises for estimation, mental computation, pencil and paper computation, application, and calculator skills are provided, as well as opportunities for students to challenge their abilities, master communications skills, and participate in group problem solving.

- **Category A** exercises, organized by section objective, are those that most students should be able to solve mentally, without pencil, paper, or calculator. Mentally working problems improves students' estimating abilities. These can often be used in class as oral exercises.
- **Category B** exercises, also organized by objective, are similar except for level of difficulty. All students should be able to master Category B.
- **Category C** exercises contain applications and more difficult exercises. Since these are not categorized by objective, the student must decide on the strategy needed to set up and solve the problem. These applications are drawn from business, health and nutrition, environment, consumer, sports, and science fields. Both professional and daily-life uses of mathematics are incorporated.

Exercises 1.7

BASIC
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OBJECTIVE 1

Find the average of a set of whole numbers.

A Find the average.

- | | | | |
|----------------|------------------|---------------------|---------------------|
| 1. 8, 12 | 2. 9, 17 | 3. 12, 18 | 4. 21, 31 |
| 5. 9, 15, 18 | 6. 11, 15, 19 | 7. 7, 11, 12, 14 | 8. 9, 9, 17, 17 |
| 9. 10, 8, 5, 5 | 10. 20, 15, 3, 2 | 11. 9, 11, 6, 8, 11 | 12. 15, 7, 3, 31, 4 |

Find the missing number to make the average correct.

13. The average of 10, 13, 15, and ? is 13. 14. The average of 12, 17, 21, and ? is 17.

B Find the average.

- | | | |
|------------------------|----------------------------|-----------------------|
| 15. 22, 26, 40, 48 | 16. 22, 43, 48, 67 | 17. 31, 41, 51, 61 |
| 18. 22, 19, 34, 63, 52 | 19. 14, 17, 25, 34, 50, 82 | 20. 93, 144, 221, 138 |

State Your Understanding exercises require a written response, usually no more than two or three sentences. These responses may be kept in a journal by the student. Maintaining a journal allows students to review concepts as they have written them. These writing opportunities facilitate student writing in accordance with standards endorsed by AMATYC and NCTM.

STATE YOUR UNDERSTANDING

88. Explain what is meant by the average of two or more numbers.
89. Explain how to find the average (mean) of 2, 4, 5, 5, and 9. What does the average of a set of numbers tell you about the set?

Challenge exercises stretch the content and are more demanding computationally and conceptually.

CHALLENGE

90. A patron of the arts estimates that the average donation to a fund-raising drive will be \$72. She will donate \$150 for each dollar by which she misses the average. The 150 donors made the contributions listed in the table.

Contributions to the Arts

Number of Donors	Donation
5	\$153
13	\$125
24	\$110
30	\$100
30	\$ 75
24	\$ 50
14	\$ 25
10	\$ 17

Group Work exercises and **Group Projects** provide opportunities for small groups of students to work together to solve problems and create reports. While the use of these is optional, the authors suggest the assignment of two or three of these per semester or term to furnish students with an environment for exchanging ideas. **Group Work** exercises encourage cooperative learning as recommended by AMATYC and NCTM guidelines.

GROUP WORK

91. Divide 35, 68, 120, 44, 56, 75, 82, 170, and 92 by 2 and 5. Which ones are divisible by 2 (the division has no remainder)? Which ones are divisible by 5? See if your group can find simple rules for looking at a number and telling whether or not it is divisible by 2 and/or 5.
92. Using the new car ads in the newspaper, find four advertised prices for the same model of a car. What is the average price, to the nearest 10 dollars?

Group Project (1–2 weeks) CHAPTER 1

OPTIONAL

All tables, graphs, and charts should be clearly labeled and computer-generated if possible. Written responses should be typed and checked for spelling and grammar.

1. Go to the library and find the population and area for each state in the United States. Organize your information by geographic region. Record your information in a table.
2. Calculate the total population and the total area for each region. Calculate the population density (number of people per square mile, rounded to the nearest whole person) for each region, and put this and the other regional totals in a regional summary table. Then make three separate graphs, one for regional population, one for regional area, and the third for regional population density.
3. Calculate the average population per state for each region, rounding as necessary. Put this information in a bar graph. What does this information tell you about the regions? How is it different from the population density of the region?
4. How did your group decide on the makeup of the regions? Explain your reasoning.

Maintain Your Skills exercises continually reinforce mastery of skills and concepts from previous sections. The problems are specially chosen to review topics that will be needed in the next section.

MAINTAIN YOUR SKILLS

- | | |
|---|---|
| <p>80. Round 56,857 to the nearest thousand and nearest ten thousand.</p> <p>82. Divide: $792 \div 66$</p> <p>84. Find the perimeter of a square that is 14 cm on a side.</p> <p>86. Multiply 12 by 1, 2, 3, 4, 5, and 6.</p> <p>88. Multiply 123 by 1, 2, 3, 4, 5, and 6.</p> | <p>81. Round 5,056,857 to the nearest ten thousand and nearest hundred thousand.</p> <p>83. Divide: $1386 \div 66$</p> <p>85. Find the area of a square that is 14 cm on a side.</p> <p>87. Multiply 13 by 1, 2, 3, 4, 5, and 6.</p> <p>89. Multiply 1231 by 1, 2, 3, 4, 5, and 6.</p> |
|---|---|

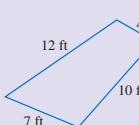
Key Concepts recap the important concepts and skills covered in the chapter. The **Key Concepts** can serve as a quick review of the chapter material.

Key Concepts CHAPTER 1

Section 1.1 Whole Numbers and Tables: Writing, Rounding, and Inequalities

Definitions and Concepts	Examples														
The whole numbers are 0, 1, 2, 3, and so on.	238 two hundred thirty-eight 6,198,349 six million, one hundred ninety-eight thousand, three hundred forty-nine														
One whole number is smaller than another if it is to the left on the number line.	$3 < 6$														
One whole number is larger than another if it is to the right on the number line.	$14 > 2$														
To round a whole number:	$6,745 \approx 7,000$ (nearest thousand) $6,745 \approx 6,700$ (nearest hundred)														
Tables are a method of organizing information or data in rows and columns.	Enrollment by Gender at River CC														
	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Males</th> <th style="text-align: center;">Females</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">English</td> <td style="text-align: center;">52</td> </tr> <tr> <td style="text-align: center;">Math</td> <td style="text-align: center;">67</td> </tr> <tr> <td style="text-align: center;">Science</td> <td style="text-align: center;">64</td> </tr> <tr> <td style="text-align: center;">History</td> <td style="text-align: center;">75</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;">63</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;">59</td> </tr> </tbody> </table>	Males	Females	English	52	Math	67	Science	64	History	75		63		59
Males	Females														
English	52														
Math	67														
Science	64														
History	75														
	63														
	59														
	There are 71 males taking math and 75 females taking science.														

Section 1.2 Adding and Subtracting Whole Numbers

Definitions and Concepts	Examples
To add whole numbers, write the numbers in columns so the place values are aligned. Add each column starting with the ones. Carry as necessary. <i>addend + addend = sum</i>	$\begin{array}{r} 372 \\ +594 \\ \hline 966 \end{array}$ $\begin{array}{r} 11 \\ 36 \\ +785 \\ \hline 821 \end{array}$
To subtract whole numbers, write the numbers in columns so the place values are aligned. Subtract, starting with the ones column. Borrow if necessary.	$\begin{array}{r} 4597 \\ -362 \\ \hline 4235 \end{array}$ $\begin{array}{r} 214412 \\ 3452 \\ -735 \\ \hline 2717 \end{array}$
The answer to a subtraction problem is called the <i>difference</i> .	
The perimeter of a polygon is the distance around the outside.	
To calculate the perimeter, add the lengths of the sides.	 $\begin{aligned} \text{Perimeter} &= 12 + 4 + 10 + 7 \\ &= 33 \\ P &= 33 \text{ ft} \end{aligned}$

Chapter Review Exercises provide a student with a set of exercises, usually 8–10 per section, to verify mastery of the material in the chapter prior to taking an exam.

Review Exercises CHAPTER 1

Section 1.1

Write the word name for each of these numbers.

1. 607,321 2. 9,070,800

Write the place value name for each of these numbers.

3. Sixty-two thousand, three hundred thirty-seven 4. Five million, four hundred forty-four thousand, nineteen

Insert > or < between the numbers to make a true statement.

5. 347 351 6. 76 69 7. 809 811

Round to the nearest ten, hundred, thousand, and ten thousand.

8. 79,437 9. 183,659

Cumulative Reviews are included at the end of Chapters 3, 5, and 7. Each review covers all of the material in the text that precedes it, allowing students to maintain their skills as the term progresses.

Cumulative Review CHAPTERS 1–3

Write the word name for each of the following.

1. 6,091 2. 110,532

Write the place value name.

3. One million three hundred ten 4. Sixty thousand two hundred fifty-seven

Round to the indicated place value.

5. 654,785 (hundred) 6. 43,949 (ten thousand)

Insert > or < between the numbers to make a true statement.

7. 6745 6739 8. 11,899 11,901

Add or subtract.

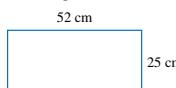
9.
$$\begin{array}{r} 76,843 \\ 34,812 \\ 12,833 \\ + 9,711 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 55,304 \\ -37,478 \\ \hline \end{array}$$

11. $54 + 87 + 124 + 784 + 490 + 54$

12. $70,016 - 54,942$

13. Find the perimeter of the rectangle.



Multiply.

14.
$$\begin{array}{r} 14,654 \\ \times 251 \\ \hline \end{array}$$

15. $(341)(73)$

16. Find the area.



Chapter True/False Concept Review exercises require students to judge whether a statement is true or false and, if false, to rewrite the sentence to make it true. Students evaluate their understanding of concepts and also gain experience using the vocabulary of mathematics.

True/False Concept Review CHAPTER 1 ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. All whole numbers can be written using nine digits.	1. _____
2. In the number 8425, the digit 4 represents 400.	2. _____
3. The word <i>and</i> is not used when writing the word names of whole numbers.	3. _____
4. The symbols, $7 < 23$, can be read “seven is greater than twenty-three.”	4. _____
5. $2567 < 2566$	5. _____
6. To the nearest thousand, 7398 rounds to 7000.	6. _____
7. It is possible for the rounded value of a number to be equal to the original number.	7. _____
8. The expanded form of a whole number shows the plus signs that are usually not written.	8. _____

Chapter Test exercises end the chapter. Written to imitate a 50-minute exam, each chapter test covers all of the chapter content. Students can use the chapter test as a self-test before the classroom test.

Test CHAPTER 1 ANSWERS

1. Divide: $72 \overline{)15,264}$	1. _____
2. Subtract: $9615 - 6349$	2. _____
3. Simplify: $55 \div 5 + 6 \cdot 4 - 7$	3. _____
4. Multiply: $37(428)$	4. _____
5. Insert $<$ or $>$ to make the statement true: $368 \quad 371$	5. _____
6. Multiply: 55×10^6	6. _____
7. Multiply: $608(392)$	7. _____
8. Write the place value name for seven hundred thirty thousand sixty-one.	8. _____
9. Find the average of 3456, 812, 4002, 562, and 1123.	9. _____
10. Multiply: $65(5733)$. Round the product to the nearest hundred.	10. _____

Changes in the Ninth Edition

Instructors who have used a previous edition of *Fundamentals of Mathematics* will see changes and improvements in format, pedagogy, exercises, and sectioning of content. Many of these changes are in response to comments and suggestions offered by users and

reviewers of the manuscript. We continue to make changes in line with math reform standards and to give the instructor the chance to follow educational guidelines recommended by AMATYC and NTCM.

- Thirty to fifty percent of routine exercises are new to each section.
- Applications have been updated and new ones have been added. These are in keeping with the emphases on real-world data.
- New examples have been added to the Examples and Warm-Ups.
- Estimating has been rewritten for all operations and is included in whole numbers and decimals.
- Finding the greatest common factor (GCF) has been deleted.
- In Chapter 1, pictorial graphs have been de-emphasized due to decline in usage in the media.
- Chapter 6 has undergone a major reorganization:
 - a. Routine conversions of fractions-decimals-percents have been consolidated.
 - b. Applications of percent have been reorganized and significantly expanded.
 - c. The three new sections of applications are:

Section 6.6: General applications of percent and percent of increase and percent of decrease.

Section 6.7: Sales tax, discount, and commissions.

Section 6.8: Simple and compound interest as applied to savings and on loans, credit card payments, and balances.

- In Chapter 7, the conversion tables for measurement have been standardized to four decimal places.
- Icons  **BASIC MATHEMATICS \Rightarrow Now™** point students to material combined on ThomsonNow and on the Interactive Video Skillbuilder CD-ROM.

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*James Van Dyke
James Rogers
Hollis Adams*

Strategies for Success

Are you afraid of math? Do you panic on tests or “blank out” and forget what you have studied, only to recall the material after the test? Then you are just like many other students. In fact, research studies estimate that as many as 50% of you have some degree of math anxiety.

What is math anxiety? It is a learned fear response to math that causes disruptive, debilitating reactions to tests. It can be so encompassing that it becomes a dread of doing **anything** that involves numbers. Although some anxiety at test time is beneficial—it can motivate and energize you, for example—numerous studies show that too much anxiety results in poorer test scores. Besides performing poorly on tests, you may be distracted by worrisome thoughts and be unable to concentrate and recall what you’ve learned. You may also set unrealistic performance standards for yourself and imagine catastrophic consequences for your failure to be successful in math. Your physical signs could be muscle tightness, stomach upset, sweating, headache, shortness of breath, shaking, or rapid heartbeat.

The good news is that anxiety is a learned behavior and therefore can be unlearned. If you want to stop feeling anxious, the choice is up to you. You can choose to learn behaviors that are more useful to achieve success in math. You can learn and choose the ways that work best for you.

To achieve success, you can focus on two broad strategies. First, you can study math in ways **proven** to be effective in learning mathematics and taking tests. Second, you can learn to physically and mentally **relax**, to manage your anxious feelings, and to think rationally and positively. Make a time commitment to practice relaxation techniques, study math, and record your thought patterns. A commitment of 1 or 2 hours a day may be necessary in the beginning. Remember, it took time to learn your present study habits and to be anxious. It will take time to unlearn these behaviors. After you become proficient with these methods, you can devote less time to them.

Begin now to learn your strategies for success. Be sure you have read *To the Student* at the beginning of this book. The purpose of this section is to introduce you to the authors’ plan for this text. *To the Student* will help you to understand the authors’ organization or “game plan” for your math experience in this course.

At the beginning of each chapter, you will find more Good Advice for Studying sections, which will help you study and take tests more effectively, as well as help you manage your anxiety. You may want to read ahead so that you can improve even more quickly. Good luck!

1

WHOLE NUMBERS



Dreamworks/The Kobal Collection

SECTIONS

- 1.1** Whole Numbers and Tables: Writing, Rounding, and Inequalities
- 1.2** Adding and Subtracting Whole Numbers
- 1.3** Multiplying Whole Numbers
- 1.4** Dividing Whole Numbers
- 1.5** Whole-Number Exponents and Powers of 10
- 1.6** Order of Operations
- 1.7** Average, Median, and Mode
- 1.8** Drawing and Interpreting Graphs

BASIC
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APPLICATION

The top ten grossing movies in the United States for 2004 are given in Table 1.1.

Table 1.1 Top Grossing Movies for 2004

<i>Shrek 2</i>	\$436,471,036
<i>Spider-Man 2</i>	\$373,377,893
<i>The Passion of the Christ</i>	\$370,274,604
<i>Meet the Fockers</i>	\$273,488,020
<i>The Incredibles</i>	\$258,938,368
<i>Harry Potter and the Prisoner of Azkaban</i>	\$249,358,727
<i>The Day After Tomorrow</i>	\$186,739,919
<i>The Bourne Supremacy</i>	\$176,049,130
<i>National Treasure</i>	\$169,378,371
<i>The Polar Express</i>	\$162,458,888

Source: Internet Movie Database.

Group Discussion

1. How many of the top grossing movies for 2004 were animated? How many were suitable for children 12 and under?
2. Which movies were comedies? Which were action-adventure?
3. How many of the top grossing movies won major Academy Awards? What is the relationship between top grossing movies and award-winning movies?

1.1 Whole Numbers and Tables: Writing, Rounding, and Inequalities

OBJECTIVES

1. Write word names from place value names and place value names from word names.
2. Write an inequality statement about two numbers.
3. Round a given whole number.
4. Read tables.

VOCABULARY

The **digits** are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

The **natural numbers (counting numbers)** are 1, 2, 3, 4, 5, and so on.

The **whole numbers** are 0, 1, 2, 3, 4, 5, and so on. Numbers larger than 9 are written in **place value name** by writing the digits in positions having standard **place value**.

Word names are written words that represent numerals. The word name of 213 is two hundred thirteen.

The symbols **less than**, $<$, and **greater than**, $>$, are used to compare two whole numbers that are not equal. So, $11 < 15$, and $21 > 5$.

To **round** a whole number means to give an approximate value. The symbol \approx means "approximately equal to."

A **table** is a method of displaying data in an **array** using a horizontal and vertical arrangement to distinguish the type of data. A **row** of a table is a horizontal line of a table and reads left to right across the page. A **column** of a table is a vertical line of a table and reads up or down the page. For example, in Table 1.2 the number "57" is in row 3 and column 2.

Column 2

	134	56	89	102
	14	116	7	98
Row 3	65	57	12	67
	23	56	7	213

Table 1.2

How & Why

OBJECTIVE 1

Write word names from place value names and place value names from word names.

In our written whole number system (called the Hindu-Arabic system), digits and commas are the only symbols used. This system is a positional base 10 (decimal) system. The location of the digit determines its value, from right to left. The first three place value names are one, ten, and hundred. See Figure 1.1.

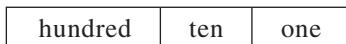


Figure 1.1

For the number 583,

3 is in the ones place, so it contributes 3 ones, or 3, to the value of the number,

8 is in the tens place, so it contributes 8 tens, or 80, to the value of the number,

5 is in the hundreds place, so it contributes 5 hundreds, or 500, to the value of the number.

So 583 is 5 hundreds + 8 tens + 3 ones or $500 + 80 + 3$. These are called **expanded forms** of the number. The word name is five hundred eighty-three.

For numbers larger than 999, we use commas to separate groups of three digits. The first four groups are unit, thousand, million, and billion (Figure 1.2). The group on the far left may have one, two, or three digits. All other groups must have three digits. Within each group the names are the same (hundred, ten, and one).

hundred	ten	one	hundred	ten	one	hundred	ten	one	hundred	ten	one
billion			million			thousand			(unit)		

Figure 1.2

For 63,506,345,222 the group names are:

63 506 345 222
billion million thousand unit

The number is read “63 billion, 506 million, 345 thousand, 222.” The word *units* for the units group is not read. The complete word name is sixty-three billion, five hundred six million, three hundred forty-five thousand, two hundred twenty-two.

To write the word name from a place value name

1. From left to right, write the word name for each set of three digits followed by the group name (except units).
2. Insert a comma after each group name.

CAUTION

The word *and* is not used to write names of whole numbers. So write: three hundred ten, **NOT** three hundred and ten, also one thousand, two hundred twenty-three, **NOT** one thousand and two hundred twenty-three.

To write the place value name from the word name of a number, we reverse the previous process. First identify the group names and then write each group name in the place value name. Remember to write a 0 for each missing place value. Consider

three billion, two hundred thirty-five million, nine thousand, four hundred thirteen

three *billion*, two hundred thirty-five *million*, Identify the group names.
nine *thousand*, four hundred thirteen (*Hint*: Look for the commas.)

3 billion, 235 million, 9 thousand, 413 Write the place value name for each group.

3,235,009,413 Drop the group names. Keep all commas. Zeros must be inserted to show that there are no hundreds or tens in the thousands group.

To write a place value name from a word name

1. Identify the group names.
2. Write the three-digit number before each group name, followed by a comma. (The first group, on the left, may have fewer than three digits.) It is common to omit the comma in 4-digit numerals.

Numbers like 81,000,000,000, with all zeros following a single group of digits, are often written in a combination of place value notation and word name. The first set of digits on the left is written in place value notation followed by the group name. So 81,000,000,000 is written 81 billion.

Warm-Ups A–B

- A. Write the word name for 43,733,061.

- B. Write the word name for 8,431,619.

Examples A–B



DIRECTIONS: Write the word name.

STRATEGY: Write the word name of each set of three digits, from left to right, followed by the group name.

- A. Write the word name for 19,817,583.

19 Nineteen million,
817 eight hundred seventeen thousand
583 five hundred eighty-three

Write the word name for each group followed by the group name.

CAUTION

Do not write the word *and* when reading or writing a whole number.

The word name is nineteen million, eight hundred seventeen thousand, five hundred eighty-three.

- B. Write the word name for 9,382,059.

Nine million, three hundred eighty-two thousand, fifty-nine.

Warm-Ups C–F

- C. Write the place value name for twenty-two million, seventy-seven thousand, four hundred eleven.

- D. Write the place value name for 74 thousand.

- E. Write the place value name for seven thousand fifteen.

- F. The purchasing agent for the Russet Corporation also received a bid of twenty-one thousand, five hundred eighteen dollars for a supply of paper. What is the place value name of the bid that she will include in her report to her superior?

Examples C–F



BASIC Mathematics Now™

DIRECTIONS: Write the place value name.

STRATEGY: Write the 3-digit number for each group followed by a comma.

- C. Write the place value name for four million, seventy-six thousand, two hundred sixty-five.

4, Millions group.
076, Thousands group. (Note that a zero is inserted on the left to fill out the three digits in the group.)
265 Units group.

The place value name is 4,076,265.

- D. Write the place value name for 346 million.

The place value name is 346,000,000. Replace the word *million* with six zeros.

- E. Write the place value name for four thousand fifty-three. Note that the comma is omitted.

The place value name is 4053.

- F. The purchasing agent for the Russet Corporation received a telephone bid of forty-three thousand fifty-one dollars as the price of a new printing press. What is the place value name of the bid that she will include in her report to her superior?

forty-three thousand, fifty-one
43, 051

The place value name she reports is \$43,051.

Answers to Warm-Ups

- A. forty-three million, seven hundred thirty-three thousand, sixty-one
B. eight million, four hundred thirty-one thousand, six hundred nineteen
C. 22,077,411 D. 74,000 E. 7015
F. The place value name she reports is \$21,518

How & Why

OBJECTIVE 2

Write an inequality statement about two numbers.

If two whole numbers are not equal, then the first is either *less than* or *greater than* the second. Look at the number line (or ruler) in Figure 1.3.

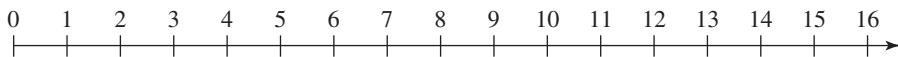


Figure 1.3

Given two numbers on a number line or ruler, the number on the right is the larger. For example,

- | | |
|----------|---|
| $9 > 7$ | 9 is to the right of 7, so 9 is greater than 7. |
| $11 > 1$ | 11 is to the right of 1, so 11 is greater than 1. |
| $14 > 8$ | 14 is to the right of 8, so 14 is greater than 8. |
| $13 > 0$ | 13 is to the right of 0, so 13 is greater than 0. |

Given two numbers on a number line or ruler, the number on the left is the smaller. For example,

- | | |
|-----------|---|
| $3 < 9$ | 3 is to the left of 9, so 3 is less than 9. |
| $5 < 12$ | 5 is to the left of 12, so 5 is less than 12. |
| $1 < 9$ | 1 is to the left of 9, so 1 is less than 9. |
| $10 < 14$ | 10 is to the left of 14, so 10 is less than 14. |

For larger numbers, imagine a longer number line. Notice how the points in the symbols $<$ and $>$ point to the smaller of the two numbers. For example,

$$\begin{aligned}181 &< 715 \\87 &> 56 \\5028 &> 5026\end{aligned}$$

To write an inequality statement about two numbers

1. Insert $<$ between the numbers if the number on the left is smaller.
2. Insert $>$ between the numbers if the number on the left is larger.

Examples G–H

BASIC
Mathematics Now™

DIRECTIONS: Insert $<$ or $>$ to make a true statement.

STRATEGY: Imagine a number line. The smaller number is on the left. Insert the symbol that points to the smaller number.

- G. Insert the appropriate inequality symbol: $62 \quad 83$

$$62 < 83$$

- H. Insert the appropriate inequality symbol: $3514 \quad 2994$

$$3514 > 2994$$

Warm-Ups G–H

- G. Insert the appropriate inequality symbol:
 $164 \quad 191$

- H. Insert the appropriate inequality symbol:
 $6318 \quad 6269$

Answers to Warm-Ups

G. $<$ H. $>$

How & Why

OBJECTIVE 3

Round a given whole number.

Many numbers that we see in daily life are approximations. These are used to indicate the approximate value when it is believed that the exact value is not important to the discussion. So attendance at a political rally may be stated at 15,000 when it was actually 14,783. The amount of a deficit in the budget may be stated as \$2,000,000 instead of \$2,067,973. In this chapter, we use these approximations to estimate the outcome of operations with whole numbers. The symbol \approx , read “approximately equal to,” is used to show the approximation. So $\$2,067,973 \approx \$2,000,000$.

We approximate numbers by **rounding**. The number line can be used to see how whole numbers are rounded. Suppose we wish to round 57 to the nearest ten. See Figure 1.4

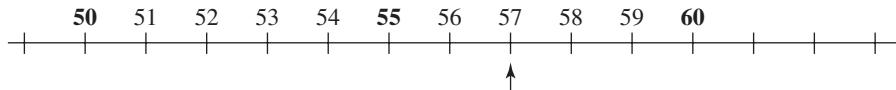


Figure 1.4

The arrow under the 57 is closer to 60 than to 50. We say “to the nearest ten, 57 rounds to 60.”

We use the same idea to round any number, although we usually make only a mental image of the number line. The key question is: Is this number closer to the smaller rounded number or to the larger one? Practically, we need to determine only if the number is more or less than half the distance between the rounded numbers.

To round 47,472 to the nearest thousand without a number line, draw an arrow under the digit in the thousands place.

47,472
↑

Because 47,472 is between 47,000 and 48,000, we must decide which number it is closer to. Because 47,500 is halfway between 47,000 and 48,000 and because $47,472 < 47,500$, we conclude that 47,472 is less than halfway to 48,000.

Whenever the number is less than halfway to the larger number, we choose the smaller number.

$47,472 \approx 47,000$ 47,472 is closer to 47,000 than to 48,000.

To round a number to a given place value

1. Draw an arrow under the given place value.
2. If the digit to the right of the arrow is 5, 6, 7, 8, or 9, add one to the digit above the arrow. (Round to the larger number.)
3. If the digit to the right of the arrow is 0, 1, 2, 3, or 4, do not change the digit above the arrow. (Round to the smaller number.)
4. Replace all the digits to the right of the arrow with zeros.

Examples I–J



DIRECTIONS: Round to the indicated place value.

STRATEGY: Choose the larger number if the digit to the right of the round-off place is 5 or greater, otherwise, choose the smaller number.

- I. Round 347,366 to the nearest ten thousand.

347,366 Draw an arrow under the ten-thousands place.
↑

350,000 The digit to the right of the arrow is 7. Because $7 \geq 5$, choose the larger number.

So $347,366 \approx 350,000$.

- J. Round the numbers to the indicated place value.

Number	Ten	Hundred	Thousand
862,548	862,550	862,500	863,000
35,632	35,630	35,600	36,000

Warm-Ups I–J

- I. Round 89,457 to the nearest ten.

- J. Round the numbers to the indicated place value.

Number	Ten	Hundred	Thousand
725,936			
68,478			

How & Why

OBJECTIVE 4

Read tables.

Data are often displayed in the form of a *table*. We see tables in the print media, in advertisements, and in business presentations. Reading a table involves finding the correct *column* and *row* that describes the needed information, and then reading the data at the intersection of that column and that row.

Table 1.3 Student Course Enrollment

Class	Mathematics	English	Science	Humanities
Freshman	950	1500	500	1200
Sophomore	600	700	650	1000
Junior	450	200	950	1550
Senior	400	250	700	950

For example, in Table 1.3, to find the number of sophomores who take English, find the column headed English and the row headed Sophomore and read the number at the intersection.

The number of sophomores taking English is 700.

We can use the table to compare enrollments by class. For instance, are more seniors or sophomores taking science? From the table we see that 650 sophomores are taking science and 700 seniors are taking science. Since $700 > 650$, more seniors than sophomores are taking science.

Answers to Warm-Ups

I. 89,460

J.

Number	Ten	Hundred	Thousand
725,936	725,940	725,900	726,000
68,478	68,480	68,500	68,000

Warm-Up K

- K. Use the table in Example K to answer the questions.
- Which location has the lowest-priced home sold?
 - Round the average price of a home sold in S.E. Portland to the nearest thousand.
 - Which location has the higher price for a home sold, N.E. Portland or S.E. Portland?

Example K BASIC Mathematics Now™

DIRECTIONS: Answer the questions associated with the table.

STRATEGY: Examine the rows and columns of the table to determine the values that are related.

- K. This table shows the value of homes sold in the Portland metropolitan area for a given month.

Values of Houses Sold

Location	Lowest	Highest	Average
N. Portland	\$86,000	\$258,500	\$184,833
N.E. Portland	\$78,000	\$220,000	\$165,091
S.E. Portland	\$82,000	\$264,000	\$173,490
Lake Oswego	\$140,000	\$1,339,000	\$521,080
W. Portland	\$129,500	\$799,000	\$354,994
Beaverton	\$98,940	\$665,000	\$293,737

- In which location was the highest-priced home sold?
- Which area has the highest average sale price?
- Round the highest price of a house in Beaverton to the nearest hundred thousand.

- We look at the Highest column for the largest entry. It is \$1,339,000, which is in the fourth row. So Lake Oswego is the location of the highest-priced home sold.
- Looking at the Average column for the largest entry, we find \$521,080 in the fourth row. So Lake Oswego has the largest average sale price.
- Looking at the Highest column and the sixth row, we find \$665,000. So the highest price of a house in Beaverton is \$700,000, rounded to the nearest hundred thousand.

Answers to Warm-Ups

- K. 1. N.E. Portland has the lowest-priced home sold.
2. The rounded price is \$173,000.
3. S.E. Portland has the higher price.

Exercises 1.1

OBJECTIVE 1

Write word names from place value names and place value names from word names.

A Write the word names of each of these numbers.

1. 574

2. 391

3. 890

4. 340

5. 7020

6. 66,086

Write the place value name.

7. Fifty-seven

8. Thirty-four

9. Nine thousand, five hundred

10. Nine thousand, five

11. 100 million

12. 493 thousand

B Write the word name of each of these numbers.

13. 27,690

14. 27,069

15. 207,690

16. 270,069

17. 45,000,000

18. 870,000

Write the place value name.

19. Three hundred fifty-nine thousand, eight hundred

20. Three hundred fifty-nine thousand, eight

21. Twenty-two thousand, five hundred seventy

22. Twenty-three thousand, four hundred seventy-seven

23. Seventy-six billion

24. Nine hundred thousand, nine

OBJECTIVE 2

Write an inequality statement about two numbers.

A Insert $<$ or $>$ between the numbers to make a true statement.

25. 15 31

26. 53 49

27. 72 45

28. 72 81

B

29. 246 251

30. 212 208

31. 7470 7850

32. 2751 2693

OBJECTIVE 3

Round a given whole number.

A Round to the indicated place value.

33. 836 (ten)

34. 684 (ten)

35. 1468 (hundred)

36. 3450 (hundred)

B

Number	Ten	Hundred	Thousand	Ten Thousand
--------	-----	---------	----------	--------------

37. 607,546

38. 689,377

39. 7,635,753

40. 4,309,498

OBJECTIVE 4

Read tables.

A Exercises 41–45. The percent of people who do and do not exercise regularly, broken down by income levels, is shown in the table below (Source: Centers for Disease Control and Prevention).

Regular Exercises by Income Level

Income	Does Exercise	Does Not Exercise
\$0–\$14,999	35%	65%
\$15–\$24,999	40%	60%
\$25–\$50,000	45%	55%
Over \$50,000	52%	48%

41. What percent of people in the income level of \$15–\$24,999 exercise regularly?
42. Which income level has the highest percent of regular exercisers?
43. Which income level has the highest percent of nonexercisers?
44. Which income level(s) have more than 50% nonexercisers?
45. Use words to describe the trend indicated in the table.

B Exercises 46–50. A profile of the homeless in 27 selected cities, according to data compiled by the U.S. Conference of Mayors for 2004, is given in the following table.

Composition of Homeless in Selected Cities

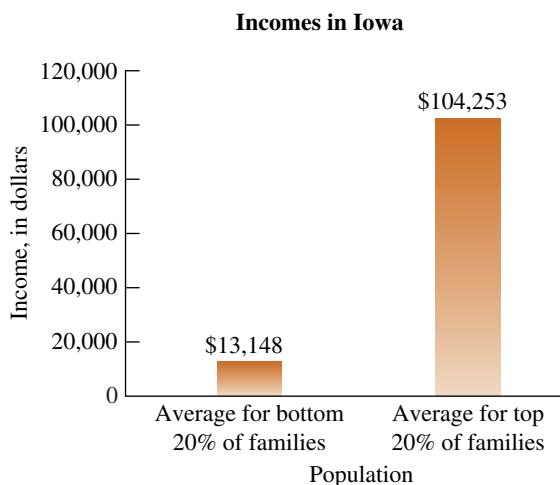
	% of Homeless in 1994	% of Homeless in 2004
Single men	48	41
Substance abusers	43	30
Families with children	39	40
Veterans	23	10
Unaccompanied youth	3	5
Severely mentally ill	26	23
Employed	19	17
Single women	11	14

- 46.** What was the decrease in the percent of homeless who are veterans over the 10-year period?
- 47.** Which of the categories increased over the 10-year period and which decreased?
- 48.** Explain why each column does not add up to 100%.
- 49.** What percent of the homeless were single men or single women in 1994? Did this percent increase or decrease in 2004?
- 50.** Did the total number of homeless increase or decrease over the 10-year period?

C Write the place value name.

- 51.** Six hundred fifty-six million, seven hundred thirty-two thousand, four hundred ten.
- 52.** Nine hundred five million, seven hundred seventy-seven

Exercises 53–54. The average income of the top 20% of the families and the bottom 20% of the families in Iowa is shown in the following figure.



- 53.** Write the word name for the average salary for the poor in Iowa.

- 54.** Write the word name for the average salary of the rich in Iowa.

Insert < or > between the numbers to make a true statement.

55. 4553 4525

56. 21,186 21,299

- 57.** What is the smallest 4-digit number?

- 58.** What is the largest 6-digit number?

Round to the indicated place value.

59. 81,634,981 (hundred thousand)

60. 62,078,991 (ten thousand)

- 61.** Round 63,749 to the nearest hundred. Round 63,749 to the nearest ten and then round your result to the nearest hundred. Why did you get a different result the second time? Which method is correct?

- 62.** Hazel bought a plasma flat screen television set for \$2495. She wrote a check to pay for it. What word name did she write on the check?

- 63.** Kimo bought a used Toyoto Camry for \$11,475 and wrote a check to pay for it. What word name did he write on the check?
- 65.** The Wisconsin Department of Natural Resources estimates that 276,400 mallard ducks stayed in the state to breed in 2004. Write the word name for the number of ducks.
- 67.** The purchasing agent for Print-It-Right received a telephone bid of thirty-six thousand, four hundred seven dollars as the price for a new printing press. What is the place value name for the bid?
- 69.** Ten thousand shares of the Income Fund of America sold for \$185,200. What is the value of the sale, to the nearest thousand dollars?
- 64.** The U.S. Fish and Wildlife Service estimates that salmon runs could be as high as 213,510 fish by 2007 on the Rogue River if new management practices are used in logging along the river. Write the word name for the number of fish.
- 66.** The U.S. Census Bureau estimates that the world population will exceed 6 billion, 815 million, 9 hundred thousand by 2010. Write the place value name for the world population.
- 68.** The Oak Ridge Missionary Baptist Church in Kansas City took out a building permit for \$2,659,500. Round the building permit price of the church to the nearest hundred thousand dollars.

Exercises 70–72. The table gives emissions estimates for volatile organic compounds, according to the Environmental Protection Agency.

Estimated Emissions of Volatile Organic Compounds
(in thousands of short tons)

1970	1980	1990	2000	2003
34,659	31,106	24,116	19,704	15,429

- 71.** Write the place value name for the number of short tons of emission of volatile organic compounds in 2003.

- 70.** Write the place value name for the number of short tons of emission of volatile organic compounds in 1980.

- 72.** What is the general trend in emissions of volatile organic compounds over the past 30 years?

Exercises 73–76. The following table gives the per capita personal income in the New England states according to the U.S. Bureau of Economic Analysis.

Per Capita Personal Income, 2003

Connecticut	43,173
Maine	28,831
Massachusetts	39,815
New Hampshire	34,702
Rhode Island	26,132
Vermont	33,671

- 76.** Does Vermont or New Hampshire have a larger per capita personal income?

- 73.** Write the word name of the per capita personal income in Maine.

- 74.** Round the per capita personal income in Massachusetts to the nearest thousand.

- 75.** Which state has the smallest per capita personal income?

- 77.** The distance from Earth to the sun was measured and determined to be 92,875,328 miles. To the nearest million miles, what is the distance?

- 78.** According to the National Cable Television Association, the top five pay-cable services for 2002–2003 were:

Network	Subscribers
The Disney Channel	84,000,000
HBO/Cinemax	39,000,000
Showtime/The Movie Channel	34,800,000
Encore	21,900,000
Starz!	12,300,000

Have any of these numbers been rounded? If so, explain how you know. Revise the table, rounding all figures to the nearest million.

Exercises 79–80. The number of marriages each month for a recent year, according to data from the U.S. Census Bureau is given in the table.

Number of Marriages Per Month

Month	Number of Marriages, in Thousands	Month	Number of Marriages, in Thousands
January	110	July	235
February	155	August	239
March	118	September	225
April	172	October	231
May	241	November	171
June	242	December	184

- 79.** Rewrite the information ordering the months from most number of marriages to least number of marriages. Use place value notation when writing the number of marriages.

- 80.** Do you think the number of marriages have been rounded? If so to what place value?

Exercises 81–82. The six longest rivers in the United States are as follows:

Arkansas	1459 miles
Colorado	1450 miles
Mississippi	2340 miles
Missouri	2315 miles
Rio Grande	1900 miles
Yukon	1079 miles

- 81.** List the rivers in order of increasing length.

- 82.** Do you think any of the river lengths have been rounded? If so, which ones?

- 83.** The state motor vehicle department estimated the number of licensed automobiles in the state to be 2,376,000, to the nearest thousand. A check of the records indicated that there were actually 2,376,499. Was their estimate correct?

- 84.** The total land area of Earth is approximately 52,425,000 square miles. What is the land area to the nearest million square miles?

Exercises 85–86. The following figure lists some nutritional facts about two brands of peanut butter.

Skippy® Super Chunk	
Nutrition Facts	
Serving Size 2 tbsp (32g) Servings Per Container about 15	
Amount Per Serving	% Daily Values
Calories 190	Calories from Fat 140
Total Fat 17g	26%
Saturated Fat 3.5g	17%
Cholesterol 0mg	0%
Sodium 140mg	6%
Total Carbohydrate 7g	2%
Dietary Fiber 2g	8%
Sugars 3g	
Protein 7g	

Jif® Creamy	
Simply Jif contains 2g sugar per serving. Regular Jif contains 3g sugar per serving	
Nutrition Facts	
Serving Size 2 tbsp (31g)	
Servings Per Container about 16	
Amount Per Serving	% Daily Values
Calories 190	Calories from Fat 130
Total Fat 16g	25%
Saturated Fat 3g	16%
Cholesterol 0mg	0%
Sodium 65mg	3%
Total Carbohydrate 6g	2%
Dietary Fiber 2g	9%
Sugars 2g	
Protein 8g	

- 85.** List the categories of nutrients for which Jif has fewer of the nutrients than Skippy.
- 86.** Round the sodium content in each brand to the nearest hundred. Do the rounded numbers give a fair comparison of the amount of sodium in the brands?

Exercises 87–90 relate to the chapter application. See Table 1.1, page 1.

- 87.** Write the word name for the dollar amount taken in by *Shrek 2* in 2004.
- 88.** Round the amount taken in by *The Bourne Supremacy* to the nearest hundred thousand.
- 89.** Round the amount taken in by *The Incredibles* to the nearest million dollars.
- 90.** Do the numbers in Table 1.1 appear to be rounded?

STATE YOUR UNDERSTANDING

- 91.** Explain why “base ten” is a good name for our number system.
- 92.** Explain what the digit 9 means in 295,862.
- 93.** What is rounding? Explain how to round 87,452 to the nearest thousand and to the nearest hundred.

CHALLENGE

- 94.** What is the place value for the digit 5 in 3,456,709,230,000?
- 95.** Write the word name for 5,326,901,570,000.
- 96.** Arrange the following numbers from smallest to largest: 1234, 1342, 1432, 1145, 1243, 1324, and 1229.
- 98.** Round 967,345 to the nearest hundred thousand.
- 97.** What is the largest value of X that makes $2X56 > 2849$ false?
- 99.** Round 49,774 to the nearest hundred thousand.

GROUP WORK

- 100.** Two other methods of rounding are called the “odd/even method” and “truncating.” Find these methods and be prepared to explain them in class.
(Hint: Try the library or talk to science and business instructors.)

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1.2 Adding and Subtracting Whole Numbers

VOCABULARY

Addends are the numbers that are added. In $9 + 20 + 3 = 32$, the addends are 9, 20, and 3.

The result of adding is called the **sum**. In $9 + 20 + 3 = 32$, the sum is 32.

The result of subtracting is called the **difference**. So in $62 - 34 = 28$, 28 is the difference.

A **polygon** is any closed figure whose sides are line segments.

The **perimeter** of a polygon is the distance around the outside of the polygon.

OBJECTIVES

- Find the sum of two or more whole numbers.
- Find the difference of two whole numbers.
- Estimate the sum or difference of whole numbers.
- Find the perimeter of a polygon.

How & Why

OBJECTIVE 1

Find the sum of two or more whole numbers.

When Jose graduated from high school he received cash gifts of \$50, \$20, and \$25. The total number of dollars received is found by adding the individual gifts. The total number of dollars he received is 95. In this section we review the procedure for adding and subtracting whole numbers.

The addition facts and place value are used to add whole numbers written with more than one digit. Let's use this to find the sum of the cash gifts that Jose received. We need to find the sum of

$$50 + 20 + 25$$

By writing the numbers in expanded form and putting the same place values in columns it is easy to add.

$$\begin{array}{r} 50 = 5 \text{ tens} + 0 \text{ ones} \\ 20 = 2 \text{ tens} + 0 \text{ ones} \\ +25 = \underline{2 \text{ tens} + 5 \text{ ones}} \\ \hline 9 \text{ tens} + 5 \text{ ones} = 95 \end{array}$$

So, $50 + 20 + 25 = 95$. Jose received \$95 in cash gifts.

Because each place can contain only a single digit, it is often necessary to rewrite the sum of a column.

$$\begin{array}{r} 77 = 7 \text{ tens} + 7 \text{ ones} \\ +16 = \underline{1 \text{ tens} + 6 \text{ ones}} \\ \hline 8 \text{ tens} + 13 \text{ ones} \end{array}$$

Because 13 ones is a 2-digit number it must be renamed:

$$\begin{aligned} 8 \text{ tens} + 13 \text{ ones} &= 8 \text{ tens} + 1 \text{ ten} + 3 \text{ ones} \\ &= 9 \text{ tens} + 3 \text{ ones} \\ &= 93 \end{aligned}$$

So the sum of 77 and 16 is 93.

The common shortcut is shown in the following sum. To add $497 + 307 + 135$, write the numbers in a column.

$$\begin{array}{r} 497 \quad \text{Written this way, the digits in the ones, tens, and hundreds} \\ 307 \quad \text{places are aligned.} \\ +135 \\ \hline \end{array}$$

I

$$\begin{array}{r}
 497 \\
 307 \\
 +135 \\
 \hline
 9
 \end{array}$$

Add the digits in the ones column: $7 + 7 + 5 = 19$. Write 9 and carry the 1 (1 ten) to the tens column.

II

$$\begin{array}{r}
 497 \\
 307 \\
 +135 \\
 \hline
 39
 \end{array}$$

Add the digits in the tens column: $1 + 9 + 0 + 3 = 13$. Write 3 and carry the 1 (10 tens = 1 hundred) to the hundreds column.

III

$$\begin{array}{r}
 497 \\
 307 \\
 +135 \\
 \hline
 939
 \end{array}$$

Add the digits in the hundreds column: $1 + 4 + 3 + 1 = 9$.

To add whole numbers

1. Write the numbers in a column so that the place values are aligned.
2. Add each column, starting with the ones (or units) column.
3. If the sum of any column is greater than nine, write the ones digit and “carry” the tens digit to the next column.

Warm-Ups A–C

Examples A–C



BASIC
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DIRECTIONS: Add.

STRATEGY: Write the numbers in a column. Add the digits in the columns starting on the right. If the sum is greater than 9, “carry” the tens digit to the next column.

A. Add: $784 + 537$

I

$$\begin{array}{r}
 784 \\
 +537 \\
 \hline
 1221
 \end{array}$$

Add the numbers in the ones column. $4 + 7 = 11$. Because the sum is greater than 9, write 1 in the ones column and carry the 1 to the tens column. Add the numbers in the tens column. $1 + 8 + 3 = 12$. Write 2 in the tens column and carry the 1 to the hundreds column. Add the numbers in the hundreds column. $1 + 6 + 5 = 12$. Because all columns have been added there is no need to carry.

B. Add 63, 4018, 98, and 5.
Round the sum to the nearest ten.

A Add 68, 714, 7, and 1309. Round the sum to the nearest ten.

I 2

$$\begin{array}{r}
 68 \\
 714 \\
 7 \\
 +1309 \\
 \hline
 2098
 \end{array}$$

When writing in a column, make sure the place values are aligned properly.

$2098 \approx 2100$ Round to the nearest ten.

C. Add: $8361 + 6217 + 515 + 3932 + 9199$

CALCULATOR EXAMPLE



C. Add: $7659 + 518 + 7332 + 4023 + 1589$

Calculators are programmed to add numbers just as we have been doing by hand. Simply enter the exercise as it is written horizontally and the calculator will do the rest.

The sum is 21,121.

How & Why

OBJECTIVE 2

Find the difference of two whole numbers.

Marcia went shopping with \$78. She made purchases totaling \$53. How much money does she have left? Finding the difference in two quantities is called subtraction. When we subtract \$53 from \$78 we get \$25.

Subtraction can be thought of as finding the missing addend in an addition exercise. For instance, $9 - 5 = ?$ asks $5 + ? = 9$. Because $5 + 4 = 9$, we know that $9 - 5 = 4$. Similarly, $47 - 15 = ?$ asks $15 + ? = 47$. Because $15 + 32 = 47$, we know that $47 - 15 = 32$.

For larger numbers, such as $875 - 643$, we take advantage of the column form and expanded notation to find the missing addend in each column.

$$\begin{array}{r} 875 = 8 \text{ hundreds} + 7 \text{ tens} + 5 \text{ ones} \\ -643 = \underline{6 \text{ hundreds}} + 4 \text{ tens} + 3 \text{ ones} \\ \hline 2 \text{ hundreds} + 3 \text{ tens} + 2 \text{ ones} = 232 \end{array}$$

Check by adding:

$$\begin{array}{r} 643 \\ +232 \\ \hline 875 \end{array}$$

So, $875 - 643 = 232$.

Now consider the difference $672 - 438$. Write the numbers in column form.

$$\begin{array}{r} 672 = 6 \text{ hundreds} + 7 \text{ tens} + 2 \text{ ones} \\ -438 = \underline{4 \text{ hundreds}} + 3 \text{ tens} + 8 \text{ ones} \\ \hline \end{array}$$

Here we cannot subtract 8 ones from 2 ones, so we rename by “borrowing” one of the tens from the 7 tens (1 ten = 10 ones) and adding the 10 ones to the 2 ones.

$$\begin{array}{r} \text{6 tens} \quad \text{12 ones} \\ 672 = 6 \text{ hundreds} + 7 \text{ tens} + 2 \text{ ones} \\ -438 = \underline{4 \text{ hundreds}} + 3 \text{ tens} + 8 \text{ ones} \\ \hline 2 \text{ hundreds} + 3 \text{ tens} + 4 \text{ ones} = 234 \end{array}$$

Check by adding:

$$\begin{array}{r} 438 \\ +234 \\ \hline 672 \end{array}$$

We generally don’t bother to write the expanded form when we subtract. We show the shortcut for borrowing in the examples.

To subtract whole numbers

1. Write the numbers in a column so that the place values are aligned.
2. Subtract in each column, starting with the ones (or units) column.
3. When the numbers in a column cannot be subtracted, borrow 1 from the next column and rename by adding 10 to the upper digit in the current column and then subtract.

Warm-Ups D–H

Examples D–H



BASIC
Mathematics = Now™

D. Subtract: $69 - 26$

$$\begin{array}{r} 87 \\ -46 \\ \hline 41 \end{array}$$

Subtract the ones column: $7 - 6 = 1$.
Subtract the tens column: $8 - 4 = 4$.

CHECK:

$$\begin{array}{r} 41 \\ +46 \\ \hline 87 \end{array}$$

So $87 - 46 = 41$.

E. Find the difference:
 $823 - 476$

$$\begin{array}{r} 216 \\ 836 \\ -379 \\ \hline 7 \end{array}$$

In order to subtract in the ones column we borrow 1 ten (10 ones) from the tens column and rename the ones ($10 + 6 = 16$).

$$\begin{array}{r} 712 \\ 216 \\ 836 \\ -379 \\ \hline 457 \end{array}$$

Now in order to subtract in the tens column, we must borrow 1 hundred (10 tens) from the hundreds column and rename the tens ($10 + 2 = 12$).

CHECK:

$$\begin{array}{r} 379 \\ +457 \\ \hline 836 \end{array}$$

So $836 - 379 = 457$.

F. Subtract 495 from 7100.

$$\begin{array}{r} 7300 \\ -759 \\ \hline 210 \end{array}$$

We cannot subtract in the ones column, and since there are 0 tens, we cannot borrow from the tens column.

$$\begin{array}{r} 7300 \\ -759 \\ \hline 910 \end{array}$$

We borrow 1 hundred (1 hundred = 10 tens) from the hundreds place.

$$\begin{array}{r} 7300 \\ -759 \\ \hline 210 \end{array}$$

Now borrow 1 ten (1 ten = 10 ones). We can now subtract in the ones and tens columns but not in the hundreds column.

$$\begin{array}{r} 612910 \\ 210 \\ 7300 \\ -759 \\ \hline 6541 \end{array}$$

Now borrow 1 thousand (1 thousand = 10 hundreds). We can now subtract in every column.

CHECK:

$$\begin{array}{r} 6541 \\ +759 \\ \hline 7300 \end{array}$$

Let's try Example F again using a technique called "reverse adding." Just ask yourself, "What do I add to 759 to get 7300?"

Answers to Warm-Ups
D. 43 E. 347 F. 6605

$\begin{array}{r} 7300 \\ - 759 \\ \hline \end{array}$	Begin with the ones column. 9 is larger than 0, so ask "What do I add to 9 to make 10?"
$\begin{array}{r} 7300 \\ - 759 \\ \hline 1 \end{array}$	Because $1 + 9 = 10$, we write the 1 in the ones column and carry the 1 over to the 5 to make 6. Now ask "What do I add to 6 to make 10?"
$\begin{array}{r} 7300 \\ - 759 \\ \hline 41 \end{array}$	Write 4 in the tens column and carry the 1 over to the 7 in the hundreds column. Now ask "What do I add to 8 to make 13?"
$\begin{array}{r} 7300 \\ - 759 \\ \hline 6541 \end{array}$	Write the 5 in the hundreds column. Finally, ask "What do I add to the carried 1 to make 7?"

The advantage of this method is that 1 is the largest amount carried, so most people can do this process mentally.

So $7300 - 759 = 6541$.

CALCULATOR EXAMPLE



G. Subtract 49,355 from 82,979.

CAUTION

When a subtraction exercise is worded "Subtract A from B," it is necessary to reverse the order of the numbers. The difference is $B - A$.

Enter $82,979 - 49,355$.

The difference is 33,624.

H. Maxwell Auto is advertising a \$986 rebate on all new cars priced above \$15,000. What is the cost after rebate of a car originally priced at \$16,798?

STRATEGY: Because the price of the car is over \$15,000, we subtract the amount of the rebate to find the cost.

$$\begin{array}{r} 16,798 \\ - 986 \\ \hline 15,812 \end{array} \quad \text{CHECK: } \begin{array}{r} 15,812 \\ + 986 \\ \hline 16,798 \end{array}$$

The car costs \$15,812.

G. Subtract 55,766 from 67,188.

H. Maxwell Auto is also advertising a \$2138 rebate on all new cars priced above \$32,000. What is the cost after rebate of a car originally priced at \$38,971?

How & Why

OBJECTIVE 3

Estimate the sum or difference of whole numbers.

The sum or difference of whole numbers can be estimated by rounding each number to a specified place value and then adding or subtracting the rounded values. Estimating is useful to check to see if a calculated sum or difference is reasonable or when the exact sum is not needed. For instance, estimate the sum by rounding to the nearest thousand.

$$\begin{array}{r} 6359 \\ 3790 \\ 9023 \\ 4825 \\ + 899 \\ \hline \end{array} \quad \begin{array}{r} 6000 \\ 4000 \\ 9000 \\ 5000 \\ + 1000 \\ \hline 25,000 \end{array} \quad \text{Round each number to the nearest thousand.}$$

The estimate of the sum is 25,000.

Answers to Warm-Ups

G. 11,422

H. The cost of the car is \$36,833.

Another estimate can be found by rounding each number to the nearest hundred.

6359	6400	Round each number to the nearest hundred.
3790	3800	
9023	9000	
4825	4800	
+ 899	+ 900	
		24,900

We can use the estimate to see if we added correctly. If a calculated sum is not close to the estimated sum, you should check the addition by re-adding. In this case the calculated sum, 24,896, is close to the estimated sums of 25,000 and 24,900.

Estimate the difference of two numbers by rounding each number. Subtract the rounded numbers.

8967	9000	Round each number to the nearest hundred.
- 5141	- 5100	Subtract.
		3900

The estimate of the difference is 3900. We use the estimate to see if the calculated difference is correct. If the calculated difference is not close to 3900, you should check the subtraction. In this case, the calculated difference is 3826, which is close to the estimate.

Warm-Ups I–M

- I. Estimate the sum by rounding each number to the nearest hundred:

$$643 + 72 + 422 + 875 + 32 + 91$$

- J. Estimate the difference of 43,981 and 11,765 by rounding to the nearest thousand.

Examples I–M BASIC Mathematics Now™

DIRECTIONS: Estimate the sum or difference.

STRATEGY: Round each number to the specified place value. Then add or subtract.

- I. Estimate the sum by rounding each number to the nearest hundred:

$$475 + 8795 + 976 + 6745 + 5288 + 12$$

475	500	Round each number to the nearest hundred.
8795	8800	With practice, this can be done mentally.
976	1000	
6745	6700	
5288	5300	
+ 12	+ 0	
		22,300

The estimated sum is 22,300.

- J. Estimate the difference of 87,456 and 37,921 by rounding to the nearest thousand.

87,000	Round each number to the nearest thousand.	
- 38,000		
		49,000

The estimated difference is 49,000.

Answers to Warm-Ups

- I. The estimated sum is 2100.
J. The estimated difference is 32,000.

- K.** Petulia subtracts 756 from 8245 and gets a difference of 685. Estimate the difference by rounding to the nearest hundred to see if Petulia is correct.

$$\begin{array}{r} 8200 \\ - 700 \\ \hline 7400 \end{array}$$

Round each to the nearest hundred.

The estimated answer is 7400 so Petulia is not correct. Apparently she did not align the place values correctly. Subtracting we find the correct answer.

$$\begin{array}{r} 711 \\ 113 \\ 315 \\ \hline 8245 \\ - 756 \\ \hline 7489 \end{array}$$

Petulia is not correct; the correct answer is 7489.

- L.** Joan and Eric have a budget of \$1200 to buy new furniture for their living room. They like a sofa that costs \$499, a love seat at \$449, and a chair at \$399. Round the prices to the nearest hundred dollars to estimate the cost of the items. Will they have enough money to make the purchases?

Sofa: \$499	\$500	Round each price to the nearest hundred.
Love seat: 499	400	
Chair: 399	+ 400	
		\$1300

The estimated cost, \$1300, is beyond their budget, so they will have to rethink the purchase.

- M.** The population of Alabama in 2005 was about 4,631,000 and the population of Mississippi was about 2,908,000. Estimate the difference in the populations by rounding each to the nearest hundred thousand.

Alabama: 4,631,000	4,600,000	Round each population
Mississippi: 2,908,000	- 2,900,000	to the nearest hundred thousand.
		1,700,000

So the estimated difference in populations is 1,700,000.

- K.** Carl does the following addition:
 $1230 + 7020 + 81 + 334$. He gets the sum of 19,690. Estimate the sum by rounding each addend to the nearest hundred, to see if his answer is correct.

- L.** Pete has budgeted \$1500 for new golf clubs. He likes the following items: driver, \$295; set of irons, \$425; putter, \$175; wedge, \$69; fairway woods, \$412. Round the prices to the nearest hundred dollars to estimate the cost of the items. Will he have enough money to make the purchases?

- M.** The population of Missouri in 2005 was about 5,718,000 and the population of Utah was about 2,411,000. Estimate the difference in the populations by rounding each to the nearest hundred thousand.

How & Why

OBJECTIVE 4

Find the perimeter of a polygon.

A polygon is a closed figure whose sides are line segments, such as rectangles, squares, and triangles (Figure 1.5). An expanded discussion of polygons can be found in Section 7.3.

Common polygons

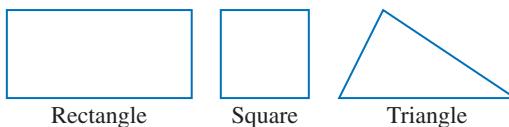


Figure 1.5

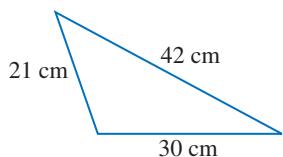
The perimeter of a polygon is the distance around the outside. To find the perimeter we add the lengths of the sides.

Answers to Warm-Ups

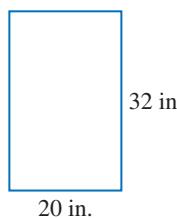
- K.** The estimated sum is 8600, so Carl is wrong. The correct answer is 8665.
L. The estimated cost is \$1400, so he should have enough money.
M. The estimated difference in populations is 3,300,000.

Warm-Ups N–O

- N. Find the perimeter of the triangle.



- O. Find the perimeter of the polygon.

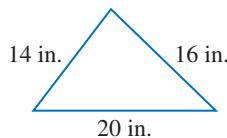


Examples N–O

DIRECTIONS: Find the perimeter of the polygon.

STRATEGY: Add up the lengths of all the sides.

- N. Find the perimeter of the triangle.

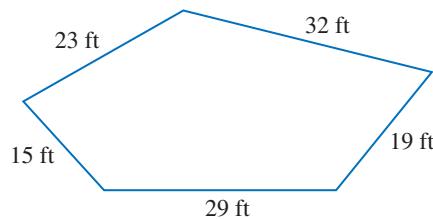


$$14 \text{ in.} + 16 \text{ in.} + 20 \text{ in.} = 50 \text{ in.}$$

Add the lengths of the sides.

The perimeter is 50 in.

- O. Find the perimeter of the polygon.



$$23 \text{ ft} + 32 \text{ ft} + 19 \text{ ft} + 29 \text{ ft} + 15 \text{ ft} = 118 \text{ ft}$$

The perimeter is 118 ft.

Answers to Warm-Ups

N. The perimeter is 93 cm.

O. The perimeter is 104 in.

Exercises 1.2

OBJECTIVE 1

Find the sum of two or more whole numbers.

A Add.

1. $75 + 38$

2. $55 + 27$

3. $748 + 231$

4. $456 + 328$

5.
$$\begin{array}{r} 212 \\ +495 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 364 \\ +537 \\ \hline \end{array}$$

7. When you add 36 and 77, the sum of the ones column is 13. You must carry the _____ to the tens column.

8. In $463 + 385$ the sum is X48. The value of X is _____.

B Add.

9. $624 + 4815 + 298$

10. $783 + 5703 + 529$

11. $7 + 85 + 607 + 5090$

12. $3 + 80 + 608 + 7050$

13. $2795 + 3643 + 7055 + 4004$ (Round sum to the nearest hundred.)

14. $6732 + 9027 + 5572 + 3428$ (Round sum to the nearest hundred.)

OBJECTIVE 2

Find the difference of two whole numbers.

A Subtract.

15.
$$\begin{array}{r} 7 \text{ hundreds} + 9 \text{ tens} + 8 \text{ ones} \\ -3 \text{ hundreds} + 9 \text{ tens} + 5 \text{ ones} \\ \hline \end{array}$$

16.
$$\begin{array}{r} 6 \text{ hundreds} + 3 \text{ tens} + 5 \text{ ones} \\ -4 \text{ hundreds} + 2 \text{ tens} + 4 \text{ ones} \\ \hline \end{array}$$

17. $406 - 72$

18. $764 - 80$

19. $876 - 345$

20. $848 - 622$

21. When subtracting $73 - 18$, you can borrow 1 from the 7. The value of the borrowed 1 is _____ ones.

22. To subtract $526 - 462$, you can borrow from the _____ column to subtract in the _____ column.

B Subtract.

23. $821 - 347$

24. $752 - 359$

25. $300 - 164$

26. $700 - 467$

27. $8769 - 4073$ (Round difference to the nearest hundred.)

28. $9006 - 6971$ (Round difference to the nearest hundred.)

OBJECTIVE 3

Estimate the sum or difference of whole numbers.

A Estimate the sum by rounding each number to the nearest hundred.

29. $345 + 782$

30. $495 + 912$

31.
$$\begin{array}{r} 3411 \\ 2001 \\ +4561 \\ \hline \end{array}$$

32.
$$\begin{array}{r} 5467 \\ 3811 \\ +2199 \\ \hline \end{array}$$

Estimate the difference by rounding each number to the nearest hundred.

33. $773 - 523$

34. $854 - 392$

35.
$$\begin{array}{r} 8678 \\ -3914 \\ \hline \end{array}$$

36.
$$\begin{array}{r} 6235 \\ -5991 \\ \hline \end{array}$$

B Estimate the sum by rounding each number to the nearest thousand.

37.
$$\begin{array}{r} 3209 \\ 7095 \\ 4444 \\ 2004 \\ +3166 \\ \hline \end{array}$$

38.
$$\begin{array}{r} 5038 \\ 4193 \\ 2121 \\ 5339 \\ +6560 \\ \hline \end{array}$$

39.
$$\begin{array}{r} 45,902 \\ 33,333 \\ 57,700 \\ +23,653 \\ \hline \end{array}$$

40.
$$\begin{array}{r} 12,841 \\ 29,671 \\ 21,951 \\ +73,846 \\ \hline \end{array}$$

Estimate the difference by rounding each number to the nearest thousand.

41.
$$\begin{array}{r} 8753 \\ -4067 \\ \hline \end{array}$$

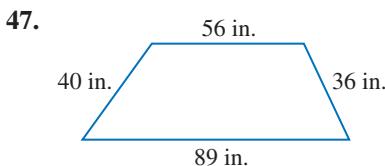
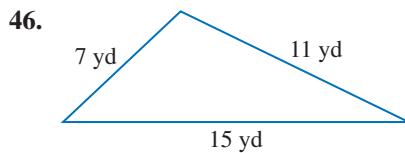
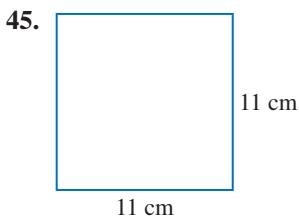
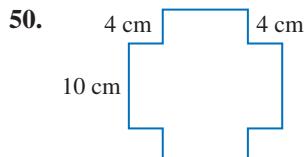
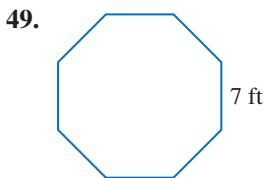
42.
$$\begin{array}{r} 7661 \\ -3089 \\ \hline \end{array}$$

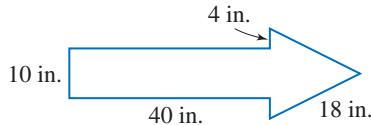
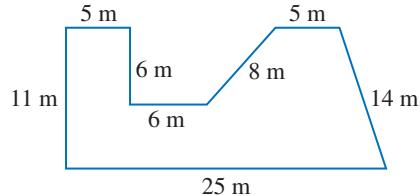
43.
$$\begin{array}{r} 65,808 \\ -32,175 \\ \hline \end{array}$$

44.
$$\begin{array}{r} 92,150 \\ -67,498 \\ \hline \end{array}$$

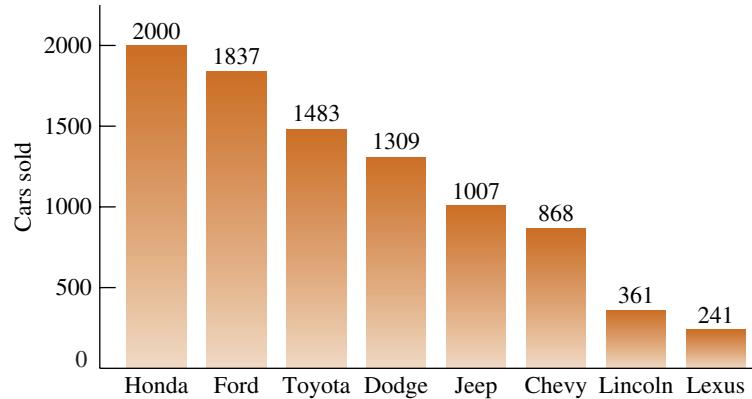
OBJECTIVE 4

Find the perimeter of a polygon.

A Find the perimeter of the following polygons.**B**

51.**52.**

C Exercises 53–58 refer to the sales chart, which gives the distribution of car sales among dealers in Wisconsin.



- 53.** What is the total number of Fords, Toyotas, and Lexuses sold?
- 54.** What is the total number of Chevys, Lincolns, Dodges, and Hondas sold?
- 55.** How many more Hondas are sold than Fords?
- 56.** How many more Toyotas are sold than Jeeps?
- 57.** What is the total number sold of the three best-selling cars?
- 58.** What is the difference in cars sold between the best-selling car and the least-selling car?
- 59.** The biologist at the Bonneville fish ladder counted the following number of coho salmon during a one-week period: Monday, 1046; Tuesday, 873; Wednesday, 454; Thursday, 1156; Friday, 607; Saturday, 541; and Sunday, 810. How many salmon went through the ladder that week? How many more salmon went through the ladder on Tuesday than on Saturday?
- 60.** Michelle works the following addition problem, $345 + 672 + 810 + 921 + 150$, and gets a sum of 1898. Estimate the answer by rounding each addend to the nearest hundred to see if Michelle's answer is reasonable. If not, find the correct sum.
- 61.** Ralph works the following subtraction problem, $10,034 - 7959$, and gets a difference of 2075. Estimate the answer by rounding each number to the nearest thousand to see if Ralph's answer is reasonable. If not, find the correct difference.
- 62.** The state of Alaska has an area of 570,374 square miles, or 365,039,104 acres. The state of Texas has an area of 267,277 square miles, or 171,057,280 acres. Estimate the difference in the areas using square miles rounded to the nearest ten thousand. Estimate the difference in the areas using acres rounded to the nearest million.
- 63.** Philipe buys a refrigerator for \$376, an electric range for \$482, a dishwasher for \$289, and a microwave oven for \$148. Estimate the cost of the items by rounding each cost to the nearest hundred dollars.

Exercises 64–66. The table gives the number of offences reported to law enforcement in Houston, TX, in 2004, according to the FBI's Uniform Crime Reports.

Violent Crimes	
Murder	131
Forcible rape	469
Robbery	4942
Aggravated assault	6018

Property Crimes	
Burglary	13,036
Larceny-theft	26,641
Motor vehicle theft	11,005
Arson	661

Exercises 67–69. A home furnace uses natural gas, oil, or electricity for the energy needed to heat the house. We humans get our energy for body heat and physical activity from calories in our food. Even when resting we use energy for muscle actions such as breathing, heartbeat, digestion, and other functions. If we consume more calories than we use up, we gain weight. If we consume less calories than we use, we lose weight. Some nutritionists recommend about 2270 calories per day for women aged 18–30 who are reasonably active.

Sasha, who is 22 years old, sets 2250 calories per day as her goal. She plans to have pasta with marinara sauce for dinner. The product labels shown here give the number of calories in each food.

Pasta	
Nutrition Facts	
Serving Size 2 oz (56g) dry (1/8 of the package)	
Servings Per Container 8	
Amount Per Serving	
Calories 200	Calories from Fat 10
	% Daily Value*
Total Fat 1 g	2%
Saturated Fat 0g	0%
Cholesterol 0mg	0%
Sodium 0mg	0%
Total Carbohydrate 41g	14%
Dietary Fiber 2g	8%
Sugars 3g	
Protein 7g	
Vitamin A 0%	• Vitamin C 0%
Calcium 0%	• Iron 10%
Thiamin 35%	• Riboflavin 15%
Niacin 15%	•
*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.	

Marinara Sauce	
Nutrition Facts	
Serving Size 1/2 cup (125g)	
Servings per Container approx 6	
Amount Per Serving	
Calories 60	Calories from Fat 20
	% Daily Value*
Total Fat 2g	3%
Saturated Fat 3g	0%
Cholesterol 0mg	0%
Sodium 370mg	15%
Total Carbohydrate 7g	2%
Dietary Fiber 2g	8%
Sugars 4g	
Protein 3g	
Vitamin A 15%	• Vitamin C 40%
Calcium 0%	• Iron 4%

67. If she eats two servings each of pasta and sauce, how many calories does she consume?

68. If Sasha has 550 more calories in bread, butter, salad, drink, and desert for dinner, how many total calories does she consume at dinner?

69. If Sasha keeps to her goal, how many calories could she have eaten at breakfast and lunch?
70. Super Bowl XIV was the highest attended Super Bowl, with a crowd of 103,985. Super Bowl XVII was the second highest attended, with a crowd of 103,667. The third highest attendance occurred at Super Bowl XI, with 103,438. What was the total attendance at all three Super Bowls? How much more was the highest attended game than the third highest attended game?
71. A forester counted 31,478 trees that are ready for harvest on a certain acreage. If Forestry Service rules require that 8543 mature trees must be left on the acreage, how many trees can be harvested?
72. The new sewer line being installed in downtown Chehalis will handle 475,850 gallons of refuse per minute. The old line handled 238,970 gallons per minute. How many more gallons per minute will the new line handle?
73. Fong's Grocery owes a supplier \$36,450. During the month, Fong's makes payments of \$1670, \$3670, \$670, and \$15,670. How much does Fong's still owe, to the nearest hundred dollars?
74. In the spring of 1989, an oil tanker hit a reef and spilled 10,100,000 gallons of oil off the coast of Alaska. The tanker carried a total of 45,700,000 gallons of oil. The oil that did not spill was pumped into another tanker. How many gallons of oil were pumped into the second tanker? Round to the nearest million gallons.
75. The median family income of a region is a way of estimating the middle income. Half the families in the region make more than the median income and the other half of the families make less. In 2004, HUD estimated that the median family income for San Francisco was \$95,000, and for Seattle it was \$71,900. What place value were these figures rounded to and how much higher was San Francisco's median income than Seattle's?
76. The Grand Canyon, Zion, and Bryce Canyon parks are found in the southwestern United States. Geologic changes over a billion years have created these formations and canyons. The chart shows the highest and lowest elevations in each of these parks. Find the change in elevation in each park. In which park is the change greatest and by how much?

Elevations at National Parks

	Highest Elevation	Lowest Elevation
Bryce Canyon	8500 ft	6600 ft
Grand Canyon	8300 ft	2500 ft
Zion	7500 ft	4000 ft

Exercises 77–80. The average number of murder victims per year in the United States who are related to the murderer, according to statistics from the FBI is given in the table.

Murder Victims Related to the Murderer

Wives	Husbands	Sons	Daughters	Fathers	Brothers	Mothers	Sisters
913	383	325	235	169	167	121	42

77. In an average year, how many more husbands killed their wives than wives killed their husbands?
78. In an average year, how many people killed their child?
79. In an average year, how many people killed a sibling?
80. In an average year, did more people kill their child or their parent?

Exercises 81–83. The table lists Ford's best sellers for 2004 according to the Blue Oval News.

Model	Units Sold
F-Series	939,511
Explorer	339,333
Taurus	248,148
Focus	208,339

81. How many more F-Series trucks were sold than Explorers?
82. What were the combined sales of the Explorer, Taurus, and Focus?
83. How many more F-Series trucks were sold than the next three top sellers combined?
84. In the National Football League, the salary cap is the absolute maximum amount that a club can spend on player salaries. For the 2004 season the salary cap was \$80,582,000, and for the 2005 season it was \$85,500,000. By how much did the salary cap increase from 2004 to 2005?

Exercises 85–86. The sub-Saharan region of Africa is the region most severely affected by AIDS. While it has only $\frac{1}{10}$ of the world's population, it has $\frac{2}{3}$ of the world's AIDS cases. The table gives statistics for people from the region living with HIV in 2003. (Source: Joint United Nations Programme on HIV/AIDS.)

Adults (15–49)	22 million
Adults and children (0–49)	25 million
Women (15–49)	13,100,000

85. How many children have HIV in sub-Saharan Africa?
86. Are there more men or women with HIV in sub-Saharan Africa?
87. Find the perimeter of a rectangular house that is 62 ft long and 38 ft wide.
88. A farmer wants to put a fence around a triangular plot of land that measures 5 km by 9 km by 8 km. How much fence does he need?
89. Blanche wants to sew lace around the edge of a rectangular tablecloth that measures 64 in. by 48 in. How much lace does she need, ignoring the corners and the seam allowances?
90. Annisa wants to trim a picture frame in ribbon. The outside of the rectangular frame is 25 cm by 30 cm. How much ribbon does she need, ignoring the corners?

STATE YOUR UNDERSTANDING

- 91.** Explain to a 6-year-old child why $15 - 9 = 6$.
- 92.** Explain to a 6-year-old child why $8 + 7 = 15$.
- 93.** Define and give an example of a sum.
- 94.** Define and give an example of a difference.

CHALLENGE

- 95.** Add the following numbers, round the sum to the nearest hundred, and write the word name for the rounded sum: one hundred sixty; eighty thousand, three hundred twelve; four hundred seventy-two thousand, nine hundred fifty-two; and one hundred forty-seven thousand, five hundred twenty-three.
- 96.** How much greater is six million, three hundred fifty-two thousand, nine hundred seventy-five than four million, seven hundred six thousand, twenty-three? Write the word name for the difference.
- 97.** Peter sells three Honda Civics for \$15,488 each, four Accords for \$18,985 each, and two Acuras for \$30,798 each. What is the total dollar sales for the nine cars? How many more dollars were paid for the four Accords than the three Civics?

Complete the sum or difference by writing in the correct digit wherever you see a letter.

$$\begin{array}{r} 5A68 \\ 241 \\ +10A9 \\ \hline B64C \end{array}$$

$$\begin{array}{r} 4A6B \\ -C251 \\ \hline 15D1 \end{array}$$

GROUP WORK

- 100.** Add and round to the nearest hundred.

$$\begin{array}{r} 14,657 \\ 3,766 \\ 123,900 \\ 569 \\ 54,861 \\ +346,780 \\ \hline \end{array}$$

- 101.** If Ramon delivers 112 loaves of bread to each store on his delivery route, how many stores are on the route if he delivers a total of 4368 loaves? (*Hint:* Subtract 112 loaves for each stop from the total number of loaves.) What operation does this perform? Make up three more examples and be prepared to demonstrate them in class.

Now round each addend to the nearest hundred and then add. Discuss why the answers are different. Be prepared to explain why this happens.

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Getting Ready for Algebra

VOCABULARY

An **equation** is a statement about numbers that says that two expressions are equal. Letters, called **variables** or **unknowns**, are often used to represent numbers.

OBJECTIVE

Solve an equation of the form $x + a = b$ or $x - a = b$, where a , b , and x are whole numbers.

How & Why

OBJECTIVE

Solve an equation of the form $x + a = b$ or $x - a = b$, where a , b , and x are whole numbers.

Examples of equations are

$$9 = 9 \quad 13 = 13 \quad 123 = 123 \quad 30 + 4 = 34 \quad 52 - 7 = 45$$

When variables are used an equation can look like this:

$$x = 7 \quad x = 10 \quad y = 18 \quad x + 5 = 13 \quad y - 8 = 23$$

An equation containing a variable can be true only when the variable is replaced by a specific number. For example,

$x = 7$ is true only when x is replaced by 7.

$x = 10$ is true only when x is replaced by 10.

$y = 18$ is true only when y is replaced by 18.

$x + 5 = 13$ is true only when x is replaced by 8, so that $8 + 5 = 13$.

$y - 8 = 23$ is true only when y is replaced by 31, so that $31 - 8 = 23$.

The numbers that make equations true are called *solutions*. Solutions of equations, such as $x - 7 = 12$, can be found by trial and error, but let's develop a more practical way.

Addition and subtraction are inverse, or opposite, operations. For example, if 14 is added to a number and then 14 is subtracted from that sum, the difference is the original number. So

$$23 + 14 = 37 \quad \text{Add 14 to 23.}$$

$$37 - 14 = 23 \quad \text{Subtract 14 from the sum, 37. The difference is the original number, 23.}$$

We use this idea to solve the following equation:

$$\begin{array}{ll} x + 21 = 35 & 21 \text{ is added to the number represented by } x. \\ x + 21 - 21 = 35 - 21 & \text{To remove the addition and have only } x \text{ on the left side of the} \\ x = 14 & \text{equal sign, we subtract 21. To keep a true equation, we must} \\ & \text{subtract 21 from both sides.} \end{array}$$

To check, replace x in the original equation with 14 and see if the result is a true statement:

$$\begin{array}{ll} x + 21 = 35 & \\ 14 + 21 = 35 & \\ 35 = 35 & \text{The statement is true, so the solution is 14.} \end{array}$$

We can also use the idea of inverses to solve an equation in which a number is subtracted from a variable (letter):

$$\begin{array}{ll} b - 17 = 12 & \text{Since 17 is subtracted from the variable, we eliminate the} \\ b - 17 + 17 = 12 + 17 & \text{subtraction by adding 17 to both sides of the equation. Recall} \\ b = 29 & \text{that addition is the inverse of subtraction.} \\ & \text{The equation will be true when } b \text{ is replaced by 29.} \end{array}$$

CHECK: $b - 17 = 12$
 $29 - 17 = 12$ Substitute 29 for b .
 $12 = 12$ True.

So the solution is $b = 29$.

To solve an equation using addition or subtraction

1. Add the same number to each side of the equation to isolate the variable, or
2. Subtract the same number from each side of the equation to isolate the variable.
3. Check the solution by substituting it for the variable in the original equation.

Warm-Ups A–E

A. $x + 15 = 32$

Examples A–E

DIRECTIONS: Solve and check.

STRATEGY: Isolate the variable by adding or subtracting the same number to or from each side.

A. $x + 7 = 23$
 $x + 7 = 23$
 $x + 7 - 7 = 23 - 7$
 $x = 16$

Because 7 is added to the variable, eliminate the addition by subtracting 7 from both sides of the equation.
Simplify.

CHECK: $x + 7 = 23$
 $16 + 7 = 23$
 $23 = 23$

Check by substituting 16 for x in the original equation.
The statement is true.

The solution is $x = 16$.

B. $y - 20 = 46$

B. $a - 24 = 50$
 $a - 24 = 50$
 $a - 24 + 24 = 50 + 24$
 $a = 74$

Because 24 is subtracted from the variable, eliminate the subtraction by adding 24 to both sides of the equation.
Simplify.

CHECK: $a - 24 = 50$
 $74 - 24 = 50$
 $50 = 50$

Check by substituting 74 for a in the original equation.
The statement is true.

The solution is $a = 74$.

C. $56 = z + 25$

C. $45 = b + 22$
In this example we do the subtraction vertically.

$$\begin{array}{r} 45 = b + 22 \\ - 22 \quad - 22 \\ \hline 23 = b \end{array}$$

Subtract 22 from both sides to eliminate the addition of 22.
Simplify.

CHECK: $45 = b + 22$
 $45 = 23 + 22$
 $45 = 45$

Substitute 23 for b .
The statement is true.

The solution is $b = 23$.

Answers to Warm-Ups

- A. $x = 17$ B. $y = 66$
C. $z = 31$

D.
$$\begin{aligned} z - 33 &= 41 \\ z - 33 &= 41 \\ z - 33 + 33 &= 41 + 33 \quad \text{Add 33 to both sides.} \\ z &= 74 \quad \text{Simplify.} \end{aligned}$$

CHECK:
$$\begin{aligned} z - 33 &= 41 \\ 74 - 33 &= 41 \quad \text{Substitute 74 for } z. \\ 41 &= 41 \quad \text{The statement is true.} \end{aligned}$$

The solution is $z = 74$.

- E. The selling price for a pair of Nike “Air Deluxe” shoes is \$139. If the markup on the shoes is \$43, what is the cost to the store?

Cost + markup = selling price.

$$\begin{array}{ll} C + M = S & \text{Since cost + markup = selling price.} \\ C + 43 = 139 & \text{Substitute 43 for the markup and 139 for the selling price.} \\ C + 43 - 43 = 139 - 43 & \text{Subtract 43 from both sides.} \\ C = 96 & \end{array}$$

CHECK: Does the cost + the markup equal \$139?

$$\begin{array}{rcl} \$96 & \text{Cost} \\ + 43 & \text{Markup} \\ \hline \$139 & \text{Selling price} \end{array}$$

So the cost of the shoes to the store is \$96.

D. $b - 43 = 51$

- E. The selling price of a set of golf clubs is \$576. If the markup is \$138, what is the cost to the store?

Answers to Warm-Ups

D. $b = 94$

E. The golf clubs cost the store \$438.

Exercises

OBJECTIVE

Solve an equation of the form $x + a = b$ or $x - a = b$, where a , b , and x are whole numbers.

Solve and check.

1. $x + 12 = 24$

2. $x - 11 = 14$

3. $x - 6 = 17$

4. $x + 10 = 34$

5. $z + 13 = 27$

6. $b - 21 = 8$

7. $c + 24 = 63$

8. $y - 33 = 47$

9. $a - 40 = 111$

10. $x + 75 = 93$

11. $x + 91 = 105$

12. $x - 76 = 43$

13. $y + 67 = 125$

14. $z - 81 = 164$

15. $k - 56 = 112$

16. $c + 34 = 34$

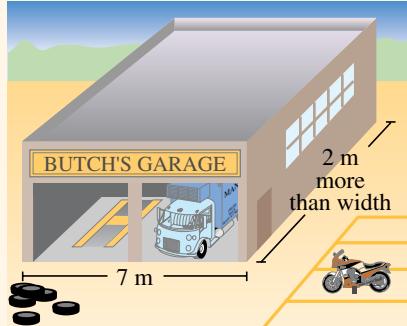
17. $73 = x + 62$

18. $534 = a + 495$

19. $87 = w - 29$

20. $373 = d - 112$

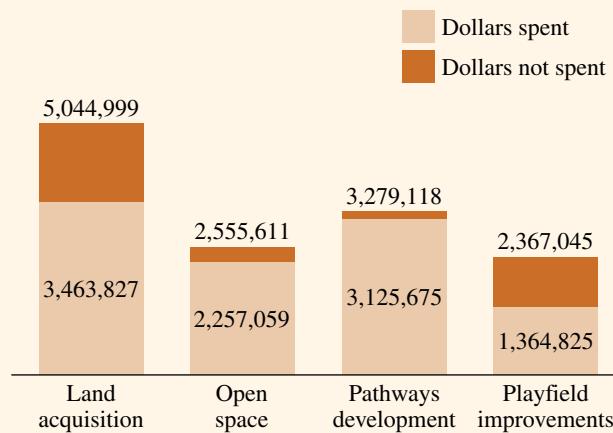
21. The selling price for a computer is \$1265. If the cost to the store is \$917, what is the markup?
22. The selling price of a trombone is \$675. If the markup is \$235, what is the cost to the store?
23. The length of a rectangular garage is 2 meters more than the width. If the width is 7 meters, what is the length?



24. The width of a rectangular fish pond is 6 feet shorter than the length. If the length is 27 feet, what is the width?
25. A Saturn with manual transmission has an EPA highway rating of 5 miles per gallon more than the EPA highway rating of a Subaru Impreza. Write an equation that describes this relationship. Be sure to define all variables in your equation. If the Saturn has an EPA highway rating of 35 mpg, find the highway rating of the Impreza.

26. In a recent year in the United States, the number of deaths by drowning was 1700 less than the number of deaths by fire. Write an equation that describes this relationship. Be sure to define all variables in your equation. If there were approximately 4800 deaths by drowning that year, how many deaths by fire were there?

Exercises 27–28. A city treasurer made the following report to the city council regarding monies allotted and dispersed from a city parks bond.



27. Write an equation that relates the total money budgeted per category to the amount of money spent and the amount of money not yet spent. Define all the variables.
28. Use your equation from Exercise 27 to calculate the amount of money not yet spent in each of the four categories.

1.3 Multiplying Whole Numbers

VOCABULARY

There are several ways to indicate multiplication. Here are examples of most of them, using 28 and 41.

$$\begin{array}{r} 28 \times 41 \\ 28 \\ \times 41 \\ \hline (28)(41) \end{array}$$

$$\begin{array}{r} 28 \cdot 41 \\ 28 \\ \times 41 \\ \hline (28)(41) \end{array}$$

$$\begin{array}{r} 28 \\ \times 41 \\ \hline (28)(41) \end{array}$$

OBJECTIVES

1. Multiply whole numbers.
2. Estimate the product of whole numbers.
3. Find the area of a rectangle.

The **factors** of a multiplication exercise are the numbers being multiplied. In $7(9) = 63$, 7 and 9 are the factors.

The **product** is the answer to a multiplication exercise. In $7(9) = 63$, the product is 63.

The **area** of a rectangle is the measure of the surface inside the rectangle.

How & Why

OBJECTIVE 1

Multiply whole numbers.

Multiplying whole numbers is a shortcut for repeated addition:

$$\underbrace{8 + 8 + 8 + 8 + 8 + 8}_{\text{6 eights}} = 48 \quad \text{or} \quad 6 \cdot 8 = 48$$

As numbers get larger, the shortcut saves time. Imagine adding 152 eights.

$$\underbrace{8 + 8 + 8 + 8 + 8 + \cdots + 8}_{\text{152 eights}} = ?$$

We multiply 8 times 152 using the expanded form of 152.

$$\begin{array}{rcl} 152 & = & 100 + 50 + 2 & \text{Write 152 in expanded form.} \\ \times 8 & = & \underline{\hspace{2cm}} & \\ \hline & = & 800 + 400 + 16 & \text{Multiply 8 times each addend.} \\ & = & 1216 & \text{Add.} \end{array}$$

The exercise can also be performed in column form without expanding the factors.

$$\begin{array}{r} 152 \\ \times 8 \\ \hline 1216 \end{array}$$

$$\begin{array}{r} 152 \\ \times 8 \\ \hline 1216 \end{array}$$

The form on the right shows the usual shortcut. The carried digit is added to the product of each column. Study this example.

$$\begin{array}{r} 635 \\ \times 47 \\ \hline \end{array}$$

First multiply 635 by 7.

$$\begin{array}{r} \overset{2}{\cancel{3}} \\ 635 \\ \times 47 \\ \hline 4445 \end{array}$$

7(5) = 35. Carry the 3 to the tens column.
7(3 tens) = 21 tens. Add the 3 tens that were carried:
(21 + 3) tens = 24 tens. Carry the 2 to the hundreds column.
7(6 hundreds) = 42 hundreds. Add the 2 hundreds that were carried: (42 + 2) hundreds = 44 hundreds.

Now multiply 635 by 40.

$$\begin{array}{r} \overset{1}{\cancel{2}} \\ \overset{2}{\cancel{3}} \\ 635 \\ \times 47 \\ \hline 4445 \\ 25400 \end{array}$$

40(5) = 200 or 20 tens. Carry the 2 to the hundreds column.
40(30) = 1200 or 12 hundreds. Add the 2 hundreds that were carried. (12 + 2) hundreds = 14 hundreds. Carry the 1 to the thousands column. 40(600) = 24,000 or 24 thousands. Add the 1 thousand that was carried: (24 + 1) thousands = 25 thousands.
Write the 5 in the thousands column and the 2 in the ten-thousands column.

$$\begin{array}{r} \overset{1}{\cancel{2}} \\ \overset{2}{\cancel{3}} \\ 635 \\ \times 47 \\ \hline 4445 \\ 25400 \\ 29845 \end{array}$$

Add the products.

Multiplication Property of Zero

Multiplication property of zero:

$$a \cdot 0 = 0 \cdot a = 0$$

Any number times zero is zero.

Multiplication Property of One

Multiplication property of one:

$$a \cdot 1 = 1 \cdot a = a$$

Any number times 1 is that number.

Two important properties of arithmetic and higher mathematics are the *multiplication property of zero* and the *multiplication property of one*.

As a result of the multiplication property of zero, we know that

$$0 \cdot 23 = 23 \cdot 0 = 0 \quad \text{and} \quad 0(215) = 215(0) = 0$$

As a result of the multiplication property of one, we know that

$$1 \cdot 47 = 47 \cdot 1 = 47 \quad \text{and} \quad 1(698) = 698(1) = 698$$

Warm-Ups A–F

A. Multiply: (671)(0)

B. Find the product: 6(7862)

Answers to Warm-Ups

A. 0 B. 47,172

Examples A–F



BASIC Mathematics Now™

DIRECTIONS: Multiply.

STRATEGY: Write the factors in columns. Start multiplying with the ones digit. If the product is 10 or more, carry the tens digit to the next column and add it to the product in that column. Repeat the process for every digit in the second factor. When the multiplication is complete, add to find the product.

A. Multiply: 1(932)

$$1(932) = 932 \quad \text{Multiplication property of one}$$

B. Find the product: 8(5437)

325

5437

$\times \quad 8$

43,496

Multiply 8 times each digit, carry when necessary, and add the number carried to the next product.

C. Multiply: $54 \cdot 49$

$$\begin{array}{r} 4 \\ 3 \\ 49 \\ \times 54 \\ \hline 196 \\ 2450 \\ \hline 2646 \end{array}$$

When multiplying by the 5 in the tens place, write a 0 in the ones column to keep the places lined up.

D. Find the product of 528 and 109.

STRATEGY: When multiplying by zero in the tens place, rather than showing a row of zeros, just put a zero in the tens column. Then multiply by the 1 in the hundreds place.

$$\begin{array}{r} 528 \\ \times 109 \\ \hline 4752 \\ 52800 \\ \hline 57552 \end{array}$$

CALCULATOR EXAMPLE



E. $3465(97)$

Most graphing calculators recognize implied multiplication but most scientific calculators do not. Be sure to insert a multiplication symbol between two numbers written with implied multiplication.

The product is 336,105.

F. General Electric ships 124 cartons of lightbulbs to Home Depot. Each carton contains 48 lightbulbs. What is the total number of lightbulbs shipped to Home Depot?

STRATEGY: To find the total number of lightbulbs, multiply the number of cartons by the number of lightbulbs in each carton.

$$\begin{array}{r} 124 \\ \times 48 \\ \hline 992 \\ 4960 \\ \hline 5952 \end{array}$$

General Electric shipped 5952 lightbulbs to Home Depot.

C. Multiply: $76 \cdot 63$

D. Find the product of 826 and 307.

E. $763(897)$

F. General Electric ships 88 cartons of lightbulbs to Lowe's. If each carton contains 36 lightbulbs, how many lightbulbs are shipped to Lowe's?

How & Why

OBJECTIVE 2

Estimate the product of whole numbers.

The product of two whole numbers can be estimated by using **front rounding**. With front rounding we round to the highest place value so that all the digits become 0 except the first one. For example, if we front round 7654 we get 8000.

So to estimate the following product of 78 and 432, front round each factor and multiply.

$$\begin{array}{r} 432 & 400 & \text{Front round each factor and multiply.} \\ \times 78 & \times 80 \\ \hline 32,000 \end{array}$$

The estimated product is 32,000, that is $(432)(78) \approx 32,000$.

Answers to Warm-Ups

- C. 4788 D. 253,582 E. 684,411
F. General Electric shipped 3168 lightbulbs to Lowe's.

One use of the estimate is to see if the product is correct. If the calculated product is not close to 32,000, you should check the multiplication. In this case the actual product is 33,696, which is close to the estimate.

Warm-Ups G–J

- G. Estimate the product.

$$\begin{array}{r} 735 \\ \times 63 \\ \hline \end{array}$$

- H. Estimate the product.

$$\begin{array}{r} 56,911 \\ \times 78 \\ \hline \end{array}$$

- I. Jerry finds the product of 380 and 32 to be 12,160. Estimate the product by front rounding, to see if Jerry is correct. If not, find the actual product.

- J. Joanna is shopping for sweaters. She finds a style she likes priced at \$78. Estimate the cost of five sweaters.

Examples G–J



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DIRECTIONS: Estimate the product.

STRATEGY: Front round both factors and multiply.

- G. $\begin{array}{r} 298 \\ \times 46 \\ \hline \end{array}$ $\begin{array}{r} 300 \\ \times 50 \\ \hline 15,000 \end{array}$ Front round and multiply.

So, $(298)(46) \approx 15,000$.

- H. $\begin{array}{r} 3,792 \\ \times 412 \\ \hline \end{array}$ $\begin{array}{r} 4,000 \\ \times 400 \\ \hline 1,600,000 \end{array}$

So $(3792)(412) \approx 1,600,000$.

- I. Paul finds the product of 230 and 47 to be 1081. Estimate the product by front rounding, to see if Paul is correct. If not, find the actual product.

$$\begin{array}{r} 230 \\ \times 47 \\ \hline 10,000 \end{array}$$

The estimate is 10,000, so Paul is not correct.

$$\begin{array}{r} 230 \\ \times 47 \\ \hline 1610 \\ 9200 \\ \hline 10,810 \end{array}$$

Paul was not correct; the correct product is 10,810.

- J. John wants to buy seven shirts that cost \$42 each. He has \$300 in cash. Estimate the cost of the shirts to see if John has enough money to buy them.

$$\begin{array}{r} \$42 \\ \times 7 \\ \hline \$280 \end{array}$$

Front round the price of one shirt
and multiply by the number of shirts.

The estimated cost of the seven shirts is \$280, so it looks like John has enough money.

How & Why

OBJECTIVE 3

Find the area of a rectangle.

Answers to Warm-Ups

- G. 42,000 H. 4,800,000
I. The estimated answer is 12,000, so Jerry's answer appears to be correct.
J. The estimated cost of the sweaters is \$400.

The area of a polygon is the measure of the space inside the polygon. We use area when describing the size of a plot of land, the living space in a house, or an amount of carpet. Area is measured in square units such as square feet or square meters. A square foot is literally a square with sides of 1 foot. The measure of the surface inside the square is 1 square foot.

When measuring the space inside a polygon, we divide the space into squares and count them. For example, consider a rug that is 2 ft by 3 ft (Figure 1.6).

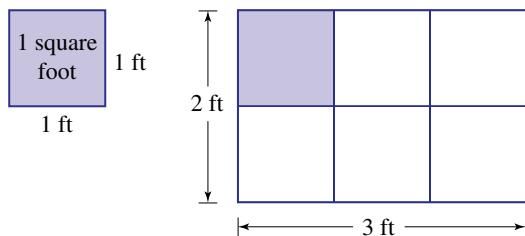


Figure 1.6

There are six squares in the subdivided rug, so the area of the rug is 6 square feet.

Finding the area of a rectangle, such as the area rug in the example, is relatively easy because a rectangle has straight sides and it is easy to fit squares inside it. The length of the rectangle determines how many squares will be in each row, and the width of the rectangle determines the number of rows. In the rug shown in Figure 1.6, there are two rows of three squares each because the width is 2 ft and the length is 3 ft. The product of the length and width gives the number of squares inside the rectangle.

$$\text{Area of a rectangle} = \text{length} \cdot \text{width}$$

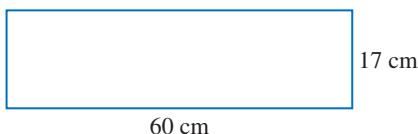
Finding the area of other shapes is a little more complicated, and is discussed in Section 7.4.

Example K

DIRECTIONS: Find the area of the rectangle.

STRATEGY: Multiply the length and width.

K. Find the area of the rectangle.



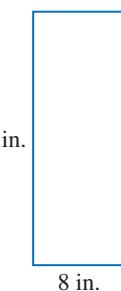
$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= 60 \cdot 17 \\ &= 1020\end{aligned}$$

The area is measured in square centimeters because the sides are measured in centimeters and so each square is a square centimeter.

The area is 1020 square centimeters.

Warm-Up K

K. Find the area of the rectangle.



Answers to Warm-Ups

K. 176 square inches

Exercises 1.3

OBJECTIVE 1

Multiply whole numbers.

A Multiply.

1.
$$\begin{array}{r} 83 \\ \times 7 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 63 \\ \times 4 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 77 \\ \times 3 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 76 \\ \times 4 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 46 \\ \times 6 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 93 \\ \times 7 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 39 \\ \times 8 \\ \hline \end{array}$$

9. $9 \cdot 48$

10. $6 \cdot 55$

11. $(105)(0)$

12. $(1)(345)$

13.
$$\begin{array}{r} 88 \\ \times 50 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 26 \\ \times 60 \\ \hline \end{array}$$

15. In 326×52 the place value of the product of 5 and 3 is _____.

16. In 326×52 the product of 5 and 6 is 30 and you must carry the 3 to the _____ column.

B Multiply.

17.
$$\begin{array}{r} 464 \\ \times 8 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 471 \\ \times 5 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 804 \\ \times 7 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 703 \\ \times 9 \\ \hline \end{array}$$

21. $(53)(67)$

22. $(49)(55)$

23. $(94)(37)$

24. $(83)(63)$

25.
$$\begin{array}{r} 315 \\ \times 400 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 582 \\ \times 700 \\ \hline \end{array}$$

27.
$$\begin{array}{r} 608 \\ \times 83 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 608 \\ \times 57 \\ \hline \end{array}$$

29.
$$\begin{array}{r} 747 \\ \times 48 \\ \hline \end{array}$$

30.
$$\begin{array}{r} 534 \\ \times 75 \\ \hline \end{array}$$

31. $(93)(362)$ Round product to the nearest hundred.

32. $(78)(561)$ Round product to the nearest thousand.

33.
$$\begin{array}{r} 312 \\ \times 50 \\ \hline \end{array}$$

34.
$$\begin{array}{r} 675 \\ \times 40 \\ \hline \end{array}$$

35.
$$\begin{array}{r} 527 \\ \times 73 \\ \hline \end{array}$$

36.
$$\begin{array}{r} 265 \\ \times 57 \\ \hline \end{array}$$

37.
$$\begin{array}{r} 475 \\ \times 39 \\ \hline \end{array}$$

38.
$$\begin{array}{r} 823 \\ \times 65 \\ \hline \end{array}$$

39. $(4321)(76)$

40. $(3510)(83)$

OBJECTIVE 2

Estimate the product of whole numbers.

A Estimate the product using front rounding.

41. 36×82

42. 64×48

43. 625×57

44. 789×29

45. 4510×53

46. 6328×27

B

47. 83×3046

48. 34×6290

49. $17,121 \times 39$

50. $52,812 \times 81$

51. $610 \times 34,560$

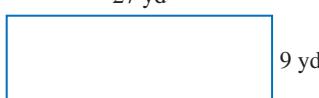
52. $459 \times 55,923$

OBJECTIVE 3

Find the area of a rectangle.

A Find the area of the following rectangles.

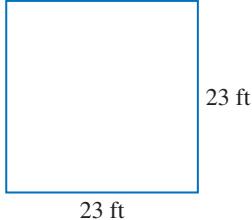
53.



54.



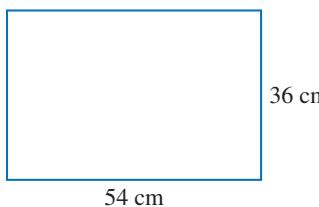
55.



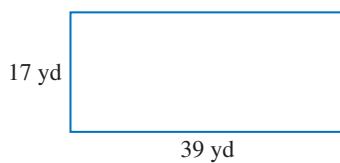
56.



57.

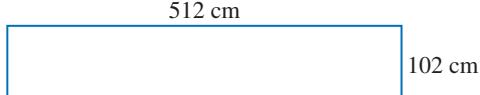
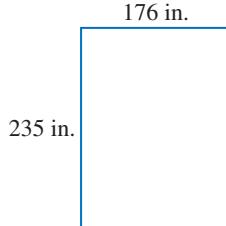
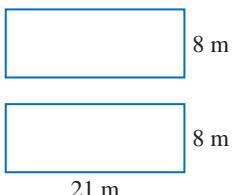
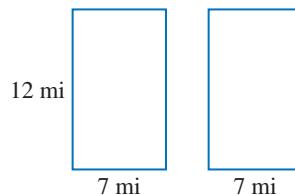
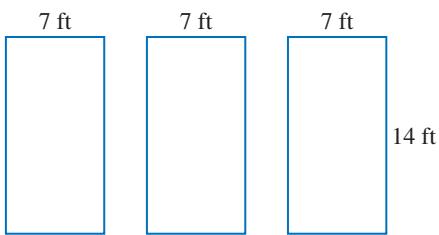
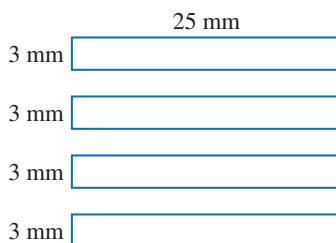


58.



59. What is the area of a rectangle that has a length of 17 ft and a width of 6 ft?

60. What is the area of a rectangle that measures 30 cm by 40 cm?

B Find the area of the following.**61.****62.****63.****64.****65.****66.**

- 67.** What is the area of three goat pens that each measure 6 ft by 11 ft?

- 68.** What is the area of five bath towels that measure 68 cm by 140 cm?

C

- 69.** Find the product of 808 and 632.

- 70.** Find the product of 808 and 792.

- 71.** Multiply and round to the nearest thousand:
 $(744)(3193)$

- 72.** Multiply and round to the nearest ten thousand:
 $(6004)(405)$

- 73.** Maria multiplies 59 times 482 and gets a product of 28,438. Estimate the product by front rounding to see if Maria's answer is reasonable.

- 74.** John multiplies 791 by 29 and gets a product of 8701. Estimate the product by front rounding to see if John's answer is reasonable.

- 75.** During the first week of the Rotary Club rose sale, 341 dozen roses are sold. The club estimates that a total of 15 times that number will be sold during the sale. What is the estimated number of dozens of roses that will be sold?

Exercises 76–79. Use the information on the monthly sales at Dick's Country Cars.

Monthly Sales at Dick's Country Cars

Car Model	Number of Cars Sold	Average Price per Sale
Durango	32	\$27,497
Grand Caravan	43	\$19,497
Dakota	26	\$20,450

- 76.** Find the gross receipts from the sale of the Durangos.
- 77.** What are the gross receipts from the sale of the Grand Caravans?
- 78.** Find the gross receipts from the sale of Dakotas.
- 79.** Find the gross receipts for the month (the sum of the gross receipts for each model) rounded to the nearest thousand dollars.
- 80.** An average of 452 salmon per day are counted at the Bonneville fish ladder during a 17-day period. How many total salmon are counted during the 17-day period?
- 81.** During 2004, the population of Washington County grew at a pace of 1874 people per month. What was the total growth in population during 2004?
- 82.** The CEO of Apex Corporation exercised his option to purchase 2355 shares of Apex stock at \$13 per share. He immediately sold the shares for \$47 per share. If broker fees came to \$3000, how much money did he realize from the purchase and sale of the shares?
- 83.** The comptroller of Apex Corporation exercised her option to purchase 1295 shares of Apex stock at \$16 per share. She immediately sold the shares for \$51 per share. If broker fees came to \$1050, how much money did she realize from the purchase and sale of the shares?
- 84.** Nyuen wants to buy radios for his seven grandchildren for Christmas. He has budgeted \$500 for these presents. The radio he likes costs \$79. Estimate the total cost, by front rounding, to see if Nyuen has enough money in his budget for these presents.
- 85.** Carmella needs to purchase 12 blouses for the girls in the choir at her church. The budget for the purchases is \$480. The blouse she likes costs \$38.35. Estimate the total cost, by front rounding, to see if Carmella has enough money in her budget for these blouses.
- 86.** A certain bacteria culture triples its size every hour. If the culture has a count of 265 at 10 A.M., what will the count be at 2 P.M. the same day?

Exercises 87–88. The depth of water is often measured in fathoms. There are 3 feet in a yard and 2 yards in a fathom.

- 87.** How many feet are in a fathom?
- 88.** How many feet are in 25 fathoms?

Exercises 89–91. A league is an old measure of about 3 nautical miles. A nautical mile is about 6076 feet.

- 89.** How many feet are in a league?
- 90.** There is famous book by Jules Verne titled *20,000 Leagues Under the Sea*. How many feet are in 20,000 leagues?
- 91.** The Mariana Trench in the Pacific Ocean is the deepest point of all the world's oceans. It is 35,840 ft deep. Is it physically possible to be 20,000 leagues under the sea?

Exercises 92–94. Because distances between bodies in the universe are so large, scientists use large units. One such unit is the light-year, which is the distance traveled by light in one year, or 5880 billion miles.

- 92.** Write the place value notation for the number of miles in a light-year.
- 93.** The star Sirius is recognized as the brightest star in the sky (other than the sun). It is 8 light-years from Earth. How many miles is Sirius from Earth?
- 94.** The star Rigel in the Orion constellation is 545 light-years from Earth. How many miles away is Rigel from Earth?

Exercises 95–96. One model of an inkjet printer can produce 20 pages per minute in draft mode, 8 pages per minute in normal mode and 2 pages per minute in best-quality mode.

- 95.** Skye is producing a large report for her group. She selects normal mode and is called away from the printer for 17 minutes. How many pages of the report were printed in that time?
- 96.** How many more pages can be produced in 25 minutes in draft mode than in 25 minutes in normal mode?

Exercises 97–98. In computers, a byte is the amount of space needed to store one character. Knowing something about the metric system, one might think a kilobyte is 1000 bytes, but actually it is 1024 bytes.

- 97.** A computer has 256 KB (kilobytes) of RAM. How many bytes is this?
- 98.** A megabyte is 1024 KB. A writable CD holds up to 700 MB (megabytes). How many bytes can the CD hold?

Exercises 99–100. A gram of fat contains about 9 calories, as does a gram of protein. A gram of carbohydrate contains about 4 calories.

- 99.** A tablespoon of olive oil has 14 g of fat. How many calories is this?
- 100.** One ounce of cream cheese contains 2 g of protein and 10 grams of fat. How many calories from fat and protein are in the cream cheese?

- 101.** The water consumption in Hebo averages 534,650 gallons per day. How many gallons of water are consumed in a 31-day month, rounded to the nearest thousand gallons?
- 102.** Ms. Munos orders two hundred twenty-five iPods for sale in her discount store. If she pays \$115 per iPod and sells them for \$198, how much do the iPods cost her and what is the net income from their sale? How much are her profits from the sale of the iPods?
- 103.** Ms. Perta orders four hundred sixty-four studded snow tires for her tire store. She pays \$48 per tire and plans to sell them for \$106 each. What do the tires cost Ms. Perta and what is her gross income from their sale? What net income does she receive from the sale of the tires?
- 104.** In 2005, Bill Gates of Microsoft was the richest person in the United States, with an estimated net worth of \$46 billion. Write the place value name for this number. A financial analyst made the observation that the average person has a hard time understanding such large amounts. She gave the example that in order to spend \$1 billion, one would have to spend \$40,000 per day for 69 years, ignoring leap years. How much money would you spend if you did this?

Exercises 105–106 relate to the chapter application. See Table 1.1, page 1.

- 105.** If the Harry Potter movie had doubled its gross earnings, would it have been the top grossing movie of 2004?
- 106.** If *National Treasure* had doubled its earnings in 2004, where would it be on the list?

STATE YOUR UNDERSTANDING

- 107.** Explain to an 8-year-old child that $3(8) = 24$.
- 108.** When 65 is multiplied by 8, we carry 4 to the tens column. Explain why this is necessary.
- 109.** Define and give an example of a product.

CHALLENGE

- 110.** Find the product of twenty-four thousand, fifty-five and two hundred thirteen thousand, two hundred seventy-six. Write the word name for the product.
- 111.** Tesfay harvests 82 bushels of wheat per acre from his 1750 acres of grain. If Tesfay can sell the grain for \$31 a bushel, what is the crop worth, to the nearest thousand dollars?

Complete the problems by writing in the correct digit wherever you see a letter,

112.
$$\begin{array}{r} 51A \\ \times \quad B2 \\ \hline 10B2 \\ 154C \\ \hline 1A5E2 \end{array}$$

113.
$$\begin{array}{r} 1A57 \\ \times \quad 42 \\ \hline B71C \\ D428 \\ \hline 569E4 \end{array}$$

GROUP WORK

114. Multiply 36, 74, 893, 627, and 1561 by 10, 100, and 1000. What do you observe? Can you devise a rule for multiplying by 10, 100, and 1000?

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1.4 Dividing Whole Numbers

VOCABULARY

There are a variety of ways to indicate division. These are the most commonly used:

$$72 \div 6 \quad 6\overline{)72} \quad \frac{72}{6}$$

The **dividend** is the number being divided, so in $54 \div 6 = 9$, the dividend is 54.

The **divisor** is the number that we are dividing by, so in $54 \div 6 = 9$, the divisor is 6.

The **quotient** is the answer to a division exercise, so in $54 \div 6 = 9$, the quotient is 9.

When a division exercise does not come out even, as in $61 \div 7$, the quotient is not a whole number.

$$\begin{array}{r} 8 \\ 7\overline{)61} \\ 56 \\ \hline 5 \end{array}$$

We call 8 the **partial quotient** and 5 the **remainder**. The quotient is written 8 R 5.

OBJECTIVE

Divide whole numbers.

How & Why

OBJECTIVE

Divide whole numbers.

The division exercise $144 \div 24 = ?$ (read “144 divided by 24”) can be interpreted in one of two ways.

How many times can 24 be subtracted from 144? This is called the “repeated subtraction” version.

What number times 24 is equal to 144? This is called the “missing factor” version.

All division problems can be done using repeated subtraction.

In $144 \div 24 = ?$, we can find the missing factor by repeatedly subtracting 24 from 144:

$$\begin{array}{r} 144 \\ - 24 \\ \hline 120 \\ - 24 \\ \hline 96 \quad \text{Six subtractions, so } 144 \div 24 = 6. \\ - 24 \\ \hline 72 \\ - 24 \\ \hline 48 \\ - 24 \\ \hline 24 \\ - 24 \\ \hline 0 \end{array}$$

The process can be shortened using the traditional method of guessing the number of 24s and subtracting from 144:

$$\begin{array}{r} 24) \overline{144} \\ \underline{72} \quad 3 \text{ twenty-fours} \\ \underline{72} \quad 48 \\ \underline{72} \quad \underline{3 \text{ twenty-fours}} \quad \text{or} \quad \underline{48} \\ \underline{0} \quad 6 \end{array} \qquad \begin{array}{r} 24) \overline{144} \\ \underline{96} \quad 4 \text{ twenty-fours} \\ \underline{48} \quad 0 \\ \underline{48} \quad 2 \text{ twenty-fours} \\ \underline{0} \quad 6 \end{array}$$

or $\begin{array}{r} 24) \overline{144} \\ \underline{144} \quad 6 \text{ twenty-fours} \\ \underline{0} \quad 6 \end{array}$

In each case, $144 \div 24 = 6$.

We see that the missing factor in $(24)(?) = 144$ is 6. Because $24(6) = 144$, consequently $144 \div 24 = 6$.

This leads to a method for checking division. If we multiply the divisor times the quotient, we will get the dividend. To check $144 \div 24 = 6$, we multiply 24 and 6.

$$(24)(6) = 144$$

So 6 is correct.

This process works regardless of the size of the numbers. If the divisor is considerably smaller than the dividend, you will want to guess a rather large number.

$$\begin{array}{r} 63) \overline{19,593} \\ \underline{6300} \quad 100 \\ 13293 \\ \underline{6300} \quad 100 \\ 6993 \\ \underline{6300} \quad 100 \\ 693 \\ \underline{630} \quad 10 \\ 63 \\ \underline{63} \quad 1 \\ 0 \end{array} \qquad \begin{array}{r} 31) \overline{17,391} \\ \underline{155} \quad 5 \\ 189 \\ \underline{186} \quad 1 \\ 31 \\ \underline{31} \quad 0 \end{array}$$

So, $19,593 \div 63 = 311$.

All divisions can be done by this method. However, the process can be shortened by finding the number of groups, starting with the largest place value on the left, in the dividend, and then working toward the right. Study the following example. Note that the answer is written above the problem for convenience.

$\begin{array}{r} 31) \overline{17,391} \\ \underline{155} \quad 5 \\ 189 \\ \underline{186} \quad 1 \\ 31 \\ \underline{31} \quad 0 \end{array}$ Working from left to right, we note that 31 does not divide 1, and it does not divide 17. However, 31 does divide 173 five times. Write the 5 above the 3 in the dividend.

$5(31) = 155$. Subtract 155 from 173. Because the difference is less than the divisor, no adjustment is necessary. Bring down the next digit, which is 9. Next, 31 divides 189 six times. The 6 is placed above the 9 in the dividend. $6(31) = 186$. Subtract 186 from 189. Again, no adjustment is necessary, since $3 < 31$. Bring down the next digit, which is 1. Finally, 31 divides 31 one time. Place the 1 above the 1 in the dividend. $1(31) = 31$. Subtract 31 from 31, the remainder is zero. The division is complete.

CHECK: $561 \times 31 = 17,391$ Check by multiplying the quotient by the divisor.

$$\begin{array}{r} \times 31 \\ \hline 561 \\ 16,830 \\ \hline 17,391 \end{array}$$

So $17,391 \div 31 = 561$.

Not all division problems come out even (have a zero remainder). In

$$\begin{array}{r} 4 \\ 21 \overline{)94} \\ 84 \\ \hline 10 \end{array}$$

we see that 94 contains 4 twenty-ones and 10 toward the next group of twenty-one. The answer is written as 4 remainder 10. The word *remainder* is abbreviated “R” and the result is 4 R 10.

Check by multiplying (21)(4) and adding the remainder.

$$\begin{aligned}(21)(4) &= 84 \\ 84 + 10 &= 94\end{aligned}$$

So $94 \div 21 = 4$ R 10.

The division, $61 \div 0 = ?$, can be restated: What number times 0 is 61? $0 \times ? = 61$. According to the multiplication property of zero we know that $0 \times (\text{any number}) = 0$, so it cannot equal 61.

CAUTION

Division by zero is not defined. It is an operation that cannot be performed.

When dividing by a single-digit number the division can be done mentally using “short division.”

$$\begin{array}{r} 423 \\ 3 \overline{)1269} \\ \quad 12 \\ \quad -6 \\ \quad \quad 6 \\ \quad \quad -9 \\ \quad \quad \quad 9 \end{array}$$

Divide 3 into 12. Write the answer, 4, above the 2 in the dividend. Now divide the 6 by 3 and write the answer, 2, above the 6. Finally divide the 9 by 3 and write the answer, 3, above the 9.

The quotient is 423.

If the “mental” division does not come out even, each remainder is used in the next division.

$$\begin{array}{r} 452 \text{ R } 2 \\ 3 \overline{)1358} \\ \quad 12 \\ \quad -15 \\ \quad \quad 5 \\ \quad \quad -4 \\ \quad \quad \quad 8 \end{array}$$

$13 \div 3 = 4$ R 1. Write the 4 above the 3 in the dividend. Now form a new number, 15, using the remainder 1 and the next digit 5. Divide 3 into 15. Write the answer, 5, above 5 in the dividend. Because there is no remainder, divide the next digit, 8, by 3. The result is 2 R 2. Write this above the 8.

The quotient is 452 R 2.

Examples A–E



BASIC
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DIRECTIONS: Divide and check.

STRATEGY: Divide from left to right. Use short division for single-digit divisors.

A. $7 \overline{)5621}$

STRATEGY: Because there is a single-digit divisor we use short division.

$$\begin{array}{r} 803 \\ 7 \overline{)5621} \end{array}$$

7 divides 56 eight times. 7 divides 2 zero times with a remainder of 2.

Now form a new number, 21, using the remainder and the next number 1.
7 divides 21 three times.

Warm-Ups A–E

A. $9 \overline{)6318}$

Answers to Warm-Ups

A. 702

CAUTION

A zero must be placed in the quotient so that the 8 and the 3 have the correct place values.

The quotient is 803.

B. Divide: $13\overline{)2028}$

B. Divide: $23\overline{)5635}$

STRATEGY: Write the partial quotients above the dividend with the place values aligned.

$$\begin{array}{r} 245 \\ 23\overline{)5635} \\ \underline{46} \quad 23(2) = 46 \\ 103 \\ \underline{92} \quad 23(4) = 92 \\ 115 \\ \underline{115} \quad 23(5) = 115 \\ 0 \end{array}$$

CHECK:

$$\begin{array}{r} 245 \\ \times 23 \\ \hline 735 \\ 4900 \\ \hline 5635 \end{array}$$

The quotient is 245.

C. Find the quotient: $\frac{233,781}{482}$

C. Find the quotient: $\frac{127,257}{482}$

STRATEGY: When a division is written as a fraction, the dividend is above the fraction bar and the divisor is below.

$$\begin{array}{r} 264 \\ 482\overline{)127,257} \\ \underline{96\ 4} \quad 482 \text{ does not divide } 1. \\ 30\ 85 \quad 482 \text{ does not divide } 12. \\ \underline{28\ 92} \quad 482 \text{ does not divide } 127. \\ 1\ 937 \quad 482 \text{ divides } 1272 \text{ two times.} \\ \underline{1\ 928} \quad 482 \text{ divides } 3085 \text{ six times.} \\ 9 \quad 482 \text{ divides } 1937 \text{ four times.} \\ \text{The remainder is } 9. \end{array}$$

CHECK: Multiply the divisor by the partial quotient and add the remainder.

$$\begin{aligned} 264(482) + 9 &= 127,248 + 9 \\ &= 127,257 \end{aligned}$$

The answer is 264 with a remainder of 9 or 264 R 9.

You may recall other ways to write a remainder using fractions or decimals. These are covered in a later chapter.

Answers to Warm-Ups

B. 156 C. 485 R 11

CALCULATOR EXAMPLE

- D. Divide 73,965 by 324.

Enter the division: $73,965 \div 324$

$$73,965 \div 324 \approx 228.28703$$

The quotient is not a whole number. This means that 228 is the partial quotient and there is a remainder. To find the remainder, multiply 228 times 324. Subtract the product from 73,965. The result is the remainder.

$$73,965 - 228(324) = 93$$

$$\text{So } 73,965 \div 324 = 228 \text{ R } 93.$$

- E. When planting Christmas trees, the Greenfir Tree Farm allows 64 square feet per tree. How many trees will they plant in 43,520 square feet?

STRATEGY: Because each tree is allowed 64 square feet, we divide the number of square feet by 64 to find out how many trees will be planted.

$$\begin{array}{r} 680 \\ 64 \overline{)43,520} \\ 384 \\ \hline 512 \\ 512 \\ \hline 0 \\ 0 \end{array}$$

There will be a total of 680 trees planted in 43,520 square feet.

- D. Divide 47,753 by 415.

- E. The Greenfir Tree Farm allows 256 square feet per large spruce tree. If there are 43,520 square feet to be planted, how many trees will they plant?

Answers to Warm-Ups

D. 115 R 28

E. They will plant 170 trees.

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Exercises 1.4

BASIC
Mathematics  Now™**OBJECTIVE**

Divide whole numbers.

A Divide.

1. $8\overline{)72}$

2. $9\overline{)99}$

3. $5\overline{)45}$

4. $5\overline{)80}$

5. $5\overline{)435}$

6. $3\overline{)327}$

7. $5\overline{)455}$

8. $9\overline{)549}$

9. $136 \div 8$

10. $276 \div 6$

11. $840 \div 21$

12. $900 \div 18$

13. $492 \div 6$

14. $1668 \div 4$

15. $32 \div 7$

16. $29 \div 6$

17. $81 \div 17$

18. $93 \div 29$

19. The division has a remainder when the last difference in the division is smaller than the _____ and is not zero.

20. For $2600 \div 13$, in the partial division $26 \div 13 = 2$, 2 has place value _____.

B Divide.

21. $16,248 \div 4$

22. $12,324 \div 6$

23. $\frac{768}{24}$

24. $\frac{664}{83}$

25. $46\overline{)2484}$

26. $38\overline{)2546}$

27. $46\overline{)4002}$

28. $56\overline{)5208}$

29. $432\overline{)28,944}$

30. $417\overline{)30,441}$

31. $355\overline{)138,805}$

32. $617\overline{)124,017}$

33. $34\overline{)8143}$

34. $49\overline{)6925}$

35. $33\overline{)591}$

36. $51\overline{)666}$

37. $(62)(?) = 3596$

38. $(?)(73) = 2555$

39. $27\overline{)345,672}$

40. $62\overline{)567,892}$

41. $64,782 \div 56$. Round quotient to the nearest ten.

42. $67,000 \div 43$. Round quotient to the nearest hundred.

43. $722,894 \div 212$. Round quotient to the nearest hundred.

44. $876,003 \div 478$. Round quotient to the nearest hundred.

- C Exercises 45–48.** The revenue department of a state had the following collection data for the first 3 weeks of March.

Taxes Collected

Number of Returns	Total Taxes Paid
Week 1—4563	\$24,986,988
Week 2—3981	\$19,315,812
Week 3—11,765	\$48,660,040

45. Find the taxes paid per return during week 1.

46. Find the taxes paid per return during week 2.

- 47.** Find the taxes paid per return during week 3. Round to the nearest hundred dollars.
- 48.** Find the taxes paid per return during the 3 weeks. Round to the nearest hundred dollars.
- 49.** A forestry survey finds that 1890 trees are ready to harvest on a 14-acre plot. On the average, how many trees are ready to harvest per acre?
- 50.** Rosebud Lumber Company replants 4824 seedling fir trees on an 18-acre plot of logged-over land. What is the average number of seedlings planted per acre?
- 51.** Ms. Munos buys 45 radios to sell in her department store. She pays \$1260 for the radios. Ms. Munos reorders an additional 72 radios. What will she pay for the reordered radios if she gets the same price per radio as the original order?
- 52.** Burkhardt Floral orders 25 dozen red roses at the wholesale market. The roses cost \$300. The following week they order 34 dozen of the roses. What do they pay for the 34 dozen roses if they pay the same price per dozen as in the original order?
- 53.** In 2005, Bill Gates of Microsoft was the richest person in the United States, with an estimated net worth of \$46 billion. How much would you have to spend per day in order to spend all of Bill Gates's \$46 billion in 90 years, ignoring leap years? Round to the nearest thousand.
- 54.** How much money would you have to spend per day, ignoring leap years, in order to spend Bill Gates's \$46 billion in 50 years? In 20 years? Round to the nearest hundred dollars.

Exercises 55–57. Use the estimated population in 2005 and the area of the country as given.

Population in 2005

Country	Estimated Population	Area, in Square Kilometers
China	1,322,273,000	9,596,960
Italy	57,253,000	301,230
United States	300,038,000	9,629,091

- 55.** What was the population density (people per square kilometer, that is, the number of people divided by the number of square kilometers) of China, rounded to the nearest whole person?
- 56.** What was the population density (people per square kilometer, that is, the number of people divided by the number of square kilometers) of Italy, rounded to the nearest whole person?
- 57.** What was the population density (people per square kilometer, that is, the number of people divided by the number of square kilometers) of the United States, rounded to the nearest whole person?

Exercises 58–59. The Humane Society estimates that there are 65 million dogs owned in the United States and 77,600,000 cats.

- 58.** The Humane Society estimates that dog owners spent \$17,095,000,000 in veterinary fees for their dogs in the last year. What is the average cost per dog?
- 59.** The average cat owner owns two cats. Approximately how many households own cats?

Exercises 60–62. The 2000 Census population and the number of House of Representative seats in the United States and two states are given below.

Population and House Representation

	Population	Number of House Seats
United States	272,171,813	435
California	33,145,121	53
Montana	882,779	2

- 60.** How many people does each House member represent in the United States?
- 61.** How many people does each representative from California represent?
- 62.** How many people does each representative from Montana represent?
- 63.** In 2004, the estimated population of Florida was 17,397,161 and the gross state product was \$571,600,000,000. What was the state product per person, rounded to the nearest dollar?
- 64.** In 2002, the estimated population of Kansas was 2,708,935 and the total personal income tax for the state was about \$4,808,361,999. What was the per capita income tax, rounded to the nearest dollar?

Exercises 65–66. A bag of white cheddar corn cakes contains 14 servings, a total of 630 calories and 1820 mg of sodium.

- 65.** How many calories are there per serving?
- 66.** How many milligrams of sodium are there per serving?
- 67.** Juan is advised by his doctor not to exceed 2700 mg of aspirin per day for his arthritis pain. If he takes capsules containing 325 mg of aspirin, how many capsules can he take without exceeding the doctor's orders?

Exercises 68–69 refer to the chapter application. See Table 1.1, page 1.

- 68.** If *Shrek 2* had grossed only half the amount it actually took in, where would it be on the list?
- 69.** Estimate how many times more money *Spider-Man 2* took in than *The Day After Tomorrow*.
- 70.** Jerry Rice of the San Francisco 49ers holds the Super Bowl record for most pass receptions. In the 1989 game, he had 11 receptions for a total of 215 yards. What was the average yardage per reception, rounded to the nearest whole yard?
- 71.** In 2003, the Baltimore Ravens had a roster of 60 players and a payroll of \$76,154,450. Find the average salary, rounded to the nearest thousand dollars.

STATE YOUR UNDERSTANDING

72. Explain to an 8-year-old child that $45 \div 9 = 5$.

73. Explain the concept of remainder.

74. Define and give an example of a quotient.

CHALLENGE

75. The Belgium Bulb Company has 171,000 tulip bulbs to market. Eight bulbs are put in a package when shipping to the United States and sold for \$3 per package. Twelve bulbs are put in a package when shipping to France and sold for \$5 per package. In which country will the Belgium Bulb Company get the greatest gross return? What is the difference in gross receipts?

Exercises 76–77. Complete the problems by writing in the correct digit wherever you see a letter.

$$\begin{array}{r} 5AB2 \\ 3)1653C \end{array}$$

$$\begin{array}{r} 21B \\ A3)4CC1 \end{array}$$

GROUP WORK

78. Divide 23,000,000 and 140,000,000 by 10, 100, 1000, 10,000, and 100,000. What do you observe? Can you devise a rule for dividing by 10, 100, 1000, 10,000, and 100,000?

Getting Ready for Algebra

How & Why

In Section 1.2, the equations involved the inverse operations addition and subtraction. Multiplication and division are also inverse operations. We can use this idea to solve equations containing those operations. For example, if 4 is multiplied by 2, $4 \cdot 2 = 8$, the product is 8. If the product is divided by 2, $8 \div 2$, the result is 4, the original number. In the same manner, if 12 is divided by 3, $12 \div 3 = 4$, the quotient is 4. If the quotient is multiplied by 3, $4 \cdot 3 = 12$, the original number. We use this idea to solve equations in which the variable is either multiplied or divided by a number.

When a variable is multiplied or divided by a number, the multiplication symbols (\cdot or \times) and the division symbol (\div) normally are not written. We write $3x$ for 3 times x and $\frac{x}{3}$ for x divided by 3.

Consider the following:

$$\begin{aligned} 5x &= 30 \\ \frac{5x}{5} &= \frac{30}{5} && \text{Division will eliminate multiplication.} \\ x &= 6 \end{aligned}$$

If x in the original equation is replaced by 6, we have

$$\begin{aligned} 5x &= 30 \\ 5 \cdot 6 &= 30 \\ 30 &= 30 && \text{A true statement.} \end{aligned}$$

Therefore, the solution is $x = 6$.

Now consider when the variable is divided by a number:

$$\begin{aligned} \frac{x}{7} &= 21 \\ 7 \cdot \frac{x}{7} &= 7 \cdot 21 && \text{Multiplication will eliminate division.} \\ x &= 147 \end{aligned}$$

If x in the original equation is replaced by 147, we have

$$\begin{aligned} \frac{147}{7} &= 21 \\ 21 &= 21 && \text{A true statement.} \end{aligned}$$

Therefore, the solution is $x = 147$.

OBJECTIVE

Solve an equation of the form $ax = b$ or $\frac{x}{a} = b$, where x , a , and b are whole numbers.

To solve an equation using multiplication or division

1. Divide both sides by the same number to isolate the variable, or
2. Multiply both sides by the same number to isolate the variable.
3. Check the solution by substituting it for the variable in the original equation.

Warm-Ups A–E

Examples A–E

DIRECTIONS: Solve and check.

STRATEGY: Isolate the variable by multiplying or dividing both sides of the equation by the same number. Check the solution by substituting it for the variable in the original equation.

A. $6y = 18$

A. $3x = 24$

$$\begin{aligned}3x &= 24 \\ \frac{3x}{3} &= \frac{24}{3} \quad \text{Isolate the variable by dividing both sides of the equation by 3.} \\ x &= 8 \quad \text{Simplify.}\end{aligned}$$

CHECK: $3x = 24$

$$\begin{aligned}3(8) &= 24 && \text{Substitute 8 for } x \text{ in the original equation.} \\ 24 &= 24 && \text{The statement is true.}\end{aligned}$$

The solution is $x = 8$.

B. $\frac{a}{5} = 10$

B. $\frac{x}{4} = 9$

$$\begin{aligned}\frac{x}{4} &= 9 \\ 4 \cdot \frac{x}{4} &= 4 \cdot 9 \quad \text{Isolate the variable by multiplying both sides by 4.} \\ x &= 36 \quad \text{Simplify.}\end{aligned}$$

CHECK: $\frac{x}{4} = 9$

$$\begin{aligned}\frac{36}{4} &= 9 && \text{Substitute 36 for } x \text{ in the original equation.} \\ 9 &= 9 && \text{The statement is true.}\end{aligned}$$

The solution is $x = 36$.

C. $\frac{b}{3} = 33$

C. $\frac{c}{7} = 12$

$$\begin{aligned}\frac{c}{7} &= 12 \\ 7 \cdot \frac{c}{7} &= 7 \cdot 12 \quad \text{Isolate the variable by multiplying both sides of the equation by 7.} \\ c &= 84 \quad \text{Simplify.}\end{aligned}$$

CHECK: $\frac{c}{7} = 12$

$$\begin{aligned}\frac{84}{7} &= 12 && \text{Substitute 84 for } c \text{ in the original equation.} \\ 12 &= 12 && \text{The statement is true.}\end{aligned}$$

The solution is $c = 84$.

Answers to Warm-Ups

- A. $y = 3$ B. $a = 50$ C. $b = 99$

D. $9y = 117$

$$9y = 117$$

$$\frac{9y}{9} = \frac{117}{9}$$

$$y = 13$$

Isolate the variable by dividing both sides of the equation by 9.

Simplify.

CHECK: $9y = 117$

$$9(13) = 117$$

$$117 = 117$$

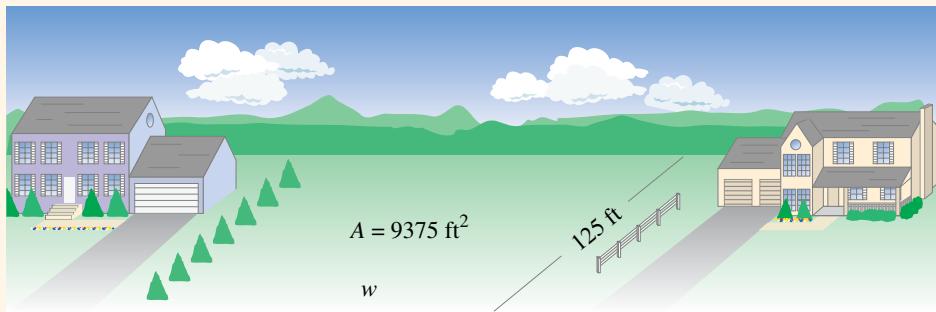
Substitute 13 for y in the original equation.

The statement is true.

The solution is $y = 13$.

- E. What is the width (w) of a rectangular lot in a subdivision if the length (ℓ) is 125 feet and the area (A) is 9375 square feet? Use the formula $A = \ell w$.

STRATEGY: To find the width of the lot, substitute the area, $A = 9375$, and the length, $\ell = 125$, into the formula and solve.



$$A = \ell w$$

$$9375 = 125w \quad A = 9375, \ell = 125$$

$$\frac{9375}{125} = \frac{125w}{125}$$

Divide both sides by 125.

$$75 = w$$

CHECK: If the width is 75 feet and the length is 125 feet, is the area 9375 square feet?

$$A = (125 \text{ ft})(75 \text{ ft}) = 9375 \text{ sq ft} \quad \text{True.}$$

The width of the lot is 75 feet.

D. $8t = 96$

- E. What is the length (ℓ) of a second lot in the subdivision if the width (w) is 90 feet and the area (A) is 10,350 square feet? Use the formula $A = \ell w$.

Answers to Warm-Ups

D. $t = 12$

E. The length of the lot is 115 feet.

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Exercises

OBJECTIVE

Solve an equation of the form $ax = b$ or $\frac{x}{a} = b$, where x , a , and b are whole numbers.

Solve and check.

1. $3x = 15$

2. $\frac{z}{4} = 5$

3. $\frac{c}{3} = 6$

4. $8x = 32$

5. $13x = 52$

6. $\frac{y}{4} = 14$

7. $\frac{b}{2} = 23$

8. $15a = 135$

9. $12x = 144$

10. $\frac{x}{14} = 12$

11. $\frac{y}{13} = 24$

12. $23c = 184$

13. $27x = 648$

14. $\frac{a}{32} = 1216$

15. $\frac{b}{12} = 2034$

16. $57z = 2451$

17. $1098 = 18x$

18. $616 = 11y$

19. $34 = \frac{w}{23}$

20. $64 = \frac{c}{33}$

21. Find the width of a rectangular garden plot that has a length of 35 feet and an area of 595 square feet. Use the formula $A = \ell w$.
23. Crab sells at the dock for \$2 per pound. A fisherman sells his catch and receives \$4680. How many pounds of crab does he sell?
25. If the wholesale cost of 18 stereo sets is \$5580, what is the wholesale cost of one set? Use the formula $C = np$, where C is the total cost, n is the number of units purchased, and p is the price per unit.

27. The average daily low temperature in Toronto in July is twice the average high temperature in January. Write an equation that describes this relationship. Be sure to define all variables in your equation. If the average daily low temperature in July is 60°F , what is the average daily high temperature in January?

22. Find the length of a room that has an area of 391 square feet and a width of 17 feet.
24. Felicia earns \$7 an hour. Last week she earned \$231. How many hours did she work last week?
26. Using the formula in Exercise 25, if the wholesale cost of 24 personal computers is \$18,864, what is the wholesale cost of one computer?
28. Car manufacturers recommend that the fuel filter in a car be replaced when the mileage is ten times the recommended mileage for an oil change. Write an equation that describes this relationship. Be sure to define all variables in your equation. If a fuel filter should be replaced every 30,000 miles, how often should the oil be changed?

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1.5 Whole-Number Exponents and Powers of 10

VOCABULARY

A **base** is a number used as a repeated factor. An **exponent** indicates the number of times the base is used as a factor and is always written as a superscript to the base. In 2^3 , 2 is the base and 3 is the exponent.

The **value** of 2^3 is 8.

An exponent of 2 is often read “**squared**” and an exponent of 3 is often read “**cubed**.”

A **power of 10** is the value obtained when 10 is written with an exponent.

OBJECTIVES

- Find the value of an expression written in exponential form.
- Multiply or divide a whole number by a power of 10.

How & Why

OBJECTIVE 1

Find the value of an expression written in exponential form.

Exponents show repeated multiplication. Whole-number *exponents* greater than 1 are used to write repeated multiplications in shorter form. For example,

5^4 means $5 \cdot 5 \cdot 5 \cdot 5$

and since $5 \cdot 5 \cdot 5 \cdot 5 = 625$ we write $5^4 = 625$. The number 625 is sometimes called the “fourth power of five” or “the *value* of 5^4 .”

EXONENT
↓
BASE → $5^4 = 625$ ← VALUE

Similarly, the value of 7^6 is

$$7^6 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 117,649.$$

The base, the repeated factor, is 7. The exponent, which indicates the number of times the base is used as a factor, is 6.

The exponent 1 is a special case.

In general, $x^1 = x$. So $2^1 = 2$, $13^1 = 13$, $7^1 = 7$, and $(413)^1 = 413$.

We can see a reason for the meaning of $6^1(6^1 = 6)$ by studying the following pattern.

$$6^4 = 6 \cdot 6 \cdot 6 \cdot 6$$

$$6^3 = 6 \cdot 6 \cdot 6$$

$$6^2 = 6 \cdot 6$$

$$6^1 = 6$$

Exponential Property of One

If 1 is used as an exponent, the value is equal to the base.

$$b^1 = b$$

To find the value of an expression with a natural number exponent

- If the exponent is 1, the value is the same as the base.
- If the exponent is greater than 1, use the base number as a factor as many times as shown by the exponent. Multiply.

Exponents give us a second way to write an area measurement. Using exponents we can write 74 square inches as 74 in^2 . The symbol 74 in^2 is still read “seventy-four square inches.” Also 65 square feet is written as 65 ft^2 .

Warm-Ups A–F

Examples A–F BASIC Mathematics Now™

A. Find the value of 14^2 .

A. Find the value of 9^4 .

$$9^4 = 9 \cdot 9 \cdot 9 \cdot 9 \quad \text{Use 9 as a factor four times.} \\ = 6561$$

The value is 6561.

B. Simplify: 72^1

B. Simplify: 29^1

$$29^1 = 29 \quad \text{If the exponent is 1, the value is the base number.}$$

The value is 29.

C. Find the value of 10^6 .

C. Find the value of 10^7 .

$$10^7 = (10)(10)(10)(10)(10)(10) \\ = 10,000,000$$

The value is 10,000,000.

Ten million.

Note that the value has seven zeros.

D. Evaluate: 7^4

D. Evaluate: 6^5

$$6^5 = 6(6)(6)(6)(6) = 7776$$

The value is 7776.

E. Find the value of 4^{11} .

CALCULATOR EXAMPLE

E. Find the value of 8^7 .

Calculators usually have an exponent key marked y^x or \wedge . If your calculator doesn't have such a key, you can multiply the repeated factors.

The value is 2,097,152.

F. Herman's vegetable garden covers a total of 700 square feet. Write the area of his garden using an exponent to show the measure.

F. Mabel needs 125 square yards of carpet to carpet her house wall to wall. Write the amount of carpet needed using an exponent to show the measure.

$$125 \text{ square yards} = 125 \text{ yd}^2 \quad \text{Square yards is written yd}^2.$$

Mabel needs 125 yd^2 of carpet.

How & Why

OBJECTIVE 2

Multiply or divide a whole number by a power of 10.

It is particularly easy to multiply or divide a whole number by a power of 10. Consider the following and their products when multiplied by 10.

$$4 \times 10 = 40 \quad 9 \times 10 = 90 \quad 17 \times 10 = 170 \quad 88 \times 10 = 880$$

The place value of every digit becomes 10 times larger when the number is multiplied by 10. So to multiply by 10, we need to merely write a zero to the right of the whole number. If a whole number is multiplied by 10 more than once, a zero is written to the right for each 10. So

$$42 \times 10^3 = 42,000 \quad \text{Three zeros are written on the right, one for each 10.}$$

Answers to Warm-Ups

- A. 196 B. 72 C. 1,000,000
- D. 2401 E. 4,194,304
- F. The area of Herman's garden is 700 ft².

Because division is the inverse of multiplication, dividing by 10 will eliminate the last zero on the right of a whole number. So,

$$650,000 \div 10 = 65,000 \quad \text{Eliminate the final zero on the right.}$$

If we divide by 10 more than once, one zero is eliminated for each 10. So,

$$650,000 \div 10^4 = 65 \quad \text{Eliminate four zeros on the right.}$$

To multiply a whole number by a power of 10

1. Identify the exponent of 10.
2. Write as many zeros to the right of the whole number as the exponent of 10.

To divide a whole number by a power of 10

1. Identify the exponent of 10.
2. Eliminate the same number of zeros to the right of the whole number as the exponent of 10.

Using powers of 10, we have a third way of writing a whole number in expanded form.

$$\begin{aligned} 8257 &= 8000 + 200 + 50 + 7, \text{ or} \\ &= 8 \text{ thousands} + 2 \text{ hundreds} + 5 \text{ tens} + 7 \text{ ones}, \text{ or} \\ &= (8 \times 10^3) + (2 \times 10^2) + (5 \times 10^1) + (7 \times 1) \end{aligned}$$

Examples G–K



BASIC
Mathematics Now™

DIRECTIONS: Multiply or divide.

STRATEGY: Identify the exponent of 10. For multiplication, write the same number of zeros to the right of the whole number as the exponent of 10. For division, eliminate the same number of zeros on the right of the whole number as the exponent of 10.

G. Multiply: $45,786 \times 10^5$

$$45,786 \times 10^5 = 4,578,600,000$$

The exponent of 10 is 5. To multiply, write 5 zeros to the right of the whole number.

The product is 4,578,600,000.

H. Simplify: 569×10^6

$$569 \times 10^6 = 569,000,000$$

The product is 569,000,000.

I. Divide: $\frac{134,000}{10^2}$

$$\frac{134,000}{10^2} = 1340 \quad \text{The exponent of 10 is 2.}$$

To divide, eliminate 2 zeros on the right of the whole number.

The quotient is 1340.

Warm-Ups G–K

G. Multiply: 9821×10^4

H. Simplify: 67×10^7

I. Divide: $\frac{72,770,000}{10^3}$

Answers to Warm-Ups

- G. 98,210,000 H. 670,000,000
I. 72,770

J. Simplify: $1,450,000 \div 10^2$

- K. A survey of 1,000,000 (10^6) people indicated that they pay an average of \$6432 in federal taxes. What was the total paid in taxes?

J. Simplify: $803,200,000 \div 10^5$

$$803,200,000 \div 10^5 = 8032 \quad \text{Eliminate 5 zeros on the right.}$$

The quotient is 8032.

- K. A recent fund-raising campaign raised an average of \$315 per donor. How much was raised if there were 100,000 (10^5) donors?

STRATEGY: To find the total raised, multiply the average donation by the number of donors.

$$315 \times 10^5 = 31,500,000 \quad \text{The exponent is 5. To multiply, write 5 zeros to the right of the whole number.}$$

The campaign raised \$31,500,000.

Answers to Warm-Ups

J. 14,500

- K. The total paid in taxes was \$6,432,000,000.

Exercises 1.5

OBJECTIVE 1

Find the value of an expression written in exponential form.

A Write in exponential form.

1. $12(12)(12)(12)(12)(12)(12)$

2. $43 \times 43 \times 43 \times 43 \times 43 \times 43 \times 43 \times 43$

Find the value.

3. 7^2

4. 6^2

5. 2^3

6. 3^3

7. 15^1

8. 20^1

9. In $17^3 = 4913$, 17 is the _____, 3 is the _____, and 4913 is the _____.10. In $6^4 = 1296$, 1296 is the _____, 6 is the _____, and 4 is the _____.**B Find the value.**

11. 6^3

12. 2^7

13. 21^2

14. 15^2

15. 10^3

16. 10^6

17. 8^3

18. 7^4

19. 3^8

20. 5^6

OBJECTIVE 2

Multiply or divide a whole number by a power of 10.

A Multiply or divide.

21. 72×10^2

22. 36×10^1

23. 21×10^5

24. 69×10^4

25. $2300 \div 10^2$

26. $49,000 \div 10^2$

27. $3,700,000 \div 10^3$

28. $346,000 \div 10^1$

29. To multiply a number by a power of 10, write as many zeros to the right of the number as the _____ of 10.

30. To divide a number by a power of 10, eliminate as many _____ on the right of the number as the exponent of 10.

B Multiply or divide.

31. 499×10^6

32. 513×10^3

33. $8,710,000 \div 10^3$

34. $92,000,000 \div 10^3$

35. 6734×10^4

36. 5466×10^5

37. $\frac{61,200}{100}$

38. $\frac{13,700}{10^2}$

39. 5620×10^8

40. 732×10^5

41. $450,000,000 \div 10^5$

42. $8,953,000,000 \div 10^6$

C

- 43.** Write in exponent form:

$$10(10)(10)(10)(10)(10)(10)(10)(10)(10)$$

- 44.** Write in exponent form:

$$4(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)(4)$$

Find the value.

45. 11^5

46. 23^4

47. 8^9

48. 7^8

Multiply or divide.

49. $47,160 \times 10^9$

50. 630×10^{13}

51. $\frac{79,000,000,000}{10^7}$

52. $\frac{3,120,000,000,000}{10^8}$

- 53.** Salvador bought a lot for his new house in a suburban subdivision. The lot consists of 10,800 square feet. Write the size of Salvador's lot using an exponent to express the measure.

- 55.** The operating budget for city parks in a large metropolitan area is approximately 84×10^6 dollars. Write this amount in place value form.

- 57.** The distance from Earth to the nearest star outside our solar system (Alpha Centauri) is approximately 255×10^{11} miles. Write this distance in place value form.

- 59.** The distance that light travels in a year is called a light-year. This distance is almost 6 trillion miles. Write the place value name for this number. Write the number as 6 times a power of 10.

- 54.** To make her niece's wedding gown, Marlene needs 25 square meters of material. Write the amount of material she needs using an exponent to express the measure.

- 56.** In 2005, President Bush proposed to increase the national debt by 2×10^{12} dollars to fund the partial privatization of Social Security. Write this amount in place value form.

- 58.** A high roller in Atlantic City places nine consecutive bets at the "Twenty-one" table. The first bet is \$4 and each succeeding bet is four times the one before. How much does she wager on the ninth bet? Express the answer as a power of 4 and as a whole number.

- 60.** The average distance from Earth to the sun is approximately 93 million miles. Write the place value name for this distance and write it as 93 times a power of 10.

Exercises 61–63. A bacteria culture is tripling in size every hour. There are three bacteria at the start of the experiment.

- 61.** How many bacteria are there after 4 hours? Write your answer in exponential form and also in place value notation.

- 63.** How many hours will it take for the number of bacteria to exceed 1000?

- 62.** How many bacteria are there after 14 hours? Write your answer in exponential form and also in place value notation.

- 64.** The distance around Earth at the equator is about 25×10^3 miles. Write this number in place value notation.

- 65.** The surface area of the Pacific Ocean is about 642×10^5 square miles. Write this number in place value notation.

- 66.** The area of the three largest deserts in the world is given below.

The Three Largest Deserts in the World

Desert	Sahara (N. Africa)	Arabian (Arabian Peninsula)	Gobi (Mongolia/China)
Approximate Area in Square Miles	3,320,000	900,000	450,000

Write each area as the product of a whole number and a power of 10.

- 67.** Disk space in a personal computer is commonly measured in gigabytes, which are billions of bytes. Write the number of bytes in 1 gigabyte as a power of 10.

- 68.** A googol is 1 followed by 100 zeros. Legend has it that the name for this number was chosen by a 9-year-old nephew of mathematician Edward Kasner. Write a googol as a power of 10.

Exercises 69–72 refer to the chapter application. See Table 1.1, page 1.

- 69.** Round the earnings of *Meet the Fockers* to the nearest million dollars and then express this amount as a product of a number and a power of 10.

- 70.** Round the earnings of *The Passion of the Christ* to the nearest hundred million dollars and then express this amount as a product of a number and a power of 10.

- 71.** What is the smallest power of 50 that is larger than the gross earnings of *The Incredibles*?

- 72.** What is the smallest power of 2 that is larger than the gross earnings of *Shrek 2*?

STATE YOUR UNDERSTANDING

- 73.** Explain what is meant by 4^{10} .

- 74.** Explain how to multiply a whole number by a power of 10. Give at least two examples.

CHALLENGE

- 75.** Mitchell's grandparents deposit \$4 on his first birthday and quadruple that amount on each succeeding birthday until he is 10. What amount did Mitchell's grandparents deposit on his 10th birthday? What is the total amount they have deposited in the account?

- 76.** Find the sum of the cubes of the digits.

- 77.** Find the difference between the sum of the cubes of 5, 10, 12, and 24 and the sum of the fifth powers of 2, 4, 6, and 7.

- 78.** The sun is estimated to weigh 2 octillion tons. Write the place value for this number. Write it as 2 times a power of 10.

GROUP WORK

79. Research your local newspaper and find at least five numbers that could be written using a power of 10. Write these as a product using a power of 10.

80. Have each member of your group work the following problem independently.

$$30 - 4 \cdot 6 - 3 + 7$$

Compare your answers. If they are different, be prepared to discuss in class why they are different.

1.6 Order of Operations

VOCABULARY

Parentheses () and **brackets** [] are used in mathematics as **grouping** symbols. These symbols indicate that the operations inside are to be performed first. Other grouping symbols that are often used are **braces** {} and the **fraction bar** —.

OBJECTIVE

Perform any combination of operations on whole numbers.

How & Why

OBJECTIVE

Perform any combination of operations on whole numbers.

Without a rule it is possible to interpret $6 + 4 \cdot 11$ in two ways:

$$\begin{aligned} 6 + 4 \cdot 11 &= 6 + 44 \\ &= 50 \end{aligned}$$

or

$$\begin{aligned} 6 + 4 \cdot 11 &= 10 \cdot 11 \\ &= 110 \end{aligned}$$

In order to decide which answer to use, we agree to use a standard set of rules. Among these is the rule that we multiply before adding. So

$$6 + 4 \cdot 11 = 50$$

The order in which the operations are performed is important because the order often determines the answer. Therefore, there is an established *order of operations*. This established order was agreed upon many years ago and is programmed into most of today's calculators and computers.

Order of Operations

To evaluate an expression with more than one operation

1. Parentheses—Do the operations within grouping symbols first (parentheses, fraction bar, etc.), in the order given in steps 2, 3, and 4.
2. Exponents—Do the operations indicated by exponents.
3. Multiply and Divide—Do multiplication and division as they appear from left to right.
4. Add and Subtract—Do addition and subtraction as they appear from left to right.

So we see that

$$15 - 8 \div 2 = 15 - 4$$

Divide first.
Subtract.

$$= 11$$

$$(18 - 11)(5) = 7(5)$$

Subtract in parentheses first.
Multiply.

$$= 35$$

$$84 \div 21 \cdot 4 = 4 \cdot 4$$

Neither multiplication nor division takes preference over the other. So do them from left to right.

$$= 16$$

As you can see, the rules for the order of operations are fairly complicated and it is important that you learn them all. A standard memory trick is to use the first letters to make an easy-to-remember phrase.

Parentheses

Exponents

Multiplication/Division

Addition/Subtraction

Consider the phrase: **Please Excuse My Dear Aunt Sally**. Note that the first letters of the words in this phrase are exactly the same (and in the same order) as the first letters for the order of operations.

Many students use “Please excuse my dear Aunt Sally” to help them remember the order for operations. Why not give it a try?

CAUTION

Brackets [] and braces { } are often used differently in calculators. Consult your calculator manual.

Exercises involving all of the operations are shown in the examples.

Warm-Ups A–G

A. Simplify:
 $7 \cdot 3 + 6 \cdot 8$

B. Simplify:
 $6 \cdot 11 - 16 \div 4 + 8 \cdot 2$

C. Simplify:
 $54 \div 6 + 15 - 2(9 - 6)$

D. Simplify:
 $6 \cdot 4^2 - 3 \cdot 5^2 + 47 - 8 \cdot 4$

Examples A–G



BASIC Mathematics Now™

DIRECTIONS: Simplify.

STRATEGY: The operations are done in this order: operations in parentheses first, exponents next, then multiplication and division, and finally, addition and subtraction.

A. Simplify: $8 \cdot 9 + 5 \cdot 2$

$$\begin{aligned} 8 \cdot 9 + 5 \cdot 2 &= 72 + 10 && \text{Multiply first.} \\ &= 82 && \text{Add.} \end{aligned}$$

B. Simplify: $33 - 9 \div 3 + 7 \cdot 4$

$$\begin{aligned} 33 - 9 \div 3 + 7 \cdot 4 &= 33 - 3 + 28 && \text{Divide and multiply.} \\ &= 30 + 28 && \text{Subtract.} \\ &= 58 && \text{Add.} \end{aligned}$$

C. Simplify: $4 \cdot 8 + 22 - 5(7 + 2)$

$$\begin{aligned} 4 \cdot 8 + 22 - 5(7 + 2) &= 4 \cdot 8 + 22 - 5(9) && \text{Add in parentheses first.} \\ &= 32 + 22 - 45 && \text{Multiply.} \\ &= 54 - 45 && \text{Add.} \\ &= 9 && \text{Subtract.} \end{aligned}$$

D. Simplify: $2 \cdot 3^4 - 5 \cdot 2^2 + 31$

$$\begin{aligned} 2 \cdot 3^4 - 5 \cdot 2^2 + 31 &= 2 \cdot 81 - 5 \cdot 4 + 31 && \text{Do exponents first.} \\ &= 162 - 20 + 31 && \text{Multiply.} \\ &= 142 + 31 && \text{Subtract.} \\ &= 173 && \text{Add.} \end{aligned}$$

Answers to Warm-Ups

- A. 69 B. 78 C. 18 D. 36

E. Simplify: $(4^2 - 4 \cdot 3)^2 + 8^2$

STRATEGY: First do the operations in the parentheses following the proper order.

$$\begin{aligned}(4^2 - 4 \cdot 3)^2 + 8^2 &= (16 - 4 \cdot 3)^2 + 8^2 && \text{Do the exponent in parentheses first.} \\ &= (16 - 12)^2 + 8^2 && \text{Multiply.} \\ &= (4)^2 + 8^2 && \text{Subtract.}\end{aligned}$$

Now that the operations inside the parentheses are complete, continue using the order of operations.

$$\begin{aligned}&= 16 + 64 && \text{Do the exponents} \\ &= 80 && \text{Add.}\end{aligned}$$

CALCULATOR EXAMPLE



F. Simplify: $2469 + 281 \cdot 11 - 3041$

Enter the numbers and operations as they appear from left to right.

The answer is 2519.

- G. Food for the Poor prepares two types of food baskets for distribution to the victims of the 2004 tsunami in Southeast Asia. The family pack contains 15 pounds of rice and the elderly pack contains 8 pounds of rice. How many pounds of rice are needed for 325 family packs and 120 elderly packs?

STRATEGY: To find the number of pounds of rice needed for the packs, multiply the number of packs by the number of pounds per pack. Then add the two amounts.

$$\begin{aligned}325(15) + 120(8) &= 4875 + 960 && \text{Multiply.} \\ &= 5835 && \text{Add.}\end{aligned}$$

Food for the Poor needs 5835 pounds of rice.

E. Simplify: $(5^3 - 220 \div 2)^2 + 7^2$

F. Simplify: $4236 + 17 \cdot 584 \div 8$

- G. The Fruit-of-the Month Club prepares two types of boxes for shipment. Box A contains 12 apples and Box B contains 16 apples. How many apples are needed for 125 orders of Box A and 225 orders of Box B?

Answers to Warm-Ups

E. 274 F. 5477

G. The orders require 5100 apples.

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Exercises 1.6

OBJECTIVE

Perform any combination of operations on whole numbers.

A Simplify.

- | | | | |
|--------------------------------|---------------------------------|------------------------------------|------------------------------------|
| 1. $4 \cdot 8 + 12$ | 2. $21 + 7 \cdot 5$ | 3. $28 - 4 \cdot 7$ | 4. $32 \cdot 5 - 12$ |
| 5. $51 + 24 \div 3$ | 6. $54 \div 6 - 7$ | 7. $53 - (23 + 2)$ | 8. $(41 - 15) - 11$ |
| 9. $84 \div 7 \times 4$ | 10. $72 \div 8 \times 3$ | 11. $34 + 6 \cdot 3 - 3$ | 12. $36 - 4 \cdot 7 + 9$ |
| 13. $2^3 - 5 + 3^3$ | 14. $6^2 \div 9 + 4^2$ | 15. $5 \cdot 7 + 3 \cdot 6$ | 16. $7 \cdot 7 - 6 \cdot 5$ |

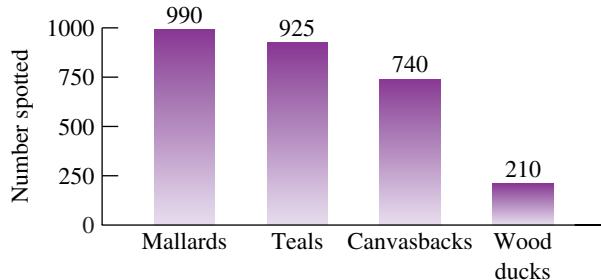
B

- | | | |
|--|---|---|
| 17. $4^2 - 5 \cdot 2 + 7 \cdot 6$ | 18. $7^2 + 30 \div 5 + 2 \cdot 8$ | 19. $72 \div 8 + 12 - 5$ |
| 20. $42 \cdot 5 \div 14 + 13 - 6$ | 21. $(34 + 17) - (51 - 42)$ | 22. $(63 - 28) - (20 + 7)$ |
| 23. $64 \div 8 \cdot 2^3 + 6 \cdot 7$ | 24. $90 \div 15 \cdot 5^2 + 4 \cdot 9$ | 25. $75 \div 15 \cdot 7$ |
| 26. $120 \div 15 \cdot 6$ | 27. $88 - 3(45 - 37) + 42 - 35$ | 28. $46(37 - 33) \div 8 - 15 + 31$ |
| 29. $4^4 + 5^3$ | 30. $3^5 - 2^4 + 7^2$ | 31. $102 \cdot 3^3 - 72 \div 6 + 15$ |
| 32. $17 \cdot 2^3 - 99 \div 9 + 35$ | | |

C

- 33.** $50 - 12 \div 6 - 36 \div 6 + 3$ **34.** $80 - 24 \div 4 + 30 \div 6 + 4$

Exercises 35–38. The graph shows the count of ducks, by species, at a lake in northern Idaho, as taken by a chapter of Ducks Unlimited.



- 35.** How many more mallards and canvasbacks were counted than teals and wood ducks?
- 36.** If twice as many canvasbacks had been counted, how many more canvasbacks would there have been than teals?

37. If four times the number of wood ducks had been counted, how many more wood ducks and mallards would there have been than teals and canvasbacks?

38. If twice the number of teals had been counted, how many more teals and wood ducks would there have been compared with mallards and canvasbacks?

Simplify.

39. $11(2^3 \cdot 3 - 20) \div 4 + 33$

41. $4(11 - 5)^3 - 7^3$

43. To begin a month, Elmo's Janitorial Service, has an inventory of 110 packages of 6-count paper towels at \$9 each, 65 cans of powdered cleanser at \$2 each, and 75 disposable mops at \$11 each. At the end of the month they have 26 packages of towels, 32 cans of cleanser, and 7 mops. What is the cost of the supplies used for the month?

45. A long-haul trucker is paid \$29 for every 100 miles driven and \$17 per stop. If he averages 2400 miles and 35 stops per week, what is his average weekly income? Assuming he takes 2 weeks off each year for vacation, what is his average yearly income?

40. $15(4^3 \cdot 5 - 61) \div 7 - 155$

42. $9(7 - 3)^3 - 21(3 - 1)^4$

44. Muriel's Appliance Store ordered television sets that cost \$135 each and DVD players at \$71 each. She sells the TVs for \$217 each and the DVD players for \$116 each. If she sells 28 TVs and 35 DVD players, how much net income is realized on the sale of the TVs and DVD players?

Exercises 46–47. The labels for bread, cereal with milk, orange juice, and jam show their nutrition facts.

Cereal		
Nutrition Facts		
Serving Size 1/2 cup (58g) Servings per Container about 12		
Amount Per Serving	Cereal with 1/2 cup Cereal	Skim milk
Calories	200	240
Calories from Fat	10	10
% Daily Values**		
Total Fat 1g*	2%	2%
Saturated Fat 3g	0%	0%
Cholesterol 0mg	0%	0%
Sodium 350mg	15%	17%
Potassium 160mg	5%	10%
Total Carbohydrate 47g	16%	18%
Dietary Fiber 5g	21%	21%
Sugars 7g		
Other Carbohydrates 35g		
Protein 6g		

Orange Juice		
Nutrition Facts		
Serving Size 8 fl oz (240 mL) Servings Per Container 8		
Amount Per Serving		
Calories 110	Calories from Fat 0	% Daily Value*
Total Fat 0g	0g	0%
Sodium 0mg	0mg	0%
Potassium 450mg	13%	
Total Carbohydrate 26g	9%	
Sugars 22g		
Protein 2g		
Vitamin C 120% • Calcium 2%		
Thiamin 10% • Niacin 4%		
Vitamin B6 6% • Folate 15%		
Not a significant source of saturated fat, cholesterol, dietary fiber, vitamin A and iron		
*Percent Daily Values are based on a 2,000 calorie diet.		

Jam		
Nutrition Facts		
Serving Size 1 Tbsp. (20g) Servings per Container about 25		
Amount Per Serving		
Calories 50	Calories from Fat 0	% Daily Value*
Total Fat 0g	0g	0%
Sodium 15mg	15mg	1%
Total Carbohydrate 13g	4%	
Sugars 13g		
Protein 0g		
*Percent Daily Values are based on a 2,000 calorie diet.		

Bread		
Nutrition Facts		
Serving Size 1 slice (49g) Servings Per Container 14		
Amount Per Serving		
Calories 120	Calories from Fat 10	% Daily Value*
Total Fat 1g	1g	2%
Saturated Fat 0g	0g	0%
Cholesterol 0mg	0mg	0%
Sodium 360mg	360mg	15%
Total Carbohydrate 24g	8%	
Dietary Fiber 1g	1g	5%
Sugars 1g		
Protein 4g		
Vitamin A 0% • Vitamin C 0%		
Thiamin 10% • Riboflavin 8%		
Calcium 2% • Iron 10%		
Niacin 8% • Folic Acid 10%		
*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.		

- 46.** How many milligrams (mg) of sodium are consumed if Marla has 3 servings of orange juice, 2 servings of cereal, and 2 slices of bread with jam for breakfast? Milk contains 62 mg of sodium per one-half cup serving.
- 48.** Ruth orders clothes from an Appleseed's catalog. She orders two turtleneck tops for \$35 each, two pairs of stretch pants for \$45 each, one suede jacket for \$130, and three Italian leather belts for \$25 each. The shipping and handling charge for her order is \$15. What is the total charge for Ruth's order?
- 50.** Ron makes the following purchases at his local auto supply store: four spark plugs at \$1 each, a can of tire sealant for \$4, two heavy duty shock absorbers at \$10 each, a case (12 quarts) of motor oil at \$1 per quart, and a gallon of antifreeze for \$5. The sales tax for his purchases is \$3. Ron has a store credit of \$25. How much does he owe the store for this transaction?
- 47.** How many calories does Marla consume when she eats the breakfast listed in Exercise 46?
- 49.** Sally orders birthday presents for her parents from a catalog. She orders each parent a 9-band AM/FM radio at \$30 each. She orders her dad three double decks of playing cards at \$3 each and a gold-clad double eagle coin for \$20. She orders her mother a deluxe touch panel telephone for \$21 and four rainbow garden flower bulb sets for \$18 each. The shipping and handling charges for the order are \$8. What is the total charge for Sally's order?
- 51.** Clay and Connie have a \$100 gift certificate to the Olive Garden. One night they order an appetizer for \$6, veal parmesan for \$17, and fettuccine alfredo for \$13. They each have a glass of wine, at \$5 each. They end the dinner with desserts for \$6 each and coffee for \$2 each. How much do they have left on their gift certificate after the meal?

Exercises 52–55. To determine the intensity of cardiovascular training, a person needs to know her maximal heart rate (MHR) and resting heart rate (RHR). The MHR may be approximated by subtracting your age from 220. The RHR can be measured by sitting quietly for about 15 minutes, then taking your pulse for 15 seconds, and multiplying it by 4. The difference between the MHR and the RHR is the heart rate reserve (HRR), the number of beats available to go from rest to all-out effort. To determine a target training rate, divide the HRR by 2 and add the results to the RHR.

- 52.** Jessie is 30 years old and has an RHR of 70 beats per minute (bpm). Determine her cardiovascular target training rate.
- 54.** Determine your own cardiovascular target training rate.
- 53.** June is 64 years old. She measures her resting heart rate as 16 beats in 15 seconds. Determine her cardiovascular target training rate.
- 55.** As you age, does your target training rate increase or decrease? Why?

Exercises 56–58 refer to the chapter application. See Table 1.1, page 1.

- 56.** Calculate the sum of the gross earnings for *Shrek 2* and *The Polar Express* and divide by 2.
- 57.** Calculate the sum of the gross earnings for *Shrek 2* and *The Polar Express* divided by 2.
- 58.** Use the order of operations to explain why the answers to Exercise 56 and Exercise 57 are not the same.

STATE YOUR UNDERSTANDING

- 59.** Which of the following is correct? Explain.

$$20 - 10 \div 2 = 10 \div 2 \quad \text{or} \quad 20 - 10 \div 2 = 20 - 5 \\ = 5 \qquad \qquad \qquad = 15$$

- 60.** Explain how to simplify $2(1 + 36 \div 3^2) - 3$ using the order of operations.

CHALLENGE

- 61.** Simplify: $(6 \cdot 3 - 8)^2 - 50 + 2 \cdot 3^2 + 2(9 - 5)^3$

- 62.** USA Video buys three first-run movies: #1 for \$185, #2 for \$143, and #3 for \$198. During the first 2 months, #1 is rented 10 times at the weekend rate of \$5 and 26 times at the weekday rate of \$3; #2 is rented 12 times at the weekend rate and 30 times at the weekday rate; and #3 is rented 8 times at the weekend rate and 18 times at the weekday rate. How much money must still be raised to pay for the cost of the three videos?

GROUP WORK

- 63.** It is estimated that hot water heaters need to be big enough to accommodate the water usage for an entire hour. To figure what size heater you need, first identify the single hour of the day in which water usage is highest. Next, identify the types of water usage during this hour. The estimates of water usage for various activities are shown in the table.

Water Usage

Activity	Gallons Used
Shower	20
Bath	20
Washing hair	4
Shaving	2
Washing hands/face	4
Dishwasher	14

Now calculate the total number of gallons of water used in your hour. Your water heater must have this capacity.

Getting Ready for Algebra

How & Why

Recall that we have solved equations involving only one operation. Let's look at some equations that involve two operations.

To solve $x - 5 = 7$, we add 5 to both sides of the equation. To solve $4x = 16$, we divide both sides of the equation by 4. The following equation requires both steps.

$$\begin{aligned} 4x - 5 &= 7 \\ 4x - 5 + 5 &= 7 + 5 \quad \text{First, eliminate the subtraction by adding 5 to both sides.} \\ 4x &= 12 \quad \text{Simplify both sides.} \\ \frac{4x}{4} &= \frac{12}{4} \quad \text{Eliminate the multiplication. Divide both sides by 4.} \\ x &= 3 \quad \text{Simplify.} \end{aligned}$$

CHECK: $4x - 5 = 7$

$$\begin{aligned} 4(3) - 5 &= 7 && \text{Substitute 3 for } x. \\ 12 - 5 &= 7 && \text{Multiply.} \\ 7 &= 7 && \text{Subtract.} \end{aligned}$$

Thus, if x is replaced by 3 in the original equation, the statement is true. So the solution is $x = 3$. Now solve

$$\begin{aligned} 5x + 11 &= 46 \\ 5x + 11 - 11 &= 46 - 11 \quad \text{First, eliminate the addition by subtracting 11 from both sides.} \\ 5x &= 35 \quad \text{Simplify.} \\ \frac{5x}{5} &= \frac{35}{5} \quad \text{Eliminate the multiplication by dividing both sides by 5.} \\ x &= 7 \quad \text{Simplify.} \end{aligned}$$

CHECK: $5x + 11 = 46$

$$\begin{aligned} 5(7) + 11 &= 46 && \text{Substitute 7 for } x \text{ in the original equation.} \\ 35 + 11 &= 46 && \text{Multiply.} \\ 46 &= 46 && \text{Add.} \end{aligned}$$

Thus, if x is replaced by 7 in the original equation, the statement is true. So the solution is $x = 7$.

Note that in each of the previous examples the operations are eliminated in the opposite order in which they are performed. That is, addition and subtraction were eliminated first and then the multiplication and division.

OBJECTIVE

Solve equations of the form $ax + b = c$, $ax - b = c$, $\frac{x}{a} + b = c$, and $\frac{x}{a} - b = c$, in which x , a , b , and c are whole numbers.

To solve an equation of the form $ax + b = c$,

$$ax - b = c, \frac{x}{a} + b = c, \text{ or } \frac{x}{a} - b = c$$

1. Eliminate the addition or subtraction by subtracting or adding the same number to both sides.
2. Eliminate the multiplication or division by dividing or multiplying by the same number on both sides.
3. Check the solution by substituting it in the original equation.

Warm-Ups A–E

Examples A–E

DIRECTIONS: Solve and check.

STRATEGY: Isolate the variable by first adding or subtracting the same number from both sides. Second, multiply or divide both sides by the same number.

A. $6x - 11 = 55$

A. $3x - 23 = 7$
 $3x - 23 = 7$
 $3x - 23 + 23 = 7 + 23$ Add 23 to both sides to eliminate the subtraction.
 $3x = 30$ Simplify.
 $\frac{3x}{3} = \frac{30}{3}$ Divide both sides by 3 to eliminate the multiplication.
 $x = 10$ Simplify.

CHECK: $3x - 23 = 7$
 $3(10) - 23 = 7$ Substitute 10 for x in the original equation.
 $30 - 23 = 7$ Simplify.
 $7 = 7$ The statement is true.

The solution is $x = 10$.

B. $\frac{a}{7} + 11 = 15$

B. $\frac{y}{9} + 3 = 13$
 $\frac{y}{9} + 3 = 13$
 $\frac{y}{9} + 3 - 3 = 13 - 3$ Subtract 3 from both sides to eliminate the addition.
 $\frac{y}{9} = 10$ Simplify.
 $9\left(\frac{y}{9}\right) = 9(10)$ Multiply both sides by 9 to eliminate the division.
 $y = 90$

CHECK: $\frac{y}{9} + 3 = 13$
 $\frac{90}{9} + 3 = 13$ Substitute 90 for y in the original equation.
 $10 + 3 = 13$ Simplify.
 $13 = 13$ The statement is true.

The solution is $y = 90$.

C. $\frac{t}{5} - 11 = 14$

C. $\frac{z}{4} - 8 = 14$
 $\frac{z}{4} - 8 = 14$
 $\frac{z}{4} - 8 + 8 = 14 + 8$ Add 8 to both sides to eliminate the subtraction.
 $\frac{z}{4} = 22$ Simplify.
 $4\left(\frac{z}{4}\right) = 4(22)$ Multiply both sides by 4 to eliminate the division.
 $z = 88$ Simplify.

Answers to Warm-Ups

- A. $x = 11$ B. $a = 28$
C. $t = 125$

CHECK: $\frac{z}{4} - 8 = 14$

$$\begin{aligned}\frac{88}{4} - 8 &= 14 && \text{Substitute 88 for } z \text{ in the original equation.} \\ 22 - 8 &= 14 && \text{Simplify.} \\ 14 &= 14 && \text{The statement is true.}\end{aligned}$$

The solution is $z = 88$.

D. $6b + 14 = 26$

$$6b + 14 = 26$$

$$\begin{aligned}6b + 14 - 14 &= 26 - 14 && \text{Subtract 14 from both sides to eliminate the addition.} \\ 6b &= 12 && \text{Simplify.} \\ \frac{6b}{6} &= \frac{12}{6} && \text{Divide both sides by 6 to eliminate the multiplication.} \\ b &= 2 && \text{Simplify.}\end{aligned}$$

CHECK: $6b + 14 = 26$

$$\begin{aligned}6(2) + 14 &= 26 && \text{Substitute 2 for } b \text{ in the original equation.} \\ 12 + 14 &= 26 && \text{Simplify.} \\ 26 &= 26 && \text{The statement is true.}\end{aligned}$$

The solution is $b = 2$.

- E. The formula for the balance of a loan (D) is $D + NP = B$, where P represents the monthly payment, N represents the number of payments, and B represents the amount of money borrowed. Find the number of payments that have been made on an original loan of \$875 with a current balance of \$425 if the payment is \$25 per month.

STRATEGY: Substitute the given values in the formula and solve.

$$\begin{aligned}D + NP &= B \\ 425 + N(25) &= 875 && \text{Substitute 425 for } D, 25 \text{ for } P \text{ and 875 for } B. \\ 425 - 425 + 25N &= 875 - 425 && \text{Subtract 425 from each side.} \\ 25N &= 450 \\ \frac{25N}{25} &= \frac{450}{25} && \text{Divide both sides by 25.} \\ N &= 18\end{aligned}$$

CHECK: If 18 payments have been made, is the balance \$425?

$$\begin{array}{r} \$875 \\ -450 \\ \hline \$425 \end{array}$$

18 payments of \$25 is \$450.
True.

Eighteen payments have been made.

D. $8c + 11 = 43$

- E. Find the monthly payment on an original loan of \$1155 if the balance after 14 payments is \$385.

Answers to Warm-Ups

D. $c = 4$

E. The monthly payment is \$55.

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Exercises

OBJECTIVE

Solve equations of the form $ax + b = c$, $ax - b = c$, $\frac{x}{a} + b = c$, and $\frac{x}{a} - b = c$, in which x , a , b , and c are whole numbers.

Solve and check.

- | | | | |
|------------------------------|-------------------------------|----------------------------|----------------------------|
| 1. $4x - 16 = 12$ | 2. $\frac{a}{4} + 9 = 13$ | 3. $\frac{y}{3} - 7 = 4$ | 4. $36 = 5x + 6$ |
| 5. $45 = 6x + 9$ | 6. $\frac{a}{9} + 5 = 10$ | 7. $\frac{c}{8} + 23 = 27$ | 8. $12x - 10 = 38$ |
| 9. $11x + 32 = 54$ | 10. $7y + 53 = 123$ | 11. $15c - 63 = 117$ | 12. $6 = \frac{w}{3} - 33$ |
| 13. $81 = \frac{a}{14} + 67$ | 14. $\frac{x}{21} + 92 = 115$ | 15. $673 = 45b - 272$ | 16. $804 = 43c + 30$ |

17. Fast-Tix charges \$43 per ticket for a rock concert plus an \$8 service charge. How many tickets did Remy buy if he was charged \$309? Use the formula $C = PN + S$, where C is the total cost, P is the price per ticket, N is the number of tickets purchased, and S is the service charge.
18. Ticket Master charges Jose \$253 for nine tickets to the Festival of Jazz. If the service charge is \$10, what is the price per ticket? Use the formula in Exercise 17.
19. Rana is paid \$40 per day plus \$8 per artificial flower arrangement she designs and completes. How many arrangements did she complete if she earned \$88 for the day? Use the formula $S = B + PN$, where S is the total salary earned, B is the base pay for the day, P is the pay per unit, and N is the number of units completed.
20. Rana's sister works at a drapery firm where the pay is \$50 per day plus \$12 per unit completed. How many units did she complete if she earned \$122 for the day?

Exercises 21–24. Several different health spa plans are shown in the table.

Health Spa Plans

Spa	Monthly Fee	Charge per Visit
B-Fit	None	\$8
Join-Us	\$24	\$6
Gym Rats	\$32	\$4

21. Jessica has \$72 budgeted for spas each month. Write an equation for the number of visits she would get from B-Fit. Let v be the number of visits per month. Let C represent Jessica's monthly spa budget. Find the number of visits that Jessica can purchase from B-Fit each month.
22. Jessica has \$72 budgeted for spas each month. Write an equation for the number of visits she would get from Join-Us. Let v be the number of visits per month. Let C represent Jessica's monthly spa budget. Find the number of visits that Jessica can purchase from Join-Us each month.

Name _____ Class _____ Date _____

- 23.** Jessica has \$72 budgeted for spas each month. Write an equation for the number of visits she would get from Gym Rats. Let v be the number of visits per month. Let C represent Jessica's monthly spa budget. Find the number of visits that Jessica can purchase from Gym Rats each month.
- 24.** Using the results of Exercises 21–23, which company will give Jessica the most visits for her \$72?

1.7 Average, Median, and Mode

VOCABULARY

The **average**, or **mean**, of a set of numbers is the sum of the set of numbers divided by the total number of numbers in the set.

The **median** of a set of numbers, ordered from smallest to largest, is either the middle number of the set or the average of the two middle numbers in the set.

The **mode** of a set of numbers is the number or numbers that appear the most often in the set.

OBJECTIVES

- Find the average of a set of whole numbers.
- Find the median of a set of whole numbers.
- Find the mode of a set of whole numbers.

How & Why

OBJECTIVE 1

Find the average of a set of whole numbers.

The *average* or *mean* of a set of numbers is used in statistics. It is one of the ways to find the middle of a set of numbers (like the average of a set of test grades). Mathematicians call the average or mean a “measure of central tendency.” The average of a set of numbers is found by adding the numbers in the set and then dividing that sum by the number of numbers in the set. For example, to find the average of 11, 21, and 28:

$$11 + 21 + 28 = 60 \quad \text{Find the sum of the numbers in the set.}$$
$$60 \div 3 = 20 \quad \text{Divide the sum by the number of numbers, 3.}$$

The average is 20.

The “central” number or average does not need to be one of the members of the set. The average, 20, is not a member of the set.

To find the average of a set of whole numbers

- Add the numbers.
- Divide the sum by the number of numbers in the set.

Examples A–E



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DIRECTIONS: Find the average.

STRATEGY: Add the numbers in the set. Divide the sum by the number of numbers in the set.

- A. Find the average of 212, 189, and 253.

$$212 + 189 + 253 = 654 \quad \text{Add the numbers in the group.}$$
$$654 \div 3 = 218 \quad \text{Divide the sum by the number of numbers.}$$

The average is 218.

- B. Find the average of 23, 57, 352, and 224.

$$23 + 57 + 352 + 224 = 656 \quad \text{Add the numbers in the group.}$$
$$656 \div 4 = 164 \quad \text{Divide the sum by the number of numbers.}$$

The average is 164.

Warm-Ups A–E

- A. Find the average of 251, 92, and 449.

- B. Find the average of 12, 61, 49, 82, and 91.

Answers to Warm-Ups

A. 264 B. 59

CALCULATOR EXAMPLE

- C. Find the average of 777, 888, 914, and 505.

- D. The average of 42, 63, 21, 39, and ? is 50. Find the missing number.

- E. The Alpenrose Dairy ships the following number of gallons of milk to local groceries: Monday, 1045; Tuesday, 1325; Wednesday, 2005; Thursday, 1810; and Friday, 2165. What is the average number of gallons shipped each day?

- C. Find the average of 673, 821, 415, and 763.

STRATEGY: Enter the sum, in parentheses, and divide by 4.

$$(673 + 821 + 415 + 763) \div 4$$

The average is 668.

- D. The average of 38, 26, 12, and ? is 28. Find the missing number.

STRATEGY: Because the average of the four numbers is 28, we know that the sum of the four numbers is $4(28)$ or 112. To find the missing number, subtract the sum of the three given numbers from 112.

$$\begin{aligned} 112 - (38 + 26 + 12) &= 112 - (76) \\ &= 36 \end{aligned}$$

So the missing number is 36.

- E. In order to help Pete lose weight the dietician has him record his caloric intake for a week. He records the following: Monday, 3120; Tuesday, 1885; Wednesday, 1600; Thursday, 2466; Friday, 1434; Saturday, 1955; and Sunday, 2016. What is Pete's average caloric intake per day?

STRATEGY: Add the calories for each day and then divide by 7, the number of days.

$$\begin{array}{r} 3120 \quad 2068 \\ 1885 \quad 7 \overline{)14476} \\ 1600 \quad 14 \\ 2466 \quad \underline{4} \\ 1434 \quad 0 \\ 1955 \quad 47 \\ 2016 \quad \underline{42} \\ \hline 14476 \quad 56 \\ \quad \quad \underline{56} \\ \quad \quad 0 \end{array}$$

Pete's average caloric intake is 2068 calories per day.

How & Why

OBJECTIVE 2

Find the median of a set of whole numbers.

Another “measure of central tendency” is called the *median*. The median of a set of numbers is a number such that half the numbers in the set are smaller than the median and half the numbers are larger than the median. If the set is ordered from smallest to largest and there is an odd number of numbers in the set, the median is literally the middle number. Consider the set of numbers

3, 35, 41, 46, 50

The median is the middle number, 41.

If the set is ordered from smallest to largest and there is an even number of numbers in the set, the median is the average (mean) of the two middle numbers. Consider the set of numbers

4, 6, 9, 13, 15, 30

The median is the average of the middle numbers, 9 and 13.

$$(9 + 13) \div 2 = 11$$

The median is 11.

Answers to Warm-Ups

C. 771 D. 85

E. Alpenrose Dairy shipped an average of 1670 gallons of milk each day.

The median is used when a set of numbers has one or two numbers that are much larger or smaller than the others. For instance, a basketball player scores the following points per game over nine games; 32, 30, 32, 5, 33, 28, 33, 7, and 35. The average of the set is 25, which is near the low end of the set. The median is the middle number, which in this case is 32, and may give us a better center point.

To find the median of a set of numbers

1. List the numbers in order from smallest to largest.
2. If there is an odd number of numbers in the set, the median is the middle number.
3. If there is an even number of numbers in the set, the median is the average (mean) of the two middle numbers.

Examples F–G



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DIRECTIONS: Find the median of the set of whole numbers.

STRATEGY: List the numbers from smallest to largest. If there is an odd number of numbers in the set, choose the middle number. If there is an even number of numbers in the set, find the average of the two middle numbers.

F. Find the median of 37, 25, 46, 39, 22, 64, and 80.

22, 25, 37, 39, 46, 64, 80 List the numbers from smallest to largest.

The median is 39.

Because there is an odd number of numbers in the set, the median is the middle number.

G. Find the median of 88, 56, 74, 40, 29, 123, 81, and 9.

9, 29, 40, 56, 74, 81, 88, 123 List the numbers from smallest to largest.

$$\frac{56 + 74}{2} = \frac{130}{2} = 65$$

Because there is an even number of numbers in the set, the median is the average of the two middle numbers, 56 and 74.

The median is 65.

Warm-Ups F–G

F. Find the median of 103, 77, 113, 94, and 55.

G. Find the median of 147, 160, 111, 85, 137, and 287.

How & Why

OBJECTIVE 3

Find the mode of a set of whole numbers.

A third “measure of central tendency” is the *mode*. The mode is the number that occurs most often in the set. For example, consider

45, 67, 88, 88, 92, 100.

The number that occurs most often is 88, so 88 is the mode.

Each set of numbers has exactly one average and exactly one median. However a set of numbers can have more than one mode or no mode at all. See Examples I and J.

To find the mode of a set of numbers

1. Find the number or numbers that occur most often.
2. If all the numbers occur the same number of times, there is no mode.

Answers to Warm-Ups

F. 94 G. 142.

Warm-Ups H–K

H. Find the mode of 46, 67, 82, 85, 82, and 56.

I. Find the mode of 6, 7, 11, 7, 9, 13, 11, and 14.

J. Find the mode of 1, 4, 6, 3, 11, 13, and 9.

K. In a class of 30 seniors, the following weights are recorded on Health Day:

Number of Students	Weight per Student
1	122 lb
4	130 lb
6	151 lb
4	173 lb
7	187 lb
3	193 lb
4	206 lb
1	208 lb

What are the average, median, and mode of the weights of the students in the class?

Answers to Warm-Ups

H. 82 I. 7 and 11 J. There is no mode.

K. The average weight of a student is 172 lb.

The median weight is 180 lb.
The mode is 187 lb.

Examples H–K



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DIRECTIONS: Find the mode of the set of numbers.

STRATEGY: Find the number or numbers that occur most often. If all the numbers occur the same number of times, there is no mode.

H. Find the mode of 21, 13, 17, 13, 15, 21, and 13.

The mode is 13. The number 13 appears three times. No other number appears three times.

I. Find the mode of 22, 56, 72, 22, 81, 72, 93, and 105.

The modes are 22 and 72. Both 22 and 72 occur twice and the other numbers appear just once.

J. Find the mode of 13, 35, 16, 35, 16, and 13.

There is no mode. All the numbers occur the same number of times, 2. So no number appears most often.

K. During the annual Salmon Fishing Derby, 46 fish are entered. The weights of the fish are recorded as shown:

Number of Fish	Weight per Fish
3	6 lb
2	8 lb
9	11 lb
12	14 lb
8	18 lb
9	23 lb
2	27 lb
1	30 lb

What are the average, median, and mode of the weights of the fish entered in the derby?

STRATEGY: To find the average, first find the total weight of all 46 fish.

$$\begin{aligned}3(6) &= 18 && \text{Multiply 3 times 6 because there are 3 fish that weigh 6 lb,} \\2(8) &= 16 && \text{for a total of 18 lb, and so on.} \\9(11) &= 99 \\12(14) &= 168 \\8(18) &= 144 \\9(23) &= 207 \\2(27) &= 54 \\1(30) &= \underline{\underline{30}} \\&& 736\end{aligned}$$

Now divide the total weight by the number of salmon, 46.

$$736 \div 46 = 16$$

The average weight per fish is 16 lb.

Because both the 23rd and 24th fish weigh 14 lb, the median weight of the fish is 14 lb.

The mode of the weight of the fish is 14 lb.

Because there are 46 fish, we need to find the average of the weights of the 23rd and the 24th fish.

14 lb occurs most often in the list.

Exercises 1.7

OBJECTIVE 1

Find the average of a set of whole numbers.

A Find the average.

- | | | | |
|-----------------------|-------------------------|----------------------------|----------------------------|
| 1. 8, 12 | 2. 9, 17 | 3. 12, 18 | 4. 21, 31 |
| 5. 9, 15, 18 | 6. 11, 15, 19 | 7. 7, 11, 12, 14 | 8. 9, 9, 17, 17 |
| 9. 10, 8, 5, 5 | 10. 20, 15, 3, 2 | 11. 9, 11, 6, 8, 11 | 12. 15, 7, 3, 31, 4 |

Find the missing number to make the average correct.

- 13.** The average of 10, 13, 15, and ? is 13. **14.** The average of 12, 17, 21, and ? is 17.

B Find the average.

- | | | |
|-------------------------------|---------------------------------------|---------------------------------------|
| 15. 22, 26, 40, 48 | 16. 22, 43, 48, 67 | 17. 31, 41, 51, 61 |
| 18. 22, 19, 34, 63, 52 | 19. 14, 17, 25, 34, 50, 82 | 20. 93, 144, 221, 138 |
| 21. 123, 133, 164, 132 | 22. 371, 749, 578, 666 | 23. 100, 151, 228, 145 |
| 24. 45, 144, 252, 291 | 25. 82, 95, 101, 153, 281, 110 | 26. 149, 82, 105, 91, 262, 217 |

Find the missing number.

- 27.** The average of 39, 86, 57, 79, and ? is 64. **28.** The average of 32, 40, 57, 106, 44, and ? is 58.

OBJECTIVE 2

Find the median of a set of whole numbers.

A Find the median.

- | | | |
|---------------------------|---------------------------|---------------------------|
| 29. 13, 56, 102 | 30. 44, 53, 67 | 31. 14, 20, 32, 40 |
| 32. 21, 26, 56, 87 | 33. 25, 62, 16, 55 | 34. 77, 9, 57, 93 |

B

- | | |
|--|--|
| 35. 27, 81, 107, 123, 142 | 36. 8, 37, 92, 41, 106 |
| 37. 97, 101, 123, 129, 133, 145 | 38. 17, 42, 18, 18, 51, 48, 67 |
| 39. 39, 77, 95, 103, 41, 123 | 40. 175, 309, 174, 342, 243, 189, 233, 94 |

OBJECTIVE 3

Find the mode of a set of whole numbers.

A Find the mode.

- 41.** 23, 28, 28, 45, 52 **42.** 7, 9, 12, 12, 14, 18 **43.** 1, 1, 2, 2, 3, 3, 3, 6, 7, 7
44. 9, 27, 44, 67, 27, 20 **45.** 23, 45, 19, 36, 22, 46, 89 **46.** 17, 32, 63, 17, 18, 34, 12
- B**
- 47.** 27, 40, 58, 36, 40, 21, 40, 58 **48.** 64, 42, 70, 64, 42, 70, 79, 42
49. 45, 85, 60, 45, 58, 115, 60 **50.** 12, 16, 18, 13, 12, 17, 18, 19
51. 19, 23, 14, 14, 19, 23 **52.** 35, 27, 88, 55, 55, 35, 88, 27

C

- 53.** On a Saturday in January 2005, the eight winning teams in the NBA scored the following number of points: 100, 115, 100, 95, 91, 110, 92, and 105. Find the average and the median number of points scored by the winning teams.
- 55.** After two rounds of the Champions Tour in Hawaii, the top 10 golfers had the following scores: 128, 131, 132, 132, 134, 134, 134, 135, 135, and 135. Find the average and the median scores after two rounds.
- 57.** A consumer magazine tests 24 makes of cars for gas mileage. The results are shown in the table.
- 54.** The following number of fish were counted at the Bonneville fish ladder during one week in July: coho, 210; shad, 567; silver salmon, 346; and sturgeon, 101. Find the average and median number of fish per species.
- 56.** A bowler has the following scores for nine games: 260, 200, 210, 300, 195, 211, 271, 200, and 205. What are the average, median, and mode scores per game?
- 58.** Big 5 Sporting Goods had a sale on shoes. The number of styles and the price are given in the table.

New Car Gas Mileage

Number of Makes	Gas Mileage Based on 200 Miles
2	19 mpg
4	22 mpg
2	24 mpg
4	30 mpg
5	34 mpg
4	40 mpg
3	48 mpg

What are the average, median, and mode of the gas mileage of the cars?

Shoes on Sale

Number of Styles	Price per Pair
3	\$15
4	\$17
5	\$21
6	\$25
6	\$40
2	\$60

What are the average, median, and mode prices per pair of shoes?

- 59.** Twenty wrestlers are weighed in on the first day of practice.

Weights of Wrestlers

Number of Players	Weight
1	99 lb
3	110 lb
2	115 lb
5	124 lb
3	130 lb
2	155 lb
2	167 lb
1	197 lb
1	210 lb

What is the average weight of the players?

- 60.** Eighty-two ladies at the Rock Creek Country Club ladies championship tournament recorded the following scores:

Ladies Championship Scores

Number of Golfers	Score
1	66
3	68
7	69
8	70
12	71
16	72
15	74
10	76
6	78
3	82
1	85

What are the average, median, and mode scores for the tournament?

- 61.** A West Coast city is expanding its mass transit system. It is building a 15-mile east–west light rail line for \$780 million and an 11-mile north–south light rail line for \$850 million. What is the average cost per mile, to the nearest million dollars, of the new lines?

- 63.** The table lists information about top-rated one-cup coffee makers, as tested by the Good Housekeeping Institute in 2004.

Rating	Best Overall	Best Deal	Best Gourmet
Brand Price	Philips Senseo \$70	Melitta One \$50	Bunn My Café \$230

What is the average price (rounded to the nearest dollar) of the three models of coffee makers? How many cost less than the average price?

- 62.** A city in Missouri has a budget of \$73,497,400 and serves a population of 35,800. A second city in Missouri has a budget of \$59,925,000 and serves a population of 79,900. What is the average cost per resident in each city? What is the average combined cost per resident in the cities, rounded to the nearest hundred dollars?

- 64.** The table lists information about the five best-ranked camera cell phones, as tested by the Good Housekeeping Institute in 2004.

Rank	1	2	3	4	5
Brand and Model	Nokia 3660	Lg VX 6000	Sanyo 8100	Sony Ericsson T610	Motorola V600
Price	\$299	\$150	\$229	\$199	\$299

What is the average price (rounded to the nearest dollar) of the top-rated camera phones? How many of them are more expensive than the average cost?

Exercises 65–68. According to the U.S. Census Bureau, the population of Nevada in 1990 was 1,198,954 and the population estimate in 2003 was 2,241,164.

- 65.** What was the average population of Nevada over the 13-year period?

- 66.** What was the average increase per year over the 13-year period?

- 67.** Use your answer to Exercise 66 to complete the table.

Population by Year in Nevada

Year	Population	Year	Population
1990	1,198,954	1997	
1991		1998	
1992		1999	
1993		2000	
1994		2001	
1995		2002	
1996		2003	

- 70.** The table gives the states with the greatest number of hazardous waste sites in 2004, according to the Environmental Protection Agency.

New Jersey	California	Pennsylvania	New York	Michigan	Florida	Washington	Texas
111	96	93	90	67	51	47	42

What is the average number of hazardous waste sites for the top eight states? Round to the nearest whole number.

Exercises 71–74. The information on the assets of the top five U.S. commercial banks in 2004 is given in the table.

Assets of the Top Five U.S. Commercial Banks

Bank	Bank of America, NA	J.P. Morgan & Co.	Citibank, NA	Wachovia Bank, NA	Wells Fargo Bank, NA
Assets (in Millions)	\$690,573	\$648,693	\$606,191	\$364,475	\$347,560

- 71.** What were the average assets for the top three banks?
Round to the nearest million dollars.

- 72.** What were the average assets for the top four banks?
Round to the nearest million dollars.

- 73.** What were the average assets for the top five banks?
Round to the nearest million dollars.

- 74.** Compare your answers for Exercises 71, 72, and 73.
Are they increasing or decreasing? Explain.

Exercises 75–80. The table lists the number of Internet users in the United States from 1997 to 2004 (Source: U.S. Department of Commerce).

Year	1997	1998	1999	2000	2001	2002	2003	2004
Users (millions)	57	85	102	117	143	158	162	189

- 75.** What is the mean number of Internet users over the 8-year period? Round to the nearest million.

- 76.** How many years had more than the mean number of users?

77. What is the median number of users over the 8-year period?
78. What is the mean number of users for the last 5 years in the table? Round to the nearest million.
79. What is the median number of users for the last 5 years in the table?
80. How does the mean number of users over the 8 year period compare to the mean number of users over the last 5 years? Explain.
81. On Jupiter Island in Florida, the median home price is about \$4 million. State in words what this means.
82. According to U.S. Census figures for 2000, the city of Boston, Massachusetts, had a population of 589,142 and a median age of 31. Approximately how many people in the city of Boston were over 31 years old?
83. In 1950, the median age of men in the United States at their first marriage was 23, whereas in 2002 the median age was 27. Explain in words what this means.

Exercises 84–87 refer to the chapter application. See Table 1.1, page 1.

84. Calculate the average earnings of the top three movies for 2004.
85. Calculate the average earnings of the top five movies for 2004. Round to the nearest dollar.
86. Calculate the median gross earnings of the top five movies for 2004.
87. Round the gross earnings for all top 10 movies of 2004 to the nearest million dollars, then calculate the average earnings of all 10 movies.

STATE YOUR UNDERSTANDING

88. Explain what is meant by the average of two or more numbers.
89. Explain how to find the average (mean) of 2, 4, 5, 5, and 9. What does the average of a set of numbers tell you about the set?

CHALLENGE

- 90.** A patron of the arts estimates that the average donation to a fund-raising drive will be \$72. She will donate \$150 for each dollar by which she misses the average. The 150 donors made the contributions listed in the table.

Contributions to the Arts

Number of Donors	Donation
5	\$153
13	\$125
24	\$110
30	\$100
30	\$ 75
24	\$ 50
14	\$ 25
10	\$ 17

How much does the patron donate to the fund drive?

GROUP WORK

- 91.** Divide 35, 68, 120, 44, 56, 75, 82, 170, and 92 by 2 and 5. Which ones are divisible by 2 (the division has no remainder)? Which ones are divisible by 5? See if your group can find simple rules for looking at a number and telling whether or not it is divisible by 2 and/or 5.
- 92.** Using the new car ads in the newspaper, find four advertised prices for the same model of a car. What is the average price, to the nearest 10 dollars?

1.8 Drawing and Interpreting Graphs

VOCABULARY

Graphs are used to illustrate sets of numerical information.

A **bar graph** uses solid lines or heavy bars of fixed length to represent numbers from a set. Bar graphs contain two **scales**, a vertical scale and a horizontal scale. The **vertical scale** represents one set of values and the **horizontal scale** represents a second set of values. These depend on the information to be presented. The bar graph in Figure 1.7 illustrates four types of cars (first set of values) and the number of each type of car sold (second set of values).

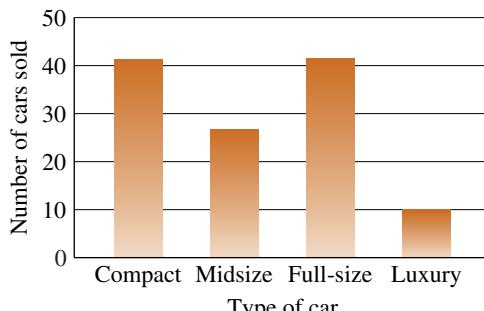


Figure 1.7

A **line graph** uses lines connecting points to represent numbers from a set. A line graph has a vertical and a horizontal scale, like a bar graph. A line graph showing the percentage of women in real estate is shown in Figure 1.8.

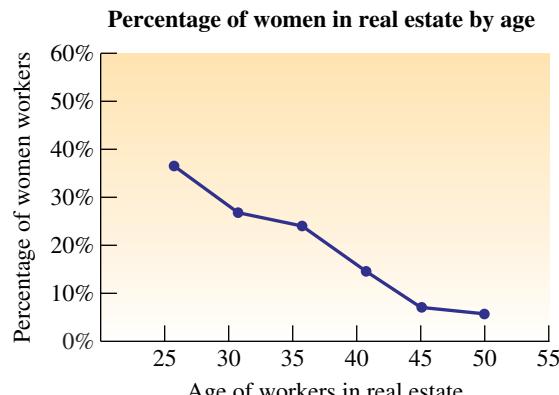


Figure 1.8

A **pictograph** uses symbols or simple drawings to represent numbers from a set. The pictograph in Figure 1.9 shows the distribution of mathematics students at a community college.

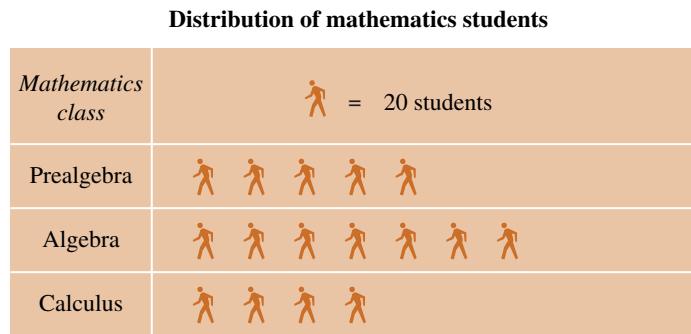


Figure 1.9

OBJECTIVES

1. Read data from bar, pictorial, and line graphs.
2. Construct a bar or line graph.

How & Why

OBJECTIVE 1

Read data from bar, pictorial, and line graphs.

A graph or chart is a picture used for presenting data for the purpose of comparison. To “read a graph” means to find values from the graph.

Examine the bar graph in Figure 1.10.

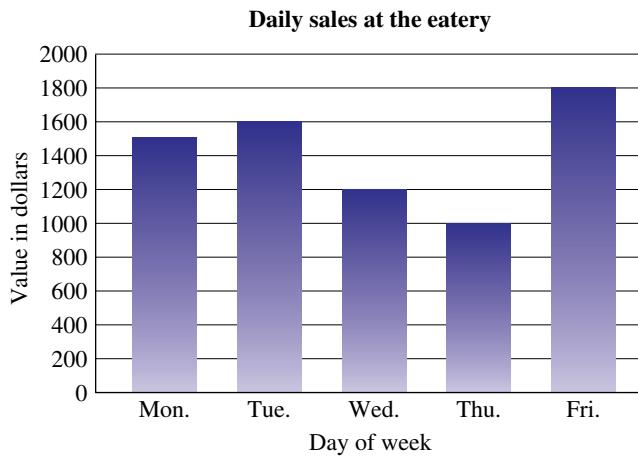


Figure 1.10

The vertical scale shows dollar values and is divided into units of \$200. The horizontal scale shows days of the week. From the graph we see that

1. Friday had the greatest sales (highest bar), with \$1800 in sales.
2. Thursday had the least sales (lowest bar), with sales of \$1000.
3. Monday's sales appear to be \$1500 (the bar falls between the scale divisions).
4. Friday had \$600 more in sales than those for Wednesday.
5. The total sales for the week were \$7100.

Some advantages of displaying data with a graph:

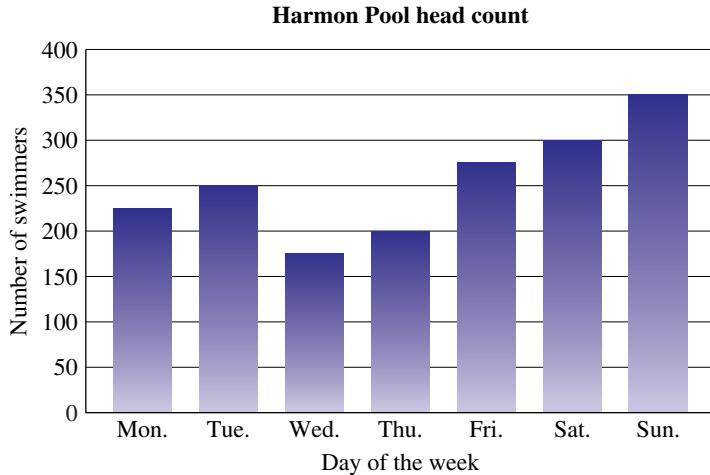
1. Each person can easily find the data most useful to him or her.
2. The visual display is easy for most people to read.
3. Some questions can be answered by a quick look at the graph. For example, “What day does the Eatery need the least staff?”

Examples A–B

DIRECTIONS: Answer the questions associated with the graph.

STRATEGY: Examine the graph to determine the values that are related.

- A. The graph shows the number of people who used the Harmon Pool during a 1-week period.



- What day had the most swimmers?
- What day had the fewest swimmers?
- How many people used the pool on Monday?
- How many people used the pool on the weekend?

- Sunday
 - Wednesday
 - 225
 - 650
- The tallest bar shows the greatest number of swimmers.
The shortest bar shows the fewest number of swimmers.
Read the vertical scale at the top of the bar for Monday. The value is estimated because the top of the bar is not on a scale line.
Add the number of swimmers for Saturday and Sunday.

- B. The total sales from hot dogs, soda, T-shirts, and buttons during an air show are shown in the pictorial graph.

Sales at air show

Item	= \$1000	= \$500
Hot dogs		
Soda		
T-shirts		
Buttons		

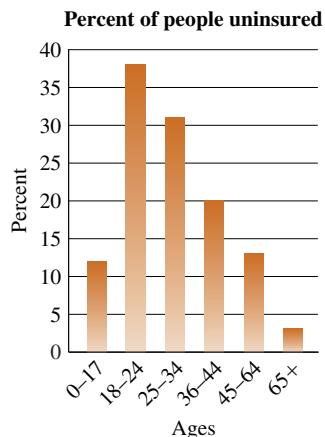
- What item has the largest dollar sales?
- What were the total sales from hot dogs and buttons?
- How many more dollars were realized from the sale of T-shirts than from buttons?

- T-shirts
 - Hot dogs: \$3000
 - Buttons: $\begin{array}{r} +\$500 \\ \hline \end{array}$
 - Total: \$3500
 - T-shirts: \$5000
 - Buttons: $\begin{array}{r} -\$500 \\ \hline \end{array}$
 - Difference: \$4500
- T-shirts have the greatest number of symbols.
- Subtract the sales of buttons from the sales of T-shirts.

The sales from T-shirts were \$4500 more than from buttons.

Warm-Ups A–B

- A. The percent of people in a Western State who did not have health insurance in 2004 is shown in the bar graph.



- Which age group had the most uninsured people?
- Which age group had the fewest uninsured people?
- What percent of the 0–17 age group was uninsured?
- What percent of the people in the 25–34 age group was uninsured?
- The number of birds spotted during a recent expedition of the Huntsville Bird Society is shown in the pictorial graph.

Birds spotted

Species	= 50 = 25
Crows	
Woodpeckers	
Wrens	
Canaries	

- Which species of bird was spotted most often?
- How many woodpeckers and wrens were spotted?
- How many more canaries were spotted than crows?

Answers to Warm-Ups

- A. 1. 18–24 2. 65+ 3. 12 percent
4. 31 percent
- B. 1. Wrens 2. 350 3. 50

How & Why

OBJECTIVE 2

Construct a bar or line graph.

Table 1.4 Heating Bills for the Morales Family

Month	Cost, in Dollars
January	200
February	175
March	150
April	100
May	75
June	25

Let us construct a bar graph to show variations in the heating bill for the Morales Family. The data are shown in Table 1.4.

To draw and label the bar graph for these data, we show the cost on the vertical scale and the months on the horizontal scale. This is a logical display because we will most likely be asked to find the highest and lowest monthly heating costs and a vertical display of numbers is easier to read than a horizontal display of numbers. This is the typical way bar graphs are displayed. Be sure to write the labels on the vertical and horizontal scales as soon as you have chosen how the data will be displayed. Now title the graph so that the reader will recognize the data it contains.

The next step is to construct the two scales of the graph. Because each monthly total is divisible by 25, we choose multiples of 25 for the vertical scale. We could have chosen one unit for the vertical scale, but the bars would be very long and the graph would take up a lot of space. If we had chosen a larger scale, for instance 100, then the graph might be too compact and we would need to find fractional values on the scale. It is easier to draw the graph if we use a scale that divides each unit of data. The months are displayed on the horizontal scale. Be sure to draw the bars with uniform width, because each of them represents the cost for one month. A vertical display of between 5 and 12 units is typical. The vertical display should start with zero. See Figure 1.11.

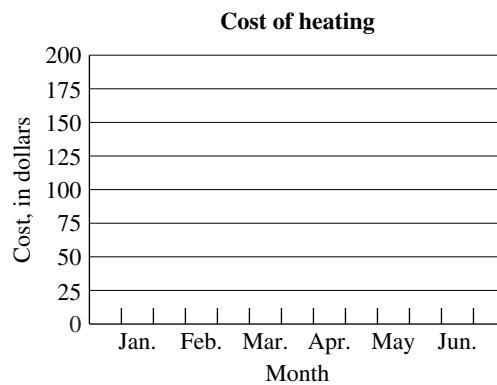


Figure 1.11

We stop the vertical scale at 200 because the maximum heating cost to be displayed is \$200. The next step is to draw the bars. Start by finding the cost for January. The cost was \$200 for January, so we draw the bar for January until the height of 200 is reached. This is the top of the bar. Now draw the solid bar for January. See Figure 1.12.

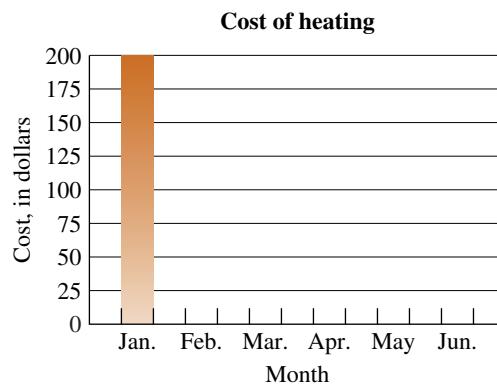


Figure 1.12

Complete the graph by drawing the bars for the other months. See Figure 1.13.

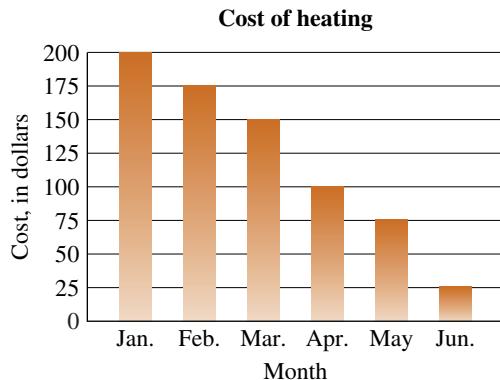


Figure 1.13

A line graph is similar to a bar graph in that it has vertical and horizontal scales. The data are represented by points rather than bars, and the points are connected by line segments. We use a line graph to display the data in Table 1.5.

Table 1.5 Property Taxes

Year	Tax Rate (per \$1000)
1985	\$12.00
1990	\$15.00
1995	\$14.00
2000	\$16.00
2005	\$20.00

The vertical scale represents the tax rate, and each unit represents \$2. This requires using a half space for the \$15.00 rate.

Another possibility is to use a vertical scale in which each unit represents \$1, but this would require 20 units on the vertical scale and would make the graph much taller. We opt to save space by using \$2 units on the vertical scale. See Figure 1.14.

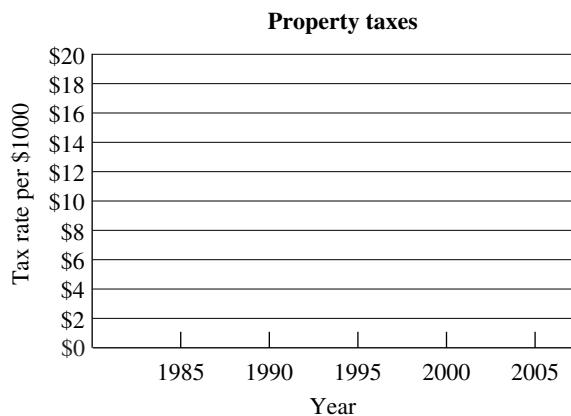


Figure 1.14

To find the points that represent the data, locate the points that are at the intersection of the horizontal line through the tax rate and the vertical line through the corresponding year. Once all the points have been located, connect them with line segments. See Figure 1.15.

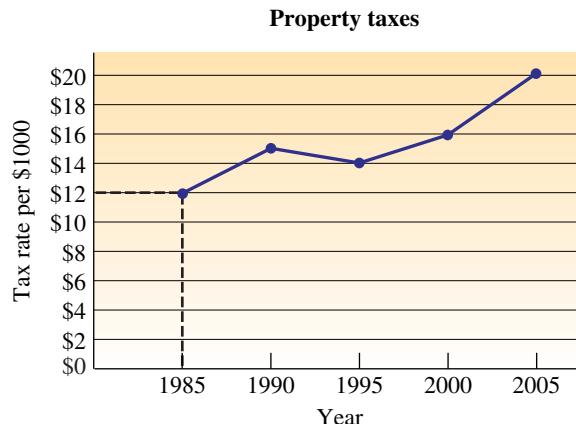


Figure 1.15

From the graph we can conclude the following:

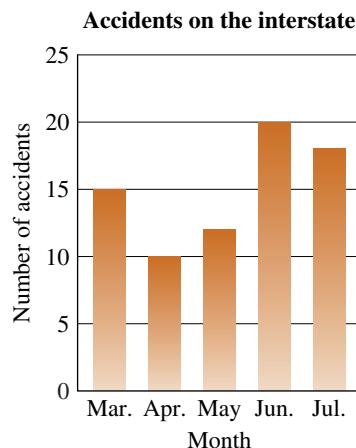
1. Only during one 5-year period (1990–1995) did the tax rate decline.
2. The largest increase in the tax rate took place from 2000 to 2005.
3. The tax rate has increased \$8 per thousand from 1985 to 2005.

Warm-Up C

- C. The number of accidents on a stretch of an interstate high-way during 5 months: March, 15; April, 10; May, 12; June, 20; July, 18.

Answers to Warm-Ups

C.

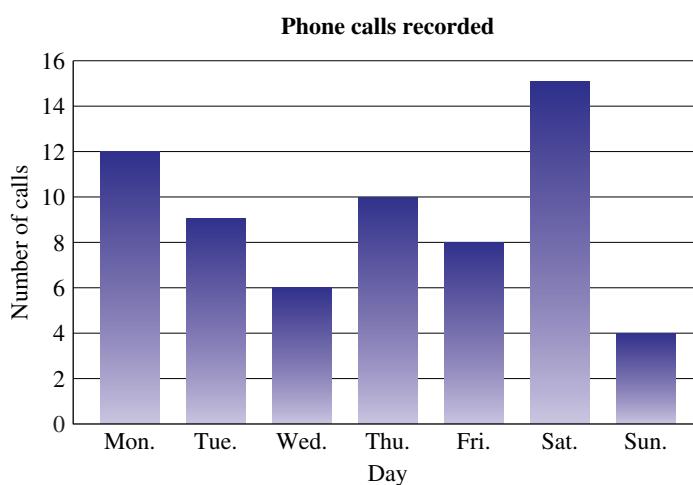


Example C BASIC Mathematics Now™

DIRECTIONS: Construct a bar graph.

STRATEGY: List the related values in pairs and draw two scales to show the pairs of values.

- C. The number of phone calls recorded during the week Mary was on vacation: Monday, 12; Tuesday, 9; Wednesday, 6; Thursday, 10; Friday, 8; Saturday, 15; Sunday, 4.



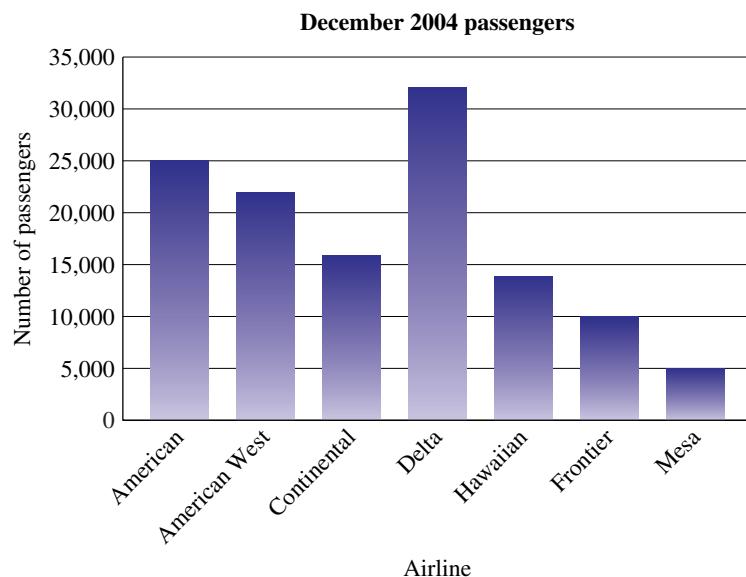
Choose a scale of 1 unit = 2 calls for the vertical scale. Divide the horizontal scale so that it will accommodate 7 days with a common space between them. Construct the graph, label the scales, and give the graph a title.

Exercises 1.8

OBJECTIVE 1

Read data from bar, pictorial, and line graphs.

- A** Exercises 1–6. The graph shows the number of passengers by airline for December 2004 at the Portland International Airport:



- Which airline had the greatest number of passengers?
- Which airline had the least number of passengers?
- How many passengers did Frontier have during December?
- How many passengers did American and Continental have during December?
- Estimate the number of passengers carried by all of the airlines in December.
- How many more passengers did American have than Frontier?

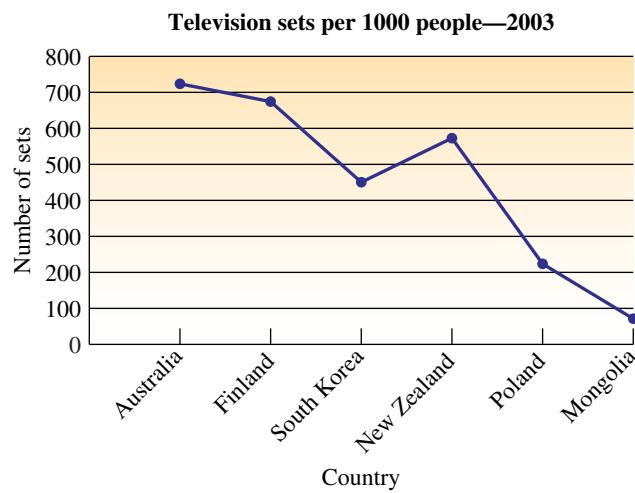
Exercises 7–12. The graph shows the number of cars in the shop for repair during a given year:

2005 repair intake record

Type of car	 = 20 cars
Compact	
Full-size	
Van	
Subcompact	

7. How many vans are in the shop for repair during the year?
9. What type of vehicle has the most cars in for repair?
11. How many vehicles are in for repair during the year?
8. How many compacts and subcompacts are in for repair during the year?
10. Are more subcompacts or compacts in for repair during the year?
12. If the average repair cost for compacts is \$210, what is the gross income on compact repairs for the year?

B Exercises 13–18. The graph shows the number of television sets per 1000 people in 2003 in selected countries.



13. Which country had the most sets per 1000 people?
14. Which country had the fewest sets per 1000 people?
15. How many sets per 1000 people did New Zealand have?
16. What was the difference in the number of sets per 1000 people in Finland and South Korea?
17. To the nearest 10 sets, what was the average number of sets per 1000 people in these countries?
18. The population of Mongolia in 2003 was approximately 2,750,000. About how many television sets were in the country in 2003? Round to the nearest thousand.

Exercises 19–24. The graph shows the amounts paid for raw materials at Southern Corporation during a production period.



19. What is the total paid for paint and lumber?
20. What is the total paid for raw materials?
21. How much less is paid for steel casting than for plastic?
22. How much more is paid for plastic than for paint?
23. If Southern Corporation decides to double its production during the next period, what will it pay for steel casting?
24. If Southern Corporation decides to double its production during the next period, how much more will it pay for lumber and steel casting than plastic and paint?

OBJECTIVE 2

Construct a bar or line graph.

A In Exercises 25–28, draw bar graphs to display the data. Be sure to title the graph and label the axes and scales.

25. Distribution of grades in a history class: A, 10; B, 12; C, 25; D, 6; F, 4.
26. The distribution of monthly income: rent, \$550; automobile, \$325; taxes, \$250; clothes, \$100; food, \$350; miscellaneous, \$200.
27. Career preference as expressed by a senior class: business, 120; law, 20; medicine, 40; science, 100; engineering, 50; public service, 80; armed service, 10.
28. In 2004, according to statistics from *Forbes Magazine*, the average value of a major league football team (NFL) was \$768 million, the average value of a major league baseball team (MLB) was \$295 million, the average value of a major league hockey team (NHL) was \$170 million, and the average value of a major league basketball team (NBA) was \$304 million. Use a scale of 1 unit = 200 million dollars on the vertical axis.

In Exercises 29–32, draw line graphs to display the data. Be sure to title the graph and label the axes and scales.

- 29.** Daily sales at the local men's store: Monday, \$1500; Tuesday, \$2500; Wednesday, \$1500; Thursday, \$3500; Friday, \$4000; Saturday, \$6000; Sunday, \$4500.
- 30.** The gallons of water used each quarter of the year by a small city in New Mexico:
- | | |
|------------|------------|
| Jan.–Mar. | 20,000,000 |
| Apr.–Jun. | 30,000,000 |
| Jul.–Sept. | 45,000,000 |
| Oct.–Dec. | 25,000,000 |
- 31.** Profits from a recent church bazaar: bingo, \$750; craft sales, \$1500; quilt raffle, \$450; bake sale, \$600; refreshments, \$900.
- 32.** Jobs in the electronics industry in a western state are shown in the table.

Year	Number of Jobs
2000	15,000
2001	21,000
2002	18,000
2003	16,000
2004	15,000
2005	12,000

B

33. Draw a bar graph to display the cost of an average three-bedroom house in Austin, Texas.

Year	Cost
1985	\$75,000
1990	\$90,000
1995	\$125,000
2000	\$150,000
2005	\$185,000

34. Draw a line graph to display the oil production from a local well over a 5-year period.

Year	Barrels Produced
2001	15,000
2002	22,500
2003	35,000
2004	32,500
2005	40,000

35. The age at which a person is eligible for full Social Security benefits is increasing. The table gives year of birth and full retirement age, according to the Social Security Administration. Make a line graph for this data.

Year of Birth	Full Retirement Age
1935	65
1940	65 yr 6 mo
1945	66
1950	66
1955	66 yr 2 mo
1960	67

36. The table gives the most prescribed drugs of 2003, in millions of prescriptions (*Source: RxList*). Draw a bar graph for the data.

Drug	Prescriptions
Hydrocodone/APAP	85,100,000
Lipitor	65,500,000
Synthroid	47,200,000
Atenolol	40,900,000
Zithromax	39,500,000

37. Draw a bar graph to display the median price of existing single-family housing in Miami, Florida, which was \$121,500 in 1998, \$138,200 in 2000, \$138,700 in 2002, and \$139,200 in 2004, according to data from the National Association of Realtors.

38. The number of veterans in the United States (in thousands) over the past century is given in the table.

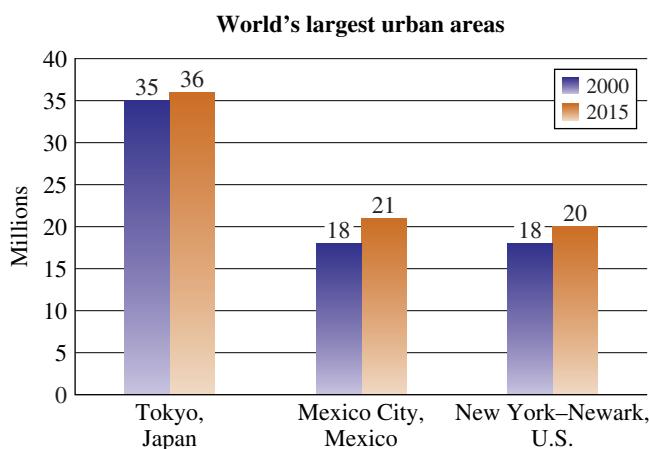
Veterans in the United States, in Thousands

1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	1998
1100	1000	5100	4700	4300	19,100	22,500	27,600	28,600	27,000	25,100

- a. Make a line graph for this information.

- b.** What historic event caused the dramatic increase in 1950?
- c.** What historic event caused the increase in 1920?
- d.** What year had the maximum number of veterans?

C Exercises 39–42 refer to the bar graph, which shows the population (and estimated population) of the three largest urban areas in the year 2000.



- 39.** Which city had the largest population in the year 2000?
- 40.** In the year 2015, is Tokyo expected to have a larger population than Mexico City and New York–Newark combined?
- 41.** Which urban area is expected to grow the most during the 15-year period?
- 42.** How many million people lived in the three largest urban areas in the year 2000?

Exercises 43–44. Use the information on zoo attendance given in the table.

Zoo Attendance

	2001	2002	2003	2004	2005
Fisher Zoo	2,367,246	2,356,890	2,713,455	2,745,111	2,720,567
Delaney Zoo	1,067,893	1,119,875	1,317,992	1,350,675	1,398,745
Shefford Garden	2,198,560	2,250,700	2,277,300	2,278,345	2,311,321
Utaki Park	359,541	390,876	476,200	527,893	654,345

- 43.** Draw a line graph to display the attendance at Utaki Park for the 5 years.
- 44.** Draw a bar graph to display the data on attendance at the zoos in 2005.

Exercises 45–46. The table lists visitor information at Lizard Lake State Park.

Visitors at Lizard Lake State Park

	May	June	July	August	September
Overnight camping	231	378	1104	1219	861
Picnics	57	265	2371	2873	1329
Boat rental	29	147	147	183	109
Hiking/climbing	48	178	178	192	56
Horse rental	22	43	43	58	27

- 45.** Draw a line graph to display the data on overnight camping for the 5 months.

- 46.** Draw a bar graph to display the data on hiking/climbing for the 5 months.

Exercise 47 refers to the chapter application. See Table 1.1, page 1.

- 47.** Make a bar graph that shows the gross earnings of the top 10 movies of 2004. Use the figures rounded to the nearest million dollars.

STATE YOUR UNDERSTANDING

48. Explain the advantages of each type of graph.
Which is preferable? Why?

CHALLENGE

49. The figures for U.S. casualties in four declared wars of the 20th century are: World War I, 321,000; World War II, 1,076,000; Korean War, 158,000; Vietnam War, 211,000. Draw a bar graph and a line graph to illustrate the information. Which of your graphs do you think does the best job of displaying the data?

GROUP WORK

50. Have each member select a country and find the most recent population statistics for that country. Put the numbers together, and have each member draw a different kind of graph of the populations.

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Key Concepts CHAPTER 1

Section 1.1 Whole Numbers and Tables: Writing, Rounding, and Inequalities

Definitions and Concepts	Examples	
The whole numbers are 0, 1, 2, 3, and so on.	238 6,198,349	two hundred thirty-eight six million, one hundred ninety-eight thousand, three hundred forty-nine
One whole number is smaller than another if it is to the left on the number line.	$3 < 6$	
One whole number is larger than another if it is to the right on the number line.	$14 > 2$	
To round a whole number:		
• Round to the larger number if the digit to the right is 5 or more. • Round to the smaller number if the digit to the right is 4 or less.	$6,745 \approx 7,000$ (nearest thousand) $6,745 \approx 6,700$ (nearest hundred)	
Tables are a method of organizing information or data in rows and columns.	Enrollment by Gender at River CC	
	Males	Females
	English Math Science History	52 67 71 64 69 75 63 59
	There are 71 males taking math and 75 females taking science.	

Section 1.2 Adding and Subtracting Whole Numbers

Definitions and Concepts	Examples	
To add whole numbers, write the numbers in columns so the place values are aligned. Add each column starting with the ones. Carry as necessary.	$\begin{array}{r} 1 \\ 372 \\ +594 \\ \hline 966 \end{array}$	$\begin{array}{r} 11 \\ 36 \\ +785 \\ \hline 821 \end{array}$
<i>addend + addend = sum</i>		
To subtract whole numbers, write the numbers in columns so the place values are aligned. Subtract, starting with the ones column. Borrow if necessary.	$\begin{array}{r} 214412 \\ 3452 \\ -362 \\ \hline 4235 \end{array}$	$\begin{array}{r} 214412 \\ 3452 \\ -735 \\ \hline 2717 \end{array}$
The answer to a subtraction problem is called the <i>difference</i> .		
The perimeter of a polygon is the distance around the outside.		
To calculate the perimeter, add the lengths of the sides.	$\begin{aligned} \text{Perimeter} &= 12 + 4 + 10 + 7 \\ &= 33 \\ P &= 33 \text{ ft} \end{aligned}$	

Section 1.3 Multiplying Whole Numbers

Definitions and Concepts	Examples
To multiply whole numbers, multiply the first factor by each digit in the second factor, keeping alignment. Add the partial products.	$\begin{array}{r} 482 \\ \times 12 \\ \hline 964 \\ 482 \\ \hline 5784 \end{array}$
$(\text{factor})(\text{factor}) = \text{product}$	2×482 1×482
The area of a rectangle is the space inside it: $A = \ell \cdot w$	 <p>Area = $\ell \cdot w$ = $6(4)$ = 24</p> <p>The area is 24 square feet.</p>

Section 1.4 Dividing Whole Numbers

Definitions and Concepts	Examples
To divide whole numbers, use long division as shown. <i>quotient</i> <i>divisordividend</i>	$\begin{array}{r} 205 \\ 32 \overline{)6578} \\ 64 \\ \hline 17 \\ 16 \\ \hline 0 \\ 178 \\ 160 \\ \hline 18 \end{array}$ <p>32(2) = 64 Subtract and bring down the 7 32(0) = 0 Subtract and bring down the 8 32(5) = 160 Subtract. The remainder is 18.</p>

Section 1.5 Whole-Number Exponents and Powers of 10

Definitions and Concepts	Examples
An exponent indicates how many times a number is used as a factor. $\text{base}^{\text{exponent}} = \text{value}$	$2^3 = (2)(2)(2) = 8$
A power of 10 is the value of 10 with some exponent.	$10^4 = (10)(10)(10)(10) = 10,000$

Section 1.6 Order of Operations

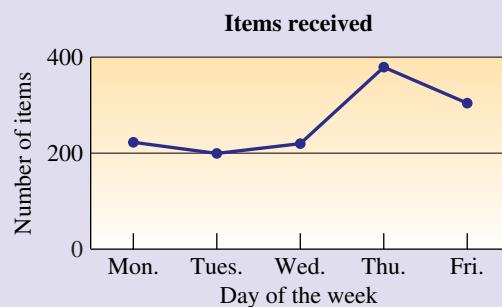
Definitions and Concepts	Examples
The order of operations is:	$2(7 + 4) - 36 \div 3^2 + 5$
Parentheses	$2(11) - 36 \div 3^2 + 5$
Exponents	$2(11) - 36 \div 9 + 5$
Multiplication and Division	$22 - 4 + 5$
Addition and Subtraction	$18 + 5$
	23

Section 1.7 Average, Median, and Mode

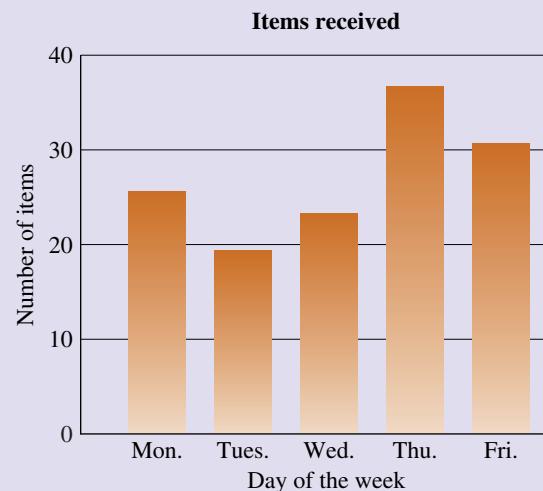
Definitions and Concepts	Examples
To find the average of a set of numbers:	4, 13, 26, 51, 51
<ul style="list-style-type: none"> • Add the numbers. • Divide by the number of numbers. 	$4 + 13 + 26 + 51 + 51 = 145$ $145 \div 5 = 29$ The average is 29.
To find the median of a set of numbers:	The median is 26.
<ul style="list-style-type: none"> • List the numbers in order from smallest to largest. • If there is an odd number of numbers in the set, the median is the middle number. • If there is an even number of numbers in the set, the median is the average (mean) of the middle two. 	
To find the mode of a set of numbers:	The mode is 51.
<ul style="list-style-type: none"> • Find the number or numbers that occur most often. • If all the numbers occur the same number of times, there is no mode. 	

Section 1.8 Drawing and Interpreting Graphs**Definitions and Concepts**

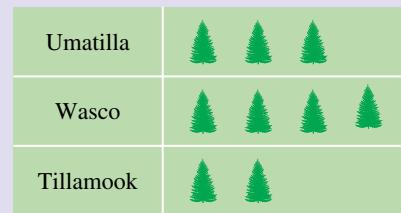
A line graph uses a line to connect data points.

Examples

A bar graph uses bars to represent data values.



A pictograph uses pictures to represent data values.

Board-feet of timber produced

= 1,000,000 board-feet

Review Exercises CHAPTER 1

Section 1.1

Write the word name for each of these numbers.

Write the place value name for each of these numbers.

- 3.** Sixty-two thousand, three hundred thirty-seven **4.** Five million, four hundred forty-four thousand, nineteen

Insert > or < between the numbers to make a true statement.

- $$\begin{array}{r} \textbf{5. } 347 \quad 351 \\ \textbf{6. } 76 \quad 69 \\ \textbf{7. } 809 \quad 811 \end{array}$$

Round to the nearest ten, hundred, thousand, and ten thousand.

Exercises 10–14. The table displays the population of Hepner by age group.

Population of Hepner by Age Group

Age, in years	Number of Residents	Age, in years	Number of Residents
Under 15	472	36–50	1098
15–25	398	51–70	602
26–35	612	Over 70	89

- 10.** Which age group is the largest?
 - 11.** How many more residents are in the under-15 group than in the 15–25 group?
 - 12.** What is the total population in the 26–50 age group?
 - 13.** How many more people are there in the under-15 age group as opposed to the over-70 age group?
 - 14.** What is the population of Hepner?

Section 1.2

Add.

- | | | | |
|--------------|--------------|--------------|--------------|
| 15. | 16. | 17. | 18. |
| 336 | 3834 | 34 | 6,891 |
| 72 | 510 | 455 | 12,055 |
| <u>+ 509</u> | <u>+ 519</u> | 17 | 51,932 |
| | | 377 | 4,772 |
| | | <u>+ 881</u> | <u>+ 492</u> |

Subtract.

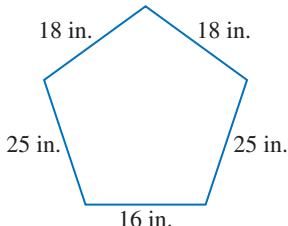
19.
$$\begin{array}{r} 943 \\ - 722 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 803 \\ - 738 \\ \hline \end{array}$$

21.
$$\begin{array}{r} 8315 \\ - 6983 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 246,892 \\ - 149,558 \\ \hline \end{array}$$

23. Find the perimeter of the polygon.



24. Estimate the sum by rounding each addend to the nearest ten thousand and to the nearest thousand.

$$\begin{array}{r} 34,683 \\ 5,278 \\ 11,498 \\ 678 \\ + 56,723 \\ \hline \end{array}$$

25. Estimate the difference by rounding each number to the nearest thousand and to the nearest hundred.

$$\begin{array}{r} 7534 \\ - 4267 \\ \hline \end{array}$$

Section 1.3*Multiply.*

26.
$$\begin{array}{r} 73 \\ \times 28 \\ \hline \end{array}$$

27.
$$\begin{array}{r} 406 \\ \times 29 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 407 \\ \times 68 \\ \hline \end{array}$$

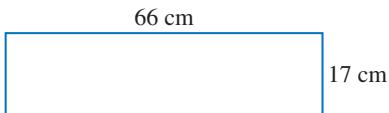
29.
$$(449)(171)$$

30. The local Girl Scout Troop sold 54 cases of cookies during the recent sale. Each case contained 24 boxes of cookies. If the cookies sold for \$4 a box, how much did they gross from the sale?

31. Estimate the product by front rounding the factors.

$$\begin{array}{r} 5,810 \\ \times 462 \\ \hline \end{array}$$

32. Find the area of the rectangle.

**Section 1.4***Divide.*

33. $14 \overline{)210}$

34. $18 \overline{)576}$

35. $176 \overline{)15,141}$

36. $274,486 \div 74$

37. $65 \overline{)345,892}$ (Round to the nearest hundred.)

38. The Candy Basket packs boxes containing 32 pieces of assorted chocolates. How many boxes can be made from 4675 chocolates? Will any pieces of candy be left over? If so how many?

Section 1.5

Find the value.

39. 11^3 40. 4^5

Multiply or divide.

41. 23×10^3 42. $78,000,000 \div 10^5$ 43. 712×10^6

44. $35,600,000 \div 10^4$

45. In 2005, President Bush's plan to privatize part of Social Security required borrowing approximately $\$34 \times 10^8$. Write this amount in place value form.

Section 1.6

Simplify.

46. $40 - 24 + 8$ 47. $6 \cdot 10 + 5$ 48. $18 \div 2 - 3 \cdot 2$

49. $94 \div 47 + 47 - 16 \cdot 2 + 6$ 50. $35 - (25 - 17) + (12 - 10)^2 + 5 \cdot 2$

Section 1.7

Find the average, median, and mode.

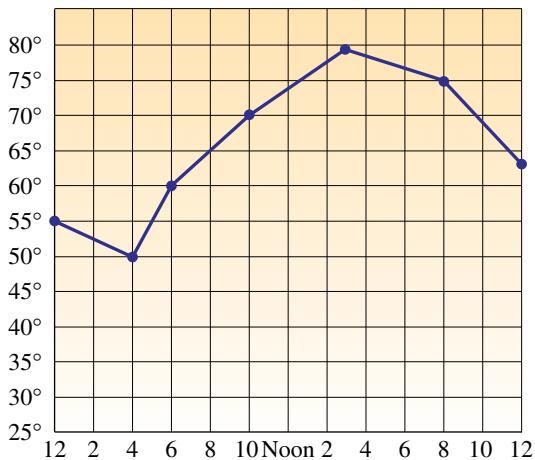
51. 41, 641, 23, 70, 87 52. 93, 110, 216, 317

53. 63, 74, 53, 63, 37, 82 54. 1086, 4008, 3136, 8312, 8312, 1474

55. The six children of Jack and Mary Barker had annual incomes of \$54,500, \$45,674, \$87,420, \$110,675, \$63,785, and \$163,782. Find the average salary of the six siblings to the nearest hundred dollars.

Section 1.8

Exercises 56–57. The graph displays the temperature readings for a 24-hour period.



56. At what times do the highest and lowest temperatures occur?
57. What is the difference in temperature from 6 A.M. and 8 P.M.?
58. The grade distribution in an algebra class is displayed in the table.

Algebra Grades

Grade	Number of Students
A	5
B	10
C	12
D	4
F	2

Construct a bar graph to display the data.

True/False Concept Review CHAPTER 1

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. All whole numbers can be written using nine digits. 1. _____
2. In the number 8425, the digit 4 represents 400. 2. _____
3. The word *and* is not used when writing the word names of whole numbers. 3. _____
4. The symbols, $7 < 23$, can be read “seven is greater than twenty-three.” 4. _____
5. $2567 < 2566$ 5. _____
6. To the nearest thousand, 7398 rounds to 7000. 6. _____
7. It is possible for the rounded value of a number to be equal to the original number. 7. _____
8. The expanded form of a whole number shows the plus signs that are usually not written. 8. _____
9. The sum of 80 and 7 is 807. 9. _____
10. The process of “carrying” when doing an addition problem with pencil and paper is based on the place value of the numbers. 10. _____
11. A line graph has at least two scales. 11. _____
12. The product of 15 and 3 is 18. 12. _____
13. It is possible to subtract 47 from 65 without “borrowing.” 13. _____
14. The number 8 is a factor of 72. 14. _____
15. The multiplication sign is sometimes omitted when writing a multiplication problem. 15. _____
16. Whenever a number is multiplied by zero, the value remains unchanged. 16. _____
17. There is more than one method for doing division problems. 17. _____

18. In $104 \div 4 = 26$, the quotient is 104. 18. _____
19. If a division exercise has a remainder, then we know that there is no whole number quotient. 19. _____
20. When zero is divided by any whole number from 25 to 91, the result is 0. 20. _____
21. The result of zero divided by zero can be either 1 or 0. 21. _____
22. The value of 7^2 is 14. 22. _____
23. The value of 222^1 is 222. 23. _____
24. One billion is a power of 10. 24. _____
25. The product of 450×10^3 is equal to 45,000. 25. _____
26. The quotient of 9000 and 10 is 900. 26. _____
27. In the order of operations, exponents are always evaluated before addition. 27. _____
28. In the order of operations, multiplication is always evaluated before subtraction. 28. _____
29. The value of $2^3 + 2^3$ is the same as the value of 2^4 . 29. _____
30. The average of three different numbers is smaller than the largest of the three numbers. 30. _____
31. The word *mean* sometimes has the same meaning as *average*. 31. _____
32. A table is a method of displaying data in an array using a horizontal and vertical arrangement to distinguish the type of data. 32. _____
33. The median of 34, 54, 14, 44, 67, 81, and 90 is 44. 33. _____
34. The distance around a polygon is called the perimeter. 34. _____

Test CHAPTER 1**ANSWERS**

1. Divide: $72 \overline{)15,264}$
2. Subtract: $9615 - 6349$
3. Simplify: $55 \div 5 + 6 \cdot 4 - 7$
4. Multiply: $37(428)$
5. Insert $<$ or $>$ to make the statement true: $368 \quad 371$
6. Multiply: 55×10^6
7. Multiply: $608(392)$
8. Write the place value name for seven hundred thirty thousand sixty-one.
9. Find the average of 3456, 812, 4002, 562, and 1123.
10. Multiply: $65(5733)$. Round the product to the nearest hundred.
11. Round 38,524 to the nearest thousand.
12. Estimate the sum of 95,914, 31,348, 68,699, and 30,341 by rounding each number to the nearest ten thousand.
13. Find the value of 9^3 .
14. Add: $84 + 745 + 56 + 7802$
15. Find the perimeter of the rectangle.
- 
16. Estimate the product by front rounding: $752(38)$
17. Subtract:
- | | |
|---------------------|--------|
| $\underline{- 895}$ | 7053 |
|---------------------|--------|
18. Write the word name for 4005.
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____

19. Simplify: $65 + 3^3 - 66 \div 11$

19. _____

20. Add:
42,888
67,911
93,467
23,567
+ 31,823

20. _____

21. Divide: $7,730,000,000 \div 10^6$

21. _____

22. Round 675,937,558 to the nearest million.

22. _____

23. Divide: $75,432 \div 65$

23. _____

24. Simplify: $95 - 8^2 + 48 \div 4$

24. _____

25. Find the area of a rectangle that measures 23 cm by 15 cm.

25. _____

26. Simplify: $(5 \cdot 3)^2 + (4^2)^2 + 11 \cdot 3$

26. _____

27. Find the average, median, and mode of 795, 576, 691, 795, 416, and 909.

27. _____

28. A secretary can type an average of 75 words per minute. If there are approximately 700 words per page, how long will it take the secretary to type 18 pages?

28. _____

29. Nine people share in a Power Ball lottery jackpot. If the jackpot is worth \$124,758,000, how much will each person receive? If each person's share is to be distributed evenly over a 20-year period, how much will each person receive per year?

29. _____

Exercises 30–32. The graph shows the home sales for a month at a local real estate office.

30. What price range had the greatest sales?

30. _____

31. What was the total number of homes sold in the top two price ranges?

31. _____

32. How many more homes were sold in the lowest range than in the highest range?

32. _____

Exercises 33–35. The table shows the number of employees by division and shift for Beaver Horseradish.

Employees by Division: Beaver Horseradish

Division	Day Shift	Night Shift
A	215	175
B	365	120
C	95	50

33. Which division has the greatest number of employees?

33. _____

34. How many more employees are in the day shift in division A as compared to the day shift in division C?

34. _____

35. How many employees are in the three divisions?

35. _____

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Group Project (1–2 WEEKS) CHAPTER 1

OPTIONAL

All tables, graphs, and charts should be clearly labeled and computer-generated if possible. Written responses should be typed and checked for spelling and grammar.

1. Go to the library and find the population and area for each state in the United States. Organize your information by geographic region. Record your information in a table.
2. Calculate the total population and the total area for each region. Calculate the population density (number of people per square mile, rounded to the nearest whole person) for each region, and put this and the other regional totals in a regional summary table. Then make three separate graphs, one for regional population, one for regional area, and the third for regional population density.
3. Calculate the average population per state for each region, rounding as necessary. Put this information in a bar graph. What does this information tell you about the regions? How is it different from the population density of the region?
4. How did your group decide on the makeup of the regions? Explain your reasoning.
5. Are your results what you expected? Explain. What surprised you?

New Habits from Old

If you are in the habit of studying math by only reading the examples to learn how to do the exercises, stop now! Instead, read the assigned section—all of it—before class. It is important that you read more than the examples so that you fully understand the concepts. Knowing how to do a problem isn't all that needs to be learned. Knowing where and when to use specific skills is also essential.

When you read, read interactively. This means that you should be both writing and thinking about what you are reading. Write down new vocabulary; perhaps start a list of new terms paraphrased in words that are clear to you. Take notes on the How & Why segments, jotting down questions you may have, for example. As you read examples, work the Warm-Up problems in the margin. Begin the exercise set only when you understand what you have read in the section. This process should make your study sessions go much faster and be more effective.

If you have written down questions during your study session, be sure to ask them at the next class session, seek help from a tutor, or discuss them with a classmate. Don't leave these questions unanswered.

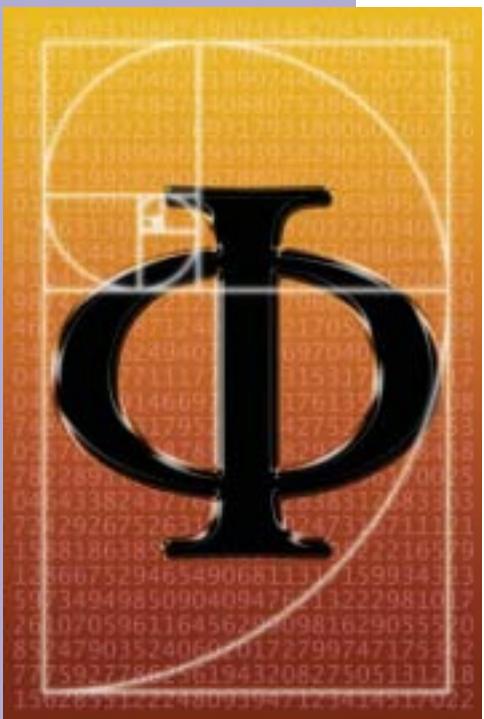
Pay particular attention to the objectives at the beginning of each section. Read these at least twice: first, when you do your reading before class and again, after attending class. Ask yourself, "Do I understand what the purpose of this section is?" Read the objectives again before test time to see if you feel that you have met these objectives.

During your study session, if you notice yourself becoming tense and your breathing shallow (light and from your throat or upper part of your lungs), follow this simple coping strategy. Say to yourself, "I'm in control. Relax and take a deep breath." Breathe deeply and properly by relaxing your stomach muscle (that's right, you have permission to let your stomach protrude!) and inhaling so that the air reaches the bottom of your lungs. Hold the air in for a few seconds; then slowly exhale, pulling your stomach muscle in as you exhale. This easy exercise not only strengthens your stomach muscle but gives your body and brain the oxygen you need to perform free from physical stress and anxiety. This deep breathing relaxation method can be done in 1 to 5 minutes. You may want to use it several times a day, especially during an exam.

These techniques can help you to start studying math more effectively and to begin managing your anxiety. Begin today.

2

PRIMES AND MULTIPLES



APPLICATION

Mathematicians have always been fascinated by numbers—their structure and their uses. All the other chapters in this textbook explain ways to use numbers. This chapter explores the structure of counting numbers. The structure of the counting numbers—their basic building blocks—is similar in nature to the concept of all molecules being made up of atoms of the basic elements. As a molecule of water is formed from two atoms of hydrogen and one atom of oxygen, H_2O , so the number 12 is made up of two factors of 2 and one factor of 3, that is $12 = 2 \cdot 2 \cdot 3$.

Throughout the history of humanity, people have studied different sets of numbers. Some groups have even attributed magical powers to certain numbers (“lucky seven”) or groups of numbers because of some special properties. We investigate here the notion of a “magic square.” Magic squares are arrangements of the numbers 14, or 19, or 146, or 125, or 1 any number squared. These numbers are written in square arrays that are 2×2 , or 3×3 , or 4×4 , and so on. The numbers are placed so that the sum of each row and of each column and of each diagonal is the same. Figure 2.1 shows an example of a 3×3 magic square, which uses the numbers from 1 to 9 (3^2).

2	9	4
7	5	3
6	1	8

Figure 2.1

SECTIONS

- 2.1** Divisibility Tests
- 2.2** Multiples
- 2.3** Divisors and Factors
- 2.4** Primes and Composites
- 2.5** Prime Factorization
- 2.6** Least Common Multiple

BASIC
Mathematics Now™

Row 1	$2 + 9 + 4 = 15$	Column 1	$2 + 7 + 6 = 15$
Row 2	$7 + 5 + 3 = 15$	Column 2	$9 + 5 + 1 = 15$
Row 3	$6 + 1 + 8 = 15$	Column 3	$4 + 3 + 8 = 15$
Diagonal	$2 + 5 + 8 = 15$	Diagonal	$4 + 5 + 6 = 15$

In medieval times, some people wore magic squares and used them as talismans. The talisman wearers considered them powerful enough to provide protection from evil spirits.

Benjamin Franklin was a big fan of magic squares. As clerk to the Pennsylvania Assembly, he admitted to creating them when he was bored with the proceedings. He created more and more complex squares as time passed. He even experimented with magic circles. Figure 2.2 shows an 8×8 square, using the numbers from 1 to 64 (8^2), that Franklin created. This square has several interesting features.

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

Figure 2.2

Group Discussion

1. What is the sum of each row and each column?
2. What is the sum of the four corners? Of the four middle squares?
3. Starting at the lower left corner, 16, move diagonally up three times, move right once, move down and right diagonally to the lower right corner, 17. What is the sum of this path?
4. Start at 50 and trace the path “parallel” to the one in Exercise 3. What is the sum of this path?
5. Find at least six other paths of eight numbers through the square that have the same sum.
6. What is the sum of the first four numbers in each row? In each column? Of the last four numbers in each row or column? What is the sum of these half rows and half columns?

2.1 Divisibility Tests

VOCABULARY

A whole number is **divisible** by another whole number if the quotient of these numbers is a natural number and the remainder is 0. The second number is said to be a **divisor** of the first. Thus, 7 is a **divisor** of 42, because $42 \div 7 = 6$.

We also say 42 is **divisible** by 7.

The **even digits** are 0, 2, 4, 6, and 8.

The **odd digits** are 1, 3, 5, 7, and 9.

OBJECTIVES

1. Determine whether a natural number is divisible by 2, 3, or 5.
2. Determine whether a natural number is divisible by 6, 9, or 10.

HOW & WHY

OBJECTIVE 1

Determine whether a natural number is divisible by 2, 3, or 5.

To ask if a number is divisible by 3 is to ask if the division of the number by 3 comes out even (has no remainder). We can answer this question by doing the division and checking to see that there is no remainder. Or, we can bypass the division by using divisibility tests. For many numbers we can answer questions about divisibility mentally. Table 2.1 provides clues for some of these tests.

Table 2.1 Patterns of Multiplying

Some Natural Numbers	Multiply by 2	Multiply by 3	Multiply by 5	Multiply by 9
1	2	3	5	9
2	4	6	10	18
3	6	9	15	27
4	8	12	20	36
5	10	15	25	45
6	12	18	30	54
10	20	30	50	90
16	32	48	80	144
24	48	72	120	216
44	88	132	220	396
66	132	198	330	594

In the second column (Multiply by 2), the ones digit of each number is an even digit, that is either 0, 2, 4, 6, or 8. Because the ones place is even, the number itself is also even.

In the third column (Multiply by 3), the sum of the digits of each number in the column is divisible by 3. For example, the sum of the digits of 48 ($16 \cdot 3$) is $4 + 8$ or 12, and 12 is divisible by 3. Likewise, the sum of the digits of 198 is divisible by 3 because $1 + 9 + 8$ is 18.

In the fourth column (Multiply by 5), the ones digit of each number is 0 or 5.

To test for divisibility of a natural number by 2, 3, or 5

If the ones-place digit of the number is even (0, 2, 4, 6, or 8), the number is divisible by 2.

If the sum of the digits of the number is divisible by 3, then the number is divisible by 3.

If the ones-place digit of the number is 0 or 5, the number is divisible by 5.

Warm-Ups A–D

A. Is 63 divisible by 2, 3, or 5?

63 is divisible by 3.

The ones-place digit is 3.

$3 + 6 = 9$, which is divisible by 3.

The ones-place digit is neither 0 nor 5.

B. Is 390 divisible by 2, 3, or 5?

390 is divisible by 2.

The ones-place digit is 0.

390 is divisible by 3.

$3 + 9 + 0 = 12$, which is divisible by 3.

390 is divisible by 5.

The ones-place digit is 0.

C. Is 235 divisible by 2, 3, or 5?

235 is not divisible by 2.

The ones-place digit is not even.

235 is not divisible by 3.

$2 + 3 + 5 = 10$, which is not divisible by 3.

235 is not divisible by 5.

The ones-place digit is neither 0 nor 5.

D. If Anna has ¥626 to divide among the children, will each child receive the same number of yen in whole numbers. Why or why not?

Examples A–D BASIC Mathematics Now™

DIRECTIONS: Determine whether the natural number is divisible by 2, 3, or 5.

STRATEGY: First check the ones-place digit. If it is even, the number is divisible by 2. If it is 0 or 5, the number is divisible by 5. Next find the sum of the digits. If the sum is divisible by 3, the number is divisible by 3.

A. Is 42 divisible by 2, 3, or 5?

42 is divisible by 2.

The ones-place digit is 2.

42 is divisible by 3.

$4 + 2 = 6$, which is divisible by 3.

42 is not divisible by 5.

The ones-place digit is neither 0 nor 5.

B. Is 210 divisible by 2, 3, or 5?

210 is divisible by 2.

The ones-place digit is 0.

210 is divisible by 3.

$2 + 1 + 0 = 3$, which is divisible by 3.

210 is divisible by 5.

The ones-place digit is 0.

C. Is 721 divisible by 2, 3, or 5?

721 is not divisible by 2.

The ones-place digit is not even.

721 is not divisible by 3.

$7 + 2 + 1 = 10$, which is not divisible by 3.

721 is not divisible by 5.

The ones-place digit is neither 0 nor 5.

D. Georgio and Anna and their three children are on a trip of Japan. Anna has a total of ¥585 to divide among the children. She wants each child to receive the same number of yen in whole numbers. Is this possible? Why or why not?

Yes, each child will receive the same number of yen in whole numbers (¥195 each) because 585 is divisible by 3.

HOW & WHY

OBJECTIVE 2

Determine whether a natural number is divisible by 6, 9, or 10.

In Table 2.1, some numbers appear in both the Multiply by 2 column and the Multiply by 3 column. These numbers are divisible by 6. Because $6 = 2 \cdot 3$, every number divisible by 6 must also be divisible by 2 and 3. For example, 132 is divisible by both 2 and 3. Therefore, it is also divisible by 6.

In the Multiply by 9 column, the sum of the digits of each number is divisible by 9. For example, the sum of the digits in 36 is $3 + 6$, or 9, which is divisible by 9. Also, the sum of the digits in 981 is $9 + 8 + 1$, or 18, which is divisible by 9.

Notice that, because 0 is even, all natural numbers ending in 0 are divisible by 2. They are also divisible by 5 because they end in 0. These numbers are also divisible by 10. These numbers appear in both the Multiply by 2 column and the Multiply by 5 column. The numbers 20, 90, and 130 are all divisible by 10.

Answers to Warm-Ups

- A. 63 is divisible by 3; 63 is not divisible by 2 or 5.
- B. 390 is divisible by 2, 3, and 5.
- C. 235 is divisible by 5; 235 is not divisible by 2 or 3.
- D. No, each child will not receive the same amount because 626 is not divisible by 3.

To test for divisibility of a natural number by 6, 9, or 10

If the number is divisible by both 2 and 3, then the number is divisible by 6.

If the sum of the digits of the number is divisible by 9, then the number is divisible by 9.

If the ones-place digit of the number is 0, then the number is divisible by 10.

Examples E–F



BASIC
Mathematics Now™

DIRECTIONS: Determine whether a natural number is divisible by 6, 9, or 10.

STRATEGY: First check whether the number is divisible by both 2 and 3. If so, the number is divisible by 6. Second, find the sum of the digits. If the sum is divisible by 9, then the number is divisible by 9. Finally, check the ones-place digit. If the digit is 0, the number is divisible by 10.

E. Is 810 divisible by 6, 9, or 10?

- | | |
|-------------------------|--|
| 810 is divisible by 6. | 810 is divisible by both 2 and 3. |
| 810 is divisible by 9. | $8 + 1 + 0 = 9$, which is divisible by 9. |
| 810 is divisible by 10. | The ones-place digit is 0. |

F. Is 1770 divisible by 6, 9, or 10?

- | | |
|-----------------------------|---|
| 1770 is divisible by 6. | 1770 is divisible by both 2 and 3. |
| 1770 is not divisible by 9. | $1 + 7 + 7 + 0 = 15$, which is not divisible by 9. |
| 1770 is divisible by 10. | The ones-place digit is 0. |

Warm-Ups E–F

E. Is 720 divisible by 6, 9, or 10?

F. Is 1152 divisible by 6, 9, or 10?

Answers to Warm-Ups

- E. 720 is divisible by 6, 9, and 10.
F. 1152 is divisible by 6 and 9; 1152 is not divisible by 10.

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Exercises 2.1

OBJECTIVE 1

Determine whether a natural number is divisible by 2, 3, or 5.

A Is each number divisible by 2?

1. 27

2. 22

3. 20

4. 75

5. 38

6. 58

Is each number divisible by 5?

7. 56

8. 23

9. 45

10. 115

11. 551

12. 710

Is each number divisible by 3?

13. 81

14. 30

15. 36

16. 37

17. 43

18. 53

B Determine whether the natural number is divisible by 2, 3, or 5.

19. 2190

20. 2670

21. 3998

22. 4578

23. 4815

24. 1845

25. 4175

26. 5280

27. 11,205

28. 11,206

OBJECTIVE 2

Determine whether a natural number is divisible by 6, 9, or 10.

A Is each number divisible by 6?

29. 114

30. 141

31. 254

32. 333

33. 444

34. 452

Is each number divisible by 9?

35. 117

36. 171

37. 376

38. 333

39. 414

40. 765

Is each number divisible by 10?

41. 233

42. 330

43. 555

44. 706

45. 1920

46. 9210

B Determine whether the natural number is divisible by 6, 9, or 10.**47.** 4980**48.** 6894**49.** 6993**50.** 3780**51.** 5555**52.** 8888**53.** 5700**54.** 7880**55.** 7290**56.** 9156**C**

57. Pedro and two friends plan to buy a used car that is priced at \$3231. Is it possible for each of them to spend the same whole number of dollars? Explain.

59. A marching band has 175 members. Can the band march in rows of 3 without any member being left over? Rows of 5? Rows of 10?

61. Lucia has 120 students in a single class at a community college. She wants to divide the students into small equal-sized groups to work on a group project. Is it possible to have groups of 5 students? Of 6 students? Of 9 students? Explain.

63. Allen has a collection of 936 comic books. He plans to divide the collection evenly between his five nieces and four nephews. Will each receive the same number of comic books?

65. Mark and Barbara attend the zoo and see a pen with peacocks and water buffalo. On the way out, Barbara remarks that the pen has 30 eyes and 44 feet. How many animals are in the pen?

67. A small circus has an act with elephants and riders. In all, there are 48 eyes and 68 feet. What is the total number of elephants plus riders?

58. Janna and four of her friends plan to run 82 miles in relays. Is it possible for each runner to run the same whole number of miles? Explain.

60. Joe has a bookshelf that is 42 in. wide. He wants to use it for a set of encyclopedias. Each volume is 2 in. thick. Is it possible for him to completely fill the shelf with volumes? Could he completely fill the shelf with 3-in. volumes?

62. Six merchants combine to build one new store in each of 318 cities. Is it possible for each merchant to oversee the building of the same number of new stores? Explain.

64. A movie theater is to have 250 seats. The manager plans to arrange them in rows with the same number of seats in each row. Is it possible to have rows of 10 seats each? Rows of 15 seats each? Determine the answers by using divisibility tests.

66. In Exercise 65, how many peacocks and how many water buffalo are in the pen?

68. In Exercise 67, how many elephants and how many riders are in the act?

STATE YOUR UNDERSTANDING

- 69.** Explain what it means to say, “This number is divisible by 5.” Give an example of a number that is divisible by 5 and one that is not divisible by 5.
- 70.** Explain why a number that is divisible by both 2 and 3 must also be divisible by 6.
- 71.** Explain the difference in the divisibility test for 2 and 3.
- 72.** Write a short statement to explain why every number divisible by 9 is also divisible by 3.

CHALLENGE

- 73.** Is 23,904 divisible by 6?
- 74.** Is 11,370 divisible by 15? Write a divisibility test for 15.
- 75.** Is 11,370 divisible by 30? Write a divisibility test for 30.
- 76.** Is 99,000,111,370 divisible by:
 - a.** 2? _____, because the _____
 - b.** 3? _____, because the _____
 - c.** 5? _____, because the _____
 - d.** 6? _____, because the _____
 - e.** 9? _____, because the _____
 - f.** 10? _____, because the _____

GROUP WORK

- 77.** As a group, find divisibility tests for 4, 8, 20, and 25. Report to the class and compare your tests with the other groups.
- 78.** Call a local recycling center and ask for the number of pounds of newspaper they collect for 1 week. Use the figure given and round to the nearest whole number. Is this amount divisible into 2-, 3-, 5-, 6-, 9-, or 10-pound bins? Which size is the most efficient? Why?
- 79.** Make up a puzzle that is similar to those in Exercises 6568. Trade puzzles with another group and solve each others’ puzzle. One student group made up a puzzle with a group of animals having 25 eyes and 37 feet. Barring deformed animals, explain why this is not possible.

MAINTAIN YOUR SKILLS

- 80.** Round 56,857 to the nearest thousand and nearest ten thousand.
- 81.** Round 5,056,857 to the nearest ten thousand and nearest hundred thousand.
- 82.** Divide: $792 \div 66$
- 83.** Divide: $1386 \div 66$
- 84.** Find the perimeter of a square that is 14 cm on a side.
- 85.** Find the area of a square that is 14 cm on a side.
- 86.** Multiply 12 by 1, 2, 3, 4, 5, and 6.
- 87.** Multiply 13 by 1, 2, 3, 4, 5, and 6.
- 88.** Multiply 123 by 1, 2, 3, 4, 5, and 6.
- 89.** Multiply 1231 by 1, 2, 3, 4, 5, and 6.

2.2 Multiples

VOCABULARY

A **multiple** of a whole number is the product of that number and a natural number. For instance,
21 is a multiple of 7 because $7(3) = 21$.
77 is a multiple of 7 because $7(11) = 77$.
98 is a multiple of 7 because $7(14) = 98$.
147 is a multiple of 7 because $7(21) = 147$.

OBJECTIVES

1. List multiples of a whole number.
2. Determine whether a given whole number is a multiple of another whole number.

HOW & WHY

OBJECTIVE 1 List multiples of a whole number.

To list the multiples of 7, we multiply 7 by each natural number. See Table 2.2.

Table 2.2 Multiples of 7

Natural Number	Multiple of 7
1	7
2	14
3	21
4	28
5	35
6	42
...	...
15	105
...	...
47	329
...	...

Table 2.2 can be continued without end. We say that the first multiple of 7 is 7, the second multiple of 7 is 14, the 15th multiple of 7 is 105, the 47th multiple of 7 is 329, and so on. To find a particular multiple of 7, say the 23rd, we multiply 7 by 23. The 23rd multiple of 7 is 161.

Examples A–E BASIC Mathematics Now™

DIRECTIONS: List the designated multiples.

STRATEGY: Multiply the natural number by the given value.

- A. List the first five multiples of 8.

$$1 \cdot 8 = 8 \quad \text{Multiply 8 by 1, 2, 3, 4, and 5.}$$

$$2 \cdot 8 = 16$$

$$3 \cdot 8 = 24$$

$$4 \cdot 8 = 32$$

$$5 \cdot 8 = 40$$

The first five multiples of 8 are 8, 16, 24, 32, and 40.

Warm-Ups A–E

- A. List the first five multiples of 6.

Answers to Warm-Ups

- A. 6, 12, 18, 24, 30

- B. List the first five multiples of 14.

- C. Find the 5th, 23rd, 28th, and 452nd multiples of 14.

- D. Find all of the multiples of 7 between 400 and 445.

- E. Maria's mathematics teacher assigns homework problems numbered from 1 to 70 that are multiples of 4. Which problems should she work?

- B. List the first five multiples of 16.

$$\begin{aligned}1 \cdot 16 &= 16 && \text{Multiply 16 by 1, 2, 3, 4, and 5.} \\2 \cdot 16 &= 32 \\3 \cdot 16 &= 48 \\4 \cdot 16 &= 64 \\5 \cdot 16 &= 80\end{aligned}$$

The first five multiples of 16 are 16, 32, 48, 64, and 80.

CALCULATOR EXAMPLE



- C. Find the 7th, 23rd, 28th, and 452nd multiples of 16.

STRATEGY: Use a calculator to multiply 7, 23, 28, and 452 by 16. The 7th multiple of 16 is 112, the 23rd multiple of 16 is 368, the 28th multiple of 16 is 448, and the 452nd multiple of 16 is 7232.

CALCULATOR EXAMPLE



- D. Find all of the multiples of 8 between 410 and 460.

STRATEGY: Use a calculator to make a quick estimate. Start with, say, the 50th multiple. Multiply 50 times 8.

$$\begin{aligned}50(8) &= 400 && \text{The product is too small.} \\51(8) &= 408 && \text{The product, 408, is still too small.} \\52(8) &= 416 && \text{This is the first multiple of 8 larger than 410.}\end{aligned}$$

The 52nd multiple is 416 so keep on multiplying.

$$\begin{array}{lll}52(8) = 416 & 53(8) = 424 & 54(8) = 432 \\55(8) = 440 & 56(8) = 448 & 57(8) = 456\end{array}$$

The multiples of 8 between 410 and 460 are 416, 424, 432, 440, 448, and 456.

- E. Jordan's mathematics teacher assigns homework problems numbered from 1 to 60 that are multiples of 6. Which problems should he work?

STRATEGY: Find the multiples of 6 from 1 to 60 by multiplying by 1, 2, 3, 4, and so on, until the product is 60 or larger.

$$\begin{array}{llll}1(6) = 6 & 2(6) = 12 & 3(6) = 18 & 4(6) = 24 \\6(6) = 36 & 7(6) = 42 & 8(6) = 48 & 9(6) = 54 \\& & & 10(6) = 60\end{array}$$

Jordan should work problems 6, 12, 18, 24, 30, 36, 42, 48, 54, and 60.

HOW & WHY

OBJECTIVE 2

Determine whether a given whole number is a multiple of another whole number.

Answers to Warm-Ups

- B. 14, 28, 42, 56, 70
C. 70, 322, 392, 6328
D. 406, 413, 420, 427, 434, 441
E. Maria should work problems 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, and 68.

If one number is a multiple of another number, the first must be divisible by the second. To determine whether 870 is a multiple of 6, we check to see if 870 is divisible by 6; that is, check to see whether 870 is divisible by both 2 and 3.

852 Divisible by 2 because the ones-place digit is even (2).
852 Divisible by 3 because $8 + 5 + 2 = 15$, which is divisible by 3.

So 852 is a multiple of 6.

If there is no divisibility test, use long division. For example, is 299 a multiple of 13? Divide 299 by 13.

$$\begin{array}{r} 23 \\ 13 \overline{)299} \\ 26 \\ \hline 39 \\ 39 \\ \hline 0 \end{array}$$

Because $299 \div 13 = 23$ with no remainder, $299 = 13 \times 23$. Therefore, 299 is a multiple of 13.

Examples F–J



BASIC Mathematics Now™

DIRECTIONS: Determine whether a given whole number is a multiple of another whole number.

STRATEGY: Use divisibility tests or long division to determine whether the first number is divisible by the second.

F. Is 873 a multiple of 9?

Use the divisibility test for 9.

$8 + 7 + 3 = 18$ The sum of the digits is divisible by 9.

So 873 is a multiple of 9.

G. Is 6138 a multiple of 6?

Use the divisibility tests for 2 and 3.

6138 is divisible by 2. The ones-place digit is even (2).

6138 is divisible by 3. The sum of the digits, $6 + 1 + 3 + 8 = 18$, is divisible by 3.

So 6138 is a multiple of 6.

H. Is 2103 a multiple of 19?

Because we have no divisibility test for 19, we use long division.

$$\begin{array}{r} 110 \\ 19 \overline{)2103} \\ 19 \\ \hline 20 \\ 19 \\ \hline 13 \end{array}$$

The remainder is not 0.

So 2103 is not a multiple of 19.

I. Is 810 a multiple of 15?

Use the divisibility tests for 3 and 5 or use long division.

810 is divisible by 3. The sum of the digits, $8 + 1 + 0 = 9$, is divisible by 3.

810 is divisible by 5. The ones-place digit is 0.

Long division gives the same result because $810 \div 15 = 54$.

So 810 is a multiple of 15.

J. Is 4 a multiple of 20?

The number 4 is not divisible by 20. The multiples of 20 are 20, 40, 60, and so on.

The smallest multiple of 20 is $20 \cdot 1 = 20$.

No, 4 is not a multiple of 20.

Warm-Ups F–J

F. Is 738 a multiple of 9?

G. Is 4144 a multiple of 6?

H. Is 481 a multiple of 13?

I. Is 675 a multiple of 45?

J. Is 10 a multiple of 70?

Answers to Warm-Ups

- F. yes G. no H. yes
I. yes J. no

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Exercises 2.2

OBJECTIVE 1

List multiples of a whole number.

A List the first five multiples of the whole number.

- | | | |
|----------------|----------------|----------------|
| 1. 3 | 2. 14 | 3. 17 |
| 4. 18 | 5. 21 | 6. 24 |
| 7. 30 | 8. 34 | 9. 50 |
| 10. 45 | | |
| B | | |
| 11. 54 | 12. 59 | 13. 64 |
| 14. 72 | 15. 85 | 16. 113 |
| 17. 157 | 18. 234 | 19. 361 |
| 20. 427 | | |

OBJECTIVE 2

Determine whether a given whole number is a multiple of another whole number.

A Is each number a multiple of 6?

- | | | |
|---------------------------------|----------------|----------------|
| 21. 54 | 22. 44 | 23. 72 |
| 24. 96 | 25. 95 | 26. 102 |
| Is each number a multiple of 9? | | |
| 27. 57 | 28. 81 | 29. 84 |
| 30. 117 | 31. 324 | 32. 378 |

B Is each number a multiple of 7?

- | | | |
|----------------|----------------|----------------|
| 33. 84 | 34. 86 | 35. 91 |
| 36. 105 | 37. 119 | 38. 167 |

Is each number a multiple of 6? of 9? of 15?

- | | | |
|----------------|----------------|----------------|
| 39. 558 | 40. 600 | 41. 660 |
| 42. 675 | 43. 690 | 44. 768 |

Is each number a multiple of 13? or 19?

45. 299

46. 304

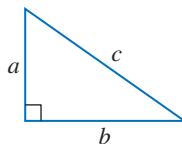
47. 741

48. 988

C

- 49.** Jean is driving home from work one evening when she comes across a police safety inspection team stopping cars. She knows that the team chooses every fourth car to inspect. She counts and determines that she is 14th in line. Will she be selected to have her car safety-checked?
- 51.** A teacher assigns problems numbered from 1 to 52 that are multiples of 4. Which problems should the students work?
- 53.** Katy reported that she counted 30 goat feet in a pen at a petting zoo. Explain how you know that she made a mistake.
- 55.** Joaquim supervises the quality-control team at a bottling plant. His team is responsible for checking the quality of a line of soft drinks. The team is assigned to check bottles numbered 500 to 700 in each batch. Joaquim decides to have his team check every bottle that is a multiple of 15. List the bottle numbers the team should check.
- 57.** A gear has 20 teeth. The gear is rotated through 240 teeth. Is it in the original position after the rotation?
- 59.** According to recent nationwide estimates by the Consumer Product Safety Commission, there were approximately 105,000 injuries treated in hospital emergency rooms related to in-line skating. During the same period, there were approximately 35,000 injuries due to skateboarding. Is the number of in-line skating injuries a multiple of the number of skateboarding injuries? If you were reporting a comparison of these injuries, how might you write the comparison so that it is easily understood?
- 50.** In Exercise 49, would Jean be selected if she were 28th in line?
- 52.** A teacher assigns problems numbered from 1 to 105 that are multiples of 8. Which problems should the students work?
- 54.** Vance reported that he counted 37 duck feet in a pen. Can he be correct?
- 56.** Minh and his crew are setting up 450 chairs in a large banquet hall for a lecture. They want to have the same number of chairs in each row. What different arrangements are possible if there must be at least 5 rows? Assume that the number of chairs in a row must be larger than the number of rows.
- 58.** A gear has 24 teeth. The gear is rotated through 212 teeth. Is it in its original position after the rotation?

Exercises 60–63 relate to the chapter application. Pythagoras (ca. 580–500 B.C.) was a Greek mathematician who contributed to number theory and geometry. The Pythagorean theorem states that the sides of a right triangle satisfy the equation $a^2 + b^2 = c^2$, where a , b , and c are the sides.



Any set of three numbers that satisfies the equation is called a Pythagorean triple. A famous Pythagorean triple is 3, 4, 5.

- 60.** Verify that 3, 4, 5 is a Pythagorean triple.
- 61.** The branch of mathematics called “number theory” informs us that all multiples of a Pythagorean triple are also Pythagorean triples. Find three different Pythagorean triples that are multiples of 3, 4, and 5, and verify that they are also Pythagorean triples.
- 62.** Another Pythagorean triple includes $a = 5$ and $b = 12$. What is the third number in this triple?
- 63.** Find three triples that are multiples of the triple 5, 12, 13 and verify that they are Pythagorean triples.

STATE YOUR UNDERSTANDING

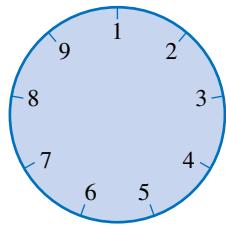
- 64.** Write a short statement to explain why *every* multiple of 12 is also a multiple of 6.
- 65.** One of the factors of 135 is 9 because $9 \cdot 15 = 135$. The number 9 is also a divisor of 135 because $135 \div 9 = 15$. Every factor of a number is also a divisor of that number. Explain in your own words why you think we have two different words, *factor* and *divisor*, for such numbers.
- 66.** Suppose you are asked to list all of the multiples of 7 from 126 to 175. Describe a method you could use to be sure all the multiples are listed.

CHALLENGE

- 67.** Is 3645 a multiple of 27?
- 68.** Is 8008 a multiple of 91?
- 69.** Find the largest number less than 6000 that is a multiple of 6, 9, and 17.
- 70.** How many multiples of 3 are there between 1000 and 5000?

GROUP WORK

- 71.** A high-school marching band is practicing. When they march in rows of 2, Elmo, the sousaphone player, has no one to march with. The band director rearranges them into rows of 3, but Elmo is still left marching alone.
- Will anyone be left over if the band marches in rows of 4? Why or why not?
 - When the band marches by 5s, Elmo is in one of the rows. Find two possible values for number of band members.
- 72.** Draw a large circle on a sheet of paper. Mark nine points that are approximately the same distance apart on the circle and number them from 1 to 9. Next list 10 consecutive multiples of your age. The first multiple and the last multiple will be associated with point number 1. Repeatedly add the digits together until you get a single digit. For example for 75: $7 + 5 = 12$, then $1 + 2 = 3$. Connect the points on the circle in the order of the sums of the digits. Make one circle for each member of the group. Compare the designs.

**MAINTAIN YOUR SKILLS**

List all of the numbers from 1 to 18 that divide the given number evenly (0 remainder).

- 73.** 12 **74.** 14 **75.** 15
- 76.** 16 **77.** 17 **78.** 18

Find the smallest number whose square is greater than the given number.

- 79.** 33 **80.** 87 **81.** 279 **82.** 500

- 83.** Is every number divisible by both 3 and 6 also divisible by 18? Explain.
- 84.** Is every number divisible by both 2 and 8 also divisible by 16? Explain.

2.3 Divisors and Factors

VOCABULARY

The product of two numbers is a multiple of each. The two numbers are called **factors**. Thus, 8 and 5 are factors of 40 because $8 \cdot 5 = 40$.

Recall that $12^2 = 12 \cdot 12 = 144$, so the **square** of 12 is 144. The number 144 is called a **perfect square**.

When two or more numbers are multiplied, each number is a factor of the product. If a number is a factor of a second number, it is also a **divisor** of the second number.

OBJECTIVES

1. Write a counting number as the product of two factors in all possible ways.
2. List all of the factors (divisors) of a counting number.

How & Why

OBJECTIVE 1

Write a counting number as the product of two factors in all possible ways.

Finding the factors of a number can be visualized using blocks, pennies, or other small objects. To illustrate, let's examine the factors of 12. Arrange 12 blocks or squares in a rectangle. The rectangle will have an area of 12 square units. The length and width of the rectangle are factors of 12. For instance, Figure 2.3 shows a rectangle with 3 rows of 4 squares.



Figure 2.3

Because $3 \times 4 = 12$, 3 and 4 are factors of 12. Figure 2.4 shows the same 12 squares arranged into a different rectangle.

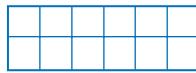


Figure 2.4

Figure 2.4 shows that $2 \times 6 = 12$, so 2 and 6 are also factors of 12.



Figure 2.5

Figure 2.5 shows a third arrangement: $1 \times 12 = 12$, so 1 and 12 are factors of 12. Figure 2.6 shows that the results of using rows of 5, 7, 8, 9, 10, or 11 do not form rectangles.

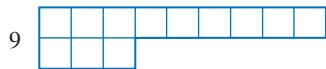
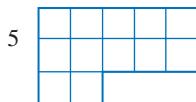


Figure 2.6

We conclude that the *only* pairs of factors of 12 are $1 \cdot 12$, $2 \cdot 6$, and $3 \cdot 4$.

To write a larger number, say 250, as the product of two factors in all possible ways, we could again draw rectangles, or divide 250 by every number smaller than 250. Either method takes too long. The following steps save time.

1. List all the counting numbers from 1 to the first number whose square is larger than 250. Because $15 \times 15 = 225$ and $16 \times 16 = 256$, we stop at 16.

1	6	11	16
2	7	12	
3	8	13	
4	9	14	
5	10	15	

We can stop at 16 because 250 divided by any number larger than 16 gives a quotient that is less than 16. But all the possible factors less than 16 are already in the chart.

2. Divide each of the listed numbers into 250. List the factors of the numbers that divide evenly. Otherwise, cross out the number.

1 • 250	6	11	16
2 • 125	7	12	
3	8	13	
4	9	14	
5 • 50	10 • 25	15	

When you find a number that is *not* a factor, you can also eliminate all the multiples of that number. For example, because 3 is not a factor of 250, we can also eliminate 6, 9, 12, and 15.

These steps give us a list of all the two-factor products. Hence, 250 written as a product in all possible ways is

$$1 \cdot 250 \quad 2 \cdot 125 \quad 5 \cdot 50 \quad 10 \cdot 25$$

To use the square method to write a counting number as the product of two factors in all possible ways

1. List all the counting numbers from 1 to the first number whose square is larger than the given number.
2. For each number on the list, test whether the number is a divisor of the given number.
3. If the number is not a divisor, cross it and all of its multiples off the list.
4. If the number is a divisor, write the indicated product of the two factors. The first factor is the tested number; the second factor is the quotient.

Warm-Ups A–C

Examples A–C BASIC Mathematics Now™

DIRECTIONS: Write the counting number as the product of two factors in all possible ways.

STRATEGY: Use the square method. Begin by testing all of the counting numbers from 1 to the first number whose square is larger than the given number.

- A. Write 98 as the product of two factors in all possible ways.

1 • 98	6
2 • 49	7 • 14
3	8
4	9
5	10

We can stop at 10 because $10^2 = 100$, which is larger than 98.

Answers to Warm-Ups

A. $1 \cdot 52$, $2 \cdot 26$, and $4 \cdot 13$

The pairs of factors whose product is 98 are $1 \cdot 98$, $2 \cdot 49$, and $7 \cdot 14$.

- B. Write 198 as the product of two factors in all possible ways.

1 • 198	6 • 33	11 • 18	We can stop at 15 because $15^2 = 225$, which is larger than 198.
2 • 99	7	12	
3 • 66	8	13	
4	9 • 22	14	
5	10	15	

The pairs of factors whose product is 198 are $1 \cdot 198$, $2 \cdot 99$, $3 \cdot 66$, $6 \cdot 33$, $9 \cdot 22$, and $11 \cdot 18$.

- C. A television station has 130 minutes of late-night programming to fill. In what ways can the time be scheduled if each program must last a whole number of minutes and if each schedule must include programs all the same length?

STRATEGY: List the pairs of factors of 130.

$$1 \cdot 130$$

[1 program that is 130 minutes long or 130 programs that are 1 minute long (probably too short)]

$$2 \cdot 65$$

[2 programs that are 65 minutes long or 65 programs that are 2 minutes long]

$$3$$

$$4$$

$$5 \cdot 26$$

[5 programs that are 26 minutes long or 26 programs that are 5 minutes long]

$$6$$

$$7$$

$$8$$

$$9$$

$$10 \cdot 13$$

[10 programs that are 13 minutes long or 13 programs that are 10 minutes long]

$$11$$

12 Stop here because $12^2 = 144$, which is larger than 130.

- B. Write 150 as the product of two factors in all possible ways.

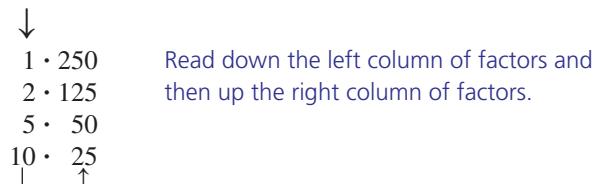
- C. A television station has 90 minutes of programming to fill. In what ways can the time be scheduled if each program must last a whole number of minutes and if each schedule must include programs all the same length?

How & Why

OBJECTIVE 2

List all of the factors (divisors) of a counting number.

The square method to find pairs of factors also gives us a list of *all* factors or divisors of a given whole number. To make a list of all factors of 250, in order, we can use the chart for all pairs of factors of 250.



Reading in the direction of the arrows, we see that the ordered list of all factors of 250 is: 1, 2, 5, 10, 25, 50, 125, and 250.

To list, in order, all the factors or divisors of a number

1. List all pairs of factors of the number in vertical form.
2. Read down the left column of factors and up the right column.

Answers to Warm-Ups

- B. $1 \cdot 150$, $2 \cdot 75$, $3 \cdot 50$, $5 \cdot 30$, $6 \cdot 25$, and $10 \cdot 15$
- C. The time can be filled with 1 program of 90 minutes or 90 programs 1 minute long, 2 programs 45 minutes long or 45 programs 2 minutes long, 3 programs 30 minutes long or 30 programs 3 minutes long, 5 programs 18 minutes long or 18 programs 5 minutes long, 6 programs 15 minutes long or 15 programs 6 minutes long, 9 programs 10 minutes long or 10 programs 9 minutes long.

Warm-Ups D–E

Examples D–E



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- D. List all the factors of 170.

$$\begin{array}{lll} 1 \cdot 168 & 6 \cdot 28 & 11 \\ 2 \cdot 84 & 7 \cdot 24 & 12 \cdot 14 \\ 3 \cdot 56 & 8 \cdot 21 & 13 \\ 4 \cdot 42 & 9 & \\ 5 & 10 & \end{array}$$



1 · 168 List the pairs and list the factors following the arrows.

$$2 \cdot 84$$

$$3 \cdot 56$$

$$4 \cdot 42$$

$$6 \cdot 28$$

$$7 \cdot 24$$

$$8 \cdot 21$$

$$12 \cdot 14$$



We can stop at 13 because $13^2 = 169$, which is larger than 168.

- E. List all the factors of 53.

- E. List all the factors of 41.

$$\begin{array}{lll} 1 \cdot 41 & 4 & 7 \\ 2 & 5 & \\ 3 & 6 & \end{array}$$

All the factors of 41 are 1 and 41.

Answers to Warm-Ups

- D. 1, 2, 5, 10, 17, 34, 85, and 170
E. 1 and 53

Exercises 2.3

OBJECTIVE 1

Write a counting number as the product of two factors in all possible ways.

A Write the whole number as the product of two factors in all possible ways.

- | | | |
|--------|--------|--------|
| 1. 16 | 2. 18 | 3. 23 |
| 4. 29 | 5. 33 | 6. 34 |
| 7. 46 | 8. 48 | 9. 49 |
| 10. 71 | 11. 72 | 12. 75 |
| 13. 80 | 14. 90 | |

B

- | | | |
|---------|---------|---------|
| 15. 95 | 16. 98 | 17. 100 |
| 18. 104 | 19. 105 | 20. 108 |
| 21. 112 | 22. 115 | 23. 116 |
| 24. 128 | 25. 333 | 26. 335 |
| 27. 339 | 28. 343 | |

OBJECTIVE 2

List all of the factors (divisors) of a counting number.

A List all of the factors (divisors) of the whole number.

- | | | |
|--------|--------|--------|
| 29. 16 | 30. 18 | 31. 29 |
| 32. 31 | 33. 33 | 34. 34 |
| 35. 46 | 36. 48 | 37. 52 |
| 38. 57 | 39. 65 | 40. 68 |

B**41.** 72**42.** 76**43.** 78**44.** 88**45.** 100**46.** 116**47.** 122**48.** 136**49.** 162**50.** 182**C** Write the counting number as a product of two factors in all possible ways.**51.** 444**52.** 555**53.** 652**54.** 672**55.** 680**56.** 772

57. In what ways can a television station schedule 120 minutes of time if each program must last a whole number of minutes, and each schedule must include programs of the same length?

58. In what ways can a television station schedule 75 minutes of time if each program must last a whole number of minutes, and each schedule must include programs of the same length?

59. Child care experts recommend that child care facilities have 1 adult for every 3 or 4 infants. The Bee Fore School Day Care has 24 infants. If they staff according to the low end of the recommendation, how many adults do they need? If they staff according to the high end of the recommendation, how many adults do they need?

60. Child care experts recommend that child care facilities have 1 caregiver for 7 to 10 preschoolers. KinderCare has 65 preschoolers. If they have 5 caregivers, do they meet the recommendations?

- 61.** A marching band has 50 members. Excluding single-file marchers, list all of the rectangular configurations possible for the band.
- 62.** Montereigh High School has a marching band with 72 members. Excluding single-file marchers, list all of the rectangular configurations possible for the band.
- 63.** The director of the Forefront Marching Band arranged the band members in rows of 4 but had one person left over. He then arranged the band in rows of 5, but still had one person left over. What do you know about the number of members in the band? What is the smallest possible number of band members?
- 64.** Jack has 36 poker chips. He is arranging them in the shape of a rectangle. How many different ways can he do this?

Exercises 65–68 relate to the chapter application.

*Numbers fascinated the ancient Greeks, in part because every Greek letter has a number associated with it. In particular, each Greek name had its own number and everyone was very interested in the properties of the number for his or her name. An exceptional individual was one whose number was a **perfect number**. A **perfect number** is a number that is the sum of all its divisors, excluding the number itself. Historical note: The Greeks did not consider a number to be a divisor of itself, although today we do.*

- 65.** List all the divisors of 6. Find the sum of the divisors that are less than 6. Is 6 a perfect number?
- 66.** Find another perfect number less than 20.
- 67.** Find a perfect number between 20 and 30.
- 68.** The third perfect number is 496. Verify that it is a perfect number.

STATE YOUR UNDERSTANDING

- 69.** Explain the difference between a factor and a divisor of a number.
- 70.** Describe how multiples, factors, and divisors are related to each other.

CHALLENGE

- 71.** Find the largest factor of 2973 that is less than 2973.
- 72.** Find the largest factor of 3381 that is less than 3381.

GROUP WORK

- 73.** Have each person in the group choose a whole number, making sure each person chooses a different whole number. Have each person with an even number divide by 2. Have each person with an odd number, multiply by 3 and add 1. Record the results. Repeat the same process with the new number—that is, divide each even number by 2 and multiply each odd number by 3 and add 1. Do this a third time. Continue until you know you have gone as far as possible. Now compare your final result with the other members of the group. You should all have ended with the same number, regardless of the number you started with. Mathematicians believe that this will always happen, yet they have been unable to prove that it is true for *all* whole numbers. This is called The Syracuse Conjecture.
- 74.** In a recent year, the average American consumed approximately 42 gallons of soft drinks, 25 gallons of milk, and 40 gallons of alcoholic beverages. Determine the number of ounces of soft drinks, milk, and alcoholic beverages each member of your group consumes in 1 week. Multiply these amounts by 52 to get an estimate of the annual consumption. Divide by 128 to find the number of gallons per category per person. Determine a group average for each category. Compare these with the given national average in a chart or graph.

MAINTAIN YOUR SKILLS*Multiply.*

75. $49(51)$

76. $68(404)$

77. $88(432)$

78. $407(702)$

Divide.

79. $78 \overline{)2574}$

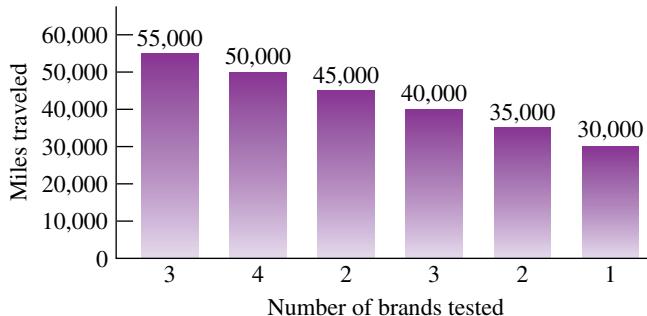
80. $82 \overline{)24,682}$

81. $306 \overline{)8265}$

82. $306 \overline{)92,110}$

- 83.** How many speakers can be wired from a spool of wire containing 1000 feet if each speaker requires 24 feet of wire? How much wire is left?

- 84.** A consumer magazine tested 15 brands of tires to determine the number of miles they could travel before the tread would be worn away. The results are shown in the graph. What is the average mileage of the 15 brands?



2.4 Primes and Composites

VOCABULARY

A **prime number** is a whole number greater than 1 with exactly two factors (divisors).

The two factors are the number 1 and the number itself.

A **composite number** is a whole number greater than 1 with more than two factors (divisors).

OBJECTIVE

Determine whether a whole number is prime or composite.

How & Why

OBJECTIVE

Determine whether a whole number is prime or composite.

The whole numbers zero (0) and one (1) are neither prime nor composite. The number 2 is the first prime number ($2 = 1 \cdot 2$), because 1 and 2 are the only factors of 2. The number 3 ($3 = 3 \cdot 1$) is also prime because 1 and 3 are its only factors. The number 4 is a composite number ($4 = 1 \cdot 4$ and $4 = 2 \cdot 2$) because 4 has more than two factors.

To determine whether a number is prime or composite, list its factors or divisors in a chart like those in Section 2.3. Then count the number of factors. For instance, the chart for 247 is

1	•	247	7	13 • 19	We would have to test up to 16 in order to find all the factors of 247, as $16^2 = 256$ and $256 > 247$. Because we do not need <i>all</i> the factors, we stop at 13 because we know that 247 has at least four factors.
2			8	14	
3			9	15	
4			10	16	
5			11		
6			12		

Therefore, 247 is a composite number.

The chart for 311 is

1	•	311	6	11	16	
2			7	12	17	
3			8	13	18	Stop here since $18 \cdot 18 = 324$ and $324 > 311$.
4			9	14		
5			10	15		

The number 311 is a prime number because it has exactly two factors, 1 and itself.

All prime numbers up to any given number may be found by a method called the Sieve of Eratosthenes. Eratosthenes (born ca. 230 B.C.) is remembered for both the prime Sieve and his method of measuring the circumference of Earth. The accuracy of his measurement, compared with modern methods, is within 50 miles, or six-tenths of 1%.

To use the famous Sieve to find all the primes from 2 to 35, list the numbers from 2 to 35.

(2)	3	4	5	6	7	8	9	The number 2 is prime, but all other multiples of 2 are not prime. They are crossed off.
10	11	12	13	14	15	16	17	
18	19	20	21	22	23	24	25	
26	27	28	29	30	31	32	33	
34	35							

The next number is 3, which is prime.

2	3	4	5	6	7	8	9	All remaining multiples of 3
10	11	12	13	14	15	16	17	are not prime, so they are
18	19	20	21	22	23	24	25	crossed off; that is 9, 15,
26	27	28	29	30	31	32	33	21, etc.
34	35							

The number 4 has already been eliminated. The next number, 5, is prime.

2	3	4	5	6	7	8	9	All remaining multiples of 5
10	11	12	13	14	15	16	17	are not prime, so they are
18	19	20	21	22	23	24	25	crossed off.
26	27	28	29	30	31	32	33	
34	35							

The multiples of the remaining numbers, except themselves, have been crossed off. We need to test divisors only up to the first number whose square is larger than 30 ($6 \cdot 6 = 36$).

The prime numbers less than 35 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31.

Looking at the Sieve, we see that we can shorten the factor chart by omitting all numbers except those that are prime. For example, is 413 prime or composite?

$$1 \cdot 413$$

$$2$$

$$3$$

$$5$$

$$7 \cdot 59 \quad \text{Stop here, since we do not need all the factors.}$$

The number 413 is composite, because it has *at least* four factors, 1, 7, 59, and 413. We know that a number is prime if no smaller prime number divides it evenly.

Keep the divisibility tests for 2, 3, and 5 in mind since they are prime numbers

To determine whether a number is prime or composite

Test every prime number whose square is less than the number.

- If the number has exactly two divisors (factors), the number 1 and itself, it is prime.
- If the number has more than two divisors (factors), it is composite.
- Remember: 0 and 1 are neither prime nor composite.

Warm-Ups A–F

Examples A–F



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DIRECTIONS: Determine whether the number is prime or composite.

STRATEGY: Test all possible prime factors of the number. If there are exactly two factors, the number 1 and itself, the number is prime.

- A. Is 71 prime or composite?

Test 1 and the prime numbers from 2 to 11. We stop at 11 because $11^2 > 71$.

1 · 71	7	The numbers 2, 3, and 5 can be crossed out using
2	11	the divisibility tests. Eliminate 7 and 11 by division.
3		
5		

The number 71 is a prime number.

Answers to Warm-Ups

- A. prime

B. Is 187 prime or composite?

Test 1 and the prime numbers from 2 to 17.

$$1 \cdot 187$$

$$2$$

$$3$$

$$5$$

$$7$$

$$11 \cdot 17$$

Stop testing at 11, because we have at least four factors.

The number 187 is a composite number.

C. Is 241 prime or composite?

Test 1 and the prime numbers from 2 to 17.

$$1 \cdot 241$$

$$2$$

$$3$$

$$5$$

$$7$$

$$11$$

$$13$$

$$17$$

Stop at 17 because $17^2 > 241$.

After testing all prime numbers in the list, we see that 241 has only two factors, 1 and itself. So 241 is a prime number.

D. Is 481 prime or composite?

Test 1 and the prime numbers from 2 to 23.

$$1 \cdot 481$$

$$2$$

$$3$$

$$5$$

$$7$$

$$11$$

$$13$$

$$17$$

We stop testing at 13 because we have at least four factors (1, 13, 37, and 481).

So 481 is a composite number.

E. Is 124,653 prime or composite?

124,653 is divisible by 3 The sum of the digits $1 + 2 + 4 + 6 + 5 + 3 = 21$, which is divisible by 3.

We have at least three factors, 1, 3, and 124,653.

So 124,653 is a composite number.

F. Christina won a math contest at her school. One of the questions in the contest was “Is 234,423 prime or composite?” What should Christina have answered?

234,423 is not divisible by 2. The ones-place digit is 3 (not even).

234,423 is divisible by 3. $2 + 3 + 4 + 4 + 2 + 3 = 18$, which is divisible by 3.

We have at least three factors, 1, 3, and 234,423.

Christina should have answered “composite.”

B. Is 119 prime or composite?

C. Is 269 prime or composite?

D. Is 493 prime or composite?

E. Is 301,755 prime or composite?

F. In the same contest Mikey was asked, “Is the number 234,425 prime or composite?” What should he have answered?

Answers to Warm-Ups

- B. composite C. prime
- D. composite E. composite
- F. composite

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Exercises 2.4

OBJECTIVE

Determine whether a whole number is prime or composite.

A Tell whether the number is prime or composite.

- | | | | |
|---------------|---------------|---------------|---------------|
| 1. 4 | 2. 7 | 3. 8 | 4. 11 |
| 5. 14 | 6. 16 | 7. 19 | 8. 21 |
| 9. 22 | 10. 23 | 11. 29 | 12. 37 |
| 13. 38 | 14. 39 | 15. 41 | 16. 43 |
| 17. 49 | 18. 50 | 19. 51 | 20. 57 |
| 21. 58 | 22. 59 | | |

B

- | | | | |
|----------------|----------------|----------------|----------------|
| 23. 60 | 24. 61 | 25. 63 | 26. 67 |
| 27. 72 | 28. 77 | 29. 79 | 30. 75 |
| 31. 73 | 32. 89 | 33. 91 | 34. 93 |
| 35. 109 | 36. 110 | 37. 123 | 38. 133 |
| 39. 137 | 40. 157 | 41. 183 | 42. 187 |
| 43. 213 | 44. 231 | 45. 305 | 46. 321 |

C

- | | | | |
|----------------|----------------|-----------------|-----------------|
| 47. 383 | 48. 389 | 49. 433 | 50. 437 |
| 51. 579 | 52. 587 | 53. 1323 | 54. 1333 |

- 55.** The year 2003 was the last year that was a prime number. What is the next year that is a prime number?
- 56.** How many prime numbers are there between 1 and 100? How many between 100 and 200?
- 57.** The Russians launched *Sputnik*, the first artificial Earth satellite, on October 4, 1957. Is 1957 a prime number?
- 58.** Is the year of your birth a prime or composite number?

- 59.** In the motion picture *Contact*, a scientist played by Jodie Foster intercepts a message from outer space. The message begins with a series of impulses grouped according to successive prime numbers. How many impulses came in the group that followed 53?

- 60.** Vigorous physical activity can lead to accidental injury. The estimates by the Consumer Product Safety Commission of nationwide basketball injuries in recent years are given in the table. These estimates are the number of related hospital emergency room visits.

Year	1	2	3	4
Injuries	761,171	716,182	692,396	653,675

Three of the four entries are clearly not prime numbers. Identify them and tell how you know they are not prime. (Optional: Is the fourth number prime?)

- 61.** How many rectangular arrangements can be made with 19 blocks?

- 62.** One year Mr. Tyson had 73 members in his marching band. Explain why he could not arrange the band members in a rectangular shape.

Exercises 63–66 relate to the chapter application.

Many mathematicians have been fascinated by prime numbers. Computer searches for new prime numbers are common. A French mathematician named Marin Mersenne (1588–1648) identified a special form of prime numbers that bears his name. A Mersenne prime has the form $M_p = 2^p - 1$, where p is a prime number. The smallest Mersenne prime number is 3 because when $p = 2$, the smallest prime, $M_2 = 2^2 - 1 = 4 - 1 = 3$.

- 63.** Find the value of the next three Mersenne primes by substituting the prime numbers 3, 5, and 7 for p .

- 64.** Mersenne was originally hoping that all numbers with his special form were prime. He discovered that this is false with M_{11} . Calculate M_{11} by substituting 11 for p and show that M_{11} is not prime.

- 65.** Every Mersenne prime has a companion perfect number. See Exercise 55 in Section 2.3. The perfect number is expressed by $P = M_p \times (2^{p-1})$. Find the perfect number that is companion to M_3 .

- 66.** What perfect number is companion to M_5 ?

STATE YOUR UNDERSTANDING

- 67.** Explain the difference between prime numbers and composite numbers. Give an example of each.
- 68.** What is the minimum number of factors that a composite number can have?
- 69.** Is it possible for a composite number to have exactly four factors? If so, give two examples. If not, tell why not.

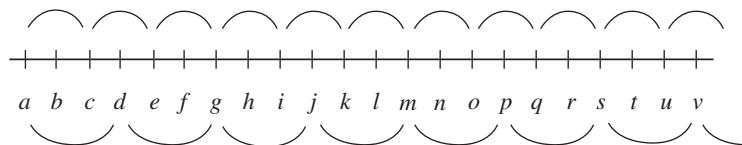
CHALLENGE

70. Is 23,341 a prime or composite number?
71. Is 37,789 a prime or composite number?
72. A sphenic number is a number that is a product of three unequal prime numbers. The smallest sphenic number is $2 \cdot 3 \cdot 5 = 30$. Is 4199 a sphenic number?

GROUP WORK

73. Mathematicians have established that every prime number greater than 3 is either one more than or one less than a multiple of 6. Use the Sieve of Eratosthenes to identify all the prime numbers less than 100. Divide up the list of these primes and verify that each of them is one away from a multiple of 6. Here is a step-by-step confirmation of this fact. As a group, supply the reasons for each step of the confirmation.
74. Have each person in your group determine the total of the ages of the people now living in their household. Are these prime numbers? Report back to your class and compute the class total. Is this a prime number?

Consider the portion of the number line shown below. The first point, a , is at an even number.



Statement

- If a is even, then so are $c, e, g, k, l, m, o, q, s$, and u .
- If a is a multiple of 3, then so are d, g, j, m, p, s , and v .
- a, g, m , and s are multiples of 6.
- None of the points in steps a, b, or c can be prime.
- The only possible places for prime numbers to occur are at b, f, h, l, n, r , and t .
- All primes are one unit from a multiple of 6.

Reason

-
-
-
-
-
-

MAINTAIN YOUR SKILLS

75. Because the last digit of the number 160 is 0, 160 is divisible by 2. Is the quotient of 160 and 2 divisible by 2? Continue dividing by 2 until the quotient is not divisible by 2. What is the final quotient?
76. Because the last digit of the number 416 is 6, 416 is divisible by 2. Is the quotient of 416 and 2 divisible by 2? Continue dividing by 2 until the quotient is not divisible by 2. What is the final quotient?

77. Because the last digit of the number 928 is 8, 928 is divisible by 2. Is the quotient of 928 and 2 divisible by 2? Continue dividing by 2 until the quotient is not divisible by 2. What is the final quotient?
79. Because the sum of the digits of 1029 is 12, 1029 is divisible by 3. Is the quotient of 1029 and 3 divisible by 3? Continue dividing by 3 until the quotient is not divisible by 3. What is the final quotient?
81. Because the last digit of the number 2880 is 0, 2880 is divisible by 5. Is the quotient of 2880 and 5 divisible by 5? Continue dividing by 5 until the quotient is not divisible by 5. What is the final quotient?
83. The number 1859 is divisible by 13. Is the quotient of 1859 and 13 divisible by 13? Continue dividing by 13 until the quotient is not divisible by 13. What is the final quotient?
78. Because the sum of the digits of 1701 is 9, 1701 is divisible by 3. Is the quotient of 1701 and 3 divisible by 3? Continue dividing by 3 until the quotient is not divisible by 3. What is the final quotient?
80. Because the last digit of the number 2875 is 5, 2875 is divisible by 5. Is the quotient of 2875 and 5 divisible by 5? Continue dividing by 5 until the quotient is not divisible by 5. What is the final quotient?
82. The number 1029 is divisible by 7. Is the quotient of 1029 and 7 divisible by 7? Continue dividing by 7 until the quotient is not divisible by 7. What is the final quotient?
84. Because the last digit of the number 17,408 is 8, 17,408 is divisible by 2. Is the quotient of 17,408 and 2 divisible by 2? Continue dividing by 2 until the quotient is not divisible by 2. What is the final quotient?

2.5 Prime Factorization

VOCABULARY

The **prime factorization** of a counting number is the indicated product of prime numbers. There are two ways of asking the same question.

1. “What is the prime factorization of this number?”
2. “Write this number in prime-factored form.”

$51 = 3 \cdot 17$ and $66 = 2 \cdot 3 \cdot 11$ are prime factorizations.

$30 = 3 \cdot 10$ is not a prime factorization because 10 is not a prime number.

Recall that exponents show repeated factors. This can save space in writing.

$2 \cdot 2 \cdot 2 = 2^3$ and $3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7 = 3^4 \cdot 7^3$

OBJECTIVES

1. Find the prime factorization of a counting number by repeated division.
2. Find the prime factorization of a counting number using the Tree Method.

How & Why

OBJECTIVE 1

Find the prime factorization of a counting number by repeated division.

In chemistry, we learn that every compound in the world is made up of a particular combination of basic elements. For instance, salt is NaCl and water is H₂O. This means that every molecule of salt contains one atom of sodium (Na) and one atom of chlorine (Cl). Sodium and chlorine are two of the basic elements. Similarly, every molecule of water is made up of two atoms of hydrogen (H) and one of oxygen (O), which are also basic elements. The chemical formula tells how many units of each basic element are needed to make the compound.

In mathematics, the basic elements are the prime numbers. Every counting number (except 1) is either prime or a unique combination of prime factors. Finding the prime factorization of a composite number is comparable to finding the chemical formula for a compound. The prime factorization simply allows us to see the basic elements of a number.

To find a prime factorization, repeatedly divide by prime numbers until the quotient is 1. The prime factorization is the product of the prime divisors. We begin by dividing the given number by 2 repeatedly, until the quotient is odd. Then we divide by 3, 5, 7, 11 . . . to check for other prime factors. To save time and space, we do not rewrite the division problem each time. Instead, divide each quotient, starting at the top and dividing down until the quotient is 1. The division is shown for the number 108.

$$\begin{array}{r} 2)108 \\ 2)54 \\ 3)27 \quad 108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 2^2 \cdot 3^3 \\ 3)9 \\ 3)3 \\ 1 \end{array}$$

If there is a large prime factor, you will find it when you have divided by each prime whose square is smaller than the number. Consider the number 970.

2) 970

5) 485

- 97) 97 97 is not divisible by 2, 3, 4, 7, or 11; also $11^2 = 121 > 97$.
1 Therefore, 97 is prime.

So $970 = 2 \cdot 5 \cdot 97$

In each case, the goal is to keep dividing until the quotient is 1.

To find the prime factorization of a counting number using repeated division

1. Divide the counting number and each following quotient by a prime number until the quotient is 1. Begin with 2 and divide until the quotient is odd, then divide by 3. Divide by 3 until the quotient is not a multiple of 3. Continue with 5, 7, and so on, testing the prime numbers in order.
2. Write the indicated product of all the divisors.

Warm-Ups A–D

- A. Find the prime factorization of 54.

- B. Find the prime factorization of 140.

- C. Find the prime factorization of 540.

Examples A–D



BASIC Mathematics Now™

DIRECTIONS: Find the prime factorization of the counting number.

STRATEGY: Use repeated division by prime numbers.

- A. Find the prime factorization of 165.

STRATEGY: The last digit is not even, so start dividing by 3.

3) 165

165 is divisible by 3.

5) 55

55 is divisible by 5.

11) 11
1

The number 11 is prime and the quotient is 1.

CHECK: Multiply all of the divisors.

$$3 \cdot 5 \cdot 11 = 165$$

The prime factorization of 165 is $3 \cdot 5 \cdot 11$.

- B. Find the prime factorization of 984.

STRATEGY: 984 is even, so start dividing by 2. Continue until 2 is no longer a divisor. Then try another prime number as a divisor.

2) 984

2) 492

2) 246

3) 123

41) 41
1

The number 41 is prime since it is not divisible by 3, 5, or 7.

CHECK: $2 \cdot 2 \cdot 2 \cdot 3 \cdot 41 = 984$.

The prime factorization of 984 is $2^3 \cdot 3 \cdot 41$.

CALCULATOR EXAMPLE



- C. Find the prime factorization of 5040.

STRATEGY: Record the number of times you divide by each prime number.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$$

The prime factorization of 5040 is $2^4 \cdot 3^2 \cdot 5 \cdot 7$.

Answers to Warm-Ups

A. $2 \cdot 3^3$

B. $2^2 \cdot 5 \cdot 7$

C. $2^2 \cdot 3^3 \cdot 5$

- D. When she was adding two fractions, Christine needed to find the least common denominator of $\frac{5}{18}$ and $\frac{11}{36}$; therefore, she needed to write the prime factorizations of 18 and 36. What are the prime factorizations of 18 and 36?

STRATEGY: Because 18 and 36 are both divisible by 2, start dividing by 2.

$$\begin{array}{r} 2)18 \\ 3)9 \\ 3)3 \\ \hline 1 \end{array} \quad \begin{array}{r} 2)36 \\ 2)18 \\ 3)9 \\ \hline 1 \end{array}$$

The prime factorizations of 18 and 36 are

$$18 = 2 \cdot 3 \cdot 3 \quad \text{or} \quad 2 \cdot 3^2$$

$$36 = 2 \cdot 2 \cdot 3 \cdot 3 \quad \text{or} \quad 2^2 \cdot 3^2$$

- D. Chris needs to know the least common denominator of $\frac{7}{24}$ and $\frac{11}{30}$. What are the prime factorizations of 24 and 30?

How & Why

OBJECTIVE 2

Find the prime factorization of a counting number using the Tree Method.

Another method for finding the prime factorization is the Tree Method. We draw “factor branches” from a given number using *any* two factors of the number. We then draw additional branches from the end of each original branch by using factors of the number at the end of the branch. A branch stops splitting when it ends in a prime number. Figure 2.7 shows the Tree Method to prime factor 140.

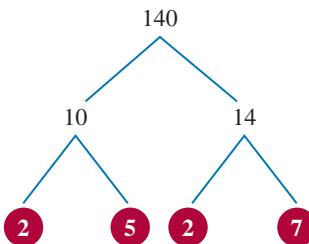


Figure 2.7

There are often different trees for the same number. Examine the two trees in Figure 2.8 used to find the prime factorization of 140.

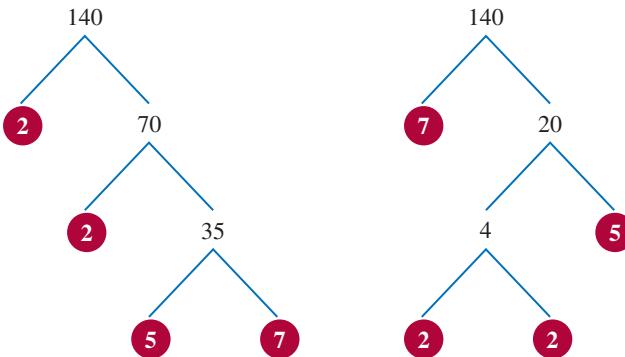


Figure 2.8

Answers to Warm-Ups

- D. The prime factorization of 24 is $2^3 \cdot 3$ and the prime factorization of 30 is $2 \cdot 3 \cdot 5$.

In each case, we can see that $140 = 2^2 \cdot 5 \cdot 7$. It does not matter which tree we use. As long as you keep branching until you come to a prime number, you will get the same prime factorization. In fact, *any* tree results in the same prime factorization as repeated division.

To find the prime factorization of a counting number using the Tree Method

Draw factor branches starting with any two factors of the number. Form additional branches by using factors of the number at the end of each branch. The factoring is complete when the number at the end of each branch is a prime number.

Warm-Ups E–F

- E. Find the prime factorization of 420.

- F. Find the prime factorization of 980.

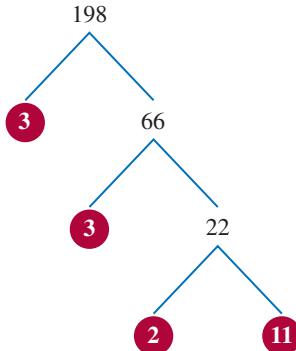
Examples E–F BASIC Mathematics Now™

DIRECTIONS: Find the prime factorization of the counting number.

STRATEGY: Use the Tree Method to write the given counting number as a product of primes.

- E. Find the prime factorization of 198.

STRATEGY: We start with $3 \cdot 66$ as the pair of factors of 198.



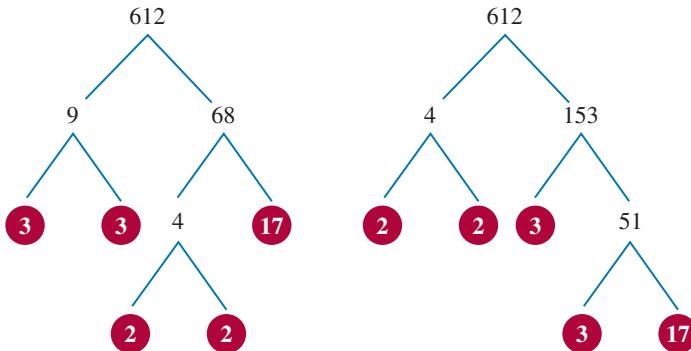
We could have used 2 and 99, or 6 and 33, or 11 and 18 as the first pairs of factors. The same prime factors would have been discovered.

The circled number at the end of each branch is a prime number.

The prime factorization of 198 is $2 \cdot 3^2 \cdot 11$.

- F. Find the prime factorization of 612.

STRATEGY: Start with any two factors whose product is 612. In this case, we develop two trees, one showing factors $9 \cdot 68$, and the other $4 \cdot 153$. The tree is shorter if larger factors are used first.



The same set of factors are shown in each tree. The factors just appear in a different order.

Answers to Warm-Ups

E. $2^2 \cdot 3 \cdot 5 \cdot 7$

F. $2^2 \cdot 5 \cdot 7^2$

Exercises 2.5

OBJECTIVE 1

Find the prime factorization of a counting number by repeated division.

OBJECTIVE 2

Find the prime factorization of a counting number using the Tree Method.

A Find the prime factorization of the counting number using either method.

- | | | | |
|---------------|---------------|---------------|---------------|
| 1. 12 | 2. 14 | 3. 15 | 4. 18 |
| 5. 21 | 6. 22 | 7. 24 | 8. 25 |
| 9. 28 | 10. 30 | 11. 34 | 12. 38 |
| 13. 39 | 14. 44 | 15. 45 | 16. 52 |
| 17. 76 | 18. 84 | 19. 85 | 20. 88 |

B

- | | | | |
|----------------|----------------|----------------|----------------|
| 21. 90 | 22. 91 | 23. 92 | 24. 104 |
| 25. 105 | 26. 106 | 27. 131 | 28. 137 |
| 29. 156 | 30. 160 | 31. 161 | 32. 162 |
| 33. 180 | 34. 190 | 35. 200 | 36. 207 |
| 37. 310 | 38. 315 | 39. 345 | 40. 348 |

C

- | | | | |
|----------------|-----------------|-----------------|-----------------|
| 41. 450 | 42. 515 | 43. 323 | 44. 391 |
| 45. 459 | 46. 460 | 47. 465 | 48. 470 |
| 49. 625 | 50. 1024 | 51. 1190 | 52. 1204 |

- 53.** Determine the prime factorization of the highest temperature in the United States for today.

- 54.** Determine the prime factorization of the sum of the digits in today's numerical date (month, day, and year).

- 55.** Determine the prime factorization of your birth year.

Two numbers are said to be **relatively prime** if they do not have any common factors. The numbers 14 and 15 are relatively prime because $14 = 2 \cdot 7$ and $15 = 3 \cdot 5$.

- 56.** Are the numbers 12 and 15 relatively prime? Explain
- 57.** Are the numbers 119 and 143 relatively prime? Explain.

- 58.** Find five different numbers that are relatively prime to 12.

- 59.** What is the largest number less than 100 that is relatively prime to 180?

STATE YOUR UNDERSTANDING

- 60.** Explain how to find the prime factorization of 990.
- 61.** Explain how to determine whether a number is written in prime-factored form.

CHALLENGE

- 62.** Find the prime factorization of 1492.
- 63.** Find the prime factorization of 2190.
- 64.** Find the prime factorization of 1547.
- 65.** Find the prime factorization of 1792.

GROUP WORK

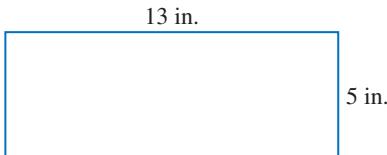
- 66.** The Goldbach conjecture states that every even number greater than 4 can be written as the sum of two prime numbers. Mathematicians believe this to be true, but as yet no one has proved that it is true for *all* even numbers. Complete the table.

Even Number	Sum of Primes	Even Number	Sum of Primes
6	$3 + 3$	26	
8	$3 + 5$	28	
10		30	
12		32	
14		34	
16		36	
18		38	
20		40	
22		42	
24		44	

Do you believe the Goldbach conjecture?

MAINTAIN YOUR SKILLS

67. Find the perimeter of the figure.



68. Find the area of the figure in Exercise 67.

69. Is 3003 a multiple of *both* 3 and 13?

70. Is 4004 a multiple of *both* 2 and 13?

71. Is 1001 a multiple of *both* 3 and 13?

72. Is 5005 a multiple of *both* 5 and 13?

73. Is 3003 a multiple of *both* 3 and 7?

74. Is 4004 a multiple of *both* 2 and 7?

75. Find two numbers that are multiples of both 3 and 5.

76. Find two numbers that are multiples of both 6 and 10.

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2.6 Least Common Multiple

VOCABULARY

The **least common multiple** of two or more whole numbers is

1. the smallest natural number that is a multiple of each whole number, and
2. the smallest natural number that has each whole number as a factor, and
3. the smallest natural number that has each whole number as a divisor, and
4. the smallest natural number that each whole number will divide evenly.

The abbreviation of least common multiple is **LCM**.

OBJECTIVES

1. Find the least common multiple (LCM) of two or more whole numbers using the Individual Prime-Factoring Method.
2. Find the least common multiple (LCM) of two or more whole numbers using the Group Prime-Factoring Method.

How & Why

OBJECTIVE 1

Find the least common multiple (LCM) of two or more whole numbers using the Individual Prime-Factoring Method.

The factorizations of this chapter are most often used to simplify fractions and find LCMs. LCMs are used to compare, add, and subtract fractions. In algebra, LCMs are useful in equation solving.

We can find the LCM of 21 and 35 by listing the multiples of both and finding the smallest value common to both lists:

Multiples of 21: 21, 42, 63, 84, 105, 126, 147, 168, 189, 210, 231, ...

Multiples of 35: 35, 70, 105, 140, 175, 210, 245, 280, 315, 650, ...

The LCM of 12 and 35 is 105 because it is the smallest multiple in both lists. This fact can be stated in four equivalent ways.

1. 105 is the smallest natural number that is a multiple of both 21 and 35.
2. 105 is the smallest natural number that has both 21 and 35 as factors.
3. 105 is the smallest natural number that has both 21 and 35 as divisors.
4. 105 is the smallest natural number that both 21 and 35 will divide evenly.

Finding the LCM by this method has a big drawback: you might have to list hundreds of multiples to find it. For this reason we look for a shortcut.

To find the LCM of 18 and 24, write the prime factorization of each. Write these prime factors in columns, so that the prime factors of 24 are under the prime factors of 18. Leave blank spaces for prime factors that do not match.

Primes with the Largest Exponents

$$\begin{array}{rcl} 18 = 2 \cdot & 3 \cdot 3 = 2 \cdot & \cdot (3^2) \\ 24 = 2 \cdot 2 \cdot 2 \cdot 3 & = (2^3) \cdot 3^1 & 2^3 \text{ and } 3^2 \end{array}$$

Because 18 must divide the LCM, the LCM must have $2 \cdot 3 \cdot 3$ as part of its factorization, and because 24 must divide the LCM, the LCM must also have $2 \cdot 2 \cdot 2 \cdot 3$ as part of its factorization. Thus, the LCM is

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ or 72. Observe that the LCM is the product of the highest power of each prime factor.

Find the LCM of 16, 10, and 24.

Primes with the Largest Exponents

$$\begin{array}{rcl} 16 = 2 \cdot 2 \cdot 2 \cdot 2 & = (2^4) \\ 10 = 2 & \cdot 5 = 2^1 \cdot (5^1) & 2^4, 3^1, \text{ and } 5^1 \\ 24 = 2 \cdot 2 \cdot 2 & \cdot 3 = 2^3 \cdot (3^1) \end{array}$$

$$\begin{aligned}\text{The LCM} &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^4 \cdot 3^1 \cdot 5^1 \\ &= 16 \cdot 15 = 240\end{aligned}$$

To write the least common multiple of two or more whole numbers using the Individual Prime-Factoring Method

- Find the prime factorization of each number in exponent form.
- Find the product of the highest power of each prime factor.

Warm-Ups A–C

A. Find the LCM of 12 and 18.

B. Find the LCM of 10, 15, and 45.

C. Find the LCM of 8, 9, 12, and 24.

Examples A–C



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DIRECTIONS: Write the least common multiple (LCM) of two or more whole numbers.

STRATEGY: Use the Individual Prime-Factoring Method.

A. Find the LCM of 21 and 35.

STRATEGY: Find the prime factorization of 21 and 35, in exponent form.

$$\begin{array}{ll} 21 = 3 \cdot 7 = 3^1 \cdot 7^1 & \text{The different prime factors are 3, 5, and 7. The largest} \\ 35 = 5 \cdot 7 = 5^1 \cdot 7^1 & \text{exponent of each is 1. Multiply the powers.} \end{array}$$

The LCM of 21 and 35 is $3 \cdot 5 \cdot 7 = 105$.

B. Find the LCM of 12, 15, and 20.

STRATEGY: Find the prime factorization of 12, 15, and 20 in exponent form.

$$\begin{array}{ll} 12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3^1 & \text{The different prime factors are 2, 3 and 5. The largest} \\ 10 = 2 \cdot 5 = 2^1 \cdot 5^1 & \text{exponent of 2 is 2, of 3 is 1, and of 5 is 1. Multiply the} \\ 20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5^1 & \text{powers.} \end{array}$$

The LCM of 12, 15, and 20 is $2^2 \cdot 3 \cdot 5 = 60$.

C. Find the LCM of 9, 16, 18, and 24.

STRATEGY: Find the prime factorization of each in exponent form.

$$\begin{array}{ll} 9 = 3 \cdot 3 = 3^2 & \text{The different prime factors are 2 and 3. The} \\ 16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4 & \text{largest exponent of 2 is 4 and of 3 is 2.} \\ 18 = 2 \cdot 3 \cdot 3 = 2^1 \cdot 3^2 & \text{Multiply the powers.} \\ 24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3^1 & \end{array}$$

The LCM of 9, 16, 18, and 36 is $2^4 \cdot 3^2 = 144$.

How & Why

OBJECTIVE 2

Find the least common multiple (LCM) of two or more whole numbers using the Group Prime-Factoring Method.

A second method for finding the LCM is sometimes referred to as the Group Prime-Factoring Method. To use this method with two numbers, find a prime number that will divide both. If there is none, multiply the two numbers. The product is the LCM of the two numbers. Continue dividing by primes until no prime will divide the quotients. The product of all of the divisors and the remaining quotients is the LCM. For example, find the LCM of 18 and 24.

Answers to Warm-Ups

A. 36 B. 90 C. 72

$$\begin{array}{r} 3) \underline{9} \quad 12 \\ \quad 3 \quad 4 \end{array}$$

Divide both numbers by 3.
The quotients 3 and 4 have no common factors.

The LCM is the product of the divisors and remaining quotients. The LCM of 18 and 24 is $2 \cdot 3 \cdot 3 \cdot 4 = 72$.

To find the LCM of three or more numbers, find a prime number that will divide *at least* two of the numbers. Divide all the numbers if possible. Bring down the numbers that are not multiples of the prime divisor. Continue until no common factors remain. For example, find the LCM of 18, 36, and 60.

$$\begin{array}{r} 2) \underline{18} \quad 36 \quad 60 \\ 2) \underline{9} \quad 18 \quad 30 \\ 3) \underline{9} \quad 9 \quad 15 \\ 3) \underline{3} \quad 3 \quad 5 \\ 1 \quad 1 \quad 5 \end{array}$$

Divide each number by 2.
Bring down the 9. Divide 18 and 30 by 2.
Divide each number by 3.
Bring down the 5. Divide by 3 again.
The remaining quotients have no common factors.

The LCM is the product of the divisors and the remaining quotients. The LCM of 12, 18, and 45 is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$.

To find the LCM of two or more numbers using the Group Prime-Factoring Method

1. Divide at least two of the numbers by any common prime number. Continue dividing the remaining quotients in the same manner until no two quotients have a common divisor. When a number cannot be divided, bring it down as a remaining quotient.
2. Write the product of the divisors and the remaining quotients.

Examples D–G



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DIRECTIONS: Find the LCM of two or more whole numbers.

STRATEGY: Use the Group Prime-Factoring Method.

- D. Find the LCM of 16 and 60.

$$\begin{array}{r} 2) \underline{16} \quad 60 \\ 2) \underline{8} \quad 30 \\ 4 \quad 15 \end{array}$$

Divide both numbers by 2.
Divide both numbers by 2 again.
The quotients 4 and 15 have no common factors.
 $2 \cdot 2 \cdot 4 \cdot 15 = 240$ Multiply the divisors and remaining quotients.

The LCM of 16 and 60 is 240.

- E. Find the LCM of 12, 20, 25, and 48.

$$\begin{array}{r} 2) \underline{12} \quad 20 \quad 25 \quad 48 \\ 2) \underline{6} \quad 10 \quad 25 \quad 24 \\ 3) \underline{3} \quad 5 \quad 25 \quad 12 \\ 5) \underline{1} \quad 5 \quad 25 \quad 4 \\ 1 \quad 1 \quad 5 \quad 4 \end{array}$$

Divide 12, 20, and 48 by 2. Bring down 25.
Divide 6, 10, and 24 by 2. Bring down 25.
Divide 3 and 12 by 3. Bring down 5 and 25.
Divide 5 and 25 by 5. Bring down 1 and 4.
The quotients 5 and 4 have no common factors.
 $2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 4 = 1200$ Multiply the divisors and remaining quotients.

The LCM of 12, 20, 25, and 48 is 1200.

Warm-Ups D–G

- D. Find the LCM of 14 and 30.

- E. Find the LCM of 12, 24, 25, and 50.

Answers to Warm-Ups

D. 210 E. 600

- F. Find the LCM of the denominators of $\frac{5}{2}$, $\frac{5}{3}$, $\frac{5}{12}$, and $\frac{7}{10}$.

- F. Find the LCM of the denominators of $\frac{5}{6}$, $\frac{5}{9}$, $\frac{7}{12}$, and $\frac{5}{18}$.

The denominators are 6, 9, 12, and 18.

$$2) \begin{array}{r} 6 \\ 3 \\ 1 \end{array} \quad \begin{array}{r} 9 \\ 3 \\ 1 \end{array} \quad \begin{array}{r} 12 \\ 6 \\ 2 \end{array} \quad \begin{array}{r} 18 \\ 9 \\ 3 \\ 2 \\ 1 \end{array}$$

$$3) \begin{array}{r} 3 \\ 1 \end{array} \quad \begin{array}{r} 9 \\ 3 \end{array} \quad \begin{array}{r} 6 \\ 2 \end{array} \quad \begin{array}{r} 9 \\ 3 \end{array}$$

$$3) \underline{1} \quad \begin{array}{r} 3 \\ 1 \end{array} \quad \begin{array}{r} 2 \\ 1 \end{array} \quad \begin{array}{r} 3 \\ 1 \end{array}$$

$$\begin{array}{r} 1 \\ 1 \end{array} \quad \begin{array}{r} 2 \\ 1 \end{array} \quad \begin{array}{r} 1 \\ 1 \end{array}$$

$$2 \cdot 3 \cdot 3 \cdot 2 = 36$$

Divide 6, 12, and 18 by 2. Bring down 9.

Divide 3, 9, 6, and 9 by 3.

Divide both 3s by 3. Bring down 2.

The remaining quotients have no common factors other than 1.

Multiply the divisors and remaining quotients.

The LCM of 6, 9, 12, and 18 is 36, so the common denominator of the fractions is 36.

- G. If Jane had saved nickels and Robin had saved quarters, what is the least each could pay for the same item if neither girl receives change?

- G. Jane and Robin have each been saving coins. Jane saved dimes and Robin saved quarters. The girls went shopping together and each bought the same item. What is the least amount the item could cost if neither girl receives change?

STRATEGY: The least amount the item could cost is the smallest number that is divisible by both 10 and 25. Find the LCM of 10 and 25.

$$5) \begin{array}{r} 10 \\ 2 \end{array} \quad \begin{array}{r} 25 \\ 5 \end{array}$$

$$2 \quad 5$$

Divide both numbers by 5.

The quotients 2 and 5 have no common factors.

$$5 \cdot 2 \cdot 5 = 50$$

Multiply the divisors and remaining quotients.

The LCM of 10 and 25 is 50.

The least cost of the item is 50¢—five dimes or two quarters.

Answers to Warm-Ups

F. 60

G. The least each could have paid is 25¢.

Exercises 2.6

OBJECTIVE 1

Find the least common multiple of two or more whole numbers using the Individual Prime-Factoring Method.

OBJECTIVE 2

Find the least common multiple of two or more whole numbers using the Group Prime-Factoring Method.

A Find the LCM of each group of whole numbers using either method.

- | | | | |
|--------------|--------------|--------------|--------------|
| 1. 4, 12 | 2. 6, 12 | 3. 7, 21 | 4. 7, 28 |
| 5. 3, 30 | 6. 4, 20 | 7. 8, 16 | 8. 9, 18 |
| 9. 4, 6 | 10. 6, 9 | 11. 9, 12 | 12. 8, 12 |
| 13. 3, 6, 12 | 14. 4, 6, 8 | 15. 3, 6, 9 | 16. 3, 4, 9 |
| 17. 2, 6, 10 | 18. 2, 8, 12 | 19. 2, 5, 10 | 20. 3, 8, 12 |

B

- | | | | |
|----------------|----------------|------------------|-------------------|
| 21. 12, 18 | 22. 10, 15 | 23. 16, 24 | 24. 22, 33 |
| 25. 12, 20 | 26. 8, 20 | 27. 12, 16 | 28. 10, 16 |
| 29. 18, 24 | 30. 28, 42 | 31. 8, 12, 16 | 32. 6, 8, 10 |
| 33. 9, 12, 15 | 34. 8, 12, 15 | 35. 2, 6, 12, 24 | 36. 4, 12, 10, 15 |
| 37. 12, 16, 24 | 38. 20, 30, 40 | 39. 21, 24, 56 | 40. 18, 24, 36 |

C

- | | | |
|--------------------|-------------------------|------------------------|
| 41. 7, 14, 28, 32 | 42. 8, 14, 28, 32 | 43. 12, 17, 51, 68 |
| 44. 14, 35, 49, 56 | 45. 35, 50, 56, 70, 175 | 46. 15, 20, 30, 40, 50 |

Find the least common denominator for each set of fractions.

- | | | | |
|---|---|---|---|
| 47. $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$ | 48. $\frac{5}{6}, \frac{4}{5}, \frac{1}{9}$ | 49. $\frac{7}{15}, \frac{15}{16}, \frac{5}{12}$ | 50. $\frac{15}{16}, \frac{25}{32}, \frac{17}{56}$ |
|---|---|---|---|

51. A gear with 36 teeth is engaged with another gear that has 12 teeth. How many turns of the first gear are necessary in order for the two gears to return to their original positions?

52. A gear with 36 teeth is engaged with another gear that has 19 teeth. How many turns of the first gear are necessary in order for the two gears to return to their original positions?

- 53.** A gear with 36 teeth is engaged with another gear that has 24 teeth. How many turns of the first gear are necessary in order for the two gears to return to their original positions?
- 55.** Melissa withdraws some 20-dollar bills from a bank. Sean withdraws some 50-dollar bills from the same bank. Luigi, too, goes to the bank and withdraws some 5-dollar bills. If they all intend to buy the same item with their money, what is the least price they could pay without receiving change?
- 54.** A gear with 36 teeth is engaged with another gear that has 45 teeth. How many turns of the first gear are necessary in order for the two gears to return to their original positions?
- 56.** Find the least common multiple for the highest and lowest temperatures in the United States yesterday.

STATE YOUR UNDERSTANDING

- 57.** Explain how to find the LCM of 20, 24, and 45.
- 58.** Identify the error in the following problem and correct it. Find the prime factorization of 100, 75, and 45 and determine the LCM of the three numbers.
- $$\begin{aligned}100 &= 2 \cdot 2 \cdot 5 \cdot 5 \\75 &= 3 \cdot 5 \cdot 5 \\45 &= 3^2 \cdot 5\end{aligned}$$
- The LCM is $2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 450$.

CHALLENGE

- 59.** Find the LCM of 144, 240, and 360.
- 60.** Find the LCM of 64, 128, and 192.
- 61.** Find the LCM of 144, 216, and 324.
- 62.** Find the LCM of 1200, 1500, and 1800.

GROUP WORK

- 63.** Here is a classic Hindu puzzle from the seventh century. A horse galloping by frightens a woman carrying a basket of eggs. She drops the basket and breaks all the eggs. Concerned passersby ask how many eggs she lost, but she can't remember. She does remember that there was one egg left over when she counted by

2s, 2 eggs left over when she counted by 3s, 3 eggs left over when she counted by 4s, and 4 eggs left over when she counted by 5s. How many eggs did she have? Show calculations that justify your answer by showing it fits all the conditions of the puzzle.

MAINTAIN YOUR SKILLS

- 64.** List all the factors of 425.
- 65.** List all the factors of 460.
- 66.** List all the divisors of 530.
- 67.** List all the divisors of 560.
- 68.** Is 3080 divisible by 35?
- 69.** Is 3080 divisible by 56?
- 70.** Is 3080 divisible by 77?
- 71.** Is 3080 divisible by 21?

72. The Won-Stawp General Store made a profit of \$112 so far this week. If their profit on each appliance sold is \$14, how many more appliances must they sell so that the profit for the week will be more than \$198?

73. A marketing researcher checked the weekly attendance at 14 multiplex theaters. The results are shown in the table. What was the average attendance at the 14 theaters?

Theater Attendance

Number of Theaters	Attendance
1	900
2	1000
2	1200
3	1300
2	1400
2	1600
1	1800
1	1900

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Key Concepts CHAPTER 2

Section 2.1 Divisibility Tests

Definitions and Concepts	Examples
A whole number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.	34, 876, and 9078 are divisible by 2. 35, 879, and 9073 are not divisible by 2.
A whole number is divisible by 3 if the sum of the digits is divisible by 3.	876 is divisible by 3 because $8 + 7 + 6 = 21$. 116 is not divisible by 3 because $1 + 1 + 6 = 8$.
A whole number is divisible by 5 if it ends in 0 or 5.	9345 is divisible by 5 because it ends in a 5. 8762 is not divisible by 5 because it ends in a 2.

Section 2.2 Multiples

Definitions and Concepts	Examples
A multiple of a number is the product of that number and a natural number.	Multiples of 7 are: $(7)(1) = 7$ $(7)(2) = 14$ $(7)(3) = 21$ and so on

Section 2.3 Divisors and Factors

Definitions and Concepts	Examples
When one number is a multiple of a second number, the second number is a divisor or factor of the first number. To find all the factors of a number: <ul style="list-style-type: none">• List all counting numbers to the first number whose square is larger than the number.• Test each number to see if it is a factor and record each pair.	$3(8) = 24$ so 3 and 8 are factors and divisors of 24. Factors of 24: $1 \cdot 24$ $4 \cdot 6$ $2 \cdot 12$ $\cancel{5}$ $3 \cdot 8$ The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Section 2.4 Primes and Composites

Definitions and Concepts	Examples
A prime number is a counting number with exactly two factors, itself and 1.	2, 3, 5, 7, 11, 13, 17, and 19 are the primes less than 20.
A composite number is a counting number with more than two factors.	4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 are the composite numbers less than 20.
To determine if a number is prime: <ul style="list-style-type: none"> • Systematically search for factor pairs up to the number whose square is larger than the number. • If none are found then the number is prime. 	Is 53 prime? 1) 53 <u>6</u> 2) <u>7</u> 3) <u>8</u> 4) Stop because $8^2 = 64 > 53$. 5) 53 is a prime number.

Section 2.5 Prime Factorization

Definitions and Concepts	Examples
The prime factorization of a number is the number written as the product of primes.	$40 = 2^3 \cdot 5$
To find the prime factorization of a number: <ul style="list-style-type: none"> • Divide the number and each succeeding quotient by a prime number until the quotient is 1. • Write the indicated product of all the primes. 	$ \begin{array}{r} 2) \underline{60} \\ 2) \underline{30} \\ 3) \underline{15} \\ 5) \underline{5} \\ 1 \end{array} $ $60 = 2^2 \cdot 3 \cdot 5$

Section 2.6 Least Common Multiple

Definitions and Concepts	Examples
The least common multiple (LCM) of two or more numbers is the smallest natural number that is a multiple of each of the numbers.	The LCM of 6 and 8 is 24.
To find the least common multiple: <ul style="list-style-type: none"> • Write each number in prime-factored form. • The LCM is the product of the highest power of each prime factor. 	Find the LCM of 20 and 150. $ \begin{array}{r} 20 = 2^2 \cdot 5 \\ 150 = 2 \cdot 3 \cdot 5^2 \\ \text{LCM} = 2^2 \cdot 3 \cdot 5^2 = 300 \end{array} $

Review Exercises CHAPTER 2

Section 2.1

1. Which of these numbers is divisible by 2?
6, 36, 63, 636, 663
2. Which of these numbers is divisible by 3?
6, 36, 63, 636, 663
3. Which of these numbers is divisible by 5?
15, 51, 255, 525, 552
4. Which of these numbers is divisible by 6?
6, 36, 63, 636, 663
5. Which of these numbers is divisible by 9?
6, 36, 63, 636, 663
6. Which of these numbers is divisible by 10?
50, 55, 505, 550, 555
7. Which of these numbers is divisible by both 2 and 3?
444, 555, 666, 777, 888, 999
8. Which of these numbers is divisible by both 2 and 9?
444, 555, 666, 777, 888, 999
9. Which of these numbers is divisible by both 3 and 5?
445, 545, 645, 745, 845
10. Which of these numbers is divisible by both 3 and 10?
440, 550, 660, 770, 880

Determine whether the number is divisible by 6, 7, or 9.

11. 567

12. 576

13. 756

14. 765

Determine whether the number is divisible by 4, 5, or 10.

15. 560

16. 575

Determine whether the number is divisible by 6 or 15.

17. 690

18. 975

19. Ed and his four partners made a profit of \$2060 in their stereo installation business. Can the profits be divided evenly in whole dollars among them? Explain.

20. Bobbie and her five partners incurred expenses of \$3060 in their satellite dish installation enterprise. Can the expenses be divided evenly in whole dollars among them? Explain.

Section 2.2

List the first five multiples of the whole number.

21. 8

22. 19

23. 51

24. 64

25. 85

26. 97

27. 122

28. 125

29. 141

30. 252

*Is each number a multiple of 6? of 9? of 15?***31.** 72**32.** 135**33.** 150**34.** 290**35.** 465**36.** 540*Is each number a multiple of 12? of 16?***37.** 288**38.** 348

- 39.** The designer of new production-line equipment at the Bredworthy Jelly plant predicts that 55 small jars of jelly can be produced per minute. The next day, a total of 605 jars are produced in 11 minutes. Is the designer's prediction accurate?

- 40.** In Exercise 39, if the plant produced 880 jars in 16 minutes, is the prediction accurate?

Section 2.3

*Write the whole number as the product of two factors in all possible ways.***41.** 15**42.** 24**43.** 38**44.** 136**45.** 236**46.** 336**47.** 338**48.** 341**49.** 343*List all of the factors (divisors) of the whole number.***50.** 33**51.** 39**52.** 42**53.** 60**54.** 78**55.** 97**56.** 99**57.** 102**58.** 110

- 59.** In a recent year, the average single-family detached residence consumed \$1590 in energy costs. Find the possible costs of energy consumption (whole number amounts) per person. Assume that each household has from 1 to 10 people.

- 60.** A rectangular floor area requires 180 one-square-foot tiles. List all the possible whole-number dimensions that this floor could measure that would require all of the tiles.

Section 2.4

*Tell whether the number is prime or composite.***61.** 13**62.** 25**63.** 47**64.** 49**65.** 51**66.** 61

67. 71**68.** 73**69.** 77**70.** 81**71.** 83**72.** 91**73.** 337**74.** 339**75.** 341**76.** 343**77.** 347**78.** 479**79.** The year 2003 is a prime number. What is the next year that is a prime number?**80.** What was the last year before 1997 that was a prime number?**Section 2.5***Write the prime factorization of each number.***81.** 27**82.** 36**83.** 38**84.** 42**85.** 52**86.** 64**87.** 222**88.** 232**89.** 252**90.** 256**91.** 258**92.** 259**93.** 260**94.** 261**95.** 263**96.** 264**97.** 265**98.** 266**99.** Write the prime factorization of the next several years.**100.** Write the prime factorization of the years your parents were born.**Section 2.6***Find the LCM of each group of numbers.***101.** 6, 8**102.** 8, 10**103.** 10, 12**104.** 12, 16**105.** 16, 18**106.** 18, 27**107.** 20, 25**108.** 28, 35**109.** 30, 35**110.** 3, 6, 8**111.** 6, 8, 12**112.** 8, 10, 12**113.** 8, 12, 18**114.** 18, 24, 36**115.** 15, 45, 60**116.** 32, 40, 60**117.** 40, 60, 105**118.** 30, 35, 40**119.** Find two numbers that have an LCM of 98.**120.** Find two numbers that have an LCM of 102.

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True/False Concept Review CHAPTER 2

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. Every multiple of 3 ends with the digit 3. 1. _____
2. Every multiple of 10 ends with the digit 0. 2. _____
3. Every multiple of 11 is divisible by 11. 3. _____
4. Every multiple of 5 is the product of 5 and some natural number. 4. _____
5. Every natural number, except the number 1, has at least two different factors. 5. _____
6. Every factor of 300 is also a divisor of 300. 6. _____
7. Every multiple of 300 is also a factor of 300. 7. _____
8. The square of 25 is 50. 8. _____
9. Every natural number ending in 6 is divisible by 6. 9. _____
10. Every natural number ending in 6 is divisible by 2. 10. _____
11. Every natural number ending in 9 is divisible by 3. 11. _____
12. The number 3192 is divisible by 7. 12. _____
13. The number 77,773 is divisible by 3. 13. _____
14. The number 123,321,231 is divisible by 9. 14. _____
15. The number 111,111,115 is divisible by 5. 15. _____
16. All prime numbers are odd. 16. _____
17. All numbers that end with the digit 8 are composite. 17. _____
18. Every prime number has exactly two multiples. 18. _____
19. It is possible for a composite number to have exactly seven factors. 19. _____

Name _____ Class _____ Date _____

20. All of the prime factors of a composite number are smaller than the number. 20. _____
21. The least common multiple (LCM) of three different numbers is the product of the three numbers. 21. _____
22. The product of two prime numbers is also the LCM of the two numbers. 22. _____
23. The largest divisor of the least common multiple (LCM) of three numbers is the largest of the three numbers. 23. _____
24. It is possible for the LCM of three natural numbers to be one of the three numbers. 24. _____

Test CHAPTER 2**ANSWERS**

1. Is 411,234 divisible by 6? 1. _____
2. List all of the factors (divisors) of 112. 2. _____
3. Is 8617 divisible by 7? 3. _____
4. Is 2,030,000 divisible by 3? 4. _____
5. What is the LCM of 12 and 32? 5. _____
6. Write 75 as the product of two factors in as many ways as possible. 6. _____
7. Write the prime factorization of 280. 7. _____
8. Find the LCM of 18, 42, and 84. 8. _____
9. Is 15,075 a multiple of 15? 9. _____
10. Write all multiples of 13 between 200 and 250. 10. _____
11. Is 200 a multiple of 400? 11. _____
12. Is 109 a prime or a composite number? 12. _____
13. Is 111 a prime or a composite number? 13. _____
14. Write the prime factorization of 605. 14. _____
15. What is the LCM of 18, 21, and 56? 15. _____
16. What is the smallest prime number? 16. _____
17. What is the largest composite number that is less than 300? 17. _____

Name _____ Class _____ Date _____

- 18.** What is the smallest natural number that 6, 18, 24, and 30 will divide evenly? **18.** _____
- 19.** Can two different numbers have the same prime factorization? Explain. **19.** _____
- 20.** List two sets of three different numbers whose LCM is 42. **20.** _____

Group Project (2–3 WEEKS) CHAPTER 2

OPTIONAL

In the early 1200s, the Italian mathematician Leonardo de Pisa, also known as Fibonacci, was interested in a set of numbers that bear his name. The set begins {1, 1, 2, 3, 5, . . .} and continues infinitely using the rule that the next number in the set is the sum of the previous two.

1. Mathematicians often name numbers of a set according to the order in which they occur. For the Fibonacci numbers, it is customary to designate F_n as the n th element of the set. So $F_3 = 2$ and $F_4 = 3$. Calculate all the Fibonacci numbers less than 500. Make a table that lists them in order along with their name.
2. The Fibonacci numbers are strongly related to items in nature that spiral, such as the cells on pineapples or the petals on pinecones. The number of spirals in normal specimens is always a Fibonacci number, although the number changes from species to species. Find two whole pineapples and two pinecones from different kinds of trees. Look at each item from the top and count the spirals that move clockwise. Then count the spirals that move counterclockwise. Record your results in the table below. Write a paragraph that summarizes your findings.

Item	Number of Clockwise Spirals	Number of Counterclockwise Spirals
Pineapple 1		
Pineapple 2		
Pinecone 1		
Pinecone 2		

3. Just as each number has a unique prime factorization, each number can also be written as the sum of Fibonacci numbers. Always start with the largest Fibonacci number less than the given number, then go to the largest Fibonacci number less than the difference, and continue until the whole number is accounted for. For example,

$$\begin{aligned} 46 &= 34 + 12 \\ &= 34 + 8 + 4 \\ &= 34 + 8 + 3 + 1 \end{aligned}$$

Make a table that gives each number from 1 to 50 as the sum of Fibonacci numbers.

4. The game of Fibonacci Nim is an easy game in which players take turns removing chips from a central pot. You may start with any number of chips. Each player must take at least one chip out of the pile but not more than twice the number just removed by his opponent. (The first player may not remove all the chips on the first move.) The player who takes the last chip(s) wins. Start with 20 chips and play Nim five times. What do you notice about the number of chips remaining just before the game was won?
5. A winning strategy is to take the smallest Fibonacci number in the sum of Fibonacci numbers that makes the number of chips remaining. For example, if 46 chips are remaining, then one chip should be taken, because $46 = 34 + 8 + 3 + 1$. Play Nim five more times starting with 20 chips and using this strategy. Did it work? If both players use the strategy, which player will win? Write a paragraph summarizing how to win at Nim.

Managing Anxiety

For those of you who become anxious as you begin to study, we recommend that you devote a section in your math notebook to record your thoughts and feelings each time you study. Record your thoughts in the first person, as "I-statements." For example, "Nobody ever uses this stuff in real life," rephrased as an I-statement, it would be: "I would like to know where I would use this in my life." Research shows that a positive attitude is the single most important key to your success in mathematics. By recognizing negative thoughts and replacing them with positive thoughts, you are beginning to work on changing your attitude. The first step is to become aware of your self-talk.

Negative self-talk falls into three categories of irrational beliefs that you have about yourself and how you view the world. You think that (1) Worrying helps. Wrong. Worrying leads to excessive anxiety, which is distracting and hinders performance. (2) Your worth as a person is determined by your performance. Wrong. Not being able to solve math problems doesn't mean you won't amount to anything. If you think it does, you are a victim of "catastrophizing." (3) You are the only one who feels anxious. Wrong. By thinking that other students have some magical coping skills that allow them to avoid anxiety, you are comparing yourself to some irrational mythical norm.

As you begin your first section, ask yourself such questions as What am I saying to myself now? What is triggering these thoughts? How do I feel physically? and What emotions am I feeling now? Your answers will likely reveal a pattern to your thoughts and feelings. You need to analyze your statements and change them into more positive and rational self-talk. You can use a simple technique called "rational emotive therapy."¹ This method, if practiced regularly, can quickly and effectively change the way you think and feel about math. When you find yourself getting upset, watch for words such as "should," "must," "ought to," "never," "always," and "I can't." They are clues to negative self-talk and signals for you to direct your attention back to math. Use the following ABCD model.

- A. Triggering event: You start to do your math homework and your mind goes blank.
- B. Negative self-talk in response to the trigger: "I can't do math. I'll never pass. My life is ruined!"
- C. Anxiety caused by the negative self-talk: panic, anger, tight neck, etc.
- D. Positive self-talk to cope with anxiety: "This negative self-talk is distracting. It doesn't help me solve these problems. Focus on the problems."; or say, "I may be uncomfortable, but I'm working on it."

Recognize that negative self-talk (B) is the culprit. It causes the anxiety (C) and must be restructured to positive rational statements (D). Practice with the model using your own self-talk statements (B) and then complete the remaining steps.

¹R.E.T. was created by Albert Ellis.

3

FRACTIONS AND MIXED NUMBERS



Ryan McVay / Photodisc / Getty Images

SECTIONS

- 3.1** Proper and Improper Fractions; Mixed Numbers
- 3.2** Simplifying Fractions
- 3.3** Multiplying and Dividing Fractions
- 3.4** Multiplying and Dividing Mixed Numbers
- 3.5** Building Fractions; Listing in Order; Inequalities
- 3.6** Adding Fractions
- 3.7** Adding Mixed Numbers
- 3.8** Subtracting Fractions
- 3.9** Subtracting Mixed Numbers
- 3.10** Order of Operations; Average

APPLICATION

Shane is building a cedar deck in back of his house. His plans include stairs at one end of the deck and planter boxes on one side. See Figure 3.1.

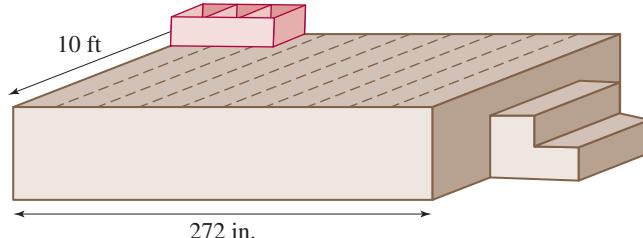


Figure 3.1

Shane would like to keep the sawing to a minimum, as he has only a hand-held circular saw, so he decides that he will design the deck so that he can use standard lengths of lumber. When he goes to Home Depot to purchase the lumber, the first thing he learns is that the standard sizes of lumber do not coincide with the actual size of the boards. For instance, a 10 ft 2-by-4 would seem to imply that its dimensions are 10 ft long, 4 in. wide, and 2 in. deep. However, the actual dimensions are 10 ft long, $3\frac{1}{2}$ in. wide, and $1\frac{1}{2}$ in. deep. (The reduction in dimensions happens when the rough cut lumber is “surfaced.”) This means Shane will have to recalculate what he needs. He had been planning to use 2-by-4s for the deck boards. Now that he knows they are only $3\frac{1}{2}$ in. wide instead of 4 in. wide, he will need more of them than he originally planned. Table 3.1 shows the actual sizes of all the lumber Shane needs for the deck.

Table 3.1 Lumber Sizes

Standard Size (in.)	Actual Size (in.)
4 × 4 posts	$3\frac{1}{2} \times 3\frac{1}{2}$
2 × 12 joists	$1\frac{1}{2} \times 11\frac{1}{4}$
2 × 4 deck boards	$1\frac{1}{2} \times 3\frac{1}{2}$
2 × 6 stair treads	$1\frac{1}{2} \times 5\frac{1}{2}$
1 × 10 trim	$\frac{3}{4} \times 9\frac{1}{4}$

Group Discussion

Use a tape measure to determine the dimensions of the room you are in using inches. Make a drawing of all four walls, including doors and windows. Measure as accurately as you can (specifying fractions of an inch). When you have finished, compare your measurements to those of another group. Are they identical? Why or why not? How can you determine which group has the most accurate measurements?

3.1 Proper and Improper Fractions; Mixed Numbers

OBJECTIVES

1. Write a fraction to describe the parts of a unit.
2. Select proper or improper fractions from a list of fractions.
3. Change improper fractions to mixed numbers.
4. Change mixed numbers to improper fractions.

VOCABULARY

A **fraction** (for example, $\frac{5}{9}$) is a name for a number. The upper numeral (5) is the **numerator**. The lower numeral (9) is the **denominator**.

A **proper fraction** is one in which the numerator is less than the denominator (for example, $\frac{5}{9}$).

An **improper fraction** is one in which the numerator is not less than the denominator (for example, $\frac{9}{5}$ or $\frac{7}{7}$).

A **mixed number** is the sum of a whole number and a fraction ($4 + \frac{3}{5}$) with the addition sign omitted ($4\frac{3}{5}$). The fraction part is usually a proper fraction.

How & Why

OBJECTIVE 1

Write a fraction to describe the parts of a unit.

A unit (here we use a rectangle) may be divided into smaller parts of equal size in order to picture a fraction. The rectangle in Figure 3.2 is divided into seven parts, and six of the parts are shaded. The fraction $\frac{6}{7}$ represents the shaded part. The denominator (7) tells the number of parts in the unit. The numerator (6) tells the number of shaded parts. The fraction $\frac{1}{7}$ represents the part that is not shaded.

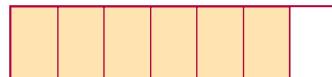


Figure 3.2

Because fractions are another way of writing a division problem and because division by zero is not defined, the denominator can never be zero. There will always be at least one part in a unit.

The unit may also be shown on a ruler. The fraction $\frac{6}{10}$ represents the distance from 0 to the arrowhead in Figure 3.3.

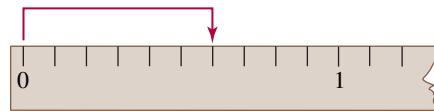


Figure 3.3

To write a fraction to describe the parts of a unit

Write the fraction:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{number of shaded parts}}{\text{total number of parts in one unit}}$$

To write a fraction from a ruler or a number line

Write the fraction:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{number of spaces between zero and end of arrow}}{\text{number of spaces between zero and one}}$$

Examples A–E



BASIC Mathematics Now™

DIRECTIONS: Write the fraction represented by the figure.

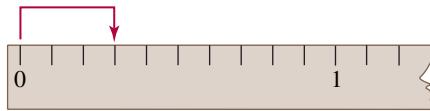
STRATEGY: First count the number of parts that are shaded or marked. This number is the numerator. Now count the total number of parts in the unit. This number is the denominator.

A. Write the fraction represented by



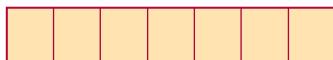
The figure represents $\frac{2}{3}$. The count of shaded parts is 2. The total count is 3.

B. Write the fraction represented by



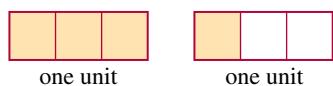
The figure represents $\frac{3}{10}$. There are 3 spaces between 0 and the arrowhead. There are 10 spaces between 0 and 1.

C. Write the fraction represented by



The figure represents $\frac{7}{7}$ or 1. The number of shaded parts is the same as the total number of parts. The whole unit, or 1, is shaded.

D. Write the fraction represented by



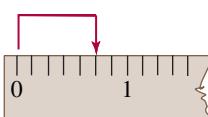
We have two units. The denominator is the number of parts in one unit. The figure represents $\frac{4}{3}$. Four parts are shaded and there are 3 parts in each unit.

Warm-Ups A–E

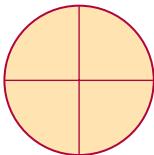
A. Write the fraction represented by



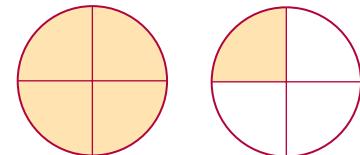
B. Write the fraction represented by



C. Write the fraction represented by



D. Write the fraction represented by



Answers to Warm-Ups

- A. $\frac{2}{4}$ B. $\frac{5}{7}$ C. $\frac{4}{4}$ D. $\frac{5}{4}$

- E. In a second low-cholesterol margarine spread, one tablespoon contains 10 grams of fat. Each fat gram has 9 calories. The total number of calories in a tablespoon is 97. Write a fraction that represents the portion of calories from fat.

E. One tablespoon of a low-cholesterol margarine spread contains 11 grams of fat. Each fat gram has 9 calories, so there are 99 calories from fat in a tablespoon of margarine. The total number of calories in a tablespoon of margarine is 100. Write a fraction that represents the portion of calories from fat.

The fraction is the number of calories from fat over the total number of calories.

The fraction is $\frac{99}{100}$. 99 calories are from fat, and total number of calories is 100.

How & Why

OBJECTIVE 2

Select proper or improper fractions from a list of fractions.

Fractions are called “proper” if the numerator is smaller than the denominator. If the numerator is equal to or greater than the denominator, the fractions are called “improper.” So in the list

$$\frac{5}{6}, \frac{11}{9}, \frac{19}{19}, \frac{10}{11}, \frac{14}{16}, \frac{18}{11}, \frac{24}{29}$$

the proper fractions are $\frac{5}{6}, \frac{10}{11}, \frac{14}{16}$, and $\frac{24}{29}$. The improper fractions are $\frac{11}{9}, \frac{19}{19}$, and $\frac{18}{11}$. If the numerator and the denominator are equal, as in $\frac{19}{19}$, the value of the fraction is 1.

This is easy to see from a picture because the entire unit is shaded. Improper fractions have a value that is greater than or equal to 1. Proper fractions have a value that is less than 1 because some part of the unit is not shaded. See Table 3.2.

Table 3.2 Regions That Show Proper and Improper Fractions

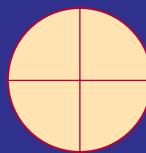
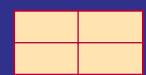
Proper Fractions

Value less than 1

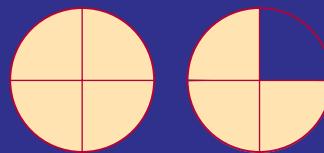
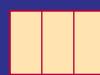


Improper Fractions

Value equal to 1



Value greater than 1



To determine if a fraction is proper or improper

1. Compare the size of the numerator and the denominator.
2. If the numerator is smaller, the fraction is proper. Otherwise the fraction is improper.

Answers to Warm-Ups

- E. The fraction of calories from fat

is $\frac{90}{97}$.

Example F

DIRECTIONS: Identify the proper and improper fractions in the list.

STRATEGY: Compare the numerator and the denominator. If the numerator is smaller, the fraction is proper. If not, the fraction is improper.

- F. Identify the proper and improper fractions: $\frac{5}{6}, \frac{4}{5}, \frac{5}{5}, \frac{7}{6}, \frac{19}{20}, \frac{21}{21}, \frac{25}{23}, \frac{25}{30}$

The proper fractions are:

$$\frac{5}{6}, \frac{4}{5}, \frac{19}{20}$$

The numerators are smaller than the denominators.

The improper fractions are

$$\frac{5}{5}, \frac{7}{6}, \frac{21}{21}, \frac{25}{23}$$

The numerators are not smaller than the denominators.

Warm-Up F

- F. Identify the proper and improper fractions:

$$\frac{8}{8}, \frac{13}{16}, \frac{5}{10}, \frac{11}{8}, \frac{8}{4}, \frac{1}{21}, \frac{25}{1}, \frac{11}{11}$$

How & Why

OBJECTIVE 3

Change improper fractions to mixed numbers.

An improper fraction is equal to a whole number or to a mixed number. A mixed number is the sum of a whole number and a fraction. Figures 3.4 and 3.5 show the conversions.

An improper fraction changed to a whole number:

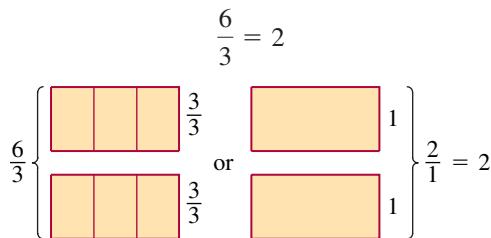


Figure 3.4

An improper fraction changed to a mixed number:

$$\frac{9}{7} = 1\frac{2}{7}$$

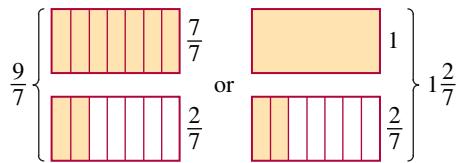


Figure 3.5

The shortcut for changing an improper fraction to a mixed number is to divide.

$$\frac{24}{4} = 24 \div 4 = 6$$

$$\frac{11}{5} = 11 \div 5 = 5 \overline{)11}^2 = 2\frac{1}{5}$$

$\frac{10}{1}$

Answers to Warm-Ups

- F. The proper fractions are $\frac{13}{16}, \frac{5}{10}, \frac{1}{21}$.

The improper fractions are

$$\frac{8}{8}, \frac{11}{8}, \frac{8}{4}, \frac{25}{1}, \frac{11}{11}$$

To change an improper fraction to a mixed number

1. Divide the numerator by the denominator.
2. If there is a remainder, write the whole number and then write the fraction: $\frac{\text{remainder}}{\text{divisor}}$.

CAUTION

Do not confuse the process of changing an improper fraction to a mixed number with “simplifying.” Simplifying fractions is a different procedure. See Section 3.2.

Warm-Ups G–I

G. Change $\frac{20}{7}$ to a mixed number.

H. Change $\frac{117}{3}$ to a mixed number.

I. Change $\frac{319}{14}$ to a mixed number.

Answers to Warm-Ups

G. $2\frac{6}{7}$ H. 39 I. $22\frac{11}{14}$

Examples G–I



DIRECTIONS: Change the improper fraction to a mixed number.

STRATEGY: Divide the numerator by the denominator to find the whole number. If there is a remainder, write it over the denominator to form the fraction part.

G. Change $\frac{11}{3}$ to a mixed number.

$$\begin{array}{r} \overset{3}{3)11} \\ \underline{-9} \\ 2 \end{array}$$

$$\text{So } \frac{11}{3} = 3\frac{2}{3}$$

H. Change $\frac{114}{6}$ to a mixed number.

$$\begin{array}{r} \overset{19}{6)114} \\ \underline{-6} \\ 54 \\ \underline{-54} \\ 0 \end{array}$$

$$\text{So } \frac{114}{6} = 19.$$

CALCULATOR EXAMPLE



I. Change $\frac{348}{7}$ to a mixed number.

If your calculator has a key for fractions, refer to the manual to see how you can use it to change fractions to mixed numbers. If your calculator does not have a fraction

key, divide 348 by 7. The quotient $\frac{348}{7} \approx 49.714$, so the whole-number part is 49.

Now subtract to find the remainder: $348 - 7(49) = 5$.

$$\begin{array}{r} 49 \\ 7 \overline{)348} \\ 343 \\ \hline 5 \end{array}$$

$$348 \div 7 = 49 \text{ R } 5 \text{ so } \frac{348}{7} = 49\frac{5}{7}$$

How & Why

OBJECTIVE 4

Change mixed numbers to improper fractions.

Despite the value judgment attached to the name “improper,” in many cases improper fractions are a more convenient and useful form than mixed numbers. Thus, it is important to be able to convert mixed numbers to improper fractions.

Every mixed number can be changed to an improper fraction. See Figure 3.6.

$$1\frac{3}{7} = \frac{10}{7}$$

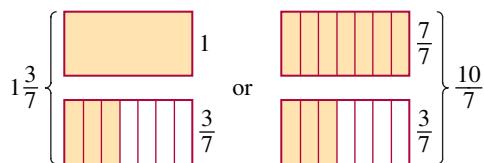


Figure 3.6

The shortcut uses multiplication and addition:

$$1\frac{3}{7} = \frac{1(7) + 3}{7} = \frac{7 + 3}{7} = \frac{10}{7} \quad \text{and} \quad 4\frac{5}{7} = \frac{4(7) + 5}{7} = \frac{33}{7}$$

To change a mixed number to an improper fraction

1. Multiply the denominator times the whole number.
2. Add the numerator to the product from step 1.
3. Write the sum from step 2 over the denominator.

Warm-Ups J–L

J. Change $4\frac{2}{5}$ to an improper fraction.

K. Change $1\frac{14}{15}$ to an improper fraction.

L. Change 6 to an improper fraction.

Examples J–L



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DIRECTIONS: Change each mixed number to an improper fraction.

STRATEGY: Multiply the whole number times the denominator. Add the numerator. Write the sum over the denominator.

J. Change $2\frac{4}{7}$ to an improper fraction.

$$2\frac{4}{7} = \frac{2(7) + 4}{7}$$
$$= \frac{18}{7}$$

Multiply the whole number times the denominator and add the numerator. Write the sum over the denominator.

K. Change $6\frac{6}{11}$ to an improper fraction.

$$6\frac{6}{11} = \frac{6(11) + 6}{11}$$
$$= \frac{72}{11}$$

Multiply the whole number times the denominator and add the numerator. Write the sum over the denominator.

L. Change 12 to an improper fraction.

First, rewrite the whole number as a mixed number. Use the fraction $\frac{0}{1}$.

$$12 = 12\frac{0}{1} = \frac{12(1) + 0}{1}$$
$$= \frac{12}{1}$$

Note: Any fraction that equals 0 can be used.

As Example L illustrates, any whole number can be written as an improper fraction by using a denominator of 1.

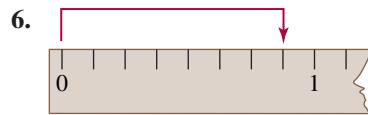
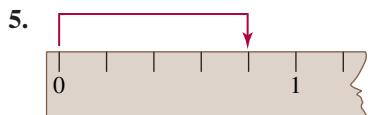
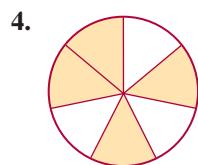
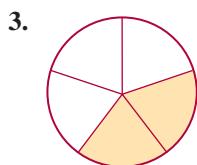
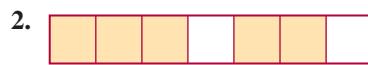
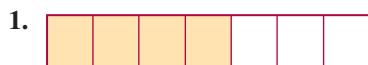
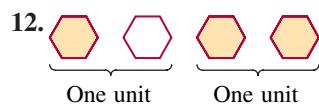
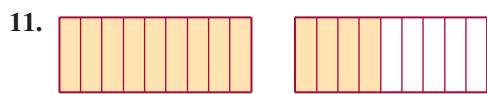
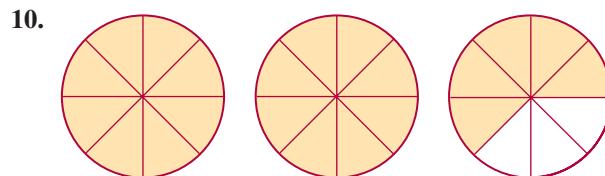
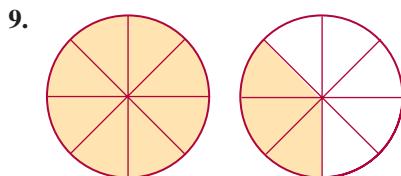
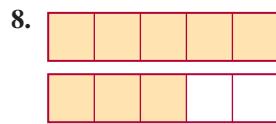
Answers to Warm-Ups

J. $\frac{22}{5}$ K. $\frac{29}{15}$ L. $\frac{6}{1}$

Exercises 3.1

OBJECTIVE 1

Write a fraction to describe the parts of a unit.

A Write the fraction represented by the shaded part of the figure.**B**

OBJECTIVE 2

Select proper or improper fractions from a list of fractions.

A Identify the proper and improper fractions from the list.

13. $\frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{11}{11}, \frac{12}{11}, \frac{13}{11}$

14. $\frac{5}{6}, \frac{8}{7}, \frac{14}{15}, \frac{16}{18}, \frac{17}{17}, \frac{23}{25}, \frac{28}{26}$

15. $\frac{17}{16}, \frac{18}{17}, \frac{29}{19}, \frac{30}{21}, \frac{23}{23}$

16. $\frac{7}{12}, \frac{11}{12}, \frac{13}{12}, \frac{12}{13}, \frac{7}{13}$

17. $\frac{6}{11}, \frac{10}{8}, \frac{11}{6}, \frac{10}{12}, \frac{9}{9}$

18. $\frac{7}{4}, \frac{10}{11}, \frac{13}{13}, \frac{20}{19}, \frac{3}{5}$

B

19. $\frac{4}{4}, \frac{5}{4}, \frac{6}{10}, \frac{12}{12}, \frac{15}{13}, \frac{17}{17}, \frac{18}{20}, \frac{23}{20}, \frac{24}{25}$

20. $\frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{10}{10}, \frac{11}{12}, \frac{12}{13}, \frac{13}{13}, \frac{15}{15}$

OBJECTIVE 3

Change improper fractions to mixed numbers.

A Change to a mixed number

21. $\frac{19}{4}$

22. $\frac{19}{5}$

23. $\frac{89}{7}$

24. $\frac{99}{7}$

25. $\frac{112}{9}$

26. $\frac{103}{6}$

B

27. $\frac{89}{13}$

28. $\frac{98}{13}$

29. $\frac{329}{22}$

30. $\frac{400}{13}$

31. $\frac{331}{15}$

32. $\frac{421}{15}$

OBJECTIVE 4

Change mixed numbers to improper fractions.

A Change to an improper fraction.

33. $4\frac{5}{9}$

34. $6\frac{7}{8}$

35. 15

36. 21

37. $7\frac{3}{4}$

38. $6\frac{5}{6}$

B

39. $27\frac{1}{4}$

40. $37\frac{3}{4}$

41. $40\frac{2}{5}$

42. $42\frac{3}{8}$

43. $45\frac{5}{6}$

44. $46\frac{6}{7}$

C Exercises 45–46. Fill in the boxes so the statement is true. Explain your answer.

45. The fraction $\frac{0}{\square}$ is a proper fraction.

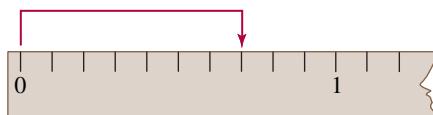
46. $30\frac{\square}{9} = \frac{278}{9}$

47. Find the error(s) in the statement: $4\frac{2}{3} = \frac{8}{12}$. Correct the statement.

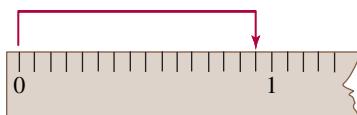
48. Find the error(s) in the statement: $\frac{37}{8} = 3\frac{7}{8}$. Correct the statement.

Write the fraction represented by the figure.

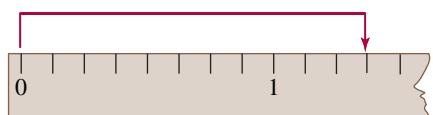
49.



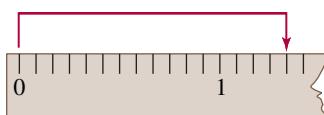
50.



51.



52.



53. Draw a rectangular unit divided into equal parts that shows $\frac{2}{7}$.

54. Draw rectangular units divided into equal parts that shows $\frac{7}{4}$.

55. In a class of 31 students there are 17 women. What fraction represents the part of the class that is female?

56. In a geology class there are 37 students; 25 of them are men. What fraction represents the part of the class that is male?

57. The U.S. Postal Service defines nonstandard mail as anything that is longer than $11\frac{1}{2}$ in., taller than $6\frac{1}{8}$ in., and/or thicker than $\frac{1}{4}$ in. Which of these dimensions can be changed to improper fractions and which cannot? Rewrite the appropriate dimensions as improper fractions.

58. The Adams family budgets \$1155 for food and housing. They spend \$772 per month for housing. What fractional part of their food and housing budget is spent for food?

- 59.** What fraction of a full tank of gas is indicated by the gas gauge?

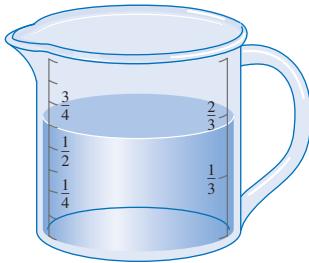


- 61.** A weight scale is marked with a whole number at each pound. What whole number mark is closest to a weight of $\frac{73}{12}$ lb?

- 63.** Beverly is installing a mosaic tile backsplash in her bathroom. She is using tiles that are $\frac{1}{2}$ inch square. If the backsplash is $49\frac{1}{2}$ inches long, how many tiles does she need in each row of tiles in the backsplash?

- 65.** In clothing, junior sizes are designed for young women who are 5'7" or less. What fractional part of a foot is 7"? Express the height 5'7" as a mixed number of feet.

- 67.** The figure shows a measuring cup that contains oil for a zucchini bread recipe. The oil is what fractional part of a cup?



Exercises 68–70 relate to the chapter application. See page 195.

- 68.** A joist labeled "2 × 12" is actually $1\frac{1}{2} \times 11\frac{1}{4}$ in. Convert these measurements to improper fractions.

- 70.** Change the length of a joist, $\frac{272}{12}$ ft, to a mixed number.

- 60.** What fraction of a full tank of gas is indicated by the gas gauge?



- 62.** A ruler is marked with a whole number at each centimeter. What whole-number mark is closest to a length of $\frac{97}{10}$ cm?

- 64.** Cedale worked a 5 hour 43 minute shift at Walgreens one afternoon. What fractional part of a day (24 hours) did Cedale work?

- 66.** Linh awoke from a bad dream at 3:51 A.M. What fractional part of an hour is 51 minutes? Express the time 3:51 A.M. as a mixed number of hours after midnight.

- 69.** According to Shane's plans for his deck, each joist must be 272 in. long. However, lumber is sold by the foot, not the inch. So Shane must convert 272 in. into feet. Because he knows there are 12 in. in a foot, he knows that $272 \text{ in.} = \frac{272}{12} \text{ ft}$. Is this a proper or improper fraction? How do you know?

STATE YOUR UNDERSTANDING

- 71.** Explain how to change $\frac{34}{5}$ to a mixed number. Explain how to change $7\frac{3}{8}$ to an improper fraction.
- 72.** Tell why mixed numbers and improper fractions are both useful. Give examples of the use of each.
- 73.** Explain why a proper fraction cannot be changed into a mixed number.

CHALLENGE

- 74.** Write the whole number 13 as an improper fraction with (a) the numerator 117 and (b) the denominator 117.
- 75.** Write the whole number 16 as an improper fraction with (a) the numerator 144 and (b) the denominator 144.
- 76.** The Swete Tuth candy company packs 30 pieces of candy in its special Mother's Day box. Write, as a mixed number, the number of special boxes that can be made from 67,892 pieces of candy. The company then packs 25 of the special boxes in a carton for shipping. Write, as a mixed number, the number of cartons that can be filled. If it costs Swete Tuth \$45 per carton to ship the candy, what is the shipping cost for the number of full cartons that can be shipped?
- 77.** Jose has $21\frac{3}{4}$ yd of rope to use in a day care class. If the rope is cut into $\frac{1}{4}$ -yd pieces, how many pieces will there be? If there are 15 children in the class, how many pieces of rope will each child get? How many pieces will be left over?

GROUP WORK

- 78.** Often data about a population are presented in a pie chart. Pie charts are based on fractions. If the fractions are easy to draw, a pie chart can be drawn quickly. Sometimes a drawing that is a reasonable estimate is adequate. A survey revealed that half of a class preferred pepperoni pizza, one-quarter preferred cheese pizza, one-eighth preferred Canadian bacon pizza, and one-eighth preferred sausage pizza. Draw a pie chart that illustrates this survey. Explain your strategy.
- 79.** According to a radio advertising survey, 77 of 100 people in the United States say they listen to the radio daily. Make a pie chart that illustrates this survey. Explain your strategy.

- 80.** Using the following figure, write a fraction to name each of the division marks. Now divide each section in half and write new names for each mark. Divide the new sections in half and again write new fraction names for the marks. You now have three names for each of the original marks. What conclusion can you make about these names?



- 81.** Write $\frac{45678}{37}$ as a mixed number with the aid of a calculator. Use the calculator to find the whole-number part and the numerator of the fraction part of the mixed number. Be prepared to show the class how to do this.

MAINTAIN YOUR SKILLS

- 82.** Divide: $78 \div 3$
- 83.** Divide: $78 \div 2$
- 84.** Divide: $390 \div 6$
- 85.** Divide: $390 \div 26$
- 86.** Is 271 prime or composite?
- 87.** Is 273 prime or composite?
- 88.** Find the prime factorization of 275.
- 89.** Find the prime factorization of 279.
- 90.** Bonnie averages 40 miles per gallon of gasoline on her vintage motorcycle. On a recent trip she traveled 680 miles. How many gallons of gas did she use?
- 91.** After a tune-up Bonnie traveled 221 miles, 203 miles, 192 miles, and 188 miles on four tanks of gas. If her tank holds 4 gallons of gas, what was her average number of miles per tank? To the nearest whole number, what was her average number of miles per gallon?

3.2 Simplifying Fractions

VOCABULARY

Equivalent fractions are fractions that are the different names for the same number.

Simplifying a fraction is the process of renaming it by using a smaller numerator and denominator.

A fraction is **completely simplified** when its numerator and denominator have no common factors other than 1. For instance, $\frac{12}{18} = \frac{2}{3}$.

OBJECTIVE

Simplify a fraction.

How & Why

OBJECTIVE

Simplify a fraction.

Fractions are *equivalent* if they represent the same quantity. When we compare the two units in Figure 3.7, we see that each is divided into four parts. The shaded part on the left is named $\frac{2}{4}$, whereas the shaded part on the right is labeled $\frac{1}{2}$. It is clear that the two are the same size, and therefore we say $\frac{2}{4} = \frac{1}{2}$.

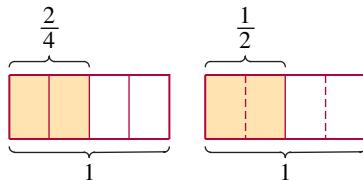


Figure 3.7

The arithmetical way of showing that $\frac{2}{4} = \frac{1}{2}$ is to eliminate the common factors by dividing:

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

The division can also be shown by eliminating the common factors.

$$\frac{2}{4} = \frac{1 \cdot \cancel{2}}{\cancel{2} \cdot 2} = \frac{1}{2}$$

This method works for all fractions. To simplify $\frac{28}{21}$:

$$\frac{28}{21} = \frac{4 \cdot \cancel{7}}{3 \cdot \cancel{7}} = \frac{4}{3} \quad \text{Eliminate the common factors.}$$

or

$$\frac{28}{21} = \frac{28 \div 7}{21 \div 7} = \frac{4}{3} \quad \text{Divide out the common factors.}$$

When all common factors have been eliminated (divided out), the fraction is completely simplified.

$$\frac{24}{40} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5} \quad \text{Completely simplified.}$$

If the common factors are not discovered easily, they can be found by writing the numerator and denominator in prime-factored form. See Example E.

To simplify a fraction completely

Eliminate all common factors, other than 1, in the numerator and the denominator.

Warm-Ups A–H

A. Simplify: $\frac{22}{33}$

B. Simplify: $\frac{18}{45}$

C. Simplify: $\frac{48}{32}$

Examples A–H



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DIRECTIONS: Simplify completely.

STRATEGY: Eliminate the common factors in the numerator and the denominator.

A. Simplify: $\frac{27}{45}$

$$\frac{27}{45} = \frac{\cancel{3} \cdot 9}{\cancel{5} \cdot 9} = \frac{3}{5}$$

The common factor is 9. Eliminate the common factor by dividing.

B. Simplify: $\frac{16}{24}$

$$\begin{aligned}\frac{16}{24} &= \frac{\cancel{2} \cdot 8}{\cancel{2} \cdot 12} = \frac{8}{12} && \text{A common factor of 2 is eliminated.} \\ &= \frac{\cancel{2} \cdot 4}{\cancel{2} \cdot 6} = \frac{4}{6} && \text{There is still a factor of 2 in the numerator and the denominator.} \\ &= \frac{\cancel{2} \cdot 2}{\cancel{2} \cdot 3} = \frac{2}{3} && \text{Again, a common factor of 2 is eliminated by division.}\end{aligned}$$

or

$$\frac{16}{24} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3}$$

Rather than divide by 2 three times, divide by 8 once, and the fraction is simplified completely.

C. Simplify: $\frac{51}{34}$

$$\frac{51}{34} = \frac{51 \div 17}{34 \div 17} = \frac{3}{2}$$

Divide both numerator and denominator by 17.

or

$$\frac{51}{34} = \frac{\cancel{51}}{\cancel{34}} = \frac{3}{2}$$

Divide 51 and 34 by 17 mentally.

Answers to Warm-Ups

A. $\frac{2}{3}$ B. $\frac{2}{5}$ C. $\frac{3}{2}$

D. Simplify: $\frac{100}{600}$

$$\frac{100}{600} = \frac{100 \div 100}{600 \div 100} = \frac{1}{6}$$
 Divide both numerator and denominator by 100.

or

$$\frac{100}{600} = \frac{1}{6}$$
 Divide by 100 mentally using the shortcut for dividing by powers of 10: $100 = 10^2$.

E. Simplify: $\frac{126}{144}$

STRATEGY: Because the numbers are large, write them in prime-factored form.

$$\begin{aligned}\frac{126}{144} &= \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 7}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{3}} && \text{Eliminate the common factors.} \\ &= \frac{7}{2 \cdot 2 \cdot 2} \\ &= \frac{7}{8} && \text{Multiply.}\end{aligned}$$

F. Simplify: $\frac{18}{35}$

$$\frac{18}{35} = \frac{2 \cdot 3 \cdot 3}{5 \cdot 7} = \frac{18}{35}$$
 There are no common factors. The fraction is already completely simplified.

CALCULATOR EXAMPLE



G. Simplify: $\frac{493}{551}$

$$\frac{493}{551} = \frac{17}{19}$$
 Use the fraction key on your calculator.

- H. Mel washes cars on Saturday to earn extra money. On a particular Saturday, he has 12 cars to wash. After he has washed 8 of them, what fraction of the total has he washed? Simplify the fraction completely.

STRATEGY: Form the fraction: $\frac{\text{number of cars washed}}{\text{total number of cars}}$

$$\frac{8}{12}$$
 Eight cars out of 12 are washed.

$$\frac{8}{12} = \frac{2 \cdot \cancel{4}}{3 \cdot \cancel{4}} = \frac{2}{3}$$
 Simplify.

So $\frac{2}{3}$ of the cars are washed.

D. Simplify: $\frac{4000}{14,000}$

E. Simplify: $\frac{150}{240}$

F. Simplify: $\frac{25}{81}$

G. Simplify: $\frac{703}{851}$

- H. In Example H, if Mel washes only 4 cars, what fraction of the total is washed?

Answers to Warm-Ups

D. $\frac{2}{7}$ E. $\frac{5}{8}$ F. $\frac{25}{81}$

G. $\frac{19}{23}$ H. He washed $\frac{1}{3}$ of the cars.

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Exercises 3.2

OBJECTIVE

Simplify a fraction.

A Simplify completely.

1. $\frac{8}{12}$

2. $\frac{9}{15}$

3. $\frac{12}{18}$

4. $\frac{15}{18}$

5. $\frac{10}{25}$

6. $\frac{16}{18}$

7. $\frac{30}{50}$

8. $\frac{40}{70}$

9. $\frac{15}{20}$

10. $\frac{21}{28}$

11. $\frac{8}{20}$

12. $\frac{18}{22}$

13. $\frac{35}{40}$

14. $\frac{36}{40}$

15. $\frac{30}{16}$

16. $\frac{66}{44}$

17. $\frac{14}{18}$

18. $\frac{28}{36}$

19. $\frac{21}{35}$

20. $\frac{25}{45}$

21. $\frac{108}{12}$

22. $\frac{198}{22}$

B

23. $\frac{63}{27}$

24. $\frac{60}{35}$

25. $\frac{14}{42}$

26. $\frac{30}{45}$

27. $\frac{12}{36}$

28. $\frac{20}{36}$

29. $\frac{27}{36}$

30. $\frac{32}{36}$

31. $\frac{31}{37}$

32. $\frac{27}{38}$

33. $\frac{50}{75}$

34. $\frac{30}{75}$

35. $\frac{45}{75}$

36. $\frac{60}{75}$

37. $\frac{300}{900}$

38. $\frac{600}{900}$

39. $\frac{45}{80}$

40. $\frac{65}{80}$

41. $\frac{72}{96}$

42. $\frac{36}{39}$

43. $\frac{72}{12}$

44. $\frac{96}{16}$

45. $\frac{85}{105}$

46. $\frac{32}{120}$

47. $\frac{96}{126}$

48. $\frac{72}{100}$

49. $\frac{99}{132}$

50. $\frac{84}{120}$

C

- 51.** Which of these fractions is completely simplified?

$$\frac{21}{35}, \frac{21}{34}, \frac{34}{51}, \frac{36}{48}$$

Simplify completely.

53. $\frac{84}{144}$

54. $\frac{108}{144}$

55. $\frac{196}{210}$

56. $\frac{268}{402}$

57. $\frac{546}{910}$

58. $\frac{630}{1050}$

- 59.** In clothing, petite sizes are designed for women who are 5'4" or less. What fractional part of a foot is 4"? Express the height 5'4" as a mixed number of feet.

- 61.** In a recent NHL game, Martin Brodeur, the goalie for the New Jersey Devils, took 32 shots on goal and made 30 saves. What fraction of the total number of shots did Brodeur save? How many points did the opposing team score on Brodeur?

- 63.** Maria performs a tune-up on her automobile. She finds that two of the six spark plugs are fouled. What fraction represents the number of fouled plugs? Simplify.

- 65.** Gyrid completes 35 hours out of her weekly shift of 40 hours. What fraction of her weekly shift remains?

- 67.** A district attorney successfully prosecutes 44 cases and is unsuccessful on another 20 cases. What fraction of the attorney's cases is successfully prosecuted? Simplify.

- 69.** One hundred forty elk are tallied at the Florence Refuge. Thirty-five of the elk are bulls. What fraction of the elk are cows? Simplify.

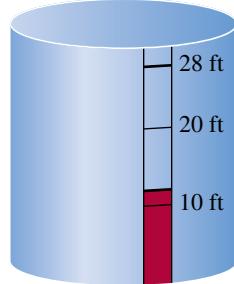
- 52.** Which of these fractions is completely simplified?

$$\frac{26}{91}, \frac{44}{76}, \frac{102}{119}, \frac{56}{145}$$

- 60.** One season, Fred Hoiberg of the Minnesota Timberwolves made 76 three-point shots out of 172 attempts. What fraction of his three-point attempts did he make? What fraction did he miss?

- 62.** In 2003, the three largest import categories in the United States were motor vehicles at \$173 billion, petroleum at \$130 billion, and electrical machinery and appliances at \$83 billion. Petroleum was what fraction of the top three import categories?

- 64.** The float on a tank registers 12 feet. If the tank is full when it registers 28 feet, what fraction of the tank is full? Simplify.



- 66.** The Bonneville Dam fish counter shows 4320 salmon passing over the dam. Of this number, 220 are cohos. What fraction of the salmon are cohos? Simplify.

- 68.** The energy used to produce 1 lb of virgin rubber is 15,800 BTUs. Producing 1 lb of recycled rubber requires only 4600 BTUs. What fraction of the BTUs needed to produce 1 lb of virgin rubber is used to produce 1 lb of recycled rubber? Simplify.

Exercises 70–71 relate to the chapter application.

- 70.** The joists on Shane's deck are each $\frac{272}{12}$ ft long.

Write this as a simplified fraction. Write as a simplified mixed number.

- 71.** Shane is considering making his deck 270 in. long instead of 272 in. Write a simplified fraction that is the number of feet in 270 in. Convert this to a mixed number.

STATE YOUR UNDERSTANDING

- 72.** Draw a picture that illustrates that $\frac{9}{12} = \frac{6}{8} = \frac{3}{4}$.

- 73.** Explain how to simplify $\frac{525}{1125}$.

- 74.** Explain why $\frac{12}{16}$ and $\frac{15}{20}$ are equivalent fractions.

CHALLENGE

- 75.** Are these four fractions equivalent? Justify your answer.

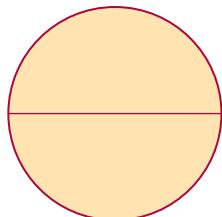
$$\frac{495}{1188} \quad \frac{665}{1596} \quad \frac{1095}{2628} \quad \frac{890}{2136}$$

- 76.** Are these four fractions equivalent? Justify your answer.

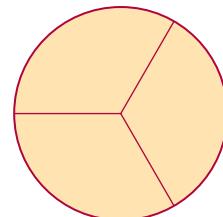
$$\frac{1170}{2925} \quad \frac{864}{2160} \quad \frac{672}{1440} \quad \frac{1134}{2430}$$

GROUP WORK

- 77.** In the figure, the circle is divided into halves. Divide each of the halves in half. Now use the figure to answer the question “What is $\frac{1}{2}$ of $\frac{1}{2}$? ” Devise a rule for finding the product without using the circle.



- 78.** In the figure, the circle is divided into thirds. Divide each of the thirds in half. Now use the figure to answer the question “What is $\frac{1}{2}$ of $\frac{1}{3}$? ” Devise a rule for finding the product without using the circle.



MAINTAIN YOUR SKILLS*Multiply.*

79. $8(77)$

80. $9(88)$

81. $10(99)$

82. $7(333)$

83. $8(444)$

Divide.

84. $270 \div 6$

85. $270 \div 15$

86. $270 \div 45$

87. $336 \div 21$

88. $336 \div 28$

3.3 Multiplying and Dividing Fractions

VOCABULARY

A **product** is the answer to a multiplication problem. If two fractions have a product of 1, each fraction is called the **reciprocal** of the other. For example, $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$. A **quotient** is the answer to a division exercise. The quotient of 33 and 3 is 11.

OBJECTIVES

1. Multiply fractions.
2. Find the reciprocal of a number.
3. Divide fractions.

How & Why

OBJECTIVE 1

Multiply fractions.

The word *of* often indicates multiplication. For example, what is $\frac{1}{2}$ of $\frac{1}{3}$? In other words, $\frac{1}{2} \cdot \frac{1}{3} = ?$

See Figure 3.8. The rectangle is divided into three parts. One part, $\frac{1}{3}$, is shaded yellow. To find $\frac{1}{2}$ of the yellow shaded region, divide each of the thirds into two parts (halves). Figure 3.9 shows the rectangle divided into six parts. So, $\frac{1}{2}$ of the shaded third is $\frac{1}{6}$ of the rectangle, which is shaded blue.

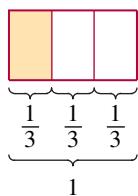


Figure 3.8

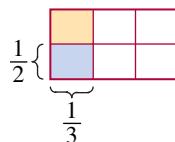
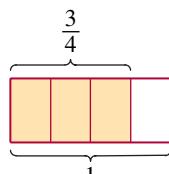


Figure 3.9

$$\frac{1}{2} \text{ of } \frac{1}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} = \frac{\text{number of parts shaded blue}}{\text{total number of parts}}$$

What is $\frac{1}{4}$ of $\frac{3}{4}$? ($\frac{1}{4} \cdot \frac{3}{4} = ?$) In Figure 3.10 the rectangle has been divided into four parts, and $\frac{3}{4}$ is represented by the parts that are shaded yellow. To find $\frac{1}{4}$ of the yellow shaded regions, divide each of the fourths into four parts. The rectangle is now divided into 16 parts, and $\frac{1}{4}$ of each of the three original yellow regions is shaded blue. The blue shaded region represents $\frac{3}{16}$. See Figure 3.11.



$$\frac{1}{4} \text{ of } \frac{3}{4} = \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}$$

Figure 3.10

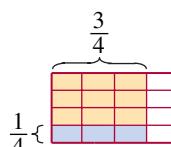


Figure 3.11

We have seen that $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$ and that $\frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}$. The mathematical method is to multiply the numerators and multiply the denominators.

$$\begin{aligned}\frac{6}{25} \cdot \frac{35}{24} &= \frac{210}{600} \\&= \frac{210 \div 10}{600 \div 10} && \text{Simplify by dividing both the numerator and the denominator by 10.} \\&= \frac{21}{60} \\&= \frac{21 \div 3}{60 \div 3} && \text{Simplify by dividing both the numerator and the denominator by 3.} \\&= \frac{7}{20} && \text{The fraction is completely simplified.}\end{aligned}$$

Multiplying two or more fractions like those just given can often be done more quickly by simplifying before multiplying.

$$\begin{aligned}\frac{6}{25} \cdot \frac{35}{24} &= \frac{\cancel{6}^1}{\cancel{25}^5} \cdot \frac{35}{\cancel{24}^4} && \text{Simplify by dividing 6 and 24 by 6.} \\&= \frac{\cancel{6}^1}{\cancel{25}^5} \cdot \frac{\cancel{35}^7}{\cancel{24}^4} && \text{Divide 25 and 35 by 5.} \\&= \frac{7}{20} && \text{Multiply.}\end{aligned}$$

The next example shows all of the simplifying done in one step.

$$\begin{aligned}\frac{20}{36} \cdot \frac{18}{25} &= \frac{\cancel{20}^1}{\cancel{36}^9} \cdot \frac{\cancel{18}^2}{\cancel{25}^5} && \text{Divide 20 and 36 by 4, then divide 5 and 25 by 5.} \\& & & \text{Next divide 18 and 9 by 9.} \\&= \frac{2}{5} && \text{Then multiply.}\end{aligned}$$

If the numbers are large, find the prime factorization of each numerator and denominator. See Example E.

To multiply fractions

1. Simplify.
2. Write the product of the numerators over the product of the denominators.

CAUTION

Simplifying before doing the operation works only for multiplication because it is based on multiplying by 1. It does **not** work for addition, subtraction, or division.

Examples A–G



DIRECTIONS: Multiply. Simplify completely.

STRATEGY: Simplify and then multiply.

A. Multiply and simplify: $\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{4}{7}$

$$\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{4}{7} = \frac{8}{105}$$

There are no common factors. Multiply the numerators and multiply the denominators.

B. Multiply and simplify: $\frac{7}{3} \cdot 2$

$$\frac{7}{3} \cdot 2 = \frac{7}{3} \cdot \frac{2}{1}$$

$$= \frac{14}{3} \quad \text{or} \quad 4\frac{2}{3}$$

Write the product of the numerators over the product of the denominators.

C. Multiply and simplify: $\frac{5}{6} \cdot \frac{3}{4}$

$$\frac{5}{6} \cdot \frac{3}{4} = \frac{5}{6} \cdot \frac{\cancel{3}}{\cancel{4}}$$

Eliminate the common factor of 3 in 6 and 3.

$$= \frac{5}{8}$$

Multiply.

D. Multiply and simplify: $\frac{7}{9} \cdot \frac{18}{5} \cdot \frac{10}{21}$

$$\frac{7}{9} \cdot \frac{18}{5} \cdot \frac{10}{21} = \frac{7}{9} \cdot \frac{\cancel{18}}{\cancel{5}} \cdot \frac{\cancel{10}}{\cancel{21}}$$

Eliminate the common factors of 9, 5, and 7.

$$= \frac{4}{3}$$

Multiply.

E. Multiply and simplify: $\frac{20}{30} \cdot \frac{15}{88}$

STRATEGY: Finding the prime factorization makes the common factors easier to detect.

$$\frac{20}{30} \cdot \frac{15}{88} = \frac{2 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 5} \cdot \frac{3 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 11}$$

$$= \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{5}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{5}} \cdot \frac{\cancel{3} \cdot \cancel{5}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 11}$$

Eliminate the common factors.

$$= \frac{5}{44}$$

Multiply.

Warm-Ups A–G

A. Multiply and simplify:

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{7}$$

B. Multiply and simplify: $3 \cdot \frac{5}{7}$

C. Multiply and simplify:

$$\frac{7}{10} \cdot \frac{4}{9}$$

D. Multiply and simplify:

$$\frac{8}{16} \cdot \frac{12}{9} \cdot \frac{15}{18}$$

E. Multiply and simplify:

$$\frac{48}{55} \cdot \frac{33}{30}$$

Answers to Warm-Ups

- A. $\frac{15}{56}$ B. $\frac{15}{7}$ or $2\frac{1}{7}$ C. $\frac{14}{45}$
D. $\frac{5}{9}$ E. $\frac{24}{25}$

CALCULATOR EXAMPLE

- F. Multiply and simplify:

$$\frac{18}{21} \cdot \frac{28}{30}$$

- G. During 1 year, $\frac{4}{5}$ of all the tires sold by Tredway Tires were highway tread tires. If $\frac{1}{50}$ of the highway tread tires were recalled, what fraction of all tires sold was highway treads that were recalled?

- F. Multiply and simplify: $\frac{16}{75} \cdot \frac{45}{56}$

$$\frac{16}{75} \cdot \frac{45}{56} = \frac{6}{35}$$

Use the fraction key. The calculator will automatically simplify the product.

- G. In a 1-year period, $\frac{9}{10}$ of all cars sold by Trustum Used Cars had automatic transmissions. Of the cars sold with automatic transmissions, $\frac{1}{18}$ had their transmissions rebuilt before they were sold. What fraction of all cars sold had rebuilt automatic transmissions?

$$\begin{aligned}\frac{9}{10} \cdot \frac{1}{18} &= \frac{\cancel{9}^1}{\cancel{10}^2} \cdot \frac{1}{\cancel{18}^2} \\ &= \frac{1}{20}\end{aligned}$$

So $\frac{1}{20}$ of all cars sold had rebuilt automatic transmissions.

How & Why

OBJECTIVE 2

Find the reciprocal of a number.

Finding the reciprocal of a fraction is often called “inverting” the fraction. For instance, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$. We check by showing that the product of the fraction and its reciprocal is 1:

$$\frac{3}{8} \cdot \frac{8}{3} = \frac{24}{24} = 1$$

To find the reciprocal of a fraction

Interchange the numerator and denominator.

CAUTION

The number zero (0) does not have a reciprocal.

Answers to Warm-Ups

F. $\frac{4}{5}$

- G. $\frac{2}{125}$ of the highway tread tires were recalled.

Examples H–I



DIRECTIONS: Find the reciprocal.

STRATEGY: Interchange the numerator and the denominator; that is, “invert” the fraction.

H. Find the reciprocal of $\frac{5}{12}$.

$$\frac{12}{5} \quad \text{Exchange the numerator and denominator.}$$

CHECK: $\frac{5}{12} \cdot \frac{12}{5} = \frac{60}{60} = 1$

The reciprocal of $\frac{5}{12}$ is $\frac{12}{5}$, or $2\frac{2}{5}$.

I. Find the reciprocal of $1\frac{4}{9}$.

STRATEGY: First write $1\frac{4}{9}$ as an improper fraction.

$$1\frac{4}{9} = \frac{13}{9}$$

$$\frac{9}{13} \quad \text{Invert the fraction.}$$

CHECK: $\frac{13}{9} \cdot \frac{9}{13} = \frac{117}{117} = 1$

The reciprocal of $1\frac{4}{9}$ is $\frac{9}{13}$.

Warm-Ups H–I

H. Find the reciprocal of $\frac{14}{5}$.

I. Find the reciprocal of $2\frac{5}{6}$.

How & Why

OBJECTIVE 3

Divide fractions.

In Chapter 1, you learned that division is the inverse of multiplication; that is, the quotient of a division problem is the number that is multiplied times the divisor (second number) to get the dividend. For example, the quotient of $21 \div 3$ is 7. When 7 is multiplied by the divisor, 3, the product is 21, the dividend.

Answers to Warm-Ups

H. $\frac{5}{14}$ I. $\frac{6}{17}$

Another way of thinking of division is to ask “How many groups of a given size are contained in a number?”

	Think	Answer
$21 \div 3$	How many 3s in 21?	7
$\frac{4}{5} \div \frac{1}{10}$	How many $\frac{1}{10}$ s in $\frac{4}{5}$?	See Figure 3.12

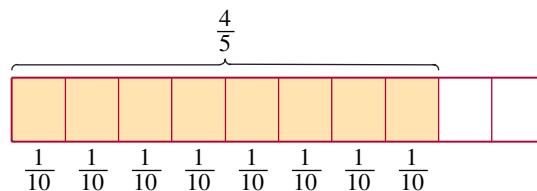


Figure 3.12

Figure 3.12 shows that there are eight $\frac{1}{10}$ s in $\frac{4}{5}$. Therefore, we write $\frac{4}{5} \div \frac{1}{10} = 8$.

Check by multiplying. Because $8 \cdot \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$, we know that the quotient is correct.

The quotient can also be found by multiplying $\frac{4}{5}$ by the reciprocal of $\frac{1}{10}$. This is the customary shortcut.

$$\frac{4}{5} \div \frac{1}{10} = \frac{4}{5} \cdot \frac{10}{1} = \frac{40}{5} = 8$$

To divide fractions

Multiply the first fraction by the reciprocal of the divisor; that is, invert the divisor and multiply.

CAUTION

Do not simplify the fractions before changing the division to multiplication; that is, invert before simplifying.

Warm-Ups J–N

J. Divide: $\frac{5}{12} \div \frac{7}{12}$

Answers to Warm-Ups

J. $\frac{5}{7}$

Examples J–N



DIRECTIONS: Divide. Simplify completely.

STRATEGY: Multiply by the reciprocal of the divisor.

J. Divide: $\frac{8}{17} \div \frac{11}{17}$

$$\frac{8}{17} \div \frac{11}{17} = \frac{8}{17} \cdot \frac{17}{11} \quad \text{Multiply by the reciprocal of the divisor.}$$

$$= \frac{8}{11}$$

K. Divide: $\frac{9}{4} \div \frac{3}{7}$

$$\frac{9}{4} \div \frac{3}{7} = \frac{9}{4} \cdot \frac{7}{3}$$

Invert the divisor and multiply.

$$= \frac{21}{4} \text{ or } 5\frac{1}{4}$$

L. Divide: $\frac{1}{12} \div \frac{3}{5}$

$$\frac{1}{12} \div \frac{3}{5} = \frac{1}{12} \cdot \frac{5}{3}$$

Invert the divisor and multiply.

$$= \frac{5}{36}$$

CALCULATOR EXAMPLE 

M. Divide and simplify: $\frac{8}{21} \div \frac{24}{77}$

STRATEGY: Use the fraction key. The calculator will automatically invert the divisor and simplify the quotient.

$$\frac{8}{21} \div \frac{24}{77} = \frac{11}{9}$$

N. The distance a nut moves one turn on a bolt is $\frac{3}{16}$ inch. How many turns will it take to move the nut $\frac{3}{4}$ inch?

STRATEGY: To find the number of turns required to move the nut $\frac{3}{4}$ inch, divide $\frac{3}{4}$ by the distance the nut moves in one turn.

$$\frac{3}{4} \div \frac{3}{16} = \frac{3}{4} \cdot \frac{16}{3} = 4$$

Invert the divisor and multiply.

It will take 4 turns to move the nut $\frac{3}{4}$ inch.

K. Divide: $\frac{5}{3} \div \frac{10}{7}$

L. Divide: $\frac{5}{11} \div \frac{2}{3}$

M. Divide and simplify:

$$\frac{9}{64} \div \frac{27}{80}$$

N. Suppose in Example N that the distance the nut moves on the bolt with one turn is

$\frac{3}{32}$ inch. How many turns will it take to move the nut $\frac{3}{4}$ inch?

Answers to Warm-Ups

K. $\frac{7}{6}$ or $1\frac{1}{6}$ L. $\frac{15}{22}$ M. $\frac{5}{12}$

N. Eight turns are needed to move the nut $\frac{3}{4}$ inch.

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Exercises 3.3

OBJECTIVE 1

Multiply fractions.

A Multiply. Simplify completely.

1. $\frac{1}{5} \cdot \frac{2}{5}$

2. $\frac{5}{6} \cdot \frac{1}{7}$

3. $\frac{3}{2} \cdot \frac{5}{14}$

4. $\frac{3}{7} \cdot \frac{6}{5}$

5. $\frac{5}{4} \cdot \frac{7}{10}$

6. $\frac{4}{9} \cdot \frac{3}{8}$

7. $\frac{2}{3} \cdot \frac{3}{10}$

8. $\frac{5}{2} \cdot \frac{2}{15}$

9. $14 \cdot \frac{8}{35}$

10. $18 \cdot \frac{7}{54}$

B

11. $\frac{4}{30} \cdot \frac{10}{15} \cdot \frac{9}{6}$

12. $\frac{2}{15} \cdot \frac{5}{6} \cdot \frac{3}{4}$

13. $8 \cdot \frac{1}{5} \cdot \frac{15}{16}$

14. $\frac{14}{3} \cdot 4 \cdot \frac{2}{7}$

15. $\frac{11}{3} \cdot \frac{3}{4} \cdot \frac{4}{11}$

16. $\frac{21}{16} \cdot 2 \cdot \frac{8}{7}$

17. $\frac{36}{55} \cdot \frac{33}{54} \cdot \frac{35}{24}$

18. $\frac{32}{39} \cdot \frac{24}{96} \cdot \frac{52}{72}$

19. $\frac{7}{6} \cdot \frac{8}{23} \cdot \frac{5}{8} \cdot \frac{0}{9}$

20. $\frac{7}{3} \cdot \frac{5}{0} \cdot \frac{20}{21} \cdot \frac{8}{3}$

OBJECTIVE 2

Find the reciprocal of a number.

A Find the reciprocal.

21. $\frac{7}{10}$

22. $\frac{7}{5}$

23. 3

24. 0

25. $\frac{2}{0}$

B

26. $2\frac{3}{8}$

27. $3\frac{9}{14}$

28. 19

29. $\frac{1}{17}$

30. 12

OBJECTIVE 3

Divide fractions.

A Divide. Simplify completely.

31. $\frac{7}{20} \div \frac{14}{15}$

32. $\frac{8}{13} \div \frac{2}{13}$

33. $\frac{4}{9} \div \frac{3}{7}$

34. $\frac{5}{7} \div \frac{9}{8}$

35. $\frac{8}{3} \div \frac{8}{9}$

36. $\frac{5}{6} \div \frac{5}{3}$

37. $\frac{6}{55} \div \frac{10}{33}$

38. $\frac{5}{12} \div \frac{25}{6}$

39. $\frac{10}{16} \div \frac{1}{4}$

40. $\frac{3}{4} \div \frac{9}{16}$

B

41. $\frac{21}{40} \div \frac{9}{28}$

42. $\frac{30}{49} \div \frac{10}{21}$

43. $\frac{20}{24} \div \frac{20}{18}$

44. $\frac{9}{12} \div \frac{12}{7}$

45. $\frac{6}{5} \div \frac{9}{25}$

46. $\frac{7}{3} \div \frac{35}{33}$

47. $\frac{90}{55} \div \frac{9}{5}$

48. $\frac{28}{35} \div \frac{7}{5}$

49. $\frac{39}{65} \div \frac{3}{5}$

50. $\frac{45}{105} \div \frac{3}{7}$

C Multiply. Simplify completely.

51. $\frac{48}{90} \cdot \frac{80}{144} \cdot \frac{120}{200}$

52. $\frac{21}{14} \cdot \frac{15}{42} \cdot \frac{18}{15}$

53. $\frac{40}{44} \cdot \frac{24}{160} \cdot \frac{77}{35}$

54. $\frac{32}{675} \cdot \frac{250}{16} \cdot \frac{135}{50}$

Divide. Simplify completely.

55. $\frac{205}{10} \div \frac{30}{15}$

56. $\frac{8}{108} \div \frac{16}{81}$

57. $\frac{16}{81} \div \frac{8}{108}$

58. $\frac{75}{90} \div \frac{50}{72}$

Fill in the boxes with a single number so the statement is true. Explain your answer.

59. $\frac{3}{4} \cdot \frac{\square}{6} = \frac{7}{8}$

60. $\frac{4}{\square} \cdot \frac{11}{20} = \frac{11}{50}$

61. Find the error in the statement: $\frac{3}{8} \cdot \frac{4}{5} = \frac{15}{32}$. Correct the statement. Explain how you would avoid this error.

62. Find the error in the statement: $\frac{3}{8} \div \frac{4}{5} = \frac{32}{15}$. Correct the statement. Explain how you would avoid this error.

- 63.** For families with four children, there are 16 possible combinations of gender and birth order. Half of these have a boy as firstborn. How many of the possible combinations have a boy as firstborn? How many have a girl as firstborn?
- 65.** For families with four children, there are 16 possible combinations of gender and birth order. One-fourth of the possible combinations have three girls and one boy, and one-fourth of these (3 girl–1 boy combinations) have a firstborn boy followed by three girls. How many of the possible combinations have a firstborn boy followed by three girls?
- 67.** A container for a household cleaner holds $\frac{7}{5}$ gallons. How much does it hold when it is $\frac{3}{4}$ full?
- 69.** Underinflation of car tires can waste up to $\frac{1}{20}$ of a car's fuel due an increase in "rolling resistance." If Becky uses 820 gallons of gas in a year, how many gallons can she save with proper tire inflation?
- 71.** According to the National Cancer Institute, 6 of 10 people who are diagnosed with cancer will survive at least 5 years. The American Cancer Society forecast about 1,361,190 new cancer cases in 2005. How many of these patients are expected to survive for 5 years?
- 73.** According to statistics gathered by the United Nations, 73 of 100 people in Mexico have access to safe drinking water. The population of Mexico in 2004 was about 104,960,000. How many Mexicans did not have access to safe drinking water that year?
- 64.** For families with four children, there are 16 possible combinations of gender and birth order. One-fourth of these have three girls and one boy. How many of the possible combinations have three girls?
- 66.** In 2003, $\frac{43}{50}$ of all heart transplant recipients in the United States survived at least 1 year. If there were about 2050 heart transplants that year, how many people survived?
- 68.** The National Zoo has T-shirts that regularly sell for \$27. The shirts are marked $\frac{1}{3}$ off during a Labor Day sale. What is the sale price of a T-shirt?
- 70.** Underinflation of truck tires can waste up to $\frac{1}{15}$ of a truck's fuel due an increase in "rolling resistance." If Melvin uses 500 gallons of gas while unaware that the tires are low, how many gallons might be wasted?
- 72.** In 2004, slightly more than 1 in every 5 commercial radio stations had a country format. About how many radio stations had a country format if there were about 9990 commercial stations?

STATE YOUR UNDERSTANDING

- 74.** Explain how to find the product of $\frac{35}{24}$ and $\frac{40}{14}$.
- 75.** Explain how to find the quotient of $\frac{35}{24}$ and $\frac{40}{14}$.
- 76.** Evalynne's supervisor tells her that her salary is to be divided by one-half. Should she quit her job? Explain.

CHALLENGE

77. Simplify. $\left(\frac{81}{75} \cdot \frac{96}{99} \cdot \frac{55}{125}\right) \div \frac{128}{250}$

78. The In-n-Out Grocery has a standard workweek of 40 hours. Jan works $\frac{3}{4}$ of a standard week, Jose works $\frac{5}{8}$ of a standard week, Aria works $\frac{9}{8}$ of a standard week, and Bill works $\frac{6}{5}$ of a standard week. How many hours did each employee work? In-n-Out pays an average salary of \$11 an hour. What is the total week's payroll for these four employees?

GROUP WORK

For 80–82. To make accurate pie charts, it is necessary to use a measuring instrument for angles. One such instrument is called a protractor. It is also necessary to convert the fractions of the components into equivalent fractions with denominators of 360 because there are 360° in a complete circle. Starting with any radius, use the protractor to measure the correct angle for each component.

79. Divide your group into two subgroups. Have the two subgroups take opposite sides to discuss the following statement: Since computers and calculators use decimals more often than fractions, the use of fractions will eventually disappear. Have each subgroup write down their arguments for or against the statement.

81. A moderate investment strategy allocates $\frac{3}{5}$ of a portfolio to stocks, $\frac{3}{10}$ to bonds, and $\frac{1}{10}$ to money market funds. Make an accurate pie chart for this strategy.

80. An aggressive investment strategy allocates $\frac{3}{4}$ of a portfolio to stocks, $\frac{1}{5}$ to bonds, and $\frac{1}{20}$ to money market funds. Make an accurate pie chart for this strategy.

82. A conservative investment strategy allocates $\frac{2}{5}$ of a portfolio to stocks, $\frac{9}{20}$ to bonds, and $\frac{3}{20}$ to money market funds. Make an accurate pie chart for this strategy.

MAINTAIN YOUR SKILLS

Change each improper fraction to a mixed number.

83. $\frac{18}{5}$

84. $\frac{18}{7}$

85. $\frac{29}{16}$

86. $\frac{29}{15}$

Change each mixed number to an improper fraction.

87. $4\frac{1}{2}$

88. $3\frac{2}{3}$

89. $3\frac{1}{5}$

90. $2\frac{5}{8}$

91. Kevin needs a load of gravel for a drainage field.

His truck can safely haul a load of $1\frac{3}{4}$ tons (3500 lb).

If the gravel is sold in 222-lb scoops, how many scoops can Kevin haul?

3.4 Multiplying and Dividing Mixed Numbers

How & Why

OBJECTIVE 1

Multiply mixed numbers.

In Section 3.1, we changed mixed numbers to improper fractions. For instance,

$$4\frac{3}{7} = \frac{4(7) + 3}{7} = \frac{31}{7}$$

To multiply mixed numbers we change them to improper fractions and then multiply.

$$\begin{aligned} \left(3\frac{1}{2}\right)\left(4\frac{2}{3}\right) &= \left(\frac{7}{2}\right)\left(\frac{14}{3}\right) && \text{Change to improper fractions and simplify.} \\ &= \frac{49}{6} && \text{Multiply.} \\ &= 16\frac{1}{3} && \text{Write as a mixed number.} \end{aligned}$$

Products may be left as improper fractions or as mixed numbers; either is acceptable. In this text, we write mixed numbers. In algebra, improper fractions are often preferred.

OBJECTIVES

1. Multiply mixed numbers.
2. Divide mixed numbers.

To multiply mixed numbers and/or whole numbers

1. Change to improper fractions.
2. Simplify and multiply.

Examples A–D



BASIC Mathematics Now™

DIRECTIONS: Multiply. Write as a mixed number.

STRATEGY: Change the mixed numbers and whole numbers to improper fractions. Simplify and multiply. Write the answer as a mixed number.

A. Multiply: $\frac{3}{5}\left(2\frac{1}{4}\right)$

$$\begin{aligned} \frac{3}{5}\left(2\frac{1}{4}\right) &= \frac{3}{5} \cdot \frac{9}{4} & 2\frac{1}{4} &= \frac{9}{4} \\ &= \frac{27}{20} && \text{Multiply.} \\ &= 1\frac{7}{20} && \text{Write as a mixed number.} \end{aligned}$$

B. Multiply: $\left(2\frac{1}{3}\right)\left(2\frac{4}{7}\right)$

$$\begin{aligned} \left(2\frac{1}{3}\right)\left(2\frac{4}{7}\right) &= \frac{7}{3} \cdot \frac{18}{7} \\ &= \frac{7}{3} \cdot \frac{6}{7} && \text{Simplify.} \\ &= 6 && \text{Multiply and write as a whole number.} \end{aligned}$$

Warm-Ups A–D

A. Multiply: $\frac{3}{5}\left(1\frac{1}{2}\right)$

B. Multiply: $\left(1\frac{1}{2}\right)\left(5\frac{1}{3}\right)$

Answers to Warm-Ups

A. $\frac{9}{10}$ B. 8

C. Multiply: $4\left(3\frac{3}{5}\right)\left(\frac{5}{9}\right)$

C. Multiply: $6\left(\frac{3}{4}\right)\left(2\frac{1}{3}\right)$

CAUTION

$6\left(\frac{3}{4}\right)$ means $6 \cdot \frac{3}{4}$, but $6\frac{3}{4}$ means $6 + \frac{3}{4}$.

D. Multiply: $5\frac{3}{8}\left(6\frac{1}{2}\right)$

$$\begin{aligned}6\left(\frac{3}{4}\right)\left(2\frac{1}{3}\right) &= \frac{\cancel{6}}{1} \cdot \frac{\cancel{3}}{\cancel{4}} \cdot \frac{7}{\cancel{3}} \\&= \frac{21}{2} = 10\frac{1}{2} \quad \text{Multiply and write as a mixed number.}\end{aligned}$$

CALCULATOR EXAMPLE



D. Multiply: $4\frac{2}{3}\left(8\frac{5}{6}\right)$

$$4\frac{2}{3}\left(8\frac{5}{6}\right) = 41\frac{2}{9}$$

On a calculator with fraction keys, it is not necessary to change the mixed numbers to improper fractions. The calculator is programmed to operate with simple fractions or with mixed numbers.

How & Why

OBJECTIVE 2

Divide mixed numbers.

Division of mixed numbers is also done by changing to improper fractions first.

$$\begin{aligned}\left(6\frac{1}{6}\right) \div \left(2\frac{1}{2}\right) &= \left(\frac{37}{6}\right) \div \left(\frac{5}{2}\right) \quad \text{Change to improper fractions.} \\&= \left(\frac{37}{6}\right)\left(\frac{2}{5}\right) \quad \text{Multiply by the reciprocal of the divisor and simplify.} \\&= \frac{37}{15} \quad \text{Multiply.} \\&= 2\frac{7}{15} \quad \text{Write as a mixed number.}\end{aligned}$$

To divide mixed numbers and/or whole numbers

1. Change to improper fractions.
2. Divide.

Answers to Warm-Ups

C. 8 D. $34\frac{15}{16}$

Examples E–I



BASIC Mathematics Now™

DIRECTIONS: Divide. Write as a mixed number.

STRATEGY: Change the mixed numbers and whole numbers to improper fractions. Divide and simplify completely. Write the answer as a mixed number.

E. Divide: $2\frac{1}{2} \div 4\frac{7}{8}$

$$\begin{aligned}2\frac{1}{2} \div 4\frac{7}{8} &= \frac{5}{2} \div \frac{39}{8} & 2\frac{1}{2} = \frac{5}{2} \text{ and } 4\frac{7}{8} = \frac{39}{8} \\&= \frac{5}{2} \cdot \frac{8}{39} & \text{Invert the divisor and multiply.} \\&= \frac{1}{2} \cdot \frac{8}{39} \\&= \frac{20}{39}\end{aligned}$$

F. Divide: $10\frac{5}{16} \div 16\frac{1}{2}$

$$\begin{aligned}10\frac{5}{16} \div 16\frac{1}{2} &= \frac{165}{16} \div \frac{33}{2} \\&= \frac{165}{16} \cdot \frac{2}{33} & \text{Invert the divisor and multiply.} \\&= \frac{5}{8}\end{aligned}$$

G. Divide: $6 \div 4\frac{2}{3}$

$$\begin{aligned}6 \div 4\frac{2}{3} &= \frac{6}{1} \div \frac{14}{3} \\&= \frac{6}{1} \cdot \frac{3}{14} & \text{Invert the divisor and multiply.} \\&= \frac{9}{7} = 1\frac{2}{7}\end{aligned}$$

CALCULATOR EXAMPLE



H. Divide: $25\frac{1}{4} \div 30\frac{3}{4}$

$$25\frac{1}{4} \div 30\frac{3}{4} = \frac{101}{123}$$

On a calculator with fraction keys, it is not necessary to change the mixed numbers to improper fractions. The calculator is programmed to operate with simple fractions or with mixed numbers.

Warm-Ups E–I

E. Divide: $3\frac{1}{6} \div 2\frac{5}{12}$

F. Divide: $12\frac{1}{2} \div 15\frac{5}{8}$

G. Divide: $6\frac{3}{5} \div 6$

H. Divide: $13\frac{4}{5} \div 12\frac{3}{10}$

Answers to Warm-Ups

- E. $1\frac{9}{29}$ F. $\frac{4}{5}$
 G. $1\frac{1}{10}$ H. $1\frac{5}{41}$

- I. The Sillvary Aluminum Works also produces ingots that are $7\frac{1}{2}$ in. thick. What is the height in feet of a stack of 18 ingots?

- I. The Sillvary Aluminum Works produces ingots that are $6\frac{3}{4}$ in. thick. What is the height in feet of a stack of 18 ingots?

$$\begin{aligned}18 \left(6\frac{3}{4} \right) &= \frac{18}{1} \cdot \frac{27}{4} \\&= \frac{\cancel{18}^9}{1} \cdot \frac{27}{\cancel{4}^2} \\&= \frac{243}{2} = 121\frac{1}{2}\end{aligned}$$

The height of the stack is $121\frac{1}{2}$ in. To find the height in feet, divide by 12 since there are 12 in. in 1 ft.

$$\begin{aligned}121\frac{1}{2} \div 12 &= \frac{243}{2} \div \frac{12}{1} \\&= \frac{\cancel{243}^{81}}{2} \cdot \frac{1}{\cancel{12}^4} \\&= \frac{81}{8} = 10\frac{1}{8}\end{aligned}$$

The stack of ingots is $10\frac{1}{8}$ ft high.

Answers to Warm-Ups

- I. The height is $11\frac{1}{4}$ ft.

Exercises 3.4

BASIC
Mathematics  Now™**OBJECTIVE 1**

Multiply mixed numbers.

A Multiply. Simplify completely and write as a mixed number if possible.

1. $\left(\frac{3}{4}\right)\left(1\frac{3}{4}\right)$

2. $\left(\frac{2}{7}\right)\left(2\frac{5}{7}\right)$

3. $\left(1\frac{3}{4}\right)\left(2\frac{1}{2}\right)$

4. $\left(2\frac{3}{5}\right)\left(1\frac{1}{4}\right)$

5. $2\left(3\frac{2}{3}\right)$

6. $3\left(4\frac{1}{4}\right)$

7. $\left(3\frac{3}{5}\right)\left(1\frac{1}{4}\right)(5)$

8. $\left(\frac{3}{5}\right)\left(2\frac{1}{3}\right)(3)$

9. $\left(7\frac{1}{2}\right)\left(3\frac{1}{5}\right)$

10. $\left(4\frac{1}{6}\right)\left(4\frac{1}{5}\right)$

11. $\left(3\frac{4}{7}\right)\left(\frac{14}{15}\right)$

12. $\left(4\frac{4}{9}\right)\left(\frac{12}{25}\right)$

B

13. $5\left(3\frac{3}{4}\right)$

14. $6\left(4\frac{4}{5}\right)$

15. $\left(7\frac{3}{4}\right)\left(\frac{2}{3}\right)(0)$

16. $\left(3\frac{5}{9}\right)\left(6\frac{5}{7}\right)(0)$

17. $4\left(3\frac{3}{4}\right)\left(3\frac{2}{3}\right)$

18. $3\left(2\frac{1}{4}\right)\left(2\frac{1}{3}\right)$

19. $\left(2\frac{2}{3}\right)\left(\frac{11}{12}\right)\left(6\frac{3}{4}\right)$

20. $\left(2\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(5\frac{5}{8}\right)$

21. $\left(1\frac{1}{5}\right)\left(4\frac{1}{3}\right)\left(6\frac{1}{6}\right)$

22. $\left(12\frac{1}{4}\right)\left(1\frac{1}{7}\right)\left(2\frac{1}{3}\right)$

23. $(14)\left(6\frac{1}{2}\right)\left(1\frac{2}{13}\right)$

24. $12\left(1\frac{4}{15}\right)\left(6\frac{1}{4}\right)$

OBJECTIVE 2

Divide mixed numbers.

A Divide. Simplify completely and write as a mixed number if possible.

25. $4 \div 1\frac{1}{4}$

26. $5 \div 1\frac{1}{5}$

27. $2\frac{7}{8} \div 3\frac{5}{6}$

28. $4\frac{2}{3} \div 8\frac{2}{3}$

29. $10\frac{1}{4} \div 2\frac{1}{2}$

30. $4\frac{1}{4} \div 3\frac{1}{16}$

31. $2\frac{1}{4} \div 1\frac{5}{8}$

32. $2\frac{3}{4} \div 1\frac{5}{8}$

33. $3\frac{1}{3} \div 2\frac{1}{2}$

34. $5\frac{1}{3} \div 1\frac{1}{7}$

35. $4\frac{4}{15} \div 6\frac{2}{3}$

36. $6\frac{1}{4} \div 7\frac{1}{2}$

B

37. $\frac{5}{6} \div 4\frac{1}{3}$

38. $\frac{7}{3} \div 1\frac{4}{9}$

39. $2\frac{1}{5} \div 4$

40. $5\frac{5}{8} \div 6$

41. $4\frac{5}{6} \div \frac{1}{3}$

42. $8\frac{3}{4} \div 2\frac{1}{3}$

43. $2\frac{3}{4} \div \frac{3}{16}$

44. $7\frac{1}{6} \div \frac{7}{3}$

45. $23\frac{1}{4} \div 1\frac{1}{3}$

46. $26\frac{7}{8} \div 3\frac{3}{4}$

47. $10\frac{2}{3} \div 2\frac{2}{7}$

48. $22\frac{2}{3} \div 6\frac{6}{7}$

C Multiply. Simplify completely and write as a mixed number if possible.

49. $2\left(4\frac{1}{5}\right)\left(1\frac{1}{3}\right)\left(6\frac{2}{7}\right)$

50. $3\left(12\frac{1}{4}\right)\left(1\frac{1}{7}\right)\left(2\frac{1}{3}\right)$

Divide. Simplify completely and write as a mixed number if possible.

51. $\left(1\frac{1}{2}\right) \div \left(2\frac{1}{3}\right) \div \left(1\frac{2}{7}\right)$

52. $\left(2\frac{1}{2}\right) \div \left(4\frac{1}{3}\right) \div \left(1\frac{1}{4}\right)$

53. Find the error in the statement: $1\frac{2}{3} \cdot 1\frac{1}{2} = 1\frac{1}{3}$.

54. Find the error in the statement: $6\frac{2}{9} \div 2\frac{2}{3} = 3\frac{1}{3}$.

Correct the statement. Explain how you would avoid this error.

Correct the statement. Explain how you would avoid this error.

55. A 6-ft-by-8-ft readymade storage shed has interior dimensions of $94\frac{3}{4}$ in. wide by 66 in. deep. How many square inches are in the interior of the shed?

56. The recommended foundation size for the storage shed in Exercise 55 is $97\frac{1}{2}$ in. by $68\frac{3}{4}$ in. What is the area of the recommended slab?

57. A jewelry store advertises two diamond rings. One ring is $\frac{1}{2}$ carat total weight for \$700. Another ring is $1\frac{1}{2}$ carats total weight for \$3000. What is the price per carat of each of the two rings?

58. Joe has an 8-ft board that he wants to cut into $20\frac{1}{2}$ -in. lengths to make shelves. How many shelves can Joe get from the board?

59. The iron content in a water sample at Lake Hieda is eight parts per million. The iron content in Swan Lake is $2\frac{3}{4}$ times greater than the content in Lake Hieda. What is the iron content in Swan Lake in parts per million?

60. The water pressure during a bad neighborhood grass fire is reduced to $\frac{5}{9}$ its original pressure at the hydrant. What is the reduced pressure if the original pressure was $70\frac{1}{5}$ pounds per square inch?

- 61.** Mike has a shelf in his entertainment cabinet that is $17\frac{1}{2}$ in. long. His DVD cases are each $\frac{5}{8}$ in. thick. How many DVDs can Mike store on the shelf?
- 63.** The amount of CO₂ a car emits is directly related to the amount of gas it uses. Cars give off 20 lb of CO₂ for every gallon of gas used. A car averaging 27 mpg will emit 2000 lb of CO₂ in 2700 miles. A car averaging 18 mpg will emit $1\frac{1}{2}$ times more CO₂ in the same distance. How many pounds of CO₂ does the less-efficient car emit in the 2700 miles?
- 62.** The American Heart Association estimates that a little more than $\frac{1}{4}$ of all Americans have some form of hypertension (high blood pressure), which puts them at greater risk for heart attacks and strokes. The population of South Dakota is estimated at 770,880. About how many South Dakotans would be expected to have hypertension?
- 64.** Nutritionists recommend that not more than $\frac{3}{10}$ of your daily intake in calories should come from fat. If each gram of fat is 10 calories, what is the recommended upper limit on fat for a diet of 2400 calories?

Exercises 65–67 relate to the chapter application.

- 65.** Shane read in his *How to Build Decks* book that stairs with 10-in. treads (widths) are easy to build using either 2 × 4s or 2 × 6s. Either size will make treads with a slight overhang at the front, which is recommended. If Shane wants the actual tread to be $10\frac{1}{2}$ in., how many 2 × 4s per tread will he need?
- See Table 3.1 on page 195 for the actual size of a 2 × 4.

- 67.** Shane will be using 2 × 4 deck boards. How many will he need for a 272-in. long deck?

STATE YOUR UNDERSTANDING

- 68.** Explain how to simplify $5\frac{1}{4} \div 1\frac{7}{8}$.
- 69.** When a number is multiplied by $1\frac{1}{2}$, the result is larger than the original number. But when you divide by $1\frac{1}{2}$ the result is smaller. Explain why.
- 70.** Why is it helpful to change mixed numbers to improper fractions before multiplying or dividing?

CHALLENGE

- 71.** Multiply and simplify completely:

$$\left(\frac{2}{15}\right)\left(1\frac{6}{7}\right)\left(2\frac{2}{49}\right)\left(16\frac{2}{5}\right)\left(8\frac{3}{4}\right)\left(5\frac{1}{4}\right)\left(3\frac{3}{13}\right)$$

- 72.** The Celtic Candy Company has two packs of mints that they sell in discount stores. One pack contains $1\frac{1}{4}$ lb of mints and the other contains $3\frac{1}{3}$ lb of mints. If the smaller pack sells for \$2 and the larger pack for \$5, which size should they use to get the most income from 3000 lb of mints? How much more is the income?

GROUP WORK

- 73.** Some people say that mixed numbers are no longer useful because of the ease of working with calculator approximations. See how many examples of the use of mixed numbers your group can find. Share the results with the class.

- 74.** With your group members, find a way to multiply two mixed numbers without changing them to improper fractions. Describe your method to the class.

MAINTAIN YOUR SKILLS

Change to a mixed number.

75. $\frac{65}{8}$

76. $\frac{553}{15}$

Change to an improper fraction.

77. $\frac{17}{15}$

78. $\frac{37}{15}$

79. $\frac{66}{5}$

Multiply.

80. $7200\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)$

81. $60\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{5280}{1}\right)$

82. $25\left(\frac{1}{12}\right)\left(\frac{1}{12}\right)\left(\frac{15}{1}\right)$

- 83.** To be eligible for a drawing at the Flick Film Mall, your ticket stub number must be a multiple of 3. If Jean's ticket number is 234572, is she eligible for the drawing?

- 84.** Sales for the Fireyear Company totaled \$954,000 last year. During the first 6 months of last year, monthly sales were \$72,400, \$68,200, \$85,000, \$89,500, \$92,700, and \$87,200. What were the average monthly sales for the rest of the year?

Getting Ready for Algebra

How & Why

We have previously solved equations in which variables (letters) are either multiplied or divided by whole numbers. We performed the inverse operations to solve for the variable. To eliminate multiplication, we divide by the number being multiplied. To eliminate division, we multiply by the number that is the divisor. Now we solve equations in which variables are multiplied by fractions. Recall from Chapter 1 that if a number is multiplied by a variable, there is usually no multiplication sign between them. For example, $2x$ is understood to mean 2 times x , and $\frac{2}{3}x$ means $\frac{2}{3}$ times x . However, we usually do not write $\frac{2}{3}x$. Instead, we write this as $\frac{2x}{3}$. We can do this because

$$\frac{2}{3}x = \frac{2}{3} \cdot x = \frac{2}{3} \cdot \frac{x}{1} = \frac{2x}{3}$$

While we usually write $\frac{2x}{3}$ instead of $\frac{2}{3}x$, for convenience we may use either of these forms. Recall that $\frac{2x}{3}$ means the product of 2 and x divided by 3.

OBJECTIVE

Solve an equation of the form $\frac{ax}{b} = \frac{c}{d}$ where $a, b, c,$ and d are whole numbers.

To solve an equation of the form $\frac{ax}{b} = \frac{c}{d}$

1. Multiply both sides of the equation by b to eliminate the division on the left side.
2. Divide both sides by a to isolate the variable.

Examples A–C

DIRECTIONS: Solve

STRATEGY: Multiply both sides by the denominator of the fraction containing the variable. Solve as before.

A. Solve: $\frac{3x}{4} = 2$

$$4\left(\frac{3x}{4}\right) = 4(2) \quad \text{To eliminate the division, multiply both sides by 4.}$$

$$3x = 8 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{8}{3} \quad \text{To eliminate the multiplication, divide both sides by 3.}$$

$$x = \frac{8}{3}$$

CHECK: $\frac{3}{4} \cdot \frac{8}{3} = 2 \quad \text{Substitute } \frac{8}{3} \text{ for } x \text{ in the original equation. Recall that } \frac{3x}{4} = \frac{3}{4}x.$
 $2 = 2$

The solution is $x = \frac{8}{3}$ or $x = 2\frac{2}{3}$.

Warm-Ups A–C

A. Solve: $\frac{6x}{7} = 12$

Answers to Warm-Ups

A. $x = 14$

B. Solve: $\frac{5x}{8} = \frac{3}{2}$

B. Solve: $\frac{2}{3} = \frac{4x}{5}$

$$5\left(\frac{2}{3}\right) = 5\left(\frac{4x}{5}\right)$$

$$\frac{10}{3} = 4x$$

$$\frac{10}{3(4)} = \frac{4x}{4}$$

$$\frac{10}{12} = x$$

The solution is $x = \frac{5}{6}$.

To eliminate the division by 5, multiply both sides by 5.

To eliminate the multiplication by 4, divide both sides by 4.

The check is left for the student.

- C.** It requires about two-fifths the amount of energy to make “new” paper from recycled paper as from trees. If the amount of energy needed to make a given amount of paper from recycled paper is equivalent to 1500 BTUs, how much energy is needed to make the same amount from trees?

- C.** Commuting to work accounts for one-third of the mileage put on a private car. If Nancy averages 510 miles per month in commuting to work, how many miles does she put on her car in a month?

STRATEGY: First write the English statement of the equation.

One-third times the total number of miles = miles of commute

$$\frac{1}{3}x = 510 \quad \text{Let } x \text{ represent the number of total miles. Miles of commute is 510.}$$

$$3\left(\frac{1}{3}x\right) = 3(510) \quad \text{Multiply both sides by 3 to eliminate the division.}$$

$$x = 1530$$

CHECK: Is one-third of 1530 equal to 510?

$$\frac{1}{3}(1530) = 510 \quad \text{Yes}$$

Nancy puts 1530 miles on her car per month.

Answers to Warm-Ups

B. $x = \frac{12}{5}$ or $x = 2\frac{2}{5}$

- C.** It would take 3750 BTUs

Exercises

Solve.

1. $\frac{2x}{3} = \frac{1}{2}$

2. $\frac{2x}{3} = \frac{2}{5}$

3. $\frac{3y}{5} = \frac{2}{3}$

4. $\frac{3y}{4} = \frac{5}{8}$

5. $\frac{4z}{5} = \frac{1}{4}$

6. $\frac{5z}{4} = \frac{5}{6}$

7. $\frac{17}{9} = \frac{8x}{9}$

8. $\frac{29}{10} = \frac{9x}{5}$

9. $\frac{7a}{4} = \frac{5}{2}$

10. $\frac{15b}{4} = \frac{13}{5}$

11. $\frac{47}{8} = \frac{47b}{12}$

12. $\frac{13}{3} = \frac{52w}{9}$

13. $\frac{35z}{6} = \frac{35}{12}$

14. $\frac{5b}{18} = \frac{2}{9}$

15. $\frac{15a}{2} = \frac{11}{4}$

16. $\frac{119x}{12} = \frac{119}{8}$

17. Vince walks $\frac{2}{3}$ of the distance from his home to school. If he walks $\frac{1}{2}$ mi, what is the distance from his home to school?

19. In a southern city, $3\frac{1}{2}$ times more pounds of glass are recycled than tin. How many pounds of tin are recycled when 630 lb of glass are recycled?

18. Mona cut a board into seven pieces of equal length. If each piece is $1\frac{4}{7}$ ft long, what was the length of the board?

20. Washing machines use about $\frac{7}{50}$ of all the water consumed in the home. If Matthew uses 280 gallons of water per month to operate his washing machine, how many gallons of water does he use in a month?

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3.5 Building Fractions; Listing in Order; Inequalities

VOCABULARY

Recall that **equivalent fractions** are fractions that are different names for the same number. For instance, $\frac{4}{8}$ and $\frac{11}{22}$ are equivalent because both represent one-half ($\frac{1}{2}$) of a unit. Two or more fractions have a **common denominator** when they have the same denominator.

OBJECTIVES

1. Rename fractions by multiplying by 1 in the form $\frac{a}{a}$.
2. Build a fraction by finding the missing numerator.
3. List a group of fractions from smallest to largest.

How & Why

OBJECTIVE 1

Rename fractions by multiplying by 1 in the form $\frac{a}{a}$.

The process of renaming fractions is often referred to as “building fractions.” Building a fraction means renaming the fraction by multiplying both numerator and denominator by a common factor. This process is often necessary when we add and subtract fractions. We “build fractions” to a common denominator so they can be compared, added, or subtracted. Building fractions is the opposite of “simplifying” fractions.

Simplifying a Fraction	Building a Fraction
$\frac{8}{10} = \frac{4 \cdot 2}{5 \cdot 2} = \frac{4}{5}$	$\frac{4}{5} = \frac{4 \cdot 2}{5 \cdot 2} = \frac{8}{10}$

Visually, in Figure 3.13 we have

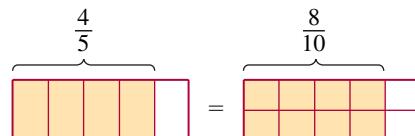


Figure 3.13

Table 3.3 shows five fractions built to equivalent fractions.

Table 3.3 Equivalent Fractions

Multiply numerator and denominator by						
	2	3	4	6	10	15
$\frac{2}{5}$	$= \frac{4}{10}$	$= \frac{6}{15}$	$= \frac{8}{20}$	$= \frac{12}{30}$	$= \frac{20}{50}$	$= \frac{30}{75}$
$\frac{1}{2}$	$= \frac{2}{4}$	$= \frac{3}{6}$	$= \frac{4}{8}$	$= \frac{6}{12}$	$= \frac{10}{20}$	$= \frac{15}{30}$
$\frac{5}{9}$	$= \frac{10}{18}$	$= \frac{15}{27}$	$= \frac{20}{36}$	$= \frac{30}{54}$	$= \frac{50}{90}$	$= \frac{75}{135}$
$\frac{7}{8}$	$= \frac{14}{16}$	$= \frac{21}{24}$	$= \frac{28}{32}$	$= \frac{42}{48}$	$= \frac{70}{80}$	$= \frac{105}{120}$
$\frac{7}{4}$	$= \frac{14}{8}$	$= \frac{21}{12}$	$= \frac{28}{16}$	$= \frac{42}{24}$	$= \frac{70}{40}$	$= \frac{105}{60}$

To rename a fraction

Multiply both the numerator and the denominator of the fraction by a common factor; that is, multiply the fraction by 1 in the form $\frac{a}{a}$, $a \neq 1$ and $a \neq 0$.

Warm-Ups A–B

A. Rename $\frac{5}{12}$, using $\frac{8}{8}$ for 1.

B. Write three fractions equivalent to $\frac{6}{7}$, using $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$.

Examples A–B



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DIRECTIONS: Rename the fraction.

STRATEGY: Multiply the fraction by 1 in the form $\frac{a}{a}$

A. Rename $\frac{7}{10}$, using $\frac{5}{5}$ for 1.

$$\frac{7}{10} \cdot \frac{5}{5} = \frac{35}{50} \quad \text{The new fraction, } \frac{35}{50}, \text{ is equivalent to } \frac{7}{10}.$$

B. Write three fractions equivalent to $\frac{5}{6}$, using $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$.

$$\frac{5}{6} \cdot \frac{3}{3} = \frac{15}{18}$$

$$\frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24}$$

$$\frac{5}{6} \cdot \frac{5}{5} = \frac{25}{30}$$

How & Why

OBJECTIVE 2

Build a fraction by finding the missing numerator.

To find the missing numerator in

$$\frac{4}{5} = \frac{?}{30}$$

divide 30 by 5 to find out what form of 1 to multiply by.

$$30 \div 5 = 6.$$

The correct factor (multiplier) is $\frac{6}{6}$. So

$$\frac{4}{5} = \frac{4}{5} \cdot \frac{6}{6} = \frac{24}{30}$$

The shortcut is to write 30, the target denominator. Then multiply the original numerator, 4, by 6, the quotient of the target denominator and the original denominator.

$$\frac{4}{5} = \frac{4 \cdot 6}{30} = \frac{24}{30}$$

The fractions $\frac{4}{5}$ and $\frac{24}{30}$ are equivalent. Either fraction can be used in place of the other.

Answers to Warm-Ups

A. $\frac{40}{96}$ B. $\frac{12}{14}, \frac{18}{21}, \frac{24}{28}$

To find the missing numerator when building fractions

1. Divide the target denominator by the original denominator.
2. Multiply this quotient by the original numerator.

Examples C–E



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DIRECTIONS: Find the missing numerator.

STRATEGY: Multiply the given numerator by the same factor used in the denominator.

C. Find the missing numerator: $\frac{3}{5} = \frac{?}{80}$

$$80 \div 5 = 16 \quad \text{Divide the denominators.}$$

$$\frac{3}{5} = \frac{3(16)}{80} = \frac{48}{80} \quad \text{Multiply the quotient, 16, by the given numerator.}$$

D. Find the missing numerator. $\frac{7}{9} = \frac{?}{54}$

$$54 \div 9 = 6 \quad \text{Divide the denominators.}$$

$$\frac{7}{9} = \frac{7(6)}{54} = \frac{42}{54} \quad \text{Multiply the quotient, 6, by the given numerator.}$$

E. Find the missing numerator: $\frac{8}{5} = \frac{?}{140}$

$$140 \div 5 = 28$$

$$\frac{8}{5} = \frac{8(28)}{140} = \frac{224}{140}$$

Warm-Ups C–E

C. Find the missing numerator:

$$\frac{3}{4} = \frac{?}{80}$$

D. Find the missing numerator:

$$\frac{3}{7} = \frac{?}{56}$$

E. Find the missing numerator:

$$\frac{19}{15} = \frac{?}{135}$$

How & Why

OBJECTIVE 3

List a group of fractions from smallest to largest.

If two fractions have the same denominator, the one with the smaller numerator has the smaller value. Figure 3.14 shows that $\frac{2}{5}$ is smaller than $\frac{4}{5}$; that is, $\frac{2}{5} < \frac{4}{5}$.

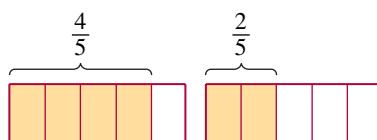


Figure 3.14

$\frac{2}{5} < \frac{4}{5}$ means “two-fifths is less than four-fifths.”

$\frac{4}{5} > \frac{2}{5}$ means “four-fifths is greater than two-fifths.”

Answers to Warm-Ups

C. 60 D. 24 E. 171

If fractions to be compared do not have a common denominator, then one or more can be renamed so that all have a common denominator. The preferred common denominator is the least common multiple (LCM) of all the denominators.

To list $\frac{3}{8}$, $\frac{5}{16}$, $\frac{1}{2}$, and $\frac{9}{16}$ from smallest to largest, we write each with a common denominator. Then we compare the numerators. The LCM of the denominators is 16. We build each fraction so that it has a denominator of 16.

$$\frac{3}{8} = \frac{6}{16} \quad \frac{5}{16} = \frac{5}{16} \quad \frac{1}{2} = \frac{8}{16} \quad \frac{9}{16} = \frac{9}{16} \quad \text{Each fraction now has a denominator of 16.}$$

We arrange the fractions with denominator 16 in order from smallest to largest according the values of the numerators.

$$\frac{5}{16} < \frac{6}{16} < \frac{8}{16} < \frac{9}{16} \quad \text{The fractions are listed in order from the smallest to the largest with common denominator, 16.}$$

Next, replace each fraction by the original, so

$$\frac{5}{16} < \frac{3}{8} < \frac{1}{2} < \frac{9}{16} \quad \text{The original fractions are listed in order from smallest to largest.}$$

To list fractions from smallest to largest

1. Build the fractions so that they have a common denominator. Use the LCM of the denominators.
2. List the fractions with common denominators so the numerators range from smallest to largest.
3. Simplify.

Warm-Ups F–K

F. Which is larger $\frac{7}{13}$ or $\frac{3}{5}$?

Answers to Warm-Ups

E. $\frac{3}{5}$

Examples F–K



BASIC Mathematics Now™

DIRECTIONS: Tell which fraction is larger.

STRATEGY: Write the fractions with a common denominator. The fraction with the larger numerator is the larger.

F. Which is larger $\frac{5}{11}$ or $\frac{1}{2}$?

STRATEGY: The LCM of 11 and 2 is 22. Build each fraction so it has 22 for a denominator.

$$\frac{5}{11} = \frac{10}{22} \quad \text{and} \quad \frac{1}{2} = \frac{11}{22}$$

$\frac{1}{2}$ is larger. $11 > 10$, so $\frac{1}{2} > \frac{5}{11}$.

DIRECTIONS: List the group of fractions from smallest to largest.

STRATEGY: Build each of the fractions to a common denominator. List the fractions from smallest to largest by the value of the numerator. Simplify.

- G. List from smallest to largest: $\frac{5}{12}$, $\frac{1}{3}$, and $\frac{3}{8}$.

$$\frac{5}{12} = \frac{10}{24} \quad \frac{1}{3} = \frac{8}{24} \quad \frac{3}{8} = \frac{9}{24}$$

The LCM of 3, 8, and 12 is 24. Build the fractions to the denominator 24.

$$\frac{8}{24}, \frac{9}{24}, \frac{10}{24}$$

List the fractions in the order of the numerators: $8 < 9 < 10$.

The list is $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{5}{12}$. Simplify.

- H. List from smallest to largest: $2\frac{3}{8}$, $2\frac{2}{5}$, and $2\frac{3}{10}$.

STRATEGY: Each mixed number has the same whole-number part. Compare the fraction parts.

$$2\frac{3}{8} = 2\frac{15}{40} \quad 2\frac{2}{5} = 2\frac{16}{40} \quad 2\frac{3}{10} = 2\frac{12}{40}$$

The LCM of 8, 5, and 10 is 40.

$$2\frac{12}{40}, 2\frac{15}{40}, 2\frac{16}{40}$$

List the numbers in the order of the numerators from smallest to largest.

The list is $2\frac{3}{10}$, $2\frac{3}{8}$, and $2\frac{2}{5}$. Simplify.

- I. The Acme Hardware Store sells bolts with diameters of $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{1}{4}$, and $\frac{7}{16}$ in. List the diameters from smallest to largest.

$$\frac{5}{16} = \frac{5}{16} \quad \frac{3}{8} = \frac{6}{16} \quad \frac{1}{2} = \frac{8}{16}$$

Write each diameter using the common denominator 16.

$$\frac{5}{8} = \frac{10}{16} \quad \frac{1}{4} = \frac{4}{16} \quad \frac{7}{16} = \frac{7}{16}$$

$$\frac{4}{16}, \frac{5}{16}, \frac{6}{16}, \frac{7}{16}, \frac{8}{16}, \frac{10}{16}$$

List the diameters in order of the numerators from smallest to largest.

From smallest to largest, the diameters are

$$\frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \text{ and } \frac{5}{8} \text{ in.}$$

Simplify.

DIRECTIONS: Tell whether the statement is true or false.

STRATEGY: Build both fractions to a common denominator and compare the numerators.

- J. True or false: $\frac{4}{5} > \frac{7}{9}$?

$$\frac{4}{5} = \frac{36}{45} \quad \frac{7}{9} = \frac{35}{45}$$

The common denominator is 45.

The statement is true. Compare the numerators.

- K. True or false: $\frac{18}{35} < \frac{12}{25}$?

$$\frac{18}{35} = \frac{90}{175} \quad \frac{12}{25} = \frac{84}{175}$$

The common denominator is 175.

The statement is false. Compare the numerators.

- G. List from smallest to largest:

$$\frac{2}{3}, \frac{5}{7}, \text{ and } \frac{9}{14}$$

- H. List from smallest to largest:

$$3\frac{4}{9}, 3\frac{2}{3}, \text{ and } 3\frac{3}{5}$$

- I. The Acme Hardware Store sells “rebar” with diameters

$$\frac{7}{8}, \frac{5}{16}, \frac{17}{32}, \frac{17}{64}, \text{ and } \frac{5}{12}$$

List the diameters from smallest to largest.

- J. True or false: $\frac{2}{15} > \frac{3}{20}$?

- K. True or false: $\frac{13}{16} < \frac{23}{27}$?

Answers to Warm-Ups

G. $\frac{9}{14}, \frac{2}{3}, \frac{5}{7}$ H. $3\frac{4}{9}, 3\frac{3}{5}, 3\frac{2}{3}$

I. $\frac{17}{64}, \frac{5}{16}, \frac{5}{12}, \frac{17}{32}, \frac{7}{8}$ J. false

K. true

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Exercises 3.5

OBJECTIVE 1

Rename fractions by multiplying by 1 in the form $\frac{a}{a}$.

A Write four fractions equivalent to each of the given fractions by multiplying by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$.

1. $\frac{2}{3}$

2. $\frac{3}{5}$

3. $\frac{1}{6}$

4. $\frac{5}{8}$

5. $\frac{4}{9}$

6. $\frac{3}{10}$

B

7. $\frac{11}{13}$

8. $\frac{11}{15}$

9. $\frac{7}{3}$

10. $\frac{6}{5}$

OBJECTIVE 2

Build a fraction by finding the missing numerator.

A Find the missing numerator.

11. $\frac{1}{2} = \frac{?}{14}$

12. $\frac{3}{4} = \frac{?}{28}$

13. $\frac{4}{7} = \frac{?}{42}$

14. $\frac{7}{12} = \frac{?}{96}$

15. $\frac{4}{5} = \frac{?}{35}$

16. $\frac{7}{8} = \frac{?}{32}$

17. $\frac{1}{12} = \frac{?}{36}$

18. $\frac{5}{6} = \frac{?}{24}$

19. $\frac{1}{5} = \frac{?}{75}$

20. $\frac{5}{9} = \frac{?}{45}$

B

21. $\frac{?}{12} = \frac{1}{3}$

22. $\frac{?}{88} = \frac{7}{22}$

23. $\frac{19}{6} = \frac{?}{12}$

24. $\frac{11}{5} = \frac{?}{100}$

25. $\frac{?}{300} = \frac{4}{75}$

26. $\frac{2}{3} = \frac{?}{108}$

27. $\frac{?}{84} = \frac{19}{42}$

28. $\frac{?}{126} = \frac{5}{6}$

29. $\frac{13}{18} = \frac{?}{144}$

30. $\frac{13}{16} = \frac{?}{144}$

OBJECTIVE 3

List a group of fractions from smallest to largest.

A List the fractions from smallest to largest.

31. $\frac{6}{13}, \frac{2}{13}, \frac{4}{13}$

32. $\frac{6}{17}, \frac{5}{17}, \frac{3}{17}$

33. $\frac{5}{8}, \frac{3}{4}, \frac{1}{2}$

34. $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$

35. $\frac{1}{2}, \frac{3}{8}, \frac{1}{3}$

36. $\frac{2}{3}, \frac{8}{15}, \frac{3}{5}$

Are the following statements true or false?

37. $\frac{2}{7} < \frac{6}{7}$

38. $\frac{7}{9} > \frac{2}{9}$

39. $\frac{11}{16} > \frac{7}{8}$

40. $\frac{9}{16} > \frac{5}{8}$

41. $\frac{11}{24} < \frac{7}{12}$

42. $\frac{11}{10} < \frac{8}{9}$

B List the fractions from smallest to largest.

43. $\frac{3}{5}, \frac{3}{7}, \frac{2}{3}$

44. $\frac{3}{8}, \frac{3}{10}, \frac{2}{5}$

45. $\frac{13}{15}, \frac{4}{15}, \frac{5}{6}, \frac{9}{10}$

46. $\frac{3}{4}, \frac{8}{9}, \frac{5}{6}, \frac{2}{3}$

47. $\frac{11}{24}, \frac{17}{36}, \frac{35}{72}$

48. $\frac{8}{25}, \frac{31}{50}, \frac{59}{100}$

49. $\frac{13}{28}, \frac{17}{35}, \frac{3}{7}$

50. $\frac{11}{15}, \frac{17}{20}, \frac{3}{4}$

51. $1\frac{9}{16}, 1\frac{13}{20}, 1\frac{5}{8}$

52. $5\frac{14}{15}, 5\frac{19}{20}, 5\frac{11}{12}$

Are the following statements true or false?

53. $\frac{47}{80} < \frac{5}{8}$

54. $\frac{8}{25} < \frac{59}{100}$

55. $\frac{3}{4} > \frac{11}{15}$

56. $\frac{19}{40} > \frac{31}{60}$

57. $\frac{11}{30} < \frac{17}{18}$

58. $\frac{11}{27} > \frac{29}{36}$

C

59. Find the LCM of the denominators of
- $\frac{1}{2}, \frac{2}{3}, \frac{1}{6}$
- , and
- $\frac{5}{8}$
- .

Build the four fractions so that each has the LCM as the denominator.

60. Find the LCM of the denominators of
- $\frac{1}{4}, \frac{4}{13}$
- , and
- $\frac{5}{26}$
- .

Build the three fractions so that each has the LCM as the denominator.

- 61.** The night nurse at Malcolm X Community Hospital finds bottles containing codeine tablets out of the usual order. The bottles contain tablets having the following strengths of codeine: $\frac{1}{8}$, $\frac{3}{32}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{9}{16}$, $\frac{1}{2}$, and $\frac{1}{4}$ grain, respectively. Arrange the bottles in order of the strength of codeine from the smallest to the largest.
- 63.** In the Republic of Niger, $\frac{2}{25}$ of the population is from the ethnic group of Tuareg and $\frac{17}{200}$ of the population is from the ethnic group of Fula. Is more of the population from Tuareg or Fula?
- 65.** According to the 2000 U.S. Census, $\frac{1}{8}$ of the U.S. population is of Hispanic origin and $\frac{129}{1000}$ of the U.S. population is African American. Which group makes up the larger share of the total population?
- 67.** Three chemistry students weigh a container of a chemical. Mary records the weight as $3\frac{1}{8}$ lb. George reads the weight as $3\frac{3}{16}$ lb. Chang reads the weight as $3\frac{1}{4}$ lb. Whose measurement is heaviest?
- 69.** Larry, Moe, and Curly bought a Subway franchise. Larry contributed $\frac{1}{4}$ of the costs, Moe contributed $\frac{7}{12}$ of the costs, and Curly contributed the remaining $\frac{1}{6}$. Which man owns the largest part of the business and which man owns the smallest?
- 62.** Joe, an apprentice, is given the task of sorting a bin of bolts according to their diameters. The bolts have the following diameters: $\frac{11}{16}$, $\frac{7}{8}$, $1\frac{1}{16}$, $\frac{3}{4}$, $1\frac{1}{8}$, and $1\frac{3}{32}$ in. How should he list the diameters from the smallest to the largest?
- 64.** According to the General Services Administration, the federal government owns about $\frac{1}{30}$ of the acreage in Alaska and about $\frac{6}{125}$ of the acreage of Kentucky. Which state has the larger portion of federally owned land?
- 66.** Islam is the second largest religion in the world, with more than a billion people practicing worldwide. Indonesia is the country with the largest number of Muslims, $\frac{22}{25}$ of its population. In Kuwait, $\frac{17}{20}$ of the population practices Islam. Which country has a larger portion of their total population practicing Islam?
- 68.** Three rulers are marked in inches. On the first ruler, the spaces are divided into tenths. On the second, they are divided into sixteenths, and on the third, they are divided into eighths. All are used to measure a line on a scale drawing. The nearest mark on the first ruler is $5\frac{7}{10}$, the nearest mark on the second is $5\frac{11}{16}$, and the nearest mark on the third is $5\frac{6}{8}$. Which is the largest (longest) measurement?

STATE YOUR UNDERSTANDING

- 70.** Explain why it is easier to compare the sizes of two fractions if they have common denominators.

- 71.** What is the difference between simplifying fractions and building fractions?

CHALLENGE

- 72.** List $\frac{12}{25}$, $\frac{14}{29}$, $\frac{29}{60}$, $\frac{35}{71}$, $\frac{39}{81}$, and $\frac{43}{98}$ from smallest to largest.

- 73.** Build $\frac{5}{7}$ so that it has denominators 70, 91, 161, 784, and 4067.

- 74.** Fernando and Filipe are hired to sell tickets for the holiday raffle. Fernando sells $\frac{14}{17}$ of his quota of 765 tickets. Filipe sells $\frac{19}{23}$ of his quota of 759 tickets. Who sells more of his quota? Who sells more tickets?

GROUP WORK

- 75.** We can see that $\frac{2}{5}$ is less than $\frac{4}{5}$ by looking at a rectangle representing each fraction. Show the sum, $\frac{2}{5} + \frac{4}{5}$, visually using a rectangle divided into five parts. Similarly, show $\frac{2}{7} + \frac{3}{7}$.

MAINTAIN YOUR SKILLS

Find the LCM of the denominators of each set of fractions.

76. $\frac{1}{2}, \frac{1}{5}$

77. $\frac{1}{8}, \frac{3}{4}$

78. $\frac{5}{8}, \frac{3}{10}$

79. $\frac{5}{6}, \frac{1}{10}$

80. $\frac{11}{96}, \frac{35}{72}$

81. $\frac{1}{32}, \frac{3}{16}, \frac{3}{4}$

- 82.** Find the prime factorization of 96

- 83.** Find the prime factorization of 72

- 84.** The Forest Service rents a two-engine plane at \$625 per hour and a single-engine plane at \$365 per hour to drop fire retardant. During a forest fire, the two-engine plane was used for 4 hr and the single-engine plane was used for 2 hr. What was the cost of using the two planes?

- 85.** Ms. Wallington is taking one capsule containing 250 mg of a drug every 8 hr. Beginning next Wednesday, her doctor's instructions are to increase the dosage to 500 mg every 6 hr. How many 250-mg capsules should the pharmacist give her for the following week (7 days)?

3.6 Adding Fractions

VOCABULARY

Like fractions are fractions with common denominators. **Unlike fractions** are fractions with different denominators.

OBJECTIVES

1. Add like fractions.
2. Add unlike fractions.

How & Why

OBJECTIVE 1

Add like fractions.

What is the sum of $\frac{1}{5}$ and $\frac{2}{5}$? The denominators tell us the number of parts in the unit. The numerators tell us how many of these parts are shaded. By adding the numerators we find the total number of shaded parts. The common denominator keeps track of the size of the parts. See Figure 3.15.

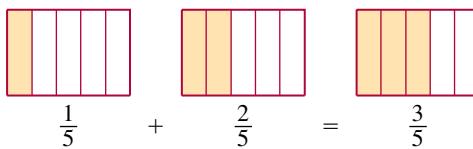


Figure 3.15

To add like fractions

1. Add the numerators.
2. Write the sum over the common denominator.
3. Simplify.

CAUTION

Do not add the denominators.

$$\frac{2}{5} + \frac{2}{5} \neq \frac{4}{10}$$

Examples A–D



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DIRECTIONS: Add and simplify.

STRATEGY: Add the numerators and write the sum over the common denominator.
Simplify.

A. Add: $\frac{3}{7} + \frac{2}{7}$

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$

Add the numerators. Keep the common denominator.

Warm-Ups A–D

A. Add: $\frac{3}{11} + \frac{5}{11}$

Answers to Warm-Ups

A. $\frac{8}{11}$

B. Add: $\frac{5}{6} + \frac{5}{6} + \frac{1}{6}$

C. Add: $\frac{1}{10} + \frac{1}{10} + \frac{3}{10}$

D. According to a report from Leed Community College, $\frac{3}{14}$ of their total revenues were from alumni contributions and $\frac{9}{14}$ were from tuition. What portion of the total revenues were from these two sources?

B. Add: $\frac{3}{4} + \frac{3}{4} + \frac{1}{4}$

$$\frac{3}{4} + \frac{3}{4} + \frac{1}{4} = \frac{7}{4}$$

$$= 1\frac{3}{4}$$

Add.

Write as a mixed number.

C. Add: $\frac{1}{9} + \frac{1}{9} + \frac{4}{9}$

$$\frac{1}{9} + \frac{1}{9} + \frac{4}{9} = \frac{6}{9}$$

$$= \frac{2}{3}$$

Add.

Simplify.

D. According to the annual report from Radd Community College, $\frac{11}{20}$ of their total revenues were from state taxes and $\frac{7}{20}$ were from tuition and fees. What portion of the total revenues were from these two sources?

$$\frac{11}{20} + \frac{7}{20} = \frac{18}{20}$$

Add.

$$= \frac{9}{10}$$

Simplify.

Taxes and tuition and fees were $\frac{9}{10}$ of the revenue for the school.

How & Why

OBJECTIVE 2

Add unlike fractions.

The sum $\frac{1}{2} + \frac{1}{5}$ cannot be found in this form. A look at Figure 3.16 shows that the parts are not the same size.

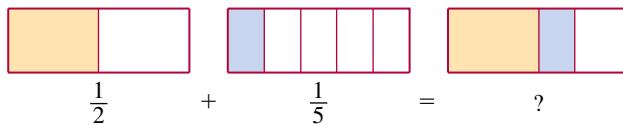


Figure 3.16

To add, rename $\frac{1}{2}$ and $\frac{1}{5}$ as like fractions. The LCM (least common multiple) of the two denominators serves as the least common denominator. The LCM of 2 and 5 is 10. Renaming the fractions we write

$$\frac{1}{2} = \left(\frac{1}{2}\right)\left(\frac{5}{5}\right) = \frac{5}{10} \quad \text{and} \quad \frac{1}{5} = \left(\frac{1}{5}\right)\left(\frac{2}{2}\right) = \frac{2}{10}$$

Figure 3.17 shows the regions with equal size parts.

Answers to Warm-Ups

B. $\frac{11}{6}$ or $1\frac{5}{6}$ C. $\frac{1}{2}$

D. Contributions and tuition were $\frac{6}{7}$ of the revenue for the school.

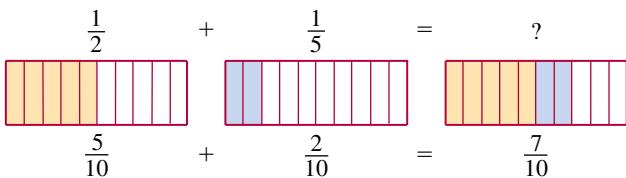


Figure 3.17

To add unlike fractions

1. Build the fractions so that they have a common denominator.
2. Add and simplify.

Examples E–J



BASIC Mathematics Now™

DIRECTIONS: Add and simplify.

STRATEGY: Build each of the fractions to a common denominator, add, and simplify.

E. Add: $\frac{1}{8} + \frac{3}{4}$

$$\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{6}{8} \quad \text{The LCM of 8 and 4 is 8. Build the fractions.}$$

$$= \frac{7}{8} \quad \text{Add.}$$

F. Add: $\frac{5}{8} + \frac{3}{10}$

$$\frac{5}{8} + \frac{3}{10} = \frac{25}{40} + \frac{12}{40} \quad \text{The LCM of 8 and 10 is 40; } \frac{5}{8} \cdot \frac{5}{5} = \frac{25}{40} \text{ and } \frac{3}{10} \cdot \frac{4}{4} = \frac{12}{40}.$$

$$= \frac{37}{40} \quad \text{Add.}$$

G. Add: $\frac{5}{6} + \frac{1}{10}$

$$\frac{5}{6} + \frac{1}{10} = \frac{25}{30} + \frac{3}{30} \quad \text{The LCM of 6 and 10 is 30.}$$

$$= \frac{28}{30} \quad \text{Add.}$$

$$= \frac{14}{15} \quad \text{Simplify.}$$

H. Add: $\frac{11}{96} + \frac{35}{72}$

STRATEGY: Find the prime factorization of the denominators to help find the LCM.

$$\frac{11}{96} + \frac{35}{72} = \frac{11(3)}{288} + \frac{35(4)}{288} \quad 96 = 2^5 \cdot 3 \text{ and } 72 = 2^3 \cdot 3^2 \text{ so the LCM is } 2^5 \cdot 3^2 = 288.$$

$$= \frac{33}{288} + \frac{140}{288}$$

$$= \frac{173}{288} \quad \text{Add.}$$

Warm-Ups E–J

E. Add: $\frac{1}{6} + \frac{5}{12}$

F. Add: $\frac{5}{12} + \frac{2}{9}$

G. Add: $\frac{3}{5} + \frac{3}{8}$

H. Add: $\frac{13}{45} + \frac{28}{75}$

Answers to Warm-Ups

- E. $\frac{7}{12}$ F. $\frac{23}{36}$ G. $\frac{39}{40}$
 H. $\frac{149}{225}$

CALCULATOR EXAMPLE

I. Add: $\frac{8}{35} + \frac{9}{28}$

- J. A nail must be long enough to reach through three thicknesses of wood and penetrate a fourth piece $\frac{1}{4}$ in. If the first piece of wood is $\frac{5}{16}$ in. thick, the second is $\frac{3}{8}$ in. thick, and the third is $\frac{9}{16}$ in. thick, how long must the nail be?

I. Add: $\frac{41}{60} + \frac{13}{84}$

$$\frac{41}{60} + \frac{13}{84} = \frac{88}{105}$$

On a calculator with fraction keys, it is not necessary to find the common denominator. The calculator is programmed to add and simplify.

- J. Sheila is assembling a composting bin for her lawn and garden debris. She needs a bolt that will reach through a $\frac{1}{32}$ -in.-thick washer, a $\frac{3}{16}$ -in.-thick plastic bushing, a $\frac{3}{4}$ -in.-thick piece of steel tubing, a second $\frac{1}{32}$ -in.-thick washer, and a $\frac{1}{4}$ -in.-thick nut. How long a bolt does she need?

STRATEGY: Add the thicknesses of each part to find the total length needed.

$$\frac{1}{32} + \frac{3}{16} + \frac{3}{4} + \frac{1}{32} + \frac{1}{4}$$

The LCM of 32, 16, and 4 is 32.

$$\frac{1}{32} + \frac{6}{32} + \frac{24}{32} + \frac{1}{32} + \frac{8}{32}$$

Build the fractions to a common denominator.

$$= \frac{40}{32}$$

Add.

$$= \frac{5}{4} = 1\frac{1}{4}$$

Simplify and write as a mixed number.

The bolt must be $1\frac{1}{4}$ in. long.

Answers to Warm-Ups

I. $\frac{11}{20}$

J. The nail must be at least $1\frac{1}{2}$ in. long.

Exercises 3.6

OBJECTIVE 1

Add like fractions.

A Add. Simplify completely.

1. $\frac{4}{11} + \frac{5}{11}$

2. $\frac{5}{12} + \frac{2}{12}$

3. $\frac{1}{9} + \frac{4}{9} + \frac{1}{9}$

4. $\frac{1}{8} + \frac{2}{8} + \frac{3}{8}$

5. $\frac{3}{4} + \frac{5}{4}$

6. $\frac{6}{7} + \frac{8}{7}$

7. $\frac{5}{12} + \frac{4}{12} + \frac{1}{12}$

8. $\frac{4}{12} + \frac{3}{12} + \frac{8}{12}$

9. $\frac{4}{13} + \frac{5}{13} + \frac{1}{13}$

10. $\frac{5}{11} + \frac{2}{11} + \frac{1}{11}$

11. $\frac{5}{12} + \frac{5}{12} + \frac{5}{12}$

12. $\frac{9}{16} + \frac{7}{16} + \frac{4}{16}$

B

13. $\frac{3}{16} + \frac{3}{16} + \frac{2}{16}$

14. $\frac{7}{32} + \frac{8}{32} + \frac{5}{32}$

15. $\frac{5}{48} + \frac{7}{48} + \frac{3}{48}$

16. $\frac{3}{16} + \frac{2}{16} + \frac{5}{16}$

17. $\frac{7}{30} + \frac{11}{30} + \frac{3}{30}$

18. $\frac{5}{24} + \frac{7}{24} + \frac{9}{24}$

19. $\frac{3}{20} + \frac{3}{20} + \frac{9}{20}$

20. $\frac{7}{18} + \frac{2}{18} + \frac{7}{18}$

OBJECTIVE 2

Add unlike fractions.

A Add. Simplify completely.

21. $\frac{1}{6} + \frac{3}{8}$

22. $\frac{5}{12} + \frac{3}{8}$

23. $\frac{1}{8} + \frac{7}{24}$

24. $\frac{7}{15} + \frac{1}{3}$

25. $\frac{7}{16} + \frac{3}{8}$

26. $\frac{4}{9} + \frac{5}{18}$

27. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10}$

28. $\frac{1}{4} + \frac{2}{5} + \frac{3}{20}$

29. $\frac{1}{5} + \frac{1}{10} + \frac{1}{2}$

30. $\frac{2}{15} + \frac{2}{5} + \frac{1}{3}$

B

31. $\frac{16}{35} + \frac{6}{21}$

32. $\frac{1}{18} + \frac{1}{24}$

33. $\frac{3}{10} + \frac{9}{20} + \frac{11}{30}$

34. $\frac{7}{8} + \frac{7}{12} + \frac{1}{6}$

35. $\frac{1}{10} + \frac{2}{5} + \frac{5}{6} + \frac{1}{15}$

36. $\frac{1}{2} + \frac{3}{10} + \frac{3}{5} + \frac{1}{4}$

37. $\frac{1}{4} + \frac{3}{8} + \frac{1}{16} + \frac{1}{32}$

38. $\frac{15}{36} + \frac{1}{6} + \frac{5}{12} + \frac{1}{18}$

39. $\frac{5}{6} + \frac{5}{8} + \frac{3}{4} + \frac{5}{12}$

40. $\frac{7}{10} + \frac{4}{5} + \frac{1}{15} + \frac{31}{35}$

41. $\frac{2}{9} + \frac{1}{3} + \frac{25}{27} + \frac{7}{9}$

42. $\frac{4}{5} + \frac{62}{75} + \frac{2}{15} + \frac{9}{25}$

43. $\frac{7}{9} + \frac{4}{5} + \frac{4}{15} + \frac{11}{30}$

44. $\frac{11}{15} + \frac{7}{12} + \frac{9}{10} + \frac{17}{20}$

C

45. $\frac{5}{48} + \frac{7}{16} + \frac{1}{8}$

46. $\frac{3}{8} + \frac{7}{24} + \frac{1}{12}$

47. $\frac{17}{30} + \frac{11}{20} + \frac{13}{75}$

48. $\frac{25}{72} + \frac{19}{144} + \frac{1}{12}$

49. $\frac{25}{36} + \frac{19}{48}$

50. $\frac{72}{85} + \frac{69}{102}$

51. According to the annual report from George Fox University, $\frac{1}{8}$ of the total revenues were from contributions and $\frac{31}{50}$ were from tuition and fees. What portion of the total revenues were from these two sources?

53. Of the worldwide adherents to the Muslim faith, slightly more than $\frac{1}{4}$ of them are in Africa, while $\frac{7}{10}$ of them reside in Asia. What portion of the world's Muslims are on these two continents?

55. Find the error in the statement: $\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$. Correct the statement. Explain how you would avoid this error.

52. North America accounts for more than $\frac{9}{32}$ of the total petroleum consumption worldwide. Central and South America together account for another $\frac{1}{16}$ of the total petroleum consumption. Do the Americas consume more or less than half of the petroleum consumed worldwide?

54. The Republic of Panama has four major ethnic groups. The largest is Mestizo. About $\frac{7}{50}$ of the population is West Indian, $\frac{1}{10}$ of the population is white, and $\frac{6}{100}$ of the population is Amerindian. What portion of Panama's population is not Mestizo?

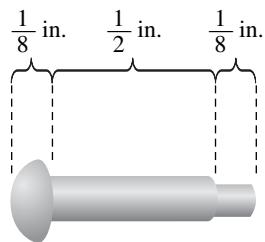
56. Find the error in the statement: $\frac{1}{2} + \frac{4}{7} = \frac{5}{14}$. Correct the statement. Explain how you would avoid this error.

- 57.** Chef Ramon prepares a punch for a wedding party.

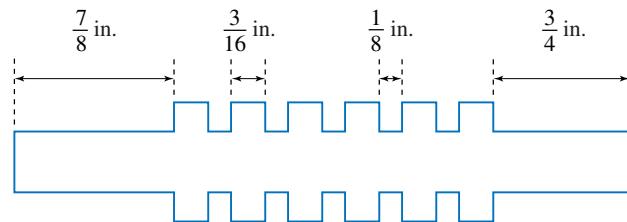
The punch calls for $\frac{1}{4}$ gallon of lemon juice, $\frac{3}{4}$ gallon of raspberry juice, $\frac{1}{2}$ gallon of cranberry juice, $\frac{1}{4}$ gallon of lime juice, $\frac{5}{4}$ gallons of 7-Up, and $\frac{3}{4}$ gallon of vodka. How many gallons of punch does the recipe make?

- 59.** Jonnie is assembling a rocking horse for his granddaughter. He needs a bolt to reach through a $\frac{7}{8}$ -in. piece of steel tubing, a $\frac{1}{16}$ -in. bushing, a $\frac{1}{2}$ -in. piece of tubing, a $\frac{1}{8}$ -in.-thick washer, and a $\frac{1}{4}$ -in.-thick nut. How long a bolt does he need?

- 61.** Find the length of this pin.



- 63.** Find the length of the rod in the figure. Assume that the grooves and teeth are uniform in length.

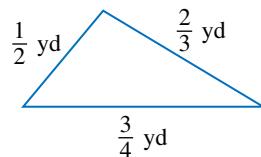


Exercise 65 relates to the chapter application. See page 195.

- 65.** One weekend Shane got his friend Mike to help him with the deck. Together they installed $\frac{1}{3}$ of the deck boards. The next weekend Shane's sister Carrie helped him and they installed $\frac{1}{2}$ of the deck boards. How much of the deck is installed after the two weekends?

- 58.** A physical therapist advises Belinda to swim $\frac{1}{4}$ mi on Monday and increase this distance by $\frac{1}{16}$ mi each day from Tuesday through Friday. What is the total number of miles she advises?

- 60.** What is the perimeter of this triangle?



- 62.** The Sandoz family spends $\frac{2}{15}$ of their income on rent, $\frac{1}{4}$ on food, $\frac{1}{20}$ on clothes, $\frac{1}{10}$ on transportation, and $\frac{5}{24}$ on taxes. What fraction of their income is spent on these costs?

- 64.** Elena walked $\frac{3}{8}$ mi from her house to the bus stop, then $\frac{1}{10}$ mi from where the bus let her off to the library. For lunch, she walked $\frac{1}{5}$ mi to a coffee shop. Elena then returned to the library, finished her research, and caught the bus home. How many miles did she walk on the entire trip?

STATE YOUR UNDERSTANDING

- 66.** Explain how to find the sum of $\frac{5}{12}$ and $\frac{3}{20}$.
- 67.** Why is it important to write fractions with a common denominator before adding?

CHALLENGE

- 68.** Find the sum of $\frac{107}{372}$ and $\frac{41}{558}$.
- 69.** Find the sum of $\frac{67}{124} + \frac{27}{868}$.
- 70.** Janet left $\frac{1}{7}$, $\frac{3}{14}$, and $\frac{1}{6}$ of her estate to Bob, Greta, and Joe Guerra. She also left $\frac{1}{8}$, $\frac{5}{16}$, and $\frac{1}{9}$ of the estate to Pele, Rhonda, and Shauna Contreras. Which family received the greater share of the estate?
- 71.** Jim is advised by his doctor to limit his fat intake. For breakfast, his fat intake is a bagel, $\frac{3}{4}$ g; a banana, $\frac{13}{16}$ g; cereal, $\frac{17}{10}$ g; milk, $\frac{9}{8}$ g; jelly, 0 g; and coffee, 0 g. Rounded to the nearest whole number, how many grams of fat does Jim consume at breakfast? If each gram of fat represents 9 calories and the total calories for breakfast is 330, what fraction represents the number of calories from fat? Use the rounded whole number of grams of fat.

GROUP WORK

- 72.** In the next section we add mixed numbers. Formulate a procedure for adding $3\frac{3}{4} + 5\frac{4}{5}$. Also, find two or three applications for which the sum of mixed numbers is necessary. Be prepared to share the results with the class.

MAINTAIN YOUR SKILLS

Add.

- | | | |
|--|---|--|
| 73. $2 + 8 + \frac{1}{4} + \frac{1}{6}$ | 74. $3 + 5 + \frac{2}{9} + \frac{7}{18}$ | 75. $1 + 7 + 10 + \frac{1}{6} + \frac{7}{10} + \frac{1}{5}$ |
| 76. $2 + 9 + 5 + \frac{2}{9} + \frac{5}{9} + \frac{2}{3}$ | 77. $3 + \frac{3}{8} + 2 + \frac{1}{8} + 1 + \frac{3}{16}$ | 78. $4 + \frac{1}{12} + 1 + \frac{3}{4} + 3 + \frac{1}{8}$ |

Perform the indicated operations.

- | | |
|---|--|
| 79. $(21 - 4)6 - 3(19 - 11)$ | 80. $(13 - 2^3)^3 - 7(11)$ |
| 81. Mrs. Teech has five classes to teach this term. The enrollment in the classes is 42, 36, 56, 32, and 34. What is the average class size? | 82. In a metal benchwork class that has 36 students, each student is allowed $11\frac{5}{8}$ in. of wire solder. How many inches of wire must the instructor provide for the class? |

3.7 Adding Mixed Numbers

How & Why

OBJECTIVE

Add mixed numbers.

What is the sum of $3\frac{1}{6}$ and $5\frac{1}{4}$? Pictorially we can show the sum by drawing rectangles such as those in Figure 3.18.

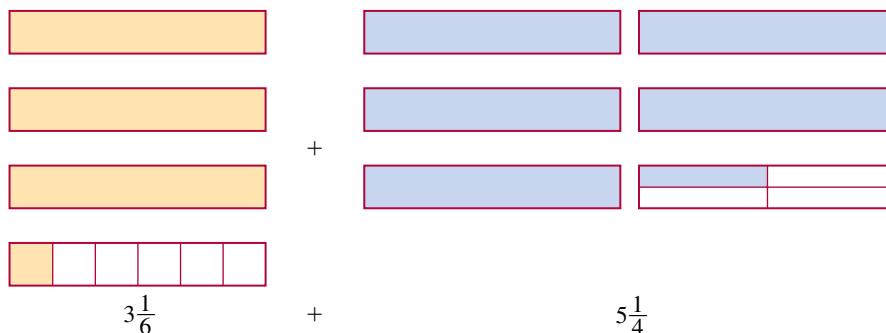


Figure 3.18

It is easy to see that the sum contains eight whole units. The sum of the fraction parts requires finding a common denominator. The LCM of 6 and 4 is 12. Figure 3.19 shows the divided rectangles.

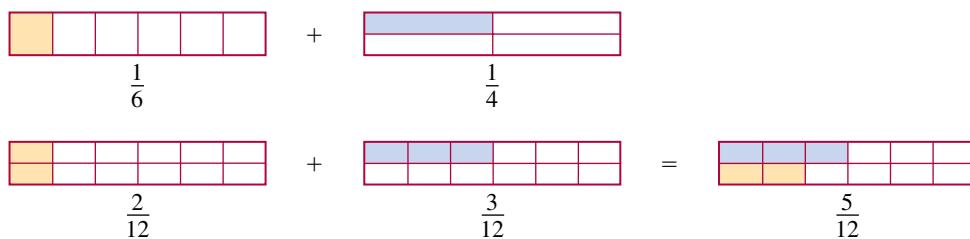


Figure 3.19

So the sum is $8\frac{5}{12}$.

Mixed numbers can be added horizontally or in columns. The sum of $3\frac{1}{6}$ and $5\frac{1}{4}$ is shown both ways.

$$\begin{aligned} \left(3 + \frac{1}{6}\right) + \left(5 + \frac{1}{4}\right) &= (3 + 5) + \left(\frac{1}{6} + \frac{1}{4}\right) \\ &= 8 + \left(\frac{2}{12} + \frac{3}{12}\right) \\ &= 8\frac{5}{12} \end{aligned}$$

When we write the sum vertically, the grouping of the whole numbers and the fractions takes place naturally.

$$\begin{array}{r} 3\frac{1}{6} = 3\frac{2}{12} \\ + 5\frac{1}{4} = 5\frac{3}{12} \\ \hline 8\frac{5}{12} \end{array}$$

OBJECTIVE

Add mixed numbers.

Sometimes the sum of the fractions is greater than 1. In this case, change the fraction sum to a mixed number and add it to the whole-number part.

$$\begin{array}{r} 17\frac{7}{10} = 17\frac{21}{30} \\ + 14\frac{8}{15} = 14\frac{16}{30} \\ \hline 31\frac{37}{30} \end{array}$$

Write the fractions with a common denominator.

Add the whole numbers and the fractions.

$$\begin{aligned} &= 31 + 1\frac{7}{30} \\ &= 32\frac{7}{30} \end{aligned}$$

Rewrite the improper fraction as a mixed number.

Add the mixed number to the whole number.

To add mixed numbers

1. Add the whole numbers.
2. Add the fractions. If the sum of the fractions is more than 1, change the fraction to a mixed number and add again.
3. Simplify.

Warm-Ups A–E

A. Add: $4\frac{1}{2} + 9\frac{5}{16}$

Examples A–E BASIC Mathematics Now™

DIRECTIONS: Add. Write as a mixed number.

STRATEGY: Add the whole numbers and add the fractions. If the sum of the fractions is an improper fraction, rewrite it as a mixed number and add to the sum of the whole numbers. Simplify.

A. Add: $3\frac{3}{4} + 2\frac{5}{12}$

STRATEGY: Write the mixed numbers in a column to group the whole numbers and to group the fractions.

$$\begin{array}{r} 3\frac{3}{4} = 3\frac{9}{12} \\ + 2\frac{5}{12} = 2\frac{5}{12} \\ \hline 5\frac{14}{12} \end{array}$$

Build the fractions to the common denominator 12.

Add.

$$\begin{aligned} 5\frac{14}{12} &= 5 + 1\frac{2}{12} \\ &= 6\frac{2}{12} \\ &= 6\frac{1}{6} \end{aligned}$$

Write the improper fraction as a mixed number.

Add the whole numbers.

Simplify.

Answers to Warm-Ups

A. $13\frac{13}{16}$

B. Add: $25\frac{7}{8} + 13\frac{5}{9} + 7\frac{1}{6}$

$$25\frac{7}{8} = 25\frac{63}{72}$$

The LCM of 8, 9, and 6 is 72.

$$13\frac{5}{9} = 13\frac{40}{72}$$

$$\begin{array}{r} + 7\frac{1}{6} \\ \hline 45\frac{115}{72} \end{array}$$

Add.

$$45\frac{115}{72} = 45 + 1\frac{43}{72}$$

$$= 46\frac{43}{72}$$

Write the improper fraction as a mixed number.

Add.

C. Add: $18 + 7\frac{4}{15}$

$$18 + 7\frac{4}{15} = 25\frac{4}{15}$$

Add the whole numbers.

CALCULATOR EXAMPLE



D. Add: $31\frac{3}{8} + 62\frac{13}{15}$

$$\begin{array}{r} 31\frac{3}{8} + 62\frac{13}{15} \\ \hline = 94\frac{29}{120} \end{array}$$

On a calculator with fraction keys, it is not necessary to find the common denominator. The calculator is programmed to add and simplify. Some calculators may not change the improper sum to a mixed number. See your calculator manual.

- E. A report by Environmental Hazards Management lists the following amounts of hazardous material that a city of 100,000 discharges into city drains each month:

$\frac{3}{4}$ tons of toilet bowl cleaner, $13\frac{3}{4}$ tons of liquid household cleaners, and $3\frac{2}{5}$ tons of motor oil. How many tons of these materials are discharged each month?

STRATEGY: Find the sum of the number of tons of hazardous material.

$$3\frac{3}{4} = 3\frac{15}{20}$$

The LCM of 4 and 5 is 20.

$$13\frac{3}{4} = 13\frac{15}{20}$$

$$\begin{array}{r} + 3\frac{2}{5} \\ \hline \end{array}$$

$$19\frac{38}{20}$$

Add.

$$19\frac{38}{20} = 19 + 1\frac{18}{20} = 20\frac{9}{10}$$

Change the improper fraction to a mixed number and simplify.

The residents discharge $20\frac{9}{10}$ tons of hazardous material each month.

B. Add: $12\frac{8}{9} + 14\frac{3}{4} + 6\frac{2}{3}$

C. Add: $21\frac{3}{11} + 19$

D. Add: $51\frac{7}{8} + 36\frac{5}{12}$

- E. The same report also records that the city recycles

$10\frac{1}{3}$ tons of paper, $3\frac{3}{8}$ tons of aluminum, and $4\frac{5}{12}$ tons of glass each month. How many tons of material are recycled each month?

Answers to Warm-Ups

B. $34\frac{11}{36}$ C. $40\frac{3}{11}$ D. $88\frac{7}{24}$

E. The city recycles $18\frac{1}{8}$ tons of material per month.

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Exercises 3.7

OBJECTIVE

Add mixed numbers.

A Add. Write the results as mixed numbers where possible.

1.
$$\begin{array}{r} 1\frac{4}{7} \\ + 3\frac{2}{7} \\ \hline \end{array}$$

2.
$$\begin{array}{r} 6\frac{1}{3} \\ + 2\frac{1}{3} \\ \hline \end{array}$$

3.
$$\begin{array}{r} 3\frac{4}{5} \\ + 7\frac{4}{5} \\ \hline \end{array}$$

4.
$$\begin{array}{r} 2\frac{5}{8} \\ + 3\frac{7}{8} \\ \hline \end{array}$$

5.
$$\begin{array}{r} 2\frac{7}{12} \\ + 3\frac{1}{4} \\ \hline \end{array}$$

6.
$$\begin{array}{r} 2\frac{5}{6} \\ + 7\frac{2}{3} \\ \hline \end{array}$$

7.
$$\begin{array}{r} 5\frac{5}{9} \\ + 2\frac{2}{3} \\ \hline \end{array}$$

8.
$$\begin{array}{r} 8\frac{5}{12} \\ + 7\frac{1}{2} \\ \hline \end{array}$$

9.
$$\begin{array}{r} 4\frac{4}{7} \\ + 2\frac{11}{14} \\ \hline \end{array}$$

10.
$$\begin{array}{r} 9\frac{7}{12} \\ + 3\frac{5}{6} \\ \hline \end{array}$$

11.
$$\begin{array}{r} 2\frac{8}{15} \\ + 7\frac{3}{5} \\ \hline \end{array}$$

12.
$$\begin{array}{r} 4\frac{7}{15} \\ + 6\frac{2}{3} \\ \hline \end{array}$$

13.
$$\begin{array}{r} 9\frac{3}{8} \\ + 5\frac{11}{16} \\ \hline \end{array}$$

14.
$$\begin{array}{r} 8\frac{5}{9} \\ + 2\frac{13}{27} \\ \hline \end{array}$$

15.
$$\begin{array}{r} 1\frac{1}{2} \\ 6\frac{1}{3} \\ + 3\frac{5}{6} \\ \hline \end{array}$$

16.
$$\begin{array}{r} 5\frac{2}{5} \\ 8\frac{7}{10} \\ + 1\frac{1}{2} \\ \hline \end{array}$$

17.
$$4\frac{1}{2} + 8\frac{3}{4} + 6 + 7\frac{3}{8}$$

18.
$$5\frac{2}{5} + 4 + 3\frac{2}{3} + 9\frac{7}{15}$$

B

19.
$$\begin{array}{r} 6\frac{3}{8} \\ + 4\frac{5}{6} \\ \hline \end{array}$$

20.
$$\begin{array}{r} 12\frac{4}{15} \\ + 3\frac{5}{6} \\ \hline \end{array}$$

21.
$$\begin{array}{r} 3\frac{1}{10} \\ 2\frac{3}{5} \\ + 4\frac{7}{15} \\ \hline \end{array}$$

22.
$$\begin{array}{r} 7\frac{1}{8} \\ 3\frac{5}{12} \\ + 8\frac{5}{6} \\ \hline \end{array}$$

23.
$$\begin{array}{r} 11\frac{7}{20} \\ + 9\frac{7}{30} \\ \hline \end{array}$$

24.
$$\begin{array}{r} 15\frac{7}{18} \\ + 21\frac{6}{27} \\ \hline \end{array}$$

25.
$$21\frac{5}{7} + 15\frac{9}{14} + 12\frac{10}{21}$$

26.
$$18\frac{3}{4} + 17\frac{7}{8} + 23\frac{1}{6}$$

27.
$$\begin{array}{r} 119\frac{11}{12} \\ + 217\frac{7}{18} \\ \hline \end{array}$$

28.
$$\begin{array}{r} 219\frac{3}{10} \\ + 308\frac{5}{6} \\ \hline \end{array}$$

29. $38\frac{2}{5} + 17\frac{1}{3} + 11\frac{1}{2}$

30. $26\frac{5}{8} + 33 + 41\frac{5}{6}$

31. $15\frac{3}{4} + 18\frac{2}{3} + 21\frac{1}{2}$

32. $39\frac{11}{12} + 49\frac{7}{8} + 59\frac{3}{4}$

33. $57 + 12\frac{7}{9} + 24\frac{4}{15}$

34. $38 + 14\frac{5}{6} + 8\frac{11}{14}$

35. $37\frac{18}{35} + 29\frac{9}{14} + 36$

36. $12\frac{11}{12} + 22\frac{5}{8} + 8$

37.
$$\begin{array}{r} 11\frac{1}{6} \\ 12\frac{3}{10} \\ 13\frac{1}{12} \\ + 14\frac{1}{20} \\ \hline \end{array}$$

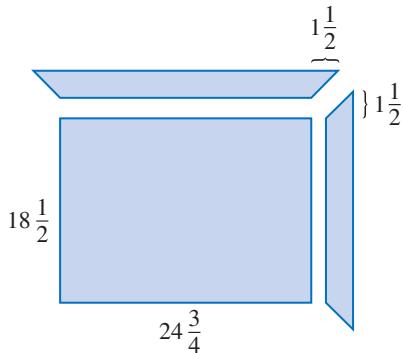
38.
$$\begin{array}{r} 11\frac{5}{18} \\ 21\frac{2}{9} \\ 31\frac{1}{3} \\ + 41\frac{1}{6} \\ \hline \end{array}$$

C

39. $3\frac{3}{5} + 2\frac{1}{6} + \frac{7}{15} + 8\frac{3}{10}$

41. $12\frac{3}{5} + 7\frac{1}{8} + 29\frac{3}{4} + 14\frac{9}{10}$

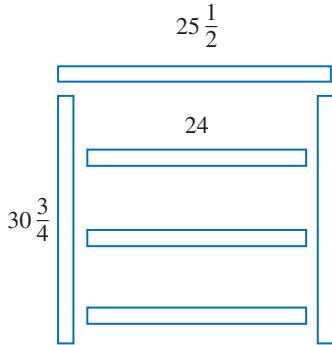
- 43.** Nancy painted a portrait of her daughters. The canvas measures $18\frac{1}{2}$ by $24\frac{3}{4}$ in. She plans to frame it with molding that will require an extra inch and a half on each end to make mitered corners, as in the figure. How much molding does Nancy need to frame her portrait?



40. $12\frac{3}{10} + 5\frac{2}{5} + 19\frac{2}{3} + \frac{11}{15}$

42. $14\frac{17}{25} + 4\frac{3}{5} + 25\frac{2}{15} + 10$

- 44.** Tom is making a bookshelf with six pieces of wood, as shown in the figure. He needs three pieces of wood for the shelves, which are 24 in. long, one piece of wood for the top, which is $25\frac{1}{2}$ in. long, and two pieces of wood for the vertical supports, which are $30\frac{3}{4}$ inches long. What is the total length of wood that Tom needs?



- 45.** Elizabeth is sewing her daughter's wedding dress. The bodice of the dress requires $1\frac{3}{8}$ yd of fabric, the skirt requires $6\frac{1}{2}$ yd, and the jacket requires $2\frac{3}{4}$ yd. How many yards of fabric does she need for the dress and jacket?

- 46.** Juanita worked the following hours at her part-time job during the month of October:

Week	Oct. 1–7	Oct. 8–14	Oct. 15–21	Oct. 22–28	Oct. 29–31
Hours	$25\frac{1}{2}$	$19\frac{2}{3}$	10	$16\frac{5}{6}$	$4\frac{3}{4}$

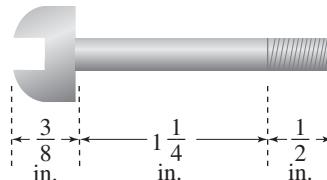
How many hours did she work during October?

- 47.** Scott is making seafood paella for a party. His recipe calls for $2\frac{1}{2}$ lb shrimp, 4 lb clams, $3\frac{1}{4}$ lb scallops, and $1\frac{1}{2}$ lb calamari. How much seafood does he need?

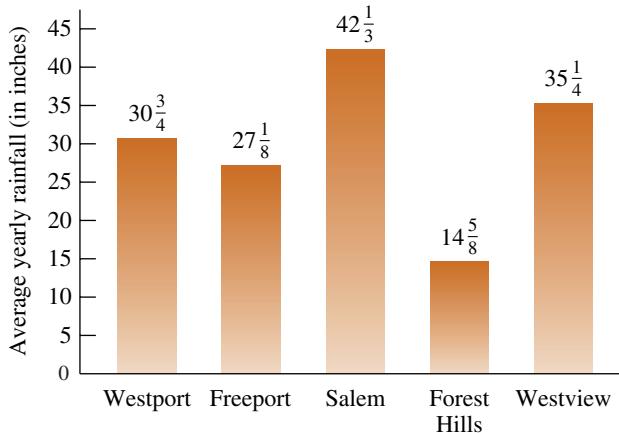
- 48.** Find the perimeter of a rectangle that has length $22\frac{1}{4}$ ft and width $16\frac{2}{3}$ ft.

- 49.** Find the perimeter of a triangle with sides $10\frac{5}{6}$ in., $14\frac{1}{4}$ in., and $17\frac{1}{12}$ in.

- 50.** What is the overall length of this bolt?



- 51.** The graph displays the average yearly rainfall for five cities.



What is the total amount of rain that falls in a given year in Salem, Forest Hills, and Westview?

- 52.** The rain gauge at the water reservoir recorded the following rainfall during a 6-month period:

- $1\frac{2}{3}$ in., $4\frac{5}{6}$ in., $3\frac{3}{4}$ in., $\frac{2}{3}$ in., $7\frac{7}{8}$ in., and $2\frac{1}{4}$ in. What is the total rainfall recorded during the 6 months? If an inch of rain means a gain of 2,400,000 gallons of water in the reservoir, what is the water gain during the 6 months?

- 53.** The State Department of Transportation must resurface parts of seven roads this summer. The distances to be paved are $6\frac{11}{16}$ mi, $8\frac{3}{5}$ mi, $9\frac{3}{4}$ mi, $17\frac{1}{2}$ mi, $5\frac{1}{8}$ mi, $12\frac{4}{5}$ mi, and $\frac{7}{8}$ mi. How many miles of highway are to be resurfaced? If it costs \$15,000 to resurface 1 mile, what is the cost of the resurfacing project? Round to the nearest thousand dollars.

Exercises 54–55 relate to the chapter application. See page 195.

The ledger board is a beam that supports the joists on one end. Shane is using 2×12 s for the joists. Beginning on one end, there is a pair of joists right next to each other that are called the rim joists (for extra strength at the edge of the deck). The rest of the joists are laid out “16 in. on center,” meaning that from the center of one joist to the center of the next joist is 16 in. See Figure 3.20.

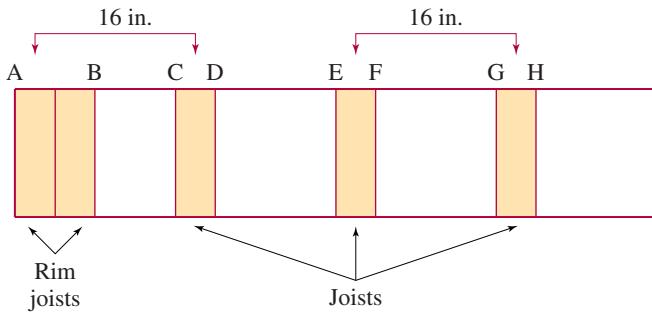


Figure 3.20 Ledger Beam Layout

- 54.** How long is it from point A to point B? See Table 3.1 on page 195 for the actual size of a 2×12 joist.

- 55.** How long is it from point A to point D?

STATE YOUR UNDERSTANDING

- 56.** Explain why it is sometimes necessary to rename the sum of two mixed numbers after adding the whole numbers and the fractional parts. Give an example in which this happens.

- 57.** Add $8\frac{4}{5} + 7\frac{3}{8}$ by the procedures of this section. Then change each mixed number to an improper fraction and add. Be sure you get the same result for both. Which method do you prefer? Why?

CHALLENGE

- 58.** Is $3\left(4\frac{1}{4}\right) + 2\frac{2}{3} + 5\left(2\frac{5}{6}\right) = 3\left(3\frac{1}{8}\right) + 7\frac{1}{12}$ a true statement?

- 59.** Is $6\left(6\frac{4}{9}\right) + 5\left(2\frac{5}{6}\right) + 7\left(4\frac{1}{3}\right) = 6\left(\frac{5}{3}\right) + 8\left(3\frac{2}{3}\right) + 7\left(3\frac{5}{42}\right)$ a true statement?

- 60.** During the month of January, the rangers at Yellowstone National Park record the following snowfall:

week 1, $6\frac{9}{10}$ in.; week 2, $8\frac{3}{4}$ in.; week 3, $13\frac{5}{6}$ in.;

and week 4, $9\frac{2}{3}$ in. How many inches of snow have

fallen during the month? If the average snowfall for the

month is $38\frac{17}{32}$ in., does this January exceed the

average?

GROUP WORK

- 61.** We know that a mixed number can be changed to an improper fraction, so $5\frac{3}{7} = \frac{38}{7}$. How many ways can you find to express $5\frac{3}{7}$ as a mixed number using improper fractions?

MAINTAIN YOUR SKILLS

Subtract.

62. $103 - 77$

63. $212 - 128$

64. $1111 - 889$

65. $2222 - 1798$

Find the missing number.

66. $\frac{7}{8} = \frac{?}{40}$

67. $\frac{3}{16} = \frac{?}{80}$

68. Simplify: $\frac{1950}{4095}$

69. Simplify: $\frac{462}{847}$

- 70.** Par on the first nine holes of the Ricochet Golf Course is 36. If Millie records scores of 5, 4, 6, 2, 3, 3, 3, 5, and 4 on the nine holes, what is her total score? Is she under or over par for the first nine holes?

- 71.** Dried prunes weigh one-third the weight of fresh prunes. How many pounds of fresh prunes are required to make 124 half-pound packages of dried prunes?

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3.8 Subtracting Fractions

How & Why

OBJECTIVE

Subtract fractions.

OBJECTIVE

Subtract fractions.

What is the difference of $\frac{2}{3}$ and $\frac{1}{3}$? In Figure 3.21 we can see that we subtract the numerators and keep the common denominator (subtract the cross-hatched region from the blue region).

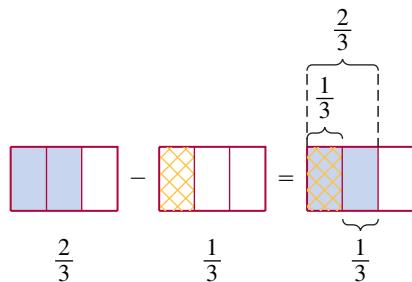


Figure 3.21

What is the value of $\frac{3}{4} - \frac{1}{3}$? See Figure 3.22.

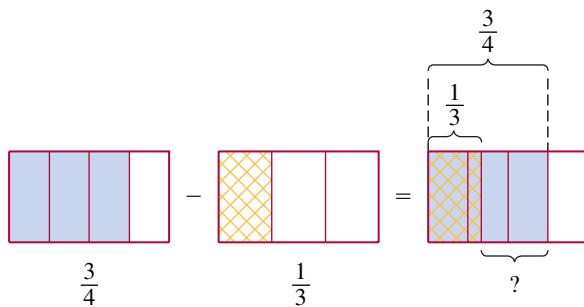


Figure 3.22

The region shaded blue with the question mark cannot be named immediately because the original parts are not the same size. If the fractions had a common denominator, we could subtract as in Figure 3.21. Using the common denominator 12, we see in Figure 3.23 that

the difference is $\frac{5}{12}$.

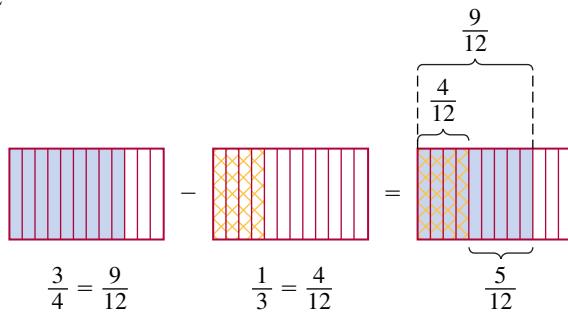


Figure 3.23

$$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

The method for subtracting fractions is similar to that for adding fractions.

To subtract fractions

1. Build each fraction to a common denominator.
2. Subtract the numerators and write the difference over the common denominator.
3. Simplify.

Warm-Ups A–F

A. Subtract: $\frac{13}{18} - \frac{7}{18}$

B. Subtract: $\frac{4}{5} - \frac{5}{12}$

C. Subtract: $\frac{13}{18} - \frac{9}{24}$

D. Subtract: $\frac{67}{72} - \frac{11}{90}$

E. Subtract: $\frac{43}{48} - \frac{23}{32}$

- Answers to Warm-Ups**
- A. $\frac{1}{3}$ B. $\frac{23}{60}$ C. $\frac{25}{72}$
D. $\frac{97}{120}$ E. $\frac{17}{96}$

Examples A–F BASIC Mathematics Now™

DIRECTIONS: Subtract and simplify.

STRATEGY: Build each fraction to a common denominator.

A. Subtract: $\frac{11}{20} - \frac{7}{20}$

$$\frac{11}{20} - \frac{7}{20} = \frac{4}{20} \quad \text{Subtract the numerators.}$$
$$= \frac{1}{5} \quad \text{Simplify.}$$

B. Subtract: $\frac{5}{8} - \frac{1}{6}$

$$\frac{5}{8} - \frac{1}{6} = \frac{15}{24} - \frac{4}{24} \quad \text{The LCM of 8 and 6 is 24.}$$
$$= \frac{11}{24} \quad \text{Subtract the numerators.}$$

C. Subtract: $\frac{13}{18} - \frac{7}{12}$

$$\frac{13}{18} - \frac{7}{12} = \frac{26}{36} - \frac{21}{36} \quad \text{The LCM of 18 and 12 is 36.}$$
$$= \frac{5}{36} \quad \text{Subtract the numerators.}$$

D. Subtract: $\frac{43}{60} - \frac{5}{48}$

$$\frac{43}{60} - \frac{5}{48} = \frac{172}{240} - \frac{25}{240} \quad \text{The LCM of 60 and 48 is 240.}$$
$$= \frac{147}{240} \quad \text{Subtract the numerators.}$$
$$= \frac{49}{80} \quad \text{Simplify.}$$

CALCULATOR EXAMPLE

E. Subtract: $\frac{39}{50} - \frac{8}{15}$

$$\frac{39}{50} - \frac{8}{15} = \frac{37}{150}$$

On a calculator with fraction keys, it is not necessary to find the common denominator. The calculator is programmed to subtract and simplify.

- F. Lumber mill operators must plan for the shrinkage of “green” (wet) boards when they cut logs. If the shrinkage for a $\frac{5}{8}$ -in.-thick board is expected to be $\frac{1}{16}$ in., what will the thickness of the dried board be?

STRATEGY: To find the thickness of the dried board, subtract the shrinkage from the thickness of the green board.

$$\begin{aligned}\frac{5}{8} - \frac{1}{16} &= \frac{10}{16} - \frac{1}{16} && \text{Build } \frac{5}{8} \text{ to a fraction with denominator 16.} \\ &= \frac{9}{16}\end{aligned}$$

The dried board will be $\frac{9}{16}$ in. thick.

- F. Mike must plane $\frac{3}{32}$ in. from the thickness of a board. If the board is now $\frac{3}{8}$ in. thick, how thick will it be after he has planed it?

Answers to Warm-Ups

- F. The board will be $\frac{9}{32}$ in. thick.

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Exercises 3.8

OBJECTIVE

Subtract fractions.

A Subtract. Simplify completely.

1. $\frac{7}{9} - \frac{2}{9}$

2. $\frac{5}{8} - \frac{3}{8}$

3. $\frac{8}{9} - \frac{5}{9}$

4. $\frac{7}{8} - \frac{3}{8}$

5. $\frac{13}{30} - \frac{3}{30}$

6. $\frac{13}{15} - \frac{8}{15}$

7. $\frac{5}{7} - \frac{3}{14}$

8. $\frac{11}{15} - \frac{2}{5}$

9. $\frac{3}{4} - \frac{5}{16}$

10. $\frac{8}{9} - \frac{5}{18}$

11. $\frac{3}{15} - \frac{2}{45}$

12. $\frac{11}{18} - \frac{4}{9}$

13. $\frac{1}{3} - \frac{1}{6}$

14. $\frac{5}{6} - \frac{1}{3}$

15. $\frac{13}{18} - \frac{2}{3}$

16. $\frac{5}{8} - \frac{5}{24}$

17. $\frac{17}{20} - \frac{1}{5}$

18. $\frac{19}{30} - \frac{1}{5}$

19. $\frac{23}{40} - \frac{1}{8}$

20. $\frac{17}{36} - \frac{1}{4}$

B

21. $\frac{7}{8} - \frac{5}{6}$

22. $\frac{2}{3} - \frac{3}{8}$

23. $\frac{7}{16} - \frac{1}{12}$

24. $\frac{7}{15} - \frac{3}{20}$

25. $\frac{3}{7} - \frac{5}{21}$

26. $\frac{2}{3} - \frac{4}{15}$

27. $\frac{9}{16} - \frac{1}{6}$

28. $\frac{5}{6} - \frac{4}{5}$

29. $\frac{8}{9} - \frac{5}{6}$

30. $\frac{4}{7} - \frac{5}{14}$

31. $\frac{5}{8} - \frac{1}{12}$

32. $\frac{7}{10} - \frac{7}{15}$

33. $\frac{7}{10} - \frac{1}{4}$

34. $\frac{8}{15} - \frac{5}{12}$

35. $\frac{7}{8} - \frac{2}{3}$

36. $\frac{13}{15} - \frac{7}{12}$

37. $\frac{18}{25} - \frac{7}{15}$

38. $\frac{21}{32} - \frac{5}{16}$

39. $\frac{13}{16} - \frac{11}{24}$

40. $\frac{17}{18} - \frac{5}{12}$

C

41. $\frac{13}{18} - \frac{7}{12}$

42. $\frac{7}{10} - \frac{5}{8}$

43. $\frac{17}{50} - \frac{13}{40}$

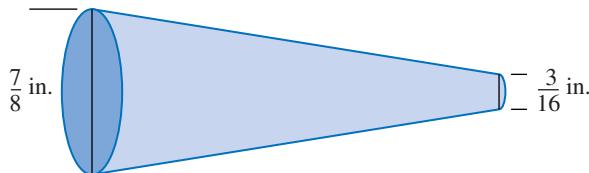
44. $\frac{32}{35} - \frac{17}{20}$

45. $\frac{7}{24} - \frac{5}{18}$

46. $\frac{14}{15} - \frac{11}{20}$

- 47.** If the shrinkage of a $\frac{6}{5}$ -in.-thick “green” board is $\frac{1}{8}$ in., what will be the thickness when the board has dried?
- 48.** On July 1, the reservoir at Bull Run watershed was at $\frac{3}{4}$ capacity. During the month, the reservoir lost $\frac{1}{20}$ of its capacity due to evaporation. What fraction of its capacity does the reservoir hold at the end of July?
- 49.** Ben got a tax refund of \$390. He spent half of it to pay off his cell phone bill and put $\frac{1}{5}$ of it in savings. What fraction of his refund does he still have left? How many dollars does he still have?
- 50.** A water sample from Lake Tuscumbia contains 21 parts per million of phosphate. A sample from Lost Lake contains 2 parts per hundred thousand. Which lake has the greater phosphate content? By how much?
- 51.** In a recent year, 16 of every 25 Americans owned stock in a publicly held company or mutual fund. What fraction of Americans did not own stock?
- 52.** In a recent year, approximately 80 million people attended a professional ice hockey, basketball, football, or baseball game. If $\frac{1}{8}$ of them attended ice hockey, $\frac{1}{8}$ of them attended basketball, and $\frac{3}{16}$ of them attended football, what fraction attended baseball?
- 53.** At the turn of the last century, according to the Recording Industry Association of America, $\frac{1}{10}$ of all the recorded music and music video sales was cassettes. In that same year, $\frac{4}{5}$ of all sales were CDs. What part of the total sales for the year was not CDs or cassettes?
- 54.** In 1820, according to figures from the U.S. Dept. of Agriculture, approximately $\frac{18}{25}$ of the U.S. labor force were farm occupations. By the 1990s, only $\frac{1}{40}$ of the labor force was farm occupations. What part of the labor force has changed from farm to non-farm occupations over this time period?
- 55.** According to the U.S. Bureau of the Census, $\frac{29}{50}$ of Americans 65 or older are female. What portion of this population is male?
- 56.** Oxygen and carbon are the two most plentiful elements in the human body. On average, $\frac{13}{20}$ of the body is oxygen and $\frac{9}{50}$ of the body is carbon. What part of the body do the remaining elements account for?

- 57.** The diameter at the large end of a tapered pin is $\frac{7}{8}$ in., and at the smaller end, it is $\frac{3}{16}$ in. What is the difference between the diameters?



- 58.** Aunt Gertrude left her estate to her four nephews. Two of them each received $\frac{1}{4}$ of the estate, and the third received $\frac{3}{8}$ of the estate. What portion of the estate did the fourth nephew receive?

- 59.** President George W. Bush proposed a federal budget to Congress with total spending of \$2,570,000,000,000. The budget is divided into categories of mandatory spending, discretionary spending, and interest payments. The mandatory spending was $\frac{11}{20}$ of the total, and the discretionary spending was $\frac{37}{100}$ of the total. What part of the budget was for interest payments?

STATE YOUR UNDERSTANDING

- 60.** Explain in writing how you would teach a child to subtract fractions.
- 61.** Explain why $\frac{3}{4} - \frac{1}{2}$ is not equal to $\frac{2}{2}$.

CHALLENGE

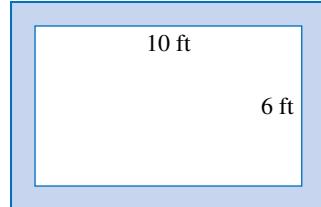
Subtract.

62. $\frac{213}{560} - \frac{109}{430}$

63. $\frac{93}{125} - \frac{247}{625}$

- 64.** A donor agrees to donate \$1000 for each foot that Skola outdistances Sheila in 13 minutes. Skola walks $\frac{19}{24}$ mi. Sheila walks $\frac{47}{60}$ mi. Does Skola outdistance Sheila? By what fraction of a mile? How much does the donor contribute? (A mile equals 5280 ft.)

- 65.** A landscaper is building a brick border, one brick wide, around a formal rose garden. The garden is a 10-ft-by-6-ft rectangle. Standard bricks are 8 in. by $3\frac{3}{4}$ in. by $2\frac{1}{4}$ in., and the landscaper is planning to use a $\frac{3}{8}$ -in.-wide mortar in the joints. How many whole bricks are needed for the project? Explain your reasoning.



GROUP WORK

- 66.** With your group, find a way to subtract $4\frac{7}{8}$ from $6\frac{3}{4}$.

Can you find more than one way? Write the procedure down and share it with the class.

MAINTAIN YOUR SKILLS

Subtract.

67. $41,695 - 3969$

68. $21,596 - 9396$

Subtract. Write as a mixed number

69. $(9 - 5) + \left(\frac{2}{3} - \frac{5}{12}\right)$

70. $(22 - 16) + \left(\frac{2}{3} - \frac{4}{15}\right)$

71. $(92 - 87) + \left(\frac{2}{3} - \frac{7}{18}\right)$

72. $(81 - 76) + \left(\frac{2}{3} - \frac{5}{21}\right)$

73. $(54 - 45) + \left(\frac{3}{4} - \frac{5}{12}\right)$

74. $(63 - 36) + \left(\frac{3}{4} - \frac{1}{20}\right)$

- 75.** Three bricklayers can each lay 795 bricks per day, on the average. How many bricks can they lay in 5 days?

- 76.** If a retaining wall requires 19,080 bricks, how many days will it take the three bricklayers in Exercise 75 to build the wall?

3.9 Subtracting Mixed Numbers

How & Why

OBJECTIVE

Subtract mixed numbers.

OBJECTIVE

Subtract mixed numbers.

A subtraction problem may be written in horizontal or vertical form. Horizontally:

$$\begin{aligned}8\frac{7}{9} - 3\frac{2}{9} &= (8 - 3) + \left(\frac{7}{9} - \frac{2}{9}\right) \\&= 5 + \frac{5}{9} = 5\frac{5}{9}\end{aligned}$$

Because the denominators are the same, subtract the whole-number parts and then subtract the fraction parts.

Vertically:

$$\begin{array}{r} 8\frac{7}{9} \\ - 3\frac{2}{9} \\ \hline 5\frac{5}{9} \end{array}$$

The process is similar to that for adding mixed numbers.

It is sometimes necessary to “borrow” from the whole number in order to subtract the fractions. For example,

$$\begin{aligned}8\frac{2}{5} - 3\frac{3}{4} &=? \quad \text{First, write in columns and build each fraction to the common denominator, 20.} \\8\frac{2}{5} &= 8\frac{8}{20} \\- 3\frac{3}{4} &= 3\frac{15}{20} \\ \hline &\end{aligned}$$

Because we cannot subtract $\frac{15}{20}$ from $\frac{8}{20}$, we need to “borrow.” To do this we rename $8\frac{8}{20}$ by “borrowing” 1 from 8.

$$\begin{aligned}8\frac{8}{20} &= 7 + 1\frac{8}{20} \quad \text{Borrow 1 from 8 and add it to the fraction part.} \\&= 7 + \frac{28}{20} \quad \text{Change the mixed number, } 1\frac{8}{20}, \text{ to an improper fraction.} \\&= 7\frac{28}{20} \quad \text{Write as a mixed number.}\end{aligned}$$

CAUTION

Do not write $\frac{18}{20}$. If we “borrow” 1 from 8, we must add 1 (that is, $\frac{20}{20}$) to $\frac{8}{20}$.

The example can now be completed.

$$\begin{aligned}8\frac{2}{5} &= 8\frac{8}{20} = 7\frac{28}{20} \quad \text{Rename by “borrowing” 1, or } \frac{20}{20}. \\- 3\frac{3}{4} &= 3\frac{15}{20} = 3\frac{15}{20} \\ \hline &4\frac{13}{20} \quad \text{Subtract the whole numbers. Subtract the fractions.}\end{aligned}$$

To subtract mixed numbers

1. Build the fractions so they have a common denominator.
2. Subtract the fractions. If the fractions cannot be subtracted, rename the first mixed number by “borrowing” 1 from the whole-number part to add to the fraction part. Then subtract the fractions.
3. Subtract the whole numbers.
4. Simplify.

Warm-Ups A–G

A. Subtract: $22\frac{3}{5} - 15\frac{2}{9}$

B. Subtract: $61\frac{9}{10} - 18$

C. Subtract: $25 - 16\frac{5}{7}$

Examples A–G BASIC Mathematics Now™

DIRECTIONS: Subtract. Write as a mixed number.

STRATEGY: Subtract the fractions and subtract the whole numbers. If necessary, borrow. Simplify.

A. Subtract: $15\frac{4}{5} - 13\frac{1}{6}$

STRATEGY: Write the mixed numbers in columns to group the whole numbers and group the fractions.

$$\begin{array}{r} 15\frac{4}{5} = 15\frac{24}{30} \\ - 13\frac{1}{6} = 13\frac{5}{30} \\ \hline & 2\frac{19}{30} \end{array} \quad \begin{array}{l} \text{Build the fraction to the common denominator, 30.} \\ \text{Subtract.} \end{array}$$

B. Subtract: $21\frac{3}{4} - 9$

$$21\frac{3}{4} - 9 = 12\frac{3}{4} \quad \begin{array}{l} \text{Subtract the whole numbers.} \end{array}$$

C. Subtract: $21 - 9\frac{3}{4}$

STRATEGY: Notice the difference between Examples B and C. Here we must also subtract the fraction. We may think of 21 as $21\frac{0}{4}$ in order to get a common denominator for the improper fraction. Or think:

$$\begin{array}{r} 21 = 20 + 1 = 20 + \frac{4}{4} = 20\frac{4}{4} \\ - 9\frac{3}{4} \\ \hline 11\frac{1}{4} \end{array}$$

Answers to Warm-Ups

A. $7\frac{17}{45}$ B. $43\frac{9}{10}$ C. $8\frac{2}{7}$

CAUTION

Do not just bring down the fraction $\frac{3}{4}$ and then subtract the whole numbers:

$21 - 9\frac{3}{4} \neq 12\frac{3}{4}$. The fraction must also be subtracted.

D. Subtract: $15\frac{5}{12} - 7\frac{7}{12}$

STRATEGY: Because $\frac{7}{12}$ cannot be subtracted from $\frac{5}{12}$, we need to borrow.

$$\begin{array}{r}
 15\frac{5}{12} = 14 + 1\frac{5}{12} = 14\frac{17}{12} \\
 - 7\frac{7}{12} = \underline{\quad} \quad \quad = 7\frac{7}{12} \\
 \hline
 \end{array}$$

Borrow 1 from 15 and change the mixed number to an improper fraction.

Subtract.

Simplify.

E. Subtract: $23\frac{5}{12} - 11\frac{7}{15}$

$$\begin{array}{r}
 23\frac{5}{12} = 23\frac{25}{60} = 22 + 1\frac{25}{60} = 22\frac{85}{60} \\
 - 11\frac{7}{15} = 11\frac{28}{60} \\
 \hline
 & & = 11\frac{28}{60} \\
 & & \hline
 & & 11\frac{57}{60} \\
 & & \hline
 & & = 11\frac{19}{20}
 \end{array}$$

The LCM of 12 and 15 is 60. Borrow 1 from 23 and change the mixed number to an improper fraction.

Subtract

Simplify.

CALCULATOR EXAMPLE



F. Subtract: $31\frac{1}{3} - 18\frac{3}{4}$

$$31\frac{1}{3} - 18\frac{3}{4} = \frac{151}{12}$$

$$= 12\frac{7}{12}$$

On a calculator with fraction keys, it is not necessary to find the common denominator. The calculator is programmed to subtract and simplify. Some calculators may not change the improper result to a mixed number.

D. Subtract: $18\frac{11}{15} - 8\frac{12}{15}$

E. Subtract: $58\frac{3}{8} - 43\frac{14}{15}$

F. Subtract: $72\frac{7}{9} - 25\frac{13}{15}$

Answers to Warm-Ups

D. $9\frac{14}{15}$ E. $14\frac{53}{120}$ F. $46\frac{41}{45}$

- G. Jamie weighs $138\frac{1}{2}$ lb and decides to lose weight. She loses a total of $5\frac{3}{4}$ lb in 2 weeks. What is her weight after the loss?

- G. Shawn brings a roast home for Sunday dinner that weighs 7 lb. He cuts off some fat and takes out a bone. The meat left weighs $4\frac{1}{3}$ lb. How many pounds of fat and bone does he trim off?

STRATEGY: Subtract the weight of the remaining meat from the original weight of the roast.

$$\begin{array}{rcl} 7 = 6 + 1\frac{0}{3} = 6\frac{3}{3} & & \text{Borrow 1 from 7 and rename it as an improper fraction.} \\ - 4\frac{1}{3} & = & 4\frac{1}{3} \\ \hline & & = 2\frac{2}{3} \end{array}$$

Shawn trims off $2\frac{2}{3}$ lb of fat and bone.

Answers to Warm-Ups

- G. Her weight is $132\frac{3}{4}$ lb.

Exercises 3.9

OBJECTIVE

Subtract mixed numbers.

A Subtract. Write the results as mixed numbers where possible.

1.
$$\begin{array}{r} 16\frac{6}{7} \\ - 9\frac{4}{7} \\ \hline \end{array}$$

2.
$$\begin{array}{r} 31\frac{5}{9} \\ - 20\frac{2}{9} \\ \hline \end{array}$$

3.
$$\begin{array}{r} 212\frac{37}{80} \\ - 210\frac{21}{80} \\ \hline \end{array}$$

4.
$$\begin{array}{r} 205\frac{6}{11} \\ - 199\frac{4}{11} \\ \hline \end{array}$$

5.
$$\begin{array}{r} 10\frac{7}{8} \\ - 5\frac{3}{4} \\ \hline \end{array}$$

6.
$$\begin{array}{r} 11\frac{4}{5} \\ - 7\frac{3}{10} \\ \hline \end{array}$$

7.
$$\begin{array}{r} 12 \\ - 9\frac{4}{7} \\ \hline \end{array}$$

8.
$$\begin{array}{r} 16 \\ - 12\frac{1}{3} \\ \hline \end{array}$$

9.
$$\begin{array}{r} 100\frac{7}{9} \\ - 22\frac{1}{2} \\ \hline \end{array}$$

10.
$$\begin{array}{r} 66\frac{5}{6} \\ - 52\frac{9}{10} \\ \hline \end{array}$$

11.
$$\begin{array}{r} 5\frac{1}{4} \\ - 2\frac{3}{4} \\ \hline \end{array}$$

12.
$$\begin{array}{r} 7\frac{3}{8} \\ - 4\frac{5}{8} \\ \hline \end{array}$$

13.
$$\begin{array}{r} 26\frac{1}{10} \\ - 11\frac{9}{10} \\ \hline \end{array}$$

14.
$$\begin{array}{r} 17\frac{3}{8} \\ - 5\frac{5}{8} \\ \hline \end{array}$$

15.
$$\begin{array}{r} 234\frac{7}{9} \\ - 62 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 300\frac{5}{8} \\ - 119 \\ \hline \end{array}$$

17. $9\frac{1}{3} - 2\frac{5}{12}$

18. $8\frac{1}{2} - 6\frac{5}{12}$

B

19.
$$\begin{array}{r} 21\frac{15}{16} \\ - 12\frac{7}{16} \\ \hline \end{array}$$

20.
$$\begin{array}{r} 24\frac{7}{12} \\ - 15\frac{11}{12} \\ \hline \end{array}$$

21.
$$\begin{array}{r} 310\frac{23}{24} \\ - 254\frac{5}{8} \\ \hline \end{array}$$

22.
$$\begin{array}{r} 118\frac{7}{12} \\ - 93\frac{1}{4} \\ \hline \end{array}$$

23.
$$\begin{array}{r} 66\frac{7}{15} \\ - 51\frac{1}{12} \\ \hline \end{array}$$

24.
$$\begin{array}{r} 19\frac{9}{16} \\ - 13\frac{5}{12} \\ \hline \end{array}$$

25.
$$\begin{array}{r} 37\frac{2}{3} \\ - 15\frac{11}{12} \\ \hline \end{array}$$

26.
$$\begin{array}{r} 28\frac{1}{3} \\ - 15\frac{7}{9} \\ \hline \end{array}$$

27.
$$\begin{array}{r} 40 \frac{1}{6} \\ -24 \frac{3}{16} \\ \hline \end{array}$$

28.
$$\begin{array}{r} 31 \frac{3}{10} \\ -17 \frac{5}{18} \\ \hline \end{array}$$

29.
$$\begin{array}{r} 34 \\ -12 \frac{3}{4} \\ \hline \end{array}$$

30.
$$\begin{array}{r} 62 \\ -16 \frac{2}{3} \\ \hline \end{array}$$

31.
$$\begin{array}{r} 3 \frac{17}{32} \\ -1 \frac{5}{16} \\ \hline \end{array}$$

32.
$$\begin{array}{r} 9 \frac{3}{10} \\ -\frac{17}{20} \\ \hline \end{array}$$

33.
$$\begin{array}{r} 10 \frac{7}{18} \\ -\frac{7}{12} \\ \hline \end{array}$$

34.
$$\begin{array}{r} 4 \frac{5}{6} \\ -\frac{7}{8} \\ \hline \end{array}$$

35.
$$\begin{array}{r} 93 \frac{7}{8} \\ -19 \\ \hline \end{array}$$

36.
$$\begin{array}{r} 55 \frac{7}{16} \\ -28 \\ \hline \end{array}$$

37.
$$\begin{array}{r} 46 \frac{14}{15} \\ -19 \frac{27}{40} \\ \hline \end{array}$$

38.
$$\begin{array}{r} 60 \frac{13}{18} \\ -42 \frac{37}{45} \\ \hline \end{array}$$

39. $75 \frac{7}{12} - 47 \frac{13}{18}$

40. $82 \frac{4}{15} - 56 \frac{7}{12}$

C

41.
$$\begin{array}{r} 34 \frac{2}{39} \\ -17 \frac{21}{26} \\ \hline \end{array}$$

42.
$$\begin{array}{r} 38 \frac{5}{24} \\ -21 \frac{3}{40} \\ \hline \end{array}$$

43. $3 \frac{11}{15} - 2 \frac{9}{10}$

44. $6 \frac{1}{3} - 5 \frac{5}{6}$

45. Find the error(s) in the statement: $16 - 13 \frac{1}{4} = 3 \frac{3}{4}$.

Correct the statement.

46. Find the error(s) in the statement: $5 \frac{1}{2} - 2 \frac{3}{4} = 3 \frac{1}{4}$.

Correct the statement.

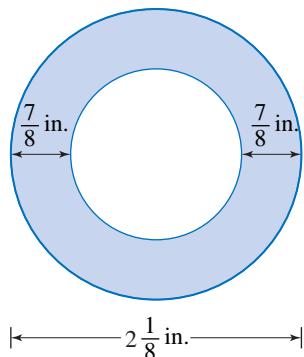
47. Han Kwong trims bone and fat from a $8 \frac{1}{2}$ -lb roast. The meat left weighs $6 \frac{1}{8}$ lb. How many pounds does she trim off?

48. Patti has a piece of lumber that measures $8 \frac{5}{12}$ ft that is to be used in a spot that calls for a length of $6 \frac{1}{2}$ ft. How much of the board must be cut off?

49. Dick harvests $30 \frac{3}{4}$ tons of wheat. He sells $18 \frac{7}{10}$ tons to the Cartwright Flour Mill. How many tons of wheat does he have left?

50. A $14 \frac{3}{4}$ -in. casting shrinks $\frac{5}{32}$ in. on cooling. Find the size when the casting is cold.

- 51.** Nancy bought a carpet remnant that is 14 ft long and 10 ft wide. Her bedroom is $9\frac{1}{3}$ ft long and 10 ft wide. What is the size of the remnant that is left over?
- 52.** According to the International Game Fish Association, the largest cubera snapper ever caught weighed $121\frac{5}{8}$ lb and was caught in Louisiana in 1982 by Mike Hebart. The largest red snapper ever caught weighed $50\frac{1}{3}$ lb and was caught in the Gulf of Mexico off Louisiana in 1996 by Doc Kennedy. How much larger was the record cubera snapper than the red snapper?
- 53.** An airline defines overweight luggage as anything over 70 lb. Amber's suitcase weighs $82\frac{7}{8}$ lbs. How much extra weight is she charged for?
- 54.** A McDonnell Douglas DC-9 seats 158 and is $147\frac{5}{6}$ ft long. A Boeing 737 seats 128 to 149 and is $109\frac{7}{12}$ ft long. How much longer is the DC-9?
- 55.** Larry and Greg set out to hike 42 mi in 2 days. At the end of the first day they have covered $22\frac{7}{10}$ mi. How many miles do they have to go?
- 56.** Frank pours $9\frac{1}{10}$ yd of cement for a fountain. Another fountain takes $6\frac{7}{8}$ yd. How much more cement is needed for the larger fountain?
- 57.** The Rodrigas family recycles an average of $147\frac{1}{5}$ lb of material per month. The Madera family recycles an average of $120\frac{7}{8}$ lb of material per month. How many more pounds of material are recycled by the Rodrigas family in a year?
- 58.** The town of Fredonia averages $35\frac{1}{5}$ in. of rain per year. The town of Wheatland averages $27\frac{13}{15}$ in. of rain per year. Over a 10-yr period, how much more rain does Fredonia get?
- 59.** Haja needs to replace a bolt that holds her headboard on the bed frame. She wants to use the washer pictured below. What is the largest-diameter bolt she can buy and use the washer?



Exercise 60 relates to the chapter application. See page 195.

- 60.** Shane is almost done installing the deck boards on his deck. He has already installed 67 boards. How many inches of his 272-in. deck still need to be covered?

STATE YOUR UNDERSTANDING

- 61.** Explain how to simplify $4\frac{1}{3} - 2\frac{5}{8}$.
- 62.** When you “borrow” 1 to subtract mixed numbers, explain the fraction form it is written in and explain why.

CHALLENGE

- 63.** Does $4\left(5\frac{5}{6}\right) - 3\left(2\frac{7}{12}\right) = 6\left(5\frac{1}{2}\right) - 5\frac{3}{4}$?
- 64.** Does $3\left(7\frac{7}{8}\right) - 13\frac{1}{10} = 6\left(3\frac{3}{16}\right) - 7\left(1\frac{8}{35}\right)$?
- 65.** A snail climbs $5\frac{1}{3}$ ft in a day and slips back $1\frac{2}{3}$ ft at night. What is the snail’s net distance in 24 hr? How many days will it take the snail to make a net gain of over 20 ft?

GROUP WORK

- 66.** Have each member of your group create an application involving subtraction of mixed numbers. Trade them around and have other members solve them. Share the best two with the rest of the class.

MAINTAIN YOUR SKILLS*Simplify.*

67. $82 - 4 \cdot 3 + 10$

68. $8^2 - 4 \cdot 3 + 10$

69. $82 - 4^3 + 10$

70. $6 \cdot 18 \div 3 \cdot 2$

71. $6 \cdot 18 \div 3^2$

72. $6 + 18 \div 3^2$

Multiply.

73. $\frac{16}{25} \cdot \frac{25}{28} \cdot \frac{14}{15}$

74. $\frac{1}{2} \cdot \frac{5}{8} \cdot \frac{3}{5} \cdot 24$

75. Last week when Karla filled the tank of her car with gasoline, the odometer read 57,832 miles. Yesterday when she filled the tank with 18 gallons of gasoline, the odometer read 58,336 miles. What is Karla's mileage; that is, how many miles to the gallon did she get?

76. A pet food canning company packs Feelein Cat Food in cans, each containing $7\frac{3}{4}$ oz of cat food. Each empty can weighs $1\frac{1}{2}$ oz. Twenty-four cans are packed in a case that weighs 10 oz empty. What is the shipping weight of five cases of the cat food?

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Getting Ready for Algebra

How & Why

We have solved equations in which whole numbers are either added to or subtracted from a variable. Now we solve equations in which fractions or mixed numbers are either added to or subtracted from the variable. We use the same procedure as with whole numbers.

OBJECTIVE

Solve equations of the form $x + \frac{a}{b} = \frac{c}{d}$, where a, b, c , and d are whole numbers.

Examples A–C

DIRECTIONS: Solve.

STRATEGY: Add or subtract the same number from each side of the equation to isolate the variable.

A. Solve: $x - \frac{4}{5} = 3\frac{1}{2}$

$$x - \frac{4}{5} + \frac{4}{5} = 3\frac{1}{2} + \frac{4}{5}$$

To eliminate the subtraction, add $\frac{4}{5}$ to each side of the equation.

$$x = 3\frac{1}{2} + \frac{4}{5}$$

Simplify the left side.

$$x = 3\frac{5}{10} + \frac{8}{10}$$

Build each fraction to the common denominator, 10.

$$x = 3\frac{13}{10}$$

Add.

$$x = 4\frac{3}{10}$$

Change the improper fraction to a mixed number and add.

CHECK: $4\frac{3}{10} - \frac{4}{5} = 3\frac{1}{2}$

Substitute $4\frac{3}{10}$ for x in the original equation.

$$3\frac{1}{2} = 3\frac{1}{2}$$

The solution is $x = 4\frac{3}{10}$.

B. Solve: $4\frac{1}{2} = x + 2\frac{2}{3}$

$$4\frac{1}{2} - 2\frac{2}{3} = x + 2\frac{2}{3} - 2\frac{2}{3}$$

To eliminate the addition, subtract $2\frac{2}{3}$ from each side of the equation.

$$4\frac{1}{2} - 2\frac{2}{3} = x$$

$$1\frac{5}{6} = x$$

CHECK: $4\frac{1}{2} = 1\frac{5}{6} + 2\frac{2}{3}$

Substitute $1\frac{5}{6}$ for x in the original equation.

$$4\frac{1}{2} = 4\frac{1}{2}$$

The solution is $x = 1\frac{5}{6}$.

Warm-Ups A–C

A. Solve: $x - \frac{5}{8} = 3\frac{7}{8}$

B. Solve: $6\frac{1}{2} = a + 5\frac{5}{8}$

Answers to Warm-Ups

A. $x = 4\frac{1}{2}$ B. $a = \frac{7}{8}$

- C. When $3\frac{7}{8}$ mi of new freeway opened last month, it bought the total length to $21\frac{3}{5}$ mi. What was the original length of the freeway?

- C. On Tuesday, $2\frac{3}{8}$ in. of rain fell on Springfield. This brought the total for the last 5 consecutive days to $14\frac{1}{2}$ in. What was the rainfall for the first 4 days?

STRATEGY: First write the English version of the equation.

$$\left(\begin{array}{c} \text{rain on first} \\ \text{4 days} \end{array} \right) + \left(\begin{array}{c} \text{rain on} \\ \text{Tuesday} \end{array} \right) = \text{total rain}$$

Let x represent the number of inches of rain on the first 4 days.

$$\begin{array}{rcl} x + 2\frac{3}{8} & = & 14\frac{1}{2} \\ -2\frac{3}{8} & & -2\frac{3}{8} \\ \hline x & = & 12\frac{1}{8} \end{array} \quad \begin{array}{l} \text{Translate to algebra.} \\ \text{Subtract } 2\frac{3}{8} \text{ from each side.} \end{array}$$

Since $12\frac{1}{8} + 2\frac{3}{8} = 14\frac{1}{2}$, $12\frac{1}{8}$ in. of rain fell during the first 4 days.

Answers to Warm-Ups

- C. The original freeway was

$17\frac{29}{40}$ mi long.

Exercises

Solve.

1. $a + \frac{1}{8} = \frac{5}{8}$

2. $y + \frac{3}{8} = \frac{7}{8}$

3. $c - \frac{3}{16} = \frac{7}{16}$

4. $w + \frac{5}{12} = \frac{11}{12}$

5. $x + \frac{2}{9} = \frac{3}{8}$

6. $x - \frac{7}{8} = \frac{3}{4}$

7. $y - \frac{5}{7} = \frac{8}{9}$

8. $y + \frac{5}{9} = \frac{9}{10}$

9. $a + \frac{9}{8} = \frac{12}{5}$

10. $a - \frac{5}{4} = \frac{3}{8}$

11. $c - 1\frac{1}{8} = 2\frac{1}{3}$

12. $c + 1\frac{1}{8} = 3\frac{3}{4}$

13. $x + 6\frac{3}{4} = 7\frac{7}{9}$

14. $x - 2\frac{5}{9} = 2\frac{5}{8}$

15. $12 = w + 8\frac{5}{6}$

16. $25 = m + 15\frac{5}{8}$

17. $a - 13\frac{5}{6} = 22\frac{11}{18}$

18. $b + 23\frac{11}{12} = 34\frac{1}{3}$

19. $c + 44\frac{13}{21} = 65\frac{5}{7}$

20. $x - 27\frac{5}{8} = 48\frac{2}{3}$

21. A native pine tree grew $1\frac{15}{16}$ ft in the past 10 years to its present height of $45\frac{1}{2}$ ft. What was the height of the tree 10 years ago?

22. Juan brought in $35\frac{3}{4}$ lb of tin to be recycled. This brings his total for the month to $122\frac{1}{4}$ lb. How many pounds had he already brought in this month?

23. Freeda purchased a supply of nails for a construction project. She has used $18\frac{2}{3}$ lb and has $27\frac{1}{3}$ lb left. How many pounds of nails did she buy?

24. For cross-country race practice, Althea has run $10\frac{7}{10}$ mi. She needs to run an additional $13\frac{3}{10}$ mi to meet the goal set by her coach. How many miles does the coach want her to run?

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3.10 Order of Operations; Average

VOCABULARY

Recall that the **mean**, or **average**, of a set of numbers is the sum of the set of numbers divided by the total number of numbers in the set.

How & Why

OBJECTIVE 1

Do any combinations of operations with fractions.

The order of operations for fractions is the same as for whole numbers.

Order of Operations

To evaluate an expression with more than one operation

1. Parentheses—Do the operations within grouping symbols first (parentheses, fraction bar, etc.) in the order given in steps 2, 3, and 4.
2. Exponents—Do the operations indicated by exponents.
3. Multiply and Divide—Do multiplication and division as they appear from left to right.
4. Add and Subtract—Do addition and subtraction as they appear from left to right.

Table 3.4 summarizes some of the processes that need to be remembered when working with fractions.

Table 3.4 Operations with Fractions

Operation	Find the LCM and Build	Change Mixed Numbers to Improper Fractions	Invert Divisor and Multiply	Simplify Answer
Add	Yes	No	No	Yes
Subtract	Yes	No	No	Yes
Multiply	No	Yes	No	Yes
Divide	No	Yes	Yes	Yes

Examples A–D



BASIC Mathematics Now™

DIRECTIONS: Perform the indicated operations.

STRATEGY: Follow the order of operations that are used for whole numbers.

A. Simplify: $\frac{5}{6} - \frac{1}{2} \cdot \frac{2}{3}$

$$\begin{aligned}\frac{5}{6} - \frac{1}{2} \cdot \frac{2}{3} &= \frac{5}{6} - \frac{1}{3} && \text{Multiplication is performed first.} \\ &= \frac{5}{6} - \frac{2}{6} && \text{Build } \frac{1}{3} \text{ to a denominator of 6.} \\ &= \frac{3}{6} && \text{Subtract.} \\ &= \frac{1}{2} && \text{Simplify.}\end{aligned}$$

Warm-Ups A–D

A. Simplify: $\frac{7}{8} - \frac{3}{4} \cdot \frac{8}{9}$

Answers to Warm-Ups

A. $\frac{5}{24}$

B. Simplify: $\frac{3}{8} \div \frac{5}{16} \cdot \frac{1}{3}$

B. Simplify: $\frac{1}{3} \div \frac{3}{4} \cdot \frac{1}{2}$

$$\begin{aligned}\frac{1}{3} \div \frac{3}{4} \cdot \frac{1}{2} &= \frac{1}{3} \cdot \frac{4}{3} \cdot \frac{1}{2} \\ &= \frac{4}{18} \\ &= \frac{2}{9}\end{aligned}$$

Division is performed first, as it appears from left to right.

Multiply from left to right.

Simplify.

C. Simplify: $\left(\frac{3}{4}\right)^2 \cdot \frac{2}{5} - \frac{1}{5}$

C. Simplify: $\left(\frac{2}{3}\right)^2 \cdot \frac{1}{2} - \frac{1}{5}$

$$\begin{aligned}\left(\frac{2}{3}\right)^2 \cdot \frac{1}{2} - \frac{1}{5} &= \frac{\cancel{4}}{9} \cdot \frac{1}{\cancel{2}} - \frac{1}{5} \\ &= \frac{2}{9} - \frac{1}{5} \\ &= \frac{10}{45} - \frac{9}{45} \\ &= \frac{1}{45}\end{aligned}$$

Exponentiation is done first, then simplify.

Multiply.

Build to a common denominator.

Subtract.

- D. Jill, Jean, and Joan have equal shares in a gift shop. Jill sells her share. She sells $\frac{1}{4}$ to Jean, and the rest to Joan. What share of the gift shop does Joan now own?

- D. Gwen, George, Carlos, and Maria have equal shares in a florist shop. George sells $\frac{3}{8}$ of his share to Gwen, $\frac{1}{2}$ to Carlos, and the rest to Maria. What share of the florist shop does Gwen now own?

STRATEGY: Since each of the four had equal shares, each of them owned $\frac{1}{4}$ of the business. To find the share Gwen now owns, add her original share, $\frac{1}{4}$, to $\frac{3}{8}$ of George's share, which was $\frac{1}{4}$.

$$\begin{aligned}\frac{1}{4} + \frac{3}{8} \cdot \frac{1}{4} &= \frac{1}{4} + \frac{3}{32} \quad \text{Multiply first.} \\ &= \frac{8}{32} + \frac{3}{32} \quad \text{Build } \frac{1}{4} \text{ to the denominator 32.} \\ &= \frac{11}{32}\end{aligned}$$

Gwen now owns $\frac{11}{32}$ of the florist shop.

How & Why

OBJECTIVE 2

Find the average of a group of fractions.

To find the average of a set of fractions, divide the sum of the fractions by the number of fractions. The procedure is the same for all numbers.

Answers to Warm-Ups

B. $\frac{2}{5}$ C. $\frac{1}{40}$

D. Joan owns $\frac{7}{12}$ of the gift shop.

To find the average of a set of numbers

- Add the numbers.
- Divide the sum by the number of numbers in the set.

Examples E–G

DIRECTIONS: Find the mean (average).

STRATEGY: Find the sum of the set of fractions, then divide by the number of fractions.

- E. Find the mean: $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{1}{2}$

$$\frac{5}{6} + \frac{2}{3} + \frac{1}{2} = \frac{5}{6} + \frac{4}{6} + \frac{3}{6} \quad \text{Add the three fractions in the set.}$$
$$= \frac{12}{6} \text{ or } 2$$

$$2 \div 3 = \frac{2}{3} \quad \text{Divide the sum by 3, the number of fractions in the set.}$$

The mean is $\frac{2}{3}$.

- F. A class of 10 students takes a 12-problem quiz. The results are listed in the table. What is the average score?

Number of Students	Fraction of Problems Correct
1	$\frac{12}{12}$
2	$\frac{11}{12}$
3	$\frac{10}{12}$
4	$\frac{9}{12}$

STRATEGY: To find the class average, add all the grades and divide by 10. There were two scores of $\frac{11}{12}$, three scores of $\frac{10}{12}$, and four scores of $\frac{9}{12}$ in addition to one perfect score of $\frac{12}{12}$.

$$\frac{12}{12} + 2\left(\frac{11}{12}\right) + 3\left(\frac{10}{12}\right) + 4\left(\frac{9}{12}\right) \quad \text{Find the sum of the 10 scores.}$$

$$\left(\frac{12}{12} + \frac{22}{12} + \frac{30}{12} + \frac{36}{12}\right) = \frac{100}{12}$$

$$\frac{100}{12} \div 10 = \frac{100}{12} \cdot \frac{1}{10} \quad \text{Divide the sum by the number of students in the class.}$$

$$= \frac{10}{12}$$

CAUTION

Do not simplify the answer, because the test scores are based on 12.

The class average is $\frac{10}{12}$ of the problems correct, or 10 problems correct.

Warm-Ups E–G

- E. Find the mean: $\frac{1}{6}$, $\frac{5}{8}$, and $\frac{3}{4}$

- F. A class of 12 students takes a 20-problem test. The results are listed in the table. What is the average score?

Number of Students	Fraction of Problems Correct
1	$\frac{20}{20}$
2	$\frac{19}{20}$
4	$\frac{16}{20}$
5	$\frac{14}{20}$

Answers to Warm-Ups

- E. $\frac{37}{72}$

- F. The class average is $\frac{16}{20}$ of the problems correct.

G. Find the average:

$$3\frac{5}{6}, 4\frac{1}{2}, \text{ and } 2\frac{2}{3}$$

G. Find the average: $7\frac{1}{2}, 6\frac{1}{4}$, and $4\frac{5}{8}$

$$7\frac{1}{2} + 6\frac{1}{4} + 4\frac{5}{8} = 18\frac{3}{8}$$

$$18\frac{3}{8} \div 3 = \frac{147}{8} \div 3$$

$$= \frac{147}{8} \cdot \frac{1}{3}$$

$$= \frac{49}{8} = 6\frac{1}{8}$$

The average is $6\frac{1}{8}$.

Answers to Warm-Ups

G. $3\frac{2}{3}$

Exercises 3.10

OBJECTIVE 1

Do any combinations of operations with fractions.

A Perform the indicated operations.

1. $\frac{3}{13} + \frac{6}{13} - \frac{2}{13}$

2. $\frac{7}{15} - \frac{2}{15} + \frac{4}{15}$

3. $\frac{5}{17} - \left(\frac{1}{17} + \frac{2}{17} \right)$

4. $\frac{5}{17} - \left(\frac{2}{17} - \frac{1}{17} \right)$

5. $\frac{5}{6} - \frac{1}{2} \cdot \frac{2}{3}$

6. $\frac{1}{6} + \frac{1}{2} \div \frac{3}{2}$

7. $\frac{1}{4} + \frac{3}{8} \div \frac{1}{2}$

8. $\frac{1}{4} \div \frac{3}{8} + \frac{1}{2}$

9. $\frac{3}{5} \cdot \frac{1}{4} - \frac{1}{8}$

10. $\frac{3}{5} \cdot \frac{1}{8} - \frac{1}{16}$

11. $\frac{5}{8} \div \left(\frac{3}{4} + \frac{3}{4} \right)$

12. $\frac{1}{3} \div \left(\frac{1}{6} + \frac{4}{9} \right)$

13. $\frac{1}{3} + \left(\frac{1}{4} \right)^2$

14. $\frac{3}{4} + \left(\frac{1}{2} \right)^2$

B

15. $\frac{1}{2} \div \frac{2}{3} \cdot \frac{5}{6}$

16. $\frac{1}{2} \div \left(\frac{2}{3} \cdot \frac{5}{6} \right)$

17. $\frac{3}{4} - \left(\frac{1}{2} \cdot \frac{2}{3} \right)$

18. $\frac{2}{3} - \left(\frac{5}{6} \cdot \frac{1}{9} \right)$

19. $\frac{3}{4} - \frac{1}{2} \div \frac{2}{3} + \frac{1}{4}$

20. $\frac{7}{8} - \frac{1}{6} \div \frac{2}{3} + \frac{5}{6}$

21. $\frac{5}{8} \cdot \frac{1}{3} + \frac{1}{2} \div \frac{1}{3} - \frac{7}{8}$

22. $\frac{1}{9} \div \frac{1}{2} - \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{9}$

23. $\frac{7}{12} + \left(\frac{3}{4} \right)^2 - \frac{5}{9} \cdot \frac{33}{16}$

24. $\frac{3}{4} \cdot \frac{4}{5} - \frac{3}{0} + \left(\frac{2}{3} \right)^2$

25. $\frac{15}{16} - \frac{5}{8} + \frac{1}{4} \div \left(\frac{4}{3} \right)^2$

26. $\frac{19}{25} - \left(\frac{2}{5} \right)^2 + \frac{3}{5} \div \frac{2}{3}$

27. $\frac{3}{4} - \left(\frac{1}{3} \div \frac{2}{5} - \frac{1}{4} \right)$

28. $\frac{2}{3} + \left(\frac{3}{4} \cdot \frac{4}{9} + \frac{1}{2} \right)$

OBJECTIVE 2

Find the average of a group of fractions.

A Find the average.

29. $\frac{1}{9}$ and $\frac{7}{9}$

30. $\frac{3}{10}$ and $\frac{7}{10}$

31. $\frac{2}{7}, \frac{4}{7}$, and $\frac{5}{7}$

32. $\frac{3}{5}, \frac{4}{5}$, and $\frac{2}{5}$

33. $\frac{1}{11}, \frac{2}{11}$, and $\frac{9}{11}$

34. $\frac{1}{5}, \frac{3}{5}$, and $\frac{8}{5}$

35. $\frac{1}{5}, \frac{2}{5}$, and $\frac{7}{15}$

36. $\frac{1}{2}, \frac{3}{4}$, and $\frac{11}{12}$

37. $1\frac{1}{4}, 2\frac{1}{2}$, and $3\frac{1}{4}$

38. $2\frac{2}{3}, 4\frac{1}{3}$, and $5\frac{5}{6}$

B

39. $\frac{3}{8}, \frac{1}{4}, \frac{1}{2}$, and $\frac{3}{4}$

40. $\frac{1}{6}, \frac{5}{12}, \frac{1}{4}$, and $\frac{1}{2}$

41. $\frac{2}{3}, \frac{5}{12}, \frac{1}{2}, \frac{3}{4}$, and $\frac{5}{6}$

42. $\frac{2}{5}, \frac{1}{10}, \frac{3}{10}, \frac{1}{2}$, and $\frac{4}{5}$

43. $3\frac{1}{6}, 2\frac{1}{3}$, and $4\frac{1}{6}$

44. $5\frac{3}{8}, 5\frac{3}{4}$, and $7\frac{1}{2}$

45. $5\frac{1}{3}, 6\frac{2}{5}$, and $9\frac{13}{15}$

46. $2\frac{1}{2}, 1\frac{1}{6}, 1\frac{7}{9}$, and $2\frac{1}{3}$

C Perform the indicated operations.

47. $\frac{8}{9} - \left(\frac{1}{2} + \frac{1}{3} \div \frac{3}{4} \right) \cdot \frac{1}{2}$

48. $\frac{1}{2} - \left(\frac{1}{2} - \frac{3}{4} \div \frac{12}{5} \right) \cdot \frac{1}{6}$

49. $\left(\frac{3}{5} + \frac{7}{10} \cdot \frac{2}{3} \right) \left(\frac{3}{2} \right)^2$

50. $\frac{7}{6} \div \frac{5}{3} \cdot \frac{5}{14} \div \left(\frac{1}{2} \right)^3$

Find the average.

51. $3\frac{2}{3}, 4\frac{5}{6}$, and $2\frac{5}{9}$

52. $6\frac{7}{8}, 8\frac{3}{4}$, and $8\frac{1}{2}$

53. $12\frac{11}{12}, 6, \frac{1}{3}$, and 5

54. $5, \frac{8}{9}, 12$, and $\frac{5}{6}$

- 55.** Wayne catches six salmon. The salmon measure $23\frac{1}{4}$ in., $31\frac{5}{8}$ in., $42\frac{3}{4}$ in., $28\frac{5}{8}$ in., $35\frac{3}{4}$ in., and 40 in. in length. What is the average length of the salmon?

- 56.** Karla, a nurse at Kaiser Hospital, weighs five new babies. They weigh $6\frac{1}{2}$ lb, $7\frac{3}{4}$ lb, $9\frac{3}{8}$ lb, $7\frac{1}{2}$ lb, and $8\frac{7}{8}$ lb. What is the average weight of the babies?

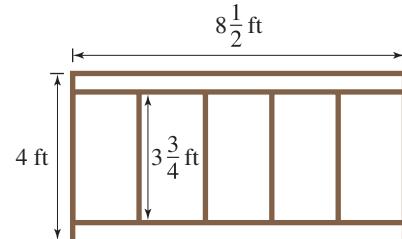
- 57.** The mothers of a swim team are making Rice Krispie treats for a big meet for the team. Each batch calls for 6 cups of cereal, $\frac{1}{4}$ cup of butter, $1\frac{3}{4}$ cups of chocolate chips, and $3\frac{1}{2}$ cups of marshmallows. They are also making Gorp, which uses $2\frac{1}{2}$ cups of cereal, 4 cups of pretzels, 1 cup of marshmallows, 1 cup of raisins, and $2\frac{1}{2}$ cups of chocolate chips. How much of each ingredient do they need if they intend to make 5 batches of Rice Krispie treats and 10 batches of Gorp?

- 59.** The results of the Womens Shot Put for the last five Olympiads are given in the table. What is the length of the average winning throw over the past 20 years?

Year	Winner	Distance
1988	Natalya Lisovskaya, U.S.S.R.	$72' 11\frac{1}{2}''$
1992	Svetlana Krivaleva, Unified Team	$69' 1\frac{1}{4}''$
1996	Astrid Kumbernuss, Germany	$67' 5\frac{1}{2}''$
2000	Yanina Korolchik, Belarus	$67' 5''$
2004	Irina Korzhanenko, Russia	$69' 1\frac{1}{8}''$

- 61.** Kohough Inc. packs a variety carton of canned seafood. Each carton contains three $3\frac{1}{2}$ -oz cans of smoked sturgeon, five $7\frac{3}{4}$ -oz cans of albacore tuna, four $5\frac{1}{2}$ -oz cans of salmon, and four 16-oz cans of solid white tuna. How many ounces of seafood are in the carton? If the carton sells for \$82, to the nearest cent what is the average cost per ounce?

- 58.** Nellie is making the trellis shown below out of $\frac{1}{2}$ -in. copper pipe, which comes in 10-ft lengths. How many 10-ft lengths does she need and how much will she have left over?



- 60.** According to industry analysts, the table gives the portion of IRA plans invested in mutual funds over the past 4 years. What is the average portion of an IRA invested in mutual funds?

Year	Portion in Mutual Funds
2001	$\frac{47}{100}$
2002	$\frac{47}{100}$
2003	$\frac{21}{50}$
2004	$\frac{22}{50}$

- 62.** In a walk for charity, seven people walk $3\frac{1}{8}$ mi, six people walk $2\frac{7}{8}$ mi, nine people walk $3\frac{1}{4}$ mi, and five people walk $6\frac{1}{2}$ mi. What is the total number of miles walked? If the charity raises \$2355, what is the average amount raised per mile, rounded to the nearest dollar?

Exercises 63–64 relate to the chapter application. See page 195.

- 63.** Now that Shane has finished his deck, he wants to build planter boxes along one end. Each planter box is 2 ft wide and 4 ft long, and he will build them using $2 \times 12s$. See Figure 3.24.
- 64.** How many cubic inches of potting soil are needed to fill one planter box?

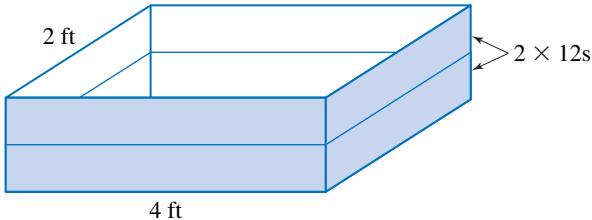


Figure 3.24

How many $2 \times 12s$ will Shane need to construct the sides of one planter box? Assume that the $2 \times 12s$ come in 10-ft lengths.

STATE YOUR UNDERSTANDING

- 65.** Write out the order of operations for fractions. How is it different from the order of operations for whole numbers?
- 66.** Must the average of a group of numbers be larger than the smallest number and smaller than the largest number? Why?

CHALLENGE

Perform the indicated operations.

67. $2\frac{5}{8} \left(4\frac{1}{5} - 3\frac{5}{6} \right) \div 2\frac{1}{2} \left(3\frac{1}{7} + 2\frac{1}{5} \right)$

68. $1\frac{2}{5} \left(5\frac{1}{5} - 4\frac{3}{4} \right) \div 4\frac{1}{2} \left(3\frac{1}{7} - 2\frac{1}{3} \right)$

- 69.** The Acme Fish Company pays \$1500 per ton for crab.

Jerry catches $3\frac{2}{5}$ tons; his brother Joshua catches $1\frac{1}{2}$ times as many as Jerry. Their sister Salicita catches $\frac{7}{8}$ the amount that Joshua does. What is the

total amount paid to the three people by Acme Fish Company to the nearest dollar?

GROUP WORK

- 70.** Prepare for the chapter exam by having each member of the group make up 10 exercises, one from each section of the chapter. Exchange these with another member, work each other's exercises, and check all of your answers. Discuss with the whole group the most common errors and how to avoid them.

MAINTAIN YOUR SKILLS

Perform the indicated operations.

71. $5\frac{1}{3} + \frac{7}{9}$

72. $5\frac{1}{3} - \frac{7}{9}$

73. $5\frac{1}{3}\left(\frac{7}{9}\right)$

74. $5\frac{1}{3} \div \frac{7}{9}$

75. $\frac{15}{28} \cdot \frac{21}{45} \cdot \frac{20}{35}$

76. $\frac{9}{15} \cdot \frac{3}{4} \cdot \frac{35}{6}$

- 77.** Find the prime factorization of 650.

- 78.** Find the prime factorization of 975.

- 79.** A coffee table is made of a piece of maple that is $\frac{3}{4}$ in. thick, a piece of chipboard that is $\frac{3}{8}$ in. thick, and a veneer that is $\frac{1}{8}$ in. thick. How thick is the tabletop?

- 80.** Felicia works a 5-day week for the following hours: $6\frac{3}{4}$ hr, $7\frac{1}{3}$ hr, $6\frac{2}{3}$ hr, $9\frac{3}{4}$ hr, and $7\frac{1}{2}$ hr. How many hours does she work for the week? What is her pay if the rate is \$12 per hour?

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Key Concepts CHAPTER 3

Section 3.1 Proper and Improper Fractions; Mixed Numbers

Definitions and Concepts	Examples
A fraction has the form $\frac{\text{numerator}}{\text{denominator}}$.	$\frac{4}{81}, \frac{20}{3}, \frac{25}{25}$
A proper fraction has a smaller numerator than denominator.	$\frac{4}{81}$ is a proper fraction.
An improper fraction has a numerator that is not smaller than the denominator.	$\frac{20}{3}$ and $\frac{25}{25}$ are improper fractions.
A mixed number is the sum of a whole number and a fraction.	$4\frac{1}{6}$
To change a mixed number to an improper fraction:	
<ul style="list-style-type: none"> • Multiply the whole number by the denominator and add the numerator. • Place the sum over the denominator. 	$4\frac{1}{6} = \frac{4 \cdot 6 + 1}{6} = \frac{25}{6}$
To change an improper fraction to a mixed number:	
<ul style="list-style-type: none"> • Divide the numerator by the denominator. • The mixed number is the whole number plus the remainder over the divisor. 	$\frac{38}{7} = 5\frac{3}{7}$ because $7 \overline{)38} \begin{array}{r} 5 \\ 35 \\ \hline 3 \end{array}$

Section 3.2 Simplifying Fractions

Definitions and Concepts	Examples
A fraction is completely simplified when its numerator and denominator have no common factors.	$\frac{6}{7}$ is completely simplified.
To simplify a fraction, eliminate all common factors of the numerator and denominator.	$\frac{10}{12}$ is not completely simplified because both 10 and 12 have a factor of 2. $\frac{36}{72} = \frac{9 \cdot 4}{9 \cdot 8} = \frac{4}{8} = \frac{4 \cdot 1}{4 \cdot 2} = \frac{1}{2}$

Section 3.3 Multiplying and Dividing Fractions

Definitions and Concepts	Examples
To multiply fractions, simplify if possible and then multiply numerators and multiply denominators.	$\frac{8}{15} \cdot \frac{5}{7} = \frac{8}{21}$
Two fractions are reciprocals if their product is 1.	$\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals because $\frac{3}{4} \cdot \frac{4}{3} = 1$.
To divide fractions, multiply the first fraction by the reciprocal of the divisor.	$\frac{4}{9} \div \frac{5}{3} = \frac{4}{9} \cdot \frac{3}{5} = \frac{4}{15}$

Section 3.4 Multiplying and Dividing Mixed Numbers

Definitions and Concepts	Examples
To multiply or divide mixed numbers, change them to improper fractions first. Then multiply or divide.	$3\frac{1}{5} \cdot 2\frac{3}{4} = \frac{16}{5} \cdot \frac{11}{4} = \frac{44}{5} = 8\frac{4}{5}$

Section 3.5 Building Fractions; Listing in Order; Inequalities

Definitions and Concepts	Examples
Building a fraction is writing an equivalent fraction with a different denominator.	$\frac{1}{2} = \frac{15}{30}$
To build a fraction, multiply both its numerator and denominator by the same factor.	$\frac{3}{5} = \frac{3 \cdot 6}{5 \cdot 6} = \frac{18}{30}$
To list fractions in order:	<p>List $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{5}{8}$ in order from smallest to largest.</p> <ul style="list-style-type: none"> Rewrite each fraction with a common denominator. Order the fractions according to their numerators. $\frac{3}{5} = \frac{24}{40}, \quad \frac{7}{10} = \frac{28}{40}, \quad \frac{5}{8} = \frac{25}{40}$ $\frac{24}{40} < \frac{25}{40} < \frac{28}{40}, \text{ so}$ $\frac{3}{5} < \frac{5}{8} < \frac{7}{10}$

Section 3.6 Adding Fractions

Definitions and Concepts	Examples
Like fractions have common denominators.	$\frac{3}{54}$ and $\frac{18}{54}$ are like fractions.
Unlike fractions have different denominators.	$\frac{1}{4}$ and $\frac{2}{5}$ are unlike fractions.
To add fractions: <ul style="list-style-type: none"> Rewrite with common denominators (if necessary). Add the numerators and keep the common denominator. Simplify. 	$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

Section 3.7 Adding Mixed Numbers

Definitions and Concepts	Examples
To add mixed numbers: <ul style="list-style-type: none"> Add the whole numbers. Add the fractions. If this sum is more than 1, change to a mixed number and add again. Simplify. 	$ \begin{array}{r} 5\frac{1}{2} = 5\frac{2}{4} \\ + 2\frac{3}{4} = 2\frac{3}{4} \\ \hline 7\frac{5}{4} = 7 + 1\frac{1}{4} = 8\frac{1}{4} \end{array} $

Section 3.8 Subtracting Fractions

Definitions and Concepts	Examples
To subtract fractions: <ul style="list-style-type: none"> Rewrite with common denominators (if necessary). Subtract the numerators and keep the common denominator. Simplify. 	$ \begin{aligned} \frac{71}{72} - \frac{31}{90} &= \frac{355}{360} - \frac{124}{360} \\ &= \frac{231}{360} \\ &= \frac{77}{120} \end{aligned} $

Section 3.9 Subtracting Mixed Numbers

Definitions and Concepts	Examples
To subtract mixed numbers: <ul style="list-style-type: none"> • Subtract the fractions. If the fractions cannot be subtracted, borrow 1 from the whole-number part and add it to the fractional part. Then subtract the fractions. • Subtract the whole numbers. • Simplify. 	$\begin{array}{r} 17\frac{3}{8} = 17\frac{45}{120} = 16 + 1\frac{45}{120} = 16\frac{165}{120} \\ - 12\frac{14}{15} = \underline{12\frac{112}{120}} \\ \hline 4\frac{53}{120} \end{array}$

Section 3.10 Order of Operations; Average

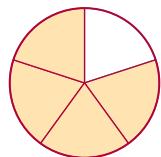
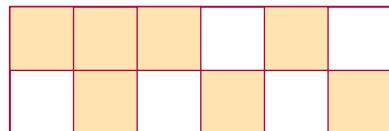
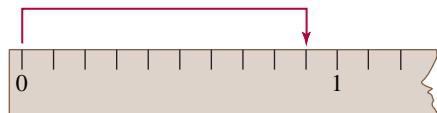
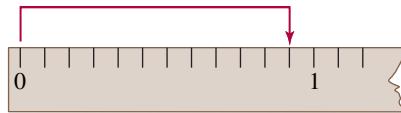
Definitions and Concepts	Examples
The order of operations for fractions is the same as that for whole numbers: <ul style="list-style-type: none"> • Parentheses • Exponents • Multiplication/Division • Addition/Subtraction 	$\begin{aligned} \left(\frac{1}{2} + \frac{2}{5}\right) \div \left(\frac{2}{3}\right)^2 &= \left(\frac{5}{10} + \frac{4}{10}\right) \div \left(\frac{2}{3}\right)^2 \\ &= \left(\frac{9}{10}\right) \div \left(\frac{2}{3}\right)^2 \\ &= \frac{9}{10} \div \left(\frac{4}{9}\right) \\ &= \frac{9}{10} \cdot \frac{9}{4} \\ &= \frac{81}{40} = 2\frac{1}{40} \end{aligned}$
Finding the average of a set of fractions is the same as for whole numbers: <ul style="list-style-type: none"> • Add the fractions. • Divide by the number of fractions. 	Find the average of $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{6}$. $\begin{aligned} \frac{1}{4} + \frac{1}{3} + \frac{1}{6} &= \frac{3}{12} + \frac{4}{12} + \frac{2}{12} = \frac{9}{12} = \frac{3}{4} \\ \frac{3}{4} \div 3 &= \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4} \\ \text{The average is } \frac{1}{4}. \end{aligned}$

Review Exercises

CHAPTER 3

Section 3.1

Write the fraction represented by the figure.

1.**2.****3.****4.**

Identify the proper fractions from each list.

5. $\frac{8}{3}, \frac{11}{12}, \frac{9}{9}, \frac{22}{19}, \frac{3}{20}$

6. $\frac{1}{13}, \frac{11}{15}, \frac{8}{10}, \frac{5}{2}, \frac{12}{18}$

Change to a mixed number.

7. $\frac{73}{12}$

8. $\frac{76}{9}$

9. $\frac{344}{3}$

10. $\frac{344}{7}$

Change to an improper fraction.

11. $6\frac{5}{12}$

12. $3\frac{2}{9}$

13. $12\frac{5}{6}$

14. $9\frac{2}{3}$

15. 17

16. 21

- 17.** A food wholesaler packs 24 cans of beans in a case for shipping. Write as a mixed number the number of cases that can be packed if she has 64,435 cans of beans.

- 18.** A food wholesaler packs 16 cans of fruit juice in a case for shipping. Write as a mixed number the number of cases that can be packed if he has 22,827 cans of fruit juice.

Section 3.2*Simplify.*

19. $\frac{20}{28}$

20. $\frac{10}{25}$

21. $\frac{40}{70}$

22. $\frac{18}{24}$

23. $\frac{21}{35}$

24. $\frac{25}{45}$

25. $\frac{102}{6}$

26. $\frac{132}{33}$

27. $\frac{14}{42}$

28. $\frac{30}{45}$

29. $\frac{88}{92}$

30. $\frac{75}{125}$

31. $\frac{26}{130}$

32. $\frac{84}{144}$

33. $\frac{268}{402}$

34. $\frac{630}{1050}$

35. Cardis completes 12 hr out of his weekly part time job of 30 hr. What fraction of his weekly shift remains?

36. On a math test, a student answers 42 items correctly and 18 incorrectly. What fraction of the items are answered correctly? Simplify.

Section 3.3*Multiply. Simplify completely.*

37. $\frac{1}{5} \cdot \frac{2}{5}$

38. $\frac{7}{8} \cdot \frac{1}{6}$

39. $\frac{3}{5} \cdot \frac{6}{11}$

40. $7 \cdot \frac{1}{4} \cdot \frac{8}{21}$

41. $\frac{21}{5} \cdot \frac{5}{4} \cdot \frac{4}{21}$

42. $\frac{24}{30} \cdot \frac{3}{8} \cdot \frac{4}{9}$

Find the reciprocal.

43. $\frac{3}{8}$

44. 5

Divide. Simplify completely.

45. $\frac{7}{20} \div \frac{14}{15}$

46. $\frac{8}{13} \div \frac{2}{13}$

47. $\frac{15}{18} \div \frac{30}{27}$

48. $\frac{12}{15} \div \frac{15}{8}$

49. $\frac{9}{100} \div \frac{3}{14}$

50. $\frac{32}{45} \div \frac{8}{9}$

51. Lois spends half of the family income on rent, utilities, and food. She pays $\frac{2}{7}$ of this amount for rent. What fraction of the family income goes for rent?

52. As part of his job at a pet store, Perry feeds each gerbil $\frac{1}{8}$ cup of seeds each day. If the seeds come in packages of $\frac{5}{4}$ cups, how many gerbils can be fed from one package?

Section 3.4*Multiply. Simplify completely and write as a mixed number if possible.*

53. $\left(\frac{3}{4}\right)\left(1\frac{3}{4}\right)$

54. $\left(\frac{3}{7}\right)\left(2\frac{5}{7}\right)$

55. $\left(4\frac{1}{2}\right)\left(2\frac{2}{3}\right)$

56. $\left(3\frac{1}{3}\right)\left(1\frac{4}{5}\right)$

57. $\left(3\frac{4}{7}\right)\left(\frac{14}{15}\right)$

58. $\left(4\frac{4}{9}\right)\left(\frac{12}{25}\right)$

59. $\left(3\frac{2}{3}\right)\left(\frac{15}{22}\right)\left(7\frac{1}{2}\right)$

60. $\left(4\frac{3}{4}\right)\left(3\frac{1}{5}\right)\left(5\frac{5}{8}\right)$

Divide. Simplify completely and write as a mixed number if possible.

61. $3 \div 1\frac{1}{2}$

62. $4 \div 1\frac{1}{4}$

63. $3\frac{3}{8} \div 3$

64. $2\frac{1}{6} \div 4$

65. $8\frac{2}{5} \div 2\frac{1}{3}$

66. $2\frac{1}{2} \div 1\frac{1}{5}$

67. $31\frac{1}{3} \div 1\frac{1}{9}$

68. $21\frac{3}{7} \div 8\frac{1}{3}$

69. A wheat farmer in Iowa averages $55\frac{3}{4}$ bushels of wheat per acre on 150 acres of wheat. How many bushels of wheat does she harvest?

70. A wildlife survey in a water fowl preserve finds that there are $3\frac{1}{3}$ times as many brant geese in the preserve as there are Canada geese. If the survey counts 7740 Canada geese, how many brant geese are there?

Section 3.5*Write four fractions equivalent to each of the given fractions by multiplying by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{5}{5}$, and $\frac{8}{8}$.*

71. $\frac{2}{3}$

72. $\frac{3}{5}$

73. $\frac{3}{14}$

74. $\frac{4}{11}$

Find the missing numerator.

75. $\frac{3}{4} = \frac{?}{24}$

76. $\frac{6}{7} = \frac{?}{56}$

77. $\frac{5}{6} = \frac{?}{144}$

78. $\frac{5}{8} = \frac{?}{224}$

List the fractions from smallest to largest.

79. $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$

80. $\frac{1}{3}, \frac{5}{12}, \frac{2}{5}$

81. $\frac{2}{9}, \frac{1}{5}, \frac{3}{11}$

82. $\frac{10}{9}, \frac{4}{3}, \frac{7}{6}, \frac{19}{18}$

83. $\frac{3}{5}, \frac{8}{25}, \frac{31}{50}, \frac{59}{100}$

84. $7\frac{3}{4}, 7\frac{7}{8}, 7\frac{5}{6}$

Are the following statements true or false?

85. $\frac{3}{14} < \frac{5}{14}$

86. $\frac{11}{8} > \frac{9}{8}$

87. $\frac{11}{14} > \frac{17}{21}$

88. $\frac{13}{15} < \frac{22}{25}$

89. Four pickup trucks are advertised in the local car ads.

The load capacities listed are $\frac{3}{4}$ ton, $\frac{5}{8}$ ton, $\frac{7}{16}$ ton, and $\frac{1}{2}$ ton. Which capacity is the smallest and which is the largest?

90. During 1 week on her diet, Samantha ate five servings of chicken, each containing $\frac{3}{16}$ oz of fat. During the same period her brother ate four servings of beef, each containing $\frac{6}{25}$ oz of fat. Who ate the greatest amount of fat from these entrees?

Section 3.6

Add. Simplify completely.

91. $\frac{5}{11} + \frac{2}{11}$

92. $\frac{7}{12} + \frac{4}{12}$

93. $\frac{2}{9} + \frac{2}{9} + \frac{2}{9}$

94. $\frac{3}{16} + \frac{2}{16} + \frac{3}{16}$

95. $\frac{7}{32} + \frac{8}{32} + \frac{5}{32}$

96. $\frac{5}{24} + \frac{7}{24} + \frac{9}{24}$

97. $\frac{4}{15} + \frac{1}{3}$

98. $\frac{7}{24} + \frac{3}{8}$

99. $\frac{3}{35} + \frac{8}{21}$

100. $\frac{11}{30} + \frac{9}{20} + \frac{3}{10}$

101. $\frac{1}{6} + \frac{7}{8} + \frac{7}{12}$

102. $\frac{7}{15} + \frac{11}{30} + \frac{5}{6}$

103. An elephant ear bamboo grew $\frac{1}{2}$ in. on Tuesday, $\frac{3}{8}$ in. on Wednesday, and $\frac{1}{4}$ in. on Thursday. How much did the bamboo grow in the 3 days?

104. In order to complete a project, Preston needs $\frac{1}{10}$ in. of foam, $\frac{3}{10}$ in. of metal, $\frac{4}{10}$ in. of wood, and $\frac{7}{10}$ in. of plexiglass. What will be the total thickness of this project when these materials are piled up?

Section 3.7*Add. Write the results as mixed numbers where possible.*

105.
$$\begin{array}{r} 2 \frac{4}{7} \\ + 3 \frac{11}{14} \\ \hline \end{array}$$

106.
$$\begin{array}{r} 17 \frac{5}{12} \\ + 1 \frac{5}{6} \\ \hline \end{array}$$

107.
$$\begin{array}{r} 2 \frac{7}{15} \\ + 13 \frac{2}{5} \\ \hline \end{array}$$

108.
$$\begin{array}{r} 4 \frac{7}{15} \\ + 6 \frac{2}{3} \\ \hline \end{array}$$

109.
$$\begin{array}{r} 7 \frac{3}{8} \\ + 5 \frac{5}{6} \\ \hline \end{array}$$

110.
$$\begin{array}{r} 15 \frac{4}{15} \\ + 5 \frac{5}{6} \\ \hline \end{array}$$

111.
$$\begin{array}{r} 14 \frac{7}{20} \\ + 11 \frac{3}{16} \\ \hline \end{array}$$

112.
$$\begin{array}{r} 11 \frac{7}{24} \\ + 32 \frac{7}{18} \\ \hline \end{array}$$

113. $28 \frac{1}{4} + 39 \frac{1}{3} + 12 \frac{5}{12}$

114. $18 \frac{3}{4} + 19 + 25 \frac{7}{12}$

115. $25 \frac{2}{3} + 16 \frac{1}{6} + 18 \frac{3}{4}$

116. $29 \frac{7}{8} + 19 \frac{5}{12} + 32 \frac{3}{4}$

- 117.** Russ rode his bicycle $3 \frac{3}{8}$ mi on Monday, $1 \frac{2}{3}$ mi on Tuesday, $1 \frac{7}{8}$ mi on Wednesday, $\frac{1}{2}$ mi on Thursday, and $4 \frac{1}{8}$ mi on Friday. What was his total mileage for the week?

- 118.** On a fishing excursion Roona caught four fish weighing $6 \frac{3}{4}$ lb, $1 \frac{3}{5}$ lb, $2 \frac{2}{3}$ lb, and $5 \frac{1}{2}$ lb. What was the total weight of her catch?

Section 3.8*Subtract.*

119. $\frac{5}{7} - \frac{3}{14}$

120. $\frac{5}{18} - \frac{2}{9}$

121. $\frac{5}{6} - \frac{1}{3}$

122. $\frac{17}{20} - \frac{1}{5}$

123. $\frac{19}{30} - \frac{1}{5}$

124. $\frac{7}{15} - \frac{3}{20}$

125. $\frac{5}{6} - \frac{4}{5}$

126. $\frac{7}{10} - \frac{1}{4}$

127. $\frac{18}{25} - \frac{7}{15}$

128. $\frac{21}{32} - \frac{5}{16}$

129. $\frac{47}{60} - \frac{17}{24}$

130. $\frac{13}{15} - \frac{9}{20}$

- 131.** Wanda finds $\frac{3}{4}$ oz of gold during a day of panning along the Snake River. She gives a $\frac{1}{3}$ oz nugget to Jose, her guide. What fraction of an ounce of gold does she have left?

- 132.** A carpenter planes the thickness of a board from $\frac{13}{16}$ to $\frac{5}{8}$ in. How much is removed?

Section 3.9

Subtract. Write the results as mixed numbers where possible.

$$\begin{array}{r} 145\frac{2}{3} \\ - 27\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 6\frac{5}{6} \\ - 3\frac{3}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 26\frac{1}{10} \\ - 10\frac{9}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 19\frac{3}{8} \\ - 8\frac{5}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 7\frac{1}{4} \\ - 5\frac{5}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 76\frac{7}{15} \\ - 50\frac{1}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 9\frac{9}{16} \\ - 3\frac{5}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 30\frac{7}{16} \\ - 22\frac{5}{6} \\ \hline \end{array}$$

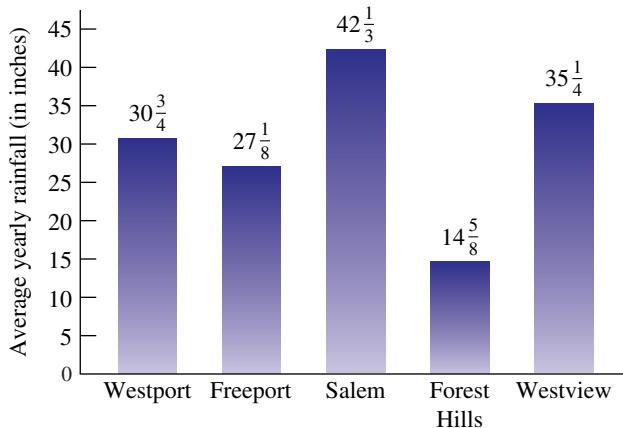
$$141. \quad 9\frac{9}{16} - 2\frac{5}{6}$$

$$142. \quad 33\frac{17}{30} - 25\frac{7}{9}$$

$$143. \quad 5\frac{31}{32} - \frac{3}{16}$$

$$144. \quad 8\frac{11}{20} - \frac{9}{10}$$

- 145.** The graph displays the average yearly rainfall for five cities.



- a. How much more rain falls in Westport during a year than in Freeport?
- b. In a 10-yr period, how much more rain falls in Salem than in Forest Hills?

- 146.** Using the graph in Exercise 145, if the average rain fall in Westview doubles, how much more rain would it receive than Salem?

Section 3.10

Perform the indicated operations.

147. $\frac{1}{4} + \frac{3}{8} \div \frac{1}{2}$

148. $\frac{1}{4} \div \frac{3}{8} + \frac{1}{2}$

149. $\frac{5}{8} \div \frac{3}{4} + \frac{3}{4}$

150. $\frac{1}{3} \div \frac{1}{6} + \frac{4}{9}$

151. $\frac{3}{4} - \left(\frac{1}{2}\right)^2$

152. $\frac{3}{4} - \left(\frac{1}{4}\right)^2$

153. $\left(\frac{9}{8}\right)^2 - \left(\frac{1}{2} \div \frac{4}{5} - \frac{3}{8}\right)$

154. $\left(\frac{1}{2}\right)^2 + \left(\frac{4}{5} \cdot \frac{5}{8} + \frac{2}{3}\right)$

Find the average.

155. $\frac{3}{8}, \frac{1}{4}, \frac{1}{2}$, and $\frac{3}{4}$

156. $\frac{3}{8}, \frac{3}{4}, \frac{1}{6}$, and $\frac{5}{8}$

157. $1\frac{2}{3}, 1\frac{5}{12}, 2\frac{1}{2}, \frac{3}{4}$, and $\frac{5}{6}$

158. $\frac{2}{3}, \frac{5}{12}, \frac{1}{2}, 1\frac{3}{4}$, and $1\frac{5}{6}$

- 159.** A class of 20 students took a 10-problem quiz. Their results were as follows.

Number of Students	Fraction of Problems Correct
3	$\frac{10}{10}$ (all correct)
1	$\frac{9}{10}$
3	$\frac{8}{10}$
5	$\frac{7}{10}$
4	$\frac{6}{10}$
3	$\frac{5}{10}$
1	$\frac{3}{10}$

What is the class average?

- 160.** What is the average of the top six scores in Exercise 159?

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True/False Concept Review CHAPTER 3

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. It is possible to picture an improper fraction using unit regions. 1. _____
2. The fraction $\frac{7}{8}$ written as a mixed number is $1\frac{7}{8}$. 2. _____
3. The whole number 1 can also be written as a proper fraction. 3. _____
4. A fraction is another way of writing a division problem. 4. _____
5. When a fraction is completely simplified, its value is less than 1. 5. _____
6. Every improper fraction can be changed to a mixed number or a whole number. 6. _____
7. Some fractions with large numerators and denominators cannot be further simplified. 7. _____
8. Two mixed numbers can be subtracted without first changing them to improper fractions. 8. _____
9. The reciprocal of an improper fraction is greater than 1. 9. _____
10. The quotient of two nonzero fractions can always be found by multiplication. 10. _____
11. Simplifying fractions is the opposite of building fractions. 11. _____
12. The primary reason for building fractions is so that they will have a common denominator. 12. _____
13. Like fractions have the same numerators. 13. _____
14. Mixed numbers must be changed to improper fractions before adding them. 14. _____
15. It is sometimes necessary to use “borrowing” to subtract mixed numbers as we do when subtracting some whole numbers. 15. _____
16. The order of operations for fractions is the same as the order of operations for whole numbers. 16. _____

Name _____ Class _____ Date _____

17. The average of three nonequivalent fractions is smaller than at least one of the fractions.

17. _____

18. The product of two fractions is sometimes smaller than the two fractions.

18. _____

Test CHAPTER 3**ANSWERS**

1. Change $\frac{61}{3}$ to a mixed number.

1. _____

2. Add: $\frac{7}{8} + \frac{5}{12}$

2. _____

3. Change $8\frac{7}{9}$ to an improper fraction

3. _____

4. List these fractions from the smallest to the largest: $\frac{2}{5}, \frac{3}{8}, \frac{3}{7}$.

4. _____

5. Change 11 to an improper fraction.

5. _____

6. Find the missing numerator: $\frac{3}{8} = \frac{?}{72}$

6. _____

7. Add:

$$\begin{array}{r} 5\frac{3}{10} \\ + 3\frac{5}{6} \\ \hline \end{array}$$

7. _____

8. Multiply. Write the result as a mixed number. $\left(3\frac{2}{3}\right)\left(5\frac{1}{9}\right)$

8. _____

9. Perform the indicated operations: $\frac{1}{2} - \frac{3}{8} \div \frac{3}{4}$

9. _____

10. Simplify $\frac{68}{102}$ completely.

10. _____

11. Subtract: $17\frac{4}{5} - 11$

11. _____

12. Multiply: $\frac{4}{5} \cdot \frac{7}{8} \cdot \frac{15}{21}$

12. _____

13. Subtract: $\frac{2}{3} - \frac{4}{9}$

13. _____

14. Divide: $1\frac{2}{9} \div 3\frac{2}{3}$

14. _____

15. Multiply: $\frac{3}{7} \cdot \frac{4}{5}$

15. _____

16. Subtract:

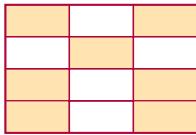
$$\begin{array}{r} 11\frac{7}{12} \\ - 4\frac{14}{15} \\ \hline \end{array}$$

17. Simplify $\frac{220}{352}$ completely.**18.** Add: $\frac{1}{35} + \frac{5}{14} + \frac{2}{5}$ **19.** What is the reciprocal of $3\frac{1}{5}$?**20.** What is the reciprocal of $\frac{8}{21}$?**21.** Which of these fractions are proper?

$$\frac{7}{8}, \frac{8}{8}, \frac{9}{8}, \frac{7}{9}, \frac{9}{7}, \frac{8}{9}, \frac{9}{9}$$

22. Divide: $\frac{7}{3} \div \frac{8}{9}$ **23.** Subtract:

$$\begin{array}{r} 11\frac{7}{10} \\ - 3\frac{3}{8} \\ \hline \end{array}$$

24. Write the fraction for the shaded part of this figure.**25.** Subtract: $11 - 3\frac{5}{11}$ **26.** Add: $\frac{4}{15} + \frac{8}{15}$ **27.** Find the average of $1\frac{3}{8}$, $\frac{1}{4}$, $3\frac{1}{2}$, and $2\frac{3}{8}$.**16.** _____**17.** _____**18.** _____**19.** _____**20.** _____**21.** _____**22.** _____**23.** _____**24.** _____**25.** _____**26.** _____**27.** _____

Name _____ Class _____ Date _____

28. Multiply: $\left(\frac{8}{25}\right)\left(\frac{9}{16}\right)$

28. _____

29. True or false? $\frac{5}{7} > \frac{11}{16}$

29. _____

30. Which of the fractions represent the number 1?

30. _____

$\frac{6}{5}, \frac{5}{5}, \frac{7}{6}, \frac{6}{6}, \frac{7}{7}, \frac{6}{7}, \frac{5}{7}$

31. A rail car contains $126\frac{1}{2}$ tons of baled hay. A truck that is being used to unload the hay can haul $5\frac{3}{4}$ tons in one load. How many truckloads of hay are in the rail car?

31. _____

32. Jill wants to make up 20 bags of homemade candy for the local bazaar. Each bag will contain $1\frac{1}{4}$ lb of candy. How many pounds of candy must she make?

32. _____

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Group Project (2 WEEKS) CHAPTER 3

OPTIONAL

One of the major applications of statistics is their value in predicting future occurrences. Before the future can be predicted, statisticians study what has happened in the past and look for patterns. If a pattern can be detected, and it is reasonable to assume that nothing will happen to interrupt the pattern, then it is a relatively easy matter to predict the future simply by continuing the pattern. Insurance companies, for instance, study the occurrences of traffic accidents among various groups of people. Once they have identified a pattern, they use this to predict future accident rates, which in turn are used to set insurance rates. When a group, such as teenaged boys, is identified as having a higher incidence of accidents, their insurance rates are set higher.

Dice

While predicting accident rates is a very complicated endeavor, there are other activities for which the patterns are relatively easy to find. Take, for instance, the act of rolling a die. The die has six sides, marked 1 to 6. Theoretically, each side has an equal chance of ending in the up position after a roll. Fill in the following table by rolling a die 120 times.

Side Up	1	2	3	4	5	6
Times Rolled						

Theoretically, each side will be rolled the same number of times as the others. Since you rolled the die 120 times and there were six possible outcomes, each side should come up $120 \div 6 = 20$ times. How close to 20 are your outcomes in the table? What do you suppose are reasons for not getting a perfectly distributed table?

Mathematicians are likely to express the relationships in this situation using the concept of *probability*, which is a measure of the likelihood of a particular event occurring. We describe the probability of an event with a fraction. The numerator of the fraction is the number of different ways the desired event can occur and the denominator of the fraction is the total number of possible outcomes. So the probability of rolling a 2 on the die is $\frac{1}{6}$ because there is only one way to roll a 2 but there are 6 possible outcomes when rolling a die. What is the probability of rolling a 5? What is the probability of rolling a 6? Nonmathematicians are more likely to express this relationship using the concept of *odds*. They would say that the odds of rolling a 2 are 1 in 6. This means that for every six times you roll a die, you can expect one of them to result in a 2.

Coin Toss

Suppose you and a friend each flip a coin. What are all the possible joint outcomes? What is the probability of getting two heads? What is the probability of getting two tails? What is the probability of getting one head and one tail? What does it mean if the probability of an event is $\frac{3}{3}$? Is it possible for the probability of an event to be $\frac{5}{4}$? Explain.

Cards

Suppose you pick a card at random out of a deck of playing cards. What is the probability that the card will be the queen of hearts? What is the probability that the card will be a queen? What is the probability that the card will be a heart?

Fill out the table below and try to discover the relationship among these three probabilities.

Probability of a Queen	Probability of a Heart	Probability of the Queen of Hearts

For a card to be the queen of hearts, two conditions must hold true at the same time. The card must be a queen *and* the card must be a heart. Make a guess about the relationship of the probabilities when two conditions must occur simultaneously. Test your guess by considering the probability of drawing a black 7. What are the two conditions that must be true in order for the card to be a black 7? What are their individual probabilities? Was your guess correct?

What two conditions must be true when you draw a red face card? What is the probability of drawing a red face card?

Suppose you pick a card at random out of a deck of playing cards. What is the probability that the card will be a 3 or a 4? What is the probability that the card will be a 3? A 4? Fill in the table to try to discover the relationship between these probabilities.

Probability of a 3	Probability of a 4	Probability of a 3 or a 4

A card is a 3 *or* a 4 if either condition holds. Make a guess about the relationship of the probabilities when either of two conditions must be true. Test your guess by calculating the probability that a card will be a heart or a club. Was your guess correct?

Sometimes a complicated probability is easier to calculate using a back-door approach. For instance, suppose you needed to calculate the probability that a card drawn is an ace or a 2 or a 3 or a 4 or a 5 or a 6 or a 7 or an 8 or a 9 or a 10 or a jack or a queen. You can certainly add the individual probabilities (what do you get?). However, another way to look at the situation is to ask what is the probability of not getting a king. We reason that if you do not get a king, then you do get one of the desired cards. We calculate this by subtracting the probability of getting a king from 1. This is because 1 must be the sum of all the probabilities that totally define the set (in this case, the sum of the probabilities of getting a king and the probability of getting one of the other cards). Verify that you get the same probability using both methods.

Cumulative Review CHAPTERS 1–3

Write the word name for each of the following.

1. 6,091 2. 110,532

Write the place value name.

3. One million three hundred ten 4. Sixty thousand two hundred fifty-seven

Round to the indicated place value.

5. 654,785 (hundred) 6. 43,949 (ten thousand)

Insert > or < between the numbers to make a true statement.

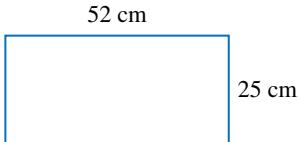
7. 6745 6739 8. 11,899 11,901

Add or subtract.

<p>9. 76,843 34,812 12,833 + 9,711</p>	<p>10. 55,304 —37,478</p>
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11. $54 + 87 + 124 + 784 + 490 + 54$ 12. $70,016 - 54,942$

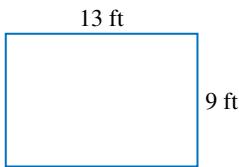
13. Find the perimeter of the rectangle.



Multiply.

<p>14. $\begin{array}{r} 14,654 \\ \times \quad 251 \\ \hline \end{array}$</p>	<p>15. $(341)(73)$</p>
--	-----------------------------------

16. Find the area.



Divide.

<p>17. $76 \overline{) 19,532}$</p>	<p>18. $16,702 \div 95$</p>	<p>19. Write in exponential form: $(7)(7)(7)(7)(7)(7)(7)(7)$</p>
--	--	---

Multiply or divide.

20. $54,000,000 \div 10^4$

21. 321×10^3

Simplify.

22. $8 \cdot 2^3 - 10 \div 2 + 6 \cdot 10 \div 12$

23. $80 \div 16 \cdot 3^2 - (5 \cdot 4 - 7)$

Find the average and median.

24. 345, 672, 801, 943, 144

25. 10,504, 13,654, 92,230, 65,236

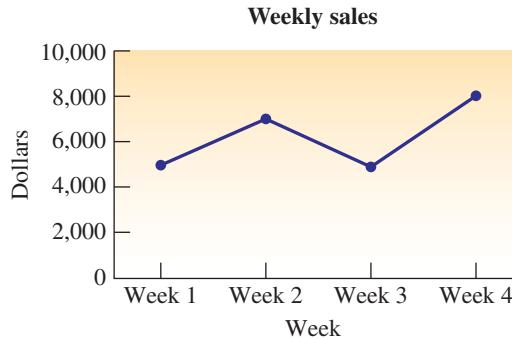
*Exercises 26–28 refer to the table.***Golf Scores**

	Round 1	Round 2	Round 3	Round 4
Tiger Woods	66	70	68	66
Ernie Els	70	65	73	69
Phil Mickelson	67	68	70	64

26. Find the average, median, and mode of all the scores.

27. Who scored the lowest score for a single round of golf? What was his score?

28. Who had the best score for the 72 holes (all four rounds)?

Exercises 29–30. The graph gives the weekly sales at the local convenience mart.

29. Which week had the highest weekly sales?

30. What is the approximate total sales for weeks 1 and 2?

31. Holli decided to redecorate her living room. She bought a new painting for \$235, a mirror for \$410, a love seat for \$895, and new carpeting for \$1864. What is the total amount she spent on the redecorating?

32. Russ left an estate valued at \$568,950. He designated that \$150,000 would go to the local community college foundation for scholarships. The remaining money was to be divided equally among his seven nieces and nephews. How much money did each niece and nephew receive?

- 33.** For four performances of *Guys and Dolls, A Semi-Staged Musical*, the Portland Symphony sold 2143 tickets at \$60 per ticket, 1867 tickets at \$47 per ticket, 976 tickets at \$36 per ticket, and 561 tickets at \$19 per ticket. What was the total revenue generated by the ticket sales?

Determine whether the natural number is divisible by 2, 3, or 5.

- 34.** 2850 **35.** 4436 **36.** 4515 **37.** 35,742

List the first five multiples of the whole number.

- 38.** 34 **39.** 140 **40.** Is 3584 a multiple of 7? **41.** Is 1340 a multiple of 8?

Write the whole number as a product of two factors in all possible ways.

- 42.** 224 **43.** 495 **44.** 280

List all the factors (divisors) of the whole number.

- 45.** 120 **46.** 165 **47.** 183

Tell whether the number is prime or composite.

- 48.** 45,873 **49.** 167 **50.** 11,112

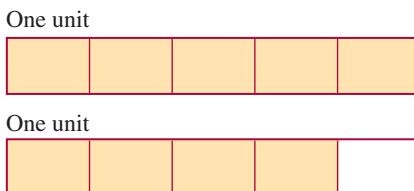
Write the prime factorization of the whole number.

- 51.** 420 **52.** 11,850

Find the least common multiple of the group of whole numbers.

- 53.** 7, 21, 30 **54.** 24, 28, 35, 45

- 55.** Write the fraction represented by the figure.



- 56.** Select the proper and improper fractions from the list.

$$\frac{6}{5}, \frac{8}{9}, \frac{12}{11}, \frac{18}{18}, \frac{19}{20}, \frac{23}{24}, \frac{31}{30}$$

Change to a mixed number.

- 57.** $\frac{27}{8}$ **58.** $\frac{165}{31}$

Change to an improper fraction.

59. $7\frac{1}{6}$

60. $17\frac{9}{11}$

Simplify completely.

61. $\frac{66}{102}$

62. $\frac{216}{360}$

Multiply or divide. Simplify completely.

63. $\frac{12}{35} \cdot \frac{49}{28}$

64. $\frac{40}{57} \cdot \frac{38}{48}$

65. $\frac{15}{16} \div \frac{18}{20}$

66. $\frac{25}{32} \div \frac{75}{40}$

Multiply or divide. Simplify completely and write as a mixed number if possible.

67. $\left(4\frac{8}{9}\right)\left(3\frac{7}{8}\right)$

68. $\left(11\frac{5}{7}\right)\left(9\frac{5}{8}\right)$

69. $8\frac{3}{5} \div 5\frac{3}{5}$

70. $13\frac{1}{8} \div 15\frac{2}{5}$

71. Write four fractions equivalent to $\frac{6}{7}$ by multiplying by $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$, and $\frac{5}{5}$.*Find the missing numerator.*

72. $\frac{4}{7} = \frac{?}{35}$

73. $\frac{9}{22} = \frac{?}{66}$

74. List the fractions from smallest to largest: $\frac{3}{5}, \frac{6}{25}, \frac{11}{15}, \frac{1}{6}$ 75. True or false: $\frac{21}{40} < \frac{18}{35}$ 76. True or false: $\frac{9}{25} > \frac{13}{40}$ *Add or subtract. Simplify completely and write as mixed number if possible.*

77. $\frac{4}{11} + \frac{1}{11} + \frac{5}{11}$

78. $\frac{8}{9} + \frac{5}{12} + \frac{2}{3}$

79. $\frac{17}{35} - \frac{8}{21}$

80. $5\frac{2}{3} + 1\frac{5}{6} + 3\frac{3}{4}$

81.
$$\begin{array}{r} 19\frac{4}{15} \\ - 14\frac{7}{12} \\ \hline \end{array}$$

82. $\frac{7}{10} + \frac{8}{15} + \frac{9}{20} + \frac{5}{6}$

Perform the indicated operations.

83. $\frac{7}{8} - \frac{5}{6} \cdot \frac{8}{15}$

84. $\frac{3}{5} + \left(\frac{2}{3}\right)^2 - \frac{4}{9} \div \frac{8}{3}$

85. Find the average and median of $\frac{1}{6}, \frac{7}{18}, \frac{7}{12}, \frac{5}{18}, \frac{3}{4}$, and $\frac{8}{9}$.

86. Find the perimeter of the rectangle.

$6\frac{3}{4}$ in.



87. Find the area of the rectangle in Exercise 86.

88. Ersula is making a batch of cookies. The recipe calls for $2\frac{2}{3}$ cups of flour. If Ersula doubles the recipe, how many cups of flour will she need?

89. The Perez family spends $\frac{3}{20}$ of their income on rent, $\frac{1}{6}$ on food, $\frac{1}{15}$ on transportation, and $\frac{1}{5}$ on taxes. What fraction of their income is spent on these items?

90. During the past 6 months, the following amounts of rain were recorded: $2\frac{4}{5}$ in., $1\frac{1}{2}$ in., $3\frac{1}{3}$ in., $5\frac{2}{5}$ in., $2\frac{3}{5}$ in., and $4\frac{1}{2}$ in. Find the average monthly rainfall during these 6 months.

Planning Makes Perfect

Now is the time to formalize a study plan. Set aside a time of day, every day, to focus all of your attention on math. For some students, finding a quiet place in the library to study regularly for 1 hour is far more efficient than studying 2 hours at home, where there are constant distractions. For others, forming a study group where you can talk about what you have learned is helpful. Decide which works best for you.

Try to schedule time as close to the class session as possible while the concepts are fresh in your mind. If you wait several hours to practice what seemed clear during class, you may find that what was clear earlier may no longer be meaningful. This may mean planning a schedule of classes that includes an hour after class to study.

If, on some days, you cannot devote 1 or 2 hours to math, find at least a few minutes and review one thing—perhaps read your notes, reread the section objectives, or if you want to do a few problems, do the section Warm-Ups. This helps to keep the concepts fresh in your memory.

If you choose to form or join a study group with other math-anxious students, don't use your time together to gripe. Instead, use it to discuss and recognize the content of your negative self-talk and to write positive coping statements.

Plan, too, for your physical health. Notice how your anxious thought patterns trigger physical tension. When you wrinkle your forehead, squint your eyes, make a trip to the coffee machine, or light up a cigarette, you are looking for a way to release these tensions. Learning relaxation techniques, specifically progressive relaxation, is a healthier alternative to controlling body tension. Briefly, relaxation training involves alternately tensing and relaxing all of the major muscles in the body with the goal of locating your specific muscle tension and being able to relax it away. Use a professionally prepared progressive relaxation tape, or take a stress management class to properly learn this technique. Allow at least 20 to 30 minutes for this exercise daily. The time it takes for you to deeply relax will become briefer as you become more skilled. Soon relaxation will be as automatic as breathing, and when you find yourself feeling math anxious, you can stop, take control, and relax.

4

DECIMALS



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APPLICATION

Sports hold a universal attraction. People all over the world enjoy a good game. For some sports, it is relatively easy to determine which athlete is the best. In track, downhill skiing, and swimming, for instance, each contestant races against the clock and the fastest time wins. In team sports, it is easy to tell which team wins, but sometimes difficult to determine how the individual athletes compare with one another. In order to make comparisons more objective, we often use sports statistics.

The simplest kind of statistic is to count how many times an athlete performs a particular feat in a single game. In basketball, for instance, it is usual to count the number of points scored, the number of rebounds made, and the number of assists for each player.

Consider the following playoff statistics for members of the Detroit Pistons in their win of the 2005 NBA championship.

Player	Points Scored	Rebounds	Assists
Billups, Chauncey	141	36	56
Hamilton, Richard	165	26	39
Prince, Tayshaun	128	56	37
Wallace, Ben	95	106	8
Wallace, Rasheed	123	54	12

Group Discussion

1. Which player had the best overall statistics? Justify your answer.
2. Which is more important in basketball, rebounds or assists? Explain.
3. Between Billups and Hamilton, which had the best overall performance? Explain.

SECTIONS

- 4.1** Decimals: Reading, Writing, and Rounding
- 4.2** Changing Decimals to Fractions; Listing in Order
- 4.3** Adding and Subtracting Decimals
- 4.4** Multiplying Decimals
- 4.5** Multiplying and Dividing by Powers of 10; Scientific Notation
- 4.6** Dividing Decimals; Average, Median, and Mode
- 4.7** Changing Fractions to Decimals
- 4.8** Order of Operations; Estimating

4.1 Decimals: Reading, Writing, and Rounding

OBJECTIVES

1. Write word names from place value names and place value names from word names.
2. Round a given decimal.

VOCABULARY

Decimal numbers, more commonly referred to as **decimals**, are another way of writing fractions and mixed numbers. The digits used to write whole numbers and a period called a **decimal point** are used to write place value names for these numbers.

The **number of decimal places** is the number of digits to the right of the decimal point. **Exact decimals** are decimals that show exact values.

Approximate decimals are rounded values.

How & Why

OBJECTIVE 1

Write word names from place value names and place value names from word names.

Decimals are written by using a standard place value in the same way we write whole numbers. Numbers such as 12.65, 0.45, 0.795, 1306.94, and 19.36956 are examples of decimals.

In general, the place value for decimals is

1. The same as whole numbers for digits to the left of the decimal point, and
2. A fraction whose denominator is 10, 100, 1000, and so on, for digits to the right of the decimal point.

The digits to the right of the decimal point have place values of

$$\frac{1}{10^1} = \frac{1}{10} = 0.1$$

$$\frac{1}{10^2} = \frac{1}{10 \cdot 10} = \frac{1}{100} = 0.01$$

$$\frac{1}{10^3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1000} = 0.001$$

$$\frac{1}{10^4} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10} = \frac{1}{10,000} = 0.0001$$

and so on, in that order from left to right.

Using the ones place as the central position, the place values of a decimal are shown in Figure 4.1.

Whole-number part					Decimal point	Fractional part				
Ten thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousands	Ten-thousandths	Hundred-thousandths
...	10,000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$
...	10^4	10^3	10^2	10^1	1	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	$\frac{1}{10^5}$

Figure 4.1

$$71.961 = \underbrace{71}_{\text{Whole-number part}} + \underbrace{.961}_{\text{Fraction part}}$$

If the decimal point is not written, as in the case of a whole number, the decimal point is understood to follow the ones place; thus,

$$87 = 87. \quad 7 = 7. \quad 725 = 725.$$

We can write an expanded form of the decimal using fractions with denominators that are powers of 10. So

$$71.961 = 70 + 1 + \frac{9}{10} + \frac{6}{100} + \frac{1}{1000}.$$

The expanded form can also be written:

7 tens + 1 one + 9 tenths + 6 hundredths + 1 thousandth.

Table 4.1 shows how to write the word names for 237.58 and 0.723.

Table 4.1 Word Names for Decimals

	Number to Left of Decimal Point	Decimal Point	Number to Right of Decimal Point	Place Value of Last Digit
Place value name	237	.	58	$\frac{1}{100}$
Word name of each	Two hundred thirty-seven	and	fifty-eight	hundredths
Word name for decimal	Two hundred thirty-seven and fifty-eight hundredths			
Place value name	0	.	723	$\frac{1}{1000}$
Word name of each	Omit	Omit	Seven hundred twenty-three	thousandths
Word name for decimal	Seven hundred twenty-three thousandths			

For numbers greater than zero and less than one (such as 0.639), it is preferable to write the digit 0 in the ones place.

To write the word name for a decimal

1. Write the name for the whole number to the left of the decimal point.
2. Write the word *and* for the decimal point.
3. Write the whole number name for the number to the right of the decimal point.
4. Write the place value of the digit farthest to the right.

If the decimal has only zero or no digit to the left of the decimal point, omit steps 1 and 2.

Table 4.2 shows some other numbers and their corresponding word names.

Table 4.2 Word Names for Decimals

Number	Word Name
31.45	Thirty-one and forty-five hundredths
0.795	Seven hundred ninety-five thousandths
0.00082	Eighty-two hundred-thousandths
6.004	Six and four thousandths

To write the place value name for a decimal

1. Write the whole number. (The number before the word *and*.)
2. Write a decimal point for the word *and*.
3. Ignoring the place value name, write the name for the number following the word *and*. Insert zeros, if necessary, between the decimal point and the digits following it to ensure that the place on the far right has the correct (given) place value.

So the place value notation for three hundred thirteen and forty-two thousandths is

- 313 First write the whole number to the left of the word *and*.
313. Write a decimal point for the word *and*.
313.042 The “whole number” after the word *and* is 42. A zero is inserted to place the 2 in the thousandths place.

Warm-Ups A–D

Examples A–D

DIRECTIONS: Write the word name.

STRATEGY: Write the word name for the whole number to the left of the decimal point. Then write the word *and* for the decimal point. Finally, write the word name for the number to the right of the decimal point followed by the place value of the digit farthest to the right.

- A. Write the word name for 0.48.

Ninety-seven

Write the word name for the number right of the decimal point.

Ninety-seven hundredths

Next, write the place value of the digit 7. The word name for 0 in the ones place may be written or omitted. “Zero and ninety-seven hundredths” is correct but unnecessary.

- B. Write the word name for 0.091.

Twenty-seven ten-thousandths

- C. Write the word name for 123.053.

C. Write the word name for 556.43.

Five hundred fifty-six

Write the word name for the whole number left of the decimal point.

Write the word *and* for the decimal point.

Write the word name for the number right of the decimal point.

Write the place value of the digit 3.

Answers to Warm-Ups

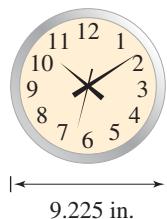
- A. forty-eight hundredths
B. ninety-one thousandths
C. one hundred twenty-three and fifty-three thousandths

Five hundred fifty-six and

Five hundred fifty-six and forty-three

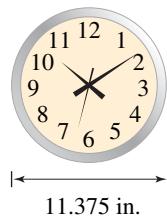
Five hundred fifty-six and forty-three hundredths

- D. Janet called an employee to find the measurement of the outside diameter of a new wall clock the company is manufacturing. She asked the employee to check the plans. What is the word name the employee will read to her? The clock is shown below.



The employee will read “Nine and two hundred twenty-five thousandths inches.”

- D. The measurement of the outside diameter of another clock is shown in the diagram below. What word name will the employee read?



Examples E–F BASIC Mathematics Now™

DIRECTIONS: Write the place value name.

STRATEGY: Write the digit symbols for the corresponding words. Replace the word *and* with a decimal point. If necessary, insert zeros.

- E. Write the place value name for thirty-eight ten-thousandths.

38 First, write the number for thirty-eight.

.0038 The place value “ten-thousandths” indicates four decimal places, so write two zeros before the numeral thirty-eight and then a decimal point. This puts the numeral 8 in the ten-thousandths place.

0.0038 Since the number is between zero and one, we write a 0 in the ones place.

- F. Write the place value name for “four hundred five and four hundred five ten-thousandths.”

405 The whole number part is 405.

405. Write the decimal point for *and*.

405.0405 The “whole number” after the *and* is 405. A zero is inserted so the numeral 5 is in the ten-thousandths place.

Warm-Ups E–F

- E. Write the place value name for seventy-six hundred-thousandths.

- F. Write the place value name for seven hundred three and three hundred seven ten-thousandths.

How & Why

OBJECTIVE 2

Round a given decimal.

Decimals can be either *exact* or *approximate*. For example, decimals that count money are exact. The figure \$56.35 shows an exact amount. Most decimals that describe measurements are approximations. For example, 6.1 ft shows a person’s height to the nearest tenth of a foot and 1.9 m shows the height to the nearest tenth of a meter, but neither is an exact measure.

Answers to Warm-Ups

- D. The employee will read “Eleven and three hundred seventy-five thousandths inches.”
E. 0.00076 F. 703.0307

Decimals are rounded using the same procedure as for whole numbers. Using a ruler (see Figure 4.2), we round 2.563.

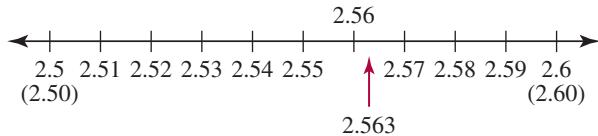


Figure 4.2

To the nearest tenth, 2.563 is rounded to 2.6, because it is closer to 2.6 than to 2.5. Rounded to the nearest hundredth, 2.563 is rounded to 2.56, because it is closer to 2.56 than to 2.57.

To round 21.8573 to the nearest hundredth, without drawing a number line, draw an arrow under the hundredths place to identify the round-off place.

21.8573
↑

We must choose between 21.85 and 21.86. Because the digit to the right of the round-off position is 7, the number is more than halfway to 21.86. So we choose the larger number.

$$21.8573 \approx 21.86$$

CAUTION

Do not replace the dropped digits with zeros if the round-off place is to the right of the decimal point. $21.8573 \approx 21.8600$ indicates a round-off position of ten-thousandths.

To round a decimal number to a given place value

1. Draw an arrow under the given place value. (After enough practice, you will be able to round mentally and will not need the arrow.)
2. If the digit to the right of the arrow is 5, 6, 7, 8, or 9, add 1 to the digit above the arrow; that is, round to the larger number.
3. If the digit to the right of the arrow is 0, 1, 2, 3, or 4, keep the digit above the arrow; that is, round to the smaller number.
4. Write whatever zeros are necessary after the arrow so that the number above the arrow has the same place value as the original. See Example H.

This method is sometimes called the “four-five” rule. Although this rounding procedure is the most commonly used, it is not the only way to round. Many government agencies round by *truncation*; that is, by dropping the digits after the decimal point. Thus, $\$87.32 \approx \87 . It is common for retail stores to round up for any amounts smaller than one cent. Thus, $\$3.553 \approx \3.56 . There is also a rule for rounding numbers in science, which is sometimes referred to as the “even/odd” rule. You might learn and use a different round-off rule depending on the kind of work you are doing.

Examples G–J



DIRECTIONS: Round as indicated.

STRATEGY: Draw an arrow under the given place value. Examine the digit to the right of the arrow to determine whether to round up or down.

- G. Round 0.7539 to the nearest hundredth.

$0.7539 \approx 0.75$ The digit to the right of the round-off place is 3, so round down.
↑

- H. Round 7843.9 to the nearest thousand.

$7843.9 \approx 8000$ Three zeros must be written after the 8 to keep it
↑ in the thousands place.

- I. Round 419.8 to the nearest unit.

$419.8 \approx 420$ The number to the right of the round-off place is 8,
 so we round up by adding $419 + 1 = 420$.

- J. Round 83.5738 and 9.9976 to the nearest unit, the nearest tenth, the nearest hundredth, and the nearest thousandth.

Unit	Tenth	Hundredth	Thousandth
$83.5738 \approx 84$	≈ 83.6	≈ 83.57	≈ 83.574
$9.9976 \approx 10$	≈ 10.0	≈ 10.00	≈ 9.998

CAUTION

The zeros following the decimal in 10.0 and 10.00 are necessary to show that the original number was rounded to the nearest tenth and hundredth, respectively.

Warm-Ups G–J

- G. Round 0.56892 to the nearest thousandth.

- H. Round 883.89 to the nearest ten.

- I. Round 7859.6 to the nearest unit.

- J. Round 57.6737 to the nearest unit, the nearest tenth, the nearest hundredth, and the nearest thousandth.

Answers to Warm-Ups

- G. 0.569 H. 880 I. 7860
J. 58; 57.7; 57.67; 57.674

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Exercises 4.1

OBJECTIVE 1

Write word names from place value names and place value names from word names.

A Write the word name.

- | | | | |
|----------|-----------|----------|----------|
| 1. 0.26 | 2. 0.82 | 3. 0.267 | 4. 0.943 |
| 5. 3.007 | 6. 6.0708 | 7. 11.92 | 8. 32.03 |

Write the place value name.

- | | |
|--|---------------------------------------|
| 9. Forty-two hundredths | 10. Sixty-nine hundredths |
| 11. Four hundred nine thousandths | 12. Five hundred nineteen thousandths |
| 13. Seven and thirty-three thousandths | 14. Ten and four hundredths |
| 15. One hundred five ten-thousandths | 16. Twelve ten-thousandths |

B Write the word name.

- | | | |
|------------|-------------|-------------|
| 17. 0.805 | 18. 8.05 | 19. 80.05 |
| 20. 8.005 | 21. 37.0705 | 22. 39.0071 |
| 23. 90.003 | 24. 900.030 | |

Write the place value name.

- | | |
|---|---|
| 25. Thirty-five ten-thousandths | 26. Thirty-five thousand |
| 27. One thousand eight hundred and twenty-eight thousandths | 28. Three hundred and fifteen thousandths |
| 29. Five hundred five and five thousandths | 30. Five and five hundred five thousandths |
| 31. Sixty-seven and eighty-three hundredths | 32. Five hundred two and five hundred two thousandths |

OBJECTIVE 2

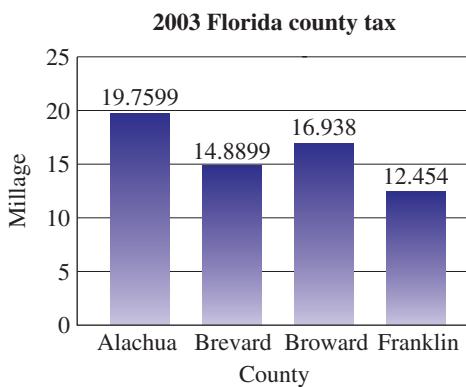
Round a given decimal.

A Round to the nearest unit, tenth, and hundredth.**Unit Tenth Hundredth****33.** 35.777**34.** 73.788**35.** 819.735**36.** 922.444**37.** 0.7359**38.** 0.67382

Round to the nearest cent.

39. \$67.4856**40.** \$27.6372**41.** \$548.7235**42.** \$375.7545**B** Round to the nearest ten, hundredth, and thousandth.**Ten Hundredth Thousandth****43.** 35.7834**44.** 61.9639**45.** 86.3278**46.** 212.7364**47.** 0.9165**48.** 0.6137

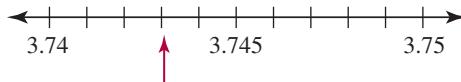
Round to the nearest dollar.

49. \$72.49**50.** \$38.51**51.** \$5647.49**52.** \$7356.73**C** Exercises 53–56. The graph shows the 2003 tax for four counties in Florida by millage.**53.** Write the word name for the millage in Brevard County.**54.** Write the word name for the millage in Franklin County.**55.** Round the millage for Broward County to the nearest hundredth.**56.** Round the millage for Alachua County to the nearest thousandth.

- 57.** Dan Ngo buys a ~~fit~~ screen monitor for his computer that has a marked price of \$275.85. What word name does he write on the check?

- 58.** Fari Alhadet makes a down payment on a used Honda Accord. The down payment is \$816.95. What word name does she write on the check?

Exercises 59–62. Use the graph of the number line.



- 59.** What is the position of the arrow to the nearest hundredth?
- 60.** What is the position of the arrow to the nearest thousandth?
- 61.** What is the position of the arrow to the nearest tenth?
- 62.** What is the position of the arrow to the nearest unit?

Write the word name.

- 63.** 567.9023
- 64.** 6055.6055
- 65.** The computer at Grant's savings company shows that his account, including the interest he has earned, has a value of \$3478.59099921. Round the value of the account to the nearest cent.
- 66.** In doing her homework, Catherine's calculator shows that the answer to a division exercise is 25.69649912. If she is to round the answer to the nearest thousandth, what answer does she report?

Write the place value name.

- 67.** Two hundred thirteen and one thousand, one hundred one ten-thousandths
- 68.** Eleven thousand five and one thousand fifteen hundred-thousandths
- 69.** In 2004, Kuwait had an estimated population density of 117.631 people per square kilometer. Round the population density to the nearest whole person per square kilometer.
- 70.** One text lists the mean (average) distance from Earth to the sun as 92,960,000 miles. To what place does this number appear to have been rounded?
- 71.** The number of pet birds per household in the United states is estimated at 0.10. To what place does this number appear to have been rounded?

Round to the indicated place value.

Thousand	Hundredth	Ten-thousandth
----------	-----------	----------------

- 72.** 3742.80596
- 73.** 7564.35926
- 74.** 2190.910053
- 75.** 78,042.38875

Exercises 76–79 relate to the chapter application.

- 76.** In January of 1906 a Stanley car with a steam engine set a 1-mile speed record by going 127.659 miles per hour. Round this rate to the nearest tenth of a mile per hour.
- 78.** In October 1970 a Blue Flame set a 1-mile speed record by going 622.407 mph. Explain why it is incorrect to round the rate to 622.5 mph.
- 77.** In March 1927 a Sunbeam auto set a 1-mile speed record by going 203.790 mph. What place value was this rate rounded to?
- 79.** On October 15, 1997, Andy Green recorded the first supersonic land 1-mile record in a Thrust SSC with a speed of 763.035 mph. Round this speed to the nearest whole mile per hour and to the nearest tenth of a mile per hour.

STATE YOUR UNDERSTANDING

- 80.** Explain the difference between an exact decimal value and an approximate decimal value. Give an example of each.
- 81.** Explain in words, the meaning of the value of the 4s in the numerals 43.29 and 18.64. Include some comment on how and why the values of the digit 4 are alike and how and why they are different.
- 82.** Consider the decimal represented by abc.defg. Explain how to round this number to the nearest hundredth.

CHALLENGE

- 83.** Round 8.28282828 to the nearest thousandth. Is the rounded value less than or greater than the original value? Write an inequality to illustrate your answer.
- 85.** Write the place value name for five hundred twenty-two hundred-millionths.
- 84.** Round 7.7777777 to the nearest unit. Is the rounded value less than or greater than the original value? Write an inequality to illustrate your answer.
- 86.** Write the word name for 40,715.300519.

GROUP ACTIVITY

- 87.** Discuss with the members of the group cases in which you think “truncating” is the best way to round. Rounding by truncating means to drop all digit values to the right of the rounding position. For example
- $2.77 \approx 2$, $\$19.33 \approx \19 , $34,999 \approx 34,000$. Can each of the groups think of a situation in which such rounding is actually used?

MAINTAIN YOUR SKILLS*Find the missing numerator.*

88. $\frac{3}{7} = \frac{?}{28}$

89. $\frac{7}{12} = \frac{?}{132}$

90. $\frac{42}{125} = \frac{?}{10,000}$

91. $\frac{19}{40} = \frac{?}{1000}$

True or false.

92. $3971 < 3969$

93. $30,951 > 30,899$

List the numbers from smallest to largest.

94. 367, 401, 392, 363, 390

95. 6227, 6223, 6218, 6209, 6215

96. During a recent candy sale, Mia sold 21 boxes of candy each containing 24 candy bars. Chia sold 16 bags of candy, each containing 30 candy bars. Who sold more candy bars?

97. Kobe Bryant hits 17 of 20 field goals attempted. If in the next game he attempts 40 field goals, how many must he hit to have the same shooting percentage?

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4.2 Changing Decimals to Fractions; Listing in Order

How & Why

OBJECTIVE 1

Change a decimal to a fraction.

The word name of a decimal is also the word name of a fraction.

Consider 0.615.

READ: six hundred fifteen thousandths

WRITE: $\frac{615}{1000}$

$$\text{So, } 0.615 = \frac{615}{1000} = \frac{123}{200}$$

Some other examples are shown in the Table 4.3.

OBJECTIVES

1. Change a decimal to a fraction.
2. List a set of decimals from smallest to largest.

Table 4.3 Decimals to Fractions

Place Value Name	Word Name	Fraction
0.77	Seventy-seven hundredths	$\frac{77}{100}$
0.9	Nine tenths	$\frac{9}{10}$
0.225	Two hundred twenty-five thousandths	$\frac{225}{1000} = \frac{9}{40}$
0.48	Forty-eight hundredths	$\frac{48}{100} = \frac{12}{25}$

To change a decimal to a fraction

1. Read the decimal word name.
2. Write the fraction that has the same value.
3. Simplify.

Notice that because of place value, the number of decimal places in a decimal tells us the number of zeros in the denominator of the fraction. This fact can be used as another way to write the fraction or to check that the fraction is correct:

$$4.\overbrace{58}^{\text{Two decimal places}} = 4\overbrace{\frac{58}{100}}^{\text{Two zeros}} = 4\frac{29}{50} \quad \text{or} \quad 4.58 = \frac{458}{100} = \frac{229}{50} = 4\frac{29}{50}$$

Warm-Ups A–C

Examples A–C



BASIC Mathematics Now™

A. Change 0.83 to a fraction.

Forty-three hundredths Say the word name.

$$\frac{43}{100}$$
 Write as a fraction.

B. Write 0.675 as a fraction.

Two hundred ninety-six thousandths Say the word name.

$$\frac{296}{1000} = \frac{37}{125}$$
 Write as a fraction and simplify.

C. Write as a mixed number:

45.15

19.44

$$19\frac{44}{100} = 19\frac{11}{25}$$
 Write as a mixed number and simplify.

Nineteen and forty-four hundredths Say the word name.

How & Why

OBJECTIVE 2

List a set of decimals from smallest to largest.

Fractions can be listed in order when they have a common denominator by ordering the numerators. This idea can be extended to decimals when they have the same number of decimal places. For instance, $0.36 = \frac{36}{100}$ and $0.47 = \frac{47}{100}$ have a common denominator

when written in fractional form. Because $\frac{36}{100} < \frac{47}{100}$, we know that 0.36 is less than 0.47; or $0.36 < 0.47$.

The decimals 0.7 and 0.59 do not have a common denominator. But we can make common denominators by placing a zero after the 7. Thus,

$$0.7 = \frac{7}{10} = \frac{7}{10} \cdot \frac{10}{10} = \frac{70}{100} = 0.70$$

so that

$$0.7 = \frac{70}{100} \quad \text{and} \quad 0.59 = \frac{59}{100}$$

Then, since $\frac{70}{100} > \frac{59}{100}$, we conclude that $0.7 > 0.59$.

We can also use a number line to order decimals. The number line shows that $2.6 < 3.8$ because 2.6 is to the left of 3.8. See Figure 4.3.

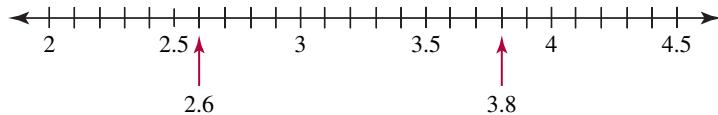


Figure 4.3

Answers to Warm-Ups

- A. $\frac{83}{100}$ B. $\frac{27}{40}$ C. $45\frac{3}{20}$

Many forms for decimal numbers are equivalent. For example,

$$6.3 = 6.30 = 6.300 = 6.3000 = 6.30000$$

$$0.85 = 0.850 = 0.8500 = 0.85000 = 0.850000$$

$$45.982 = 45.9820 = 45.98200 = 45.982000 = 45.9820000$$

The zeros to the right of the decimal point following the last nonzero digit do not change the value of the decimal. Usually these extra zeros are not written, but they are useful when operating with decimals.

To list a set of decimals from smallest to largest

1. Make sure that all numbers have the same number of decimal places to the right of the decimal point by writing zeros to the right of the last digit when necessary.
2. Write the numbers in order as if they were whole numbers.
3. Remove the extra zeros.

Examples D–F



BASIC
Mathematics Now™

DIRECTIONS: Is the statement true or false?

STRATEGY: Write each numeral with the same number of decimal places. Compare the values without regard to the decimal point.

D. True or false: $0.84 > 0.81$?

$84 > 81$ is true. Compare the numbers without regard to the decimal points.

So, $0.84 > 0.81$ is true.

E. True or false: $0.743 < 0.74$?

$0.743 < 0.740$ Write with the same number of decimal places.

$743 < 740$ is false. Without regard to the decimal point, 743 is larger.

So, $0.743 < 0.74$ is false.

F. True or false: $32.008 > 32.09$?

$32.008 > 32.090$ Write with the same number of decimal places.

$32008 > 32090$ is false.

So, $32.008 > 32.09$ is false.

Warm-Ups D–F

D. True or false:
 $0.54 < 0.53$?

E. True or false:
 $0.31 < 0.3199$?

F. True or false:
 $66.7 > 66.683$?

Answers to Warm-Ups

D. false E. true F. true

Warm-Ups G–I

Examples G–I



- G. List 0.56, 0.558, 0.5601, and 0.559 from smallest to largest.

- H. List 3.03, 3.0033, 3.0333, and 3.0303 from smallest to largest.

- I. Roberto and Rachel measure the same coin. Roberto measures 0.916 and Rachel measures 0.9153. Whose measure is larger?

DIRECTIONS: List the decimals from smallest to largest.

STRATEGY: Write zeros on the right so that all numbers have the same number of decimal places. Compare the numbers as if they were whole numbers and then remove the extra zeros.

- G. List 0.48, 0.472, 0.4734, and 0.484 from smallest to largest.

0.4800 First, write all numbers with the same number of decimal places by
0.4720 inserting zeros on the right.
0.4734
0.4840

0.4720, 0.4734, 0.4800, 0.4840 Second, write the numbers in order as if they
were whole numbers.

0.472, 0.4734, 0.48, 0.484 Third, remove the extra zeros.

- H. List 8.537, 8.631, 8.5334, and 8.538 from smallest to largest.

8.537 = 8.5370 Step 1
8.631 = 8.6310
8.5334 = 8.5334
8.538 = 8.5380

8.5334, 8.5370, 8.5380, 8.6310 Step 2

8.5334, 8.537, 8.538, 8.631 Step 3

- I. Using a micrometer to measure the diameter of a foreign coin, Mike measures 0.6735 and Mildred measures 0.672. Whose measure is larger?

0.6735, 0.6720 Write with the same number of decimal places and compare
as whole numbers.

Mike's measure is larger since $6735 > 6720$.

Answers to Warm-Ups

- G. 0.558, 0.559, 0.56, 0.5601
H. 3.0033, 3.03, 3.0303, 3.0333
I. Roberto's measure is larger.

Exercises 4.2

OBJECTIVE 1

Change a decimal to a fraction.

A Change each decimal to a fraction and simplify if possible.**1.** 0.91**2.** 0.37**3.** 0.65**4.** 0.6**5.** Four hundred twenty-nine thousandths**6.** Three hundred one thousandths**7.** 0.78**8.** 0.32**9.** 0.48**10.** 0.55**B** Change the decimal to a fraction or mixed number and simplify.**11.** 7.23**12.** 36.39**13.** 0.125**14.** 0.575**15.** 9.16**16.** 47.64**17.** 11.344**18.** 5.228**19.** Eight hundred thousandths**20.** Twenty-five hundred-thousandths**OBJECTIVE 2**

List a set of decimals from smallest to largest.

A List the set of decimals from smallest to largest.**21.** 0.8, 0.5, 0.2**22.** 0.04, 0.03, 0.035**23.** 0.27, 0.19, 0.38**24.** 0.46, 0.48, 0.29**25.** 3.26, 3.185, 3.179**26.** 7.18, 7.183, 7.179

Is the statement true or false?

27. $0.38 < 0.3$ **28.** $0.49 < 0.50$ **29.** $10.48 > 10.84$ **30.** $7.78 > 7.87$ **B** List the set of decimals from smallest to largest.**31.** 0.0477, 0.047007, 0.047, 0.046, 0.047015**32.** 1.006, 1.106, 0.1006, 0.10106**33.** 0.888, 0.88799, 0.8881, 0.88579**34.** 5.48, 5.4975, 5.4599, 5.4801**35.** 25.005, 25.051, 25.0059, 25.055**36.** 92.0728, 92.0278, 92.2708, 92.8207

Is the statement true or false?

37. $3.1231 < 3.1213$

38. $6.3456 > 6.345$

39. $74.6706 < 74.7046$

40. $21.6043 > 21.6403$

C

41. The probability that a flipped coin will come up heads four times in a row is 0.0625. Write this as a reduced fraction.

42. The probability that a flipped coin will come up heads twice and tails once out of three flips is 0.375. Write this as a reduced fraction.

43. The Alpenrose Dairy bids \$2.675 per gallon to provide milk to the local school district. Tillamook Dairy puts in a bid of \$2.6351, and Circle K Dairy makes a bid of \$2.636. Which is the best bid for the school district?

44. Larry loses 3.135 pounds during the week. Karla loses 3.183 pounds and Mitchell loses 3.179 pounds during the same week. Who loses the most weight this week?

Exercises 45–47. The following free-throw records are established in the National Basketball Association: highest percentage made in a season: 0.832, Boston Celtics in 1989–90; lowest percentage made in a season: 0.635, Philadelphia in 1967–68; lowest percentage made by both teams in a single game: 0.410, Los Angeles vs. Chicago in 1968.

45. Write a simplified fraction to show the highest percentage of free throws made in a season.

46. Write a simplified fraction to show the lowest percentage of free throws made in a season.

47. Write a simplified fraction to show the lowest percentage of free throws made in a game by both teams.

Change the decimal to a fraction or mixed number and simplify.

48. 0.1775

49. 0.3875

50. 403.304

51. 65.075

52. Gerry may choose a 0.055 raise in pay or a $\frac{1}{18}$ increase.

53. A chemistry class requires 0.667 mg of soap for each student. Hoa has 0.67 mg of soap. Does she need more or less soap?

Which value will yield more money? Compare in fraction form.

List the decimals from smallest to largest.

54. 0.00829, 0.0083001, 0.0082, 0.0083, 0.0083015

55. 5.0009, 5.001, 5.00088, 5.00091, 5.00101

56. 49.457, 49.449, 49.501, 49.576, 49.491, 49.5011

57. 37.96, 38.01, 37.95, 37.59, 38.15, 38.25, 37.90, 37.79

58. Lee is pouring a concrete patio in his back yard. Lee needs 2.375 cubic yards of concrete for his patio. Change the amount of concrete to a mixed number and simplify.

59. For a bow for a prom dress, Maria may choose 0.725 yd or $\frac{5}{7}$ yd for the same price. Which should she choose to get the most ribbon? Compare in fraction form.

- 60.** One synodic day on Jupiter (midday to midday) is about 9.925933 hours, while one sidereal day (measured by apparent star movements) is about 9.925 hours. Which is longer?
- 62.** Betty Crocker cake mixes, when prepared as directed, have the following decimal fraction of the calories per slice from fat: Apple Cinnamon, 0.36; Butter Pecan, 0.4; Butter Recipe/Chocolate, 0.43; Chocolate Chip, 0.42; Spice, 0.38; and Golden Vanilla, 0.45. If each slice contains 280 calories, which cake has the most calories from fat? Fewest calories from fat?
- 61.** The population density of Israel is about 277.595 people per square kilometer and the population density of the West Bank is about 276.0575 people per square kilometer. Which geographic area has fewer people per square kilometer?

Exercises 64–67 relate to the chapter application.

- 64.** At one point in the 20042005 NBA season, the listed teams had won the given decimal fraction of their games: Seattle, 0.700; Phoenix, 0.759; San Antonio, 0.774; Detroit, 0.627; Miami, 0.741; New York, 0.556; Sacramento, 0.623; and Cleveland, 0.588. Rank these teams from best record to the worst record.
- 66.** The table displays the batting champions in each league for 20002004.
- 65.** For the 20032004 season, Shaquille O’Neal of the LA Lakers had the top field goal percentage in the NBA. That year O’Neal made 0.584 of his field goal attempts. Express this record as a fraction. What fraction of his field goals did he miss?
- 67.** Sort the table in Exercise 66, so that the averages are displayed from highest to lowest by league.

National League			
Year	Name	Team	Average
2000	Todd Hetton	Colorado	0.372
2001	Larry Walker	Colorado	0.350
2002	Barry Bonds	San Francisco	0.370
2003	Albert Pujols	St. Louis	0.359
2004	Barry Bonds	San Francisco	0.362

American League			
Year	Name	Team	Average
2000	Nomar Garciaparra	Boston	0.372
2001	Ichiro Suzuki	Seattle	0.350
2002	Manny Ramirez	Boston	0.349
2003	Bill Mueller	Boston	0.326
2004	Ichiro Suzuki	Seattle	0.372

Which player had the highest batting average in the 5-year period?

STATE YOUR UNDERSTANDING

- 68.** Explain how the number line can be a good visual aid for determining which of two numerals has the larger value.

CHALLENGE

- 69.** Change 0.44, 0.404, and 0.04044 to fractions and reduce.
- 70.** Determine whether each statement is true or false.
- | | |
|----------------------------------|-----------------------------------|
| a. $7.44 < 7\frac{7}{18}$ | b. $8.6 > 8\frac{5}{9}$ |
| c. $3\frac{2}{7} < 3.285$ | d. $9\frac{3}{11} > 9.271$ |

GROUP WORK

- 71.** Find a pattern, a vehicle manual, and/or a parts list whose measurements are given in decimal form. Change the measurements to fraction form.
- 72.** Have each member of the group write one fraction and one decimal each with values between 3 and 4. Then as a group, list all the fractions and decimal values from smallest to largest.

MAINTAIN YOUR SKILLS

Add or subtract.

73. $479 + 3712 + 93 + 7225$

74. $75,881 + 3007 + 45,772 + 306$

75. $34,748 - 27,963$

76. $123,007 - 17,558$

77. $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$

78. $\frac{1}{3} + \frac{7}{12} + \frac{5}{6}$

79. $\frac{25}{64} - \frac{3}{8}$

80. $\frac{17}{20} - \frac{7}{12}$

- 81.** Pedro counted the attendance at the seven-screen MetroPlex Movie Theater for Friday evening. He had the following counts by screen: #1, 456; #2, 389; #3, 1034; #4, 672; #5, 843; #6, 467; #7, 732. How many people attended the theater that Friday night?

- 82.** Joanna has \$1078 in her bank account. She writes checks for \$54, \$103, \$152, \$25, and \$456. What balance does she now have in her account?

4.3 Adding and Subtracting Decimals

How & Why

OBJECTIVE 1

Add decimals.

What is the sum of $27.3 + 42.5$? We make use of the expanded form of the decimal to explain addition.

$$\begin{array}{r} 27.3 = 2 \text{ tens} + 7 \text{ ones} + 3 \text{ tenths} \\ + 42.5 = 4 \text{ tens} + 2 \text{ ones} + 5 \text{ tenths} \\ \hline 6 \text{ tens} + 9 \text{ ones} + 8 \text{ tenths} = 69.8 \end{array}$$

We use the same principle for adding decimals that we use for whole numbers—that is, we add like units. The vertical form gives us a natural grouping of the tens, ones, and tenths. By inserting zeros so all the numbers have the same number of decimal places, we write the addition $6.4 + 23.9 + 7.67$ as $6.40 + 23.90 + 7.67$.

$$\begin{array}{r} 6.40 \\ 23.90 \\ + 7.67 \\ \hline 37.97 \end{array}$$

OBJECTIVES

1. Add decimals.
2. Subtract decimals.

To add decimals

1. Write in columns with the decimal points aligned. Insert extra zeros to help align the place values.
2. Add the decimals as if they were whole numbers.
3. Align the decimal point in the sum with those above.

Examples A–C



BASIC Mathematics Now™

DIRECTIONS: Add.

STRATEGY: Write each numeral with the same number of decimal places, align the decimal points, and add.

A. Add: $8.2 + 56.93 + 38 + 0.08$

$$\begin{array}{r} 8.20 \quad \text{Write each numeral with two decimal places. The extra zeros} \\ 56.93 \quad \text{help line up the place values.} \\ 38.00 \\ + 0.08 \\ \hline 103.21 \end{array}$$

CALCULATOR EXAMPLE



B. Add: $3.8756 + 0.0338 + 12.2 + 36.921$

STRATEGY: The extra zeros do not need to be inserted. The calculator will automatically align the like place values when adding.

The sum is 53.0304.

Warm-Ups A–C

A. Add:

$$7.3 + 82.51 + 66 + 0.06$$

B. Add: $15.6327 + 0.0078 + 12.94 + 83.556$

Answers to Warm-Ups

A. 155.87 B. 112.1365

- C. Cheryl goes to Nordstrom Rack and buys the following: shoes, \$56.29; hose, \$15.95; pants, \$29.95; and jacket, \$47.83. How much does Cheryl spend at Nordstrom?

- C. Wanda goes to Target and buys the following: greeting cards, \$4.65; Diet Coke, \$2.45; lamp, \$25.99; camera, \$42.64; and dishwashing soap, \$3.86. What is the total cost of Wanda's purchase?

STRATEGY: Add the prices of each item.

$$\begin{array}{r} \$4.65 \\ 2.45 \\ 25.99 \\ 42.64 \\ + 3.86 \\ \hline \$79.59 \end{array}$$

Wanda spends \$79.59 at Target.

How & Why

OBJECTIVE 2

Subtract decimals.

What is the difference $8.68 - 4.37$? To find the difference, we write the numbers in column form, aligning the decimal points. Now subtract as if they are whole numbers.

$$\begin{array}{r} 8.68 \\ -4.37 \\ \hline 4.31 \end{array}$$

The decimal point in the difference is aligned with those above.

When necessary, we can regroup, or borrow, as with whole numbers. What is the difference $7.835 - 3.918$?

$$\begin{array}{r} 7.835 \\ -3.918 \\ \hline \end{array}$$

We need to borrow 1 from the hundredths column (1 hundredth = 10 thousandths) and we need to borrow 1 from the ones column (1 one = 10 tenths).

$$\begin{array}{r} 6\ 18\ 2\ 15 \\ 7.\ 8\ 3\ 5 \\ -3.\ 9\ 1\ 8 \\ \hline 3.\ 9\ 1\ 7 \end{array}$$

So the difference is 3.917.

Sometimes it is necessary to write zeros on the right so the numbers have the same number of decimal places. See Example E.

To subtract decimals

1. Write the decimals in columns with the decimal points aligned. Insert extra zeros to align the place values.
2. Subtract the decimals as if they are whole numbers.
3. Align the decimal point in the difference with those above.

Answers to Warm-Ups

- C. Cheryl spends \$150.02 at Nordstrom.

Examples D–H



DIRECTIONS: Subtract.

STRATEGY: Write each numeral with the same number of decimal places, align the decimal points, and subtract.

- D. Subtract: $21.573 - 5.392$

$$\begin{array}{r} 21.573 \\ - 5.392 \\ \hline \end{array}$$

Line up the decimal points so the place values are aligned.

$$\begin{array}{r} 21.573 \\ - 5.392 \\ \hline 16.181 \end{array}$$

Borrow 1 tenth from the 5 in the tenths place.
(1 tenth = 10 hundredths)

$$\begin{array}{r} 21.573 \\ - 5.392 \\ \hline 16.181 \end{array}$$

Borrow 1 ten from the 2 in the tens place.
(1 ten = 10 ones)

CHECK: $5.392 + 16.181 = 21.573$ Check by adding.

$$\begin{array}{r} +16.181 \\ 21.573 \\ \hline \end{array}$$

The difference is 16.181.

- E. Subtract 6.83 from 9.

$$\begin{array}{r} 9.00 \\ - 6.83 \\ \hline \end{array}$$

We write 9 as 9.00 so that both numerals will have the same number of decimal places.

$$\begin{array}{r} 9.00 \\ - 6.83 \\ \hline 2.17 \end{array}$$

We need to borrow to subtract in the hundredths place.
Since there is a 0 in the tenths place, we start by borrowing 1 from the ones place. (1 one = 10 tenths)

$$\begin{array}{r} 9.00 \\ - 6.83 \\ \hline 2.17 \end{array}$$

Now borrow 1 tenth to add to the hundredths place.
(1 tenth = 10 hundredths)
Subtract.

CHECK: $2.17 + 6.83 = 9.00$

$$\begin{array}{r} +6.83 \\ 9.00 \\ \hline \end{array}$$

The difference is 2.17.

- F. Find the difference of 9.271 and 5.738. Round to the nearest tenth.

$$\begin{array}{r} 9.271 \\ - 5.738 \\ \hline 3.533 \end{array}$$

The check is left for the student.

The difference is 3.5 to the nearest tenth

CAUTION

Do not round before subtracting. Note the difference if we do:

$$9.3 - 5.7 = 3.6$$

Warm-Ups D–H

- D. Subtract: $17.535 - 9.476$

- E. Subtract 4.642 from 7.

- F. Find the difference of 8.493 and 3.736. Round to the nearest tenth.

Answers to Warm-Ups

D. 8.059 E. 2.358 F. 4.8

- G. Subtract:
 $934.4466 - 345.993$

- H. Mickey buys a DVD for \$24.79. She gives the clerk a \$10 bill and a \$20 bill to pay for it. How much change does she get?

CALCULATOR EXAMPLE

G. Subtract: $759.3471 - 569.458$

STRATEGY: The calculator automatically lines up the decimal points.

The difference is 189.8891.

- H. Marta purchases antibiotics for her son. The antibiotics cost \$34.86. She gives the clerk two \$20 bills. How much change does she get?

STRATEGY: Since two \$20 bills are worth \$40, subtract the cost of the antibiotics from \$40.

$$\begin{array}{r} \$40.00 \\ - 34.86 \\ \hline \$5.14 \end{array}$$

Marta gets \$5.14 in change.

Clerks sometimes make change by counting backward, that is, by adding to \$34.86 the amount necessary to equal \$40.

$$\begin{aligned} \$34.86 + 4 \text{ pennies} &= \$34.90 \\ \$34.90 + \text{a dime} &= \$35.00 \\ \$35.00 + 5 \text{ dollars} &= \$40.00 \end{aligned}$$

So the change is $\$0.04 + \$0.10 + \$5 = \5.14 .

Answers to Warm-Ups

- G. 588.4536
H. Mickey gets \$5.21 in change.

Exercises 4.3

OBJECTIVE 1

Add decimals.

A Add.

1. $0.8 + 0.5$

2. $0.6 + 0.5$

3. $3.7 + 2.2$

4. $8.5 + 3.6$

5. $4.2 + 1.8 + 7.2$

6. $6.7 + 2.3 + 4.6$

7. $34.8 + 5.29$

8. $12.6 + 9.34$

9. To add 7.6, 6.7821, 9.752, and 61, first rewrite each with _____ decimal places.

10. The sum of 6.7, 10.56, 5.993, and 45.72 has _____ decimal places.

B

11.
$$\begin{array}{r} 21.3 \\ + 34.567 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 37.8 \\ + 9.45 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 4.15 \\ 0.73 \\ 24.8 \\ + 4.64 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 25.86 \\ 6.29 \\ 21.8 \\ + 8.94 \\ \hline \end{array}$$

15. $2.337 + 0.672 + 4.056$

16. $9.445 + 5.772 + 0.822$

17. $0.0017 + 1.007 + 7 + 1.071$

18. $1.0304 + 1.4003 + 1.34 + 0.403$

19. $59.045 + 47.009 + 5.43 + 7.368$

20. $63.008 + 8.93 + 7.043 + 51.77$

21. $0.0781 + 0.00932 + 0.07639 + 0.00759$

22. $7.006 + 0.9341 + 0.003952 + 4.0444$

23. $0.067 + 0.456 + 0.0964 + 0.5321 + 0.112$

24. $4.005 + 0.875 + 3.96 + 7.832 + 4.009$

25.
$$\begin{array}{r} 7.8 \\ 35.664 \\ + 76.9236 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 15.07 \\ 189.981 \\ + 6904.4063 \\ \hline \end{array}$$

27.
$$\begin{array}{r} 75.995 \\ 24.9 \\ + 694.447 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 314.143 \\ 712.217 \\ + 333.444 \\ \hline \end{array}$$

29. Find the sum of 11.08, 8.1, 0.346, and 9.07.

30. Find the sum of 2.6933, 5.895, 0.99, and 5.328.

OBJECTIVE 2

Subtract decimals.

A Subtract.

31. $0.7 - 0.4$

32. $5.8 - 5.6$

33. $9.5 - 5.2$

34. $0.75 - 0.42$

35.
$$\begin{array}{r} 6.45 \\ -2.35 \\ \hline \end{array}$$

36.
$$\begin{array}{r} 36.29 \\ -5.17 \\ \hline \end{array}$$

37.
$$\begin{array}{r} 21.56 \\ -18.49 \\ \hline \end{array}$$

38.
$$\begin{array}{r} 37.81 \\ -29.63 \\ \hline \end{array}$$

39. Subtract 11.14 from 32.01.

40. Find the difference of 23.465 and 9.9.

B Subtract.

41.
$$\begin{array}{r} 0.723 \\ -0.457 \\ \hline \end{array}$$

42.
$$\begin{array}{r} 7.403 \\ -3.625 \\ \hline \end{array}$$

43.
$$\begin{array}{r} 2.712 \\ -1.148 \\ \hline \end{array}$$

44.
$$\begin{array}{r} 8.554 \\ -3.477 \\ \hline \end{array}$$

45. $0.831 - 0.462$

46. $0.067 - 0.049$

47. $33.456 - 29.457$

48. $7.598 - 4.7732$

49. $216.47 - 134.563$

50. $708.7087 - 563.563$

51.
$$\begin{array}{r} 0.0764 \\ -0.03621 \\ \hline \end{array}$$

52.
$$\begin{array}{r} 0.0056982 \\ -0.003781 \\ \hline \end{array}$$

53. $41.8341 - 34.6152$

54. $7.9342 - 2.78932$

55. $0.075 - 0.0023$

56. $0.00675 - 0.000984$

57. Subtract 56.78 from 61.02.

58. Subtract 6.607 from 11.5.

59. Find the difference of 9.453 and 9.278.

60. Find the difference of 82.606 and 65.334.

C Perform the indicated operations.

61. $0.0643 + 0.8143 + 0.513 - (0.4083 + 0.7114)$

62. $7.619 + 13.048 - (1.699 + 2.539 + 4.87)$

63. $7.045 - (3.24 + 1.893) - 0.0561$

64. $13.095 - (6.334 - 2.556) + 5.231$

65. On a vacation trip, Manuel stopped for gas four times. The first time, he bought 19.2 gallons. At the second station he bought 21.9 gallons, and at the third, he bought 20.4 gallons. At the last stop, he bought 23.7 gallons. How much gas did he buy on the trip?

66. Heather wrote five checks in the amounts of \$63.78, \$44.56, \$394.06, \$11.25, and \$67.85. She has \$595.94 in her checking account. Does she have enough money to cover the five checks?

67. Find the sum of 457.386, 423.9, 606.777, 29.42, and 171.874. Round the sum to the nearest tenth.

68. Find the sum of 641.85, 312.963, 18.4936, 29.0049, and 6.1945. Round the sum to the nearest hundredth.

Exercises 69–71. The table shows the estimated gross sales in billions of dollars for selected business sectors in New Mexico for 2004.

Building Materials	\$0.42	Motor Vehicles	\$0.51
Department Stores	\$1.32	Restaurants	\$1.63
Retail Food	\$2.24	General Merchandise	\$1.16

- 69.** Find the total sales for the six business sectors
- 70.** How many more dollars were spent on retail food than in restaurants?
- 71.** How many dollars were spent in the nonfood sectors?
- 72.** Doris makes a gross salary (before deductions) of \$3565 per month. She has the following monthly deductions: federal income tax, \$320.85; state income tax, \$192.51; Social Security, \$196.07; Medicare, \$42.78; retirement contribution, \$106.95; union dues, \$45; and health insurance, \$214.35. Find her actual take-home (net) pay.
- 73.** Jack goes shopping with \$95 in cash. He pays \$8.99 for a T-shirt, \$8.64 for a CD, and \$29.50 for a sweater. On the way home, he buys \$21.58 worth of gas. How much money does he have left?
- 74.** In 1996, the average interest on a 30-year home mortgage was 7.56 percent. In 2004, the average interest was 6.15 percent. What was the drop in interest rate?
- 75.** What is the total cost of a cart of groceries that contains bread for \$2.18, bananas for \$2.37, cheese for \$4.97, cereal for \$4.28, coffee for \$5.16, and meat for \$11.89?

Exercises 76–77. The table shows the lengths of railway tunnels in various countries.

World's Longest Railway Tunnels

Tunnel	Length (km)	Country
Seikan	53.91	Japan
English Channel Tunnel	49.95	UK–France
Dai-shimizu	22.53	Japan

- 76.** How much longer is the longest tunnel than the second longest tunnel?
- 77.** What is the total length of the Japanese tunnels?

Exercises 78–80. The table shows projections for the number of families without children under 18.

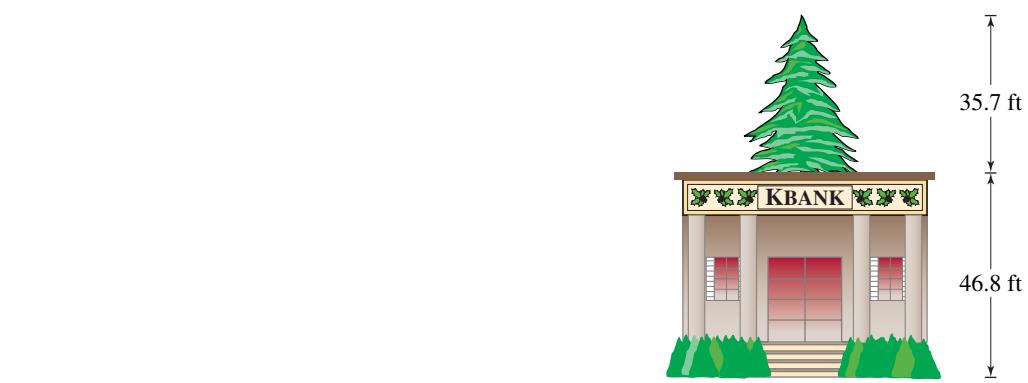
Projected Number of Families without Children under 18

	Year			
	1995	2000	2005	2010
Families without Children under 18 (in millions)	35.8	38.6	42.0	45.7

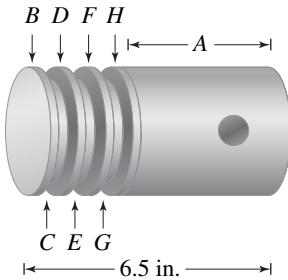
(Source: U.S. Census Bureau)

- 78.** What is the projected change in the number of families in the United States without children under 18 between 1995 and 2010?
- 79.** Which 5-year period is projected for the largest change?

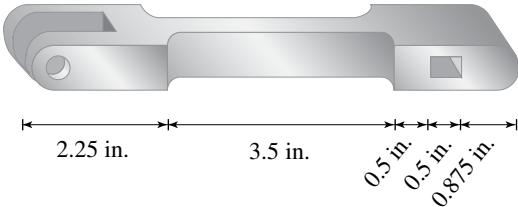
80. What could explain the increase indicated in the table?



82. Find the length of the piston skirt (A) shown below if the other dimensions are as follows: $B = 0.3125$ in., $C = 0.250$ in., $D = 0.3125$ in., $E = 0.250$ in., $F = 0.3125$ in., $G = 0.375$ in., $H = 0.3125$.



84. Find the total length of the connecting bar shown below.

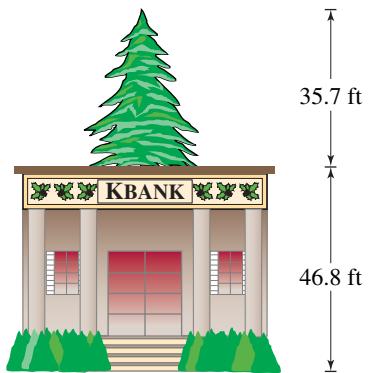


Exercises 85–89 relate to the chapter application.

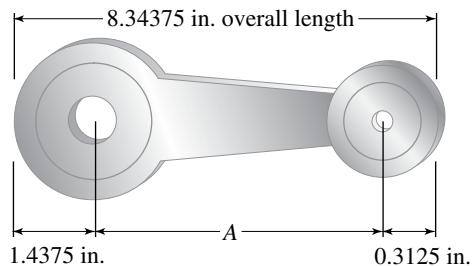
85. Muthoni runs a race in 12.16 seconds, whereas Sera runs the same race in 11.382 seconds. How much faster is Sera?

87. A college men's 4- \times -100-m relay track team has runners with individual times of 9.35 sec, 9.91 sec, 10.04 sec, and 9.65 sec. What is the time for the relay?

81. How high from the ground level is the top of the tree shown below? Round to the nearest foot.



83. What is the center-to-center distance, A , between the holes in the diagram?



86. A skier posts a race time of 1.257 minutes. A second skier posts a time of 1.32 minutes. The third skier completes the race in 1.2378 minutes. Find the difference between the fastest and the slowest times.

88. A high-school girls' swim team has a 200-yd freestyle relay, in which swimmers have times of 21.79 sec, 22.64 sec, 22.38 sec, and 23.13 sec. What is the time for the relay?

- 89.** A high-school women's track coach knows that the rival school's team in the 4- \times -100-m relay has a time of 52.78 sec. If the coach knows that her top three sprinters have times of 12.83, 13.22, and 13.56 sec, how fast does the fourth sprinter need to be in order to beat the rival school's relay team?

STATE YOUR UNDERSTANDING

- 90.** Explain the procedure for adding 2.005, 8.2, 0.00004, and 3.
- 91.** Explain the similarities between subtracting decimals and subtracting fractions.
- 92.** Copy the table and fill it in.

Operation on Decimals	Procedure	Example
Addition		
Subtraction		

CHALLENGE

- 93.** How many 5.83s must be added to have a sum that is greater than 150?
- 94.** Find the missing number in the sequence: 0.4, 0.8, 1.3, _____, 2.6, 3.4, 4.3, 5.3.
- 95.** Find the missing number in the sequence: 0.2, 0.19, 0.188, _____, 0.18766, 0.187655.
- 96.** Which number in the following group is 11.1 less than 989.989: 999.999, 989.999, 988.889, 979.889, or 978.889?
- 97.** Write the difference between $6\frac{7}{16}$ and 5.99 in decimal form.
- 98.** Round the sum of 9.8989, 8.9898, 7.987, and 6.866 to the nearest tenth.

GROUP WORK

- 99.** As a group, review the multiplication of fractions. Have each member make up a pair of fractions whose denominators are in the list: 10, 100, 1000, and 10,000.

Find the product of each pair and change it to decimal form. In group discussion, make up a rule for multiplying decimals.

MAINTAIN YOUR SKILLS*Multiply.*

100. $62(217)$

102.
$$\begin{array}{r} 6921 \\ \times 415 \\ \hline \end{array}$$

104. $\frac{1}{5} \cdot \frac{9}{10}$

106. $4\frac{2}{5} \cdot 2\frac{4}{5}$

- 108.**
- A nursery plants one seedling per square foot of ground. How many seedlings can be planted in a rectangular plot of ground that measures 310 ft by 442 ft?

101. $703(557)$

103. $(83)(27)(19)$

105. $\frac{36}{75} \cdot \frac{15}{16} \cdot \frac{40}{27}$

107. $\left(4\frac{1}{2}\right)\left(5\frac{3}{5}\right)$

- 109.**
- Harry and David puts 24 pears in its Royal Golden Pear Box. How many pears are needed to fill an order for 345 Royal Golden Pear Boxes?

Getting Ready for Algebra

How & Why

We solve equations that involve addition and subtraction of decimals in the same way as equations with whole numbers and fractions.

To solve an equation using addition or subtraction

1. Add the same number to both sides of the equation to isolate the variable, or
2. Subtract the same number from both sides of the equation to isolate the variable.

OBJECTIVE

Solve equations that involve addition and subtraction of decimals.

Examples A–E

DIRECTIONS: Solve.

STRATEGY: Isolate the variable by adding or subtracting the same number to or from both sides.

A. $7.8 = x + 5.6$

$$\begin{array}{rcl} 7.8 & = & x + 5.6 \\ 7.8 - 5.6 & = & x + 5.6 - 5.6 \\ 2.2 & = & x \end{array}$$

CHECK: $7.8 = 2.2 + 5.6$
 $7.8 = 7.8$

The solution is $x = 2.2$.

B. $z - 14.9 = 32.7$

$$\begin{array}{rcl} z - 14.9 & = & 32.7 \\ + 14.9 & = & + 14.9 \\ \hline z & = & 47.6 \end{array}$$

CHECK: $47.6 - 14.9 = 32.7$
 $32.7 = 32.7$

The solution is $z = 47.6$.

C. $b + 17.325 = 34.6$

$$\begin{array}{rcl} b + 17.325 & = & 34.6 \\ b + 17.325 - 17.325 & = & 34 - 17.325 \\ b & = & 17.275 \end{array}$$

CHECK: $17.275 + 17.325 = 34.6$
 $34.6 = 34.6$

The solution is $b = 17.275$.

Warm-Ups A–E

A. $11.7 = p + 4.2$

B. $t - 13.6 = 29.5$

C. $c + 56.785 = 62$

Answers to Warm-Ups

- A. $p = 7.5$ B. $t = 43.1$
C. $c = 5.215$

D. $w - 33.17 = 12.455$

- E. A farmer practicing “sustainable” farming reduced his soil erosion by 1.58 tons in 1 year. If he lost 3.94 tons of topsoil this year to erosion, how many tons did he lose last year?

D. $y - 6.233 = 8.005$

$$\begin{array}{rcl} y - 6.233 & = & 8.005 \\ + 6.233 & = & + 6.233 \\ \hline y & = & 14.238 \end{array}$$

Add 6.233 to both sides and simplify.

CHECK: $14.238 - 6.233 = 8.005$ Substitute 14.238 for y in the original equation.
 $8.005 = 8.005$ True.

The solution is $y = 14.238$.

- E. The price of a graphing calculator decreased by \$19.30 over the past year. What was the price a year ago if the calculator now sells for \$81.95?

First write the English version of the equation:

$$(\text{cost last year}) - (\text{decrease in cost}) = \text{cost this year}$$

Let x represent the cost last year.

$$\begin{array}{ll} x - 19.30 = 81.95 & \text{Translate to algebra.} \\ x - 19.30 + 19.30 = 81.95 + 19.30 & \text{Add 19.30 to both sides.} \\ x = 101.25 & \text{Simplify.} \end{array}$$

Because $101.25 - 19.30 = 81.95$, the cost of the calculator last year was \$101.25.

Answers to Warm-Ups

- D. $w = 45.625$
E. The farmer lost 5.52 tons of topsoil to erosion.

Exercises

Solve.

- | | | |
|--|---|----------------------------------|
| 1. $16.3 = x + 5.2$ | 2. $6.904 = x + 3.5$ | 3. $y - 0.64 = 13.19$ |
| 4. $w - 0.08 = 0.713$ | 5. $t + 0.03 = 0.514$ | 6. $x + 14.7 = 28.43$ |
| 7. $x - 7.3 = 5.21$ | 8. $y - 9.3 = 0.42$ | 9. $7.33 = w + 0.13$ |
| 10. $14 = x + 7.6$ | 11. $t - 8.37 = 0.08$ | 12. $w - 0.03 = 0.451$ |
| 13. $5.78 = a + 1.94$ | 14. $55.9 = w - 11.8$ | 15. $6.6 = x - 9.57$ |
| 16. $7 = 5.9 + x$ | 17. $a + 82.3 = 100$ | 18. $b + 45.76 = 93$ |
| 19. $s - 2.5 = 4.773$ | 20. $r - 6.7 = 5.217$ | 21. $c + 432.8 = 1029.16$ |
| 22. $d - 316.72 = 606.5$ | | |
| 23. The price of an energy-efficient hot-water heater decreased by \$52.75 over the past 2 years. What was the price 2 years ago if the heater now sells for \$374.98? | 24. In one state the use of household biodegradable cleaners increased by 2444.67 lb per month because of state laws banning phosphates. How many pounds of these cleaners were used before the new laws if the average use now is 5780.5 lb? | |
| 25. The selling price of a personal computer is \$1033.95. If the cost is \$875.29, what is the markup? | 26. The selling price of a new tire is \$128.95. If the markup on the tire is \$37.84, what is the cost (to the store) of the tire? | |
| 27. A shopper needs to buy a bus pass and some groceries. The shopper has \$61 with which to make both purchases. If the bus pass costs \$24, write and solve an equation that represents the shopper's situation. How much can the shopper spend on groceries? | 28. In a math class, the final grade is determined by adding the test scores and the homework scores. If a student has a homework score of 18 and it takes a total of 90 to receive a grade of A, what total test score must the student have to receive a grade of A? Write an equation and solve it to determine the answer. | |

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4.4 Multiplying Decimals

How & Why

OBJECTIVE

Multiply decimals.

OBJECTIVE

Multiply decimals.

The “multiplication table” for decimals is the same as for whole numbers. In fact, decimals are multiplied the same way as whole numbers with one exception: the location of the decimal point in the product. To discover the rule for the location of the decimal point, we use what we already know about multiplication of fractions. First, change the decimal form to fractional form to find the product. Next, change the product back to decimal form and observe the number of decimal places in the product. Consider the examples in Table 4.4. We see that the product in decimal form has the same number of decimal places as the total number of places in the decimal factors.

Table 4.4 Multiplication of Decimals

Decimal Form	Fractional Form	Product of Fractions	Product as a Decimal	Number of Decimal Places in Product
0.3×0.8	$\frac{3}{10} \times \frac{8}{10}$	$\frac{24}{100}$	0.24	Two
11.2×0.07	$\frac{112}{10} \times \frac{7}{100}$	$\frac{784}{1000}$	0.784	Three
0.02×0.13	$\frac{2}{100} \times \frac{13}{100}$	$\frac{26}{10,000}$	0.0026	Four

The shortcut is to multiply the numbers and insert the decimal point. If necessary, insert zeros so that there are enough decimal places. The product of 0.2×0.3 has two decimal places, because tenths multiplied by tenths yields hundredths.

$$0.2 \times 0.3 = 0.06 \quad \text{because} \quad \frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$$

To multiply decimals

1. Multiply the numbers as if they were whole numbers.
2. Locate the decimal point by counting the number of decimal places (to the right of the decimal point) in both factors. The total of these two counts is the number of decimal places the product must have.
3. If necessary, zeros are inserted to the *left of the numeral* so there are enough decimal places (see Example D).

When multiplying decimals, it is not necessary to align the decimal points in the decimals being multiplied.

Warm-Ups A–F

Examples A–F



BASIC Mathematics Now™

A. Multiply: $(8)(0.7)$

B. Find the product of 0.4 and 0.76.

C. Find the product of 32.8 and 0.46.

D. Multiply 0.03 times 0.091.

E. Find the product:
 $49.54(37.603)$

F. If 12 strips, each 6.45 centimeters wide, are to be cut from a piece of sheet metal, what is the narrowest piece of sheet metal that can be used?

Answers to Warm-Ups

- A. 5.6 B. 0.304 C. 15.088
- D. 0.00273 E. 1862.85262
- F. The sheet metal must be 77.4 centimeters wide.

DIRECTIONS: Multiply.

STRATEGY: First multiply the numbers, ignoring the decimal points. Place the decimal point in the product by counting the number of decimal places in the two factors. Insert zeros if necessary to produce the number of required places.

A. Multiply: $(0.9)(17)$

$$(0.9)(17) = 15.3 \quad \text{Multiply 9 and 17. The total number of decimal places in both factors is one (1), so there is one decimal place in the product.}$$

$$\text{So, } (0.9)(17) = 15.3.$$

B. Find the product of 0.8 and 0.57.

$$(0.8)(0.57) = 0.456 \quad \text{There are three decimal places in the product because the total number of places in the factors is three.}$$

So the product of 0.8 and 0.57 is 0.456.

C. Find the product of 9.73 and 6.8.

$$\begin{array}{r} 9.73 \\ \times 6.8 \\ \hline 7784 \\ 58380 \\ \hline 66.164 \end{array} \quad \text{Multiply the numbers as if they were whole numbers. There are three decimal places in the product.}$$

So the product of 9.63 and 6.8 is 66.164.

D. Multiply 7.9 times 0.0004.

$$\begin{array}{r} 7.9 \\ \times 0.0004 \\ \hline 0.00316 \end{array} \quad \text{Because 7.9 has one decimal place and 0.0004 has four decimal places, the product must have five decimal places. We must insert two zeros to the left so there are enough places in the answer.}$$

So 7.9 times 0.0004 is 0.00316.

CALCULATOR EXAMPLE



E. Find the product: $(38.56)(71.238)$

STRATEGY: The calculator will automatically place the decimal point in the correct position.

The product is 2746.93728.

F. If exactly eight strips of metal, each 3.875 inches wide, are to be cut from a piece of sheet metal, what is the smallest (in width) piece of sheet metal that can be used?

STRATEGY: To find the width of the piece of sheet metal we multiply the width of one of the strips by the number of strips needed.

$$\begin{array}{r} 3.875 \\ \times 8 \\ \hline 31.000 \end{array} \quad \text{The extra zeros can be dropped.}$$

The piece must be 31 inches wide.

Exercises 4.4

BASIC
Mathematics Now™

OBJECTIVE

Multiply decimals.

A Multiply.

1.
$$\begin{array}{r} 0.6 \\ \times 8 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 0.8 \\ \times 3 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 1.9 \\ \times 5 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 3.4 \\ \times 7 \\ \hline \end{array}$$

5. 6×0.07

6. 0.03×3

7. 0.8×0.8

8. 0.7×0.5

9. 0.04×0.8

10. 0.9×0.007

11. $0.18 \times (0.7)$

12. $1.7 \times (0.07)$

13. The number of decimal places in the product of 3.511 and 6.2 is _____.
-
- _____

14. In the product
- $0.34 \times ? = 0.408$
- , the number of decimal places in the missing factor is _____.
-
- _____

B Multiply.

15.
$$\begin{array}{r} 7.72 \\ \times 0.008 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 3.47 \\ \times 0.0065 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 3.87 \\ \times 3.9 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 6.23 \\ \times 5.8 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 6.84 \\ \times 0.42 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 4.99 \\ \times 0.37 \\ \hline \end{array}$$

21.
$$\begin{array}{r} 0.476 \\ \times 8.3 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 0.092 \\ \times 8.7 \\ \hline \end{array}$$

23.
$$\begin{array}{r} 42.7 \\ \times 0.53 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 38.5 \\ \times 0.21 \\ \hline \end{array}$$

25.
$$\begin{array}{r} 0.356 \\ \times 0.067 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 0.832 \\ \times 0.041 \\ \hline \end{array}$$

27.
$$\begin{array}{r} 0.0416 \\ \times 4.02 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 0.00831 \\ \times 6.73 \\ \hline \end{array}$$

29.
$$\begin{array}{r} 0.825 \\ \times 0.0054 \\ \hline \end{array}$$

30.
$$\begin{array}{r} 0.575 \\ \times 0.00378 \\ \hline \end{array}$$

31. Find the product of 8.54 and 3.78.
-
- _____

32. Find the product of 6.68 and 4.33.
-
- _____

33. Multiply:
- $5.3(0.7)(0.56)$
-
- _____

34. Multiply:
- $8.1(6.5)(0.23)$
-
- _____

Multiply and round as indicated.

- 35.
- $32(0.846)$
- to the nearest tenth.
-
- _____

- 36.
- $680(0.0731)$
- to the nearest hundredth.
-
- _____

- 37.
- $75.96(23.39)$
- to the nearest tenth.
-
- _____

- 38.
- $3789.1(34.54)$
- to the nearest ten.
-
- _____

- 39.
- $16.93(31.47)$
- to the nearest hundredth.
-
- _____

- 40.
- $0.046(0.9523)$
- to the nearest thousandth.
-
- _____

C Multiply. Round the product to the nearest hundredth.

- 41.
- $(34.06)(23.75)(0.134)$
-
- _____

- 42.
- $(0.056)(67.8)(21.115)$
-
- _____

Exercises 43–47. The table shows the amount of gas purchased by Grant and the price he paid per gallon for five fill-ups.

Gasoline Purchases

Number of Gallons	Price per Gallon
19.7	\$2.399
21.4	\$2.419
18.6	\$2.559
20.9	\$2.399
18.4	\$2.629

43. What is the total number of gallons of gas that Grant purchased?
44. To the nearest cent, how much did he pay for the second fill-up?
45. To the nearest cent, how much did he pay for the fifth fill-up?
46. To the nearest cent, what is the total amount he paid for the five fill-ups?
47. At which price per gallon did he pay the least for his fill-up?

Multiply.

48. $(469.5)(7.12)$

49. $(89.76)(4.61)$

50. $(313.17)(8.73)$

51. $(10.5)(760.02)$

Multiply.

52. $(7.85)(3.52)(27.89)$ Round to the nearest hundredth.

53. $(3.58)(165.9)(11.053)$ Round to the nearest thousandth.

54. $(7.4)(5.12)(0.88)(13.2)$ Round to the nearest tenth.

55. $(10.6)(0.53)(6.07)(90.5)$ Round to the nearest hundredth.

56. Joe earns \$16.45 per hour. How much does he earn if he works 38.5 hours in 1 week? Round to the nearest cent.

57. Safeway has a sale on Rancher's Reserve Angus beef petite sirloin steaks at \$2.99 per pound. Mary buys 6.225 pounds of the steak for her dinner party. What did she pay for the steak? The store rounds prices to the nearest cent.

Exercises 58–61. The table shows the cost of renting a car from a local agency.

Cost of Car Rentals

Type of Car	Cost per Day	Price per Mile Driven over 150 Miles per Day
Intermediate	\$51.93	\$0.20
Full size	\$55.93	\$0.23
Minivan	\$56.97	\$0.25

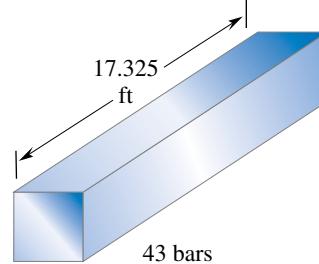
58. What does it cost to rent an intermediate car for 4 days if it is driven 435 miles?
60. What does it cost to rent a minivan for 5 days if it is driven 1050 miles?
62. Tiffany can choose any of the following ways to finance her new car. Which method is the least expensive in the long run?
 \$750 down and \$315.54 per month for 6 years
 \$550 down and \$362.57 per month for 5 years
 \$475 down and \$435.42 per month for 4 years
59. What does it cost to rent a full-size car for 3 days if it is driven 710 miles?
61. Which costs less, renting a full-size car for 3 days and driving it 425 miles or an intermediate car for 2 days and driving it 625 miles? How much less does it cost?
63. A new freezer-refrigerator is advertised at three different stores as follows:
 Store 1: \$80 down and \$91.95 per month for 18 months
 Store 2: \$125 down and \$67.25 per month for 24 months
 Store 3: \$350 down and \$119.55 per month for 12 months
 Which store is selling the freezer-refrigerator for the least total cost?

Exercises 64–66. The table shows calories expended for some physical activities.

Calorie Expenditure for Selected Physical Activities

Activity	Step Aerobics	Running (7 min/mile)	Cycling (10 mph)	Walking (4.5 mph)
Calories per pound of body weight per minute	0.070	0.102	0.050	0.045

64. Vanessa weighs 145 lb and does 75 min of step aerobics per week. How many calories does she burn per week?
66. Sephra weighs 143 lb and likes to walk daily at 4.5 mph. Her friend Dana weighs 128 lb and prefers to cycle daily at 10 mph. If both women exercise the same amount of time, who burns more calories?
65. Steve weighs 187 lb and runs 25 min five times per week at a 7 min/mi pace. How many calories does he burn per week?
67. An order of 43 bars of steel is delivered to a machine shop. Each bar is 17.325 ft long. Find the total linear feet of steel in the order.



68. From a table in a machinist's handbook, it is determined that hexagon steel bars 1.325 in. across weigh 4.3 lb per running foot. Using this constant, find the weight of a 1.325 in. hexagon steel bar that is 22.56 ft long.
69. In 1970, the per capita consumption of red meat was 132 pounds. In 1990 the consumption was 112.3 pounds. In 2003, the amount consumed was 111.9 pounds per person. Compute the total weight of red meat consumed by a family of four using the rates for each of these years. Discuss the reasons for the change in consumption.

- 70.** The fat content in a 3-oz serving of meat and fish is as follows: beef rib, 7.4 g; beef top round, 3.4 g; beef top sirloin, 4.8 g; dark meat chicken without skin, 8.3 g; white meat chicken without skin, 3.8 g; pink salmon, 3.8 g; and Atlantic cod, 0.7 g. Which contains the most grams of fat: 3 servings of beef ribs, 6 servings of beef top round, 4 servings of beef top sirloin, 2 servings of dark meat chicken, 6 servings of white meat chicken, 5 servings of pink salmon, or 25 servings of Atlantic cod?
- 72.** The Camburns live in Las Vegas, Nevada. Their house is assessed at \$344,780. The property tax rate for state, county, and schools is \$2.1836 per thousand dollars of assessed evaluation for 2004–2005. Find what they owe in property taxes.
- 71.** Older models of toilets use 5.5 gallons of water per flush. Models made in the 1970s use 3.5 gallons per flush. The new low-flow models use 1.55 gallons per flush. Assume each person flushes the toilet an average of five times per day. Determine the amount of water used in a town with a population of 41,782 in 1 day for each type of toilet. How much water is saved using the low-flow model as opposed to the pre-1970s model?
- 73.** Find the property tax on the Gregory estate, which is assessed at \$3,980,700. The tax rate in the area is \$2.775 per thousand dollars of assessment. Round to the nearest dollar.

Exercises 74–76 relate to the chapter application. In Olympic diving, seven judges rate each dive using a whole or half number between 0 and 10. The high and low scores are thrown out and the remaining scores are added together. The sum is then multiplied by 0.6 and then by the difficulty factor of the dive to obtain the total points awarded.

- 74.** A diver does a reverse $1\frac{1}{2}$ somersault with $2\frac{1}{2}$ twists, a dive with a difficulty factor of 2.9. She receives scores of 6.0, 6.5, 6.5, 7.0, 6.0, 7.5, and 7.0. What are the total points awarded for the dive?
- 75.** Another diver also does a reverse $1\frac{1}{2}$ somersault with $2\frac{1}{2}$ twists, a dive with a difficulty factor of 2.9. This diver receives scores of 7.5, 6.5, 7.5, 8.0, 8.0, 7.5, and 8.0. What are the total points awarded for the dive?
- 76.** A cut-through reverse $1\frac{1}{2}$ somersault has a difficulty factor of 2.6. What is the highest number of points possible with this dive?

STATE YOUR UNDERSTANDING

- 77.** Explain how to determine the number of decimal places needed in the product of two decimals.
- 78.** Suppose you use a calculator to multiply $(0.006)(3.2)(68)$ and get 13.056. Explain, using placement of the decimal point in a product, how you can tell that at least one of the numbers was entered incorrectly.

CHALLENGE

- 79.** What is the smallest whole number you can multiply 0.74 by to get a product that is greater than 82?
- 81.** Find the missing number in the following sequence:
2.1, 0.42, 0.126, 0.0504, _____.
- 80.** What is the largest whole number you can multiply 0.53 by to get a product that is less than 47?
- 82.** Find the missing number in the following sequence:
3.1, 0.31, _____, 0.0000031, 0.0000000031.

GROUP WORK

- 83.** Visit a grocery store or use newspaper ads to “purchase” the items listed. Have each member of your group use a different store or chain. Which members of your group “spent” the most? Least? Which group “spent” the most? Least?

Three 12-packs of Diet Pepsi
 Eight gallons of 2% milk
 Five pounds of hamburger
 Four cans of the store-brand creamed corn
 Five 4-roll packs of toilet paper
 Seven pounds of butter
 72 hamburger rolls
 12 large boxes of Cheerios

MAINTAIN YOUR SKILLS

Multiply or divide as indicated.

- 84.** $337(100)$
- 85.** $82(10,000)$
- 86.** $235,800 \div 100$
- 87.** $22,000,000 \div 10,000$
- 88.** $48(1,000,000)$
- 89.** 692×10^3
- 90.** $55,000 \div 10^3$
- 91.** $4,760,000 \div 10^4$
- 92.** 84×10^8
- 93.** $4,210,000,000 \div 10^6$

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4.5 Multiplying and Dividing by Powers of 10; Scientific Notation

VOCABULARY

Recall that a **power of 10** is the value obtained when 10 is written with an exponent.

Scientific notation is a special way to write numbers as a product using a number between 1 and 10 and a power of 10.

OBJECTIVES

1. Multiply or divide a number by a power of 10.
2. Write a number in scientific notation or change a number in scientific notation to its place value name.

How & Why

OBJECTIVE 1

Multiply or divide a number by a power of 10.

The shortcut used in Section 1.5 for multiplying and dividing by a power of 10 works in a similar way with decimals. Consider the following products:

$$\begin{array}{r} 0.8 \\ \times 10 \\ \hline 8.0 = 8 \end{array} \quad \begin{array}{r} 0.63 \\ \times 10 \\ \hline 6.30 = 6.3 \end{array} \quad \begin{array}{r} 9.36 \\ \times 10 \\ \hline 93.60 = 93.6 \end{array}$$

Note in each case that multiplying a decimal by 10 has the effect of moving the decimal point one place to the right.

Because $100 = 10 \cdot 10$, multiplying by 100 is the same as multiplying by 10 two times in succession. So, multiplying by 100 has the effect of moving the decimal point two places to the right. For instance,

$$(0.42)(100) = (0.42)(10 \cdot 10) = (0.42 \cdot 10) \cdot 10 = 4.2 \cdot 10 = 42$$

Because $1000 = 10 \cdot 10 \cdot 10$, the decimal point will move three places to the right when multiplying by 1000. Because $10,000 = 10 \cdot 10 \cdot 10 \cdot 10$, the decimal point will move four places to the right when multiplying by 10,000, and so on in the same pattern:

$$(0.05682)(10,000) = 568.2$$

Zeros may have to be placed on the right in order to move the correct number of decimal places:

$$(6.3)(1000) = 6.\underbrace{300} = 6300$$

In this problem, two zeros are placed on the right.

Because multiplying a decimal by 10 has the effect of moving the decimal point one place to the right, dividing a number by 10 must move the decimal point one place to the left. Again, we are using the fact that multiplication and division are inverse operations. Division by 100 will move the decimal point two places to the left, and so on. Thus,

$$739.5 \div 100 = 739.\underbrace{5} = 7.395$$

$$0.596 \div 10,000 = 0.0000596$$

Four zeros are placed on the left so that the decimal point may be moved four places to the left.

To multiply a number by a power of 10

Move the decimal point to the right. The number of places to move is shown by the number of zeros in the power of 10.

To divide a number by a power of 10

Move the decimal point to the left. The number of places to move is shown by the number of zeros in the power of 10.

Warm-Ups A–G

Examples A–G **BASIC Mathematics Now™**

A. Multiply: $9.21(10)$

B. Multiply: $0.72(100)$

C. Find the product of 34.78 and 10^3 .

D. Divide: $662 \div 10$

E. Divide: $49.16 \div 100$

F. Find the quotient:
 $82.113 \div 10^5$

DIRECTIONS: Multiply or divide as indicated.

STRATEGY: To multiply by a power of 10, move the decimal point to the right, inserting zeros as needed. To divide by a power of 10, move the decimal point to the left, inserting zeros as needed. The exponent of 10 specifies the number of places to move the decimal point.

A. Multiply: $67.445(10)$

$67.445(10) = 674.45$ Multiplying by 10 moves the decimal point one place to the right.

So, $67.445(10) = 674.45$.

B. Multiply: $0.094(100)$

$0.094(100) = 9.4$ Multiplying by 100 moves the decimal point two places to the right.

So, $0.094(100) = 9.4$.

C. Find the product of 8.57 and 10^4 .

$8.57(10^4) = 85,700$ Multiplying by 10^4 moves the decimal point four places to the right. Two zeros must be inserted on the right to make the move.

So the product of 8.57 and 10^4 is $85,700$.

D. Divide: $9.02 \div 10$

$9.02 \div 10 = 0.902$ Dividing by 10 moves the decimal point one place to the left.

So, $9.02 \div 10 = 0.902$.

E. Divide: $760.1 \div 100$

$760.1 \div 100 = 7.601$ Dividing by 100 moves the decimal point two places to the left.

So, $760.1 \div 100 = 7.601$.

F. Find the quotient: $55.6 \div 10^4$

$55.6 \div 10^4 = 0.00556$ Move the decimal point four places to the left. Two zeros are inserted on the left so we can make the move.

So, $55.6 \div 10^4 = 0.00556$.

Answers to Warm-Ups

- A. 92.1 B. 72 C. 34,780
D. 66.2 E. 0.4916
F. 0.00082113

- G. Bi-Mart orders 1000 boxes of chocolates for their Valentine's Day sales. The total cost to Bi-Mart was \$19,950. What did Bi-Mart pay per box of chocolates?

$$\$19,950 \div 1000 = \$19.95 \quad \text{To find the cost paid per box, divide the total cost by the number of boxes.}$$

Bi-Mart paid \$19.95 per box of chocolates.

- G. Ten thousand sheets of clear plastic are 5 in. thick. How thick is each sheet? (This is the thickness of some household plastic wrap.)

How & Why

OBJECTIVE 2

Write a number in scientific notation or change a number in scientific notation to its place value name.

Scientific notation is widely used in science, technology, and industry to write large and small numbers. Every “scientific calculator” has a key for entering numbers in scientific notation. This notation makes it possible for a calculator or computer to deal with much larger or smaller numbers than those that take up 8, 9, or 10 spaces on the display.

For example, see Table 4.5.

Table 4.5 Scientific Notation

Word Form	Place Value (Numeral Form)	Scientific Notation	Calculator or Computer Display		
One million	1,000,000	1×10^6	1. 06	or	1. E 6
Five billion	5,000,000,000	5×10^9	5. 09	or	5. E 9
One trillion, three billion	1,003,000,000,000	1.003×10^{12}	1.003 12	or	1.003 E 12

Small numbers are shown by writing the power of 10 using a negative exponent. (This is the first time that we have used negative numbers. You probably have run into them before. For instance, when reporting temperatures, a reading of 10 degrees above zero is written +10. While a reading of 10 degrees below zero is written -10. You will learn more about negative numbers in Chapter 8.) For now, remember that multiplying by a negative power of 10 is the same as dividing by a power of 10, which means you will be moving the decimal point to the left. See Table 4.6.

Table 4.6 Scientific Notation

Word Name	Place Value Name	Scientific Notation	Calculator or Computer Display
Eight thousandths	0.008	8×10^{-3}	8. -03 or 8. E -3
Seven ten-millionths	0.0000007	7×10^{-7}	7. -07 or 7. E -7
Fourteen hundred-billionths	0.00000000014	1.4×10^{-10}	1.4 -10 or 1.4 E -10

The shortcut for multiplying by a power of 10 is to move the decimal to the right, and the shortcut for dividing by a power of 10 is to move the decimal point to the left.

To write a number in scientific notation

- Move the decimal point right or left so that only one digit remains to the left of the decimal point. The result will be a number between 1 and 10. If the choice is 1 or 10 itself, use 1.
- Multiply the decimal found in step 1 by a power of 10. The exponent of 10 to use is one that will make the new product equal to the original number.
 - If you had to move the decimal to the left, multiply by the same number of 10s as the number of places moved.
 - If you had to move the decimal to the right, divide (by writing a negative exponent) by the same number of 10s as the number of places moved.

Scientific Notation

A number in scientific notation is written as the product of two numbers. The first number is between 1 and 10 (including 1 but not 10) and the second number is a power of 10. The multiplication is indicated using an “×” symbol.

To change from scientific notation to place value name

1. If the exponent of 10 is positive, multiply by as many 10s (move the decimal point to the right as many places) as the exponent shows.
2. If the exponent of 10 is negative, divide by as many 10s (move the decimal point to the left as many places) as the exponent shows.

For numbers larger than 1:

Place value name:	15,000	7,300,000	18,500,000,000	
Numbers between 1 and 10:	1.5	7.3	1.85	Move the decimal (which is after the units place) to the left until the number is between 1 and 10 (one digit to the left of the decimal).
Scientific notation:	1.5×10^4	7.3×10^6	1.85×10^{10}	Multiply each by a power of 10 that shows how many places left the decimal moved, or how many places you would have to move to the right to recover the original number.

For numbers smaller than 1:

Place value name:	0.000074	0.00000009	0.0000000000267	
Numbers between 1 and 10:	7.4	9.	2.67	Move the decimal to the right until the number is between 1 and 10.
Scientific notation:	7.4×10^{-5}	9×10^{-8}	2.67×10^{-11}	Divide each by a power of 10 that shows how many places right the decimal moved. Show this division by a negative power of 10.

It is important to note that scientific notation is not rounding. The scientific notation has exactly the same value as the original name.

Warm-Ups H–J

H. $123,000,000$

I. 0.0000903

Answers to Warm-Ups

H. 1.23×10^8

I. 9.03×10^{-5}

Examples H–J

DIRECTIONS: Write in scientific notation.

STRATEGY: Move the decimal point so that there is one digit to the left. Multiply or divide this number by the appropriate power of 10 so the value is the same as the original number.

H. $46,700,000$

4.67 is between 1 and 10.

Move the decimal until the number is between 1 and 10.

$4.67 \times 10,000,000$ is $46,700,000$.

Moving the decimal left is equivalent to dividing by 10 for each place.

$46,700,000 = 4.67 \times 10^7$

To recover the original number, we multiply by 10 seven times.

I. 0.000000045

4.5 is between 1 and 10.

Move the decimal until the number is between 1 and 10.

$4.5 \div 100,000,000$ is 0.000000045 .

Moving the decimal right is equivalent to multiplying by 10 for each place.

$0.000000045 = 4.5 \times 10^{-8}$

To recover the original number, we divide by 10 eight times.

- J. Approximately 1,500,000 people in the United States suffer from autism. Write this number in scientific notation.

1.5 is between 1 and 10.

$$1.5 \times 1,000,000 = 1,500,000$$
$$1,500,000 = 1.5 \times 10^6$$

In scientific notation the number of people with autism is 1.5×10^6 .

- J. The age of a 25-year-old student is approximately 789,000,000 seconds. Write this number in scientific notation.

Examples K–L



DIRECTIONS: Write the place value name.

STRATEGY: If the exponent is positive, move the decimal point to the right as many places as shown in the exponent. If the exponent is negative, move the decimal point to the left as many places as shown by the exponent.

- K. Write the place value name for 5.72×10^{-7} .

$$5.72 \times 10^{-7} = 0.000000572$$

The exponent is negative, so move the decimal point seven places to the left. That is, divide by 10 seven times.

- L. Write the place value name for 1.004×10^8 .

$$1.004 \times 10^8 = 100,400,000$$

The exponent is positive, so move the decimal point eight places to the right; that is, multiply by 10 eight times.

Warm-Ups K–L

- K. Write the place value name for 8.08×10^{-6} .

- L. Write the place value name for 4.62×10^9 .

Answers to Warm-Ups

- J. The age of a 25-year-old student is approximately 7.89×10^8 seconds.
K. 0.00000808 L. 4,620,000,000

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Exercises 4.5

OBJECTIVE 1

Multiply or divide a number by a power of 10.

A Multiply or divide.

- | | | |
|-------------------------|--------------------------|------------------------|
| 1. $4.95 \div 10$ | 2. $18.65 \div 10$ | 3. $92.6(100)$ |
| 4. $5.12(100)$ | 5. $(1.3557)(1000)$ | 6. $(0.0421)(1000)$ |
| 7. $\frac{345.8}{100}$ | 8. $\frac{456.71}{1000}$ | 9. $\frac{8325}{100}$ |
| 10. $\frac{2421}{1000}$ | 11. 0.107×10^4 | 12. 7.32×10^5 |

13. To multiply 4.56 by 10^4 , move the decimal point four places to the _____. 14. To divide 4.56 by 10^5 , move the decimal point five places to the _____.

B Multiply or divide.

- | | | |
|-----------------------------|-------------------------------|-----------------------|
| 15. $(78.324)(1000)$ | 16. $17.66(100)$ | 17. $42.6 \div 10$ |
| 18. $672.86 \div 1000$ | 19. $57.9(1000)$ | 20. $0.0735(10)$ |
| 21. $\frac{9077.5}{10,000}$ | 22. $\frac{6351.42}{100,000}$ | 23. $17.55(100,000)$ |
| 24. $16.33(1,000,000)$ | 25. $6056.32 \div 100$ | 26. $8.045 \div 1000$ |
| 27. $32.76 \div 100,000$ | 28. $134.134 \div 1,000,000$ | |

OBJECTIVE 2

Write a number in scientific notation or change a number in scientific notation to its place value name.

A Write in scientific notation.

- | | | |
|----------------|------------|--------------|
| 29. 750,000 | 30. 19,300 | 31. 0.000091 |
| 32. 0.00000723 | 33. 4195.3 | 34. 82710.3 |

Write in place value form.

- | | | |
|------------------------|------------------------|------------------------|
| 35. 8×10^4 | 36. 3×10^6 | 37. 4×10^{-3} |
| 38. 7×10^{-7} | 39. 9.43×10^5 | 40. 8.12×10^5 |

B Write in scientific notation.

41. 43,700

42. 81,930,000

43. 0.00000141

44. 0.0000642

45. 0.000000000684

46. 0.00000000555

47. 975.002

48. 3496.701

Write in place value notation.

49. 7.341×10^{-5}

50. 9.37×10^{-6}

51. 1.77×10^9

52. 7.43×10^8

53. 3.11×10^{-8}

54. 5.6×10^{-9}

55. 2.7754×10^3

56. 5.11166×10^6

C

57. Max's Tire Store buys 100 tires that cost \$39.68 each. What is the total cost of the tires?
59. Ms. James buys 100 acres of land at a cost of \$3100 per acre. What is the total cost of her land?
61. The total land area of Earth is approximately 52,000,000 square miles. What is the total area written in scientific notation?
63. The length of a red light ray is 0.000000072 cm. Write this length in scientific notation.
65. The speed of light is approximately 1.116×10^7 miles per minute. Write this speed in place value notation.
67. The shortest wavelength of visible light is approximately 4×10^{-5} cm. Write this length in place value form.
69. A family in the Northeast used 3.467×10^8 BTUs of energy during 2004. A family in the Midwest used 3.521×10^8 BTUs in the same year. A family in the South used 2.783×10^8 BTUs, and a family in the West used 2.552×10^8 BTUs. Write the total energy usage for the four families in place value form.
71. The population of Cabot Cove was approximately 100,000 in 2005. During the year, the community consumed a total of 2,290,000 gallons of milk. What was the per capita consumption of milk in Cabot Cove in 2005?

58. If Mae's Tire Store buys 100 tires for a total cost of \$5278, what is the cost of each tire?
60. If 1000 concrete blocks weigh 11,100 lb, how much does each block weigh?
62. A local computer store offers a small computer with 260 MB (2,662,240 bytes) of memory. Write the number of bytes in scientific notation.
64. The time it takes light to travel 1 kilometer is approximately 0.0000033 second. Write this time in scientific notation.
66. Earth is approximately 1.5×10^8 kilometers from the sun. Write this distance in place value form.
68. A sheet of paper is approximately 1.3×10^{-3} in. thick. Write the thickness in place value form.
70. In 2004, the per capita consumption of fish was 15.1 pounds. In the same year, the per capita consumption of poultry was 82.6 pounds and of red meat was 118.3 pounds. Write the total amount in each category consumed by 100,000 people in scientific notation.
72. In 1980, \$24,744,000,000 was spent on air pollution abatement. Ten years later, \$26,326,000,000 was spent. In scientific notation, how much more money was spent in 1990 than in 1980? What is the average amount of increase per year during the period?

Exercises 73–75 relate to the chapter application. In baseball, a hitter's batting average is calculated by dividing the number of hits by the number of times at bat. Mathematically, this number is always between zero and 1.

- 73.** In 1923, Babe Ruth led Major League Baseball with a batting average of 0.393. However, players and fans would say that Ruth has an average of "three hundred ninety-three." Mathematically, what are they doing to the actual number?
- 75.** The major league player with the highest season batting average in the past century was Roger Hornsby of St. Louis. In 1924 he batted .424. Change this to the mathematically calculated number of his batting average.
- 74.** Explain why the highest possible batting average is 1.0.

STATE YOUR UNDERSTANDING

- 76.** Find a pair of numbers whose product is larger than 10 trillion. Explain how scientific notation makes it possible to multiply these factors on a calculator. Why is it not possible without scientific notation?

CHALLENGE

- 77.** A parsec is a unit of measure used to determine distance between stars. One parsec is approximately 206,265 times the average distance of Earth from the sun. If the average distance from Earth to the sun is approximately 93,000,000 miles, find the approximate length of one parsec. Write the length in scientific notation. Round the number in scientific notation to the nearest hundredth.

Simplify.

$$\mathbf{79.} \frac{(3.25 \times 10^{-3})(2.4 \times 10^3)}{(4.8 \times 10^{-4})(2.5 \times 10^{-3})}$$

- 78.** Light will travel approximately 5,866,000,000,000 miles in 1 year. Approximately how far will light travel in 11 years? Write the distance in scientific notation. Round the number in scientific notation to the nearest thousandth.

$$\mathbf{80.} \frac{(3.25 \times 10^{-7})(2.4 \times 10^6)}{(4.8 \times 10^4)(2.5 \times 10^{-3})}$$

GROUP WORK

- 81.** Find the 2000 population for the 10 largest and the 5 smallest cities in your state. Round these numbers to the nearest thousand. Find the total number of pounds of fruit, at the rate of 92.3 pounds per person, and the

total number of pounds of vegetables, at the rate of 11.2 pounds per person, consumed in each of these 15 cities.

MAINTAIN YOUR SKILLS*Divide.*

82. $42 \overline{)7938}$

83. $59 \overline{)18,408}$

84. $216 \overline{)66,744}$

85. $\frac{745}{12}$

86.
$$\begin{array}{r} 5936 \\ \hline 37 \end{array}$$

88. Find the quotient of 146,457 and 416.

90. Find the perimeter of a rectangular field that is 312 ft long and 125 ft wide.

87. Find the quotient of 630,828 and 243.

89. Find the quotient of 6,542,851 and 711. Round to the nearest ten.

91. The area of a rectangle is 1008 in
- ²
- . If the length of the rectangle is 4 ft, find the width.

4.6 Dividing Decimals; Average, Median, and Mode

How & Why

OBJECTIVE 1

Divide decimals.

Division of decimals is the same as division of whole numbers, with one difference. The difference is the location of the decimal point in the quotient.

As with multiplication, we examine the fractional form of division to discover the method of placing the decimal point in the quotient. First, change the decimal form to fractional form to find the quotient. Next, change the quotient to decimal form. Consider the information in Table 4.7.

OBJECTIVES

1. Divide decimals.
2. Find the average, median, or mode of a set of decimals.

Table 4.7 Division by a Whole Number

Decimal Form	Fractional Form	Division Fractional Form	Division Decimal Form
$3\overline{)0.36}$	$\frac{36}{100} \div 3$	$\frac{36}{100} \div 3 = \frac{36}{100} \cdot \frac{1}{3} = \frac{12}{100}$	$3\overline{)0.36}$
$8\overline{)0.72}$	$\frac{72}{100} \div 8$	$\frac{72}{100} \div 8 = \frac{72}{100} \cdot \frac{1}{8} = \frac{9}{100}$	$8\overline{)0.72}$
$5\overline{)0.3}$	$\frac{3}{10} \div 5$	$\frac{3}{10} \div 5 = \frac{3}{10} \cdot \frac{1}{5} = \frac{3}{50} = \frac{6}{100}$	$5\overline{)0.3}$

We can see from Table 4.7 that the decimal point for the quotient of a decimal and a whole number is written directly above the decimal point in the dividend. It may be necessary to insert zeros to do the division. See Example B.

When a decimal is divided by 7, the division process may not have a remainder of zero at any step:

$$\begin{array}{r} 0.97 \\ 7\overline{)6.85} \\ 63 \\ \hline 55 \\ 49 \\ \hline 6 \end{array}$$

At this step we can write zeros to the right of the digit 5, since $6.85 = 6.850 = 6.8500 = 6.85000 = 6.850000$.

$$\begin{array}{r} 0.97857 \\ 7\overline{)6.85000} \\ 63 \\ \hline 55 \\ 49 \\ \hline 60 \\ 56 \\ \hline 40 \\ 35 \\ \hline 50 \\ 49 \\ \hline 1 \end{array}$$

It appears that we might go on inserting zeros and continue endlessly. This is indeed what happens. Such decimals are called “nonterminating, repeating decimals.” For example, the quotient of this division is sometimes written

$$0.97857142857142\dots \quad \text{or} \quad 0.\overline{97857142}$$

The bar written above the sequence of digits 857142 indicates that these digits are repeated endlessly.

In practical applications we stop the division process one place value beyond the accuracy required by the situation and then round. Therefore,

$$\begin{array}{r} 0.97 \\ 7)6.85 \\ \underline{63} \\ 55 \\ \underline{49} \\ 6 \quad \text{Stop} \end{array}$$

$6.85 \div 7 \approx 1.0$ rounded to the nearest tenth.

$$\begin{array}{r} 0.9785 \\ 7)6.8500 \\ \underline{63} \\ 55 \\ \underline{49} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 5 \quad \text{Stop} \end{array}$$

$6.85 \div 7 \approx 0.979$ rounded to the nearest thousandth.

Now let's examine division when the divisor is also a decimal. We will use what we already know about division with a whole-number divisor. See Table 4.8.

Table 4.8 Division by a Decimal

Decimal Form	Conversion to a Whole-Number Divisor	Decimal Form of the Division
$0.\overline{3} \mid 0.36$	$\frac{0.36}{0.3} \cdot \frac{10}{10} = \frac{3.6}{3} = 1.2$	$0.\overline{3} \mid 0.\overline{36}$
$0.\overline{4} \mid 1.52$	$\frac{1.52}{0.4} \cdot \frac{10}{10} = \frac{15.2}{4} = 3.8$	$0.\overline{4} \mid 1.\overline{52}$
$0.0\overline{8} \mid 0.72$	$\frac{0.72}{0.08} \cdot \frac{100}{100} = \frac{72}{8} = 9$	$0.0\overline{8} \mid 0.\overline{72}$
$0.2\overline{5} \mid 0.3$	$\frac{0.3}{0.25} \cdot \frac{100}{100} = \frac{30}{25} = 1.2$	$0.2\overline{5} \mid 0.\overline{300}$
$0.00\overline{6} \mid 4.8$	$\frac{4.8}{0.006} \cdot \frac{1000}{1000} = \frac{4800}{6} = 800$	$0.00\overline{6} \mid 4.\overline{800}$

We see from the Table 4.8 that we move the decimal point in both the divisor and the dividend the number of places to make the divisor a whole number. Then divide as before.

To divide two numbers

1. If the divisor is not a whole number, move the decimal point in both the divisor and dividend to the right the number of places necessary to make the divisor a whole number.
2. Place the decimal point in the quotient above the decimal point in the dividend.
3. Divide as if both numbers are whole numbers.
4. Round to the given place value. (If no round-off place is given, divide until the remainder is zero or round as appropriate in the problem. For instance, in problems with money, round to the nearest cent.)

Examples A–G



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DIRECTIONS: Divide. Round as indicated.

STRATEGY: If the divisor is not a whole number, move the decimal point in both the divisor and the dividend to the right the number of places necessary to make the divisor a whole number. The decimal point in the quotient is found by writing it directly above the decimal (as moved) in the dividend.

A. Divide: $32 \overline{)43.0592}$

$$\begin{array}{r} 1.3456 \\ 32 \overline{)43.0592} \\ \underline{32} \\ 110 \\ \underline{96} \\ 145 \\ \underline{128} \\ 179 \\ \underline{160} \\ 192 \\ \underline{192} \\ 0 \end{array}$$

The numerals in the answer are lined up in columns that have the same place value as those in the dividend.

CHECK:
$$\begin{array}{r} 1.3456 \\ \times \quad 32 \\ \hline 26\ 912 \\ 403\ 68 \\ \hline 43.0592 \end{array}$$

So the quotient is 1.3456.

CAUTION

Write the decimal point for the quotient directly above the decimal point in the dividend.

Warm-Ups A–G

A. Divide: $14 \overline{)94.962}$

Answers to Warm-Ups

A. 6.783

- B. Find the quotient of 42.39 and 18.

- B. Find the quotient of 7.41 and 6.

STRATEGY: Recall that the quotient of a and b can be written $a \div b$, $\frac{a}{b}$, or $b\overline{)a}$.

$$\begin{array}{r} 1.23 \\ 6 \overline{)7.41} \\ \underline{6} \\ 14 \\ \underline{12} \\ 21 \\ \underline{18} \\ 3 \end{array}$$

$$\begin{array}{r} 1.235 \\ 6 \overline{)7.410} \\ \underline{6} \\ 14 \\ \underline{12} \\ 21 \\ \underline{18} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

The quotient is 1.235.

- C. Divide 634.7 by 56 and round the quotient to the nearest hundredth.

- C. Divide 847.9 by 37 and round the quotient to the nearest hundredth.

$$\begin{array}{r} 22.916 \\ 37 \overline{)847.900} \\ \underline{74} \\ 107 \\ \underline{107} \\ 0 \\ 37 \\ \underline{33} \\ 39 \\ \underline{33} \\ 60 \\ 37 \\ \underline{230} \\ 222 \\ \underline{8} \end{array}$$

It is necessary to place two zeros on the right in order to round to the hundredths place, since the division must be carried out one place past the place to which you wish to round.

The quotient is approximately 22.92.

- D. Divide $13.61 \div 0.13$ and round the quotient to the nearest hundredth.

- D. Divide $12.451 \div 0.11$ and round the quotient to the nearest hundredth.

$$0.11 \overline{)12.451}$$

First move both decimal points two places to the right so the divisor is the whole number 11. The same result is obtained by multiplying both divisor and dividend by 100.

$$\frac{12.451}{0.11} \times \frac{100}{100} = \frac{1245.1}{11}$$

$$\begin{array}{r} 113.1 \\ 11 \overline{)1245.1} \\ \underline{11} \\ 14 \\ \underline{11} \\ 35 \\ \underline{33} \\ 21 \\ \underline{11} \\ 10 \end{array}$$

Answers to Warm-Ups

- B. 2.355 C. 11.33 D. 104.69

$$\begin{array}{r} 113.190 \\ 11 \overline{)1245.100} \\ 11 \\ \hline 14 \\ 11 \\ \hline 35 \\ 33 \\ \hline 21 \\ 11 \\ \hline 100 \\ 99 \\ \hline 10 \\ 0 \\ \hline 10 \end{array}$$

The number of zeros you place on the right depends on either the directions for rounding or your own choice of the number of places. Here we find the approximate quotient to the nearest hundredth.

The quotient is approximately 113.19.

- E. Divide 0.57395 by 0.067 and round the quotient to the nearest thousandth.

$$0.067 \overline{)0.57395}$$

$$\begin{array}{r} 8.5664 \\ 67 \overline{)573.9500} \\ 536 \\ \hline 379 \\ 335 \\ \hline 445 \\ 402 \\ \hline 430 \\ 402 \\ \hline 280 \\ 268 \\ \hline 12 \end{array}$$

Move both decimals three places to the right. It is necessary to insert two zeros on the right in order to round to the thousandths place.

The quotient is approximately 8.566.

CALCULATOR EXAMPLE

- F. Find the quotient of 1134.7654 and 32.16 and round to the nearest thousandth.

$$1134.7654 \div 32.16 \approx 35.284993$$

The quotient is 35.285, to the nearest thousandth.

- G. What is the cost per ounce of a 15.25-oz can of fruit that costs \$1.09? This is called the “unit price” and is used for comparing prices. Many stores are required to show this price for the food they sell.

STRATEGY: To find the unit price (cost per ounce), we divide the cost by the number of ounces. Round to the nearest tenth of a cent.

$$\begin{array}{r} 0.0714 \\ 15.25 \overline{)1.090000} \\ 10675 \\ \hline 2250 \\ 1525 \\ \hline 7250 \\ 6100 \\ \hline 1150 \end{array}$$

The fruit costs approximately \$0.071 or 7.1¢ per ounce.

- E. Divide 0.85697 by 0.083 and round the quotient to the nearest thousandth.

- F. Find the quotient of 708.7083 and 23.237 and round to the nearest thousandth.

- G. What is the unit price of 3.8 oz of instant coffee if it costs \$3.69? Round to the nearest tenth of a cent.

Answers to Warm-Ups

E. 10.325 F. 30.499

G. The unit price of coffee is \$0.971 or 97.1¢.

How & Why

OBJECTIVE 2

Find the average, median, or mode of a set of decimals.

The method for finding the average, median, or mode of a set of decimals is the same as that for whole numbers and fractions.

To find the average (mean) of a set of numbers

1. Add the numbers.
2. Divide the sum by the number of numbers in the set.

To find the median of a set of numbers

1. List the numbers in order from smallest to largest.
2. If there is an odd number of numbers in the set, the median is the middle number.
3. If there is an even number of numbers in the set, the median is the average (mean) of the two middle numbers.

To find the mode of a set of numbers

1. Find the number or numbers that occur most often.
2. If all the numbers occur the same number of times, there is no mode.

Warm-Ups H–J

- H. Find the average of 7.3, 0.66, 10.8, 4.11, and 1.32.

Examples H–J

STRATEGY: Use the same procedures as for whole numbers and fractions.

- H. Find the average of 0.75, 0.43, 3.77, and 2.23.

$$0.75 + 0.43 + 3.77 + 2.23 = 7.18$$

First add the numbers.

$$7.18 \div 4 = 1.795$$

Second, divide by 4, the number of numbers.

So the average of 0.75, 0.43, 3.77, and 2.23 is 1.795.

Answers to Warm-Ups

- H. 4.838

- I. Pedro's grocery bills for the past 5 weeks were:

Week 1: \$155.72

Week 2: \$172.25

Week 3: \$134.62

Week 4: \$210.40

Week 5: \$187.91

What are the average and median costs of Pedro's groceries per week for the 5 weeks?

Average:

$$\begin{array}{r} 155.72 \\ 172.25 \\ 134.62 \quad \text{Add the weekly totals and divide by 5, the number of weeks.} \\ 210.40 \\ +187.91 \\ \hline 860.90 \end{array}$$

$$860.90 \div 5 = 172.18$$

Median:

134.62, 155.72, 172.25, 187.91, 210.40 List the numbers from smallest to largest.

172.25 The median is the middle number.

Pedro's average weekly cost for groceries is \$172.18, and the median cost is \$172.25.

- J. During a 6-month period, the highest recorded price for a share of Intel in each month was: \$22.95, \$26.72, \$28.14, \$26.72, \$29.45, and \$30.15. Find the average, median, and mode for the 6-month period.

Average:

$$22.95 + 26.72 + 28.14 + 26.72 + 29.45 + 30.15 = 164.13 \quad \text{Add the numbers.}$$

$$164.13 \div 6 \approx 27.36 \quad \text{Divide by the number of numbers and round to the nearest cent.}$$

Median:

22.95, 26.72, 26.72, 28.14, 29.45, 30.15 List the numbers from smallest to largest.

$$(26.72 + 28.14) \div 2 = 27.43 \quad \text{Find the average of the two middle numbers and round to the nearest cent.}$$

Mode:

26.72 The number that appears most often.

The average highest price for a share of Intel is about \$27.36, the median price is \$27.43, and the mode price is \$26.72.

- I. Mary's weekly car expenses, including parking, for the past 6 weeks were:

Week 1: \$37.95

Week 2: \$43.15

Week 3: \$28.65

Week 4: \$59.14

Week 5: \$61.72

Week 6: \$50.73

What are the average and median weekly expenses for Mary during the 6 weeks?

- J. Over a 7-month period, Jose was able to save the following amount each month: \$123.25, \$110.60, \$145.85, \$123.25, \$150.10, \$162.34, and \$132.15. Find the average, median, and mode for the 7-month period.

Answers to Warm-Ups

- I. Mary's average weekly car expense is \$46.89, and the median expense is \$46.94.
- J. Jose's average savings is about \$135.36, the median savings is \$132.15, and the mode of the savings is \$123.25.

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Exercises 4.6

OBJECTIVE 1

Divide decimals.

A Divide.

1. $8\overline{)6.4}$

2. $6\overline{)5.4}$

3. $4\overline{)19.6}$

4. $5\overline{)35.5}$

5. $0.1\overline{)32.67}$

6. $0.01\overline{)8.12}$

7. $393.9 \div 0.13$

8. $55.22 \div 0.11$

9. $60\overline{)331.8}$

10. $60\overline{)172.8}$

11. $28.35 \div 36$

12. $5.238 \div 36$

13. To divide 27.8 by 0.6 we first multiply both the dividend and the divisor by 10 so we are dividing by a _____.

14. To divide 0.4763 by 0.287 we first multiply both the dividend and the divisor by _____.

B Divide.

Divide and round to the nearest tenth.

15. $6\overline{)7.23}$

16. $7\overline{)0.5734}$

17. $1.3\overline{)12.863}$

18. $6.9\overline{)49.381}$

Divide and round to the nearest hundredth.

19. $6\overline{)0.5934}$

20. $8\overline{)0.0693}$

21. $0.3\overline{)2.462}$

22. $0.6\overline{)5.723}$

23. $0.793 \div 0.413$

24. $0.6341 \div 0.0285$

25. $34 \div 0.0756$

26. $76 \div 0.08659$

Divide and round to the nearest thousandth.

27. $4.3\overline{)67.37}$

28. $41.6\overline{)83.126}$

29. $4.16\overline{)0.06849}$

30. $3.46\overline{)0.5699}$

31. $0.13 \div 0.009$

32. $0.39 \div 0.0087$

OBJECTIVE 2

Find the average, median, or mode of a set of decimals.

A Find the average.

33. 4.6, 6.6

34. 3.6, 8.2

35. 5.7, 10.2

36. 21.7, 36.3

37. 7.5, 5.3, 4.3

38. 7.2, 8.8, 0.8

39. 12.1, 12.5, 12.6

40. 7.9, 15.2, 8.7

41. 9.1, 5.2, 11.5, 7.3

42. 14.3, 20.6, 16.7, 11.2

B Find the average, median, and the mode.

43. 7.8, 9.08, 3.9, 5.7

44. 4.87, 6.93, 4.1, 9.6

45. 21.5, 21.5, 11.75, 13.5, 23.27

46. 9.4, 6.48, 12.2, 6.48, 8.6

47. 14.3, 15.4, 7.6, 17.4, 21.6**48.** 57.8, 36.9, 48.9, 51.9, 63.7**49.** 0.675, 0.431, 0.662, 0.904**50.** 0.261, 0.773, 0.663, 0.308**51.** 0.4523, 0.8674, 0.8674, 0.9234, 0.4535**52.** 2.67, 11.326, 17.53, 22.344, 22.344**C**

53. The common stock of Microsoft Corporation closed at \$25.25, \$24.375, \$22.15, \$23.85, and \$24.325 during 1 week in 2005. What was the average closing price of the stock?

54. A consumer watchdog group priced a box of a certain type of cereal at six different grocery stores. They found the following prices: \$4.19, \$4.42, \$4.25, \$3.99, \$4.05, and \$4.59. What are the average and median selling prices of a box of cereal? Round to the nearest cent.

55. Find the quotient of 17.43 and 0.19, and round to the nearest hundredth.

56. Find the quotient of 1.706 and 77, and round to the nearest hundredth.

Exercises 57–62. The table shows some prices from a grocery store.

Grocery Prices

Item	Quantity	Price	Item	Quantity	Price
Apples	4 lb	\$3.89	Potatoes	10 lb	\$4.19
Strawberries	3 pt	\$5.25	Rib steak	2.38 lb	\$14.21
BBQ sauce	18 oz	\$1.09	Turkey	15.6 lb	\$13.88

57. Find the unit price (price per pound) of apples. Round to the nearest tenth of a cent.

58. Find the unit price (price per ounce) of BBQ sauce. Round to the nearest tenth of a cent.

59. Find the unit price of rib steak. Round to the nearest tenth of a cent.

60. Find the unit price of potatoes. Round to the nearest tenth of a cent.

61. Using the unit price, find the cost of a 21.8-lb turkey. Round to the nearest cent.

62. Using the unit price, find the cost of 11 pt of strawberries. Round to the nearest cent.

63. Two hundred fifty-six alumni of Miami University donated \$245,610 to the university. To the nearest cent, what was the average donation?

64. The Adams family had the following natural gas bills for last year:

January	\$176.02	July	\$ 10.17
February	69.83	August	14.86
March	43.18	September	18.89
April	38.56	October	23.41
May	12.85	November	63.19
June	29.55	December	161.51

The gas company will allow them to make equal payments this year equal to the monthly average of last year. How much will the payment be? Round to the nearest cent.

- 65.** The average daily temperature by month in Orlando, Florida, measured in degrees Fahrenheit is:
- | | | | | | |
|----------|----|--------|----|-----------|----|
| January | 72 | May | 88 | September | 90 |
| February | 73 | June | 91 | October | 84 |
| March | 78 | July | 92 | November | 78 |
| April | 84 | August | 92 | December | 73 |

To the nearest tenth, what is the average daily temperature over the year? What are the median and mode of the average daily temperatures?

- 66.** Tim Raines, the fullback for the East All-Stars, gained the following yards in six carries: 8.5 yd, 12.8 yd, 3.2 yd, 11 yd, 9.6 yd, and 4 yd. What was the average gain per carry? Round to the nearest tenth of a yard.

- 67.** The price per gallon of the same grade of gasoline at eight different service stations is 2.489, 2.599, 2.409, 2.489, 2.619, 2.599, 2.329, and 2.479. What are the average, median, and mode for the price per gallon at the eight stations? Round to the nearest thousandth.

Exercises 68–72. The table gives the high and low temperatures in cities in the Midwest.

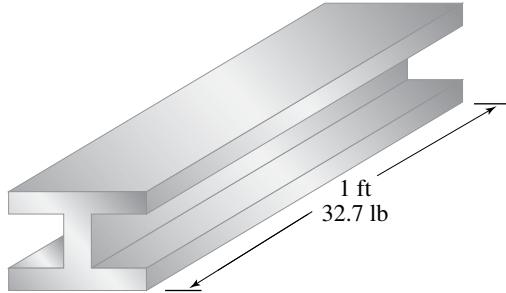
Temperatures in Midwest Cities, 2/28/05

City	High (°F)	Low (°F)
Detroit	37	26
Cincinnati	43	31
Chicago	35	30
St. Louis	44	28
Kansas City	33	24
Minneapolis	27	17
Milwaukee	33	28
Rapid City	40	18

- 68.** What was the average high temperature for the cities, to the nearest tenth?
- 69.** What was the average low temperature for the cities, to the nearest tenth?
- 70.** What was the average daily range of temperature for the cities, to the nearest tenth? Did any of the cities have the average daily range?
- 71.** What was the mode for the high temperatures?
- 72.** What was the median of the low temperatures?
- 73.** June drove 766.5 miles on 15.8 gallons of gas in her hybrid car. What is her mileage (miles per gallon)? Round to the nearest mile.
- 74.** A 65-gallon drum of cleaning solvent in an auto repair shop is being used at the rate of 1.94 gallons per day. At this rate, how many full days will the drum last?
- 75.** The Williams Construction Company uses cable that weighs 2.75 pounds per foot. A partly filled spool of the cable is weighed. The cable itself weighs 867 pounds after subtracting the weight of the spool. To the nearest foot, how many feet of cable are on the spool?

- 76.** A plumber connects the sewers of four buildings to the public sewer line. The total bill for the job is \$7358.24. What is the average cost for each connection?

- 77.** The 1-ft I beam, shown below, weighs 32.7 lb. What is the length of a beam weighing 630.6 lb? Find the length to the nearest tenth of a foot.



- 78.** Allowing 0.125 in. of waste for each cut, how many bushings, which are 1.45 in. in length, can be cut from a 12-in. length of bronze? What is the length of the piece that is left?

Exercises 79–81. The table shows population and area facts for Scandinavia.

Population and Area in Scandinavia

Country	Area in Square Miles	Population
Denmark	16,639	5,336,394
Finland	130,100	5,167,486
Norway	125,200	4,481,162
Sweden	173,732	8,873,052

- 79.** Which country has the smallest area and which has the smallest population?

- 80.** Population density is the number of people per square mile. Calculate the population density for each country, rounded to the nearest hundredth. Add another column to the table with this information.

- 81.** What do you conclude about how crowded the countries are?

Exercises 82–86 relate to the chapter application. In baseball, a pitcher's earned run average (ERA) is calculated by dividing the number of earned runs by the quotient of the number of innings pitched and 9. The lower a pitcher's ERA the better.

- 82.** Suppose a pitcher allowed 34 earned runs in 85 innings of play. Calculate his ERA and round to the nearest hundredth.
- 84.** A runner's stolen base average is the quotient of the number of bases stolen and the number of attempts. As with the batting averages, this number is usually rounded to the nearest thousandth. Calculate the stolen base average of a runner who stole 18 bases in 29 attempts.
- 86.** The combined height of the NBA's 348 players at the start of the 2000–2001 season was 2294.67 feet, or about twice the height of the Empire State Building. Find the average height of an NBA player. Round to the nearest tenth.
- 83.** A pitcher allows 20 earned runs in 110 innings. Calculate his ERA, rounding to the nearest hundredth.
- 85.** A good stolen base average is 0.700 or higher. Express this as a fraction and say in words what the fraction represents.

STATE YOUR UNDERSTANDING

- 87.** Describe a procedure for determining the placement of the decimal in a quotient. Include an explanation for the justification of the procedure.
- 88.** Explain how to find the quotient of $4.1448 \div 0.0012$.
- 89.** Copy the table and fill it in.

Division Procedure

Operation on Decimals	Procedure	Example
Division		

CHALLENGE

- 90.** What will be the value of \$3000 invested at 6% interest compounded quarterly at the end of 1 year? (Compounded quarterly means that the interest earned for the quarter, the annual interest divided by four, is

added to the principal and then earns interest for the next quarter.) How much more is earned by compounding quarterly instead of annually?

GROUP WORK

- 91.** Determine the distance each member of your group travels to school each day. Find the average distance to the nearest hundredth of a mile for your group. Compare these results with the class. Find the average distance for the entire class. Recalculate the class average after throwing out the longest and the shortest distances. Are the averages different? Why?

- 92.** Go to the library and use the daily newspaper to provide the data for you to calculate the following:

- average daily high temperature
- average daily low temperature
- average daily rainfall
- average daily minutes of daylight

Use data from your town over the past 7 days.

MAINTAIN YOUR SKILLS

Simplify.

93. $\frac{95}{114}$

94. $\frac{168}{216}$

Write as an improper fraction.

95. $4\frac{8}{11}$

96. $18\frac{5}{7}$

Write as a mixed number.

97. $\frac{215}{12}$

98. $\frac{459}{25}$

Find the missing numerator.

99. $\frac{17}{25} = \frac{?}{100}$

100. $\frac{9}{40} = \frac{?}{1000}$

Write as a fraction or mixed number and simplify.

101. $24 \div 40$

102. $135 \div 30$

Getting Ready for Algebra

How & Why

We solve equations that involve multiplication and division of decimals in the same way as equations with whole numbers and fractions.

OBJECTIVE

Solve equations that involve multiplication and division of decimals.

To solve an equation using multiplication or division

1. Divide both sides of the equation by the same number to isolate the variable, or
2. Multiply both sides of the equation by the same number to isolate the variable.

Examples A–E

DIRECTIONS: Solve.

STRATEGY: Isolate the variable by multiplying or dividing both sides by the same number.

A. $1.7x = 86.7$

$$\frac{1.7x}{1.7} = \frac{86.7}{1.7}$$

Because x is multiplied by 1.7, we divide both sides by 1.7. The division is usually written in fractional form. Because division is the inverse of multiplication, the variable is isolated.

$$x = 51$$

Simplify.

CHECK: $17.1(51) = 86.7$

$$86.7 = 86.7$$

The solution is $x = 51$.

Substitute 51 for x in the original equation and simplify.

True.

B. $9.8 = \frac{a}{11.6}$

$$11.6(9.8) = 11.6\left(\frac{a}{11.6}\right)$$

$$113.68 = a$$

Because a is divided by 11.6, we multiply both sides by 11.6. Because multiplication is the inverse of division, the variable is isolated.

CHECK: $9.8 = \frac{113.68}{11.6}$

$$9.8 = 9.8$$

The solution is $a = 113.68$.

Substitute 113.68 for a in the original equation and simplify.

True.

C. $34.6y = 186.84$

$$\frac{34.6y}{34.6} = \frac{186.84}{34.6}$$

$$y = 5.4$$

Divide both sides by 34.6 to eliminate the multiplication and simplify.

CHECK: $34.6(5.4) = 186.84$

$$186.84 = 186.84$$

The solution is $y = 5.4$.

Substitute 5.4 for y in the original equation and simplify.

True.

Warm-Ups A–E

A. $1.2t = 96$

B. $13.5 = \frac{r}{9.7}$

C. $0.18a = 1.6632$

Answers to Warm-Ups

- A. $t = 80$ B. $130.95 = r$
C. $a = 9.24$

D. $\frac{x}{0.481} = 7.2$

D. $\frac{c}{0.508} = 3.1$

$$0.508\left(\frac{c}{0.508}\right) = 0.508(3.1) \quad \text{Multiply both sides by 0.508 and simplify.}$$
$$c = 1.5748$$

CHECK: $\frac{1.5748}{0.508} = 3.1$ Substitute 1.5748 for c in the original equation and simplify.

$$3.1 = 3.1 \quad \text{True.}$$

The solution is $c = 3.1$.

- E. Use the formula in Example E to find the number of calories per serving if there is a total of 3253.6 calories in 28 servings.

- E. The total number of calories T is given by the formula $T = sC$, where s represents the number of servings and C represents the number of calories per serving. Find the number of calories per serving if 7.5 servings contain 739.5 calories.

First substitute the known values into the formula.

$$\begin{aligned} T &= sC \\ 739.5 &= 7.5C \quad \text{Substitute 739.5 for } T \text{ and 7.5 for } s. \\ \frac{739.5}{7.5} &= \frac{7.5C}{7.5} \quad \text{Divide both sides by 7.5.} \\ 98.6 &= C \end{aligned}$$

Since $7.5(98.6) = 739.5$, the number of calories per serving is 98.6.

Answers to Warm-Ups

D. $x = 3.4632$

E. There are 116.2 calories per serving.

Exercises

Solve.

1. $2.7x = 18.9$

2. $2.3x = 0.782$

3. $0.04y = 12.34$

4. $0.06w = 0.942$

5. $0.9476 = 4.12t$

6. $302.77 = 13.7x$

7. $3.3m = 0.198$

8. $0.008p = 12$

9. $0.016q = 9$

10. $11 = 0.025w$

11. $9 = 0.32h$

12. $2.6x = 35.88$

13. $\frac{y}{9.5} = 0.28$

14. $0.07 = \frac{b}{0.73}$

15. $0.312 = \frac{c}{0.65}$

16. $\frac{w}{0.12} = 1.35$

17. $0.0325 = \frac{x}{32}$

18. $0.17 = \frac{t}{8.23}$

19. $\frac{s}{0.07} = 0.345$

20. $\frac{y}{16.75} = 2.06$

21. $\frac{z}{21.02} = 4.08$

22. $\frac{c}{10.7} = 2.055$

23. The total number of calories T is given by the formula $T = sC$, where s represents the number of servings and C represents the number of calories per serving. Find the number of servings if the total number of calories is 3617.9 and there are 157.3 calories per serving.

25. Ohm's law is given by the formula $E = IR$, where E is the voltage (number of volts), I is the current (number of amperes), and R is the resistance (number of ohms). What is the current in a circuit if the resistance is 22 ohms and the voltage is 209 volts?

27. Find the length of a rectangle that has a width of 13.6 ft and an area of 250.24 ft².

29. Each student in a certain instructor's math classes hands in 20 homework assignments. During the term, the instructor has graded a total of 3500 homework assignments. How many students does this instructor have in all her classes? Write and solve an equation to determine the answer.

24. Use the formula in Exercise 23 to find the number of servings if the total number of calories is 10,628.4 and there are 312.6 calories per serving.

26. Use the formula in Exercise 25 to find the current in a circuit if the resistance is 16 ohms and the voltage is 175 volts.

28. Find the width of a rectangular plot of ground that has an area of 3751.44 m² and a length of 127.6 m.

30. Twenty-four plastic soda bottles were recycled and made into one shirt. At this rate, how many shirts can be made from 910 soda bottles? Write and solve an equation to determine the answer.

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4.7 Changing Fractions to Decimals

How & Why

OBJECTIVE

Change fractions to decimals.

Every decimal can be written as a whole number times the place value of the last digit on the right:

$$0.81 = 81 \times \frac{1}{100} = \frac{81}{100}$$

The fraction has a power of 10 for the denominator. Any fraction that has only prime factors of 2 and/or 5 in the denominator can be written as a decimal by building the denominator to a power of 10.

$$\frac{3}{5} = \frac{3}{5} \cdot \frac{2}{2} = \frac{6}{10} = 0.6$$

$$\frac{11}{20} = \frac{11}{20} \cdot \frac{5}{5} = \frac{55}{100} = 0.55$$

Every fraction can be thought of as a division problem $\left(\frac{3}{5} = 3 \div 5\right)$. Therefore, a second method for changing fractions to decimals is division. As you discovered in the previous section, many division problems with decimals do not have a zero remainder at any point. If the denominator of a simplified fraction has prime factors other than 2 or 5, the quotient will be a nonterminating decimal. The fraction $\frac{5}{6}$ is an example:

$$\frac{5}{6} = 0.83333333\dots = 0.\overline{83}$$

The bar over the 3 indicates that the decimal repeats the number 3 forever. Expressing the decimal form of a fraction using a repeat bar is an exact conversion and is indicated with an equal sign (=). In the exercises for this section, round the division to the indicated decimal place or use the repeat bar as directed.

CAUTION

Be careful to use an equal sign (=) when your conversion is exact and an approximately equal sign (\approx) when you have rounded.

To change a fraction to a decimal

Divide the numerator by the denominator.

To change a mixed number to a decimal

Change the fractional part to a decimal and add to the whole-number part.

OBJECTIVE

Change fractions to decimals.

Warm-Ups A–G

Examples A–G BASIC Mathematics Now™

DIRECTIONS: Change the fraction or mixed number to a decimal.

STRATEGY: Divide the numerator by the denominator. Round as indicated. If the number is a mixed number, add the decimal to the whole number.

- A. Change $\frac{19}{20}$ to a decimal.

- A. Change $\frac{37}{40}$ to a decimal.

$$\begin{array}{r} 0.925 \\ 40) 37.000 \\ \underline{36} \quad 0 \\ 1 \quad 00 \\ \underline{80} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

Divide the numerator, 37, by the denominator, 40.

$$\text{Therefore, } \frac{37}{40} = 0.925.$$

- B. Change $13\frac{14}{25}$ to a decimal.

- B. Change $11\frac{13}{50}$ to a decimal.

$$\begin{array}{r} 0.26 \\ 50) 13.00 \\ \underline{10} \quad 0 \\ 3 \quad 00 \\ \underline{3} \quad 00 \\ 0 \end{array}$$

Divide the numerator, 13, by the denominator, 50.

$$11\frac{13}{50} = 11.26 \quad \text{Add the decimal to the whole number.}$$

or

$$\frac{13}{50} = \frac{13}{50} \cdot \frac{2}{2} = \frac{26}{100} = 0.26$$

A fraction with a denominator that has only 2s or 5s for prime factors can be changed to a fraction with a denominator that is a power of 10. This fraction can then be written as a decimal.

$$\text{So } 11\frac{13}{50} = 11 + 0.26 = 11.26.$$

- C. Change $\frac{21}{32}$ to a decimal.

- C. Change $\frac{23}{64}$ to a decimal.

$$\begin{array}{r} 0.359375 \\ 64) 23.000000 \\ \underline{19} \quad 2 \\ 3 \quad 80 \\ \underline{3} \quad 20 \\ 600 \\ \underline{576} \\ 240 \\ \underline{192} \\ 480 \\ \underline{448} \\ 320 \\ \underline{320} \\ 0 \end{array}$$

This fraction can be changed by building to a denominator of 1,000,000 but the factor is not easily recognized, unless we use a calculator.

$$\frac{23}{64} = \frac{23}{64} \cdot \frac{15,625}{15,625} = \frac{359,375}{1,000,000} = 0.359375$$

$$\text{So } \frac{23}{64} = 0.359375.$$

Answers to Warm-Ups

- A. 0.95 B. 13.56 C. 0.65625

CAUTION

Most fractions cannot be changed to terminating decimals because the denominators contain factors other than 2 and 5. In these cases we round to the indicated place value or use a repeat bar.

- D. Change $\frac{23}{29}$ to a decimal rounded to the nearest hundredth.

$$\begin{array}{r} 0.793 \\ 29 \overline{)23.000} \\ 20\ 3 \\ \hline 2\ 70 \\ 2\ 61 \\ \hline 90 \\ 87 \\ \hline 3 \end{array}$$

So $\frac{23}{29} \approx 0.79$.

- E. Change $\frac{7}{11}$ to an exact decimal.

$$\begin{array}{r} .6363 \\ 11 \overline{)7.0000} \\ 6\ 6 \\ \hline 40 \\ 33 \\ \hline 70 \\ 66 \\ \hline 40 \\ 33 \\ \hline 7 \end{array}$$

So $\frac{7}{11} = 0.\overline{63}$.

CALCULATOR EXAMPLE



- F. Change $\frac{773}{923}$ to a decimal rounded to the nearest ten-thousandth.

$773 \div 923 \approx 0.8374864$

So $\frac{773}{923} \approx 0.8375$ to the nearest ten-thousandth.

- D. Change $\frac{8}{13}$ to a decimal rounded to the nearest hundredth.

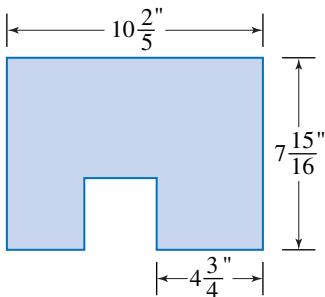
- E. Change $\frac{7}{9}$ to an exact decimal.

- F. Change $\frac{604}{673}$ to a decimal rounded to the nearest thousandth.

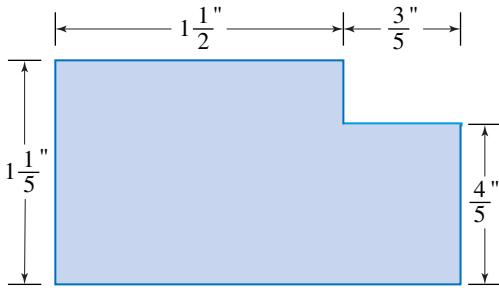
Answers to Warm-Ups

D. 0.62 E. $0.\bar{7}$ F. 0.897

- G. Change the measurements on the given pattern to the nearest tenth for use with a ruler marked in tenths.



- G. Jan needs to make a pattern of the shape as shown. Her ruler is marked in tenths. Change all the measurements to tenths so she can make an accurate pattern.



So that Jan can use her ruler for more accurate measure, each fraction is changed to a decimal rounded to the nearest tenth.

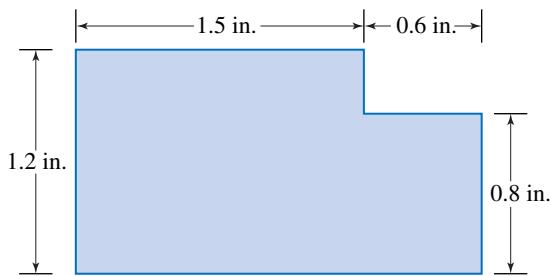
$$1\frac{1}{2} = 1\frac{5}{10} = 1.5$$

$$\frac{3}{5} = \frac{6}{10} = 0.6$$

$$1\frac{1}{5} = 1\frac{2}{10} = 1.2$$

$$\frac{4}{5} = \frac{8}{10} = 0.8$$

Each fraction and mixed number can be changed by either building each to a denominator of 10 as shown or by dividing the numerator by the denominator. The measurements on the drawing can be labeled:



Answers to Warm-Ups

- G. The decimal measures are

$$10\frac{2}{5} \text{ in.} = 10.4 \text{ in.}; 7\frac{15}{16} \text{ in.} \approx 7.9 \text{ in.};$$

and $4\frac{3}{4}$ in. ≈ 4.8 in.

Exercises 4.7

OBJECTIVE

Change fractions to decimals.

A Change the fraction or mixed number to a decimal.

1. $\frac{3}{4}$

2. $\frac{7}{10}$

3. $\frac{1}{8}$

4. $\frac{7}{8}$

5. $\frac{13}{16}$

6. $\frac{23}{32}$

7. $3\frac{7}{20}$

8. $6\frac{13}{20}$

9. $56\frac{73}{125}$

10. $48\frac{43}{50}$

B Change to a decimal rounded to the indicated place value.

Tenth	Hundredth
-------	-----------

11. $\frac{3}{7}$

12. $\frac{8}{9}$

13. $\frac{9}{11}$

14. $\frac{5}{13}$

15. $\frac{9}{13}$

16. $\frac{11}{14}$

17. $\frac{2}{15}$

18. $\frac{9}{19}$

19. $7\frac{7}{18}$

20. $46\frac{11}{17}$

Change each of the following fractions to decimals. Use the repeat bar.

21. $\frac{9}{11}$

22. $\frac{7}{22}$

23. $\frac{5}{12}$

24. $\frac{11}{15}$

C Change each of the following fractions to decimals to the nearest indicated place value.

	Hundredth	Thousandth
25. $\frac{28}{65}$		
26. $\frac{17}{49}$		
27. $\frac{59}{72}$		
28. $\frac{83}{99}$		

Change to a decimal. Use the repeat bar.

29. $\frac{5}{13}$ 30. $\frac{7}{33}$ 31. $\frac{6}{7}$ 32. $\frac{23}{26}$
33. A piece of blank metal stock is $2\frac{3}{8}$ in. in diameter. A micrometer measures in decimal units. If the stock is measured with the micrometer, what will the reading be?
35. Convert the measurements in the figure to decimals.
-
36. Stephen needs $6\frac{17}{20}$ in. of chain to secure his garden gate. What is the decimal equivalent?

Change to a decimal. Round as indicated.

	Hundredth	Thousandth	Ten-thousandth
37. $\frac{33}{57}$			
38. $27\frac{19}{71}$			
39. $33\frac{79}{165}$			
40. $\frac{888}{2095}$			

41. A remnant of material $1\frac{3}{4}$ yards long costs \$7.14. Find the cost per yard of the fabric using fractions. Recalculate the same cost using decimals. Which is easier? Why?

42. An electronics lobbyist works $34\frac{3}{4}$ hours during 1 week. If she is paid \$49.40 per hour, compute her gross wages for the week. Did you use decimals or fractions to do the calculation? Why?

- 43.** In a pole vault meet, where the results are communicated by telephone, the highest jump in Iowa is $23\frac{7}{16}$ ft. The highest jump in Texas is 23.439 ft. Which state has the winning jump?
- 45.** On May 1 in Portland, the sun rose at 5:45 A.M. and set at 8:12 P.M. Express the number of daylight hours as a decimal.

Exercises 46–47 relate to the chapter application.

- 46.** In the 2000 Summer Olympics, Virgilijus Alekna of Lithuania won the discuss throw with a toss of 227 ft 4 in. Convert this distance to a mixed number of feet, then convert your distance to decimal form.
- 47.** On April 13, 2003, Paula Radcliffe of Great Britain set the world's record for the women's marathon with a time of 2 hr 15 min 25 sec. Convert this time to a mixed number of hours and then convert the time to decimal form.

STATE YOUR UNDERSTANDING

- 48.** Write a short paragraph on the uses of decimals and of fractions. Include examples of when fractions are more useful and when decimals are more fitting.

CHALLENGE

- 49.** Which is larger, 0.0012 or $\frac{7}{625}$?
- 50.** Which is larger, 2.5×10^{-4} or $\frac{3}{2000}$?
- 51.** First decide whether the fraction $\frac{1.23}{80}$ is more or less than 0.1. Then change the fraction to a decimal. Were you correct in your estimate?
- 52.** First decide whether the fraction $\frac{62}{0.125}$ is more or less than 100. Then change the fraction to a decimal. Were you correct in your estimate?

GROUP WORK

- 53.** Select 10 stocks from the NYSE and assume you have 1000 shares of each. From today's paper, calculate the current value of your holdings. Using the reported changes, calculate the value of your holdings yesterday. How much money did you make or lose?

MAINTAIN YOUR SKILLS

Perform the indicated operations.

54. $7 \cdot 12 \div 4 + 2 - 5$

55. $(9 - 5) \cdot 5 - 14 + 6 \div 2$

56. $6^2 \cdot 4 - 3 \cdot 7 + 15$

57. $(7 - 4)^2 - 9 \div 3 + 7$

58. Estimate the sum of 34, 75, 82, and 91 by rounding to the nearest ten.

60. Estimate the product of 56 and 72 by front rounding both factors.

62. Mr. Lewis buys 350 books for \$60 at an auction. He sells two-fifths of them for \$25, 25 books at \$1.50 each, 45 books at \$1 each, and gives away the rest. How many books does he give away? What is his total profit if his handling cost is \$15?

59. Estimate the difference of 345 and 271 by rounding to the nearest ten.

61. Estimate the product of 265 and 732 by front rounding both factors.

63. John C. Scott Realty sold six houses last week at the following prices: \$145,780, \$234,700, \$195,435, \$389,500, \$275,000, and \$305,677. What was the average sale price of the houses?

4.8 Order of Operations; Estimating

How & Why

OBJECTIVE 1

Do any combination of operations with decimals.

The order of operations for decimals is the same as that for whole numbers and fractions.

Order of Operations

To simplify an expression with more than one operation follow these steps

1. Parentheses—Do the operations within grouping symbols first (parentheses, fraction bar, etc.), in the order given in steps 2, 3, and 4.
2. Exponents—Do the operations indicated by exponents.
3. Multiply and Divide—Do multiplication and division as they appear from left to right.
4. Add and Subtract—Do addition and subtraction as they appear from left to right.

Examples A–E BASIC Mathematics Now™

DIRECTIONS: Perform the indicated operations.

STRATEGY: Use the same order of operations as for whole numbers and fractions.

A. Simplify: $0.87 - 0.32(0.35)$

$$\begin{aligned}0.87 - 0.32(0.35) &= 0.87 - 0.112 && \text{Multiplication is done first.} \\&= 0.758 && \text{Subtract.}\end{aligned}$$

So $0.87 - 0.32(0.35) = 0.758$.

B. Simplify: $0.66 \div 0.22(4.05)$

$$\begin{aligned}0.66 \div 0.22(4.05) &= 3(4.05) && \text{Division is done first, because it occurs first.} \\&= 12.15 && \text{Multiply.}\end{aligned}$$

So $0.66 \div 0.22(4.05) = 12.15$.

C. Simplify: $(3.4)^2 - (2.6)^2$

$$\begin{aligned}(3.4)^2 - (2.6)^2 &= 11.56 - 6.76 && \text{Exponents are done first.} \\&= 4.8 && \text{Subtract.}\end{aligned}$$

So $(3.4)^2 - (2.6)^2 = 4.8$.

CALCULATOR EXAMPLE

D. Simplify: $8.736 \div 2.8 + (4.57)(5.9) + 12.67$

STRATEGY: All but the least expensive calculators have algebraic logic. The operations can be entered in the same order as the exercise.

So $8.736 \div 2.8 + (4.57)(5.9) + 12.67 = 42.753$.

OBJECTIVES

1. Do any combination of operations with decimals.
2. Estimate the sum, difference, product, and quotient of decimals.

Warm-Ups A–E

A. Simplify: $0.93 - 0.45(0.62)$

B. Simplify: $0.43 \div 0.5(2.55)$

C. Simplify: $(5.4)^2 - (1.6)^2$

D. Simplify: $102.92 \div 8.3 + (0.67)(34.7) - 21.46$

Answers to Warm-Ups

- A. 0.651 B. 2.193
C. 26.6 D. 14.189

- E. Nuyen buys the following tickets for upcoming Pops concerts at the local symphony: 3 tickets at \$45.75 each; 2 tickets at \$39.50 each; 4 tickets at \$42.85 each; 3 tickets at \$48.50; and 5 tickets at \$40.45. For buying more than 10 tickets, Nuyen gets \$3.00 off each ticket purchased. What is Nuyen's total cost for the tickets?

- E. Ellen buys the following items at the grocery store: 3 cans of soup at \$1.23 each; 2 cans of peas at \$0.89 each; 1 carton of orange juice at 2 for \$5.00; 3 cans of salmon at \$2.79 each; and 1 jar of peanut butter at \$3.95. Ellen had a coupon for \$2.00 off when you purchase 3 cans of salmon. What did Ellen pay for the groceries?

STRATEGY: Find the sum of the cost of each item and then subtract the coupon savings. To find the cost of each type of food, multiply the unit price by the number of items.

$$3(1.23) + 2(0.89) + 1(5.00 \div 2) \\ + 3(2.79) + 1(3.95) - 2.00$$

To find the unit price of the orange juice, we must divide the price for two by 2.

$$3.69 + 1.78 + 1(2.50) + 8.37 \\ + 3.95 - 2.00$$

Multiply and divide.

$$3.69 + 1.78 + 2.50 + 8.37 \\ + 3.95 - 2.00$$

Multiply.

$$18.29$$

Add and subtract.

Ellen spent \$18.29 for the groceries.

How & Why

OBJECTIVE 2

Estimate the sum, difference, product, and quotient of decimals.

To estimate the sum or difference of decimals, we round the numbers to a specified place value. We then add or subtract these rounded numbers to get the estimate. For example, to estimate the sum of $0.345 + 0.592 + 0.0067$, round each to the nearest tenth.

$$\begin{array}{r} 0.345 \approx 0.3 \\ 0.592 \approx 0.6 \\ + 0.0067 \approx 0.0 \\ \hline 0.9 \end{array}$$

So 0.9 is the estimate of the sum. We usually can do the estimation mentally and it serves as a check to see if our actual sum is reasonable. Here the actual sum is 0.9437.

Similarly, we can estimate the difference of two numbers. For instance, Jane found the difference of 0.00934 and 0.00367 to be 0.008973. To check, we estimate the difference by rounding each number to the nearest thousandth,

$$\begin{array}{r} 0.00934 \approx 0.009 \\ - 0.00367 \approx 0.004 \\ \hline 0.005 \end{array}$$

So 0.005 is the estimate of the difference. This is not close to Jane's answer, so she needs to subtract again.

$$\begin{array}{r} 0.00934 \\ - 0.00367 \\ \hline 0.00567 \end{array}$$

This answer is close to the estimate. Jane may not have aligned the decimal points properly.

Answers to Warm-Ups

- E. Nuyen pays \$684.40 for the tickets.

Examples F–I

BASIC
Mathematics = Now™

DIRECTIONS: Estimate the sum or difference.

STRATEGY: Round each number to a specified place value and then add or subtract.

- F. Estimate the sum by rounding to the nearest hundredth: $0.012 + 0.067 + 0.065$

$$0.01 + 0.07 + 0.07 = 0.15 \quad \text{Round each number to the nearest hundredth and add.}$$

So the estimated sum is 0.15.

- G. Estimate the sum by rounding to the nearest tenth: $0.0012 + 0.56 + 0.0035 + 2.06$

$$0.0 + 0.6 + 0.0 + 2.1 = 2.7 \quad \text{Round each number to the nearest tenth and add.}$$

The estimated sum is 2.7.

- H. Estimate the difference by rounding to the nearest tenth: $0.781 - 0.472$

$$0.8 - 0.5 = 0.3 \quad \text{Round each number to the nearest tenth and subtract.}$$

So the estimated difference is 0.3.

- I. Use estimation to see if the following answer is reasonable:

$$0.0067 - 0.0034 = 0.0023$$

$$0.007 - 0.003 = 0.004 \quad \text{Round each number to the nearest thousandth and subtract.}$$

The estimated sum is 0.004, and therefore the answer is not reasonable. So we subtract again.

$0.0067 - 0.0034 = 0.0033$, which is correct.

To estimate the product of decimals **front round** each number and then multiply. For instance to find the estimated product, $(0.067)(0.0034)$, round to the product, $(0.07)(0.003)$, and then multiply. The estimated product is $(0.07)(0.003) = 0.00021$. If the estimate is close to our calculated product we will feel comfortable that we have the product correct. In this case our calculated product is 0.0002278.

We estimate a division problem only to verify the correct place value in the quotient. If we front round and then divide the numbers, it could result in an estimate that is as much as 3 units off the correct value. However, the place value will be correct. Find the correct place value of the first nonzero digit in 0.000456 divided by 0.032.

$$\overline{0.03)0.0005}$$

$$\begin{array}{r} .01 \\ 3 \overline{)0.05} \\ \end{array} \quad \text{Multiply the divisor and the dividend by 100 so we are dividing by a whole number. Find a partial quotient.}$$

We see that the quotient will have its first nonzero digit in the hundreds place. So given a choice of answers, 0.1425, 0.01425, 0.001425, or 1.425, we choose 0.01425 because the first nonzero digit is in the hundredths place.

Warm-Ups F–I

- F. Estimate the sum by rounding to the nearest hundredth: $0.045 + 0.013 + 0.007$

- G. Estimate the sum by rounding to the nearest hundredth: $0.035 + 0.00056 + 0.004 + 0.067$

- H. Estimate the difference by rounding to the nearest hundredth: $0.072 - 0.0346$

- I. Use estimation to see if the following answer is reasonable:
 $0.843 - 0.05992 = 0.78308$

Answers to Warm-Ups

- F. 0.07 G. 0.11 H. 0.04
I. The answer is reasonable.

Warm-Ups J–M

- J. Estimate the product:
 $(0.0556)(0.0032)$

- K. Marta calculated
 $(0.3892)(0.50231)$ and got
0.1954499052. Estimate the
product to determine if this
is a reasonable answer.

- L. Use estimation to decide if
the quotient $0.00264 \div 0.033$
is (a) 0.8, (b) 0.08, (c) 8,
(d) 0.008, or (e) 0.0008.

- M. Pete has \$100 on the books
at Rock Creek Country Club.
He wants to buy the
following items: 2 dozen
golf balls at \$21.95 a dozen;
3 bags of tees at \$2.08 each;
glove, \$5.65; towel, \$10.75;
cap, \$14.78; and 3 pairs of
socks at \$4.15 each. Round
to the nearest dollar to
estimate the cost.

Examples J–M

DIRECTIONS: Estimate the product or quotient.

STRATEGY: Front round each number and then multiply or divide.

- J. Estimate the product: $(0.0632)(0.0043)$

$$(0.06)(0.004) = 0.00024$$

Front round and multiply. Note: there are
5 decimal places in the factors so there
must be 5 decimal places in the product.

So the estimated product is 0.00024.

- K. Nick calculated $(0.879)(0.03784)$ and got 0.332614. Estimate the product to determine if this is a reasonable answer.

$$(0.9)(0.04) = 0.036$$

So the estimated product is 0.036, which is not close to Nick's answer. His answer is not reasonable. The actual product is 0.03326136.

- L. Use estimation to decide if the quotient $0.209 \div 0.0076$ is (a) 2.75, (b) 0.0275, (c) 27.5, (d) 0.275, or (e) 0.00275.

$$0.2 \div 0.008 = ?$$

Front round.

$$200 \div 8 \approx 20$$

Move the decimal point three places to the right in each number so we are dividing by a whole number and divide.

From the estimated quotient, we see that the first nonzero digit is in the tens place. So the quotient is c, or 27.5.

- M. Jane goes to the store to buy the following items: eggs, \$1.29; cereal, \$2.89; 3 cans of soup at \$.89 each; hamburger, \$3.49; 2 cans of fruit at \$1.19 each; milk, \$2.15; potatoes, \$0.79; and bread, \$2.79. Jane has \$20 to spend, so she will estimate the cost to see if she can afford the items. Can Jane afford all of the items?

Round each price to the nearest dollar and keep a running total:

Item	Actual Cost	Estimated Cost	Running Total
Eggs	\$1.29	\$1	\$1
Cereal	\$2.89	\$3	\$4
Soup	$3 \times \$0.89$	$3 \times \$1 = \3	\$7
Hamburger	\$3.49	\$3	\$10
Fruit	$2 \times \$1.19$	$2 \times \$1 = \2	\$12
Milk	\$2.15	\$2	\$14
Potatoes	\$0.79	\$1	\$15
Bread	\$2.79	\$3	\$18

Multiply the rounded cost by 3, the number of cans of soup.
Multiply the rounded cost by 2, the number of cans of fruit.

Jane estimates the cost at \$18 (the actual cost is \$18.45), so she can afford the items.

Answers to Warm-Ups

- J. 0.00018
K. Marta's answer is reasonable.
L. b, or 0.08
M. The estimated cost is \$94, so Pete
can afford the items.

Exercises 4.8

OBJECTIVE 1

Do any combination of operations with decimals.

A Perform the indicated operations.

- | | | |
|-------------------------|-----------------------|-------------------------|
| 1. $0.9 - 0.7 + 0.3$ | 2. $0.8 - 0.2 + 0.4$ | 3. $0.36 \div 9 - 0.02$ |
| 4. $0.56 \div 4 + 0.13$ | 5. $2.4 - 3(0.7)$ | 6. $3.6 + 3(0.2)$ |
| 7. $4(1.2) + 2(5.3)$ | 8. $5(1.3) - 0.4(10)$ | 9. $0.21 + (0.3)^2$ |
| 10. $0.52 - (0.4)^2$ | | |

B

- | | |
|---------------------------------|---------------------------------|
| 11. $9.35 - 2.54 + 6.91 - 3.65$ | 12. $0.89 + 6.98 - 5.67 + 0.09$ |
| 13. $7.8 \div 3.9(11.3)$ | 14. $64.4 \div 9.2(0.55)$ |
| 15. $(15.6)(2.5) \div (0.3)$ | 16. $(7.5)(3.42) \div 0.15$ |
| 17. $(5.3)^2 - 5.7(2.4)$ | 18. $(6.2)^2 + 2.22 \div 0.37$ |
| 19. $(6.7)(1.4)^3 \div 0.7$ | |
| 20. $(3.1)^3 - (0.8)^2 + 4.5$ | |

OBJECTIVE 2

Estimate the sum, difference, product, and quotient of decimals.

A Estimate the sum or difference by rounding to the specified place value.

- | | |
|---|---|
| 21. $0.0635 + 0.0982 + 0.0278$, hundredth | 22. $0.0056 + 0.00378 + 0.00611$, thousandth |
| 23. $0.945 - 0.472$, tenth | 24. $0.00562 - 0.00347$, thousandth |
| 25. $3.895 + 4.0012 + 0.78 + 0.0059$, ones | 26. $0.67 + 0.345 + 0.0021 + 0.8754$, tenth |
| 27. $7.972 - 6.7234$, ones | |
| 28. $0.0573 - 0.0109$, hundredth | |

Estimate the product by front rounding the factors.

- | | |
|-----------------------|------------------------|
| 29. $0.00789(0.346)$ | 30. $17.982(3.465)$ |
| 31. $0.0076(1.95567)$ | 32. $0.000782(.00194)$ |

Using front rounding to determine the place value of the first nonzero digit in each of the quotients.

- | | |
|-----------------------------|-------------------------|
| 33. $4.95 \div 0.0341$ | 34. $0.0675 \div 0.451$ |
| 35. $0.0000891 \div 3.78$ | |
| 36. $0.000678 \div 0.00451$ | |

B Use estimation to see if the following answers are reasonable.

37. $0.0384 + 0.0752 + 0.06901 = 0.18261$

38. $0.00921 - 0.00348 = 0.0573$

39. $0.00576(0.0491) = 0.000282816$

40. $0.0135 \div 0.000027 = 500$

C

41. Elmer goes shopping and buys 3 cans of cream-style corn at 89¢ per can, 4 cans of tomato soup at \$1.09 per can, 2 bags of corn chips at \$2.49 per bag, and 6 candy bars at 59¢ each. How much does Elmer spend?

43. Using estimation, determine if the answer to $0.0023452 \div 0.572$ is
 (a) 0.041 (b) 4.1 (c) 0.00041 (d) 0.0041 or (e) 0.41

Perform the indicated operations.

45. $(7.3)(6.5) - (4.8)(5.4) + (5.6)^2$

42. Christie buys school supplies for her children. She buys 6 pads of paper at \$1.49 each, 5 pens at \$1.19 each, 4 erasers at 59¢ each, and 4 boxes of crayons at \$2.49 each. How much does she spend?

47. $21.075 - [(9.4)(1.26) + 5.15]$

46. $14.7 + 2.49(3.1) - 6.8(1.33) + 34$

49. $7.6(2.77 + 5.98 - 4.35) - 3.25(1.71)$

48. $11.3 - [(2.1)^2 - 3.89]$

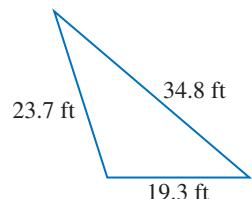
51. Alex multiplies 0.00674 by 0.134 and gets the product 0.00090316. Estimate the product to determine if Alex's answer is reasonable.

52. Catherine divides 0.0064 by 0.0125 and gets the quotient 5.12. Estimate the place value of the largest nonzero place value to see if Catherine's answer is reasonable.

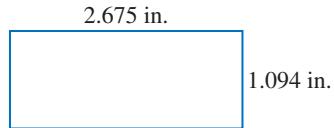
53. Estelle goes to the store to buy a shirt for each of her six grandsons. She finds a style she likes that costs \$23.45 each. Estelle has budgeted \$120 for the shirts. Using estimation, determine if she has enough money to buy the 6 shirts.

54. Pedro goes to the candy store to buy chocolates for his wife, his mother, and his mother-in-law for Mother's Day. The 3-lb box of chocolates costs \$27.85 each. Pedro has \$80 to buy the chocolates. Estimate the cost to see if Pedro has enough money to buy the boxes of chocolates.

55. Estimate the perimeter of the triangle by rounding each measurement to the nearest foot.



56. Estimate the perimeter of the rectangle by rounding each side to the nearest tenth of an inch.



- 57.** Jane goes to the store to buy the following items: egg substitute, \$2.39; cereal, \$4.59; 3 cans of soup at \$1.09 each; hamburger, \$3.15; 2 cans of gravy at \$1.39 each; orange juice, \$3.15; lettuce, \$1.59; and bread, \$1.79. Jane has \$20 to spend, so she will estimate the cost to see if she can afford the items. Can Jane afford all of the items?
- 59.** Matthew purchased the following items at a big box store in preparation for a fishing trip: fishing pole, \$14.88; 4 jars of power bait at \$2 each; a fishing rod holder, \$12.99; a fishing vest, \$16.88; 3 life vests at \$12.97 each; 6 packages of snelled hooks at \$0.88 each; 3 spools of trilene fishing line at \$4.88 each; and 4 fishing lures at \$4.88 each. Matthew has a coupon for \$14.50 off his purchases. How much did he pay for the items?
- 58.** Pete has a \$120 credit at Bi-Mart. He wants to buy the following items: 2 ivy leaf park benches at \$32.97 each; 2 bags of outdoor planting mix at \$2.29 each; 3 zonal geraniums at \$2.49 each; copier paper, \$3.49; St. Joseph Aspirin, \$2.59; shower curtain, \$21.79; and 2 scented candles at \$5.15 each. Estimate the cost by rounding to the nearest dollar.
- 60.** The wholesale cost of shampoo is \$1.11 per bottle, while the wholesale cost of conditioner is \$0.89. The Fancy Hair Beauty Salon sells the shampoo for \$8.49 a bottle and the conditioner for \$8.19 a bottle. What is the net income on the sale of a case, 24 bottles, of each product?

Exercises 61–64 relate to the chapter application.

- 61.** At the 2005 Byron Nelson PGA Championship, Ted Purdy won, and he received \$1,116,000. Second place was won by Sean O'Hair and he received \$669,600. Three players tied for third and each received \$322,400. Four players tied for sixth place and each received \$200,725.
- How much in prize winnings did these nine golfers receive?
 - If the total purse for the tournament was \$5,800,000 and there were 63 other golfers who won money, what was the average earnings of these 63 golfers? Round to the nearest dollar.
- 62.** The New York Yankees list the following salaries for the six highest paid players: Alex Rodriguez, \$25,705,118; Derek Jeter, \$19,600,000; Mike Mussina, \$19,000,000; Kevin Brown, \$15,714,286; Randy Johnson, \$15,419,815; and Jason Giambi, \$13,428,571.
- What is the average salary of these six players? Round to the nearest dollar.
 - If the total Yankee payroll is \$205,938,439 and there are another 18 players on the roster, what is the average pay of these 18 players? Round to the nearest dollar.

Exercises 63–64. The table gives a summary of the 2004 World Series between the St. Louis Cardinals and the Boston Red Sox.

Game	Team	Runs	Hits
1	Boston Red Sox	11	13
	St. Louis Cardinals	9	11
2	Boston Red Sox	6	8
	St. Louis Cardinals	2	5
3	Boston Red Sox	4	9
	St. Louis Cardinals	1	4
4	Boston Red Sox	3	9
	St. Louis Cardinals	0	4

- 63.** How many runs per game did the Red Sox average over the entire series? How many runs did the Cardinals average?
- 64.** How many hits per game did the Red Sox average over the entire series? How many hits did the Cardinals average?

STATE YOUR UNDERSTANDING

- 65.** Explain the difference between evaluating $0.3(5.1)^2 + 8.3 \div 5$ and $[0.3(5.1)^2 + 8.3] \div 5$. How do the symbols indicate the order of the operations?

CHALLENGE

Insert grouping symbols to make each statement true.

66. $2 \cdot 8.1 \div 5 - 1 = 4.05$

67. $3.62 \div 0.02 + 72.3 \cdot 0.2 = 0.25$

68. $3.62 \div 0.02 + 8.6 \cdot 0.51 = 96.696$

69. $1.4^2 - 0.8^2 = 1.3456$

- 70.** The average of 4.56, 8.23, 16.5, and a missing number is 8.2975. Find the missing number.

GROUP WORK

- 71.** The body-mass index (BMI) is a technique used by health professionals to assess a person's excess fat and associated risk for heart disease, stroke, hypertension, and diabetes. The BMI is calculated by multiplying a

person's weight (in pounds) by 705 and dividing the result by the square of the person's height in inches. The table gives the degree of risk of disease for various BMI values.

BMI	Disease Risk
< 20.00	Moderate to very high
20.00 to 21.99	Low
22.00 to 24.99	Very low
25.00 to 29.99	Low
30.00 to 34.99	Moderate
35.00 to 39.99	High
40 or higher	Very high

Source: *Lifetime Physical Fitness and Wellness* by Hoeger and Hoeger.

Calculate the BMI for everyone in your group. Round your calculations to the nearest hundredth. Why are large BMI values associated with more risk for disease?

Why are very low values of BMI also associated with more risk for disease?

MAINTAIN YOUR SKILLS*Change to a decimal.*

72. $\frac{13}{16}$

73. $\frac{27}{32}$

74. $\frac{29}{80}$

75. $\frac{58}{25}$

Change to a fraction or mixed number and simplify.

76. 0.68

77. 0.408

78. 2.435

79. 6.84

80. The sale price of a upright vacuum cleaner is \$69.75. If the sale price was marked down \$30.24 from the original price, what was the original price?
81. The price of a Panasonic 17" LCD TV is \$588.88. The store is going to put it on sale at a discount of \$98.50. What price should the clerk put on the TV for the sale?

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Getting Ready for Algebra

How & Why

We solve equations that require more than one operation in the same way as equations with whole numbers and fractions.

OBJECTIVE

Solve equations that require more than one operation.

To solve an equation that requires more than one operation

1. Eliminate the addition or subtraction by performing the inverse operation.
2. Eliminate the multiplication by dividing both sides by the same number; that is, perform the inverse operation.

Examples A–C

DIRECTIONS: Solve.

STRATEGY: Isolate the variable by performing the inverse operations.

A. $2.6x + 4.8 = 25.6$

$$2.6x + 4.8 - 4.8 = 25.6 - 4.8 \quad \text{Eliminate the addition by subtracting 4.8 from both sides.}$$

$$2.6x = 20.8$$

$$\frac{2.6x}{2.6} = \frac{20.8}{2.6} \quad \text{Eliminate the multiplication by dividing both sides by 2.6.}$$

$$x = 8$$

CHECK: $2.6(8) + 4.8 = 25.6$

$$20.8 + 4.8 = 25.6$$

$$25.6 = 25.6$$

The solution is $x = 8$.

B. $8.3 = 1.25x + 4.65$

$$\begin{array}{r} 8.3 = 1.25x + 4.65 \\ -4.65 = -4.65 \\ \hline 3.65 = 1.25x \end{array} \quad \text{Subtract 4.65 from both sides.}$$

$$\frac{3.65}{1.25} = \frac{1.25x}{1.25} \quad \text{Divide both sides by 1.25.}$$

$$2.92 = x$$

CHECK: $8.3 = 1.25(2.92) + 4.65$

$$8.3 = 3.65 + 4.65$$

$$8.3 = 8.3$$

The solution is $x = 2.92$.

Warm-Ups A–C

A. $0.07y - 3.8 = 0.4$

B. $5.72 = 3.25t + 5.33$

Answers to Warm-Ups

A. $y = 60$

B. $0.12 = t$

- C. Use the formula in Example C to find the Celsius temperature that corresponds to 122.9°F.

- C. The formula relating temperature measured in degrees Fahrenheit and degrees Celsius is $F = 1.8C + 32$. Find the Celsius temperature that corresponds to 58.19°F.

First substitute the known values into the formula.

$$F = 1.8C + 32$$

$$58.19 = 1.8C + 32$$

$$58.19 - 32 = 1.8C + 32 - 32$$

$$26.19 = 1.8C$$

$$\frac{26.19}{1.8} = \frac{1.8C}{1.8}$$

$$14.55 = C$$

Substitute $F = 58.19$.

Subtract 32 from both sides.

Divide both sides by 1.8.

Because $1.8(14.55) + 32 = 58.19$, the temperature is 14.55°C.

Answers to Warm-Ups

- C. The temperature is 50.5°C.

Exercises

Solve.

1. $2.5x - 7.6 = 12.8$
2. $0.25x - 7.3 = 0.95$
3. $1.8x + 6.7 = 12.1$
4. $15w + 0.006 = 49.506$
5. $4.115 = 2.15t + 3.9$
6. $10.175 = 1.25y + 9.3$
7. $0.03x - 18.7 = 3.53$
8. $0.08r - 5.62 = 72.3$
9. $7x + 0.06 = 2.3$
10. $13x + 14.66 = 15.7$
11. $3.65m - 122.2 = 108.115$
12. $22.5t - 657 = 231.75$
13. $5000 = 125y + 2055$
14. $3700 = 48w + 1228$
15. $60p - 253 = 9.5$
16. $17.8 = 0.66y + 7.9$
17. $8.551 = 4.42 + 0.17x$
18. $14 = 0.25w - 8.6$
19. $45 = 1.75h - 1.9$
20. $4000 = 96y + 1772.8$
21. $1375 = 80c + 873$
22. $7632 = 90t - 234$
23. The formula relating temperatures measured in degrees Fahrenheit and degrees Celsius is $F = 1.8C + 32$. Find the Celsius temperature that corresponds to 248°F .
25. The formula for the balance of a loan D is $D + NP = B$, where P represents the monthly payment, N represents the number of payments made, and B represents the amount of money borrowed. Find the number of the monthly payments Gina has made if she borrowed \$1764, has a remaining balance of \$661.50, and pays \$73.50 per month.
27. Catherine is an auto mechanic. She charges \$36 per hour for her labor. The cost of parts needed is in addition to her labor charge. How many hours of labor result from a repair job in which the total bill (including \$137.50 for parts) is \$749.50? Write and solve an equation to determine the answer.
26. Use the formula in Exercise 23 to find the Celsius temperature that corresponds to 45.5°F .
28. A car rental agency charges \$28 per day plus \$0.27 per mile to rent one of their cars. Determine how many miles were driven by a customer after a 3-day rental that cost \$390.45. Write and solve an equation to determine the answer.

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Key Concepts CHAPTER 4

Section 4.1 Decimals: Reading, Writing, and Rounding

Definitions and Concepts	Examples
Decimal numbers are another way of writing fractions and mixed numbers.	1.3 One and three tenths 2.78 Two and seventy-eight hundredths 5.964 Five and nine hundred sixty-four thousandths
To round a decimal to a given place value: <ul style="list-style-type: none"> • Mark the given place value. • If the digit on the right is 5 or more, add 1 to the marked place and drop all digits on the right. • If the digit on the right is 4 or less, drop all digits on the right. • Write zeros on the right if necessary so that the marked digit still has the same place value. 	Round 4.792 to the nearest tenth \uparrow $4.792 \approx 4.8$ Round 4.792 to the nearest hundredth \uparrow $4.792 \approx 4.79$ Round 563.79 to the nearest ten \uparrow $563.79 \approx 560$

Section 4.2 Changing Decimals to Fractions; Listing in Order

Definitions and Concepts	Examples
To change a decimal to a fraction: <ul style="list-style-type: none"> • Read the decimal word name. • Write the fraction that has the same name. • Simplify. 	0.45 is read “forty-five hundredths” $\begin{aligned} 0.45 &= \frac{45}{100} \\ &= \frac{9}{20} \end{aligned}$
To list decimals in order: <ul style="list-style-type: none"> • Insert zeros on the right so that all the decimals have the same number of decimal places. • Write the numbers in order as if they were whole numbers. • Remove the extra zeros. 	List 1.46, 1.3, and 1.427 in order from smallest to largest $1.46 = 1.460$ $1.3 = 1.300$ $1.427 = 1.427$ $1.300 < 1.427 < 1.460$ So, $1.3 < 1.427 < 1.46$.

Section 4.3 Adding and Subtracting Decimals

Definitions and Concepts	Examples	
To add or subtract decimals:	2.67 + 10.9	8.5 – 3.64
• Write in columns with the decimal points aligned. Insert zeros on the right if necessary.	2.67 + 10.90 _____ 13.57	8.50 – 3.64 _____ 4.86
• Add or subtract.		
• Align the decimal point in the answer with those above.		

Section 4.4 Multiplying Decimals

Definitions and Concepts	Examples	
To multiply decimals:	4.2 × 0.12	0.03 × 0.007
• Multiply the numbers as if they were whole numbers.		
• Count the number of decimal places in each factor. The total of the decimal places is the number of decimal places in the product. Insert zeros on the left if necessary.	$ \begin{array}{r} 4.2 \\ \times 0.12 \\ \hline 84 \\ 42 \\ \hline 0.504 \end{array} $ <p>(Three decimal places needed)</p>	$ \begin{array}{r} 0.03 \\ \times 0.007 \\ \hline 0.00021 \end{array} $ <p>(Five decimal places needed)</p>

Section 4.5 Multiplying and Dividing by Powers of 10; Scientific Notation

Definitions and Concepts	Examples
To multiply by a power of 10:	
• Move the decimal point to the right the same number of places as there are zeros in the power of 10.	$3.45 (10,000) = 34,500$ (Move four places right.)
To divide by a power of 10:	
• Move the decimal point to the left the same number of places as there are zeros in the power of 10.	$3.45 \div 1000 = 0.00345$ (Move three places left.)
Scientific notation is a special way to write numbers as a product of a number between 1 and 10 and a power of 10.	$34,500 = 3.45 \times 10^4$ $0.00345 = 3.45 \times 10^{-3}$

Section 4.6 Dividing Decimals; Average, Median, and Mode

Definitions and Concepts	Examples
To divide decimals:	
<ul style="list-style-type: none"> If the divisor is not a whole number, move the decimal point in both the divisor and the dividend to the right as many places as necessary to make the divisor a whole number. Place the decimal point in the quotient above the decimal point in the dividend. Divide as if both numbers are whole numbers. Round as appropriate. 	$\begin{array}{r} 0.04 \overline{)5.3} \\ 4 \end{array} = \begin{array}{r} 004 \overline{)530.0} \\ 13 \end{array}$ $\begin{array}{r} 12 \\ 10 \end{array}$ $\begin{array}{r} 8 \\ 20 \end{array}$ $\begin{array}{r} 20 \\ 0 \end{array}$
	Move two places right.
Finding the average of a set of decimals is the same as for whole numbers:	Find the average of 5.8, 6.12, and 7.394. $5.8 + 6.12 + 7.394 = 19.314$ $19.314 \div 3 = 6.438$ <p>The average is 6.438.</p>
Finding the median of a set of decimals is the same as for whole numbers:	Find the median of 5.8, 6.12, 7.394, 9.6, and 7.01. $5.8, 6.12, 7.01, 7.394, 9.6$ <p>The median is 7.01.</p>
Finding the mode of a set of decimals is the same as for whole numbers:	Find the mode of 5.8, 6.12, 7.03, 6.12, and 8.2. <p>The mode is 6.12.</p>

Section 4.7 Changing Fractions to Decimals

Definitions and Concepts	Examples
To change a fraction to a decimal, divide the numerator by the denominator. Round as appropriate.	Change $\frac{5}{8}$ to a decimal. $\begin{array}{r} 0.625 \\ 8 \overline{)5.000} \\ 48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \end{array}$

Section 4.8 Order of Operations; Estimating

Definitions and Concepts	Examples
The order of operations for decimals is the same as that for whole numbers:	$\begin{aligned} 14.8 - 0.2(8.3 + 4.76) \\ = 14.8 - 0.2(13.06) \\ = 14.8 - 2.612 \\ = 12.188 \end{aligned}$
To estimate sums or differences, round all numbers to a specified place value.	$\begin{aligned} 0.352 + 0.063 &\approx 0.4 + 0.1 \\ &\approx 0.5 \end{aligned}$
To estimate products, front round each number and multiply.	$\begin{aligned} (0.352)(0.063) &\approx (0.4)(0.06) \\ &\approx 0.024 \end{aligned}$

Review Exercises

CHAPTER 4

Section 4.1

Write the word name.

- | | |
|-----------|--------------|
| 1. 6.12 | 2. 0.843 |
| 3. 15.058 | 4. 0.0000027 |

Write the place value name.

- | | |
|-------------------------------------|---|
| 5. Twenty-one and five hundredths | 6. Four hundred nine ten-thousandths |
| 7. Four hundred and four hundredths | 8. One hundred twenty-five and forty-five thousandths |

Exercises 9–11. Round the numbers to the nearest tenth, hundredth, and thousandth.

Tenth	Hundredth	Thousandth
9. 34.7648		
10. 7.8736		
11. 0.467215		
12. The display on Mary's calculator shows 91.457919 as the result of a division exercise. If she is to round the answer to the nearest thousandth, what answer does she report?		

Section 4.2

Change the decimal to a fraction or mixed number and simplify.

- | | | | |
|----------|-----------|-------------|------------|
| 13. 0.76 | 14. 7.035 | 15. 0.00256 | 16. 0.0545 |
|----------|-----------|-------------|------------|

List the set of decimals from smallest to largest.

- | | |
|----------------------|-------------------------|
| 17. 0.95, 0.89, 1.01 | 18. 0.09, 0.093, 0.0899 |
|----------------------|-------------------------|

- | | |
|--------------------------------|--------------------------------------|
| 19. 7.017, 7.022, 0.717, 7.108 | 20. 34.023, 34.103, 34.0204, 34.0239 |
|--------------------------------|--------------------------------------|

Is the statement true or false?

- | | |
|-----------------------|-------------------------|
| 21. $6.1774 < 6.1780$ | 22. $87.0309 > 87.0319$ |
|-----------------------|-------------------------|

Section 4.3*Add.*

23.
$$\begin{array}{r} 11.356 \\ 0.67 \\ 13.082 \\ + 9.6 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 12.0678 \\ 7.012 \\ 56.0921 \\ + 0.0045 \\ \hline \end{array}$$

Subtract.

25.
$$\begin{array}{r} 22.0816 \\ - 8.3629 \\ \hline \end{array}$$

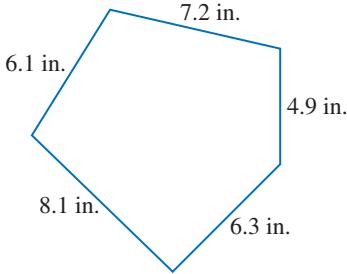
26.
$$\begin{array}{r} 54.084 \\ - 23.64936 \\ \hline \end{array}$$

27. Find the sum of 3.405, 8.12, 0.0098, 0.3456, 11.3, and 24.9345.

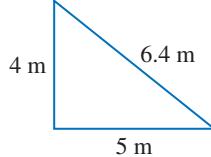
28. Find the difference of 56.7083 and 21.6249.

Find the perimeter of the following figures.

29.



30.



31. Hilda makes a gross salary (before deductions) of \$6475 per month. She has the following monthly deductions: federal income tax, \$1295; state income tax, \$582.75; Social Security, \$356.12; Medicare, \$82.24, retirement contribution, \$323.75; union dues, \$45; and health insurance, \$325.45. Find her actual take-home (net) pay.

32. Mary buys a new television that had a list price of \$785.95 for \$615.55. How much does she save from the list price?

Section 4.4*Multiply.*

33.
$$\begin{array}{r} 8.07 \\ \times 3.5 \\ \hline \end{array}$$

34.
$$\begin{array}{r} 11.24 \\ \times 3.5 \\ \hline \end{array}$$

35.
$$\begin{array}{r} 0.00678 \\ \times 3.59 \\ \hline \end{array}$$

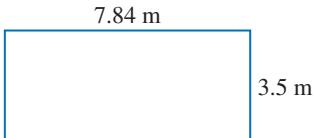
36.
$$\begin{array}{r} 12.057 \\ \times 8.08 \\ \hline \end{array}$$

37. Multiply: 0.074(2.004). Round to the nearest thousandth.

38. Multiply: (0.0098)(42.7). Round to the nearest hundredth.

39. Multiply: (0.03)(4.12)(0.015). Round to the nearest ten-thousandth.

40. Find the area of the rectangle.



- 41.** Millie selects an upholstery fabric that costs \$52.35 per yd. How much will Millie pay for 23.75 yd? Round to the nearest cent.

- 42.** Agnelo can choose any of the following ways to finance his new car. Which method is the least expensive in the long run?

\$850 down and \$401.64 per month for 5 years
\$475 down and \$443.10 per month for 54 months
\$600 down and \$495.30 per month for 4 years

Section 4.5

Multiply or divide.

43. $13.765 \div 10^3$

44. 7.023×10^6

45. $0.7321(100,000)$

46. $9.503 \div 100$

Write in scientific notation.

47. 0.0078

48. 34.67

49. 0.0000143

50. 65,700.8

Write the place value name.

51. 7×10^7

52. 8.13×10^{-6}

53. 6.41×10^{-2}

54. 3.505×10^3

- 55.** Home Run Sports buys 1000 softball bats for \$37,350. What is the average price of a bat?

- 56.** During the bear market of 2001, the stock market at one time was down \$50 billion. Write this loss in scientific notation.

Section 4.6

Divide.

57. $0.3 \overline{)0.0111}$

58. $75 \overline{)40.5}$

59. $56.7 \div 0.32$

60. $0.17 \overline{)0.01003}$

61. $0.456 \overline{)0.38304}$

62. $6.3271 \div 2.015$

Divide and round to the nearest hundredth.

63. $4.7 \overline{)332.618}$

64. $0.068 \overline{)0.01956}$

- 65.** Two hundred ten employees of Shepard Enterprises donated \$13,745.50 to the United Way. To the nearest cent, what was the average donation?

- 66.** Carol drove 375.9 miles on 12.8 gallons of gas. What is her mileage (miles per gallon)? Round to the nearest mile per gallon.

Find the average and median.

67. 4.56, 11.93, 13.4, 1.58, 8.09

68. 61.78, 50.32, 86.3, 95.04

69. 0.5672, 0.6086, 0.3447, 0.5555

70. 14.6, 18.95, 12.9, 23.5, 16.75

- 71.** Tony goes shopping and buys a 3-oz jar of Nescafé Instant Vanilla Roast Coffee for \$4.69. What is the unit price of the coffee, rounded to the nearest tenth of a cent?

- 72.** The Metropolis Police Department reported the following number of robberies for the week:

Monday 12 Tuesday 21 Wednesday 5 Thursday 18
Friday 46 Saturday 67 Sunday 17

To the nearest tenth, what is the average number of robberies reported per day?

Section 4.7

Change the fraction or mixed number to a decimal.

73. $\frac{9}{16}$

74. $\frac{7}{20}$

75. $17\frac{47}{125}$

Change to a decimal rounded to the indicated place value.

76. $\frac{11}{37}$, tenth

77. $\frac{57}{93}$, hundredth

78. $\frac{54}{61}$, thousandth

Change to a decimal. Use the repeat bar.

79. $\frac{9}{13}$

80. $\frac{7}{48}$

- 81.** The value of a share of Microsoft is $24\frac{9}{32}$. What is the value in decimal form? Round to the nearest hundredth.

- 82.** In a shot put meet where the results were communicated by telephone, the longest put in Georgia was $60\frac{11}{16}$ ft. The longest put in Idaho was 60.799 ft. Which state had the winning put?

Section 4.8

Perform the indicated operations.

83. $0.65 + 4.29 - 2.71 + 3.04$

84. $13.8 \div 0.12 \times 4.03$

85. $(6.7)^2 - (4.4)(2.93)$

86. $(5.5)(2.4)^3 \div 9.9$

87. $(6.3)(5.08) - (2.6)(0.17) + 2.42$

88. $6.2(3.45 - 2.07 + 0.98) - 3.1(1.45)$

- 89.** Jose did the following addition: $3.67 + 4.874 + 0.0621 + 0.00045 + 1.134 = 9.74055$. Estimate the sum by rounding each addend to the nearest tenth to determine if Jose's answer is reasonable.

- 90.** Sally did the following subtraction: $0.0672 - 0.037612 = 0.0634388$. Estimate the difference by rounding each number to the nearest hundredth to determine if Sally's answer is reasonable.

- 91.** Louise did the following multiplication: $0.00562(4.235) = 0.0238007$. Estimate the product by front rounding each factor to determine if Louise's answer is reasonable.

- 92.** Use estimating by front rounding to determine if the quotient of 0.678 and 0.0032 is

- a. 21.1875 b. 0.211875 c. 211.875
d. 2.11875 e. 2118.75

- 93.** Ron is given the task of buying plaques for the nine retiring employees of Risk Corporation. The budget for the plaques is \$325. Ron finds a plaque he likes at a price of \$31.95. Estimate the cost of the nine plaques to see if Ron has enough money in the budget for them.

- 94.** Millie has \$356 on the books at Michelbook Country Club. She wants to buy 4 dozen golf balls at \$48.50 per dozen, a glove for \$12.35, 4 bags of tees at \$1.25 each, a putter for \$129.75, and a driving range card at \$75. Estimate the cost of Millie's purchases by rounding to the nearest dollar.

True/False Concept Review CHAPTER 4

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. The word name for 0.709 is “seven hundred and nine thousandths.” 1. _____
2. 0.348 and .348 name the same number. 2. _____
3. To write 0.85 in expanded form we write $\frac{85}{100}$. 3. _____
4. Since 0.265 is read “two hundred sixty-five thousandths,” we write $\frac{265}{1000}$ and reduce to change the decimal to a fraction. 4. _____
5. True or false: $0.732687 > 0.74$ 5. _____
6. Because $4.6 > 3.9$ is true, 3.9 is to the left of 4.6 on the number line. 6. _____
7. To list a group of decimals in order, we need to write or think of all the numbers as having the same number of decimal places. 7. _____
8. Decimals are either exact or approximate. 8. _____
9. To round 356.7488 to the nearest tenth, we write 356.8, because the 4 in the hundredths place rounds up to 5 because it is followed by an 8. 9. _____
10. The sum of 0.6 and 0.73 is 1.33. 10. _____
11. $9.7 - 0.2 = 5.7$ 11. _____
12. The answer to a multiplication problem will always contain the same number of decimal places as the total number of places in the two numbers being multiplied. 12. _____
13. To multiply a number by a power of 10 with a positive exponent, move the decimal point in the number to the right the same number of places as the number of zeros in the power of 10. 13. _____

14. To divide a number by a power of 10 with a positive exponent, move the decimal the same number of places to the right as the exponent indicates. 14. _____
15. To change 3.57×10^{-5} to place value form, move the decimal five places to the right. 15. _____
16. To divide a number by a decimal, first change the decimal to a whole number by moving the decimal point to the right. 16. _____
17. All fractions can be changed to exact terminating decimals. 17. _____
18. $\frac{4}{11} \approx 0.\overline{36}$. 18. _____
19. The order of operations for decimals is the same as for whole numbers. 19. _____
20. To find the average of a group of decimals, find their sum and divide by the number of decimals in the group. 20. _____

Test CHAPTER 4**ANSWERS**

1. Divide. Round the answer to the nearest thousandth: $0.87\overline{)4.7441}$ 1. _____
2. List the following decimals from the smallest to the largest: 0.678, 0.682, 0.6789, 0.6699, 0.6707 2. _____
3. Write the word name for 75.032. 3. _____
4. Multiply: $6.84(4.93)$ 4. _____
5. Write as a decimal: $\frac{23}{125}$ 5. _____
6. Round to the nearest hundredth: 57.896 6. _____
7. Subtract: $87 - 14.837$ 7. _____
8. Change to a mixed number with the fraction part simplified: $18.\overline{725}$ 8. _____
9. Write in scientific notation: 0.000000723 9. _____
10. Write as an approximate decimal to the nearest thousandth: $\frac{17}{23}$ 10. _____
11. Round to the nearest hundred: 72,987.505 11. _____
12. Perform the indicated operations: $2.277 \div 0.33 \times 1.5 + 11.47$ 12. _____
13. Subtract:
$$\begin{array}{r} 305.634 \\ -208.519 \\ \hline \end{array}$$
 13. _____
14. Change to place value notation: 5.94×10^{-5} 14. _____
15. Write the place value name for nine thousand forty-five and sixty-five thousandths. 15. _____
16. Multiply: $0.000917(100,000)$ 16. _____
17. Write in scientific notation: 309,720 17. _____

18. Add: $17.98 + 1.467 + 18.92 + 8.37$

18. _____

19. Multiply: $34.4(0.00165)$

19. _____

20. Divide: $72 \overline{)0.02664}$

20. _____

21. For the first 6 months of 2005, the offering at St. Pius Church was \$124,658.95, \$110,750.50, \$134,897.70, \$128,934.55, \$141,863.20 and \$119,541.10. What was the average monthly offering? Round to the nearest cent.

21. _____

22. Add:

$$\begin{array}{r} 911.84 \\ 45.507 \\ 6003.62 \\ 7.2 \\ 35.78 \\ + 891.361 \\ \hline \end{array}$$

22. _____

23. Grant buys 78 assorted flower plants from the local nursery. If the sale price is four plants for \$3.48, how much does Grant pay for the flower plants?

23. _____

24. On April 15, 2005, Allen Iverson had the best scoring average per game, with 30.8. How many games had he played in if he scored a total of 2214 points (to the nearest game)?

24. _____

25. In baseball, the slugging percentage is calculated by dividing the number of total bases (a double is worth two bases) by the number of times at bat and then multiplying by 1000. What is the slugging percentage of a player who has 201 bases in 293 times at bat? Round to the nearest whole number.

25. _____

26. Harold and Jerry go on diets. Initially, Harold weighed 267.8 lb and Jerry weighed 209.4 lb. After 1 month of the diet, Harold weighed 254.63 lb and Jerry weighed 196.2 lb. Who lost the most weight and by how much?

26. _____

Group Project (2–3 WEEKS) CHAPTER 4

OPTIONAL

The NFL keeps many statistics regarding its teams and players. Since quarterbacks play an important part in the overall team effort, much time and attention have been given to keeping statistics on quarterbacks. But all these statistics do not necessarily make it easy to decide which quarterback is the best. Consider the following statistics from the 2004 season.

Highest-Ranked Players in 2004 NFL Season

Player	Passes Attempted	Passes Completed	Yards Gained	Touchdown Passes	Interceptions
Daunte Culpepper, Minnesota	548	379	4717	39	11
Trent Green, Kansas City	556	369	4591	27	17
Peyton Manning, Indiana	497	336	4557	49	10
Jake Plummer, Denver	521	303	4089	27	20
Brett Favre, Green Bay	540	346	4088	30	17

- Which quarterback deserved to be rated as the top quarterback of the year? Justify your answer.

The NFL has developed a rating system for quarterbacks that combines all of the statistics in the table and gives each quarterback a single numeric “grade” so they can easily be compared. While the exact calculations used by the NFL are complicated, Randolph Taylor of Las Positas College in Livermore, California, has developed the following formula that closely approximates the NFL ratings.

Let A = the number of passes attempted

C = the number of passes completed

Y = the number of yards gained passing

T = the number of touchdowns passed

I = the number of interceptions

$$\text{Rating} = \frac{5}{6} \left(\frac{C}{A} \cdot 100 \right) + \frac{25}{6} \left(\frac{Y}{A} \right) + \frac{10}{3} \left(\frac{T}{A} \cdot 100 \right) - \frac{25}{6} \left(\frac{I}{A} \cdot 100 \right) + \frac{25}{12}$$

- Use the rating formula to calculate ratings for the quarterbacks in the table. Use your calculator and do not round except at the end, rounding to the nearest hundredth.

- Explain why everything in the formula is added except $\frac{25}{6} \left(\frac{I}{A} \cdot 100 \right)$.

- According to your calculations, who was the best quarterback for the 2004 season?

- In the 2004 season, Clinton Portis of the Washington Redskins made two attempts at a pass and completed one for 15 yards and a touchdown. He had no interceptions. Calculate his rating and comment on how he compares with the quarterbacks in the table.

- What are the drawbacks to using the rating as the sole measure of a quarterback’s performance?

- (Optional) Have your group compile a list of the five all-time best quarterbacks. Find statistics for each of the quarterbacks on your list (use almanacs or the web) and compute their ratings. Comment on your results.

Learning to Learn Math

Learning mathematics is a building process. For example, if you have not mastered fractions, rational expressions are difficult to learn because they require an understanding of the rules for fractions. Therefore, if you are having difficulty with the current topic, you may not have mastered a previous skill that you need. It will be necessary to go back and learn/relearn this skill before you can continue.

Learning math also means learning not just skills, but how and where and when to **apply** the skills. For example, if it takes 16 gallons of gas to travel 320 miles, how many miles to the gallon are you getting? What skill would you use to solve this problem? (Answer: dividing) Reading the application problems and thinking of situations where you have used or could use these concepts help integrate the concept into your experience.

Learning mathematics is learning something basic to daily life and to virtually every field of science and business. The examples in the book state the problem and a **strategy** for solving the problem. This strategy applies to several related problems. Learning mathematics is learning strategies to solve related problems.

The more you begin to appreciate mathematics as relevant to your life, the more you will see mathematics as worthy of your time, and the more committed you will feel to studying mathematics. Here is an activity that may give you fresh opportunities to see how much mathematics relates to your life. First, create a simple “web” or “map” with “math” at the center and spokes out from this center naming areas in life where math comes up—areas where math is useful. Capture as many areas as you can in a 5-minute period.

Second, turn the page over and construct a second “web,” but this time choose one of your “math areas” for the center and create spokes that capture subtopics or subheadings in this particular math area. Take another 5 minutes for this second web. Next, create a problem that you believe to be solvable, from your own experience, and that might be enticing for someone else to solve.

If you are working with a partner, trade problems and see if you can solve each other’s problems. Talk about how you might approach the problems and whether they are stated clearly and believably. Here are some criteria for a good problem:

- Enough data and information to solve the problem
- Clear statement of what you need to find
- Not too many questions included
- Appropriate reading level and clearly written
- Appears solvable and not too scary
- Makes the reader care about wanting to solve the problem

Going through this activity may help you become more aware of mathematics in your daily life and give you a greater understanding of problems and problem solving.

5

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SECTIONS

- 5.1** Ratio and Rate
- 5.2** Solving Proportions
- 5.3** Applications of Proportions

BASIC
Mathematics Now™

RATIO AND PROPORTION

APPLICATION

From the earliest times, humans have drawn maps to represent the geography of their surroundings. Some maps depict features encountered on a journey, like rivers and mountains. The most useful maps incorporate the concept of scale, or proportion. Simply put, a scaled map accurately preserves relative distances. So if the distance from one city to another is twice the distance from the city to a river in real life, the distance between the cities is twice the distance from the city to a river on the map as well.

The scale of a map depends on how large an area the map covers. In the United States, the scale is often stated as “one inch represents _____.” For a street map of a city, the scale could be “one inch represents 600 yards.” The map of an entire state could have a scale of “one inch represents 45 miles.” The map of an entire country could have a scale of “one inch represents 500 miles.” Specific information about the scale is usually given in a corner of the map.

Group Activity

Go to the library and find maps with five different scales. Summarize your findings in the table below.

Map Subject	Scale	Width of map (inches)	Width of Map Subject (miles)

5.1 Ratio and Rate

OBJECTIVES

1. Write a fraction that shows a ratio comparison of two like measurements.
2. Write a fraction that shows a rate comparison of two unlike measurements.
3. Write a unit rate.

VOCABULARY

A **ratio** is a comparison of two measurements by division.

Like measurements have the same unit of measure.

Unlike measurements have different units of measure.

A **rate** is a comparison of two unlike measurements by division.

A **unit rate** is a rate with a denominator of one unit.

How & Why

OBJECTIVE 1

Write a fraction that shows a ratio comparison of two like measurements.

Two numbers can be compared by subtraction or division. If we compare 30 and 10 by subtraction, $30 - 10 = 20$, we can say that 30 is 20 more than 10. If we compare 30 and 10 by division, $30 \div 10 = 3$, we can say that 30 is 3 times larger than 10.

The indicated division, $30 \div 10$, is called a **ratio**. These are common ways to write the ratio to compare 30 and 10:

$$30:10 \quad 30 \div 10 \quad 30 \text{ to } 10 \quad \frac{30}{10}$$

Because we are comparing 30 to 10, 30 is written first or placed in the numerator of the fraction.

Here we write ratios as fractions. Because a ratio is a fraction, it can often be simplified. The ratio $\frac{12}{16}$ is simplified to $\frac{3}{4}$. If the ratio contains two like measurements, it can be simplified as a fraction.

$$\frac{\$7}{\$70} = \frac{7}{10} \quad \text{The units, \$ (or dollars), are dropped because they are the same.}$$

$$\frac{15 \text{ miles}}{35 \text{ miles}} = \frac{3}{7} \quad \text{The units, miles, are dropped and the fraction is simplified.}$$

Warm-Ups A–C

- A. Write the ratio of 70 to 112.

Answers to Warm-Ups

A. $\frac{5}{8}$

Examples A–C BASIC Mathematics Now™

DIRECTIONS: Write a ratio in simplified form.

STRATEGY: Write the ratio as a simplified fraction.

- A. Write the ratio of 88 to 110.

$$\frac{88}{110} = \frac{4}{5} \quad \text{Write 88 in the numerator and simplify.}$$

The ratio of 88 to 110 is $\frac{4}{5}$.

- B. Write the ratio of the length of a room to its width if the room is 30 ft by 24 ft.

$$\frac{30 \text{ ft}}{24 \text{ ft}} = \frac{30}{24} = \frac{5}{4}$$

Write 30 ft in the numerator, drop the common units, and simplify.

The ratio of the length of the room to its width is $\frac{5}{4}$.

- C. Write the ratio of 5 dimes to 5 quarters. Compare in cents.

$$\frac{5 \text{ dimes}}{5 \text{ quarters}} = \frac{50 \text{ cents}}{125 \text{ cents}} = \frac{50}{125} = \frac{2}{5}$$

The ratio of 5 dimes to 5 quarters is $\frac{2}{5}$.

- B. Write the ratio of the length of a meeting room to its width if the meeting room is 45 ft by 35 ft.

- C. Write the ratio of 42 feet to 35 yards. Compare in feet.

How & Why

OBJECTIVE 2

Write a fraction that shows a rate comparison of two unlike measurements.

Fractions are used to compare unlike measurements as well as like measurements. Such a comparison is called a rate. The *rate* of $\frac{27 \text{ children}}{10 \text{ families}}$ compares the unlike measurements

“27 children” and “10 families.” A familiar application of a rate occurs in the computation of gas mileage. For example, if a car travels 192 miles on 8 gallons of gas, we compare miles to gallons by writing $\frac{192 \text{ miles}}{8 \text{ gallons}}$. This rate can be simplified but the units are *not dropped* because they are unlike.

$$\frac{192 \text{ miles}}{8 \text{ gallons}} = \frac{96 \text{ miles}}{4 \text{ gallons}} = \frac{24 \text{ miles}}{1 \text{ gallon}} = 24 \text{ miles per gallon} = 24 \text{ mpg.}$$

CAUTION

When measurement units are different, they are not dropped.

Examples D–F



BASIC Mathematics Now™

DIRECTIONS: Write a rate in simplified form.

STRATEGY: Write the simplified fraction and keep the unlike units.

- D. Write the rate of 12 chairs to 11 people.

$$\frac{12 \text{ chairs}}{11 \text{ people}}$$

The units must be kept because they are different.

- E. Write the rate of 14 cars to 10 homes.

$$\frac{14 \text{ cars}}{10 \text{ homes}} = \frac{7 \text{ cars}}{5 \text{ homes}}$$

The rate is $\frac{7 \text{ cars}}{5 \text{ homes}}$.

Warm-Ups D–F

- D. Write the rate of 11 people to 7 tables.

- E. Write the rate of 24 TV sets to 10 homes.

Answers to Warm-Ups

B. $\frac{9}{7}$ C. $\frac{2}{5}$ D. $\frac{11 \text{ people}}{7 \text{ tables}}$

E. The rate is $\frac{12 \text{ TV sets}}{5 \text{ homes}}$.

- F. The following spring, the urban committee repeated the tree program. This time they sold 70 oak trees and 440 birch trees.

1. What is the rate of oak trees to birch trees sold?
2. What is the rate of birch trees to the total number of trees sold?

- F. An urban environmental committee urged the local citizens to plant deciduous trees around their homes as a means of conserving energy. The campaign resulted in 825 oak trees and 675 birch trees being sold.

1. What is the rate of the number of oak trees to the number of birch trees sold?
2. What is the rate of the number of oak trees to the total number of trees sold?

- 1. STRATEGY:** Write the first measurement, 825 oak trees, in the numerator, and the second measurement, 675 birch trees, in the denominator.

$$\frac{825 \text{ oak trees}}{675 \text{ birch trees}} = \frac{11 \text{ oak trees}}{9 \text{ birch trees}} \quad \text{Simplify.}$$

The rate is $\frac{11 \text{ oak trees}}{9 \text{ birch trees}}$; that is, 11 oak trees were sold for every 9 birch trees sold.

- 2. STRATEGY:** Write the first measurement, 825 oak trees, in the numerator, and the second measurement, total number of trees, in the denominator.

$$\frac{825 \text{ oak trees}}{1500 \text{ trees total}} = \frac{11 \text{ oak trees}}{20 \text{ trees total}} \quad \text{Simplify.}$$

The rate is $\frac{11 \text{ oak trees}}{20 \text{ trees total}}$; that is, 11 out of every 20 trees sold were oak trees.

How & Why

OBJECTIVE 3

Write a unit rate.

When a rate is re-written so that the denominator is a 1-unit measurement, then we have a *unit rate*.

For example,

$$\frac{204 \text{ miles}}{8 \text{ gallons}} = \frac{51 \text{ miles}}{2 \text{ gallons}} = \frac{25.5 \text{ miles}}{1 \text{ gallon}} \quad \text{Read this as "25.5 miles per gallon."}$$

Re-writing rates as unit rates can lead to statements such as “There are 2.6 children per family in the state,” because

$$\frac{26 \text{ children}}{10 \text{ families}} = \frac{2.6 \text{ children}}{1 \text{ family}}$$

The unit rate is a comparison, not a fact, because no family has 2.6 children.

To write a unit rate given a rate

1. Do the indicated division.
2. Keep the unlike units.

Answers to Warm-Ups

- F. 1. The rate is $\frac{7 \text{ oak trees}}{44 \text{ birch trees}}$.
 2. The rate is $\frac{44 \text{ birch trees}}{51 \text{ trees total}}$.

Examples G–I



DIRECTIONS: Write as a unit rate.

STRATEGY: Do the indicated division so that the denominator is a 1-unit measurement.

G. Write the unit rate for $\frac{\$2.04}{3 \text{ cans of peas}}$.

STRATEGY: Do indicated division so that the denominator is 1 unit.

$$\frac{\$2.04}{3 \text{ cans of peas}} = \frac{\$0.68}{1 \text{ can of peas}}$$

The unit rate is 68¢ per can.

H. Write the unit rate for $\frac{802.5 \text{ miles}}{12.5 \text{ hours}}$.

$$\frac{502.5 \text{ miles}}{12.5 \text{ hours}} = \frac{64.2 \text{ miles}}{1 \text{ hour}}$$
 Divide numerator and denominator by 12.5.

The unit rate is 64.2 miles per hour

CALCULATOR EXAMPLE



I. The population density of a region is a unit rate. The rate is the number of people per 1 square mile of area.

- Find the population density of the city of Cedar Crest in Granite County if the population is 4100 and the area of the city is 52 square miles. Round to the nearest tenth.
- Find the population density of Granite County if the population is 13,650 and the area of the county is 1600 square miles. Round to the nearest tenth.

1. STRATEGY: Write the rate and divide the numerator by the denominator using your calculator.

$$\begin{aligned} \text{Density} &= \frac{4100 \text{ people}}{52 \text{ square miles}} \\ &\approx \frac{78.8461538 \text{ people}}{1 \text{ square mile}} \quad \text{Divide.} \\ &\approx \frac{78.8 \text{ people}}{1 \text{ square mile}} \quad \text{Round to the nearest tenth.} \end{aligned}$$

The density is 78.8 people per square mile, to the nearest tenth.

2. STRATEGY: Write the rate and divide the numerator by the denominator using your calculator.

$$\begin{aligned} \text{Density} &= \frac{13,650 \text{ people}}{1600 \text{ square miles}} \\ &\approx \frac{8.53125 \text{ people}}{1 \text{ square mile}} \quad \text{Divide.} \\ &\approx \frac{8.5 \text{ people}}{1 \text{ square mile}} \quad \text{Round to the nearest tenth.} \end{aligned}$$

The density is 8.5 people per square mile, to the nearest tenth.

Warm-Ups G–I

G. Write the unit rate for

$$\frac{528 \text{ pounds}}{12 \text{ square inches}}$$

H. Write the unit rate for

$$\frac{485 \text{ miles}}{25 \text{ gallons}}$$

- I. 1. Find the approximate population density of the city of Los Angeles in 2003 if the population was estimated at 3,800,000 and the area was 489 square miles. Round to the nearest whole number.

2. What will be the approximate population density of the county of Los Angeles in 2010 if the population is estimated at 10,461,000 and the area is 1110 square miles. Round to the nearest whole number.

Answers to Warm-Ups

G. The unit rate is 44 lb per square inch.

H. The unit rate is 19.4 miles per gallon.

- I. 1. The population density was 7771 people per square mile.
2. The population density will be 9424 people per square mile.

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Exercises 5.1

OBJECTIVE 1

Write a fraction that shows a ratio comparison of two like measurements.

A Write as a ratio in simplified form.

- | | | |
|---------------------|-------------------------|-------------------------|
| 1. 14 to 56 | 2. 6 to 48 | 3. 14 ft to 42 ft |
| 4. 24 tsp to 18 tsp | 5. 20 cents to 25 cents | 6. 32 dimes to 48 dimes |

B

- | | | |
|---|--|---|
| 7. 2 dimes to 8 nickels
(compare in cents) | 8. 4 quarters to 8 dimes
(compare in cents) | 9. 3 ft to 40 in.
(compare in inches) |
| 10. 2 yd to 8 ft
(compare in feet) | 11. 100 min to 4 hr
(compare in minutes) | 12. 200 yd to 1000 in.
(compare in inches) |

OBJECTIVE 2

Write a fraction that shows a rate comparison of two unlike measurements.

A Write a rate and simplify.

- | | |
|----------------------------|--------------------------------|
| 13. 12 cars to 18 families | 14. 22 children to 11 families |
| 15. 110 mi in 2 hr | 16. 264 km in 3 hr |
| 17. 92 mi to 4 gal | 18. 110 km to 5 gal |
| 19. 253 trees in 22 rows | 20. \$280 in 16 hr |

B

- | | |
|-------------------------------|--|
| 21. 10 trees to 35 ft | 22. 164 DVDs to 6 houses |
| 23. 38 books to 95 students | 24. 750 people for 3000 tickets |
| 25. 774 students to 516 rooms | 26. 178 satellite dishes to 534 houses |
| 27. 345 pies to 46 sales | 28. \$17.68 per 34 lb of apples |

OBJECTIVE 3

Write a unit rate.

A Write a unit rate.

29. 500 mi to (per) 25 gal

30. 315 km to 3 hr

31. 36 ft to 9 sec

32. 75 m to 3 min

33. \$2.30 to 10 lb potatoes

34. 36 lb to \$18

35. 4 qt to 500 mi

36. \$17.28 per 12 dozen eggs

B Write a unit rate. Round to the nearest tenth.

37. 36 children to 15 families

38. 52 cars to 27 families

39. 1000 ft to 12 sec

40. 1000 yd to 15 min

41. 13,150 lb to 45 mi²

42. 5486 kg to 315 cm²

43. 2225 gal per 3 hr

44. 4872 plants in 78 rows

C

45. A Jackson and Perkins catalog advertised miniature roses for \$10.95 each or a special deal of three roses for \$29.95.

- a. If Carol orders the three-rose special, what is the price per rose? Round to the nearest cent.
- b. How much savings is this compared with buying three separate roses?

46. A Jackson and Perkins catalog advertised mixed color foxgloves at either 6 for \$24.95 or 12 for \$39.95.

- a. What is the price per plant if you buy six? Round to the nearest cent.
- b. What is the price per plant if you buy 12?
- c. How much would Ted save if he bought the 12-foxglove package as compared with buying two 6-foxglove packages?

It is often difficult to compare the prices of food items, frequently because of the packaging. Is a 14-oz can of pears for \$0.89 a better buy than a 16-oz can of pears for \$1.00? To help consumers compare, unit pricing is often posted. Mathematically, we write the information as a rate and rewrite as a 1-unit comparison.

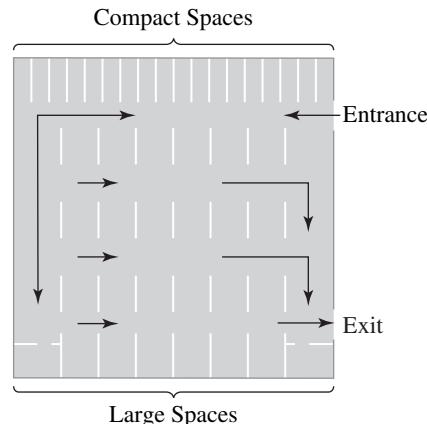
47. Write a ratio for a 14-oz can of pears that sells for \$0.89 and rewrite it as a unit price (price per 1 oz of pears). Do the same with the 16-oz can of pears for \$1.00. Which is the better buy?

48. Which is the best buy: a 15-oz box of Cheerios for \$2.49, a 20-oz box for \$3.29, or a 2-lb 3-oz box for \$5.39?

49. Which is the better buy: 5 lb of granulated sugar on sale for \$4.95 or 25 lb of sugar for \$24.90?

Exercises 50–53. Some food items have the same unit price regardless of the quantity purchased. Other food items have a decreasing unit price as the size of the container increases. In order to determine which category a food falls into, find the unit price for each item.

- 50.** Is the unit price of frozen orange juice the same if a 12-oz can costs \$1.09 and a 16-oz can costs \$1.30?
- 51.** Healthy Eats Market sells three 16-oz jars of salsa for \$10.77.
- What is the price per ounce for the salsa? Round to the nearest cent.
 - If the market puts the salsa on sale for 2 jars for \$5, what is the price per ounce?
 - Using the unit prices calculated in parts a and b, how much can Jerry save by buying 4 jars of salsa for the sale price?
- 52.** Ralph's Good Foods sells three 9-oz packages of tortilla chips for \$7.77.
- What is the price per ounce for the tortilla chips? Round to the nearest hundredth of a dollar.
 - Ralph's puts the chips on sale for 2 packages for \$2.95. What is the price per ounce?
 - Using the unit prices calculated in parts a and b, how much can Roger save if he buys 5 packages of tortilla chips at the sale price for a family picnic?
- 53.** List five items that usually have the same unit price regardless of the quantity purchased and five that do not. What circumstances could cause an item to change categories?
- 54.** Hot Wheels are scaled at 1:64. How many Hot Wheel Mustangs would line up end to end to equal the length of an actual Mustang?
- 55.** The parking lot in the lower level of the Senter Building has 18 spaces for compact cars and 24 spaces for larger cars.
- What is the ratio of compact spaces to larger spaces?
 - What is the ratio of compact spaces to the total number of spaces?
- 56.** The Reliable Auto Repair Service building has eight stalls for repairing automobiles and four stalls for repairing small trucks.
- What is the ratio of the number of stalls for small trucks to the number of stalls for automobiles?
 - What is the ratio of the number of stalls for small trucks to the total?
- 57.** A portable DVD player is regularly priced at \$599.88, but during a sale its price is \$449.91. What is the ratio of the sale price to the regular price?



- 58.** In Exercise 57, what is the ratio of the discount to the regular price?
- 59.** What is the population density of Dryton City if there are 22,450 people and the area is 230 square miles? Write as a unit comparison, rounded to the nearest tenth.
- 60.** What is the population density of Struvaria if 975,000 people live there and the area is 16,000 square miles? Write as a unit comparison, rounded to the nearest tenth.
- 61.** What was the population density of your city in 2000?
- 62.** What was the population density of your state in 2000?
- 63.** In the United States, four people use an average of 250 gallons of water per day. One hundred gallons are used to flush toilets, 80 gallons in baths or showers, 35 gallons doing laundry, 15 gallons washing dishes, 12 gallons for cooking and drinking, and 8 gallons in the bathroom sink.
- Write the ratio of laundry use to toilet use.
 - Write the ratio of bath or shower use to dishwashing use.
- 64.** Use Exercise 63.
- Write the ratio of cooking and drinking use to dishwashing use.
 - Write the ratio of laundry use per person.
- 65.** Drinking water is considered to be polluted when a pollution index of 0.05 mg of lead per liter is reached. At that rate, how many milligrams of lead are enough to pollute 25 L of drinking water?
- 66.** Available figures indicate that 3 of every 20 rivers in the United States showed an increase in water pollution in a recent 10-year period. Determine how many rivers are in your state. At the same rate, determine how many of those rivers had an increased pollution level during the same period.
- 67.** A Quantum Professional PR600C fishing reel can retrieve 105 in. of fishing line in 5 turns of the handle. Write this as a ratio and then calculate the retrieval rate (measured in inches per turn).
- 68.** A Quantum Professional PR600CX fishing reel can retrieve 127 in. of fishing line in 5 turns of the handle. Write this as a ratio and then calculate the retrieval rate.
- 69.** Full-time equivalency (FTE) is a method by which colleges calculate enrollment. The loads of all students are added together and then divided into theoretical full-time students. Three River Community College requires its full-time students to take 15 credits per term. One term there were 645 students enrolled, taking a total of 4020 credits. Find the average number of credits per student and the FTE for the term.

Exercises 70–71 relate to the chapter application.

70. A map has a scale for which 1 inch stands for 2.4 miles. Does this give information about a rate or a ratio? Why? What does it tell you about what 1 inch on the map represents?
71. A map has a scale of 1:150,000. Does this give information about a rate or a ratio? Why? What does it tell you about what 1 inch on the map represents?

STATE YOUR UNDERSTANDING

72. Write a short paragraph explaining why ratios are useful ways to compare measurements.
73. Explain the difference between a ratio, a rate, and a unit rate. Give an example of each.

CHALLENGE

74. Give an example of a ratio that is not a rate. Give an example of a rate that is not a ratio.
75. The ratio of noses to persons is $\frac{1}{1}$ or 1-to-1. Find three examples of 2-to-1 ratios and three examples of 3-to-1 ratios.
76. Each gram of fat contains 9 calories. Chicken sandwiches at various fast-food places contain the following total calories and grams of fat.

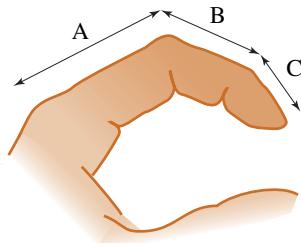
	Total Calories	Grams of Fat
a. RB's Light Roast Chicken Sandwich	276	7
b. KB's Boiler Chicken Sandwich	267	8
c. Hard B's Chicken Filet	370	13
d. LJS's Baked Chicken Sandwich	130	4
e. The Major's Chicken Sandwich	482	27
f. Mickey's Chicken	415	19
g. Tampico's Soft Chicken Taco	213	10
h. Winston's Grilled Chicken Sandwich	290	7

Find the ratio of fat calories to total calories for each sandwich.

GROUP WORK

- 77.** Have each member of your group select a country other than the United States. Each member is to use the library, the Internet, or other resource to find the population and area of the country selected. Calculate the population density for each country and compare your findings. Which country has the greatest population density? The least?

- 78.** The golden ratio of 1.618 to 1 has been determined by artists to be very pleasing aesthetically. The ratio has been discovered to occur in nature in many places, including the human body. In particular, the ratio applies to successive segments of the fingers.



Measure as accurately as possible at least three fingers of everyone in the group. Calculate the ratio of successive segments, and fill in the table.

Name	Finger	A	B	C	$\frac{A}{B}$	$\frac{B}{C}$

Whose fingers come closest to the golden ratio? Can you find other body measures that have this ratio?

MAINTAIN YOUR SKILLS

Simplify.

79. $\frac{3}{8} \cdot \frac{4}{21}$

80. $\frac{5}{12} \div \frac{15}{28}$

81. $6.5(0.03)$

82. $12.85 \div 2.5$

83. $\frac{7}{8}(0.6)$

84. $1\frac{3}{8} + 4\frac{3}{10}$

85. $9 - 4\frac{7}{15}$

86. $31.2 \div 1000$

Which is larger?

87. $\frac{3}{4}$ or $\frac{94}{125}$

88. $\frac{7}{10}$ or $\frac{29}{40}$

5.2 Solving Proportions

VOCABULARY

A **proportion** is a statement that two ratios are equal.

In a proportion, **cross multiplication** means multiplying the numerator of each ratio times the denominator of the other.

Cross products are the products obtained from cross multiplication.

Solving a proportion means finding a missing number, usually represented as a letter or **variable**, that will make the proportion true.

OBJECTIVES

1. Determine whether a proportion is true or false.
2. Solve a proportion.

How & Why

OBJECTIVE 1

Determine whether a proportion is true or false.

A *proportion* states that two rates or ratios are equal. The statement $\frac{21}{12} = \frac{28}{16}$ is a proportion. To check whether the proportion is true or false we use “cross multiplication.”

The proportion $\frac{21}{12} = \frac{28}{16}$ is true if the cross products are equal.

$$\begin{array}{r} 21 \quad ? \quad 28 \\ \cancel{12} \quad \cancel{16} \\ 21(16) \stackrel{?}{=} 12(28) \end{array}$$

Find the cross products.
The cross products are equal.

The proportion is true.

The cross-multiplication test is actually a shortcut for converting both fractions to equivalent fractions with common denominators and checking that the numerators match. Let’s examine the same proportion using the formal method.

$$\begin{aligned} \frac{21}{12} &\stackrel{?}{=} \frac{28}{16} \\ \frac{16}{16} \cdot \frac{21}{12} &\stackrel{?}{=} \frac{28}{16} \cdot \frac{12}{12} \quad \text{A common denominator is } 16 \cdot 12 = 192. \\ \frac{336}{192} &= \frac{336}{192} \quad \text{Multiply. The numerators are the same.} \end{aligned}$$

The proportion is true.

The cross products in the shortcut (cross multiplication) are the numerators in the formal method. This is the reason that checking the cross products is a valid procedure for determining the truth of a proportion.

To check whether a proportion is true or false

1. Check that the ratios or rates have the same units.
2. Cross multiply.
3. If the cross products are equal, the proportion is true.

Warm-Ups A–C

A. Is $\frac{7}{9} = \frac{56}{72}$ true or false?

B. Is $\frac{2.2}{2.5} = \frac{2}{5}$ true or false?

C. Is $\frac{1.20 \text{ dollars}}{2 \text{ quarters}} = \frac{72 \text{ nickels}}{15 \text{ dimes}}$ true or false?

Examples A–C



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DIRECTIONS: Determine whether a proportion is true or false.

STRATEGY: Check the cross products. If they are equal the proportion is true.

A. Is $\frac{6}{5} = \frac{78}{65}$ true or false?

$$\begin{array}{r} \frac{6}{5} \stackrel{?}{=} \frac{78}{65} \\ 6(65) \stackrel{?}{=} 5(78) \quad \text{Find the cross products.} \\ 390 = 390 \quad \text{True.} \end{array}$$

The proportion is true.

B. Is $\frac{2.1}{7.1} = \frac{2}{7}$ true or false?

$$\begin{array}{r} \frac{2.1}{7.1} \stackrel{?}{=} \frac{2}{7} \\ 2.1(7) \stackrel{?}{=} 7.1(2) \quad \text{Find the cross products.} \\ 14.7 \neq 14.2 \end{array}$$

The proportion is false.

C. Is $\frac{1 \text{ dollar}}{3 \text{ quarters}} = \frac{8 \text{ dimes}}{12 \text{ nickels}}$ true or false?

STRATEGY: The units in the rates are not the same. We change all units to cents and simplify.

$$\begin{array}{r} \frac{1 \text{ dollar}}{3 \text{ quarters}} \stackrel{?}{=} \frac{8 \text{ dimes}}{12 \text{ nickels}} \\ \frac{100 \text{ cents}}{75 \text{ cents}} \stackrel{?}{=} \frac{80 \text{ cents}}{60 \text{ cents}} \\ \frac{100}{75} \stackrel{?}{=} \frac{80}{60} \quad \text{Like units may be dropped.} \\ 100(60) \stackrel{?}{=} 75(80) \\ 6000 = 6000 \quad \text{True.} \end{array}$$

The proportion is true.

How & Why

OBJECTIVE 2

Solve a proportion.

Proportions are used to solve many problems in science, technology, and business. There are four numbers or measures in a proportion. If three of the numbers are known, we can find the missing number. For example,

$$\begin{array}{r} \frac{x}{6} \stackrel{?}{=} \frac{14}{21} \\ 21x = 6(14) \quad \text{Cross multiply.} \\ 21x = 84 \end{array}$$

Every multiplication fact can be written as a related division fact. The product divided by one factor gives the other factor. So $21x = 84$ can be written as $x = 84 \div 21$.

$$x = 84 \div 21 \quad \text{Rewrite as division.}$$

$$x = 4$$

Answers to Warm-Ups

- A. true B. false C. true

CHECK: $\frac{4}{6} = \frac{14}{21}$ Substitute 4 for x in the original proportion.
 $4(21) = 6(14)$ Cross multiply (or observe that both fractions simplify to $\frac{2}{3}$).
 $84 = 84$

The missing number is 4.

To solve a proportion

1. Cross multiply.
2. Do the related division problem to find the missing number.

Examples D–G



BASIC
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DIRECTIONS: Solve the proportion.

STRATEGY: Cross multiply, then write the related division and simplify.

D. Solve: $\frac{4}{15} = \frac{8}{x}$

$$\begin{aligned} 4x &= 15(8) && \text{Cross multiply.} \\ 4x &= 120 && \text{Simplify.} \\ x &= 120 \div 4 && \text{Rewrite as division.} \\ x &= 30 && \text{Simplify.} \end{aligned}$$

The missing number is 30.

E. Solve: $\frac{0.8}{z} = \frac{0.4}{1.4}$

$$\begin{aligned} 0.8(1.4) &= 0.4z && \text{Cross multiply.} \\ 1.12 &= 0.4z && \text{Simplify.} \\ 1.12 \div 0.4 &= z && \text{Rewrite as division.} \\ z &= 2.8 && \text{Simplify.} \end{aligned}$$

The missing number is 2.8.

F. Solve: $\frac{\frac{3}{4}}{\frac{5}{8}} = \frac{1}{w}$

$$\begin{aligned} \frac{3}{4}w &= \frac{5}{8} \left(\frac{1}{2} \right) && \text{Cross multiply.} \\ \frac{3}{4}w &= \frac{5}{16} && \text{Simplify.} \\ w &= \frac{5}{16} \div \frac{3}{4} && \text{Rewrite as division.} \\ w &= \frac{5}{16} \cdot \frac{4}{3} && \text{Invert the divisor.} \\ w &= \frac{5}{12} && \text{Simplify.} \end{aligned}$$

The missing number is $\frac{5}{12}$.

Warm-Ups D–G

D. Solve: $\frac{5}{8} = \frac{10}{y}$

E. Solve: $\frac{0.6}{c} = \frac{0.5}{0.75}$

F. Solve: $\frac{\frac{3}{4}}{1\frac{2}{3}} = \frac{1}{x}$

Answers to Warm-Ups

- D. $y = 16$ E. $c = 0.9$
F. $x = \frac{10}{9}$ or $1\frac{1}{9}$

CALCULATOR EXAMPLE

G. Solve $\frac{8}{v} = \frac{2.82}{7.31}$ and round to the nearest hundredth.

G. Solve $\frac{2}{t} = \frac{5.8}{6.52}$ and round to the nearest hundredth.

$$\begin{array}{ll} 2(6.52) = 5.8t & \text{Cross multiply.} \\ 2(6.52) \div 5.8 = t & \text{Rewrite as division.} \\ 2.24827568 \approx t & \text{Simplify using a calculator.} \\ 2.25 \approx t & \text{Round.} \end{array}$$

The missing number is 2.25 to the nearest hundredth.

Answers to Warm-Ups

G. $v \approx 20.74$

Exercises 5.2

OBJECTIVE 1

Determine whether a proportion is true or false.

A True or false?

1. $\frac{3}{21} = \frac{11}{77}$

2. $\frac{6}{4} = \frac{27}{18}$

3. $\frac{3}{2} = \frac{9}{4}$

4. $\frac{3}{4} = \frac{9}{16}$

5. $\frac{4}{10} = \frac{5}{20}$

6. $\frac{3}{11} = \frac{9}{33}$

B

7. $\frac{18}{12} = \frac{15}{10}$

8. $\frac{16}{10} = \frac{24}{15}$

9. $\frac{35}{30} = \frac{22}{20}$

10. $\frac{24}{36} = \frac{32}{38}$

11. $\frac{30}{27} = \frac{60}{45}$

12. $\frac{45}{36} = \frac{25}{20}$

13. $\frac{13}{4} = \frac{9.75}{3}$

14. $\frac{25}{6} = \frac{12.5}{3}$

OBJECTIVE 2

Solve a proportion.

A Solve.

15. $\frac{7}{9} = \frac{a}{18}$

16. $\frac{3}{13} = \frac{b}{52}$

17. $\frac{2}{6} = \frac{c}{18}$

18. $\frac{2}{9} = \frac{x}{18}$

19. $\frac{28}{y} = \frac{14}{5}$

20. $\frac{15}{z} = \frac{10}{12}$

21. $\frac{14}{28} = \frac{5}{c}$

22. $\frac{8}{12} = \frac{6}{d}$

23. $\frac{16}{12} = \frac{3}{x}$

24. $\frac{16}{y} = \frac{24}{1}$

25. $\frac{p}{8} = \frac{2}{32}$

26. $\frac{q}{4} = \frac{2}{24}$

B

27. $\frac{x}{5} = \frac{23}{10}$

28. $\frac{y}{6} = \frac{3}{8}$

29. $\frac{2}{z} = \frac{5}{11}$

30. $\frac{12}{x} = \frac{16}{3}$

31. $\frac{13}{6} = \frac{w}{2}$

32. $\frac{7}{12} = \frac{y}{9}$

33. $\frac{15}{16} = \frac{12}{a}$

34. $\frac{28}{7} = \frac{50}{b}$

35. $\frac{1.2}{c} = \frac{0.2}{0.1}$

36. $\frac{0.2}{z} = \frac{0.1}{0.25}$

37. $\frac{\frac{3}{5}}{b} = \frac{8}{5}$

38. $\frac{\frac{2}{3}}{c} = \frac{\frac{8}{9}}{1\frac{7}{9}}$

39. $\frac{0.9}{4.5} = \frac{0.09}{x}$

40. $\frac{2.8}{3.5} = \frac{1.5}{y}$

41. $\frac{6}{1.2} = \frac{w}{0.02}$

42. $\frac{0.8}{0.3} = \frac{b}{2.4}$

43. $\frac{y}{4} = \frac{8}{\frac{1}{4}}$

44. $\frac{s}{30} = \frac{\frac{2}{3}}{5}$

45. $\frac{t}{24} = \frac{\frac{3}{2}}{10\frac{1}{2}}$

46. $\frac{w}{4\frac{1}{4}} = \frac{\frac{3}{3}}{2\frac{1}{2}}$

Solve. Round to the nearest tenth.

47. $\frac{3}{11} = \frac{w}{5}$

48. $\frac{3}{11} = \frac{x}{15}$

49. $\frac{8}{25} = \frac{10}{y}$

50. $\frac{8}{25} = \frac{18}{z}$

Solve. Round to the nearest hundredth.

51. $\frac{2.5}{4.5} = \frac{a}{0.6}$

52. $\frac{2.5}{4.5} = \frac{b}{2.6}$

53. $\frac{\frac{3}{7}}{c} = \frac{9}{20}$

54. $\frac{\frac{3}{7}}{d} = \frac{9}{32}$

C Fill in the boxes to make the statements true. Explain your answers.

55. If $\frac{\square}{120} = \frac{x}{12}$, then $x = 1$.

56. If $\frac{\square}{25} = \frac{y}{20}$, then $y = 4$.

57. Find the error in the statement: If $\frac{2}{5} = \frac{x}{19}$, then $2x = 5(19)$.

58. Find the error in the statement: If $\frac{3}{w} = \frac{7}{9}$, then $3w = 7(9)$.

59. Current recommendations from the American Heart Association include that the ratio of total cholesterol to HDL (“good” cholesterol) for women be no more than 4.5 to 1. Fran has a total cholesterol level of 214. The proportion $\frac{214}{H} = \frac{4.5}{1}$ gives the minimum allowable level, H , of HDL for Fran to stay within the guidelines. What is this level, rounded to the nearest whole number?

61. In a recent year, the average pupilteacher ratio in public elementary schools was 23 to 1. Eastlake Elementary has 725 students. The proportion $\frac{725}{t} = \frac{23}{1}$ gives the expected number of teachers, t , in keeping with the national average. How many teachers would you expect Eastlake to have, rounded to the nearest whole teacher?

60. The American Heart Association’s recommendation for men is a ratio of total cholesterol to HDL of no more than 4.0 to 1. Jim’s total cholesterol level is 168, and the proportion $\frac{168}{H} = \frac{4.0}{1}$ gives his minimum allowable HDL level, H . What is Jim’s minimum allowable HDL level?

62. The highest marriage rate in the past century in the United States occurred in 1946, when the rate was 118 marriages per 1000 unmarried women per year. In a city with 9250 unmarried women in 1946, the proportion $\frac{m}{9250} = \frac{118}{1000}$ gives m , the number of expected marriages. How many marriages were expected in the city in 1946?

- 63.** For every 10 people in the United States, it is estimated that 7 suffer from some form of migraine headache.
- Find the number of migraine sufferers in a group of 350 people. To determine the number of sufferers, solve the proportion $\frac{7}{10} = \frac{N}{350}$, where N represents the number of migraine sufferers in the group.
 - If three times as many women as men suffer from migraines, find the number of men that suffer migraines in a group in which 306 women are affected. To determine the number of men, solve the proportion $\frac{1}{3} = \frac{M}{306}$, where M represents the number of men in the group.
- c.** Some researchers are of the opinion that headaches in 8 of every 40 migraine sufferers are related to diet. At this rate, determine the number of migraine headaches that may be related to diet in a group of 350 sufferers. To find the number, solve the proportion $\frac{8}{40} = \frac{D}{350}$, where D represents the number of headaches related to diet.

Exercises 64–66 relate to the chapter application.

- 64.** A state atlas has several maps that are marked both as “1 inch represents 4.8 miles” and with the ratio 1:300,000. Convert 4.8 miles to inches, then set up the related proportion. Is the proportion true or false? Explain.
- 65.** A map of greater London is marked both as “1 inch represents 3.1 miles” and with the ratio 1:200,000. Convert 3.1 miles to inches and set up the related proportion. Is the proportion true or false? Explain.
- 66.** You are making a map of your neighborhood and you have chosen a scale of “1 inch represents 100 feet.” If the width of your street is 30 feet, how wide is it on your map?

STATE YOUR UNDERSTANDING

- 67.** Explain how to solve $\frac{3.5}{\frac{1}{4}} = \frac{7}{y}$.
- 68.** Look up the word *proportion* in the dictionary and write two definitions that differ from the mathematical definition used in this section. Write three sentences using the word *proportion* that illustrate each of the meanings.

CHALLENGE*Solve.*

69. $\frac{7+5}{7+8} = \frac{4}{a}$

70. $\frac{6(10) - 5(5)}{7(9) - 7(3)} = \frac{4}{a}$

Solve. Round to the nearest thousandth.

71. $\frac{7}{w} = \frac{18.92}{23.81}$

72. $\frac{7}{t} = \frac{18.81}{23.92}$

GROUP WORK

- 73.** Five ounces of decaffeinated coffee contain approximately 3 mg of caffeine, whereas 5 oz of regular coffee contain an average of 120 mg of caffeine. Five ounces of tea brewed for 1 min contain an average of 21 mg of caffeine. Twelve ounces of regular cola contain an average of 54 mg of caffeine. Six ounces of cocoa contain an average of 11 mg of caffeine. Twelve ounces of iced tea contain an average of 72 mg of caffeine. Determine the total amount of caffeine each member of your

group consumed yesterday. Make a chart to illustrate this information. Combine this information with that of other groups in your class to make a class amount. Make a class chart to illustrate the totals. Determine the average amount of caffeine consumed by each member of the group and then by each member of the class. Compare these averages by making ratios. Discuss the similarities and the differences.

MAINTAIN YOUR SKILLS

- 74.** Find the difference of 620.3 and 499.9781.
- 75.** Find the average of 1.8, 0.006, 17, and 8.5.
- 76.** Find the average of 6.45, 7.13, and 5.11.
- 77.** Multiply 4.835 by 10,000.
- 78.** Divide 4.835 by 10,000.
- 79.** Multiply 0.875 by 29.
- 80.** Multiply 12.75 by 8.09.
- 81.** Divide 0.70035 by 0.35.
- 82.** Divide 3.6 by 0.72.
- 83.** If gasoline is \$2.489 per gallon, how much does Quan pay for 12.8 gallons? Round your answer to the nearest cent.

5.3 Applications of Proportions

How & Why

OBJECTIVE

Solve word problems using proportions.

If the ratio of two quantities is constant, the ratio can be used to find the missing part of a second ratio. For instance, if 2 lb of bananas cost \$0.48, what will 12 lb of bananas cost? Table 5.1 organizes this information.

OBJECTIVE

Solve word problems using proportions.

	Case I	Case II
Pounds of Bananas	2	12
Cost in Dollars	0.48	

In Table 5.1 the cost in Case II is missing. Call the missing value y , as in Table 5.2.

	Case I	Case II
Pounds of Bananas	2	12
Cost in Dollars	0.48	y

Write the proportion using the ratios shown in Table 5.2.

$$\frac{2 \text{ lb of bananas}}{\$0.48} = \frac{12 \text{ lb of bananas}}{\$y}$$

The units are the same on each side of the equation so we can drop them.

$$\begin{aligned}\frac{2}{0.48} &= \frac{12}{y} \\ 2y &= 0.48(12) \quad \text{Cross multiply.} \\ 2y &= 5.76 \\ y &= 5.76 \div 2 \quad \text{Rewrite as division.} \\ y &= 2.88 \quad \text{Simplify.}\end{aligned}$$

So 12 lb of bananas will cost \$2.88.

Using a table forces the units of a proportion to match. Therefore, we usually do not write the units in the proportion itself. We *always* use the units in the answer.

To solve word problems involving proportions

1. Write the two ratios and form the proportion. A table with three columns and three rows will help organize the data. The proportion will be shown in the boxes of the table.
2. Solve the proportion.
3. Write the solution, including the appropriate units.

Warm-Ups A–E

Examples A–E



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DIRECTIONS: Solve the following problems using proportions.

STRATEGY: Make a table with three columns and three rows. Label the last two columns “Case I” and “Case II.” Label the last two rows with the units in the problem. Fill in the table with the quantities given and assign a variable to the unknown quantity. Write the proportion shown in the table and solve it. Write the solution, including units of measure.

- A. A pro shop advertises golf balls at 6 for \$7.25. At this rate, what will 4 dozen balls cost?

- B. A house has a property tax of \$1260 and is valued at \$45,000. At the same rate, what will be the property tax on a house valued at \$65,000?

- C. In Example C, the recipe calls for $3\frac{1}{4}$ oz of peanuts in 10 oz of mix. How many ounces of peanuts should Judith use?

- A. If 3 cans of tuna fish sell for \$3.58, what is the cost of 48 cans of tuna fish?

	Case I	Case II
Cans of Tuna Fish	3	48
Cost in Dollars	3.58	C

Make a table.

$$\frac{3}{3.58} = \frac{48}{C} \quad \text{Write the proportion.}$$

$3C = (3.58)(48)$ Cross multiply.

$$3C = 171.84$$

$$C = 57.28 \quad \text{Divide.}$$

The cost of 48 cans of tuna fish is \$57.28.

- B. Mary Alice pays \$2250 in property tax on her house, which is valued at \$123,000. At the same rate, what is the property tax on a house valued at \$164,000?

	Case I	Case II
Tax Value	\$2250	T
	\$123,000	\$164,000

Make a table.

$$\frac{2250}{123,000} = \frac{T}{164,000} \quad \text{Write the proportion.}$$

$2250(164,000) = 123,000T$ Cross multiply.

$$369,000,000 = 123,000T$$

$$3000 = T \quad \text{Divide.}$$

The tax on the \$164,000 house is \$3000.

- C. Judith is making a trail mix for a large group of scouts. Her recipe calls for $2\frac{1}{2}$ oz of raisins in 10 oz of mix. How many ounces of raisins should she use to make 35 oz of mix?

	Case I	Case II
Raisins	$2\frac{1}{2}$	R
Trail Mix	10	35

Make a table.

Answers to Warm-Ups

- A. The cost of 4 dozen golf balls is \$58.
B. The property tax will be \$1820.
C. Judith should use $11\frac{3}{8}$ oz of peanuts.

$$\frac{2\frac{1}{2}}{10} = \frac{R}{35}$$

Write the proportion.

$$\frac{1}{2}(35) = 10R$$

Cross multiply.

$$\frac{5}{2}(35) = 10R$$

Change to an improper fraction.

$$\frac{175}{2} = 10R$$

Simplify.

$$\frac{175}{2} \div 10 = R$$

$$\frac{175}{2} \cdot \frac{1}{10} = R$$

$$\frac{35}{4} = R$$

$$8\frac{3}{4} = R$$

Divide.

Judith should use $8\frac{3}{4}$ oz of raisins.

- D. The city fire code requires a school classroom to have at least 50 ft^2 of floor space for every 3 students. What is the minimum number of square feet needed for 30 students?

	Case I	Case II
Number of Students	3	30
Square Feet of Space	50	S

$$\frac{3}{50} = \frac{30}{S}$$

Write the proportion.

$$3S = 50(30)$$

Cross multiply.

$$3S = 1500$$

$$S = 500$$

The room must have at least 500 ft^2 for 30 students.

- E. A veterinarian recommends that cat food contain two parts lamb meal, five parts fish product, and nine parts other ingredients. How much lamb meal is needed to make 500 lb of the cat food?

STRATEGY: Add the number of parts to get the total number of components in the cat food. Set up a proportion using “total” as one of the comparisons.

The total number of components is $2 + 5 + 9 = 16$

	Case I	Case II
Lamb Meal	2	M
Cat Food (Total)	16	500

$$\frac{2}{16} = \frac{M}{500}$$

Write the proportion.

$$2(500) = 16M$$

Cross multiply.

$$1000 = 16M$$

$$62.5 = M$$

To make 500 lb of cat food, 62.5 lb of lamb meal is needed.

- D. In another city, the fire code requires a school classroom to have at least 86 ft^2 for every 5 students. What is the minimum area needed for 30 students?

- E. The veterinarian also advises that an alternative cat food contain four parts poultry by-products, six parts lamb meal, and ten parts other ingredients. How much poultry by-product is needed for 500 lb of the cat food?

Answers to Warm-Ups

- D. The minimum area needed is 516 ft^2 .
E. For 500 lb of cat food, 100 lb of poultry is needed.

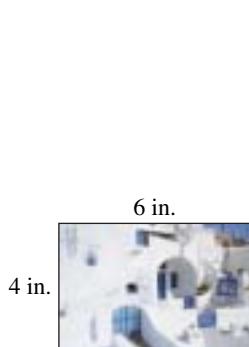
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Exercises 5.3

OBJECTIVE

Solve word problems using proportions.

A Exercises 1–6. A photograph that measures 6 in. wide and 4 in. high is to be enlarged so the width will be 15 in. What will be the height of the enlargement?



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	Case I	Case II
Width (in.)	(a)	(c)
Height (in.)	(b)	(d)

1. What goes in box (a)?
 2. What goes in box (b)?
 3. What goes in box (c)?

 4. What goes in box (d)?
 5. What is the proportion for the problem?
 6. What is the height of the enlargement?

Exercises 7–12. If a fir tree is 30 ft tall and casts a shadow of 18 ft, how tall is a tree that casts a shadow of 48 ft?

	First Tree	Second Tree
Height (ft)	(1)	(3)
Shadow	(2)	(4)

- 7.** What goes in box (1)? **8.** What goes in box (2)? **9.** What goes in box (3)?

10. What goes in box (4)? **11.** What is the proportion for the problem? **12.** How tall is the second tree?

Exercises 13–18. Jean and Jim are building a fence around their yard. From past experience they know that they are able to build 30 ft in 6 hr. If they work at the same rate, how many hours will it take them to complete the job if the perimeter of the yard is 265 ft?

	Case I	Case II
Time (hr)	(5)	(7)
Length of Fence (ft)	(6)	(8)

13. What goes in box (5)? 14. What goes in box (6)? 15. What goes in box (7)?

16. What goes in box (8)? 17. What is the proportion for the problem? 18. How many hours will it take to build the fence?

B Exercises 19–23. The Centerburg Junior High School expects a fall enrollment of 910 students. The district assigns teachers at the rate of 3 teachers for every 65 students. The district currently has 38 teachers assigned to the school. How many teachers does the district need to assign to the school?

	Case I	Case II
Teachers	3	(e)
Students	65	(f)

- 19.** What goes in box (e)?

20. What goes in box (f)?

21. What is the proportion for the problem?

22. How many teachers will be needed at the school next year?

23. How many additional teachers will need to be assigned?

Exercises 24–26. The average restaurant in Midvale produces 36 lb of garbage in 1½ days. How many pounds of garbage do they produce in 2 weeks? Let x represent the missing number of pounds.

	Case I	Case II
Days	1	1
Garbage (lb)	1	1

- 24.** What goes in each of the four boxes? **25.** What is the proportion for the problem?

26. How many pounds of garbage do they have at the end of 2 weeks?

C

- 27.** Idamae is knitting a sweater. The knitting gauge is 8 rows to the inch. How many rows must she knit to complete $12\frac{1}{2}$ in. of the sweater?

28. Merle is knitting a scarf. The knitting is six rows to the inch. How many rows must she knit to complete $11\frac{1}{2}$ in. of the scarf?

	Case I	Case II
Rows	10	10
Inches	10	10

- 29.** The Kingdom of Bahrain leads the world in its proportion of males, where 61 of every 100 people are male. With an estimated population of 677,886 in 2004, how many of them are male? Round to the nearest whole number.

30. In 1900 in the United States, there were 40 deaths from diphtheria for every 100,000 people. How many diphtheria deaths would be expected in 1900 in a town of 60,000 people? (Zero cases of diphtheria were reported in 1995.)

- 31.** In 1950 in the United States, families spent \$3 of every \$10 of family income on food. What would you expect a family to spend on food in 1950 if their income was \$30,000?
- 32.** Nutritionists recommend that frozen dinners should contain no more than 3 g of total fat per 100 calories and no more than 1 g of saturated fat per 100 calories.
- A Swanson Hungry Man Sports Grill of Pulled Pork has 920 calories and 44 g of total fat, 18 g of which are saturated. Does this fall within the guidelines? Explain.
 - A Swanson Mesquite Grilled Chicken dinner has 380 calories and 10 g of total fat, 2.5 g of which are saturated. Does this fall within the guidelines? Explain.
- 33.** If 30 lb of fertilizer will cover 1500 ft² of lawn, how many square feet will 50 lb of fertilizer cover?

Exercises 34–36. The Logan Community College basketball team won 11 of its first 15 games. At this rate how many games will they win if they play a 30-game schedule?

	Case I	Case II
Games Won		
Games Played		

- 34.** What goes in each of the four boxes?
- 35.** What is the proportion for the problem?
- 36.** How many games should they win in a 30-game schedule?
- 37.** Dawn sells cars at the Quality Used Car Co. If she sells a car for \$2600, she earns \$90. If she sells a car for \$10,920, how much does she earn?
- 38.** If gasoline sells for \$2.419 per gallon, how many gallons can be purchased for \$48.38?
- 39.** At Jeena's stand in the Farmer's Market, onions are priced at 2 lb for \$0.59. If Mike buys 5 lb, what does he pay?
- 40.** Twenty-five pounds of tomatoes cost \$22.70 at the farmer's market. At this rate, what is the cost of 10 lb?
- 41.** A car is driven 451 mi in 8.2 hr. At the same rate, how long will it take to drive 935 mi?
- 42.** Celia earns a salary of \$1200 per month, from which she saves \$50 each month. Her salary is increased to \$1260 per month. If she keeps the same rate of savings how much will she save per month?
- 43.** Ginger and George have a room in their house that needs a new carpet. It will take 33 yd² of carpet to cover the floor. Hickson's Carpet Emporium will install 33 yd² of carpet for \$526.35. Ginger and George decide to have a second room of their house carpeted. This room requires 22 yd² of carpet. At the same rate, how much will it cost to have the second room carpeted?

44. A 16-oz can of pears costs \$0.98 and a 29-oz can costs \$1.69. Is the price per ounce the same in both cases? If not, what should be the price of the 29-oz can be to make the price per ounce equivalent?

46. During the first 665.6 miles on their vacation road trip, the Scaberys used 32 gallons of gas. At this rate, how many gallons are needed to finish the remaining 530.4 miles?

48. The ratio of girls to boys taking a math class is 5 to 4. How many girls are in a class of 81 students?

Hint: Complete the table.

	Case I	Case II
Number of Girls		
Number of Students	9	81

50. A local health-food store is making a cereal mix that has nuts to cereal in a ratio of 2 to 7. If they make 126 oz of the mix, how many ounces of nuts will they need?

52. A concrete mix contains 3 bags of cement, 2 bags of sand, and 3 bags of gravel. How many bags of cement are necessary for 68 bags of the concrete mix?

54. If \$1 is worth 0.72€ (European currency, euros) and a used refrigerator costs \$247, what is the cost in euros?

56. If \$1 is worth 98.15 yen (Chinese currency) and a pair of shoes costs 6203.084 yen, what is the cost in dollars?

58. In 1970, only 7.1 lb of every 100 lb of waste were recovered. In 1980, this rose to 9.7 lb. By 1990, the amount was 13.1 lb. In 2000 the amount was up to 15.2 lb. Determine the amount of waste recovered from 56,000,000 lb of waste in each of these years.

60. A 14-lb bag of dog food is priced at three bags for \$21. A 20-lb bag is \$9. The store manager wants to put the smaller bags on sale so they are the same unit price as the larger bags. What price should the smaller bags be marked?

45. Jim's doctor gives instructions to Ida, a nurse, to prepare a hypodermic containing 8 mg of a drug. The drug is in a solution that contains 20 mg in 1 cm³ (1 cc). How many cubic centimeters should Ida use for the injection?

47. If Nora sells half of a ton of blueberries for \$650, how much does she receive for 4.25 tons?

49. Betty prepares a mixture of nuts that has cashews and peanuts in a ratio of 3 to 7. How many pounds of each will she need to make 40 lb of the mixture?

51. Debra is making green paint by using 3 quarts of blue paint for every 4 quarts of yellow paint. How much blue paint will she need to make 98 quarts of green paint?

53. Lucia makes meatballs for her famous spaghetti sauce by using 10 lb of ground round to 3 lb of spice additives. How many pounds of ground round will she need for 84.5 lb of meatballs?

55. If \$1 is worth £0.50 (British pound) and a computer costs \$1300, what is the cost in pounds?

57. Auto batteries are sometimes priced proportionally to the number of years they are expected to last. If a \$35.85 battery is expected to last 36 months, what is the comparable price of a 60-month battery?

59. The quantity of ozone contained in 1 m³ of air may not exceed 235 mg or the air is judged to be polluted. What is the maximum quantity of ozone that can be contained in 12 m³ of air before the air is judged to be polluted?

61. A large box of brownie mix that makes four batches of brownies costs \$4.79 at a warehouse outlet. A box of brownie mix that makes one batch costs \$1.29 in a grocery store. By how much should the grocery store reduce each box so that its price is competitive with the warehouse outlet?

Exercises 62–64 relate to the chapter application.

62. A street map of St. Louis has a scale of 1 in. represents 1900 ft. If two buildings are $5\frac{3}{4}$ in. apart on the map, how far apart are the real buildings?
63. A street map of Washington, D.C., has a scale of $1\frac{3}{8}$ in. represents 0.5 mi. If the distance between two bridges is $8\frac{1}{2}$ in. on the map, how far apart are the actual bridges? Round to the nearest hundredth.
64. A map of the state of Washington has a scale of $2\frac{1}{4}$ in. represents 30 mi. The distance between Spokane and Seattle is 282 mi. How far apart are they on the map?

STATE YOUR UNDERSTANDING

65. What is a proportion? Write three examples of situations that are proportional.
66. Look on the label of any food package to find the number of calories in one serving. Use this information to create a problem that can be solved by a proportion. Write the solution of your problem in the same way as the Examples in this section are written.
67. From a consumer's viewpoint, explain why it is not always an advantage for costs of goods and services to be proportional.

CHALLENGE

68. In 1982, approximately 25 California condors were alive. This low population was the result of hunting, habitat loss, and poisoning. The U.S. Fish and Wildlife Service instituted a program that resulted in there being 73 condors alive in 1992. If this increase continues proportionally, predict how many condors will be alive in 2017.
69. The tachometer of a sports car shows the engine speed to be 2800 revolutions per minute. The transmission ratio (engine speed to drive shaft speed) for the car is 2.5 to 1. Find the drive shaft speed.
70. Two families rented a mountain cabin for 19 days at a cost of \$1905. The Santini family stayed for 8 days and the Nguyen family stayed for 11 days. How much did it cost each family? Round the rents to the nearest dollar.

GROUP WORK

- 71.** A $13\frac{1}{2}$ -oz bag of Cheetos costs \$2.69 and a 24-oz bag costs \$4.09. You have a coupon for \$0.50 that the store will double. Divide the group into two teams. One team will formulate an argument that using the coupon on the smaller bag results in a better value. The other team will formulate an argument that using the coupon on the larger bag is better. Present your arguments to the whole group and select the one that is most convincing. Share your results with the rest of the class.
- 72.** List all the types of recycling done by you and your group members. Determine how many people participate in each type of recycling. Determine ratios for each kind of recycling. Find the population of your city or county. Using your class ratio, determine how many people in your area are recycling each type of material. Make a chart to illustrate your findings. Contact your local recycling center to see how your ratios compare to their estimates. Explain the similarities and differences.

MAINTAIN YOUR SKILLS

- 73.** Round 37.4145 to the nearest hundredth and to the nearest thousandth.
- 75.** Compare the decimals 0.01399 and 0.011. Write the result as an inequality.
- 77.** What is the total cost of 11.9 gallons of gasoline that costs \$2.399 per gallon? Round to the nearest cent.
- 74.** Round 62.3285 to the nearest hundredth and to the nearest thousandth.
- 76.** Compare the decimals 0.06 and 0.15. Write the result as an inequality.
- 78.** A barrel of liquid weighs 429.5 lb. If the barrel weighs 22.5 lb and the liquid weighs 7.41 lb per gallon, how many gallons of liquid are in the barrel, to the nearest gallon?

Change each decimal to a simplified fraction.

79. 0.865

80. 0.01125

Change each fraction to a decimal rounded to the nearest thousandth.

81. $\frac{123}{220}$

82. $\frac{33}{350}$

Key Concepts CHAPTER 5

Section 5.1 Ratio and Rate

Definitions and Concepts	Examples
A ratio is a comparison of two like measurements by division.	The ratio of the length of a room to its width is $\frac{12 \text{ ft}}{9 \text{ ft}} = \frac{12}{9} = \frac{4}{3}$.
A rate is a comparison of two unlike measurements by division.	The rate of a biker who rides 21 mi in 2 hr is $\frac{21 \text{ mi}}{2 \text{ hr}}$.
A unit rate is a rate with a denominator of one unit.	The unit rate of a biker who rides 21 mi in 2 hr is $\frac{21 \text{ mi}}{2 \text{ hr}} = \frac{10.5 \text{ mi}}{1 \text{ hr}} = 10.5 \text{ mph}$.

Section 5.2 Solving Proportions

Definitions and Concepts	Examples
A proportion is a statement that two ratios are equal.	$\frac{6}{12} = \frac{1}{2}$ is a proportion
A proportion is true when the cross products are equal.	$\frac{6}{12} = \frac{1}{2}$ is true because $6(2) = 12(1)$.
A proportion is false when the cross products are not equal.	$\frac{3}{5} = \frac{5}{8}$ is false because $3(8) \neq 5(5)$.
To solve a proportion: <ul style="list-style-type: none"> • Cross multiply. • Do the related division problem to find the missing number. 	Solve: $\frac{3}{x} = \frac{15}{43}$ $3 \cdot 43 = 15x$ $129 = 15x$ $129 \div 15 = x$ $8.6 = x$

Section 5.3 Applications of Proportions**Definitions and Concepts**

To solve word problems involving proportions:

- Make a table to organize the information.
- Write a proportion from the table.
- Solve the proportion.
- Write the solution, including appropriate units.

Examples

If 3 cans of cat food sell for \$3.69, how much will 8 cans cost?

	Case I	Case II
Cans	3	8
Cost	\$3.69	C

$$\frac{3}{3.69} = \frac{8}{C}$$
$$3C = 29.52$$
$$C = 29.52 \div 3$$
$$C = 9.84$$

So 8 cans of cat food will cost \$9.84.

Review Exercises CHAPTER 5

Section 5.1

Write as a ratio in simplified form.

1. 7 to 35

2. 9 to 54

3. 12 m to 10 m

4. 12 km to 9 km

5. 1 dollar to 25 nickels
(compare in nickels)

6. 660 ft to 1 mi
(compare in feet)

7. 16 in. to 2 ft
(compare in inches)

8. 3 ft to 3 yd
(compare in feet)

Write a rate and simplify.

9. 9 people to 10 chairs

10. 23 miles to 3 hikes

11. 510 km to 85 people

12. 10 cars to 6 households

13. 88 lb to 33 ft

14. 36 buttons to 24 bows

15. 765 people to 27 committees

16. 8780 households to 6 cable companies

Write as a unit rate.

17. 50 mi to 2 hr

18. 60 mi to 4 minutes

19. 90¢ per 10 lb of potatoes

20. \$1.17 per 3 lb of broccoli

Write as a unit rate. Round to the nearest tenth.

21. 825 mi per 22 gal

22. 13,266 km per 220 gal

23. 225 gal per 14 min

24. 850 L per 14 min

25. One section of the country has 3500 TV sets per 1000 households. Another section has 500 TV sets per 150 households. Are the rates of the TV sets to the number of households the same in both parts of the country?

26. In Pineberg, there are 5000 automobiles per 3750 households. In Firville, there are 6400 automobiles per 4800 households. Are the rates of the number of automobiles to the number of households the same?

Section 5.2

True or false?

27. $\frac{6}{3} = \frac{72}{36}$

28. $\frac{2}{3} = \frac{26}{39}$

29. $\frac{18}{14} = \frac{12}{10}$

30. $\frac{16}{25} = \frac{10}{15}$

31. $\frac{2.1875}{3} = \frac{15}{16}$

32. $\frac{9.375}{3} = \frac{25}{8}$

Solve.

33. $\frac{1}{2} = \frac{r}{18}$

34. $\frac{1}{3} = \frac{s}{18}$

35. $\frac{2}{t} = \frac{5}{10}$

36. $\frac{8}{v} = \frac{2}{5}$

37. $\frac{f}{7} = \frac{2}{28}$

38. $\frac{g}{2} = \frac{2}{12}$

39. $\frac{16}{24} = \frac{r}{16}$

40. $\frac{s}{10} = \frac{15}{16}$

41. $\frac{8}{12} = \frac{t}{8}$

42. $\frac{7}{5} = \frac{w}{7}$

Solve. Round to the nearest tenth.

43. $\frac{5}{6} = \frac{a}{5}$

44. $\frac{7}{6} = \frac{6}{b}$

45. $\frac{16}{7} = \frac{c}{12}$

46. $\frac{16}{5} = \frac{12}{d}$

47. A box of Arm and Hammer laundry detergent that is sufficient for 33 loads of laundry costs \$3.99. What is the most that a store brand of detergent can cost if the box is sufficient for 25 loads and is more economical to use than Arm and Hammer? To find the cost, solve the

proportion $\frac{\$3.99}{33} = \frac{c}{25}$, where c represents the cost of the store brand.

48. Available figures show that it takes the use of 18,000,000 gasoline-powered lawn mowers to produce the same amount of air pollution as 3,000,000 new cars. Determine the number of gasoline-powered lawn mowers that will produce the same amount of air pollution as 50,000 new cars. To find the number of lawn mowers,

solve the proportion $\frac{18,000,000}{3,000,000} = \frac{L}{50,000}$, where L represents the number of lawn mowers.

Section 5.3

49. For every 2 hr a week that Merle is in class, she plans to spend 5 hr a week doing her homework. If she is in class 15 hr each week, how many hours will she plan to be studying each week?
51. Juan must do 25 hr of work to pay for the tuition for three college credits. If Juan intends to sign up for 15 credit hours in the fall, how many hours will he need to work to pay for his tuition?
53. Larry sells men's clothing at the University Men's Shop. For \$120 in clothing sales, Larry makes \$15. How much does he make on a sale of \$350 worth of clothing?
50. If 16 lb of fertilizer will cover 1500 ft² of lawn, how much fertilizer is needed to cover 2500 ft²?
52. In Exercise 51, if Juan works 40 hr per week, how many weeks will he need to work to pay for his tuition? (Any part of a week counts as a full week.)
54. Dissolving 1.5 lb of salt in 1 gal of water makes a brine solution. At this rate, how many gallons of water are needed to made a brine solution with 12 lb of salt?

True/False Concept Review CHAPTER 5

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. A fraction can be regarded as a ratio. 1. _____
2. A ratio is a comparison of two numbers or measures usually written as a fraction. 2. _____
3. $\frac{18 \text{ miles}}{1 \text{ gallon}} = \frac{54 \text{ miles}}{3 \text{ hours}}$ 3. _____
4. To solve a proportion, we must know the values of only two of the four numbers. 4. _____
5. If $\frac{8}{5} = \frac{t}{2}$, then $t = \frac{5}{16}$. 5. _____
6. In a proportion, two ratios are equal. 6. _____
7. Three feet and 1 yard are unlike measures. 7. _____
8. Ratios that are rates compare unlike units. 8. _____
9. To determine whether a proportion is true or false, the ratios must have the same units. 9. _____
10. If a fir tree that is 18 ft tall casts a shadow of 17 ft, how tall is a tree that casts a shadow of 25 ft? The following table can be used to solve this problem. 10. _____

	First Tree	Second Tree
Height	17	18
Shadow	x	25

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Test CHAPTER 5**ANSWERS**

1. Write a ratio to compare 12 yards to 15 yards.
1. _____

2. On a test, Ken answered 20 of 32 questions correctly. At the same rate, how many would he answer correctly if there were 72 questions on a test?
2. _____

3. Solve the proportion: $\frac{4.8}{12} = \frac{0.36}{w}$
3. _____

4. Is the following proportion true or false? $\frac{16}{35} = \frac{24}{51}$
4. _____

5. Solve the proportion: $\frac{13}{36} = \frac{y}{18}$
5. _____

6. Is the following proportion true or false? $\frac{9 \text{ in.}}{2 \text{ ft}} = \frac{6 \text{ in.}}{16 \text{ in.}}$
6. _____

7. If Mary is paid \$49.14 for 7 hr of work, how much should she be paid for 12 hr of work?
7. _____

8. Write a ratio to compare 8 hr to 3 days (compare in hours).
8. _____

9. There is a canned food sale at the supermarket. A case of 24 cans of peas is priced at \$19.68. At the same rate, what is the price of 10 cans?
9. _____

10. If 40 lb of beef contains 7 lb of bones, how many pounds of bones may be expected in 100 lb of beef?
10. _____

11. Solve the proportion: $\frac{0.4}{0.5} = \frac{0.5}{x}$
11. _____

12. A charter fishing boat has been catching an average of 3 salmon for every 4 people they take fishing. At that rate, how many fish will they catch if over a period of time they take a total of 32 people fishing?
12. _____

13. On a trip home, Jennie used 12.5 gal of gas. The trip odometer on her car registered 295 mi for the trip. She is planning a trip to see a friend who lives 236 mi away. How much gas will Jennie need for the trip?
13. _____

14. Solve the proportion: $\frac{a}{8} = \frac{4.24}{6.4}$
14. _____

15. Is the following a rate? $\frac{130 \text{ mi}}{2 \text{ hr}}$ 15. _____
16. If a 20-ft tree casts a 15-ft shadow, how long a shadow is cast by a 14-ft tree? 16. _____
17. What is the population density of a town that is 150 square miles and has 5580 people? Reduce to a 1-square-mile comparison. 17. _____
18. Solve the proportion and round your answer to the nearest hundredth: $\frac{4.78}{y} = \frac{32.5}{11.2}$ 18. _____
19. A landscape firm has a job that it takes a crew of three $4\frac{1}{2}$ hr to do. How many of these jobs could the crew of three do in 117 hrs? 19. _____
20. The ratio of males to females in a literature class is 3 to 5. How many females are in a class of 48 students? 20. _____

Group Project CHAPTER 5

OPTIONAL

The human body is the source of many common proportions. Artists have long studied the human figure in order to portray it accurately. Your group will be investigating how each member compares to the standard and how various artists have used the standards.

Most adult bodies can be divided into eight equal portions. The first section is from the top of the head to the chin. Next is from the chin to the bottom of the sternum. The third section is from the sternum to the navel, and the fourth is from the navel to the bottom of the torso. The bottom of the torso to the bottom of the knee is two sections long, and the bottom of the knee to the bottom of the foot is the last two sections. (Actually, these last two sections are a little short. Most people agree that the body is actually closer to 7.5 sections, but because this is hard to judge proportionally, we use eight sections and leave the bottom one short.)

Complete the following table for each group member.

Section	Length (in cm)	Ratio of Section to Head (Actual)	Ratio of Section to Head (Expected)
Head			
Chin to sternum			
Sternum to navel			
Navel to torso bottom			
Torso bottom to knee			
Knee to foot			

Explain how your group arrived at the values in the last column. Which member of the group comes closest to the standards? Did you find any differences between the males and females in your group? Either draw a body using the standard proportions, or get a copy of a figure from a painting and analyze how close the artist came to the standards.

A slightly different method of dividing the upper torso is to start at the bottom of the torso and divide into thirds at the waist and the shoulders. In this method, there is a pronounced difference between males and females. In females, the middle third between the waist and shoulders is actually shorter than the other two. In males, the bottom third from waist to bottom of the torso is shorter than the others. For each member of your group, fill out the following table.

Section	Length (in cm)	Ratio of Section to Entire Upper Torso (Actual)	Ratio of Section to Entire Upper Torso (Expected)
Head to shoulders			
Shoulders to waist			
Waist to bottom of torso			

Explain how your group arrived at the values in the last column (these will depend on gender). Which member of your group comes closest to the standards? Either draw a body using the standard proportions, or get a copy of a figure from a painting and analyze how close the artist came to the standards.

Children have different body proportions than adults, and these proportions change with the age of the child. Measure three children who are the same age. Use their head measurement as one unit, and compute the ratio of head to entire body. How close are the three children's ratios to each other? Before the Renaissance, artists usually depicted children as miniature adults. This means that the proportions fit those in the first table rather than those you just discovered. Find a painting from before the Renaissance that contains a child. Calculate the child's proportions and comment on them. Be sure to reference the painting you use.

Cumulative Review CHAPTERS 1–5

Add or subtract.

1. $511 + 672 + 770 + 92$

2. $5800 - 3098$

3.
$$\begin{array}{r} 9623 \\ -6489 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 98 \\ 376 \\ 295 \\ + 29 \\ \hline \end{array}$$

Multiply or divide.

5.
$$\begin{array}{r} 405 \\ \times 46 \\ \hline \end{array}$$

6. $1071 \div 17$

7. $235(81)$

8. $36 \overline{)9144}$

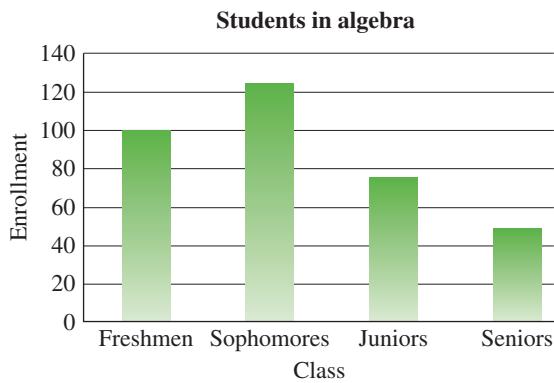
9. Find the value of 2^7 .

10. Multiply: 92×10^5

11. Simplify: $63 + 33 \times 2 \div 3 + 8$

12. Find the average, median, and mode of 35, 56, 72, 35, 18, 22, and 91.

Exercises 13–14. The graph shows the distribution of students taking algebra at a local high school.



13. Which class had the most students taking algebra?

14. How many seniors were taking algebra?

15. Draw a line graph to display the traffic count at the intersection of Elm and 3rd Avenues as given in the table.

Traffic Count at Elm and 3rd

Monday	Tuesday	Wednesday	Thursday	Friday
425	500	375	450	550

- 16.** Is 7215 divisible by 2, 3, or 5?
- 17.** Is 3060 divisible by 6, 9, or 10?
- 18.** List the first five multiples of 43.
- 19.** Is 504 a multiple of 24?
- 20.** Write 180 as a product of two factors in all possible ways.
- 21.** List all the factors of 210.
- 22.** Is 541 prime or composite?
- 23.** Is 303 prime or composite?
- 24.** Write the prime factorization of 1640.
- 25.** Find the least common multiple (LCM) of 16, 84, and 72.
- 26.** Change to a mixed number: $\frac{32}{5}$
- 27.** Change to an improper fraction: $12\frac{4}{7}$

Simplify completely.

28. $\frac{138}{150}$

29. $\frac{640}{760}$

Multiply or divide. Simplify completely.

30. $\frac{18}{35} \cdot \frac{25}{54}$

31. $\frac{56}{80} \cdot \frac{48}{49} \cdot \frac{35}{40}$

32. $\frac{24}{35} \div \frac{36}{45}$

33. $\frac{33}{34} \div \frac{88}{51}$

34. $14\frac{2}{3} \div 4\frac{5}{7}$

35. $9\frac{1}{7} \cdot 4\frac{1}{5}$

36. List the fractions from smallest to largest: $\frac{8}{17}, \frac{11}{21}, \frac{7}{15}, \frac{3}{5}$

Add and subtract. Simplify completely.

37. $\frac{12}{25} + \frac{7}{15} + \frac{1}{5}$

38. $\frac{21}{32} - \frac{7}{16}$

39. $\frac{13}{40} + \frac{11}{72}$

40. $\frac{34}{40} - \frac{18}{45}$

41. $3\frac{2}{5} + 7\frac{3}{4} + 1\frac{3}{10}$

42. $11\frac{8}{15} - 6\frac{11}{18}$

Perform the indicated operations. Simplify completely.

43. $\frac{3}{7} \cdot \frac{5}{6} + \frac{1}{7} \div \frac{8}{9}$

44. $\frac{2}{3} \left(\frac{1}{4} - \frac{1}{5} \right) + \frac{3}{4} \left(\frac{4}{5} - \frac{2}{5} \right)^2$

45. Find the average of $\frac{2}{3}, \frac{7}{10}, \frac{11}{15}, \frac{17}{20}$, and $\frac{4}{5}$.

- 46.** An article is priced to sell for \$108 at the Aquarium Gift Store. It is sale-priced at $\frac{1}{3}$ off. What is its sale price?

- 48.** At the end of one year, John has grown $\frac{5}{12}$ in. During the same time, his sister has grown $\frac{3}{8}$ in. Who has grown more? By how much?

- 47.** Jamie answers $\frac{7}{8}$ of the problems correctly on her Chapter 1 test. If there are 40 problems on the Chapter 3 test, how many must she get correct to answer the same fractional amount?

- 49.** A machinist needs a bar that is $1\frac{5}{8}$ in. thick. If he cuts it from a bar that is $1\frac{25}{32}$ in. thick, how much would be left over?

Write the word name.

50. 18.096

51. 0.000419

Write the place value name.

52. Three thousand seventy-five and three hundredths.

53. Nine hundred thousand and nine ten-thousandths.

Change each decimal to a fraction, and simplify if possible.

54. 0.55

55. 0.065

List the decimals from smallest to largest.

56. 0.34, 0.087, 0.29, 0.125, 0.031

57. 0.052, 0.63, 0.21, 0.101, 0.019

Add and subtract.

58.
$$\begin{array}{r} 456.023 \\ + 43.9 \\ + 103.045 \\ \hline + 9.32 \end{array}$$

59.
$$\begin{array}{r} 86.522 \\ - 10.777 \\ \hline \end{array}$$

60. $0.7042 - 0.349$

61. $16.45 + 8.467 + 124.009 + 0.95 + 5.832$

- 62.** Pete wrote checks in the amounts of \$34.95, \$16.75, \$234.50, \$132.60, and \$45.45. Pete has \$461.98 in his checking account. Does he have enough money to cover the checks?

- 63.** Jose goes shopping with \$124 in cash. He pays \$24.95 for a polo shirt, \$23.95 for a DVD, \$11.55 for lunch, and \$44.95 for a pair of tennis shoes. On the way home he buys \$16.50 worth of gas. How much money does he have left?

Multiply or divide.

64. 36.45
 $\times 15.7$

65. 0.0346
 $\times \underline{0.71}$

66. $3.7\overline{)32.005}$

67. $75.342 \div 0.38$ (Round to the nearest tenth.)

68. $46.789(10,000)$

69. $82.784 \div 1000$

Write in scientific notation.

70. 0.00476

71. 4600.7

- 72.** The home of Joe Tyler is assessed at \$287,000. The property tax rate in the area is \$1.414 per hundred dollars of assessment. Find the property tax that Joe pays, rounded to the nearest dollar.

- 74.** Ranee rents a backhoe for \$150 and \$34.50 a day for each day she keeps it over the initial rental period of 4 days. How much does it cost Ranee if she keeps the backhoe for 12 days?

- 73.** A 60-gallon drum of cleaning solvent at Five Star Cleaners is being used at the rate of 0.68 gallon per day. At this rate, how many days will the solvent last? Round to the nearest day.

Change the fraction or mixed number to a decimal.

75. $7\frac{12}{25}$

76. $\frac{27}{74}$ (Round to the nearest thousandth.)

Perform the indicated operations.

77. $0.4(0.6 - 0.34) + 5.5 - (3.2 + 1.18)$

78. $0.01(4.3)^2 + 7.05 \div 0.15 + 8.2$

Find the average, median, and mode of the group of numbers.

79. $14.45, 16.11, 13.05, 16.11, 20.7, 22.9$

80. $0.788, 0.142, 0.67, 1.04, 1.03$

- 81.** Andy buys school supplies for the start of the new semester. He buys three notebooks at \$3.25 each, seven pens at \$1.19 each, four packs of notepaper at \$0.98 each, and a calculator at \$74.80. How much does he spend?

- 82.** The camera system at the intersection of Hall and Cedar Hills Boulevard caught the following number of drivers going through a red light during 1 week: Monday, 56; Tuesday, 84; Wednesday, 102; Thursday, 92; Friday, 123; Saturday, 178; Sunday, 64. What was the average number of tickets issued per day? Round to the nearest tenth.

- 83.** Write as a ratio in simplified form: 9 in. to 2 ft **84.** Write as a rate and simplify: 99 lb to 44 ft

- 85.** Write as a unit rate, rounded to the nearest thousandth:
\$4.69 to 24 oz

True or false?

86. $\frac{12}{15} = \frac{32}{40}$

87. $\frac{7}{16} = \frac{12}{25}$

Solve.

88. $\frac{4}{24} = \frac{7}{a}$

89. $\frac{21}{4} = \frac{x}{12}$

Solve. Round to the nearest tenth.

90. $\frac{6}{17} = \frac{b}{23}$

91. $\frac{37}{90} = \frac{19}{c}$

Exercises 92–97. If a flagpole 60 ft tall casts a shadow of 25 ft, how tall is a flagpole that casts a shadow of 15 ft?

	First Pole	Second Pole
Height (ft)	(1)	(3)
Shadow (ft)	(2)	(4)

- 92.** What goes in box (1)?

- 93.** What goes in box (2)?

- 94.** What goes in box (3)?

- 95.** What goes in box (4)?

- 96.** What is the proportion for the problem?

- 97.** How tall is the second flagpole?

- 98.** Joanne saves \$106 a month from her take-home pay of \$3375. If her take-home pay increases to \$4050 per month, how much will she save at the same rate? Round to the nearest dollar.

- 99.** Greg sells men's clothing at the Emporium. If he sells \$1150 worth of clothes he earns \$69. At the same rate, how much does he make if he sells \$2610 worth of clothes?

- 100.** Dan buys a \$2500 worth of stock on Wednesday and sells it on Friday for \$2900. If Dan had invested \$6000 in the same stock on Wednesday, what would it have been worth on Friday?

Preparing for Tests

Testing usually causes the most anxiety for students. By studying more effectively, you can eliminate many of the causes of anxiety. But there are also other ways to prepare that will help relieve your fears.

If you are math anxious, you actually may study too much out of fear of failure and not allow enough time for resting and nurturing yourself. Every day, allow yourself some time to focus on your concerns, feelings, problems, or anything that might distract you when you try to study. Then, when these thoughts distract you, say to yourself, "I will not think about this now. I will later at ___ o'clock. Now I have to focus on math." If problems become unmanageable, make an appointment with a college counselor.

Nurturing is any activity that will help you recharge your energy. Choose an activity that makes you feel good, such as going for a walk, daydreaming, reading a favorite book, doing yard work, taking a bubble bath, or playing basketball.

Other ways to keep your body functioning effectively under stress are diet and exercise. Exercise is one of the most beneficial means of relieving stress. Try to eat healthy foods and drink plenty of water. Avoid caffeine, nicotine, drugs, alcohol, and "junk food."

Plan to have all your assignments finished two days before the test, if possible. The day before the test should be completely dedicated to reviewing and practicing for the test.

Many students can do the problems but cannot understand the instructions and vocabulary, so they do not know where to begin. Review any concepts that you have missed or any that you were unsure of or "guessed at."

When you feel comfortable with all of the concepts, you are ready to take the practice test at the end of the chapter. You should simulate the actual testing situation as much as possible. Have at hand all the tools you will use on the real test: sharpened pencils, eraser, and calculator (if your instructor allows). Give yourself the same amount of time as you'll be given on the actual test. Plan a time for your practice test when you can be sure there will be no interruptions. Work each problem slowly and carefully. Remember, if you make a mistake by rushing through a problem and have to do it over, it will take more time than doing the problem carefully in the first place.

After taking the practice test, go back and study topics referenced with the answers you missed or that you feel you do not understand. You should now know if you are ready for the test. If you have been studying effectively and did well on the practice test, you should be ready for the real test. You are prepared!

6



image 100/ Getty Images

SECTIONS

- 6.1** The Meaning of Percent
- 6.2** Changing Decimals to Percents and Percents to Decimals
- 6.3** Changing Fractions to Percents and Percents to Fractions
- 6.4** Fractions, Decimals, Percents: A Review
- 6.5** Solving Percent Problems
- 6.6** Applications of Percents
- 6.7** Sales Tax, Discounts, and Commissions
- 6.8** Interest on Loans

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PERCENT

APPLICATION

The price we pay for everyday items such as food and clothing is theoretically simple. The manufacturer of the item sets the price based on how much it costs to produce and adds a small profit. The manufacturer then sells the item to a retail store, which in turn marks it up and sells it to you, the consumer. But as you know, it is rarely as simple as that. The price you actually pay for an item also depends on the time of year, the availability of raw materials, the amount of competition among manufacturers of comparable items, the economic circumstances of the retailer, the geographic location of the retailer, and many other factors.

Group Discussion

Select a common item whose price is affected by the following factors:

1. Time of year
2. Economic circumstances of the retailer
3. Competition of comparable products

Discuss how the factor varies and how the price of the item is affected. For each factor, make a plausible bar graph that shows the change in price as the factor varies. (You may estimate price levels.)

6.1 The Meaning of Percent

OBJECTIVE

Write a percent to express a comparison of two numbers.

VOCABULARY

When ratios are used to compare numbers, the denominator is called the **base unit**. In comparing 70 to 100 (as the ratio $\frac{70}{100}$), 100 is the base unit.

The **percent comparison**, or just the **percent**, is a ratio with a base unit of 100.

The percent $\frac{70}{100} = (70)\left(\frac{1}{100}\right)$ is usually written 70%. The symbol % is read “percent,” and $\% = \frac{1}{100} = 0.01$.

How & Why

OBJECTIVE

Write a percent to express a comparison of two numbers.

The word *percent* means “by the hundred.” It is from the Roman word *percentum*. In Rome, taxes were collected by the hundred. For example, if you had 100 cattle, the tax collector might take 14 of them to pay your taxes. Hence, 14 per 100, or 14 percent, would be the tax rate.

Look at Figure 6.1 to see an illustration of the concept of “by the hundred.” The base unit is 100, and 34 of the 100 parts are shaded. The ratio of shaded parts to total parts is $\frac{34}{100} = 34 \times \left(\frac{1}{100}\right) = 34\%$. We say that 34% of the unit is shaded.

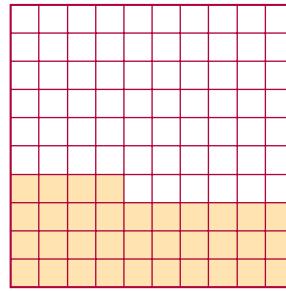


Figure 6.1

Figure 6.1 also illustrates that if the numerator is smaller than the denominator, then not all of the base unit is shaded, and hence the comparison is less than 100%. If the numerator equals the denominator, the entire unit is shaded and the comparison is 100%. If the numerator is larger than the denominator, more than one entire unit is shaded, and the comparison is more than 100%.

Any ratio of two numbers can be converted to a percent, even when the base unit is not 100. Compare 11 to 20. The ratio is $\frac{11}{20}$. Now find the equivalent ratio with a denominator of 100.

$$\frac{11}{20} = \frac{55}{100} = 55 \cdot \frac{1}{100} = 55\%.$$

If the equivalent ratio with a denominator of 100 cannot be found easily, solve as a proportion. See Example F.

To find the percent comparison of two numbers

1. Write the ratio of the first number to the base number.
2. Find the equivalent ratio with denominator 100.
3. $\frac{\text{numerator}}{100} = \text{numerator} \cdot \frac{1}{100} = \text{numerator \%}$

Examples A–C

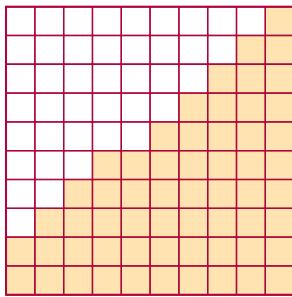


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DIRECTIONS: Write the percent of each region that is shaded.

STRATEGY: (1) Count the number of parts in each unit. (2) Count the number of parts that are shaded. (3) Write the ratio of these as a fraction and build the fraction to a denominator of 100. (4) Write the percent using the numerator in step 3.

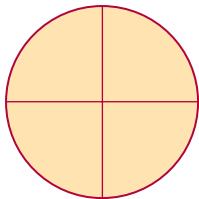
- A. What percent of the unit is shaded?



1. 100 parts in the region
2. 59 parts are shaded.
3. $\frac{59}{100}$
4. 59%

So 59% of the region is shaded.

- B. What percent of the region is shaded?

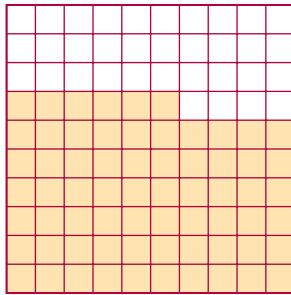


1. 4 parts in the region
2. 4 parts are shaded.
3. $\frac{4}{4} = \frac{100}{100}$
4. 100%

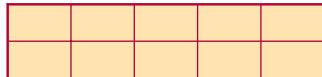
So 100% of the region is shaded.

Warm-Ups A–C

- A. What percent of the unit is shaded?



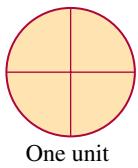
- B. What percent of the region is shaded?



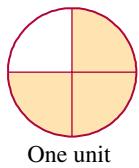
Answers to Warm-Ups

- A. 66% B. 100%

- C. What percent of the region is shaded?

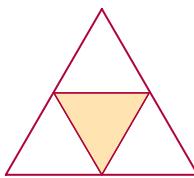
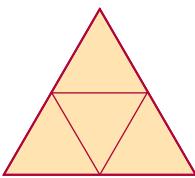


One unit



One unit

- C. What percent of the region is shaded?



1. 4 parts in each unit
2. 5 parts are shaded
3. $\frac{5}{4} = \frac{125}{100}$ Write as a fraction with a denominator of 100.
4. 125% Write as a percent.

So 125% of a unit in the region is shaded.

Warm-Ups D–H

- D. At the last soccer match of the season, of the first 100 tickets sold, 81 were student tickets. What percent were student tickets?

- E. Write the ratio of 65 to 50 as a percent.

- F. Write the ratio of 10 to 12 as a percent.

Answers to Warm-Ups

- C. 175%
D. The percent of student tickets was 81%.
E. 130% F. $83\frac{1}{3}\%$

Examples D–H BASIC Mathematics Now™

DIRECTIONS: Write the percent for the comparison.

STRATEGY: Write the comparison in fraction form. Build the fraction to hundredths or solve a proportion and write the percent using the numerator.

- D. At a football game, 32 women are among the first 100 fans to enter. What percent of the first 100 fans are women?

$$\frac{32}{100} = 32 \cdot \frac{1}{100} = 32\% \quad \text{The comparison of women to first 100 fans is 32 to 100. Write the fraction and change to a percent.}$$

So 32% of the first 100 fans are women.

- E. Write the ratio of 33 to 25 as a percent.

$$\begin{aligned} \frac{33}{25} &= \frac{132}{100} && \text{Write the ratio and build this fraction to one, with a denominator of 100.} \\ &= 132 \cdot \frac{1}{100} \\ &= 132\% && \text{Change to a percent.} \end{aligned}$$

So 33 is 132% of 25.

- F. Write the ratio of 15 to 21 as a percent.

$$\begin{aligned} \frac{15}{21} &= \frac{R}{100} && \text{Because we cannot build the fraction to one with a denominator of 100 using whole numbers, we write a proportion to find the percent.} \\ 15(100) &= 21R && \text{Cross multiply.} \end{aligned}$$

$$\frac{1500}{21} = R$$

$$71\frac{3}{7} = R$$

$$\text{So, } \frac{15}{21} = \frac{71\frac{3}{7}}{100}$$

$$\begin{aligned} &= 71\frac{3}{7} \cdot \frac{1}{100} \\ &= 71\frac{3}{7}\% \end{aligned}$$

So 15 is $71\frac{3}{7}\%$ of 21.

CALCULATOR EXAMPLE 

- G. Compare 126 to 1120 as a percent.

$$\frac{126}{1120} = \frac{R}{100}$$

Write as a proportion.

$$1120R = 126(100)$$

Solve.

$$R = 126(100) \div 1120$$

Evaluate using a calculator.

$$R = 11.25$$

So 126 is 11.25% of 1120.

- H. During a campaign to lose weight, the 180 participants lost a total of 4158 lb. If they weighed collectively 37,800 lb before the campaign, what percent of their weight was lost?

$$\frac{4158}{37,800} = \frac{11}{100}$$

Write the ratio comparison and simplify.

$$= 11 \cdot \frac{1}{100}$$

$$= 11\%$$

So 11% of the total weight of the 180 dieters was lost during the campaign. 

- G. Compare 540 to 1440 as a percent.

- H. Of 700 salmon caught during the Florence Salmon Derby, 490 were hatchery raised. What percent of the fish were hatchery raised?

Answers to Warm-Ups

G. 37.5%

H. The percent of fish that were hatchery raised was 70%.

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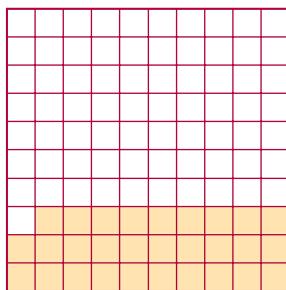
Exercises 6.1

OBJECTIVE

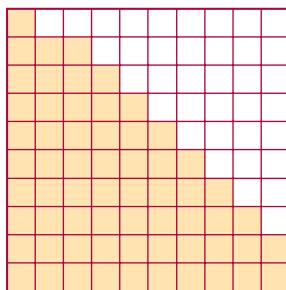
Write a percent to express a comparison of two numbers.

A What percent of each of the following regions is shaded?

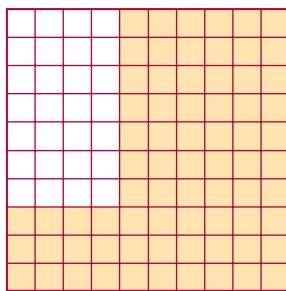
1.



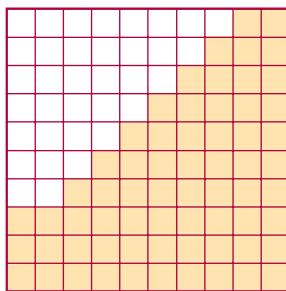
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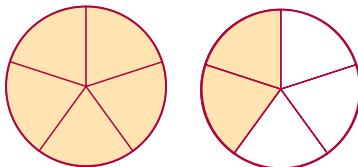
3.



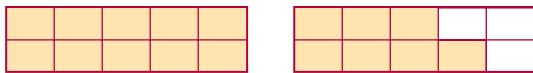
4.



5.



6.



Write an exact percent for these comparisons.

7. 62 of 100

8. 52 per 100

9. 17 to 100

10. 37 to 100

11. 28 per 50

12. 17 per 50

13. 11 of 25

14. 21 to 25

15. 11 per 20

16. 13 per 20

B

17. 19 to 10

18. 450 to 120

19. 313 of 313

20. 92 to 92

21. 28 to 16

22. 44 to 16

23. 85 to 200

24. 65 to 200

25. 9 per 15

26. 83 per 500

27. 70 per 80

28. 98 per 80

29. 35 to 42

30. 29 to 30

31. 76 to 114

32. 14 to 15

C

33. The fact that 12% of all people are blond indicates that _____ out of 100 people are blond.

34. In a recent election there was a 73% turnout of registered voters. This indicates that _____ out of 100 registered voters turned out to vote.

35. In a recent mail-in election, 82 out of every 100 eligible voters cast their ballots. What percent of the eligible voters exercised their right to vote?

36. Of the people who use mouthwash daily, 63 out of 100 report fewer cavities. Of every 100 people who report, what percent do not report fewer cavities?

Write an exact percent for these comparisons; use fractions when necessary.

- 37.** 105 to 336 **38.** 204 to 480 **39.** 165 to 25

40. 213 to 15 **41.** 319 to 600 **42.** 64 to 900

43. If a luxury tax is 11 cents per dollar, what percent is this?

45. A bank pays \$3.75 interest per year for every \$100 in savings. What is the annual interest rate?

47. Last year, Mr. and Mrs. Johanson were informed that the property tax rate on their home was \$1.48 per \$100 of the house's assessed value. What percent is the tax rate?

49. In 2002 in the United States, the number of juveniles (under age 18) arrested for property crimes was in decline. In that year, 1510 juveniles out of every 100,000 were arrested for property crimes. What percent of the juvenile population was arrested for property crimes in 2002?

51. In the year 2002, 14 out of every 25 people earning a bachelor's degree in the United States were female. What percent of the people earning bachelor's degrees in 2002 were women?

44. For every \$100 spent on gasoline in Nebraska, the state receives \$9.80 tax. What percent of the price of gasoline is the state tax?

46. James has \$500 in his savings account. Of that amount, \$35 is interest that was paid to him. What percent of the total amount is the interest?

48. Beginning in the early 1970s, women in the armed forces were treated the same as men with respect to training, pay, and rank. As a result, the number of women in the armed forces nearly tripled over the levels of the late 1960s. In the year 2001, about 7.5 out of every 50 officers were women. Express this as a percent.

50. In 2002, what percent of the juvenile population was not arrested for property crimes? (See Exercise 49.)

52. In the year 2002, what percent of the people earning bachelor's degrees were men? (See Exercise 51.)

Exercises 53–56 are related to the chapter application.

53. Carol spends \$82 on a new outfit. If she has \$100, what percent of her money does she spend on the outfit?
54. A graphing calculator originally priced at \$100 is on sale for \$78. What is the percent of discount? (Discount is the difference between the original price and the sale price.)
55. Mickie bought a TV and makes monthly payments on it. Last year, she paid a total of \$900. Of the total that she paid, \$180 was interest. What percent of the total was interest?
56. Pablo buys a suit that was originally priced at \$100. He buys it for 35% off the original price. What does he pay for the suit?

STATE YOUR UNDERSTANDING

57. What is a percent? How is it related to fractions and decimals?
58. Explain the difference in meaning of the symbols 25% and 125%. In your explanation, use diagrams to illustrate the meanings. Contrast similarities and differences in the diagrams.

CHALLENGE

59. Write the ratio of 109 to 500 as a fraction and as a percent.
60. Write the ratio of 514 to 800 as a fraction and as a percent.

GROUP WORK

61. Have the members of your group use the resource center to find some background on the percent symbol (%). Divide the task so that one member looks in a large dictionary, some look in different encyclopedias, others look in other mathematics books, and some check the Internet. Together, make a short report to the rest of the class on your findings.

MAINTAIN YOUR SKILLS*Multiply.*

62. $7.83(100)$

63. 47.335×100

64. $0.00578(1000)$

65. 207.8×1000

66. $12.45 \div 100$

67. $0.0672 \div 1000$

68. $1743 \div 10^4$

69. $0.9003 \div 10^2$

- 70.** Bill goes to the store with \$25. He uses his calculator to keep track of the money he is spending. He decides that he could make the following purchases. Is he correct?

Article	Cost
1 loaf of bread	\$3.29
2 bottles of V-8 juice	\$3.39 each
2 boxes of crackers	\$2.69 each
1 package of cheddar cheese	\$3.99
2 cartons of orange juice	\$2.00 each

- 71.** Ms. Henderson earns \$23.85 per hour and works the following hours during 1 month. How much are her monthly earnings?

Week	Hours
1	35
2	30.25
3	25
4	36.75
5	6

6.2 Changing Decimals to Percents and Percents to Decimals

How & Why

OBJECTIVE 1

Write a given decimal as a percent.

In multiplication, where one factor is $\frac{1}{100}$, the indicated multiplication can be read as a percent; that is, $75\left(\frac{1}{100}\right) = 75\%$, $0.8\left(\frac{1}{100}\right) = 0.8\%$, and $\frac{3}{4}\left(\frac{1}{100}\right) = \frac{3}{4}\%$.

To write a number as a percent, multiply by $100 \cdot \frac{1}{100}$, a name for 1. This is shown in Table 6.1.

OBJECTIVES

1. Write a given decimal as a percent.
2. Write a given percent as a decimal.

Table 6.1 Change a Decimal to a Percent

Number	Multiply by 1 $100\left(\frac{1}{100}\right) = 1$	Multiply by 100	Rename as a Percent
0.74	$0.74(100)\left(\frac{1}{100}\right)$	$\cancel{74} \cdot \left(\frac{1}{100}\right)$	74%
0.6	$0.6(100)\left(\frac{1}{100}\right)$	$\cancel{60} \cdot \left(\frac{1}{100}\right)$	60%
4	$4(100)\left(\frac{1}{100}\right)$	$400 \cdot \left(\frac{1}{100}\right)$	400%

In each case the decimal point is moved two places to the right and the percent symbol (%) is inserted.

To change a decimal to a percent

1. Move the decimal point two places to the right. (Write zeros on the right if necessary.)
2. Write the percent symbol (%) on the right.

Examples A–F



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DIRECTIONS: Change the decimal to a percent.

STRATEGY: Move the decimal point two places to the right and write the percent sign on the right.

- A. Write 0.73 as a percent.

$0.73 = 73\%$ Move the decimal point two places to the right and write the percent symbol on the right.

So $0.73 = 73\%$.

- B. Change 0.09 to a percent.

$0.09 = 009\% = 9\%$ Since the zeros are to the left of 9 we can drop them.

So $0.09 = 9\%$.

Warm-Ups A–F

- A. Write 0.27 as a percent.

- B. Change 0.03 to a percent.

Answers to Warm-Ups

- A. 27% B. 3%

- C. Change 0.0023 to a percent.
- D. Write 7 as a percent.
- E. Change $0.29\bar{6}$ to a percent.
- F. The tax code lists the tax rate on a zone 3 lot at 0.031. What is the tax rate expressed as a percent?
- C. Change 0.0011 to a percent.
 $0.0011 = 000.11\% = 0.11\%$ This is eleven hundredths of one percent.
So $0.0011 = 0.11\%$.
- D. Write 14 as a percent.
 $14 = 14.00 = 1400\%$ Insert two zeros on the right so we can move two decimal places. Fourteen hundred percent is 14 times 100%.
So $14 = 1400\%$.
- E. Change $0.84\bar{3}$ to a percent.
 $0.84\bar{3} = 84.\bar{3}\% = 84\frac{1}{3}\%$ The repeating decimal $0.\bar{3} = \frac{1}{3}$.
So $0.84\bar{3} = 84.\bar{3}\%, 84\frac{1}{3}\%$.
- F. The tax rate on a building lot is given as 0.027. What is the tax rate expressed as a percent?
 $0.027 = 002.7\% = 2.7\%$
So the tax rate expressed as a percent is 2.7% .

How & Why

OBJECTIVE 2

Write a given percent as a decimal.

The percent symbol indicates multiplication by $\frac{1}{100}$, so

$$55\% = 55 \cdot \frac{1}{100} = \frac{55}{100} = 55 \div 100$$

As we learned in Section 4.5, dividing a number by 100 is done by moving the decimal point two places to the left.

$$55\% = 55 \div 100 = 0.55$$

To change a percent to a decimal

- Move the decimal point two places to the left. (Write zeros on the left if necessary.)
- Drop the percent symbol (%).

Answers to Warm-Ups

- C. 0.23% D. 700%
E. $29.\bar{6}\%$ or $29\frac{2}{3}\%$
F. The tax rate is 3.1%.

Examples G–K



DIRECTIONS: Change the percent to a decimal.

STRATEGY: Move the decimal point two places to the left and drop the percent symbol.

G. Change 28.7% to a decimal.

$$28.7\% = 0.287 \quad \text{Move the decimal point two places left. Drop the percent symbol.}$$

$$\text{So } 28.7\% = 0.287.$$

H. Change 561% to a decimal.

$$561\% = 5.61 \quad \text{A value larger than 100% becomes a mixed number or a whole number.}$$

$$\text{So } 561\% = 5.61.$$

I. Write $43\frac{7}{10}\%$ as a decimal.

$$43\frac{7}{10}\% = 43.7\% \quad \text{Change the fraction to a decimal.}$$
$$= 0.437 \quad \text{Change the percent to a decimal.}$$

$$\text{So } 43\frac{7}{10}\% = 0.437.$$

J. Change $33\frac{7}{18}\%$ to a decimal. Round to the nearest thousandth.

$$33\frac{7}{18}\% = 33.\bar{3}8\% \quad \text{By division, } \frac{7}{18} = 0.\bar{3}8.$$
$$= 0.33\bar{3}8 \quad \text{Change to a decimal.}$$
$$\approx 0.334 \quad \text{Round to the nearest thousandth.}$$

$$\text{So } 33\frac{7}{18}\% \approx 0.334.$$

K. When ordering fresh vegetables, a grocer orders 9.3% more than is needed to allow for spoilage. What decimal is entered into the computer to calculate the amount of extra vegetables to be added to the order?

$$9.3\% = 0.093 \quad \text{Change the percent to a decimal.}$$

So the grocer will enter 0.093 in the computer.

Warm-Ups G–K

G. Change 48.3% to a decimal.

H. Change 833% to a decimal.

I. Write $72\frac{2}{5}\%$ as a decimal.

J. Change $21\frac{7}{13}\%$ to a decimal.

Round to the nearest thousandth.

K. When ordering cement, a contractor orders 3.1% more than is needed to allow for waste. What decimal will she enter into the computer to calculate the extra amount to be added to the order?

Answers to Warm-Ups

G. 0.483 **H.** 8.33

I. 0.724 **J.** 0.215

K. The contractor will enter 0.031 in the computer.

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Exercises 6.2

OBJECTIVE 1

Write a given decimal as a percent.

A Write each decimal as a percent.

- | | | | |
|------------------|------------------|-------------------|-------------------|
| 1. 0.47 | 2. 0.83 | 3. 7.53 | 4. 8.64 |
| 5. 0.08 | 6. 0.03 | 7. 1.39 | 8. 6.98 |
| 9. 19 | 10. 21 | 11. 0.0065 | 12. 0.0017 |
| 13. 0.952 | 14. 0.376 | 15. 0.517 | 16. 0.712 |

B

- | | | | |
|----------------------------|---------------------------|--------------------|--------------------|
| 17. 0.0731 | 18. 0.0716 | 19. 70 | 20. 62 |
| 21. 17.81 | 22. 4.311 | 23. 0.00029 | 24. 0.00471 |
| 25. 7.1 | 26. 2.39 | 27. 0.3954 | 28. 0.9708 |
| 29. 0.811 $\bar{6}$ | 30. 0.04 $\bar{3}$ | 31. 0.7045 | 32. 0.61609 |

OBJECTIVE 2

Write a given percent as a decimal.

A Write each of the following as a decimal.

- | | | | |
|-------------------|-------------------|--------------------|--------------------|
| 33. 37% | 34. 47% | 35. 73% | 36. 83% |
| 37. 6.41% | 38. 8.12% | 39. 908% | 40. 444% |
| 41. 327.8% | 42. 560.7% | 43. 0.0062% | 44. 0.0048% |
| 45. 0.19% | 46. 0.672% | 47. 3940% | 48. 8643% |

B

- | | | | |
|----------------------------|-----------------------------|------------------------------|------------------------------|
| 49. 0.037% | 50. 0.0582% | 51. 100% | 52. 700% |
| 53. 535% | 54. 363% | 55. $\frac{1}{2}\%$ | 56. $\frac{1}{4}\%$ |
| 57. $\frac{7}{8}\%$ | 58. $\frac{4}{5}\%$ | 59. $73\frac{3}{4}\%$ | 60. $21\frac{2}{5}\%$ |
| 61. $\frac{3}{8}\%$ | 62. $\frac{7}{16}\%$ | 63. 413.773% | 64. 491.07% |

C

- 65.** If the tax rate on a person's income in Colorado in 2005 was 0.0463, what was the rate expressed as a percent?
- 67.** A Girl Scout sold 0.36 of her quota of cookies on the first day of the sale. What percent of her cookies did she sell on the first day?
- 69.** Employees just settled their new contract and got a 3.15% raise over the next 2 years. Express this as a decimal.
- 71.** Interest rates are expressed as percents. The Credit Union charged 6.34% interest on new 48-month auto loans in 2005. What decimal will they use to compute the interest?
- 66.** A 2-year nursing program has a completion rate of 0.734. What is the rate as a percent?
- 68.** The sales tax in Illinois in 2005 was 0.0625. Express this as a percent.
- 70.** The CFO of an electronics firm adds 5.35% to the budget as a contingency fund. What decimal part is this?
- 72.** What decimal is used to compute the interest on a mortgage that has an interest rate of 5.34%?

Change to a decimal rounded to the nearest thousandth.

73. $\frac{6}{11}\%$

74. $62\frac{4}{15}\%$

75. $57\frac{9}{13}\%$

76. $\frac{47}{9}\%$

- 77.** In industrialized countries, 60% of the river pollution is due to agricultural runoff. Change this to a decimal.
- 79.** The Westview High School golf team won 0.875 of their matches. Write this as a percent.
- 81.** At the end of the 2004 season, Barry Bonds had hit 703 career home runs. This was about 0.931 of Hank Aaron's career record of 755 home runs. Express Barry Bonds's home runs as a percent of Hank Aaron's home runs.
- 83.** The Bureau of Labor Statistics expects that in the decade from 1996 to 2006 there will be about 81,000 new physical therapy jobs. The total number of physical therapy jobs will be 1.704 times the number of jobs in 1996. Express this as a percent.
- 85.** Find the interest being paid to savings accounts at three different financial institutions in your area. Express these as decimals.
- 78.** Recycling aluminum cans consumes 95% less energy than smelting new stocks of metal. Change this to a decimal.
- 80.** A WNBA basketball player makes 0.642 of her free throws. Express this as a percent.
- 82.** The highest batting average in the American League for 2004 was 0.372 by Ichiro Suzuki of the Seattle Mariners. The National League's batting leader in 2004 was Barry Bonds of the San Francisco Giants, with 0.362. Express these as percents.
- 84.** Find today's interest rates for home mortgages for 15- and 30-year fixed-rate loans. Express these as decimals.
- 86.** One yard is about 91.4% of a meter. Express this as a decimal.

- 87.** The Moscow subway system has the largest number of riders of any subway system in the world. It has 115.3% of the riders of the Tokyo subway system, the next largest system. Express this as a decimal.
- 89.** Pluto is the smallest planet in our solar system, with a diameter that is about 27.4% of the diameter of Earth. Express this as a decimal.
- 91.** The amount of Social Security paid by employees is found by multiplying the gross wages by 0.062. The Medicare payment is found by multiplying the gross wages by 0.029. Express the sum of these amounts as a percent.
- 88.** The Petronas Towers in Kuala Lumpur, Malaysia, are the tallest buildings in the world. They are about 102% of the height of the Sears Tower in Chicago, the next tallest building. Express this as a decimal.
- 90.** A nautical mile is about 1.15 times the length of a statute (land) mile. Express this as a percent.
- 92.** Cholesterol levels in Americans have dropped from a rate of 0.26 in 1981 to a rate of 0.2 in 1993. Express the difference as a percent.

Exercises 93–95 relate to the chapter application.

- 93.** The sale price of a can of beans is 0.89 of what it was before the sale. Express this as a percent. What “percent off” will the store advertise?
- 95.** A box of cereal claims to contain 125% of what it used to contain. Express this as a decimal.
- 94.** Mary spends 0.285 of her monthly income on groceries. What percent of her monthly income is spent on groceries?

STATE YOUR UNDERSTANDING

- 96.** Explain how the decimal form and the percent form of a number are related. Give an example of each form.
- 97.** When changing a percent to a decimal, how can you tell when the decimal will be greater than 1?

CHALLENGE

Write as percents.

- 98.** 0.0004507 **99.** 18,000

Write as percents without using repeating decimals.

- 100.** 0.024 and $0.02\bar{4}$ **101.** 0.425 and $0.42\bar{5}$

- 102.** Change $11\frac{4}{17}\%$ to a decimal rounded to the nearest tenth and the nearest thousandth.

- 103.** Change $56\frac{11}{12}\%$ to a decimal rounded to the nearest tenth and the nearest thousandth.

GROUP WORK

- 104.** Baseball batting averages are written as decimals. A batter with an average of .238 has hit an average of 238 times out of 1000 times at bat (.238). Find the batting averages of the top five players in the American and National Leagues for the past 5 years. Express these averages as percents.
- 105.** Research the major causes of the greenhouse effect. Find out which substances cause the greenhouse effect and the percent contributed by each. Write these percents in decimal form. Discuss ways to reduce the greenhouse effect in class and write group reports on your findings.

MAINTAIN YOUR SKILLS

Change to a decimal.

106. $\frac{7}{8}$

107. $\frac{9}{64}$

108. $\frac{19}{16}$

109. $\frac{117}{65}$

Change to a fraction and simplify.

110. 0.715

111. 0.1025

112. Round to the nearest thousandth: 3.87264

113. Round to the nearest ten-thousand: 345,891.62479

- 114.** In 1 week, Greg earns \$245. His deductions (income tax, Social Security, and so on) total \$38.45. What is his “take-home” pay?

- 115.** The cost of gasoline is reduced from \$0.695 per liter to \$0.629 per liter. How much money is saved on an automobile trip that requires 340 liters?

6.3 Changing Fractions to Percents and Percents to Fractions

How & Why

OBJECTIVE 1

Change a fraction or mixed number to a percent.

We already know how to change fractions to decimals and decimals to percents. We combine the two ideas to change a fraction to a percent.

To change a fraction or mixed number to a percent

1. Change to a decimal. The decimal is rounded or carried out as directed.
2. Change the decimal to a percent.

Unless directed to round, the division is completed or else the quotient is written as a repeating decimal.

Examples A–G



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DIRECTIONS: Change the fraction or mixed number to a percent.

STRATEGY: Change the number to a decimal and then to a percent.

- A. Change $\frac{5}{8}$ to a percent.

$$\begin{aligned}\frac{5}{8} &= 0.625 && \text{Divide } 5 \text{ by } 8 \text{ to change the fraction to a decimal.} \\ &= 62.5\% && \text{Change the decimal to a percent.}\end{aligned}$$

$$\text{So } \frac{5}{8} = 62.5\%.$$

- B. Write $\frac{11}{16}$ as a percent.

$$\begin{aligned}\frac{11}{16} &= 0.6875 && \text{Change to a decimal.} \\ &= 68.75\%\end{aligned}$$

$$\text{So } \frac{11}{16} = 68.75\%.$$

- C. Change $\frac{5}{24}$ to a percent.

$$\begin{aligned}\frac{5}{24} &= 0.208\bar{3} && \text{Write } \frac{5}{24} \text{ as a repeating decimal.} \\ &= 20.8\bar{3}\% && \text{Change to a percent.} \\ &= 20\frac{5}{6}\% && \text{The repeating decimal } 0.\bar{83} = \frac{5}{6}.\end{aligned}$$

$$\text{So } \frac{5}{24} = 20.8\bar{3}\%, \text{ or } 20\frac{5}{6}\%.$$

OBJECTIVES

1. Change a fraction or mixed number to a percent.
2. Change percents to fractions or mixed numbers.

Warm-Ups A–G

- A. Change $\frac{13}{20}$ to a percent.

- B. Write $\frac{3}{8}$ as a percent.

- C. Change $\frac{8}{11}$ to a percent.

Answers to Warm-Ups

A. 65% B. 37.5%

C. 72. $\bar{72}$ %, or $72\frac{8}{11}\%$

D. Write $1\frac{12}{25}$ as a percent.

E. Change $\frac{2}{17}$ to a percent.

Round to the nearest tenth of a percent.

D. Write $2\frac{7}{20}$ as a percent.

$$2\frac{7}{20} = 2.35 \\ = 235\%$$

$$\text{So } 2\frac{7}{20} = 235\%.$$

E. Change $\frac{5}{13}$ to a percent. Round to the nearest tenth of a percent.

CAUTION

One tenth of a percent is a thousandth; that is,

$$\frac{1}{10} \text{ of } \frac{1}{100} = \frac{1}{10} \cdot \frac{1}{100} = \frac{1}{1000} = 0.001.$$

STRATEGY: To write the percent rounded to the nearest tenth of a percent, we need to change the fraction to a decimal rounded to the nearest thousandth (that is, we round to the third decimal place).

$$\frac{5}{13} \approx 0.385 \quad \text{Write as a decimal rounded to the nearest thousandth.} \\ \approx 38.5\%$$

$$\text{So } \frac{5}{13} \approx 38.5\%.$$

CALCULATOR EXAMPLE

F. Write $5\frac{79}{230}$ as a decimal rounded to the nearest tenth of a percent.

G. An eight-cylinder motor has only seven of its cylinders firing. What percent of the cylinders are firing?

F. Write $3\frac{247}{382}$ as a percent rounded to the nearest tenth of a percent.

$$\frac{247}{382} \approx 0.6465968 \quad \text{First, convert the fraction to a decimal.}$$

$$3\frac{247}{382} \approx 3.6465968 \quad \text{Add the whole number.}$$

$$\approx 3.647 \quad \text{Round to the nearest thousandth.} \\ \approx 364.7\% \quad \text{Change to percent.}$$

$$\text{So } 3\frac{247}{382} \approx 364.7\%.$$

G. A motor that needs repair is only turning $\frac{9}{16}$ the number of revolutions per minute that is normal. What percent of the normal rate is this?

$$\frac{9}{16} = 0.5625 \\ = 56.25\%$$

So the motor is turning at 56.25% of its normal rate.

Answers to Warm-Ups

D. 148% E. 11.8% F. 534.3%

G. The percent of the cylinders firing is 87.5%.

How & Why

OBJECTIVE 2

Change percents to fractions or mixed numbers.

The expression 65% is equal to $65 \times \frac{1}{100}$. This gives a very efficient method for changing a percent to a fraction. See Example H.

To change a percent to a fraction or a mixed number

1. Replace the percent symbol (%) with the fraction $\left(\frac{1}{100}\right)$.
2. If necessary, rewrite the other factor as a fraction.
3. Multiply and simplify.

Examples H–L



DIRECTIONS: Change the percent to a fraction or mixed number.

STRATEGY: Change the percent symbol to the fraction $\frac{1}{100}$ and multiply.

H. Change 35% to a fraction.

$$\begin{aligned}35\% &= 35 \cdot \frac{1}{100} && \text{Replace the percent symbol (\%)} \text{ with } \frac{1}{100}. \\&= \frac{35}{100} && \text{Multiply.} \\&= \frac{7}{20} && \text{Simplify.}\end{aligned}$$

CAUTION

You need to multiply by $\frac{1}{100}$, not just write it down.

So $35\% = \frac{7}{20}$.

I. Change 436% to a mixed number.

$$\begin{aligned}436\% &= 436 \cdot \frac{1}{100} && \% = \frac{1}{100} \\&= \frac{436}{100} && \text{Multiply.} \\&= \frac{109}{25} && \text{Simplify.} \\&= 4\frac{9}{25} && \text{Write as a mixed number.}\end{aligned}$$

So $436\% = 4\frac{9}{25}$.

Warm-Ups H–L

H. Change 45% to a fraction.

I. Change 515% to a mixed number.

Answers to Warm-Ups

H. $\frac{9}{20}$ **I.** $5\frac{3}{20}$

J. Change $5\frac{5}{6}\%$ to a fraction.

J. Change $15\frac{1}{3}\%$ to a fraction.

$$\begin{aligned}15\frac{1}{3}\% &= 15\frac{1}{3} \cdot \frac{1}{100} \\&= \frac{46}{3} \cdot \frac{1}{100} \\&= \frac{46}{300} \\&= \frac{23}{150}\end{aligned}$$

$$\text{So } 15\frac{1}{3}\% = \frac{23}{150}.$$

K. Change 25.4% to a fraction.

$$\begin{aligned}16.7\% &= 0.167 \\&= \frac{167}{1000}\end{aligned}$$

$$\text{So } 16.7\% = \frac{167}{1000}$$

L. Greg scores 88% on a math test. What fraction of the questions does he get incorrect?

L. A biological study shows that spraying a forest for gypsy moths is 92% successful. What fraction of the moths survive the spraying?

STRATEGY: Subtract the 92% from 100% to find the percent of the moths that survived. Then change the percent that survive to a fraction.

$$100\% - 92\% = 8\%$$

$$\begin{aligned}8\% &= 8 \cdot \frac{1}{100} \\&= \frac{8}{100} \\&= \frac{2}{25}\end{aligned}$$

So $\frac{2}{25}$, or 2 out of 25 gypsy moths, survived the spraying.

Answers to Warm-Ups

J. $\frac{7}{120}$ K. $\frac{127}{500}$

L. Greg gets $\frac{3}{25}$ of the questions incorrect.

Exercises 6.3

OBJECTIVE 1

Change a fraction or mixed number to a percent.

A Change each fraction to a percent.

1. $\frac{67}{100}$

2. $\frac{37}{100}$

3. $\frac{37}{50}$

4. $\frac{8}{10}$

5. $\frac{17}{20}$

6. $\frac{22}{25}$

7. $\frac{1}{2}$

8. $\frac{3}{5}$

9. $\frac{11}{25}$

10. $\frac{9}{50}$

11. $\frac{13}{10}$

12. $\frac{27}{20}$

13. $\frac{11}{8}$

14. $\frac{21}{16}$

15. $\frac{39}{1000}$

16. $\frac{119}{1000}$

B Change each fraction or mixed number to a percent.

17. $4\frac{3}{5}$

18. $6\frac{1}{4}$

19. $\frac{2}{3}$

20. $\frac{5}{6}$

21. $\frac{29}{6}$

22. $\frac{25}{3}$

23. $7\frac{5}{12}$

24. $8\frac{11}{12}$

Change each fraction or mixed number to a percent. Round to the nearest tenth of a percent.

25. $\frac{15}{17}$

26. $\frac{17}{35}$

27. $\frac{5}{9}$

28. $\frac{7}{9}$

29. $\frac{7}{13}$

30. $\frac{13}{23}$

31. $32\frac{11}{29}$

32. $56\frac{11}{15}$

OBJECTIVE 2

Change percents to fractions or mixed numbers.

A Change each of the following percents to fractions or mixed numbers.

33. 12%

34. 20%

35. 65%

36. 15%

37. 130%

38. 180%

39. 700%

40. 300%

41. 84%

42. 68%

43. 75%

44. 90%

45. 45%

46. 67%

47. 150%

48. 225%

B

49. 45.5%

50. 24.4%

51. 6.8%

52. 3.5%

53. 60.5%

54. 16.8%

55. $\frac{1}{4}\%$

56. $\frac{5}{7}\%$

57. $7\frac{1}{2}\%$

58. $3\frac{3}{4}\%$

59. $\frac{4}{9}\%$

60. $\frac{5}{11}\%$

61. $52\frac{1}{2}\%$

62. $28\frac{3}{4}\%$

63. $331\frac{2}{3}\%$

64. $243\frac{1}{3}\%$

C

- 65.** Kobe Bryant made 17 out of 20 free throw attempts in one game. What percent of the free throws did he make?
- 67.** In the 2004 presidential election, President George W. Bush received 62,040,606 votes out of the 122,300,696 votes cast for president. What percent of the vote did he receive, to the nearest tenth of a percent?

- 69.** George and Ethel paid 18% of their annual income in taxes last year. What fractional part of their income went to taxes?

- 71.** Carmelo Anthony made 84% of his shots during an NBA game. What fractional part of his shots did Carmelo make?

- 66.** Maureen gets 19 problems correct on a 25-problem test. What percent is correct?

- 68.** In a supermarket, 2 eggs out of 11 dozen are lost because of cracks. What percent of the eggs must be discarded, to the nearest tenth of a percent?

- 70.** Ms. Nyuen was awarded scholarships that will pay for 56% of her college tuition. What fractional part of her tuition will be paid through scholarships?

- 72.** The offensive team for the Detroit Lions was on the field 55% of the time during a game with the New York Jets. What fractional part of the game were they on the field?

Change each of the following fractions or mixed numbers to a percent rounded to the nearest hundredth of a percent.

73. $\frac{67}{360}$

74. $\frac{567}{8000}$

75. $8\frac{19}{43}\%$

76. $27\frac{41}{79}\%$

Write each of the following as a fraction.

77. $4\frac{4}{9}\%$

78. $7\frac{7}{9}\%$

79. 0.875%

80. 0.325%

- 81.** A vitamin C tablet is listed as fulfilling $\frac{1}{8}$ the recommended daily allowance for vitamin C. Miguel takes 13 of these tablets per day to ward off a cold. What percent of the average recommended allowance is he taking?
- 83.** The enrollment at City Community College this year is 112% of last year's enrollment. What fraction of last year's enrollment does this represent?
- 85.** A census determines that $37\frac{1}{2}\%$ of the residents of a city are age 40 or older and that 45% are age 25 or younger. What fraction of the residents are between the ages of 25 and 40?
- 87.** The area of the island of St. Croix, one of the Virgin Islands, is 84 mi^2 . The total area of the Virgin Islands is 140 mi^2 . Write a fraction that represents the ratio of the area of St. Croix to the area of the Virgin Islands. Change this fraction to a percent.
- 89.** In 2000, one area of California had 211 smoggy days. What was the percent of smoggy days? In 2005, there were only 167 smoggy days. What was the percent that year? Compare these percents and discuss the possible reasons for this decline. Round the percents to the nearest tenth of a percent.
- 91.** The salmon run in an Oregon stream has dropped to 42% of what it was 10 years ago. What fractional part of the run was lost during the 10 years?
- 93.** According to the 2001 census, the population of South Africa is 9.58% white. Convert this to a fraction and explain its meaning.
- 82.** During the 2004 season, the New York Yankees won 101 games and lost 61. Write a fraction that gives the number of games won compared to the total games played. Convert this to a percent rounded to the nearest tenth of a percent.
- 84.** A western city had a population in 2005 that was 135% of its population in 2003. Express the percent of the 2003 population as a fraction or mixed number.
- 86.** Jorge invests $26\frac{3}{7}\%$ of his money in money market funds. The rest he puts in common stocks. What fraction of the total investment is in common stocks?
- 88.** Burger King's original Double Whopper with cheese contains 69 g of fat. Each gram of fat has 9 calories. If the entire sandwich contains 1060 calories, what percent of the calories come from the fat content? Round to the nearest percent.
- 90.** Spraying for mosquitos, in an attempt to eliminate the West Nile Virus, is found to be 85% successful. What fraction of the mosquitos are eliminated?
- 92.** The literacy rate in Vietnam is 94%. Convert this to a fraction and explain its meaning.

Exercises 94–97 relate to the chapter application.

- 94.** Consumer reports indicate that the cost of food is $1\frac{1}{12}$ what it was 1 year ago. Express this as a percent. Round to the nearest tenth of a percent.
- 96.** Sears put all its appliances on sale at 20% off. What fraction is this? What fractional part of the original price do you end up paying?
- 95.** A department store advertises one-third off the regular price on Monday and an additional one-seventh off the original price on Tuesday. What percent is taken off the original price if the item is purchased on Tuesday? Round to the nearest whole percent.
- 97.** Wendy bought a barbecue grill at WalMart that was on sale for $\frac{2}{3}$ of its original price. What percent off was the grill? Round to the nearest whole percent.

STATE YOUR UNDERSTANDING

- 98.** Explain why not all fractions can be changed to a whole-number percent. What is special about the fractions that can?
- 99.** Name two circumstances that can be described by either a percent or a fraction. Compare the advantages or disadvantages of using percents or fractions.
- 100.** Explain how to change between the fraction and percent forms of a number. Give an example of each.

CHALLENGE

- 101.** Change $2\frac{4}{13}\%$ to the nearest tenth of a percent.
- 102.** Change $\frac{6}{13}\%$ to the nearest hundredth of a percent.
- 103.** Change 0.00025 to a fraction.
- 104.** Change 150.005% to a mixed number.
- 105.** Change 180.04% to a mixed number.
- 106.** Change 0.0005% to a fraction.

GROUP WORK

- 107.** Keep a record of everything you eat for 1 day. Use exact amounts as much as possible. With a calorie and fat counter, compute the percent of fat in each item. Then find the percent of fat you consumed that day. The latest recommendations suggest that the fat content not exceed 30% per day. How did you do? Which foods have the highest and which the lowest fat content? Was this a typical day for you? Continue this exercise for 1 week and compare your daily percent of fat with others in your group. Compute a weekly average individually and as a group.
- 108.** Have each member of your group read the ads for the local department stores in the weekend paper. Record the "% off" in as many ads as you can find. Convert the percents to fractions. In which form is it easier to estimate the savings because of the sale?

MAINTAIN YOUR SKILLS

Change to a fraction or mixed number.

109. 0.84

110. 0.132

111. 4.065

112. 16.48

Change to a decimal.

113. $\frac{33}{40}$

114. $\frac{443}{640}$

Change to a percent.

115. 0.567

116. 5.007

Change to a decimal.

117. 8.13%

118. 112.8%

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6.4 Fractions, Decimals, Percents: A Review

How & Why

OBJECTIVE

Given a decimal, fraction, or percent, rewrite in an equivalent form.

Decimals, fractions, and percents can each be written in other two forms. We can:

Write a percent as a decimal and as a fraction.

Write a fraction as a percent and as a decimal.

Write a decimal as a percent and as a fraction.

For example:

$$40\% = 40 \cdot \frac{1}{100} = \frac{40}{100} = \frac{2}{5} \quad \text{and} \quad 40\% = 0.4$$

$$\frac{5}{8} = 5 \div 8 = 0.625 \quad \text{and} \quad \frac{5}{8} = 0.625 = 62.5\%$$

$$0.95 = 95\% \quad \text{and} \quad 0.95 = \frac{95}{100} = \frac{19}{20}$$

Table 6.2 shows some common fractions and their decimal and percent equivalents. Some of the decimals are repeating decimals. Remember that a repeating decimal is shown by the bar over the digits that repeat. These fractions occur often in applications of percents. They should be memorized so that you can recall the patterns when they appear.

OBJECTIVE

Given a decimal, fraction, or percent, rewrite in an equivalent form.

Table 6.2 Common Fractions, Decimals, and Percents

Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%	$\frac{1}{6}$	$0.1\bar{6}$	$16\frac{2}{3}\%$ or $16.\bar{6}\%$
$\frac{1}{3}$	$0.\bar{3}$	$33\frac{1}{3}\%$ or $33.\bar{3}\%$	$\frac{5}{6}$	$0.8\bar{3}$	$83\frac{1}{3}\%$ or $83.\bar{3}\%$
$\frac{2}{3}$	$0.\bar{6}$	$66\frac{2}{3}\%$ or $66.\bar{6}\%$	$\frac{1}{8}$	0.125	12.5%
$\frac{1}{4}$	0.25	25%	$\frac{3}{8}$	0.375	37.5%
$\frac{3}{4}$	0.75	75%	$\frac{5}{8}$	0.625	62.5%
			$\frac{7}{8}$	0.875	87.5%
$\frac{1}{5}$	0.2	20%			
$\frac{2}{5}$	0.4	40%			
$\frac{3}{5}$	0.6	60%			
$\frac{4}{5}$	0.8	80%			

Warm-Ups A–B

Fraction	Decimal	Percent
$\frac{5}{12}$		27%
	0.72	
$\frac{73}{100}$		160%
	1.3	

- B. In Example B, write the percent that is recycled as a decimal and as a fraction.

Examples A–B



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DIRECTIONS: Fill in the empty spaces with the related percent, decimal, or fraction.

STRATEGY: Use the procedures of the previous sections.

A.

Fraction	Decimal	Percent
$\frac{5}{16}$	0.62	30%

Fraction	Decimal	Percent
$\frac{3}{10}$	0.3	30%
$\frac{5}{16}$	0.3125	31.25% or $31\frac{1}{4}\%$
$\frac{31}{50}$	0.62	62%

$$30\% = 0.30 = \frac{30}{100} = \frac{3}{10}$$

$$\frac{5}{16} = 0.3125 = 31.25\% = 31\frac{1}{4}\%$$

$$0.62 = 62\% = \frac{62}{100} = \frac{31}{50}$$

- B. The average American uses about 200 lb of plastic a year. Approximately 60% of this is used for packaging and about 5% of it is recycled. Write the percent used for packaging as a decimal and as a fraction.

$$60\% = 0.60 = \frac{60}{100} = \frac{3}{5}$$

So 0.6, or $\frac{3}{5}$, of the plastic is used for packaging.

Answers to Warm-Ups

Fraction	Decimal	Percent
$\frac{5}{12}$	0.41̄	$41\frac{2}{3}\%$
$\frac{27}{100}$	0.27	27%
$\frac{18}{25}$	0.72	72%
$\frac{73}{100}$	0.73	73%
$1\frac{3}{5}$	1.6	160%
$1\frac{3}{10}$	1.3	130%

- B. Of the 200 lb of plastic, 0.05, or $\frac{1}{20}$, is recycled.

Exercises 6.4

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OBJECTIVE

Given a decimal, fraction, or percent, rewrite in an equivalent form.

1–26. Fill in the empty spaces with the related percent decimal, or fraction.

	Fraction	Decimal	Percent		Fraction	Decimal	Percent
1.	$\frac{1}{10}$			2.			30%
3.		0.75		4.	$\frac{9}{10}$		
5.			145%	6.	$\frac{3}{8}$		
7.		0.001		8.		1	
9.	$2\frac{1}{4}$			10.		0.8	
11.			$5\frac{1}{2}\%$	12.		0.875	
13.			0.5% or $\frac{1}{2}\%$	14.		0.6	
15.			$62\frac{1}{2}\%$	16.			50%
17.		0.86		18.	$\frac{5}{6}$		
19.		0.08		20.	$\frac{2}{3}$		
21.			25%	22.		0.2	
23.			40%	24.			$33\frac{1}{3}\%$
25.		0.125		26.	$\frac{7}{10}$		

27. Louis goes to the Bon Marche to buy a new swim suit. He finds one on sale for $\frac{1}{4}$ off. What percent is this?

28. Michael and Louise's new baby now weighs $66\frac{2}{3}\%$ more than his birth weight. What fraction is this?

- 29.** During the month of August, Super Value Grocery has a special on sweet corn: buy 12 ears and get $\frac{1}{6}$ more for 1 cent. During the same time period, Hank's Super Market also runs a special on sweet corn: buy 12 ears and get 25% more for 1 cent. Which store offers the better deal?
- 31.** Teresa is negotiating a business deal. Vendor A has offered an 8% increase in price, whereas Teresa's boss has authorized up to 0.0775 more. Vendor B has offered Teresa a deal that increases the price by $\frac{1}{15}$. Who has offered the better deal? Does either or both meet the boss's authorized amount?
- 33.** During one softball season, Jane got a hit 35% of the times she was at bat, Stephanie got a hit $\frac{11}{30}$ of the times she was at bat, and Ellie's batting average was 0.361. Which girl had the best batting average?
- 35.** In Peru, the literacy rate is 89%. In Jamaica, the literacy rate is 851 per 1000 people. Which country has the higher rate?
- 30.** Three multivitamins contain the following amounts of the RDA (Recommended Daily Allowance) of calcium: brand A, 15%; brand B, 0.149; and brand C, $\frac{1}{7}$. Which brand contains the most calcium?
- 32.** During the 20042005 NBA basketball season the top three field goal shooters had the following statistics: Shaquille O'Neal made 60.1% of his shots, Amare Stoudemire made 0.559 of his shots, and Yao Ming made $\frac{36}{69}$ of his shots. Who was the most accurate shooter?
- 34.** During a promotional sale, Rite-Aid advertises Coppertone sunscreen for $\frac{1}{4}$ off the suggested retail price whereas Walgreen's advertises it for 25% off. Which is the better deal?
- 36.** In Bulgaria, the death rate is about 13.3 per 1000 people. In Estonia, the death rate is about 1.31%. Which country has the lower death rate?

Exercises 37–40 relate to the chapter application.

- 37.** George buys a new VCR at a 40%-off sale. What fraction is this?
- 39.** Melinda is researching the best place to buy a computer. On the same-priced computer, Family Computers offers $\frac{1}{8}$ off, The Computer Store will give a 12% discount, and Machines Etc. will allow a 0.13 discount. Where does Melinda get the best deal?
- 38.** A local department store is having its red tag sale. All merchandise will now be 20% off the original price. What decimal is this?
- 40.** Randy is trading in his above-ground swimming pool for a larger model. Prices are the same for Model PS+ and Model PT. PS+ holds 11% more water than his old pool, whereas PT holds $\frac{1}{9}$ more water. Which should he use to get the most additional water for his new pool?

STATE YOUR UNDERSTANDING

- 41.** Write a short paragraph with examples that illustrate when to use fractions, when to use decimals, and when to use percents to show comparisons.

CHALLENGE

- 42.** Fill in the table. Be exact.

Fraction	Decimal	Percent
$\frac{14}{3}$		
	1.1875	
		6.3125%

GROUP WORK

- 43.** Have each member of your group make up a table like the one in Example A. Exchange your table with the other group members and fill in the blanks. Check your answers with the rest of the group.

MAINTAIN YOUR SKILLS

Solve the following proportions.

44. $\frac{25}{30} = \frac{x}{45}$

45. $\frac{45}{81} = \frac{23}{y}$

46. $\frac{x}{72} = \frac{40}{9}$

47. $\frac{x}{8.3} = \frac{117}{260}$

48. $\frac{\frac{1}{2}}{100} = \frac{A}{21}$

49. $\frac{85}{100} = \frac{170}{B}$

50. $\frac{R}{100} = \frac{9}{34}$; round to the nearest tenth.

51. $\frac{23}{100} = \frac{A}{41.3}$; round to the nearest tenth.

- 52.** Sean drives 536 miles and uses 10.6 gallons of gasoline. At that rate, how many gallons will he need to drive 1150 miles? Round to the nearest tenth of a gallon.

- 53.** The Bacons' house is worth \$235,500 and is insured so that the Bacons will be paid four-fifths of the value of any damage. One-third of the value of the house is destroyed by fire. How much insurance money should they collect?

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6.5 Solving Percent Problems

VOCABULARY

In the statement “ R of B is A ,”

R is the **rate** of percent.

B is the **base** unit and follows the word *of*.

A is the **amount** that is compared to B .

To solve a percent problem means to do one of the following:

1. Find A , given R and B .
2. Find B , given R and A .
3. Find R , given A and B .

OBJECTIVES

1. Solve percent problems using the formula.
2. Solve percent problems using a proportion.

How & Why

OBJECTIVE 1

Solve percent problems using the formula.

We show two methods for solving percent problems. We refer to these as

The percent formula, $R \times B = A$ (see Examples AE).

The proportion method (see Examples FH).

In each method we must identify the rate of percent (R), the base (B), and the amount (A).

To help determine these, keep in mind that

R , the rate of percent, includes the percent symbol (%).

B , the base, follows the words *of* or *percent of*.

A , the amount, sometimes called the *percentage*, is the amount compared to B and follows the word *is*.

The method you choose to solve percent problems should depend on

1. the method your instructor recommends
2. your major field of study
3. how you use percent in your day-to-day activities

What percent of B is A ? The word *of* in this context and in other places in mathematics indicates multiplication. The word *is* describes the relationship “is equal to” or “=.” Thus, we write:

R of B is A



$$R \times B = A$$

When solving percent problems, identify the rate (%) first, the base (of) next, and the amount (is) last. For example, what percent of 30 is 6?

R of B is A The rate R is unknown, the base B (B follows the word *of*) is 30, and the amount A (A follows the word *is*) is 6.

$$R \times B = A$$

$$R(30) = 6$$

$$R = 6 \div 30$$

$$= 0.2$$

$$= 20\%$$

Substitute 30 for B and 6 for A .

Divide.

Change to percent.

So, 6 is 20% of 30.

All percent problems can be solved using $R \times B = A$. However, there are two other forms that can speed up the process.

$$A = R \times B \quad R = A \div B \quad B = A \div R$$

The triangle in Figure 6.2 is a useful device to help you select the correct form of the formula to use.

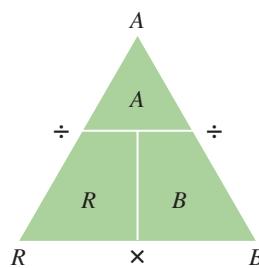


Figure 6.2

When the unknown value is covered, the positions of the uncovered (known) values help us remember which operation to use:

When A is covered, we see $R \times B$, reading from left to right.

When B is covered, we see $A \div R$, reading from top to bottom.

When R is covered, we see $A \div B$, reading from top to bottom.

For example, 34% of what number is 53.04? The rate (R) is 34%, B is unknown (B follows the word *of*), and A (A follows the word *is*) is 53.04. Fill in the triangle and cover B . See Figure 6.3.

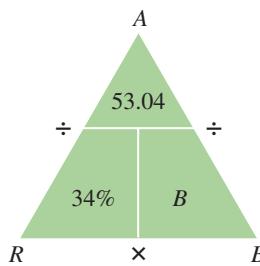


Figure 6.3

Reading from the top, we see that A is divided by R . Therefore,

$$B = A \div R$$

$$= 53.04 \div 0.34 \quad \text{Substitute 53.04 for } A \text{ and 0.34 for } R.$$

$$= 156$$

So 34% of 156 is 53.04.

CAUTION

Remember to use the decimal or fraction form of the rate R when solving percent problems.

Examples A–E



BASIC
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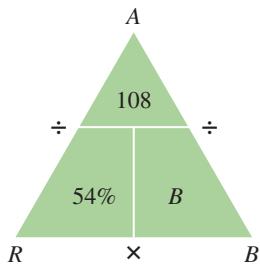
DIRECTIONS: Solve the percent problems using the formula.

STRATEGY: Identify R , B , and A . Select a formula. Substitute the known values and find the unknown value.

A. 54% of what number is 108?

54% of B is 108

The % symbol follows 54, so $R = 54\%$. The base B (following the word *of*) is unknown. A follows the word *is*, so $A = 108$.



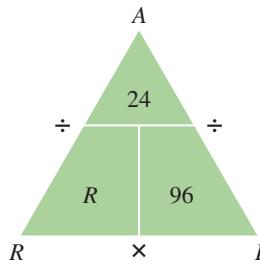
$$B = A \div R$$

$$B = 108 \div 0.54$$

$$B = 200$$

So 54% of 200 is 108.

B. 24 is what percent of 96?



$$R = A \div B$$

$$R = 24 \div 96$$

$$R = 0.25 = 25\%$$

So 24 is 25% of 96.

The rate R is unknown. B follows the word *of* so $B = 96$ and $A = 24$.

Warm-Ups A–E

A. 56% of what number is 35?

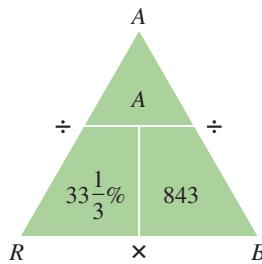
B. 37.8 is what percent of 84?

Answers to Warm-Ups

A. 62.5 **B.** 45%

- C. What is $66\frac{2}{3}\%$ of 87?

- C. What is $33\frac{1}{3}\%$ of 843?



$$A = R \times B$$

$$A = 33\frac{1}{3}\%(843)$$

$$A = \frac{1}{3}(843)$$

The % symbol follows $33\frac{1}{3}$, so $R = 33\frac{1}{3}\%$. The base B (following the word *of*) is 843. A is unknown.

Cover A in the triangle and read " $R \times B$."

Change the percent to a fraction.

$$\begin{aligned} 33\frac{1}{3}\% &= 33\frac{1}{3}\left(\frac{1}{100}\right) \\ &= \frac{100}{3}\left(\frac{1}{100}\right) \\ &= \frac{1}{3} \end{aligned}$$

$$A = 281$$

Multiply and simplify.

So 281 is $33\frac{1}{3}\%$ of 843.

- D. 132% of what number is 102.96?

- D. 128% of what number is 88.8?

STRATEGY: We use the formula $R \times B = A$.

$$R \times B = A \quad \text{Formula}$$

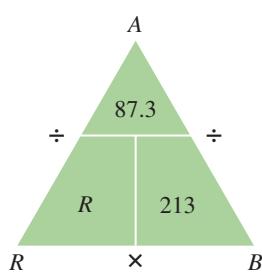
$$\begin{aligned} 1.28B &= 88.8 & R = 128\% = 1.28 \text{ and } A = 88.8 \\ &B \text{ is unknown.} & \end{aligned}$$

$$\begin{aligned} B &= 88.8 \div 1.28 & \text{Rewrite as division.} \\ &= 69.375 & \end{aligned}$$

So 128% of 69.375 is 88.8.

- E. 68.4 is what percent of 519? Round to the nearest tenth of a percent.

- E. 87.3 is what percent of 213? Round to the nearest tenth of a percent.



$A = 87.3$ and $B = 213$. R is unknown.

$$\begin{aligned} R &= A \div B \\ R &= 87.3 \div 213 \end{aligned}$$

In the percent triangle, cover R and read " $A \div B$."

$$R \approx 0.409859$$

$$R \approx 0.410$$

To round to the nearest tenth of a percent, we round the decimal to nearest thousandth.

$$R \approx 41.0\%$$

So 87.3 is about 41.0% of 213.

Answers to Warm-Ups

- C. 58 D. 78 E. 13.2%

How & Why

OBJECTIVE 2

Solve percent problems using a proportion.

Because R is a comparison of A to B and we have seen earlier that this comparison can be written as a ratio, we can write the percent ratio equal to the ratio of A and B . In writing the percent as a ratio, we let $R = X\%$. We can now write the proportion:

$$\frac{X}{100} = \frac{A}{B}$$

When any one of the values of R , A , and B is unknown it can be found by solving the proportion. For example, what percent of 225 is 99?

$A = 99$, $B = 225$, and $R = X\% = ?$

$$\frac{X}{100} = \frac{A}{B}$$

$$\frac{X}{100} = \frac{99}{225}$$

$$225X = 100(99) \quad \text{Cross multiply.}$$

$$225X = 9900$$

$$X = 9900 \div 225$$

$$X = 44$$

$$R = X\% = 44\%$$

So 44% of 225 is 99.

Examples F–H



DIRECTIONS: Solve the percent problem using a proportion.

STRATEGY: Write the proportion $\frac{X}{100} = \frac{A}{B}$, fill in the known values, and solve.

F. 85% of 76 is what number?

$$\frac{X}{100} = \frac{A}{B} \quad \text{Proportion for solving percent problems}$$

$$\frac{85}{100} = \frac{A}{76} \quad R = X\% = 85\%, \text{ so } X = 85, B = 76, \text{ and } A \text{ is unknown.}$$

$$85(76) = 100A \quad \text{Cross multiply.}$$

$$6460 = 100A$$

$$6460 \div 100 = A \quad \text{Rewrite as division.}$$

$$64.6 = A$$

So 85% of 76 is 64.6.

G. 132% of _____ is 134.64

$$\frac{X}{100} = \frac{A}{B} \quad \text{Proportion for solving percent problems}$$

$$\frac{132}{100} = \frac{134.64}{B} \quad R = X\% = 132\%, \text{ so } X = 132, B \text{ is unknown, and } A = 134.64.$$

$$132(B) = 100(134.64) \quad \text{Cross multiply.}$$

$$132(B) = 13,464$$

$$B = 13,464 \div 132 \quad \text{Rewrite as division.}$$

$$B = 102$$

So 132% of 102 is 134.64.

Warm-Ups F–H

F. 72% of 110 is what number?

G. 145% of _____ is 304.5?

Answers to Warm-Ups

F. 79.2

G. 210

- H.** 49 is what percent of 135, to the nearest tenth of a percent?

- H.** 81 is what percent of 215? Round to the nearest tenth of a percent.

$$\frac{X}{100} = \frac{A}{B}$$

$$\frac{X}{100} = \frac{81}{215}$$

$$215(X) = 100(81)$$

$$215(X) = 8100$$

$$X = 8100 \div 215$$

$$X \approx 37.67$$

$$X \approx 37.7$$

$$R \approx 37.7\%$$

$R = X\%$ is unknown, $A = 81$, and $B = 215$.

Cross multiply.

Rewrite as division.

Carry out the division to two decimal places.

Round to the nearest tenth.

$$R = X\%$$

So 81 is 37.7% of 215, to the nearest tenth of a percent.

Answers to Warm-Ups

- H.** 36.3%

Exercises 6.5

OBJECTIVE 1

Solve percent problems using the formula.

OBJECTIVE 2

Solve percent problems using a proportion.

A Solve.

1. 27 is 50% of _____. 2. 18 is 20% of _____. 3. _____ is 40% of 115.
 4. _____ is 60% of 80. 5. 6 is _____ % of 3. 6. 8 is _____ % of 2.
 7. _____ % of 88 is 44. 8. _____ % of 88 is 22. 9. 80% of _____ is 32.
 10. 30% of _____ is 18. 11. 30% of 91 is _____. 12. _____ is 55% of 72.
 13. 96 is _____ % of 120. 14. _____ % of 75 is 15. 15. 62% of _____ is 62.
 16. 62 is _____ % of 62. 17. $\frac{1}{3}\%$ of 600 is _____. 18. $\frac{1}{7}\%$ of 1400 is _____.
 19. 130% of 90 is _____. 20. 140% of 70 is _____.

B

21. 17.5% of 70 is _____. 22. 57.5% of 110 is _____. 23. 1.05 is _____ % of 42.
 24. 0.13 is _____ % of 65. 25. 289.8 is 84% of _____. 26. 162 is 18% of _____.
 27. 68.5% of 96 is _____. 28. 19.4% of 75 is _____. 29. 48% of _____ is 74.4.
 30. 73% of _____ is 83.22. 31. 124% of _____ is 328.6. 32. 132% of _____ is 285.12.
 33. 96 is _____ % of 125. 34. 135 is _____ % of 160. 35. 6.14% of 350 is _____.
 36. 12.85% of 980 is _____. 37. 2.05 is _____ % of 3.28. 38. 6.09 is _____ % of 17.4.
 39. $11\frac{1}{9}\%$ of 1845 is _____. 40. $16\frac{2}{3}\%$ of 3522 is _____.

C

41. What percent of 91 is 52? Round to the nearest tenth of a percent.
 42. What percent of 666 is 247? Round to the nearest tenth of a percent.
 43. Eighty-nine is 14.6% of what number? Round to the nearest hundredth.
 44. Forty-one is 35.2% of what number? Round to the nearest hundredth.
 45. Thirty-two and seven tenths percent of 695 is what number?
 46. Seventy-three and twelve hundredths percent of 35 is what number?

- 47.** Thirty-seven is what percent of 156? Round to the nearest tenth of a percent.

Solve.

49. $6\frac{3}{4}\%$ of $12\frac{2}{3}$ is _____. (as a fraction)

51. $8\frac{7}{15}\%$ of 1350 is _____. (as a mixed number)

53. _____ % of 34.76 is 45.87. (round to the nearest tenth of a percent)

- 48.** Two hundred thirty-two is what percent of 124? Round to the nearest tenth of a percent.

50. _____ is $9\frac{7}{8}\%$ of $61\frac{3}{5}$. (as a decimal)

52. $23\frac{5}{11}\%$ of _____ is 87.5. (as a decimal rounded to the nearest hundredth)

54. _____ % of $12\frac{4}{15}$ is $7\frac{2}{3}$. (as a mixed number)

STATE YOUR UNDERSTANDING

- 55.** Explain the inaccuracies in this statement: “Starbuck industries charges 70¢ for a part that cost them 30¢ to make. They’re making 40% profit.”

- 56.** Explain how to use the *RAB* triangle to solve percent problems.

CHALLENGE

- 57.** $\frac{1}{2}\%$ of $45\frac{1}{3}$ is what fraction?

- 58.** $\frac{3}{7}\%$ of $21\frac{1}{9}$ is what fraction?

GROUP WORK

- 59.** Divide up the task of computing these percents: 45% of 37; 37% of 45; 18% of 80; 80% of 18; 130% of 22; 22% of 130; 0.6% of 5.5; and 5.5% of 0.6. Compare your answers and together write up a statement about the answers.

- 60.** Divide up the task of computing these percents: 30% of the number that is 80% of 250; 80% of the number that is 30% of 250; 60% of the number that is 20% of 340; 20% of the number that is 60% of 230; 150% of the number that is 200% of 40; and 200% of the number that is 150% of 40. Compare your answers and together write up a statement about the answers.

MAINTAIN YOUR SKILLS

Solve the proportions.

$$61. \frac{18}{24} = \frac{x}{60}$$

$$62. \frac{6.2}{2.5} = \frac{93}{y}$$

$$63. \frac{a}{\frac{5}{8}} = \frac{\frac{1}{2}}{\frac{3}{4}}$$

$$64. \frac{1\frac{1}{2}}{t} = \frac{5\frac{5}{8}}{1\frac{2}{3}}$$

$$65. \frac{1.4}{0.21} = \frac{w}{3.03}$$

$$66. \frac{2.6}{0.07} = \frac{3.51}{t}$$

Exercises 67–70. On a certain map, $1\frac{3}{4}$ in. represents 70 mi.

$$67. \text{How many miles are represented by } 3\frac{5}{8} \text{ in.?}$$

$$68. \text{How many miles are represented by } 7\frac{11}{16} \text{ in.?}$$

$$69. \text{How many inches are needed to represent 105 mi?}$$

$$70. \text{How many inches are needed to represent 22 mi?}$$

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6.6 Applications of Percents

VOCABULARY

When a value B is increased by an amount A , the rate of percent R , or $\frac{A}{B}$, is called the **percent of increase**.

When a value B is decreased by an amount A , the rate of percent R , or $\frac{A}{B}$, is called the **percent of decrease**.

OBJECTIVES

1. Solve applications involving percent.
2. Find percent of increase and percent of decrease.
3. Read data from a circle graph or construct a circle graph from data.

How & Why

OBJECTIVE 1

Solve applications involving percent.

When a word problem is translated to the simpler form, “What percent of what is what?” the unknown value can be found using one of the methods from the previous section. For example,

The 2000 Census listed the population of Detroit at 4,043,467. The African American population was 1,011,038. What percent of the population of Detroit was African American in 2000?

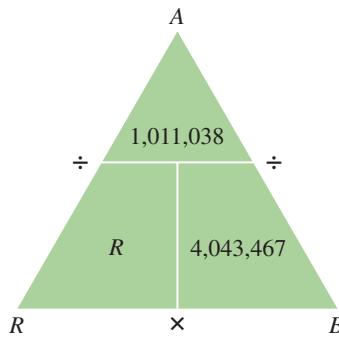
We first rewrite the problem in the form “What percent of what is what?”

What percent of the population is African American?

We know that the total population is 4,043,467 and we know the African American population is 1,011,038. Substituting these values we have:

What percent of 4,043,467 is 1,011,038?

Using the percent formula we know that R is unknown, B is 4,043,467, and that A is 1,011,038. We substitute these values into the triangle and solve for R .



$$R = A \div B$$

$$R = 1,011,038 \div 4,043,467$$

$$R \approx 0.25$$

Rounded to the nearest hundredth.

$$R \approx 25\%$$

So the population of Detroit in 2000 was about 25% African American.

- A. The cost of a certain model of Ford is 135% of what it was 5 years ago. If the cost of the automobile 5 years ago was \$20,400, what is the cost today?

- B. A list of grades in a math class revealed that 9 students received As, 14 received Bs, 27 received Cs, and 7 received Ds. What percent of the students received a grade of C?

- C. In a similar study of 615 people, 185 said they jog for exercise. What percent of those surveyed jog? Round to the nearest whole percent.

the known values and find the unknown value.

- A. This year the population of Deschutes County is 162% of its population 10 years ago. The population 10 years ago was 142,000. What is the population this year?

STRATEGY: Use the proportion.

162% of 142,000 is the current population.

$$\frac{162}{100} = \frac{A}{142,000}$$

$R = X\% = 162\%$, so $X = 162$, and $B = 142,000$.

Substitute these values into the proportion

$$\frac{X}{100} = \frac{A}{B}, \text{ and solve.}$$

$$142,000(162) = 100A$$

$$23,004,000 = 100A$$

$$23,004,000 \div 100 = A$$

$$230,040 = A$$

Cross multiply.

The population this year is 230,040.

- B. A student newspaper polls a group of students. Five of them say they walk to school, 11 say they ride the bus, 15 ride in car pools, and 4 drive their own cars. What percent of the group rides in car pools?

STRATEGY: Use the proportion.

$$5 + 11 + 15 + 4 = 35$$

What % of 35 is 15?

First, find the number in the group.

There are 35 in the group, so $B = 35$, and 15 ride in car pools so $A = 15$.

$$\frac{X}{100} = \frac{15}{35}$$

$$35(X) = 1500$$

$$X = 1500 \div 35$$

$$X \approx 43$$

$$R \approx 43\%$$

Cross multiply.

Round to the nearest whole number.

$$R = X\%.$$

Approximately 43% of the students ride in car pools.

- C. In a statistical study of 545 people, 215 said they preferred eating whole wheat bread. What percent of the people surveyed preferred eating whole wheat bread? Round to the nearest whole percent.

STRATEGY: Use the percent formula to solve.

What percent of 545 is 215?

$$B = 545 \text{ and } A = 215$$

$$R \times B = A$$

Percent formula

$$R(545) = 215$$

Substitute.

$$R = 215 \div 545$$

$$R \approx 0.39$$

$$R \approx 39\%$$

Round to the nearest hundredth.

So approximately 39% of the people surveyed preferred eating whole wheat bread.

Answers to Warm-Ups

- A. The cost of the Ford today is \$27,540.
 B. The percent of students receiving a C grade is approximately 47%.
 C. Of the 615 people, 30% jog.

How & Why

OBJECTIVE 2

Find percent of increase and percent of decrease.

To find the percent of increase or decrease, the base B is the starting number. The increase or decrease is the amount A . For instance, if the population of a city grew from 86,745 to 90,310 in 3 years, the base is 86,745 and the increase is the difference in the populations, $90,310 - 86,745 = 3565$.

To find the percent of increase in the population, we ask the question “What percent of 86,745 is 3565?” Using the percent equation $R = A \div B$, we have

$$R = 3565 \div 86,745 \quad \text{Substitute } 3565 \text{ for } A \text{ and } 86,745 \text{ for } B.$$

$$R \approx 0.0410974$$

$$R \approx 0.04$$

Round to the nearest hundredth.

$$R \approx 4\%$$

Change to a percent.

So the population increased about 4% in the 3 years.

Examples D–E



DIRECTIONS: Find the percent of increase or decrease.

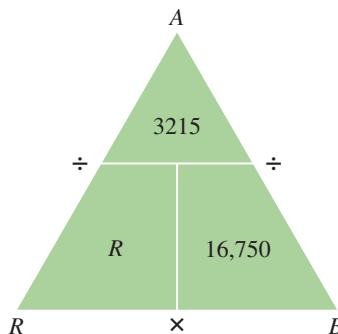
STRATEGY: Use one of the two methods to solve for R .

- D.** The cost of a new car went from \$16,750 to \$19,965 in 2 years. Find the percent of increase in the price to the nearest tenth of a percent.

STRATEGY: Use the percent formula.

$$\begin{aligned} \$19,965 - \$16,750 &= \$3215 && \text{The difference, } \$3215, \text{ is the amount of increase.} \\ &&& \text{The percent of increase is calculated from the amount of increase, } \$3215, \text{ based on the original amount,} \\ &&& \$16,750. \end{aligned}$$

What percent of \$16,750 is \$3215? $B = 16,750$ and $A = 3215$



$$R = A \div B$$

$$R = 3215 \div 16750$$

$$R \approx 0.1919402$$

$$R \approx 0.192$$

Round to the nearest thousandth.

$$R \approx 19.2\%$$

Change to a percent.

The increase in the price of the car is about 19.2%.

Warm-Ups D–E

- D.** The price of a gallon of premium gasoline went from \$2.19 to \$2.69 in 2005. Find the percent of increase in the price to the nearest tenth of a percent.

Answers to Warm-Ups

- D.** The price of a gallon of premium gasoline increased by about 22.8%.

- E. The price of an LCD television set decreased from \$6734 in 2001 to \$3989 in 2005. Find the percent of decrease in the price to the nearest tenth of a percent.

- E. John went on a diet. At the beginning of the diet, he weighed 267 lb. After 6 weeks, he weighed 216 lb. Find the percent of decrease in his weight to the nearest tenth of a percent.

STRATEGY: Use the proportion.

$$267 \text{ lb} - 216 \text{ lb} = 51 \text{ lb}$$

The difference, 51 lb, is the amount of decrease from 267 to 216. The percent of decrease is the comparison of the amount, 51 lb, to the original weight, 267 lb.

What percent of 267 is 51?

$$B = 267 \text{ and } A = 51$$

$$\frac{X}{100} = \frac{A}{B}$$

$$\frac{X}{100} = \frac{51}{267}$$

Substitute 51 for A and 267 for B .

$$267X = 51(100)$$

$$X = 5100 \div 267$$

$$X \approx 19.101123$$

$$X \approx 19.1$$

Round to the nearest tenth.

$$R \approx 19.1\%$$

$$R = X\%$$

So John had a decrease of about 19.1% in his weight.

How & Why

OBJECTIVE 3

Read data from a circle graph or construct a circle graph from data.

A circle graph or pie chart is used to show how a whole unit is divided into parts. The area of the circle represents the entire unit and each subdivision is represented by a sector. Percents are often used as the unit of measure of the subdivision. Consider the following pie chart (Figure 6.4).

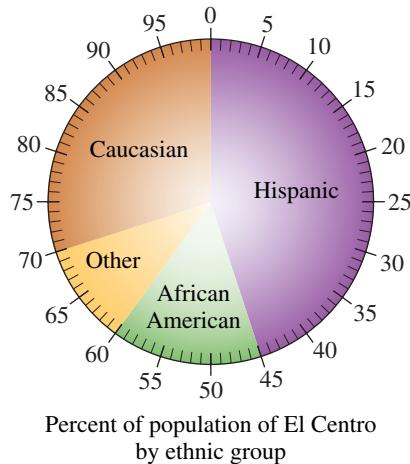


Figure 6.4

From the circle graph we can conclude:

1. The largest ethnic group in El Centro is Hispanic.
2. The Caucasian population is twice the African American population.
3. The African American and Hispanic populations are 60% of the total.

Answers to Warm-Ups

- E. The price of an LCD television set decreased about 40.8%.

If the population of El Centro is 125,000, we can also compute the approximate number in each group. For instance, the number of Hispanics is found by:

$$R \times B = A \quad R = 45\% = 0.45, B = 125,000 \\ (0.45)(125,000) = A \\ 56,250 = A$$

There are approximately 56,250 Hispanics in El Centro.

To construct a circle graph, determine what fractional part or percent each subdivision is, compared to the total. Then draw a circle and divide it accordingly. We can draw a pie chart of the data in Table 6.3.

Table 6.3 Population by Age Group			
	Age Groups		
	0–21	22–50	Over 50
Population	14,560	29,120	14,560

Begin by adding two rows and a column to Table 6.3 to create Table 6.4.

Table 6.4 Population by Age Group				
	Age Group			
	0–21	22–50	Over 50	Total
Population	14,560	29,120	14,560	58,240
Fractional part	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1
Percent	25%	50%	25%	100%

The third row is computed by writing each age group as a fraction of the total population and reducing. For example, the 0–21 age group is

$$\frac{14,560}{58,240} = \frac{1456}{5824} = \frac{364}{1456} = \frac{1}{4}, \text{ or } 25\%$$

Now draw the circle graph and label it. See Figure 6.5.

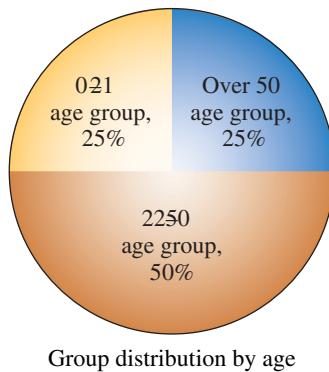
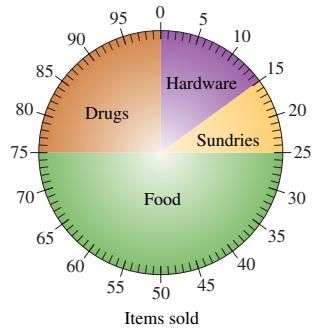


Figure 6.5

Sometimes circle graphs are drawn using 1 degree as the unit of measure for the sectors. This is left for a future course.

Warm-Up F

- F. The sales of items at Grocery Mart are displayed in the circle graph.



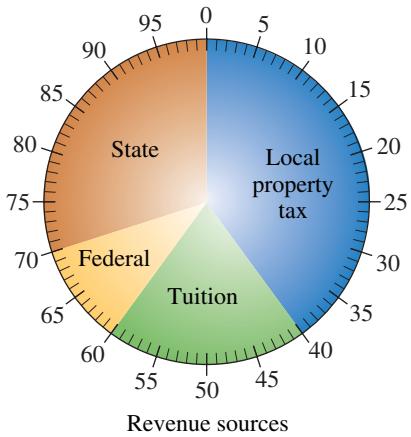
1. What is the area of highest sales?
2. What percent of total sales is from sundries and drugs?
3. What percent of total sales is from food and hardware?

Example F

DIRECTIONS: Answer the questions associated with the graph.

STRATEGY: Examine the graph to determine the size of the related sector.

- F. The sources of City Community College's revenue are displayed in the circle graph.



1. What percent of the revenue is from the federal government?
 2. What percent of the revenue is from tuition and property taxes?
 3. What percent of the revenue is from federal and state governments?
1. 10% Read directly from the graph.
 2. 60% Add the percents for tuition and property taxes.
 3. 40% Add the percents for state and federal sources.

So the percent of revenue from the federal government is 10%; the percent from tuition and property taxes is 60%; and the percent from federal and state governments is 40%.

Answers to Warm-Ups

- F. 1. The highest sales are in food.
2. Sundries and drugs account for 35% of sales.
3. Food and hardware account for 65% of sales.

Example G

DIRECTIONS: Construct a circle graph that illustrates the information.

STRATEGY: Use the information to calculate the percents. Divide the circle accordingly and label.

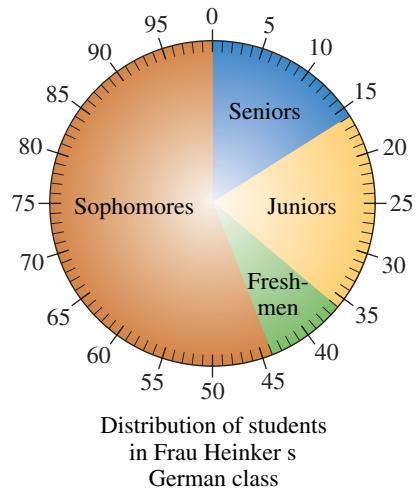
- G. Construct a circle graph to illustrate that of the 25 students in Frau Heinker's German class, 4 are seniors, 5 are juniors, 14 are sophomores, and 2 are freshmen.

$$\text{Seniors: } \frac{4}{25} = 16\% \quad \text{Compute the percents.}$$

$$\text{Juniors: } \frac{5}{25} = 20\%$$

$$\text{Sophomores: } \frac{14}{25} = 56\%$$

$$\text{Freshmen: } \frac{2}{25} = 8\%$$



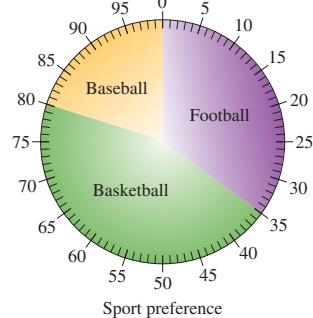
Construct and label the graph.

Warm-Up G

- G. Construct a circle graph to illustrate that in a survey of 20 people, 7 like football best, 9 like basketball best, and 4 like baseball best.

Answers to Warm-Ups

G.



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Exercises 6.6

OBJECTIVE 1

Solve applications involving percent.

1. About 16% of Yale's 5240 undergraduate students are from low-income families. To the nearest student, how many undergraduates are from low-income families?
3. In 2003, the estimated population of Los Angeles County was 9,871,506. The population was about 46.6% Hispanic or Latino. How many people of Hispanic or Latino heritage were living in Los Angeles County? Round to the nearest 10 people.
5. John got 32 problems correct on a 42-problem test. What was his percent score to the nearest whole-number percent?
7. Eddie and his family went to a restaurant for dinner. The dinner check was \$77.42. He left the waiter a tip of \$14. What percent of the check was the tip? Round to the nearest whole-number percent.
9. In preparing a mixture of concrete, Susan uses 300 pounds of gravel, 100 pounds of cement, and 200 pounds of sand. What percent of the mixture is gravel?
11. Delplanche Farms has 1180 acres of land in crops. They have 360 acres in soybeans, 410 acres in sweet corn, 225 acres in clover hay, and the rest in wheat. What percent of the acreage is in soybeans? Round to the nearest whole-number percent.
13. The city of Dallas, Texas, adopted a budget of \$2,048,293,780 for fiscal year 2004–2005. Of this budget, \$872,945,901 was the General Fund Budget. What percent of the total budget is the General Fund Budget? Round to the nearest tenth of a percent.
15. During Mickey Mantle's career in baseball, he was at bat 8102 times and got a hit 2415 times. What percent of the times at bat did he get a hit? Round to the nearest tenth of a percent.
2. Of the 1436 water fowl counted at the Jackson Bottom Wildlife Refuge, 23% were mallard ducks. How many mallard ducks were counted, to the nearest duck?
4. To pass a test to qualify for a job interview, Whitney must score at least 75%. If there are 60 questions on the test, how many must she get correct to score 75%?
6. Vera's house is valued at \$345,000 and rents for \$17,760 per year. What percent of the value of the house is the annual income from rent? Round to the nearest tenth of a percent.
8. Adams High School's lacrosse team finished the season with a record of 14 wins and 6 losses. What percent of the games played were won?
10. St. Joseph's Hospital has 8 three-bed wards, 20 four-bed wards, 12 two-bed wards, and 10 private rooms. What percent of the capacity of St. Joseph's Hospital is in three-bed wards? Round to the nearest tenth of a percent.
12. A Barnes and Noble store sold 231 fictional books, 135 books on politics, 83 self-help books, and 46 cookbooks in 1 day. What percent of the books sold are books on politics? Round to the nearest whole-number percent.
14. Texas has a total land mass of 261,914 square miles. Alaska has a land mass of 570,374 square miles. What percent of the land mass of Alaska is the land mass of Texas? Round to the nearest tenth of a percent.
16. The land mass of Ohio is 44,828 square miles. The federal government owns about 628 square miles of this land. What percent of the land in Ohio is owned by the federal government? Round to the nearest tenth of a percent.

- 17.** The total number of cars sold in Maryland in 2004 was 1,128,185. Of these cars, about 37.9% were new cars. Find the number of new cars sold in Maryland in 2004. Round to the nearest car.
- 19.** The manager of a fruit stand lost $16\frac{2}{3}\%$ of his bananas to spoilage and sold the rest. He discarded four boxes of bananas in 2 weeks. How many boxes did he have in stock?
- 21.** Maria sells candy from door to door. She keeps 15% of all her sales. How many dollars' worth of sales did Maria have if she earned \$558?
- 23.** It is claimed that in 15,000 hours, or 6 years, a gasoline engine will be down 256 hours for routine maintenance, whereas a diesel engine will be down only 104 hours. What is the difference in the percent of down times, to the nearest whole percent?
- 25.** The label in the figure shows the nutrition facts for one serving of Toasted Oatmeal. Use the information on the label to determine the recommended daily intake of (a) total fat, (b) sodium, (c) potassium, and (d) dietary fiber. Use the percentages for cereal alone. Round to the nearest whole number.
- 18.** Last year Joan had 16% of her salary withheld for taxes. If the total amount withheld was \$12,565.12, what was Joan's yearly salary?
- 20.** Frank sells magazine subscriptions. He keeps 18% of the cost of each subscription. How many dollars' worth of subscriptions must he sell to earn \$510.48?
- 22.** The town of Verboort has a population of 17,850, of which 48% is male. Of the men, 32% are 40 years or older. How many men are there in Verboort who are younger than 40?
- 24.** During a 6-year period, the cost of maintaining a diesel engine averages \$2650 and the cost of maintaining a gasoline engine averages \$4600. What is the percent of savings for the diesel compared with the gasoline engine? Round to the nearest whole percent.
- 26.** The table shows the calories per serving of the item along with the number of fat grams per serving.

Calories and Fat Grams per Serving

Item	Calories per Serving	Fat Grams per Serving
Light mayonnaise	50	4.5
Cocktail peanuts	170	14
Wheat crackers	120	4
Cream sandwich cookies	110	2.5

Each fat gram is equivalent to 10 calories. Find the percent of calories that are from fat for each item. Round to the nearest whole percent.

Nutrition Facts		
Serving Size 1 cup (49g) Servings per Container about 9		
Amount Per Serving		
	with 1/2 cup Cereal alone	Vitamin A&D Fortified Skim Milk
Calories	190	230
Calories from Fat	25	25
% Daily Values**		
Total Fat 2.5g*	4%	4%
Saturated Fat 0.5g	3%	3%
Polyunsaturated Fat 0.5g		
Monounsaturated Fat 1g		
Cholesterol 0mg	0%	0%
Sodium 220mg	9%	12%
Potassium 180mg	5%	11%
Total Carbohydrate 39g	13%	
	15%	
Dietary Fiber 3g	14%	14%
Soluble Fiber 1g		
Insoluble Fiber 2g		
Sugars 11g		
Other Carbohydrates 24g		
Protein 5g		

- 27.** In 1900, there were about 10.3 million foreign-born residents of the United States, which represented about 13.6% of the total population. What was the U.S. population in 1900?
- 29.** According to the 2000 Census figures, the city of Cleveland, Ohio, had a total of 190,725 households, of which 21% had incomes of less than \$10,000. Approximate the number of households in Cleveland that had incomes of less than \$10,000.
- 28.** In 1999, foreign-born residents were about 9.5% of the total population. If the foreign-born population was about 25.8 million, what was the total population in the United States in 1999?
- 30.** According to the 2000 Census figures (see Exercise 29) the city of Cleveland, Ohio, had 1318 households with incomes of more than \$200,000. What percent of the households are these?

OBJECTIVE 2

Find percent of increase and percent of decrease.

Fill in the table. Calculate the amount to the nearest whole number and the percent to the nearest tenth of a percent.

	Amount	New Amount	Increase or Decrease	Percent of Increase or Decrease
31.	345	415		
32.	1275	1095		
33.	764		Increase of 124	
34.	4050		Decrease of 1255	
35.	2900			Increase of 15%
36.	900			Decrease of 45%

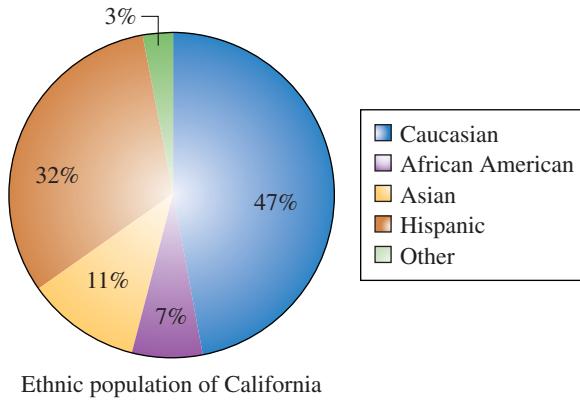
- 37.** The Portland Trail Blazers' average attendance per home game dropped from 19,420 in 20022003 to 16,594 in 20042005. What was the percent of decrease? Round to the nearest tenth of a percent.
- 38.** The Building and Services Department of a city in the South had a budget of \$26,675,000 in 20022003. The same department had a budget of \$28,021,791 for 20032004. What was the percent of increase in the budget? Round to the nearest tenth of a percent.
- 39.** On April 8, 2005, the price of a gallon of regular unleaded gasoline, self serve, hit a high of \$2.29, nationwide. This was up about 19¢ from the average price on March 18, 2005. What was the percent of increase in the price for the 3 weeks? Round to the nearest whole-number percent.
- 40.** Between January 12, 2005, and April 25, 2005, the value of a share of Thor Industries fell \$6.28 to \$28.80. What was the percent of decrease in the value of a share during this time period? Round to the nearest whole-number percent.
- 41.** Jose's Roth IRA grew, due to contributions and interest, from \$18,678 on January 1, 2005, to \$21,723 on December 31, 2005. What was the percent of increase in the IRA? Round to the nearest tenth of a percent.
- 42.** The population of Nevada grew from 1,998,257 in 2000 to 2,410,768 in 2004. What was the percent of increase in population? Round to the nearest tenth of a percent.
- 43.** The population of Washington, D.C., was 572,059 in 2000. In July 2004, the population was 553,522. What was the percent of decrease from 2000 to 2004? Round to the nearest tenth of a percent.
- 44.** Mary Ann lost 25.6 lb on her diet. She now weighs 146.5 lb. What percent of her original weight did she lose on this diet? Round to the nearest tenth of a percent.

- 45.** Wheat production in the United States fell from 2.34 billion bushels in 2003 to 2.2 billion in 2004. What was the percent of decrease in production of wheat? Round to the nearest tenth of a percent.
- 47.** Good driving habits can increase mileage and save on gas. If good driving causes a car's mileage to go from 27.5 mpg to 29.6 mpg, what is the percent of increased mileage? Round to the nearest tenth of a percent.
- 49.** By 1999, the number of foreign-born residents of the United States had increased to about 25.8 million from 10.3 million in 1900. What percent of increase is this from 1900?
- 46.** The average salary at Funco Industries went from \$34,783 in 2005 to \$36,725 in 2006. Find the percent of increase in the average salary. Round to the nearest tenth of a percent.
- 48.** According to the Bureau of Labor Statistics, there were 137,000 physical therapy jobs in the United States in 2002. Physical therapy is one of the fastest-growing occupations, with the Bureau of Labor Statistics predicting a 28% increase over 2002 levels by the year 2012. How many physical therapy jobs are predicted by the year 2012? Round to the nearest thousand.
- 50.** In 1900, tuition at Harvard was \$3000 per year. In 1999, Harvard tuition had risen to \$22,054. What percent of increase is this? In 2005, the tuition grew to \$26,066. What percent of increase is this over the 1999 tuition?

OBJECTIVE 3

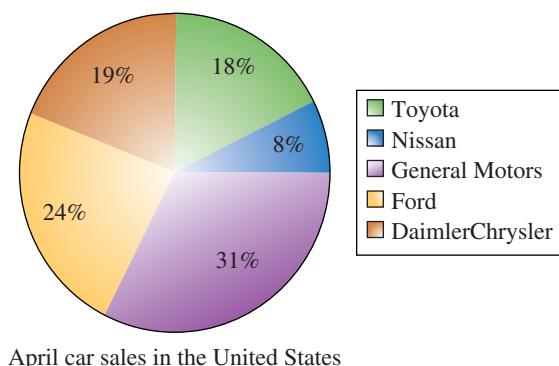
Read data from a circle graph or construct a circle graph from data.

For Exercises 51–53, the figure shows the ethnic distribution of the population of California in 2003.

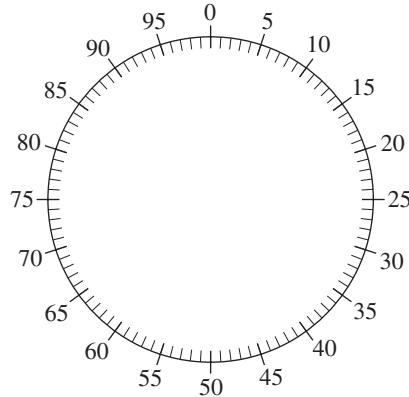
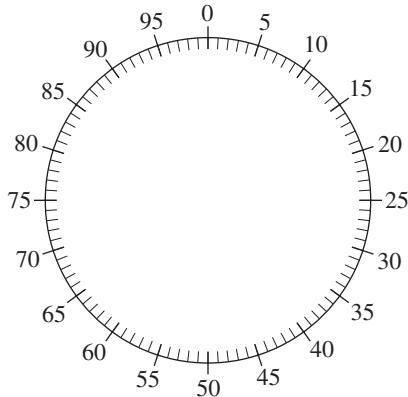


- 51.** Which identifiable ethnic group has the smallest population in California?
- 52.** What percent of the population of California is non-white?
- 53.** What is the second largest ethnic population in California?

For Exercises 54–56, the figure shows the number of new car sales by some manufacturers in April 2005.



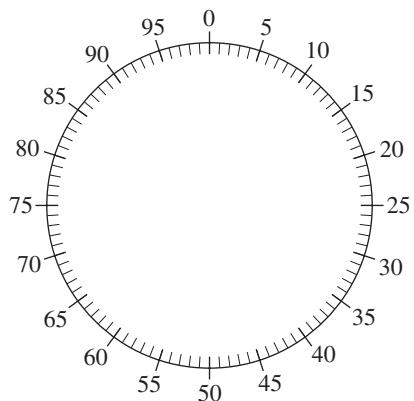
54. Which car company sold the greatest number of cars in April?
55. Which company sold more cars, Nissan or DaimlerChrysler?
56. What percent of the cars sold were not GM or Ford models?
57. In a family of three children, there are eight possibilities of boy/girl combinations. One possibility is that they are all girls. Another possibility is that they are all boys. There are three ways for the family to have two girls and a boy. There are also three ways for the family to have two boys and a girl. Use the following circle to make a circle graph to illustrate this information.
58. In 2000, Orlando, Florida, had the following percentages of households: married couples, 50%; other families, 16%; people living alone, 25%; and other non-family households, 9%. Use the circle to make a circle graph illustrating this information.



- 59.** The major causes of death worldwide are listed.

Use the following circle to make a circle graph to illustrate this information.

Infectious and parasitic diseases	32%
Heart and circulatory diseases and stroke	19%
Unknown causes	16%
Cancer	12%
Accidents and violence	8%
Infant death	6%
Chronic lung diseases	6%
Other causes	less than 1%



STATE YOUR UNDERSTANDING

- 60.** Percent comparisons go back to the Middle Ages. In your opinion, why are these kinds of comparisons still being used today?

- 62.** Explain how you know the following statement is false, without checking the math. “The population of Allensville has declined 12% over the past 10 years. In 1993 it was 500 and in 2003 it is 600.”

- 61.** When a population doubles, what is the percent of increase?

CHALLENGE

- 63.** Carol's baby weighed $7\frac{3}{4}$ lb when he was born. On his first birthday, he weighed $24\frac{3}{8}$ lb. What was the percent of increase during the year? Round to the nearest whole percent.

- 64.** Jose purchases a car for \$12,500. He makes a down payment of \$1500. His payments are \$265 per month for 48 months. What percent of the purchase cost does Jose pay for the car, including the interest? Round to the nearest tenth of a percent.

- 65.** Matilda buys the following items at Safeway: a can of peas, \$0.89; Wheaties, \$2.46; butter, \$1.78; Ivory Soap, \$1.15; broom, \$4.97; steak, \$6.78; chicken, \$3.21; milk, \$1.89; eggs, \$1.78; bread, \$1.56; peanut butter, \$2.35; stamps, \$6.80; potatoes, \$1.98; lettuce, \$2.07; and orange juice; \$2.89. What percent of the cost was in nonfood items? What percent of the cost was in meat products? Round to the nearest tenth of a percent.

GROUP WORK

- 66.** Researchers are testing the effectiveness of a new drug. There were 50,000 people in the test, half of whom received drug A (the old drug) and half of whom received drug B (the new one). Of those receiving drug A, 18,400 had relief of their symptoms. Of those receiving drug B, 18,900 had relief of their symptoms.

How would you describe the results of the test if you wanted to make it appear that drug B is much better than drug A? How would you describe the results of the test if you wanted to make it appear that drug B is only marginally better than drug A?

MAINTAIN YOUR SKILLS

- 67.** Sally and Rita are partners. How much does each receive of the income if they are to share \$12,600 in a ratio of 6 to 4 (Sally 6, Rita 4)?
- 69.** Joe buys a new car for \$2100 down and \$321 per month for 48 months. What is the total amount paid for the car?
- 71.** An engine with a displacement of 400 cubic inches develops 260 horsepower. How much horsepower is developed by an engine with a displacement of 175 cubic inches?
- 73.** Peter attended 18 of the 20 G.E.D. classes held last month. What percent of the classes did he attend?
- 75.** There are 20 problems on an algebra test. What is the percent score for 17 problems correct?
- 68.** A 9.6-m board is to be cut in two pieces in a ratio of 6 to 2. What is the length of each board after the cut?
- 70.** Pia has a savings account balance of \$3892. She deposits \$74 per month for a year. What is her account balance, not including interest earned?
- 72.** Harry pays federal income taxes of \$11,450 on an income of \$57,250. In the same tax bracket, what would be the tax on an income of \$48,000?
- 74.** A family spends \$120 for food out of a budget of \$500. What percent goes for food?
- 76.** There are 23 questions on a test for volleyball rules. What is the percent score for 18 correct answers, to the nearest percent?

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6.7 Sales Tax, Discounts, and Commissions

VOCABULARY

The **original price** of an article is the price the business sets to sell the article. When a store has a sale, the **amount of discount** is the amount subtracted from the original price (regular price). **Sales tax** is the amount charged on the final sale price to finance state and/or city programs. **Net income** is the amount left after expenses have been paid. **Commission** is the money salespeople earn based on the dollar value of the goods sold.

OBJECTIVES

1. Solve applications involving sales tax.
2. Solve applications involving discount.
3. Solve applications involving commission.

How & Why

OBJECTIVE 1

Solve applications involving sales tax.

Most states and some cities charge a tax on certain items when purchased. Stores collect this tax and send it on to the governmental unit. The amount of the sales tax is added to the purchase price to get the final cost to the buyer.

A sales tax is a percent of the purchase price. For example, a 7.5% sales tax on a purchase price of \$100 is found by taking 7.5% of \$100.

$$\text{Sales tax} = 7.5\% \text{ of } \$100 = 0.075 \times \$100 = \$7.50$$

The total cost to the customer would be

$$\begin{array}{rcl} \text{Purchase price} + \text{Sales tax} & = & \text{Total cost} \\ \$100 & + & \$7.50 = \$107.50 \end{array}$$

So the total cost to the customer is \$107.50.

Examples A–B



BASIC
Mathematics ™

DIRECTIONS: Solve sales-tax-related applications.

STRATEGY: Use the equations: Sales tax = Sales tax rate × Purchase price and Total cost = Purchase price + Sales tax.

- A. Find the sales tax and the total cost of a Sunbeam Mixmaster that has a purchase price of \$129.99 in a city where the sales tax rate is 6.8%.

$$\text{Sales tax} = \text{Sales tax rate} \times \text{Purchase price}$$

$$\begin{aligned} \text{Sales tax} &= 6.8\% \times \$129.99 \\ &= 0.068(\$129.99) \\ &= \$8.83932 \\ &\approx \$8.84 \quad \text{Round to the nearest cent.} \end{aligned}$$

So the sales tax is \$8.84.

Warm-Ups A–B

- A. Find the sales tax and the total cost of a 19-cubic-foot refrigerator that has a purchase price of \$699 in a city where the sale tax rate is 7.2%.

Answers to Warm-Ups

- A. The sales tax is \$50.33. The final price is \$749.33.

- B. Juanda buys a 30 in. stainless steel range for \$695. The cashier charges \$736.70 on her credit card. Find the sales tax and the sales tax rate.

To find the total cost to the customer, add the sales tax to the purchase price.

$$\begin{aligned}\text{Total cost} &= \$129.99 + \$8.84 \\ &= \$138.83\end{aligned}$$

So the total cost of the Mixmaster to the customer is \$138.83.

- B. Garcia buys a set of luggage for \$335. The cashier charges \$353.43 on his credit card. Find the sales tax and the sales tax rate.

To find the sales tax, subtract the purchase price from the total cost.

$$\$353.43 - \$335 = \$18.43$$

So the sales tax is \$18.43.

To find the sales tax rate we ask the question, “What percent of the purchase price is the sales tax?”

$$R \times B = A$$

$$R(\$335) = \$18.43 \quad \text{Substitute } 335 \text{ for } B \text{ and } 18.43 \text{ for } A.$$

$$R = \$18.43 \div \$335$$

$$R \approx 0.0550149$$

$$R \approx 0.055$$

Round to the nearest thousandth.

$$R \approx 5.5\%$$

The sales tax rate is 5.5%. Because all sales tax rates are exact, we can assume the approximation of the rate is due to rounding the tax to the nearest cent.

How & Why

OBJECTIVE 2

Solve applications involving discount.

MERCHANTS often discount items to move merchandise. For instance, a store might discount a dress that lists for \$125.65 by 30%. This means that the merchant will subtract 30% of the original price to determine the sale price. (Equivalently, the sale price is 70% of the original price.) To find the sale price, we first calculate the amount of the discount using the percent formula:

$R\%$ of list price is the amount of the discount.

30% of \$125.65 is the discount.

$30\% \times \$125.65 = \text{discount}$

$0.3(\$125.65) = \text{discount}$

$\$37.70 \approx \text{discount}$ Rounded to the nearest cent.

To find the sale price, subtract the discount from the original price.

$\$125.65 - \$37.70 = \text{sale price}$

$\$87.95 = \text{sale price}$

So the sale price for the dress is \$87.95.

Warm-Ups C–D

Answers to Warm-Ups

- B. The sales tax is \$41.70. The sales tax rate is 6%.

Examples C–D BASIC Mathematics Now™

DIRECTIONS: Solve discount-related applications.

STRATEGY: Use the equations: Original price – Discount = Sale price and Rate of discount × Original price = Amount of discount.

- C. Safeway offers \$20 off any purchase of \$100 or more. Jan purchases groceries totaling \$118.95. What is her total cost and what is the percent of discount?

To find the total cost to Jan, subtract \$20 from her purchases.

$$\$118.95 - \$20.00 = \$98.95$$

So the groceries cost Jan \$98.95.

To find the percent of discount answer the question:

What percent of \$118.95 is \$20.00? \$118.95 is the original cost before the discount.

Now use the formula:

$$\begin{aligned} R \times B &= A \\ R(\$118.95) &= \$20 && \text{Substitute } 118.95 \text{ for } B \text{ and } 20 \text{ for } A. \\ R &= \$20 \div \$118.95 \\ R &\approx 0.1681378 \\ R &\approx 0.17 && \text{Round to the nearest hundredth.} \\ R &\approx 17\% \end{aligned}$$

So Jan received about a 17% discount on the cost of her groceries.

- D. Supermart advertises a special Saturday morning sale. From 6 A.M. to 10 A.M., all purchases will be discounted an additional 5% off the already discounted prices. At 8:30 A.M., Carol buys a \$239 TV set that is on sale at a 25% discount. Supermart is in a city with a total sales tax of 4.5%. What does Carol pay for the TV set, including sales tax?

First find the price of the TV after the 25% discount.

25% of \$239 is the discount.

$$.25 \times \$239 = \text{discount}$$

$$\$59.75 = \text{discount}$$

$$\text{Original price} - \text{Discount} = \text{Sale price}$$

$$\$239 - \$59.75 = \$179.25$$

Now find the additional 5% discount off the sale price.

5% of the sale price is the additional discount.

$$5\% \text{ of } \$179.25 = \text{additional discount}$$

$$.05 \times \$179.25 \approx \$8.96 \quad \text{Rounded to the nearest cent.}$$

Now, to find the purchase price for Carol, subtract the additional discount from the sale price.

$$\$179.25 - \$8.96 = \$170.29$$

So Carol's purchase price for the TV is \$170.29.

Now calculate the sales tax and add it on to the purchase price to find the total cost to Carol.

$$\text{Rate of sale tax} \times \text{Purchase price} = \text{Sales tax}$$

$$4.5\% \times \$170.29 = \text{Sales tax}$$

$$\$7.66 \approx \text{Sales tax} \quad \text{Rounded to the nearest cent.}$$

$$\text{Purchase price} + \text{Sales tax} = \text{Total cost}$$

$$\$170.29 + \$7.66 = \text{Total cost}$$

$$\$177.95 = \text{Total cost}$$

Including sales tax, Carol pays \$177.95 for the TV set.

- C. Safeway offers \$20 off any purchase of \$100 or more. Gil purchases groceries totaling \$149.10. What is his total cost and what is the percent of discount?

- D. Ralph goes to the sale at the Supermart in Example D and buys a \$330 rifle that is discounted 20%. What does Ralph pay for the rifle, including the sales tax, if he buys the rifle during the 6 A.M. to 10 A.M. time period?

Answers to Warm-Ups

- C. Gil's groceries cost \$129.10. The percent of discount is about 13%.
D. Ralph pays \$262.09 for the rifle, including the sales tax.

How & Why

OBJECTIVE 3

Solve applications involving commission.

Commission is the amount of money salespeople earn based on the dollar value of goods sold. Often salespeople earn a base salary plus commission. When this happens, the total salary earned is found by adding the commission to the base salary. For instance, Nuyen earns a base salary of \$250 per week plus an 8% commission on her total sales. One week Nuyen has total sales of \$2896.75. What are her earnings for the week?

First, find the amount of her commission by multiplying the rate times the total sales.

$$\text{Commission} = \text{Rate} \times \text{Total sales}$$

$$\begin{aligned} &= 8\% \times \$2896.75 && \text{Substitute } 8\% \text{ for the rate and } \$2896.75 \\ &= 0.08 \times \$2896.75 && \text{for the total sales.} \\ &= \$231.74 \end{aligned}$$

So Nuyen earns \$231.74 in commission.

Now find her total earnings by adding the commission to her base salary.

$$\text{Total salary} = \text{Base pay} + \text{Commission}$$

$$\begin{aligned} &= \$250 + \$231.74 && \text{Substitute } \$250 \text{ for base pay and} \\ &= \$481.74 && \$231.74 \text{ for commission.} \end{aligned}$$

So Nuyen earns \$481.74 for the week.

Warm-Ups E–G

- E. Sarah works for a chemical supply company on straight commission. She earns 10% commission on all sales. Last year she had sales totaling \$1,123,450. What was her income last year?
- F. Jeremy earns a base salary of \$465 per month plus a 6.5% commission in his job at City Jewelry. In February, he had sales of \$231,745. What were his total earnings in February?

Answers to Warm-Ups

- E. Sarah's income last year was \$112,345.
- F. Jeremy earned a total of \$15,528.43 in February.

Examples E–G BASIC Mathematics Now™

DIRECTIONS: Solve commission-related applications.

STRATEGY: Use the equations: Commission = Commission rate × Total sales and Total earnings = Base pay + Commission.

- E. Monty works for a medical supply firm on straight commission. He earns 11% commission on all sales. During the month of July, his sales totaled \$234,687. How much did Monty earn during July?

$$\begin{aligned} \text{Commission} &= \text{Commission rate} \times \text{Total sales} \\ &= 11\% \times \$234,687 \\ &= 0.11 \times \$234,687 \\ &= \$25,815.57 \end{aligned}$$

Monty earned \$25,815.57 during July.

- F. Dallas earns \$900 per month plus a 4.8% commission on his total sales at Weaver's Used Cars. In November, Dallas had sales totaling \$98,650. How much did Dallas earn in November?

First find Dallas's commission earnings.

$$\begin{aligned} \text{Commission} &= \text{Commission rate} \times \text{Total sales} \\ &= 4.8\% \times \$98,650 \\ &= 0.048 \times \$98,650 \\ &= \$4735.20 \end{aligned}$$

Dallas earned \$4735.20 in commission in November.

Now find Dallas's total earnings.

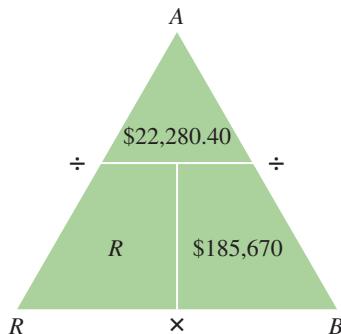
$$\begin{aligned} \text{Total earnings} &= \text{Base pay} + \text{Commission} \\ &= \$900 + \$4735.20 \\ &= \$5635.20 \end{aligned}$$

Dallas's total earnings for November were \$5635.20.

- G.** Mabel is paid a straight commission. Last month, she earned \$22,280.40 on total sales of \$185,670. What is her rate of commission?

We need to answer the question:

What percent of \$185,670 is \$22,280.40?



$$R = A \div B$$

$$R = \$22,280.40 \div \$185,670$$

$$R = 0.12$$

$$R = 12\%$$

So Mabel's rate of commission is 12%.

- G.** Tom is paid a straight commission. Last month, he earned \$4,302 on sales of \$47,800. What is his rate of commission?

Answers to Warm-Ups

- G.** Tom's rate of commission is 9%.

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Exercises 6.7

OBJECTIVE 1

Solve applications involving sales tax.

Fill in the table. Round tax rates to the nearest tenth of a percent.

	Marked Price	Sales Tax Rate	Amount of Tax	Total Cost
1.	\$238	5.8%		
2.	\$65.70	6.8%		
3.	\$90.10	3.8%		
4.	\$467	4.7%		
5.	\$821		\$69.79	
6.	\$46.30		\$3.47	
7.	\$1025			\$1090.60
8.	\$213.12			\$225.05

9. A treadmill costs \$895.89 plus a 6.8% sales tax. Find the amount of the sales tax.
10. A snowmobile costs \$4786.95 plus a 5.5% sales tax. Find the amount of the sales tax.
11. George buys a new suit at the Bon Marche. The suit costs \$476.45 plus a 6.3% sales tax. What is the total cost of the suit, including the sales tax?
12. Mildred buys a prom dress for her daughter. The dress costs \$214.50 plus a 4.6% sales tax. What is the total cost of the dress, including the sales tax?
13. Wilbur buys a new refrigerator that costs \$1075.89. When he pays for the refrigerator, the bill is \$1143.67, including the sales tax. Find the sales tax rate to the nearest tenth of a percent.
14. Helen buys an above-ground swimming pool for \$2460.61. The total bill for the pool, including sales tax, is \$2664.84. Find the sales tax rate to the nearest tenth of a percent.
15. Jim buys a new projection TV for \$2495. The store adds on a 5.6% sales tax. What is the final cost to Jim?
16. George buys a set of four new tires for his Lexus. The cost of each tire is \$119.50 plus a 6.5% sales tax. What does the set of tires cost George?
17. Bill buys a new cell phone for \$87.65, including a \$4.96 sales tax. What is the sales tax rate?
18. Marlo buys a new formal outfit for \$456.95, including a \$23.82 sales tax. What is the sales tax rate?
19. Jennifer buys a new gas barbecue grill priced at \$675.95. When she checks out, the total bill is \$726.65, including the sales tax. What is the sales tax rate?
20. Hilda buys a new self-propelled lawn mower for \$1125. When she checks out, the total bill is \$1175.63, including the sales tax. What is the sales tax rate?

OBJECTIVE 2

Solve applications involving discount.

Fill in the table.

	Original Price	Rate of Discount	Amount of Discount	Sale Price
21.	\$75.82	18%		
22.	\$25.65	20%		
23.	\$320	30%		
24.	\$587.50	25%		
25.	\$145.65			\$101.95
26.	\$82.95			\$70.51
27.			\$115.50	\$234.50
28.			\$320	\$1280

29. Joan buys an oil painting with a list price of \$189.95 at a 15% discount. What does she pay for the painting?
30. Les Schwab Tires advertises tires at a 20% discount. If a tire has a list price of \$93.65, what is the sale price?
31. Larry buys a new Lexus RX 400 SUV for 9% below the sticker price. If the sticker price is \$52,890, how much does Larry pay for the Lexus?
32. Helen's Of Course advertises leather jackets for women at a 35% discount. What will Carol pay for a jacket that has a list price of \$456.85?
33. Melvin buys a \$979.99 Honda DuraPower generator at a discount of 28%. What is the amount of the discount? What is the price Melvin pays for the generator?
34. Tran bought a new rug priced at \$785.50 at a discount of 15%. What was the amount of the discount? How much did Tran pay for the rug?
35. Raymond buys a pool table at a 14% discount from the original price plus sales tax. If the list price is \$3999.99 and Raymond's total bill is \$3629.19, what is the discount price and the sales tax rate? Find the sales tax rate to the nearest tenth of a percent.
36. Melissa buys a new lawn mower at a 20% discount from the original price plus sales tax. If the original price is \$329.99 and Melissa's final bill is \$280.89, what is the discount price? Find the sales tax rate to the nearest tenth of a percent.
37. The Galleria offers a new coat with an original price of \$167.95 for \$145.95. This weekend, they offer a discount of 10% on their already reduced price. If Catherine buys the coat this weekend, how much will she save off the original price?
38. The Garden Store sells dahlia bulbs that originally sold for \$24.75 a package at a discount price of \$21.95. This weekend, they are advertising an additional 15% off their already reduced prices. How much will Linda save off the original price for a package of dahlia bulbs this weekend?
39. The Hugh TV and Appliance Store regularly sells a TV for \$536.95. An advertisement in the paper shows that it is on sale at a discount of 25%. What is the sale price to the nearest cent?
40. A competitor of the store in Exercise 39 has the same TV set on sale. The competitor normally sells the set for \$539.95 and has it advertised at a 26% discount. To the nearest cent, what is the sale price of the TV? Which is the better buy and by how much?

- 41.** The Top Company offers a 6% rebate on the purchase of their best model of canopy. If the regular price is \$445.60, what is the amount of the rebate to the nearest cent?
- 43.** Corduroy overalls that are regularly \$42.75 are on sale for 25% off. What is the sale price of the overalls?
- 45.** A bag of Tootsie Rolls is marked “20% more free—14.5 oz for the price of 12 oz.” Assuming there has not been a change in price, is the claim accurate? Explain.
- 47.** A shoe store advertises “Buy one pair, get 50% off a second pair of lesser or equal value.” The mother of twin boys buys a pair of basketball shoes priced at \$55.99 and a pair of hikers priced at \$42.98. How much does she pay for the two pairs of shoes? What percent savings is this to the nearest tenth of a percent?
- 49.** In Exercise 48, if the salesperson says that the 10% discount can only be applied to the sale price, what is the price of the clubs?
- 42.** The Stihl Company is offering a \$24 rebate on the purchase of a chain saw that sells for \$610.95. What is the percent of the rebate to the nearest whole-number percent?
- 44.** A pair of New Balance cross trainers, which regularly sells for \$107.99, goes on sale for \$94.99. What percent off is this? Round to the nearest whole-number percent.
- 46.** A department store puts a blazer, which was originally priced at \$167.50, on sale for 20% off. At the end of the season, the store has an “additional 40% off everything that is already reduced” sale. What is the price of the blazer? What percent savings does this represent over the original price?
- 48.** The Klub House advertises on the radio that all merchandise is on sale at 25% off. When you go in to buy a set of golf clubs that originally sold for \$1375, you find that the store is giving an additional 10% discount off the original price. What is the price you will pay for the set of clubs?
- 50.** A store advertises “30% off all clearance items.” A boy’s knit shirt is on a clearance rack that is marked 20% off. How much is saved on a knit shirt that was originally priced \$25.99?

OBJECTIVE 3

Solve applications involving commission.

Fill in the table.

	Sales	Rate of Commission	Commission
51.	\$4890	9%	
52.	\$11,560	6.5%	
53.	\$67,320	8.5%	
54.	\$234,810	15%	
55.	\$1100		\$44
56.	\$1,780,450		\$62,315.75
57.		9%	\$31,212
58.		8.5%	\$1426.64

- 59.** Grant earns a 9% commission on all of his sales at a downtown department store. If Grant’s sales for the week totaled \$4365.90, what was his commission?
- 60.** Mayfair Real Estate Co. charges a 6.3% commission on homes it sells. What is the commission on the sale of a \$357,890 home? Round to the nearest dollar.

- 61.** Walt's Ticket Agency charges a 7% commission on all ticket sales. What is the commission charged for eight tickets priced at \$45.50 each?
- 63.** A salesperson at Wheremart earns \$500 per week plus a commission of 9% on all sales over \$2000. Last week, he had total sales of \$4678.50. How much did he earn last week?
- 65.** Juanita earns a 9% commission on all of her sales. How much did she earn last week if her total sales were \$7285?
- 67.** Matthew receives a weekly salary of \$210 plus a commission of 8% on his sales. Last week he earned \$625. What were his total sales for the week?
- 69.** During 1 week Ms. James sold a total of \$26,725 worth of hardware to the stores in her territory. She receives a 4% commission on sales of \$2000 or less, 5% on that portion of her sales over \$2000 and up to \$15,000, and 6% on all sales over \$15,000. What was her total commission for the week?
- 62.** A medical supplies salesperson earns an 8% commission on all sales. Last month, she had total sales of \$345,980. What was her commission?
- 64.** A salesperson at Goldman's Buick earns a base salary of \$1800 per month plus a commission of 2.5% on all sales. If she sold cars totaling \$178,740 during June, how much did she earn that month?
- 66.** Martin earns a 12.5% commission on all sales. How much did he earn last week if his total sales were \$9300?
- 68.** Amy receives a weekly salary of \$185 plus a commission of 9% on her total sales. How much did she earn last week if her total sales were \$3128?
- 70.** If Ms. James, in Exercise 69, had sales of \$22,455 the next week, how much did she earn in commissions?

STATE YOUR UNDERSTANDING

- 71.** Does a sales tax represent a percent increase or decrease in the price a consumer pays for an item? Explain.
- 72.** When a salesperson is working on commission, is it to his or her advantage to sell you a modestly priced item or an expensive item? Why?

CHALLENGE

- 73.** Marlene's sales job pays her \$1500 per month plus commissions of 9% for sales up to and including \$30,000 and 5% for all sales over \$30,000. In July Marlene had sales of \$18,700 and in August she had sales of \$49,500. What was Marlene's salary for each of the two months?
- 74.** Fransica bought a sweater that was originally priced at \$136 on Senior Day at the department store. The sweater was on sale at 30% off the original price. On Senior Day, the store offers seniors an additional 25% off the sale price. What did Fransica pay for the sweater? The next day, the store had the same sweater on sale again for 30% off the original price plus a 15% discount on the original price. Maria bought the sweater and used a \$5 coupon. What did Maria pay for the sweater? Who got the sweater for the best price and by how much?

MAINTAIN YOUR SKILLS*Add.*

75. $\frac{11}{24} + \frac{13}{36}$

76. $\frac{17}{21} + \frac{13}{28}$

Subtract.

77. $\frac{9}{16} - \frac{1}{12}$

78. $13\frac{1}{3} - 9\frac{5}{12}$

Multiply.

79. $\frac{7}{16} \cdot \frac{8}{21}$

80. $6\frac{5}{6} \cdot 3\frac{2}{3}$

Divide.

81. $\frac{11}{12} \div \frac{33}{52}$

82. $5\frac{5}{6} \div 2\frac{1}{3}$

83. Find the perimeter of the rectangle.

84. Find the area of the rectangle in Exercise 83.

 $6\frac{3}{5}$ in. $2\frac{7}{8}$ in.

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6.8 Interest on Loans

VOCABULARY

Interest is the fee charged for borrowing money. It is usually assessed as a percent of the money borrowed, or the **interest rate**. **Principal** is the amount of money borrowed. When the interest is based on borrowing the money for 1 year it is called **simple interest**. **Compound interest** occurs when interest is computed on interest already earned. **Interest** is also the money paid for use of your money. Interest is paid on savings and on investments.

OBJECTIVES

1. Calculate simple interest.
2. Calculate compound interest.
3. Solve applications related to credit card payments.

How & Why

OBJECTIVE 1

Calculate simple interest.

Simple interest is seldom used these days in the business world. You are more likely to find it when borrowing money from a family member or friend. For instance, Joyce borrows \$2000 from her uncle to pay for this year's tuition. She agrees to pay her uncle 4% interest on the money at the end of a year. To find the interest Joyce owes her uncle at the end of the year, we use the equation:

$$\text{Simple interest} = \text{Principal} \times \text{Interest rate} \times \text{Time}$$

Here, the principal is \$2000, the interest rate is 4%, and the time is 1 year. Substituting we have,

$$\begin{aligned}\text{Simple interest} &= \$2000 \times 4\% \times 1 \\ &= \$2000 \times 0.04 \times 1 \\ &= \$80\end{aligned}$$

So Joyce owes her uncle \$80 in interest at the end of 1 year. She owes a total of \$2080.

If the money is borrowed for less than 1 year, the time is expressed as a fraction of a year. See Example B.

Examples A–B



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DIRECTIONS: Find simple interest.

STRATEGY: Use the equation: Simple interest = Principal × Interest rate × Time

A. Juan borrows \$4500 at 7.5% simple interest to buy a new plasma TV. He agrees to pay back the entire amount at the end of 3 years. How much interest will Juan owe? What is the total amount he will owe at the end of 3 years?

$$\text{Simple interest} = \text{Principal} \times \text{Interest rate} \times \text{Time}$$

$$\begin{aligned}&= \$4500 \times 7.5\% \times 3 && \text{Principal is } \$4500, \text{ interest rate is } 7.5\%, \\ &= \$4500 \times 0.075 \times 3 && \text{and the time period is 3 years.} \\ &= \$1012.50\end{aligned}$$

At the end of 3 years, Juan will owe \$1012.50 in interest.

To find the total amount Juan will owe, add the interest to the principal.

$$\begin{aligned}\text{Total amount owed} &= \text{Principal} + \text{Interest} \\ &= \$4500 + \$1012.50 \\ &= \$5512.50\end{aligned}$$

So at the end of 3 years, Juan will owe \$5512.50.

Warm-Ups A–B

- A. Fife borrows \$1600 at 8% simple interest to buy a computer setup. She agrees to pay back the entire amount at the end of 2 years. How much interest will Fife owe? What is the total amount she will owe at the end of the 2 years?

Answers to Warm-Ups

- A. Fife will owe \$256 in interest and a total of \$1856.

- B. Juanita borrows \$750 at 5% simple interest to help her move into a new apartment. She agrees to pay back the entire amount at the end of 6 months. How much interest does she pay, and what is the total amount owed after the 6 months?

- B. Alex borrows \$875 at 6% simple interest to pay for his vacation. He agrees to pay back the entire amount, including interest, at the end of 9 months. How much interest does he pay? What is the total amount he owes after the 9 months?

First find the interest:

$$\text{Simple interest} = \text{Principal} \times \text{Interest rate} \times \text{Time}$$

$$\begin{aligned} &= \$875 \times 6\% \times 0.75 && \text{Nine months is } \frac{9}{12} \text{ of a year, or } \frac{3}{4} \text{ or } 0.75 \\ &= \$875 \times 0.06 \times 0.75 && \text{of a year.} \\ &\approx \$39.38 && \text{Round to the nearest cent.} \end{aligned}$$

So Alex owes \$39.38 in interest.

Now find the total amount owed:

$$\text{Total amount owed} = \text{Principal} + \text{Interest}$$

$$\begin{aligned} &= \$875 + \$39.38 \\ &= \$914.38 \end{aligned}$$

So Alex owes \$39.38 in interest and a total of \$914.38.

How & Why

OBJECTIVE 2

Calculate compound interest.

When interest is compounded, the interest earned at the end of one time period is added to the principal and earns interest during the next time period. For instance, if you invest \$2000 at 5% interest compounded semiannually your account is credited with interest in June. During the second half of the year, you will earn interest on both the principal and the interest earned during the first 6 months. To calculate the total interest we first find the simple interest earned after 6 months.

$$\begin{aligned} \text{Simple interest} &= \text{Principal} \times \text{Interest rate} \times \text{Time} \\ &= \$2000 \times 5\% \times 6 \text{ mo} \\ &= \$2000 \times 0.05 \times 0.5 && 6 \text{ mo} = 0.5 \text{ yr} \\ &= \$50 \end{aligned}$$

So \$50 is earned in interest after 6 months. This amount is added to the principal for the next time period. So for the last 6 months the principal is

$$\begin{aligned} \text{New principal} &= \$2000 + \$50 \\ &= \$2050 \end{aligned}$$

Now calculate the interest earned during the next 6 months.

$$\begin{aligned} \text{Simple interest} &= \text{Principal} \times \text{Interest rate} \times \text{Time} \\ &= \$2050 \times 5\% \times 6 \text{ mo} && \text{The new principal is } \$2050. \\ &= \$2050 \times 0.05 \times 0.5 \\ &= \$51.25 \end{aligned}$$

So an additional \$51.25 is earned during the last 6 months. To find the balance at the end of the year, add the new interest to the new principal.

$$\begin{aligned} \text{Total value at the end of 1 year} &= \$2050 + \$51.25 \\ &= \$2101.25 \end{aligned}$$

We can now find the total interest earned by subtracting the original principal from the ending balance.

$$\begin{aligned} \text{Total interest earned} &= \text{Ending balance} - \text{Original principal} \\ &= \$2101.25 - \$2000 \\ &= \$101.25 \end{aligned}$$

So the total interest earned is \$101.25.

Simple interest for the year would have been \$100, so by compounding the interest semiannually an additional \$1.25 was earned. This may not seem like a lot of money, but

Answers to Warm-Ups

- B. Juanita owes \$18.75 in interest and a total of \$768.75.

if the interest was compounded daily and over a number of years it would amount to a lot of money. For instance, \$10,000 invested at 5% simple interest will have a balance of \$15,000 at the end of 10 years. However if the \$10,000 was invested at 5% compounded daily it would grow to \$16,486.65, which is \$1486.65 more than at simple interest.

Computing compound interest can be very tedious, especially as the number of periods increase per year. To ease this burden, accountants have developed compound interest tables that provide a factor to use in calculating the ending balance. Table 6.5 gives the factors for interest rates that are compounded quarterly (4 times per year).

Table 6.5 Compound Interest Factors for Quarterly Compounding

Rate	Years					
	1	5	10	15	20	25
2%	1.0202	1.1049	1.2208	1.3489	1.4903	1.6467
3%	1.0303	1.1612	1.3483	1.5657	1.8180	2.1111
4%	1.0406	1.2202	1.4889	1.8167	2.2167	2.7048
5%	1.0509	1.2820	1.6436	2.1072	2.7015	3.4634
6%	1.0614	1.3469	1.8140	2.4432	3.2907	4.4320
7%	1.0719	1.4148	2.0016	2.8318	4.0064	5.6682

To use the table to find the ending balance, multiply the original principal by the factor in the table.

$$\text{Ending balance} = \text{Original principal} \times \text{Compound factor}$$

To calculate the value of \$12,000 at 6% interest compounded quarterly for 15 years we write:

$$\begin{aligned}\text{Ending balance} &= \text{Original principal} \times \text{Compound factor} \\ &= \$12,000 \times 2.4432 \quad \text{Multiply by the compound factor from table.} \\ &= \$29,318.40\end{aligned}$$

So \$12,000 will grow to \$29,318.40 at the end of 15 years.

The total interest earned can be found by subtracting the original principal from the ending balance. The total interest earned is

$$\$29,318.40 - \$12,000 = \$17,318.40$$

The account earned \$17,318.40 in interest.

Most banks and credit unions compute the interest daily or continuously. For these factors see Appendix D, on page A-7.

Examples C–D



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DIRECTIONS: Find compound interest and ending balances.

STRATEGY: Use the equations: Ending balance = Principal × Compound factor; Interest = Ending balance – Original principal

- C. Fred invests \$45,000 at 7% interest compounded quarterly for 20 years. Find the value of his investment at the end of the 20 years. Find the interest earned.

First find the ending balance:

$$\begin{aligned}\text{Ending balance} &= \text{Principal} \times \text{Compound factor} \\ &= \$45,000 \times 4.0064 \quad \text{Multiply by the compound factor found} \\ &\quad \text{in Table 6.5.} \\ &= \$180,288\end{aligned}$$

Warm-Ups C–D

- C. Janis invests \$20,000 at 3% interest compounded quarterly for 10 years. Find the value of her investment at the end of the 10 years. Find the interest earned.

Answers to Warm-Ups

- C. Janis's investment is now worth \$26,966, and she earned \$6966 in interest.

- D. Mitchell invests \$1850 at 5% interest compounded daily for 5 years. Find the value of his investment at the end of the 5 years and the interest earned. Use the table in Appendix D.

Now find the interest earned:

$$\begin{aligned}\text{Interest} &= \text{Ending balance} - \text{Original principal} \\ &= \$180,288 - \$45,000 \\ &= \$135,288\end{aligned}$$

So Fred's investment is now worth \$180,288, and the interest earned is \$135,288.

- D. Joan invests \$75,000 at 4% interest compounded daily for 10 years. Find the value of her investment at the end of the 10 years and the interest earned. Use the table in Appendix D.

First find the ending balance:

$$\begin{aligned}\text{Ending balance} &= \text{Principal} \times \text{Compound factor} \\ &= \$75,000 \times 1.4918 \quad \text{Multiply by the compound factor found in} \\ &= \$111,885 \quad \text{Appendix D.}\end{aligned}$$

Now find the interest earned:

$$\begin{aligned}\text{Interest} &= \text{Ending balance} - \text{Original principal} \\ &= \$111,885 - \$75,000 \\ &= \$36,885\end{aligned}$$

So the value of Joan's investment is \$111,885, and the interest earned is \$36,885.

How & Why

OBJECTIVE 3

Solve applications related to credit card payments.

In 2006, federal regulations required a change in the way credit card companies calculate the minimum payment.

Credit card companies and most major department stores charge a fixed interest rate on the unpaid balance in an account. The minimum payment is usually 2% of the unpaid balance rounded to the nearest dollar. So if a credit card company charges 19.8% on the unpaid balance and the unpaid balance is \$785.75, we can calculate the minimum payment.

$$\begin{aligned}\text{Minimum payment} &= \text{Unpaid balance} \times \text{Minimum payment rate} \\ &= \$785.75 \times 2\% \\ &= \$785.75 \times 0.02 \\ &\approx \$16 \quad \text{Rounded to the nearest dollar.}\end{aligned}$$

To find the amount of interest in the \$16 minimum payment, we need to calculate the monthly interest on the credit card balance. To do this we find the simple interest per year on the balance at 19.8% and divide it by 12 to find the monthly interest.

$$\begin{aligned}\text{Monthly interest} &= (\text{Unpaid balance} \times \text{Interest rate}) \div 12 \\ &= (\$785.75 \times 19.8\%) \div 12 \\ &= (\$785.75 \times 0.198) \div 12 \\ &= \$155.5785 \div 12 \\ &\approx \$12.96 \quad \text{Rounded to the nearest cent.}\end{aligned}$$

So the interest charged is \$12.96. The amount applied to the unpaid balance is found by subtracting the interest charge from the payment.

$$\begin{aligned}\text{Amount applied to unpaid balance} &= \text{Minimum payment} - \text{Interest charge} \\ &= \$16 - \$12.96 \\ &= \$3.04\end{aligned}$$

Now find the unpaid balance.

$$\begin{aligned}\text{Unpaid balance} &= \text{Beginning unpaid balance} - \text{Amount applied to unpaid balance} \\ &= \$785.75 - \$3.04 \\ &= \$782.71\end{aligned}$$

If no further charges are made to the account, next month's payment will be based on the new balance of \$782.71.

Answers to Warm-Ups

- D. Mitchell's investment is now worth \$2375.40, and he earned \$525.40 in interest.

If credit card companies receive the payment late they often add on late fees, which add to the unpaid balance (see Example G). Some credit card companies will also increase the interest rate on accounts when the payment is received late. For instance, a company that charges a rate of 18.5% may raise the rate to 24.5% for receiving a payment late. This rate then continues for the life of the card.

Examples E–G



BASIC
Mathematics = Now™

DIRECTIONS: Find the minimum payment, interest paid, and unpaid balance.

STRATEGY: Use the equations:

1. Fixed monthly payments for a set period of time.

$$\text{Interest paid} = (\text{Monthly payment} \times \text{Number of months}) - \text{Principal}$$

2. Credit and charge card payments.

$$\begin{aligned}\text{Minimum payment} &= \text{Unpaid balance} \times \text{Minimum payment rate} \\ &\quad (\text{rounded to the nearest dollar})\end{aligned}$$

$$\text{Monthly interest} = (\text{Unpaid balance} \times \text{Interest rate}) \div 12$$

$$\begin{aligned}\text{Amount applied} \\ \text{to unpaid balance} &= \text{Minimum payment} - \text{Interest charge}\end{aligned}$$

$$\text{Unpaid balance} = \text{Beginning unpaid balance} - \text{Amount applied to unpaid balance}$$

- E. Millie buys a refrigerator for \$1100. She makes a down payment of \$100 and agrees to monthly payments of \$62.39 for 18 months. How much interest does she pay for the refrigerator by financing the remaining \$1000?

$$\begin{aligned}\text{Interest paid} &= (\text{Monthly payment} \times \text{Number of months}) - \text{Principal} \\ &= (\$62.39 \times 18) - \$1000 \\ &= \$1123.02 - \$1000 \\ &= \$123.02\end{aligned}$$

So Millie pays \$123.02 in interest.

- F. Ivan has an unpaid balance of \$2540 on his credit card. The credit card company charges an interest rate of 15.7% and requests a minimum payment of 2% of the unpaid balance. Calculate the minimum payment. If Ivan makes the minimum payment, calculate the interest paid and the new unpaid balance.

First, find the minimum payment:

$$\begin{aligned}\text{Minimum payment} &= \text{Unpaid balance} \times \text{Minimum payment rate} \\ &= \$2540 \times 2\% \\ &\approx \$51 \quad \text{Round to the nearest dollar.}\end{aligned}$$

So the minimum payment is \$51.

Now find the amount of interest paid.

$$\begin{aligned}\text{Monthly interest} &= (\text{Unpaid balance} \times \text{Interest rate}) \div 12 \\ &= (\$2540 \times 15.7\%) \div 12 \\ &= (\$2540 \times 0.157) \div 12 \\ &\approx \$33.23 \quad \text{Round to the nearest cent.}\end{aligned}$$

So the interest paid is \$33.23.

Now find the amount applied to the unpaid balance.

$$\begin{aligned}\text{Amount applied to unpaid balance} &= \text{Minimum payment} - \text{Interest charge} \\ &= \$51 - \$33.23 \\ &= \$17.77\end{aligned}$$

Warm-Ups E–G

- E. Myron buys a 2005 Dodge Grand Caravan for \$18,964. Myron pays nothing down and agrees to make monthly payments of \$276 per month for 72 months. How much does Myron pay in interest?

- F. Lucy has an unpaid balance of \$1210 on her credit card. The company charges an interest rate of 19.6% and requests a minimum payment of 2% of the unpaid balance. If Lucy makes the minimum payment, calculate the minimum payment, the interest paid, and the new unpaid balance.

Answers to Warm-Ups

- E. Myron pays \$908 in interest.
F. Lucy makes a minimum payment of \$24; \$19.76 is interest and \$4.24 is paid on the balance. Her new unpaid balance is \$1205.76.

- G. Ruby has an unpaid balance of \$4576 on her credit card. The company charges an interest rate of 18.6% and requests a minimum payment of 2% of the unpaid balance. Ruby makes the minimum payment but is 4 days late. If the company charges a late fee of \$30, calculate the minimum payment, the interest paid, and the new unpaid balance.

Now subtract the amount paid on the unpaid balance to find the new unpaid balance.

$$\begin{aligned}\text{Unpaid balance} &= \text{Beginning unpaid balance} - \text{Amount applied to unpaid balance} \\ &= \$2540 - \$17.77 \\ &= \$2522.23\end{aligned}$$

So Ivan made a minimum payment of \$51; \$33.23 of this payment was interest and the new unpaid balance is \$2522.23.

- G. Lance has an unpaid balance of \$1680 on his credit card. The credit card company charges an interest rate of 18.6% and requests a minimum payment of 2% of the unpaid balance. Lance makes the minimum payment but is late by 5 days. If the credit card company charges a late fee of \$25, calculate the minimum payment, the interest paid, and the new unpaid balance.

First, find the minimum payment:

$$\begin{aligned}\text{Minimum payment} &= \text{Unpaid balance} \times \text{Minimum payment rate} \\ &= \$1680 \times 2\% \\ &\approx \$34 \quad \text{Round to the nearest dollar.}\end{aligned}$$

So the minimum payment is \$34. Now find the amount of interest paid.

$$\begin{aligned}\text{Monthly interest} &= (\text{Unpaid balance} \times \text{Interest rate}) \div 12 \\ &= (\$1680 \times 18.6\%) \div 12 \\ &= (\$1680 \times 0.186) \div 12 \\ &= \$26.04\end{aligned}$$

So the interest paid is \$26.04.

Now find the amount applied to the unpaid balance.

$$\begin{aligned}\text{Amount applied to unpaid balance} &= \text{Minimum payment} - \text{Interest charge} \\ &= \$34 - \$26.04 \\ &= \$7.96\end{aligned}$$

Now subtract the amount paid on the unpaid balance to find the new unpaid balance, before the late fee.

$$\begin{aligned}\text{Unpaid balance} &= \text{Beginning unpaid balance} - \text{Amount applied to unpaid balance} \\ &= \$1680 - \$7.96 \\ &= \$1672.04\end{aligned}$$

Add the \$25 late fee to the new unpaid balance.

$$\$1672.04 + \$25.00 = \$1697.04$$

So Lance made a minimum payment of \$34, \$26.04 of this payment was interest. Because of the late fee, Lance now has a balance of \$1697.04, which is more than the original balance.

Answers to Warm-Ups

- G. Ruby makes a minimum payment of \$92. She pays \$70.93 in interest and has a new unpaid balance of \$4584.93.

Exercises 6.8

OBJECTIVE 1

Calculate simple interest.

Find the interest and the total amount due on the following simple interest loans.

	Principal	Rate	Time	Interest	Total Amount Due
1.	\$10,000	5%	1 year		
2.	\$960	7%	1 year		
3.	\$5962	8%	1 year		
4.	\$23,700	3.5%	1 year		
5.	\$24,000	7.5%	4 years		
6.	\$1560	4%	5 years		
7.	\$4500	5.5%	9 months		
8.	\$850	10%	8 months		

9. Maria invests \$1500 for 1 year at 7.5% simple interest. How much interest does she earn?
10. Randolph borrows \$6700 at 6.9% simple interest. How much interest does he owe after 1 year?
11. Nancy invests \$8500 at 5.5% simple interest for 3 years. How much interest has she earned at the end of the 3 years?
12. Vince invests \$24,800 at 6% simple interest for 5 years. How much interest does he earn during this time period?
13. Janelle invests \$4500 at 4.5% simple interest for 8 months. How much interest does she earn?
14. Roberto borrows \$1700 at 7% simple interest. At the end of 5 months, he pays off the loan and interest. How much does he pay to settle the loan?
15. Janna borrows \$2300 at 6.5% simple interest. At the end of 10 months, she pays off the loan and interest. How much does she pay to settle the loan?
 - a. What was the interest paid?
 - b. What was the interest rate?
17. Ramon borrows \$45,800 at simple interest, and at the end of 1 year he pays off the loan at a cost of \$50,838.
 - a. What was the interest paid?
 - b. What was the interest rate?

OBJECTIVE 2

Calculate compound interest.

Find the ending balance in the accounts. See Appendix D.

- 18.** The account opens with \$6000, earns 5% interest compounded quarterly, and is held for 10 years.
- 20.** The account opens with \$16,000, earns 3% interest compounded monthly, and is held for 10 years.
- 21.** The account opens with \$60,000, earns 6% interest compounded monthly, and is held for 20 years.

Find the amount of compound interest earned. See Appendix D.

- 22.** The account opens with \$7500, earns 4% interest compounded quarterly, and is held for 10 years.
- 24.** The account opens with \$24,000, earns 7% interest compounded daily, and is held for 5 years.
- 25.** The account opens with \$95,000, earns 6% interest compounded daily, and is held for 15 years.
- 26.** Mary invests \$7000 at 6% compounded quarterly. Her sister Catherine invests \$7000 at 6% compounded daily. If both sisters hold their accounts for 10 years, how much more interest will Catherine's account earn?
- 28.** Jim invests \$25,000 at 5% simple interest for 10 years. Carol invests \$25,000 at 5% compounded daily for 10 years. How much more interest does Carol's account earn?
- 27.** Jose invests \$17,000 at 4% compounded quarterly. His sister Juanita invests \$17,000 at 4% compounded daily. If they both hold their accounts for 15 years, how much more interest will Juanita's account earn?
- 29.** Lucy invests \$15,000 at 3% simple interest for 5 years. Carl invests \$15,000 at 3% compounded daily for 5 years. How much more interest does Carl's account earn?

OBJECTIVE 3

Solve applications related to credit card payments.

- 30.** Felicia's credit card has a balance owed at the end of January of \$1346.59. The credit card company charges a rate of 19.8% on the unpaid balance. Felicia makes the minimum payment of \$27.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of February, assuming no additional charges were made?
- 31.** Mark's credit card has a balance owed at the end of May of \$3967.10. The credit card company charges a rate of 17.5% on the unpaid balance. Mark makes a payment of \$80.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of June, assuming no additional charges were made?

- 32.** Luis's credit card has a balance owed at the end of June of \$2476.10. The credit card company charges a rate of 19.8% on the unpaid balance. Luis makes the minimum payment of \$50.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of July, if Luis uses his card to make \$55.75 in additional charges?
- 34.** Greg's credit card has a balance owed at the end of November of \$6789.43. The credit card company charges a rate of 20.5% on the unpaid balance. Greg makes the minimum payment of \$136, but the credit card company receives it 3 days late and charges a late fee of \$30.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of December, assuming no additional charges were made?
- 33.** Debbi's credit card has a balance owed at the end of March of \$8765.25. The credit card company charges a rate of 21.5% on the unpaid balance. Debbi makes the minimum payment of \$175.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of April, if Debbi uses her card to make \$234.50 in additional charges?
- 35.** Belle's credit card has a balance owed at the end of May of \$5677.17. The credit card company charges a rate of 18.8% on the unpaid balance. Belle makes the minimum payment of \$114, but the credit card company receives it 1 day late and charges a late fee of \$25.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of June, assuming no additional charges were made?

STATE YOUR UNDERSTANDING

- 36.** What is the difference between simple interest and compound interest? Which is more advantageous to you as an investor?
- 37.** Explain how it is possible to have your credit card debt increase while making minimum monthly payments.

CHALLENGE

- 38.** Linn has a balance of \$1235.60 on her credit card. The credit card company has an interest rate of 18.6%. The company requires a minimum payment of 2% of the unpaid balance, rounded to the nearest dollar. If Linn makes no additional purchases with her card and makes the minimum payment monthly, what will be her balance at the end of 1 year? How much interest will she have paid?
- 39.** In Exercise 38, if Linn makes a \$100 payment each month and makes no additional purchases, how many months will it take to pay off the balance? How much interest will she have paid?

MAINTAIN YOUR SKILLS*Add.*

40. $456 + 387 + 1293 + 781$

41. $32.67 + 45.098 + 102.5 + 134.76$

Subtract.

42. $34,761 - 29,849$

43. $134.56 - 98.235$

Multiply.

44. $(341)(56)$

45. $(56.72)(0.023)$

Divide.

46. $9324 \div 36$

47. $76.4 \div 1.34$ Round to the nearest thousandth.

- 48.** Geri works at two jobs each week. Last week, she worked 25 hours at the job paying \$7.82 per hour and 23 hours at the job that pays \$10.42 per hour. How much did she earn last week?

- 49.** Mary divided her estate equally among her seven nieces and nephews. The executor of the estate received a 5% commission for handling the estate. How much did each niece and nephew receive if the estate was worth \$1,456,000?

Key Concepts CHAPTER 6

Section 6.1 The Meaning of Percent

Definitions and Concepts	Examples
A percent is a ratio with a base unit (the denominator) of 100.	$75\% = \frac{75}{100}$
The symbol % means $\frac{1}{100}$ or 0.01.	If 22 of 100 people are left-handed, what percent is this?
$100\% = \frac{100}{100} = 1$	$\frac{22}{100} = 22\%$

Section 6.2 Changing Decimals to Percents and Percents to Decimals

Definitions and Concepts	Examples
To change a decimal to a percent:	
<ul style="list-style-type: none"> Move the decimal point two places to the right (write zeros on the right if necessary). Write % on the right. 	$0.23 = 23\%$ $5.7 = 570\%$
To change a percent to a decimal:	
<ul style="list-style-type: none"> Move the decimal point two places to the left (write zeros on the left if necessary). Drop the percent symbol (%). 	$67\% = 0.67$ $2.8\% = 0.028$

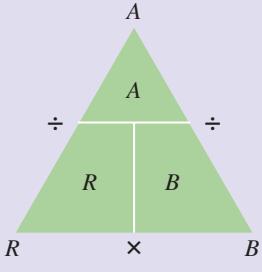
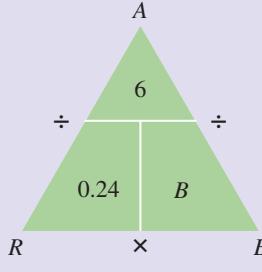
Section 6.3 Changing Fractions to Percents and Percents to Fractions

Definitions and Concepts	Examples
To change a fraction or mixed number to a percent:	
<ul style="list-style-type: none"> Change to a decimal. Change the decimal to a percent. 	$\frac{6}{7} \approx 0.857 \approx 85.7\%$ $3\frac{4}{25} = 3.16 = 316\%$
To change a percent to a fraction:	
<ul style="list-style-type: none"> Replace the percent symbol (%) with $\frac{1}{100}$. If necessary, rewrite the other factor as a fraction. Multiply and simplify. 	$45\% = 45 \cdot \frac{1}{100} = \frac{45}{100} = \frac{9}{20}$ $6.5\% = 6.5 \cdot \frac{1}{100} = \frac{13}{2} \cdot \frac{1}{100} = \frac{13}{200}$

Section 6.4 Fractions, Decimals, Percents: A Review

Definitions and Concepts	Examples		
	Fraction	Decimal	Percent
Every number has three forms: fraction, decimal, and percent.	$\frac{7}{10}$	0.7	70%
	$5\frac{3}{8}$	5.375	537.5%

Section 6.5 Solving Percent Problems

Definitions and Concepts	Examples		
The percent formula is $R \times B = A$, where R is the rate of percent, B is the base, and A is the amount.	6 is 24% of what number? $A = 6, R = 0.24, B = ?$	$B = A \div R$ $B = 6 \div 0.24$ $B = 25$	
			
To solve percent problems using a proportion, set up the proportion.	So 6 is 24% of 25. 8 is what percent of 200? $A = 8, B = 200, R = X\% = ?$		
$\frac{X}{100} = \frac{A}{B}, \text{ where } R = X\%$	$\frac{8}{200} = \frac{X}{100}$ $8 \cdot 100 = 200X$ $800 \div 200 = X$ $4 = X$ So 8 is 4% of 200.		

Section 6.6 Applications of Percents

Definitions and Concepts

To solve percent applications:

- Restate the problem as a simple percent statement.
- Identify values for A , R , and B .
- Use the percent formula or a proportion.

When a value B is increased (or decreased) by an amount A , the rate of percent R is called the percent of increase (or decrease).

Examples

Of 72 students in Physics 231, 32 are women. What percent of the students are women?

Restate: 32 is what percent of 72?
 $32 = A$, $72 = B$, $R = X\% = ?$

$$\begin{aligned}\frac{32}{72} &= \frac{X}{100} \\ 32 \cdot 100 &= 72X \\ 3200 \div 72 &= X \\ X &\approx 44.4\end{aligned}$$

So about 44.4% of the students are women.

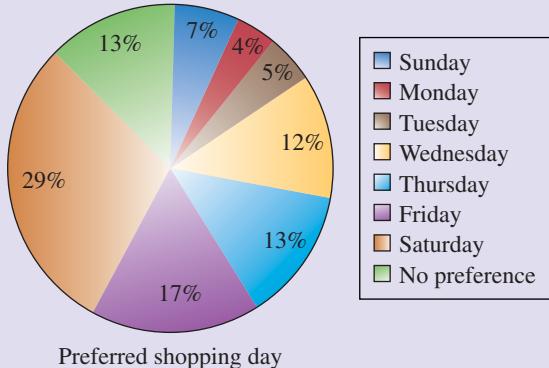
If a \$230,000 home increases in value to \$245,000 in 1 year, what was the percent of increase?

$$\text{Increase} = \$245,000 - \$230,000 = \$15,000$$

So \$15,000 is what percent of \$230,000?

$$\begin{aligned}R &= A \div B \\ R &= \$15,000 \div \$230,000 \\ R &\approx 0.065, \text{ or } 6.5\%\end{aligned}$$

Circle graphs convey information about how an entire quantity is composed of its various parts. The size of each sector indicates what percent the part is of the whole.



Most people (29%) prefer to shop on Saturday. Monday and Tuesday are the least favorite days to shop.

Section 6.7 Sales Tax, Discounts, and Commissions

Definitions and Concepts	Examples
Sales tax is a percent of the purchase price that is added to the final price.	An electric oven costs \$650. The sales tax is 6%. Find the final purchase price. Find the amount of sales tax, 6% of \$650. $\begin{aligned} \text{Sales tax} &= 6\% \times \$650 \\ &= 0.06 \times \$650 \\ &= \$39 \end{aligned}$
$\text{Sales tax} = \text{Sales tax rate} \times \text{Purchase price}$	Add the sales tax to the cost. $\$650 + \$39 = \$689$ The oven's total cost is \$689.
Total cost = Purchase price + Sales tax	
A discount is a percent of the regular price that is subtracted from the price.	A calculator that sells for \$89 is put on sale for 20% off. Find the sale price. Find the amount of discount, 20% of \$89. $\begin{aligned} \text{Amount of discount} &= 20\% \times \$89 \\ &= \$17.8 \end{aligned}$
$\text{Amount of discount} = \text{Rate of discount} \times \text{Original price}$	$\$89 - \$17.8 = \$71.2$ The calculator's sale price is \$71.20.
Sale price = Original price - Discount	
A commission is a percent of the value of goods sold that is earned by the salesperson.	Larry earns \$400 per month plus a 4% commission. One month, he sold \$12,300 worth of appliances. Find his earnings for the month. Find the amount of the commission, 4% of \$12,300. $\begin{aligned} \text{Commission} &= 4\% \times \$12,300 \\ &= 0.04 \times \$12,300 \\ &= \$492 \end{aligned}$
$\text{Commission} = \text{Commission rate} \times \text{Total sales}$	Add the commission to his salary. $\$400 + \$492 = \$892$ Larry earns \$892 for the month.
Total earnings = Base pay + Commission	

Section 6.8 Interest on Loans

Definitions and Concepts	Examples
Simple interest is paid once, at the end of the loan.	Scott borrowed \$30,000 from his rich aunt. He will pay her 4% simple interest and keep the money for 2 years. How much will he owe his aunt? $\begin{aligned} \text{Interest} &= \$30,000 \times 4\% \times 2 \\ &= \$30,000 \times 0.04 \times 2 \\ &= \$2400 \end{aligned}$ Add the interest to the principal. $\$2400 + \$30,000 = \$32,400$ Scott will owe his aunt \$32,400.
Simple interest = Principal × Interest rate × Time	Maureen invests \$5000 in an account that pays 6% compounded quarterly. How much will be in the account after 3 years?
Compound interest is paid periodically, so after the first period, interest is paid on interest earned as well as on the principal.	Multiply the principal by the compound factor. $\begin{aligned} \text{Ending balance} &= \$5000 \times 1.1956 \\ &= \$5978 \end{aligned}$ Maureen will have \$5978 in her account after 3 years.
Ending balance = Principal × Compound factor The compound factor can be found in Appendix D.	Karen has a balance of \$876 on her credit card. She must make a 2% minimum payment, and pay 15.99% per year in interest on the balance. Find her new balance.
Credit card payments are a percent of the balance. The interest owed is paid first, and the remainder is used to reduce the balance.	Find her payment, 2% of \$876. $\begin{aligned} \text{Minimum payment} &= \$876 \times 2\% \\ &= \$876 \times 0.02 \\ &= \$17.52 \end{aligned}$ Rounded to the nearest whole dollar the minimum payment is \$18.
Minimum payment = Unpaid balance × Minimum payment rate (rounded to the nearest whole dollar)	Find the interest she owes, 1/12 of 15.99% of \$876. $\begin{aligned} \text{Monthly interest} &= (\$876 \times 15.99\%) \div 12 \\ &= (\$876 \times 0.1599) \div 12 \\ &\approx \$11.67 \end{aligned}$ The difference between her total payment and the interest she owes will be used to reduce her balance.
Monthly interest = (Unpaid balance × Interest rate) ÷ 12	Amount applied to unpaid balance $\begin{aligned} &= \$18 - \$11.67 \\ &= \$6.33 \end{aligned}$ Unpaid balance = \$876 - \$6.33 $= \$869.67$
Amount applied to unpaid balance = Minimum payment – Interest charge	Karen's new balance is \$869.67
Unpaid balance = Beginning unpaid balance – Amount applied to unpaid balance	

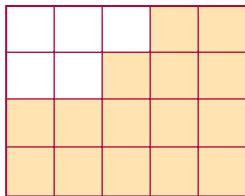
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Review Exercises CHAPTER 6

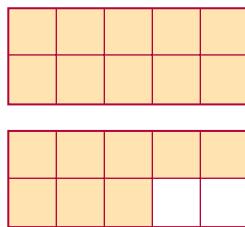
Section 6.1

What percent of each of the following regions is shaded?

1.



2.



Write an exact percent for these comparisons. Use fractions when necessary.

3. 39 per 50

4. 354 per 120

5. 44 per 77

6. Of teenagers who demonstrated violent behavior, 55 of 100 had used more than one illegal drug during the past year. What percent of the teenagers who demonstrated violent behavior had used illegal drugs?

Section 6.2

Write each decimal as a percent.

7. 0.652

8. 0.508

9. 0.00017

10. 73

11. The Phoenix Suns won 0.756 of their 82 league games in 20042005. Write this as a percent.

12. The sale price on a new stereo is 0.70 of the original price. Express this as a percent. What “percent off” will the store advertise?

Write each of the following as a decimal.

13. 48%

14. 632%

15. $\frac{1}{16}\%$

16. $81\frac{3}{4}\%$

17. What decimal number is used to compute the interest on a credit card balance that has an interest rate of 18.45%?

18. Thirty-year home mortgages are being offered at 5.72%. What decimal number will be used to compute the interest?

Section 6.3*Change each fraction or mixed number to a percent.*

19. $\frac{11}{16}$

20. $7\frac{23}{64}$

Change each fraction or mixed number to a percent. Round to the nearest tenth of a percent.

21. $\frac{13}{27}$

22. $9\frac{33}{73}$

23. A Classic League basketball team won 37 of the 46 games they played. What percent of the games did they win, rounded to the nearest hundredth of a percent?

24. Jose got 37 problems correct on a 42-problem exam. What percent of the problems did Jose get correct? Round to the nearest whole percent.

Change each of the following percents to fractions or mixed numbers.

25. 165%

26. 6.4%

27. 32.5%

28. 382%

29. The offensive team for the Chicago Bears was on the field 42% of the time during a recent game with the Green Bay Packers. What fractional part of the game was the Bears defense on the field?

30. Spraying for the gypsy moth is found to be 92.4% effective. What fraction of the gypsy moths are eliminated?

Section 6.4*Fill in the table with the related percent, decimal, or fraction.***31–38.**

Fraction	Decimal	Percent
$\frac{17}{25}$	0.74	
		1.5%
$3\frac{11}{40}$		

Section 6.5*Solve.*

39. 22% of 455 is _____.

40. 36 is 45% of _____.

41. 17 is _____ % of 80.

42. 37 is _____ % of 125.

43. 3.4% of 370 is _____.

44. 2385 is 53% of _____.

45. What percent of 677 is 123? Round to the nearest tenth of a percent.

46. Two hundred fifty-four is 154.8% of what number? Round to the nearest hundredth.

Section 6.6

47. Last year Melinda had 26.4% of her salary withheld for taxes. If the total amount withheld was \$6345.24, what was Melinda's yearly salary?

48. The population of Arlington grew from 3564 to 5721 over the past 5 years. What was the percent of increase in the population? Round to the nearest tenth of a percent.

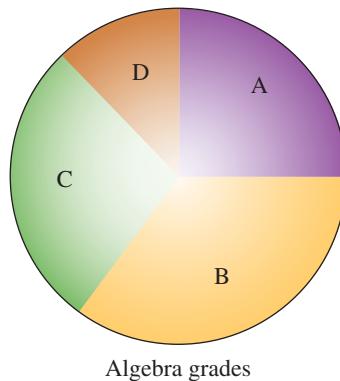
49. The work force at Omni Plastics grew by 32% over the past 3 years. If the company had 325 employees 3 years ago, how many employees do they have now?

50. Mrs. Hope's third-grade class has the following ethnic distribution: Hispanic, 13; African American, 7; Asian, 5; and Caucasian, 11. What percent of her class is African American? Round to the nearest tenth of a percent.

51. Mr. Jones bought a new Hummer H2 in 2005 for \$57,480. At the end of 1 year it had decreased in value to \$50,650. What was the percent of decrease? Round to the nearest whole-number percent.

52. To qualify for an interview, Toni had to get a minimum of 70% on a pre-employment test. Toni got 74 out of 110 questions correct. Does Toni qualify for an interview?

53–54. The figure shows the grade distribution in an algebra class.



53. Which grade was received by most students?

54. Were more A and C grades earned than B and D grades?

Section 6.7

55. Mary buys a set of golf clubs for \$465.75. The store adds on a 6.35% sales tax. What is the total cost of the clubs?
57. The May Company advertises a sale at 25% off on men's suits. The store then offers a coupon that gives an additional 20% off the sale price. What is the cost to the consumer of a suit that was originally priced at \$675.90?
59. A pair of New Balance walking shoes, which regularly sells for \$110.95, goes on sale for \$79.49. What percent off is this? Round to the nearest whole percent.
56. Rod buys a new LCD projection TV for \$3025. The store charges Rod \$3233.58, including sales tax. What is the sales tax rate? Round to the nearest hundredth of a percent.
58. A salesclerk earns a base salary of \$1500 per month plus a commission of 11.5% on all sales over \$9500. What is her salary in a month in which her sales totaled \$21,300?

Section 6.8

61. Wanda borrows \$5500 from her uncle at 6.5% simple interest for 2 years. How much does Wanda owe her uncle at the end of the 2 years?
63. Minh has a credit card balance of \$1345.60. The credit card company charges 18.6%. If Minh makes a payment of \$55 and makes no additional charges, what will be his credit card balance on the next billing?
62. Larry invests \$2000 at 8% compounded monthly. What is the value of his investment after 2 months?
64. Felicia's credit card has a balance owed at the end of September of \$4446.60. The credit card company charges a rate of 19.8% on the unpaid balance. Felicia makes the minimum payment of \$89.
- How much of the payment is interest?
 - How much of the payment goes to pay off the balance?
 - What is the unpaid balance at the end of October, assuming no additional charges were made?

True/False Concept Review CHAPTER 6

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. Percent means per 100. 1. _____
2. The symbol % is read “percent.” 2. _____
3. To change a fraction to a percent, move the decimal point in the numerator two places to the left and write the percent symbol. 3. _____
4. To change a decimal to a percent, move the decimal point two places to the left. 4. _____
5. Percent is a ratio. 5. _____
6. In percent, the base unit can be more than 100. 6. _____
7. To change a percent to a decimal, drop the percent symbol and move the decimal point two places to the left. 7. _____
8. A percent can be equal to a whole number. 8. _____
9. To solve a problem written in the form A is R of B , we can use the proportion $\frac{B}{A} = \frac{X}{100}$, where $R = X\%$. 9. _____
10. To solve the problem, “If there is a 5% sales tax on a radio costing \$64.49, how much is the tax?” the simpler word form could be, “5% of \$64.49 is what?” 10. _____
11. $2.5 = 250\%$ 11. _____
12. $4\frac{3}{4} = 4.75\%$ 12. _____
13. $0.009\% = 0.9$ 13. _____
14. If 0.4% of B is 172, then $B = 4300$. 14. _____
15. If some percent of 64 is 32, then the percent is 50%. 15. _____
16. If $2\frac{4}{5}\%$ of 300 is A , then $A = 8.4$. 16. _____

Name _____ Class _____ Date _____

17. Two consecutive decreases of 15% is the same as a decrease of 30%. 17. _____
18. If Selma is given a 10% raise on Monday but her salary is cut 10% on Wednesday, her salary is the same as it was Monday before the raise. 18. _____
19. It is possible to increase a city's population by 110%. 19. _____
20. If the price of a stock increases 100% for each of 3 years, the value of \$1 of stock is worth \$8 at the end of 3 years. 20. _____
21. A 50% growth in population is the same as 150% of the original population. 21. _____
22. $\frac{1}{2}\% = 0.5$ 22. _____
23. If interest is compounded quarterly, it means that every 3 months the interest earned is added to the principal and earns interest the next time period. 23. _____
24. If Loretta buys a new toaster for \$34.95 and pays \$37.22, including sales tax, at the checkout, the sales tax rate is 7.8%. 24. _____

Test CHAPTER 6**ANSWERS**

1. A computer regularly sells for \$1495. During a sale, the dealer discounts the price \$415.50. What is the percent of discount? Round to the nearest tenth of a percent.
2. Write as a percent: 0.03542
3. If 56 of every 100 people in a certain town are female, what percent of the population is female?
4. What percent of $8\frac{3}{8}$ is $6\frac{1}{4}$? Round to the nearest tenth of a percent.
5. Change to a percent: $\frac{27}{32}$
6. Two hundred fifty-three percent of what number is 113.85?
7. Change to a fraction: $16\frac{8}{13}\%$
8. Write as a percent: 0.0078
9. What number is 15.6% of 75?
10. Change to a percent (to the nearest tenth of a percent): $6\frac{9}{11}\%$
11. Change to a fraction or mixed number: 272%
12. Write as a decimal: 7.89%
13. The Adams family spends 17.5% of their monthly income on rent. If their monthly income is \$6400, how much do they spend on rent?

14–19. Complete the following table:

Fraction	Decimal	Percent
$\frac{13}{16}$		
	0.624	
		18.5%

20. 87.6 is ____% of 115.9 (to the nearest tenth of a percent). 20. _____

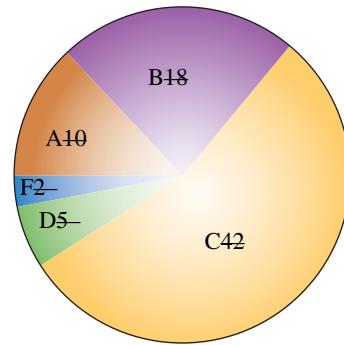
21. Write as a decimal: 4.765% 21. _____

22. The Tire Factory sells a set of tires for \$357.85 plus a sales tax of 6.3%. What is the total price charged the customer? 22. _____

23. Nordstrom offers a sale on Tommy Hilfiger jackets at a discount of 30%. What is the sale price of a jacket that originally sold for \$212.95? 23. _____

24. The population of Nevada grew from 1,998,257 people in 2000 to 2,410,758 people in 2004. What is the percent of increase in the population? Round to the nearest tenth of a percent. 24. _____

25. The following graph shows the distribution of grades in an American History class. What percent of the class received a B grade? Round to the nearest tenth of a percent. 25. _____



26. Jerry earns \$600 per month plus a 6% commission on all his sales. Last month, Jerry sold products totaling \$75,850. What were Jerry's earnings last month? 26. _____

27. Ukiah borrows \$6500 from his aunt to attend college. His aunt charges Ukiah 7% simple interest. How much will Ukiah owe his aunt at the end of 1 year? 27. _____

28. If a hamburger contains 780 calories and 37 g of fat, what percent of the calories are from fat? Assume each gram of fat contains 10 calories. Round to the nearest tenth of a percent. 28. _____

29. Greg invests \$9000 at 5% interest compounded monthly. What is his investment worth after 10 years? 29. _____

30. Loraine has a balance of \$6732.50 on her credit card at the end of April. The credit card company charges 18.5% interest and requires a minimum payment of 2% of the unpaid balance, rounded to the nearest dollar. Loraine makes the minimum payment and charges an additional \$234.50 during the next month. What is her credit card balance at the end of May? 30. _____

Group Project CHAPTER 6

OPTIONAL

Have you ever found yourself short of cash? Everyone does at some time. The business world has many “solutions” for people who need quick money, but each comes with a price in the form of fees and/or interest charged. An informed consumer chooses one option or another after careful consideration of all the costs. Let’s look at a hypothetical situation.

The transmission in LaRonda’s car needs fixing and she absolutely must have her car to get to and from work. The shop (which does not accept credit cards) gives her a bill for \$250 but she only has \$43 in her checking account. LaRonda has a good job, but it is 2 weeks until payday. LaRonda can think of three possible solutions to her dilemma.

1. Write the shop a check that she knows will bounce, and straighten everything out as soon as she gets paid.
2. Go to a Payday Loan store and have them hold her check until she gets paid.
3. Get a cash advance on her Visa card, and repay it when she gets paid.

LaRonda begins her research on each of her three options. If she bounces a check, the shop will charge her \$30 and so will her bank. In addition, the bank charges her \$7 per day for a continuous negative balance beginning on the fourth day of her negative balance. The Payday Loan store will charge her \$20 for every \$100 she borrows for a 2-week duration. Her Visa card charges her a 3% cash advance fee and then charges her 20.99% APR until she pays it back.

Assuming that LaRonda borrows \$250 and pays off her debt in 14 days, complete the following table.

Plan	Amount Borrowed	Fees and/or Interest Paid	Fees as % of Amount Borrowed	Total Payback
Bounce a check				
Payday Loan				
Cash advance				

1. Analyze the pros and cons of each option. Include considerations besides total cost.
2. What seems to be the best course of action for LaRonda and why?
3. Can you think of another option for LaRonda that would save her money? Outline your option and do a cost analysis as in the table.

The APR of a loan is a figure that allows consumers to compare loan fees. The APR is the interest rate paid if the loan is held for 1 year. Complete the table again, assuming that LaRonda borrows \$250, holds it for 1 year and then pays off her debt. Include your fourth option in the table.

Plan	Amount Borrowed	Fees and/or Interest Paid	Fees as % of Amount Borrowed	Total Payback
Bounce a check				
Payday Loan				
Cash advance				
Your solution				

4. Comment on the APRs associated with the various options. Did any of them surprise you?
5. If you need a short-term loan, what is a reasonable interest rate?

Low-Stress Tests

It's natural to be anxious before an exam. In fact, a little anxiety is actually good: it keeps you alert and on your toes. Obviously, too much stress over tests is not good. Here are some proven tips for taking low-stress tests.

1. Before going to the exam, find a place on campus where you can physically and mentally relax. Don't come into the classroom in a rush.
2. Arrive at the classroom in time to arrange all the tools you will need for the test: sharpened pencils, eraser, plenty of scratch paper, and a water bottle. Try to avoid talking with classmates about the test. Instead, concentrate on deep breathing and relaxation.
3. Before starting the test, on a separate piece of paper, write all the things you may forget while you are busy at work: formulas, rules, definitions, and reminders to yourself. Doing so relieves the load on your short-term memory.
4. Read all of the test problems and mark the easiest ones. Don't skip reading the directions. Note point values so you don't spend too much time on problems that count only a little, at the expense of problems that count a lot.
5. Do the easiest problems first; do the rest of the problems in order of difficulty.
6. Estimate a reasonable answer before you make calculations. When you finish the problem, check to see that your answer agrees with your estimate.
7. If you get stuck on a problem, mark it and come back to it later.
8. When you have finished trying all the problems, go back to the problems you didn't finish and do what you can. Show all steps because you may get partial credit even if you cannot complete a problem.
9. When you are finished, go back over the test to see that all the problems are as complete as possible and that you have indicated your final answer. Use all of the time allowed, unless you are sure there is nothing more that you can do.
10. Turn in your test and be confident that you did the best job you could. Congratulate yourself on a low-stress test!

If you find yourself feeling anxious, try a 3 × 5 "calming" card. It may include the following: (a) a personal coping statement such as "I have studied hard and prepared well for this test, I will do fine"; (b) a brief description of your peaceful scene; and (c) a reminder to stop, breathe, and relax your tense muscles.

By now, you should be closer to taking control of math instead of allowing math to control you. You are avoiding learned helplessness (believing that other people or influences control your life). Perfectionism, procrastination, fear of failure, and blaming others are also ineffective attitudes that block your power of control. Take responsibility and believe that you have the power within to control your life situation.

7

MEASUREMENT AND GEOMETRY



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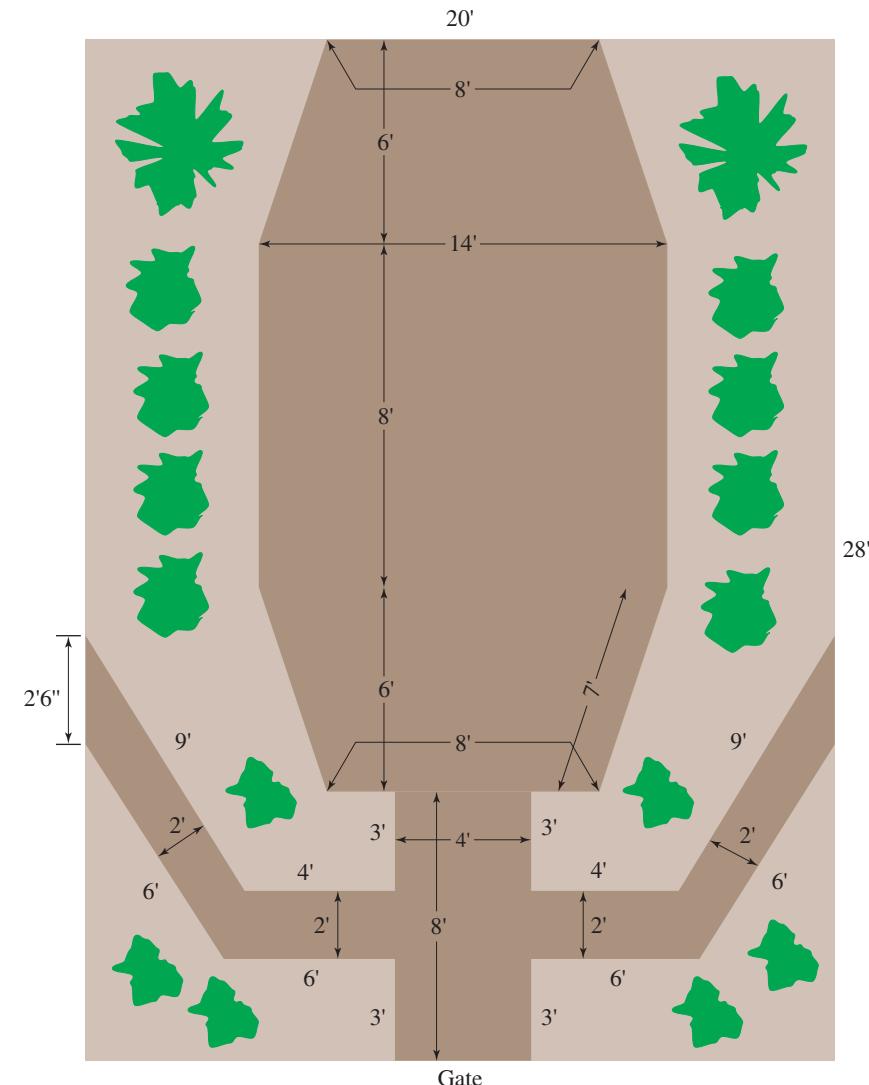
SECTIONS

- 7.1** Measuring Length
- 7.2** Measuring Capacity, Weight, and Temperature
- 7.3** Perimeter
- 7.4** Area
- 7.5** Volume
- 7.6** Square Roots and the Pythagorean Theorem

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APPLICATION

Paul and Barbara have just purchased a row house in the Georgetown section of Washington, D.C. The backyard is rather small and completely fenced. They decide to take out all the grass and put in a brick patio and formal rose garden. The plans for the patio and garden are shown here.



7.1 Measuring Length

OBJECTIVES

1. Recognize and use appropriate units of length from the English and metric measuring systems.
2. Convert units of length
 - a. Unit fractions within the same system.
 - b. Unit fractions between systems.
 - c. Moving the decimal point in the metric system.
3. Perform operations on measurements.

VOCABULARY

A **unit of measure** is the name of a fixed quantity that is used as a standard.

A **measurement** is a number together with a unit of measure.

Equivalent measurements are measures of the same amount but using different units.

A **unit fraction** is a fraction whose numerator and denominator are equivalent measurements.

The **English system** is the measurement system commonly used in the United States.

The **metric system** is the measurement system used by most of the world.

How & Why

OBJECTIVE 1

Recognize and use appropriate units of length from the English and metric measuring systems.

One of the main ways of describing an object is to give its measurements. We measure how long an object is, how much it weighs, how much space it occupies, how long it has existed, how hot it is, and so forth. Units of measure are universally defined so that we all mean the same thing when we use a measurement. There are two major systems of measurement in use in the United States. One is the English system, so named because we adopted what was used in England at the time. This system is a mixture of units from various countries and cultures, and it is the system with which most Americans are familiar. The second is the metric system, which is currently used by almost the entire world.

Measures of length answer questions such as “How long?” or “How tall?” or “How deep?” We need units of length to measure small distances, medium distances, and long distances. In the English system, we use inches to measure small distances, feet to measure medium distances, and miles to measure long distances. Other, less common units of length in the English system include yards, rods, fathoms, and light-years.

The metric system was invented by French scientists in 1799. Their goal was to make a system that was easy to learn and would be used worldwide. They based the system for length on the meter and related it to Earth by defining it as 1/10,000,000 of the distance between the North Pole and the equator.

To make the system easy to use, the scientists based all conversions on powers of 10 and gave the same suffix to all units of measure for the same characteristic. So all units of length in the metric system end in “-meter.” Furthermore, multiples of the base unit are indicated by a prefix. So any unit beginning with “kilo-” means 1000 of the base unit. Any unit beginning with “centi-” means 1/100 of the base unit, and any unit beginning with “milli-” means 1/1000 of the base unit.

Table 7.1 lists common units of length in both systems.

Table 7.1 Common Units of Length

Size	English	Metric
Large	Mile (mi)	Kilometer (km)
Medium	Foot (ft) or yard (yd)	Meter (m)
Small	Inch (in.)	Centimeter (cm)
Tiny		Millimeter (mm)

Examples A–C



DIRECTIONS: Write both an English unit and a metric unit to measure the following.

STRATEGY: Decide on the size of the object and pick the appropriate units.

- A. The distance from Baltimore, Md., to Washington, D.C.

This is a long distance, so it is measured in miles or kilometers.

- B. The width of a calculator.

This is a small distance, so it is measured in inches or centimeters.

- C. The width of a dining room.

This is a medium distance, so it is measured in feet or meters.

Warm-Ups A–C

- A. The distance from San Francisco to Los Angeles.

- B. The width of a camera.

- C. The length of a bedroom.

How & Why

OBJECTIVE 2a

Convert units of length—unit fractions within the same system.

Using Table 7.2, we can convert measurements in each system.

For a slick converter, check out <http://www.onlineconversion.com>

Table 7.2 Equivalent Length Measurements

English	Metric
12 inches (in.) = 1 foot (ft)	1000 millimeters (mm) = 1 meter (m)
3 feet (ft) = 1 yard (yd)	100 centimeters (cm) = 1 meter (m)
5280 feet (ft) = 1 mile (mi)	1000 meters (m) = 1 kilometer (km)

Because 12 in. = 1 ft, they are equivalent measurements. A fraction using one of these as the numerator and the other as the denominator is equivalent to 1 because the numerator and denominator are equal.

$$\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{1 \text{ ft}}{12 \text{ in.}} = 1$$

To convert a measurement from one unit to another, we multiply by the unit fraction: $\frac{\text{desired unit of measure}}{\text{original unit of measure}}$. Because we are multiplying by 1, the measurement is unchanged but the units are different.

For example, to convert 48 in. to feet, we choose a unit fraction that has feet, the desired units in the numerator, and inches, the original units, in the denominator. In this case, we use $\frac{1 \text{ ft}}{12 \text{ in.}}$ for the conversion.

$$\begin{aligned} 48 \text{ in.} &= \frac{48 \text{ in.}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} && \text{Multiply by the appropriate unit fraction.} \\ &= \frac{48 \text{ ft}}{12} && \text{Multiply.} \\ &= 4 \text{ ft} && \text{Simplify.} \end{aligned}$$

In some cases, it is necessary to multiply by more than one unit fraction to get the desired results. For example, to convert 5.2 km to centimeters we use the unit fraction $\frac{1000 \text{ m}}{1 \text{ km}}$ to convert the kilometers to meters, and then $\frac{100 \text{ cm}}{1 \text{ m}}$ to convert the meters to centimeters.

Answers to Warm-Ups

- A. miles or kilometers
B. inches or centimeters
C. feet or meters

$$5.2 \text{ km} = \frac{5.2 \text{ km}}{1} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{100 \text{ cm}}{1 \text{ m}}$$

Multiply by the appropriate unit fractions.

$$= 520,000 \text{ cm}$$

Simplify.

To convert units of length

1. Multiply by the unit fraction that has the desired units in the numerator and the original units in the denominator.
2. Simplify.

Warm-Ups D–E

D. Convert 27 cm to meters.

E. Convert 12 mi to inches.

Examples D–E BASIC Mathematics Now™

DIRECTIONS: Convert units of measure.

STRATEGY: Multiply the given measure by the appropriate unit fraction(s) and simplify.

D. Convert 4 mm to meters.

$$4 \text{ mm} = \frac{4 \text{ mm}}{1} \cdot \frac{1 \text{ m}}{1000 \text{ mm}}$$

Multiply by the appropriate unit fraction.

$$= \frac{4 \text{ m}}{1000}$$

Simplify.

$$= 0.004 \text{ m}$$

So $4 \text{ mm} = 0.004 \text{ m}$

E. Convert 7.8 mi to inches.

$$7.8 \text{ mi} = \frac{7.8 \text{ mi}}{1} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}$$

Convert miles to feet and then feet to inches.

$$= 494,208 \text{ in.}$$

Simplify.

So $7.8 \text{ mi} = 494,208 \text{ in.}$

How & Why

OBJECTIVE 2b

Convert units of length—unit fractions between systems.

Sometimes we need to convert lengths from one system to another. Most conversions between systems are approximations, as in Table 7.3. The method of using unit fractions for converting is the same as when converting within the same system.

Table 7.3 Length Conversions between English and Metric Systems

1 inch \approx 2.5400 centimeters	1 centimeter \approx 0.3937 inch
1 foot \approx 0.3048 meter	1 meter \approx 3.2808 feet
1 yard \approx 0.9144 meter	1 meter \approx 1.0936 yards
1 mile \approx 1.6093 kilometers	1 kilometer \approx 0.6214 mile

Because of the rounding in Table 7.3, we get approximate measurements when moving from one system to another. For everyday measurements, this is not a problem because we usually only measure to the nearest tenth or hundredth. However, in a laboratory or industrial setting, where more precision is necessary, you may need to use more

Answers to Warm-Ups

D. 0.27 m E. 760,320 in.

accurate conversions than those in Table 7.3. In changing measures from one system to another, we will always use a unit fraction with denominator of 1. For example, when converting inches to centimeters we use $\frac{2.5400 \text{ cm}}{1 \text{ in.}}$, whereas when converting from centimeters to inches we use $\frac{0.3937 \text{ in.}}{1 \text{ cm}}$. There are two reasons for this: (1) it is easier to multiply than to divide decimals when not using a calculator, and (2) it is possible for the results to vary when using different approximating unit fractions.

CAUTION

Because the conversions in Table 7.3 are all rounded to the nearest ten-thousandth, we cannot expect accuracy beyond the ten-thousandths place when using them.

Examples F–G



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DIRECTIONS: Convert the units of measure. Round to the nearest hundredth.

STRATEGY: Multiply the given measure by the appropriate unit fraction(s) and simplify.

F. Convert 2 ft to meters.

$$2 \text{ ft} \approx \frac{2 \text{ ft}}{1} \cdot \frac{0.3048 \text{ m}}{1 \text{ ft}} \quad \begin{array}{l} \text{Multiply by the appropriate unit fraction.} \\ \text{Simplify.} \end{array}$$

$$\approx 0.6096 \text{ m}$$

So $2 \text{ ft} \approx 0.61 \text{ m}$.

G. Convert 1.6 km to feet.

STRATEGY: Convert kilometers to miles and then miles to feet.

$$1.6 \text{ km} \approx \frac{1.6 \text{ km}}{1} \cdot \frac{0.6214 \text{ mi}}{1 \text{ km}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \quad \begin{array}{l} \text{Convert kilometers to miles, then miles} \\ \text{to feet.} \end{array}$$

$$\approx 5249.59 \text{ ft} \quad \begin{array}{l} \text{Simplify.} \end{array}$$

So $1.6 \text{ km} \approx 5249.59 \text{ ft}$.

Warm-Ups F–G

F. Convert 5 m to feet.

G. Convert 6 ft to centimeters.

How & Why

OBJECTIVE 2c

Convert units of length—moving the decimal point in the metric system.

The metric system is based on powers of 10 and it is easy to multiply and divide by powers of 10. Therefore, we can shortcut metric-to-metric conversions by multiplying or dividing by powers of 10.

We have already seen how kilometers, centimeters, and millimeters relate to the base unit, the meter. Other, less frequently used units are also part of the metric system. Consider the following.

Kilometer	Hecto-meter	Deka-meter	Meter	Deci-meter	Centi-meter	Millimeter
km 1000 m	hm 100 m	dam 10 m	m 1 m	dm 0.1 m	cm 0.01 m	mm 0.001 m

Answers to Warm-Ups

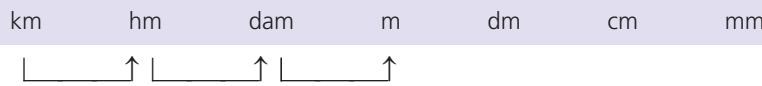
F. 16.40 ft G. 182.88 cm

Reminder

To multiply a number by a power of 10, move the decimal point to the right. The number of places to move is the number of zeros in the power of 10, or the exponent of 10.

To divide a number by a power of 10, move the decimal point to the left. The number of places to move is the number of zeros in the power of 10, or the exponent of 10.

move to get from the given unit to the desired unit. For example, to change kilometers to meters we must move three places to the right.



To change 4 km to meters, we multiply 4 km by 10^3 or 1000. So $4 \text{ km} = 4000 \text{ m}$. To change centimeters to meters, we must move two places to the left.

Kilometer	Hecto-meter	Dekameter	Meter	Deci-meter	Centi-meter	Millimeter
km	hm	dam	m	dm	cm	mm
				↑	↑	↑

To change 85 cm to meters, we must divide by 100 or 10^2 . So $85 \text{ cm} = 0.85 \text{ m}$.

To convert units within the metric system

Move the decimal point the same number of places and in the same direction as you do to go from the given units to the desired units on the chart.

Warm-Ups H–I

H. Change 69 km to meters.

I. Change 346 mm to centimeters.

Examples H–I

DIRECTIONS: Convert units as indicated.

STRATEGY: Use the chart for a shortcut.

H. Change 47 m to millimeters.

Meters to millimeters is three places right on the chart. Therefore, we move the decimal three places to the right.

$$47 \text{ m} = 47,000 \text{ mm}$$

I. Change 13 mm to meters.

Millimeters to meters is three places left on the chart. Therefore, we move the decimal three places to the left.

$$13 \text{ mm} = 0.013 \text{ m}$$

How & Why

OBJECTIVE 3

Perform operations on measurements.

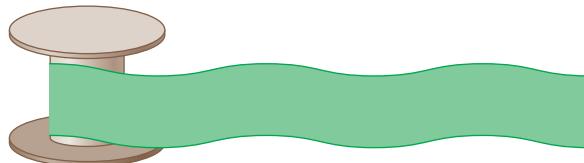
When we write “4 inches,” we mean four units that are 1 inch in length. Mathematically we can describe this as $4 \text{ inches} = 4 \cdot (1 \text{ inch})$. This way of interpreting measurements makes it easy to find multiples of measurements. Consider 3 boards, each 5 feet long. The total length of the boards is

$$\begin{aligned}3 \cdot (5 \text{ feet}) &= 3 \cdot 5 \cdot (1 \text{ foot}) \\&= 15 \cdot (1 \text{ foot}) \\&= 15 \text{ feet}\end{aligned}$$

Answers to Warm-Ups

H. 69,000 m **I.** 34.6 cm

Similarly, a bolt of ribbon has 5 yards on it. If the ribbon is cut into 10 equal pieces, how long is each piece?



$$\begin{aligned}\frac{5 \text{ yd}}{10} &= \frac{5 \cdot (1 \text{ yd})}{10} \\&= \frac{5}{10} \cdot (1 \text{ yd}) \\&= \frac{1}{2} \cdot (1 \text{ yd}) \\&= \frac{1}{2} \text{ yd}\end{aligned}$$

So each piece is $\frac{1}{2}$ yd long.

To multiply or divide a measurement by a number

Multiply or divide the two numbers and write the unit of measure.

Examples J–K BASIC Mathematics Now™

DIRECTIONS: Solve.

STRATEGY: Describe each situation with a statement involving measurements and simplify.

- J. What is the total height of three storage units stacked on one another if each unit is 60 cm tall?

STRATEGY: To find the total height, multiply the height of one unit by 3.

$$\begin{aligned}\text{Total height} &= 3 \cdot (60 \text{ cm}) \\&= 180 \text{ cm}\end{aligned}$$

The total height is 180 cm.

- K. A fence 24 ft long is to be constructed in four sections. How long is each section?

STRATEGY: To find the length of each section divide the total length by 4.

$$\begin{aligned}\frac{24 \text{ ft}}{4} &= \frac{24 \cdot (1 \text{ ft})}{4} \\&= 6 \text{ ft}\end{aligned}$$

So each section is 6 ft long.

The expression “You can’t add apples and oranges” applies to adding and subtracting measurements. Only measurements with the same units of measure may be added or subtracted.

$$\begin{aligned}10 \text{ mm} + 6 \text{ mm} &= (10 + 6) \text{ mm} \\&= 16 \text{ mm}\end{aligned}$$

Warm-Ups J–K

- J. What is the total length of six copper pipes that are each 8 ft long?

- K. An 8-km race is divided into five legs. How long is each leg?

Answers to Warm-Ups

J. 48 ft K. 1.6 km

CAUTION

$3\text{ m} + 4\text{ cm} \neq 7\text{ m}$

To add or subtract measurements

1. If the units of measure are unlike, convert to like measures.
2. Add or subtract the numbers and write the unit of measure.

Warm-Ups L–M

- L. In a bookcase, the height of the bottom shelf is 15 in., the next shelf is 12 in., and the top shelf is 10 in. What is the total height of the bookcase?

- M. One wall of a den measures 10 ft 4 in. The door is 2 ft 8 in. wide. What is the remaining width?

Examples L–M



BASIC Mathematics Now™

DIRECTIONS: Solve.

STRATEGY: Describe each situation with a statement involving measurements, and simplify.

- L. Ben is 1.88 m tall, Nate is 1.79 m tall, and Tony is 1.72 m tall. What is the total height of the three boys?

STRATEGY: To find the total height, add the three heights.

$$\begin{aligned}\text{Total height} &= 1.88\text{ m} + 1.79\text{ m} + 1.72\text{ m} \\ &= (1.88 + 1.79 + 1.72)\text{ m} \\ &= 5.39\text{ m}\end{aligned}$$

So, the total height of the three boys is 5.39 m.

- M. If a carpenter cuts a piece of board that is 2 ft 5 in. from a board that is 8 ft 3 in. long, how much board is left? (Disregard the width of the cut.)

STRATEGY: Subtract the length of the cut piece from the length of the board.

$$\begin{array}{r} 8 \text{ ft } 3 \text{ in.} \\ - 2 \text{ ft } 5 \text{ in.} \\ \hline \text{remaining board} \end{array}$$

$$\begin{array}{r} 7 \text{ ft } 1 \text{ ft } 3 \text{ in.} \quad \text{Borrow 1 ft from the 8 ft (1 ft = 12 in.).} \\ - 2 \text{ ft } \underline{5 \text{ in.}} \\ \hline 7 \text{ ft } 15 \text{ in.} \quad 1 \text{ ft } 3 \text{ in.} = 15 \text{ in.} \\ - 2 \text{ ft } \underline{5 \text{ in.}} \quad \text{Subtract.} \\ \hline 5 \text{ ft } 10 \text{ in.} \end{array}$$

There are 5 ft 10 in. of board remaining.

Answers to Warm-Ups

L. 37 in. M. 7 ft 8 in.

Exercises 7.1

OBJECTIVE 1

Recognize and use appropriate units of length from the English and metric measuring systems.

A Give both an English unit and a metric unit to measure the following.

- | | |
|--|--|
| 1. The height of a skyscraper | 2. The dimensions of a sheet of typing paper |
| 3. The thickness of a fingernail | 4. The length of a table |
| 5. The width of a slat on a mini-blind | 6. The distance from home to school |
| 7. The height of a tree | 8. The cruising altitude of a 747 jet |

B

- | | |
|---|---|
| 9. The depth of the ruins of the <i>Titanic</i> | 10. The thickness of a wire |
| 11. The length of a shoelace | 12. The width of a human hair |
| 13. The height of the ceiling in a room | 14. The distance from the moon to Earth |
| 15. The length of a swimming pool | 16. The height of Mt. Everest |

OBJECTIVE 2

Convert units of length.

A Convert as indicated.

- | | | | |
|-------------------------|--------------------------|-------------------------|---------------------|
| 17. 6 ft to inches | 18. 34 cm to millimeters | 19. 41 km to meters | 20. 2 mi to inches |
| 21. 12 yd to feet | 22. 418 cm to meters | 23. 723 m to kilometers | 24. 321 ft to yards |
| 25. 6 km to centimeters | 26. 3 mi to feet | | |

B Convert as indicated. Round to the nearest hundredth if necessary.

- | | | | |
|----------------------------|--------------------------|---------------------|--------------------------|
| 27. 3000 ft to miles | 28. 3000 ft to meters | 29. 27 in. to feet | 30. 3.2 cm to kilometers |
| 31. 157 in. to centimeters | 32. 157 mi to kilometers | 33. 120 km to miles | 34. 46 cm to inches |
| 35. 146 cm to feet | 36. 239 in. to meters | | |

OBJECTIVE 3

Perform operations on measurements.

A Do the indicated operations.

37. $7(12 \text{ ft})$

38. $19 \text{ cm} + 45 \text{ cm}$

39. $120 \text{ in.} \div 8$

40. $78 \text{ m} - 19 \text{ m}$

41. $25(8 \text{ mm})$

42. $36 \text{ mi} \div 4$

43. $13 \text{ ft} + 42 \text{ ft} + 19 \text{ ft}$

44. $81 \text{ cm} - 58 \text{ cm}$

45. $100(26 \text{ mi})$

46. $4 \text{ km} + 6 \text{ km} + 2 \text{ km}$

B Do the indicated operations. Round to the nearest hundredth if necessary.

47. $(7 \text{ ft } 1 \text{ in.}) + 27 \text{ in.}$

48. $12 \text{ m} - 318 \text{ cm}$

49. $26 \text{ mi} \div 8$

50. $44 \text{ km} + 227 \text{ m}$

51. $75(321 \text{ mm})$

52. $645 \text{ cm} \div 11$

53.
$$\begin{array}{r} 9 \text{ ft } 3 \text{ in.} \\ - 2 \text{ ft } 8 \text{ in.} \\ \hline \end{array}$$

54.
$$\begin{array}{r} 8 \text{ yd } 2 \text{ ft } 7 \text{ in.} \\ + 3 \text{ yd } 2 \text{ ft } 10 \text{ in.} \\ \hline \end{array}$$

55. $(544 \text{ mi})18$

56. $12 \text{ yd} - 133 \text{ in.}$

C

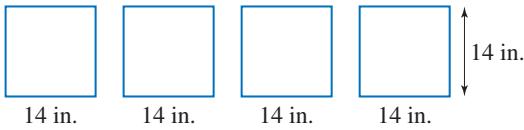
57. $(7 \text{ ft } 3 \text{ in.}) + (2 \text{ ft } 9 \text{ in.}) + (9 \text{ ft } 7 \text{ in.})$

58. $(5 \text{ yd } 1 \text{ ft } 4 \text{ in.}) - (3 \text{ yd } 2 \text{ ft } 7 \text{ in.})$

59. $(2 \text{ yd } 2 \text{ ft } 1 \text{ in.}) \cdot 3$

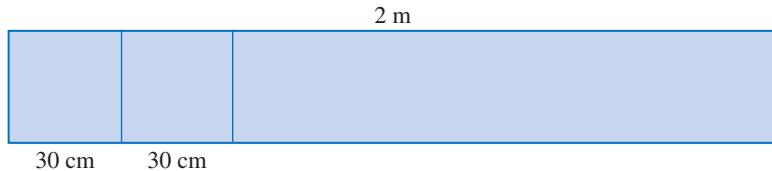
60.
$$\begin{array}{r} 3 \text{ yd } 2 \text{ ft } 5 \text{ in.} \\ + 2 \text{ yd } 1 \text{ ft } 9 \text{ in.} \\ \hline \end{array}$$

61. Martha wants to construct four picture frames that are each 14 inches square. How much molding should she buy?



62. Enrique, who lives in Seattle, is going on vacation to visit several friends who live in Atlanta, Houston, and Los Angeles. The distance between Seattle and Atlanta is 2182 mi. The distance from Atlanta to Houston is 689 mi. The distance from Houston to Los Angeles is 1374 mi, and the distance from Los Angeles to Seattle is 959 mi. How many miles does Enrique fly on his vacation?

63. Rosa buys 2 m of linen to make napkins. If she can make a napkin from each 30 cm of fabric, how many napkins can she make and how much fabric will be left over?



64. Estimate the length of your shoe in both inches and centimeters. Measure your shoe in both units.

65. Estimate the length of your small finger in both inches and centimeters. Measure your small finger in both units.

- 66.** During one round of golf, Rick made birdie putts of 5 ft 6 in., 10 ft 8 in., 15 ft 9 in., and 7 ft 2 in. What was the total length of all the birdie putts?
- 67.** The swimming pool at Tualatin Hills is 50 m long. How many meters of lane dividers should be purchased in order to separate the pool into nine lanes?
- 68.** A decorator is wallpapering. If each length of wallpaper is 7 ft 4 in., seven lengths are needed to cover a wall, and four walls are to be covered, how much total wallpaper is needed for the project?

Exercises 69–72. The table lists the longest rivers in the world.

River	Length (mi)
Nile (Africa)	4160
Amazon (South America)	4000
Chang Jiang (Asia)	3964
Ob-Irtysh (Asia)	3362
Huang (Asia)	2903
Congo (Africa)	2900

- 69.** Which, if any, of these figures appear to be estimates? Why?
- 70.** What is the total length of the five longest rivers in the world?
- 71.** The São Francisco River in South America is the twentieth longest river in the world with a length of 1988 mi. Write a sentence relating its length to that of the Amazon, using multiplication.
- 72.** Write a sentence relating the lengths of the Nile River and the Congo River using addition or subtraction.

Exercises 73–75, refer to the chapter application. See page 579.

- 73.** What are the dimensions of Paul and Barbara's back yard?
- 74.** How wide is the patio? How long is it?
- 75.** How wide are the walkways?

STATE YOUR UNDERSTANDING

- 76.** Give two examples of equivalent measures.
- 77.** Explain how to add or subtract measures.
- 78.** Explain how to multiply or divide a measure by a number.

GROUP WORK

- 79.** Measure, as accurately as you can, at least five parts of a typical desk in the classroom. Give measurements in both the English system and the metric system.

Compare your results with the other groups. Do you all agree? Give some possible reasons for the variations in measurements.

MAINTAIN YOUR SKILLS*Do the indicated operations.*

80. $4.78(10,000)$

81. $4.78 \div 100,000$

82. $\frac{3}{7} + \frac{3}{4}$

83. $\frac{12}{25} \div \frac{28}{15}$

84.
$$\begin{array}{r} 5\frac{1}{3} \\ - 2\frac{5}{6} \\ \hline \end{array}$$

85.
$$\begin{array}{r} 12\frac{9}{10} \\ + 4\frac{5}{8} \\ \hline \end{array}$$

86. $(3.63)^2$

87. $(4.8)(5.2)$

88. Find the average (mean) of 54, 78, 112, and 162.

89. Find the median of 54, 78, 112, and 162.

7.2 Measuring Capacity, Weight, and Temperature

VOCABULARY

Units of **capacity** measure liquid quantities.

Units of **weight** measure heaviness.

OBJECTIVES

- Convert and perform operations on units of capacity.
- Convert and perform operations on units of weight.
- Convert units of temperature and time.

How & Why

OBJECTIVE 1

Convert and perform operations on units of capacity.

Measures of capacity answer the question “How much liquid?” We use teaspoons to measure vanilla for a recipe, buy soda in 2-liter bottles, and measure gasoline in gallons or liters. As is true for length, there are units of capacity in both the English and metric systems. The basic unit of capacity in the metric system is the liter (L), which is slightly more than 1 quart. Table 7.4 shows the most commonly used measures of capacity in each system. Table 7.5 shows some equivalencies between the systems.

Table 7.4 Measures of Capacity

English	Metric
3 teaspoons (tsp) = 1 tablespoon (Tbsp)	1000 milliliters (mL) = 1 liter (L)
2 cups (c) = 1 pint (pt)	1000 liters (L) = 1 kiloliter (kL)
2 pints (pt) = 1 quart (qt)	
4 quarts (qt) = 1 gallon (gal)	

Table 7.5 Measures of Capacity

English–Metric	Metric–English
1 teaspoon \approx 4.9289 milliliters	1 milliliter \approx 0.2029 teaspoon
1 quart \approx 0.9464 liter	1 liter \approx 1.0567 quart
1 gallon \approx 3.7854 liters	1 liter \approx 0.2642 gallon

To convert units of capacity, we use the same technique of unit fractions that we used for units of length. See Section 7.1.

To convert units of capacity

- Multiply by the unit fraction, which has the desired units in the numerator and the original units in the denominator.
- Simplify.

CAUTION

The accuracy of conversions between systems depends on the accuracy of the conversion factors.

Warm-Ups A–B

Examples A–B



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DIRECTIONS: Convert units of measure. Round to the nearest thousandth.

STRATEGY: Multiply by the unit fraction with the desired units in the numerator and the original units in the denominator. Simplify.

- A. Convert 33 quarts (qt) to gallons (gal).

- A. Convert 259 milliliters (mL) to liters (L).

$$\begin{aligned} 259 \text{ mL} &= \frac{259 \text{ mL}}{1} \cdot \frac{1 \text{ L}}{1000 \text{ mL}} && \text{Use appropriate unit fractions.} \\ &= \frac{259 \text{ L}}{1000} && \text{Simplify.} \\ &= 0.259 \text{ L} \end{aligned}$$

So $259 \text{ mL} = 0.259 \text{ L}$

ALTERNATIVE SOLUTION: Because this is a metric-to-metric conversion, we can use a chart as a shortcut.

kL	hL	daL	L	dL	cL	mL
			↑	↑	↑	↑

To change from milliliters to liters, we move the decimal point three places left. Therefore, $259 \text{ mL} = 0.259 \text{ L}$

- B. Convert 4827 milliliters (mL) to quarts (qt).

- B. Convert 27 pints (pt) to liters (L).

$$\begin{aligned} 27 \text{ pt} &\approx \frac{27 \text{ pt}}{1} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} \cdot \frac{0.9464 \text{ L}}{1 \text{ qt}} && \text{Use appropriate unit fractions.} \\ &\approx \frac{27(0.9464)}{2} \text{ L} && \text{Simplify. Round to the nearest thousandth.} \\ &\approx 12.776 \text{ L} \end{aligned}$$

So $27 \text{ pt} \approx 12.776 \text{ L}$

Operations with units of capacity are performed using the same procedures as those for units of length. See Section 7.1.

To multiply or divide a measurement by a number

Multiply or divide the two numbers and write the unit of measure.

To add or subtract measurements

1. If the units of measure are unlike, convert to like measures.
2. Add or subtract the numbers and write the unit of measure.

Answers to Warm-Ups

- A. 8.25 gal B. 5.101 qt

Examples C–D

DIRECTIONS: Solve.

STRATEGY: Describe each situation with a statement involving measurements and simplify.

- C. A chemistry instructor has 450 mL of an acid solution that he needs to divide into equal amounts for four lab groups. How much acid does each group receive?

$$\begin{aligned}\text{Acid for one group} &= \frac{450 \text{ mL}}{4} && \text{Divide.} \\ &= \frac{450}{4} \text{ mL} \\ &= 112.5 \text{ mL}\end{aligned}$$

Each group receives 112.5 mL of acid solution.

- D. Clayton has 5 gal 3 qt of motor oil to be recycled. His friend Ernie adds another 7 qt of oil. How much oil is being recycled?

$$\begin{aligned}\text{Recycled oil} &= (5 \text{ gal } 3 \text{ qt}) + 7 \text{ qt} \\ &= 5 \text{ gal} + 3 \text{ qt} + 7 \text{ qt} \\ &= 5 \text{ gal} + 10 \text{ qt} \\ &= 5 \text{ gal} + 2 \text{ gal} + 2 \text{ qt} \\ &= 7 \text{ gal } 2 \text{ qt}\end{aligned}$$

The total amount of oil to be recycled is 7 gal 2 qt.

Warm-Ups C–D

- C. What is the total capacity of five vases, each of which holds 3.5 cups of water?

- D. Cynthia made 4 pt 1 c of strawberry jam and Bonnie made 7 c of raspberry jam. How much jam did the two women make?

How & Why

OBJECTIVE 2

Convert and perform operations on units of weight.

Units of weight answer the question “How heavy is it?” In the English system, we use ounces to measure the weight of baked beans in a can, pounds to measure the weight of a person, and tons to measure the weight of a ship. In the metric system, the basic unit that measures heaviness is the gram, which weighs about as much as a small paper clip. We use grams to measure the weight of baked beans in a can, kilograms to measure the weight of a person, and metric tons to measure the weight of a ship. Very small weights like doses of vitamins are measured in milligrams. Table 7.6 lists the most commonly used measures of weight in each system, and Table 7.7 shows some equivalencies between the systems.

Table 7.6 Measures of Weight

English System	Metric System
16 ounces (oz) = 1 pound (lb)	1000 milligrams (mg) = 1 gram (g)
2000 pounds (lb) = 1 ton	1000 grams (g) = 1 kilogram (kg)
	1000 kilograms = 1 metric ton

Table 7.7 Measures of Weight between Systems

English–Metric	Metric–English
1 ounce ≈ 28.3495 grams	1 gram ≈ 0.0353 ounces
1 pound ≈ 453.5924 grams	1 kilogram ≈ 2.2046 pounds
1 pound ≈ 0.4536 kilograms	

Answers to Warm-Ups

- C. 17.5 c D. 8 pt

(Note: The weight of an object is different from its mass. An object has the same mass everywhere in space. Weight depends on gravity, so an object has different weights on Earth and on the moon. On Earth, gravity is approximately uniform, so weight and mass are often used interchangeably. Technically, a gram is a unit of mass.)

Warm-Ups E–F

- E. Convert 200 grams (g) to ounces (oz).

- F. If 140 oz of peanut brittle is divided equally among five sacks, how much goes into each sack?

Examples E–F



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DIRECTIONS: Solve as indicated. Round the nearest hundredth.

STRATEGY: Follow the strategies given for each example.

- E. Convert 73 kilograms (kg) to pounds (lb).

STRATEGY: Multiply by the unit fraction with pounds in the numerator and kilograms in the denominator, and simplify.

$$73 \text{ kg} \approx \frac{73 \text{ kg}}{1} \cdot \frac{2.2046 \text{ lb}}{1 \text{ kg}}$$
$$\approx 160.9358 \text{ lb}$$

Use the appropriate unit fraction.

Simplify.

So $73 \text{ kg} \approx 160.94 \text{ lb}$.

- F. A can of peaches weighs 2 lb 5 oz and a can of pears weighs 1 lb 14 oz. What is the total weight of the two cans?

STRATEGY: Describe the situation using measurements, and simplify.

$$\begin{array}{r} 2 \text{ lb } 5 \text{ oz} \\ + 1 \text{ lb } 14 \text{ oz} \\ \hline 3 \text{ lb } 19 \text{ oz} = 3 \text{ lb } + 1 \text{ lb } 3 \text{ oz} \\ = 4 \text{ lb } 3 \text{ oz} \end{array}$$

The cans weigh 4 lb 3 oz.

How & Why

OBJECTIVE 3

Convert units of temperature and time.

Temperature is measured in degrees. There are two major scales for measuring temperature. The English system uses the Fahrenheit scale, which sets the freezing point of water at 32°F and the boiling point of water at 212°F. The Fahrenheit scale was developed by a physicist named Gabriel Daniel Fahrenheit in the early 1700s.

The metric system uses the Celsius scale, which sets the freezing point of water at 0°C and the boiling point of water at 100°C. The Celsius scale is named after Swedish astronomer Anders Celsius who lived in the early 1700s and invented a thermometer using the Celsius scale.

Converting between scales is often done using special conversion formulas.

To convert from Fahrenheit to Celsius

Use the formula: $C = \frac{5}{9} \cdot (F - 32)$ or

subtract 32 from the Fahrenheit temperature and multiply by $\frac{5}{9}$.

Answers to Warm-Ups

- E. 7.06 oz F. 28 oz

Example G



DIRECTIONS: Convert as indicated.

STRATEGY: Use the conversion formula.

G. What is 68°F on the Celsius scale?

$$\begin{aligned}C &= \frac{5}{9} \cdot (F - 32) \\&= \frac{5}{9} \cdot (68 - 32) \quad \text{Substitute 68 for } F. \\&= \frac{5}{9} \cdot 36 \quad \text{Simplify.} \\&= 20\end{aligned}$$

So 68°F = 20°C.

To convert from Celsius to Fahrenheit

Use the formula: $F = \frac{9}{5} \cdot C + 32$ or

multiply the Celsius temperature by $\frac{9}{5}$ and add 32.

Warm-Up G

G. What is 50°F on the Celsius scale?

Example H

DIRECTIONS: Convert as indicated.

STRATEGY: Use the conversion formula.

H. Convert 50°C to degrees Fahrenheit.

$$\begin{aligned}F &= \frac{9}{5} \cdot C + 32 \\&= \frac{9}{5} \cdot 50 + 32 \quad \text{Substitute 50 for } C. \\&= 90 + 32 \quad \text{Simplify.} \\&= 122\end{aligned}$$

So 50°C = 122°F

The same units of time are used in both the English and metric systems. We are all familiar with seconds, minutes, hours, days, weeks, and years. The computer age has contributed some new units of time that are very short, such as milliseconds ($\frac{1}{1000}$ of a second) and nanoseconds ($\frac{1}{1,000,000,000}$ of a second). Table 7.8 gives the commonly used time conversions.

Table 7.8 Time Conversions

60 seconds (sec) = 1 minute (min)
60 minutes (min) = 1 hour (hr)
24 hours (hr) = 1 day

7 days = 1 week
365 days \approx 1 year*

*Technically, a solar year (the number of days it takes Earth to make one complete revolution around the sun) is 365.2422 days. Our calendars account for this by having a leap day once every 4 years.

Warm-Up H

H. Convert 30°C to degrees Fahrenheit.

Warm-Ups I–J

- I. The three members of a Marine training team had individual times for an obstacle course of 5 min 37 sec, 4 min 56 sec, and 5 min 48 sec. What was the total time for the team?

- J. Bacteria in a lab have been growing for 12 days. How many seconds is this?

Examples I–J

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DIRECTIONS: Solve.

STRATEGY: Follow the strategies given for each example.

- I. The four members of an 800-m freestyle relay team had individual times of 1 min 54 sec, 1 min 59 sec, 2 min 8 sec, and 1 min 49 sec. What was the total time for the relay team?

STRATEGY: Add the times.

1 min 54 sec

1 min 59 sec

2 min 8 sec

1 min 49 sec

$$\begin{array}{r} \\ \\ \\ \hline 5 \text{ min } 170 \text{ sec} = 5 \text{ min} + 2 \text{ min } 50 \text{ sec} & \text{Convert seconds to minutes.} \\ = 7 \text{ min } 50 \text{ sec} \end{array}$$

Add the times.

The relay team's time was 7 min 50 sec.

- J. Jerry is 8 years old. How many minutes old is he?

STRATEGY: Use the appropriate unit fractions.

$$\begin{aligned} 8 \text{ yr} &= \frac{8 \text{ yr}}{1} \cdot \frac{365 \text{ days}}{1 \text{ yr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \\ &= 4,204,800 \text{ min} \end{aligned}$$

Note: Jerry has lived through 2 leap years, which are not accounted for in the conversion. Therefore, we must add 2 days to the total.

$$\begin{aligned} 2 \text{ days} &= \frac{2 \text{ days}}{1} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \\ &= 2880 \text{ min} \end{aligned}$$

Jerry has been alive $4,204,800 \text{ min} + 2880 \text{ min} = 4,207,680 \text{ min}$.

Answers to Warm-Ups

- I. The team's time was 16 min 21 sec.
J. The bacteria have been growing for 1,036,800 sec.

Exercises 7.2

OBJECTIVE 1

Convert and perform operations on units of capacity.

A Do the indicated operations. Round decimal answers to the nearest hundredth.

- | | | |
|------------------------------------|--------------------------------|--|
| 1. $22 \text{ lb} + 19 \text{ lb}$ | 2. $(4\text{c}) \cdot 13$ | 3. $(400 \text{ mL}) \div 16$ |
| 4. $160 \text{ L} - 114 \text{ L}$ | 5. $(80 \text{ gal}) \div 20$ | 6. $5 \text{ kL} + 9 \text{ kL} + 10 \text{ kL}$ |
| 7. Convert 5 qt to cups. | 8. Convert 4 L to milliliters. | |
| 9. Convert 21 gal to quarts. | 10. Convert 500 mL to liters. | |

B

- | | | |
|---------------------------------|--|--|
| 11. $23 \cdot (22 \text{ mL})$ | 12. $380 \text{ kL} - 175 \text{ kL}$ | 13. $33 \text{ oz} + 49 \text{ oz}$ |
| 14. $(8498 \text{ gal}) \div 7$ | 15. $45 \text{ qt} - 36 \text{ qt} - 9 \text{ qt}$ | 16. $29 \text{ mL} + 7 \text{ mL} + 19 \text{ mL} + 18 \text{ mL}$ |
| 17. Convert 5.5 gal to cups. | 18. Convert 1.3 kL to milliliters. | |
| 19. Convert 35 gal to liters. | 20. Convert 572 mL to cups. | |

OBJECTIVE 2

Convert and perform operations on units of weight.

A Do the indicated operations or conversions. Round decimal answers to the nearest hundredth.

- | | | |
|-------------------------------------|-------------------------------|--|
| 21. $132 \text{ g} - 112 \text{ g}$ | 22. $3 \cdot (18 \text{ oz})$ | 23. $37 \text{ lb} + 43 \text{ lb}$ |
| 24. $(663 \text{ mg}) \div 17$ | 25. $(15 \text{ kg})(3)$ | 26. $3 \text{ oz} + 5 \text{ oz} + 7 \text{ oz}$ |
| 27. Convert 8 kg to grams. | 28. Convert 3 lb to ounces. | |
| 29. Convert 8000 mg to grams. | 30. Convert 88 oz to pounds. | |

B

- 31.** $(2912 \text{ lb}) \div 91$ **32.** $250 \text{ kg} - 149 \text{ kg}$ **33.** $129 \text{ oz} + 4599 \text{ oz}$
- 34.** $16(24 \text{ mg})$ **35.** $234 \text{ g} + 147 \text{ g} - 158 \text{ g}$ **36.** $(3 \text{ lb } 14 \text{ oz}) \cdot 3$
- 37.** Convert 36 mg to grams. **38.** Convert 14 oz to pounds.
- 39.** Convert 125 g to ounces. **40.** Convert 140 lb to kilograms.

OBJECTIVE 3

Convert units of temperature and time.

A Convert as indicated. Round decimals to the nearest tenth.

- 41.** 32°F to degrees Celsius **42.** 30°C to degrees Fahrenheit **43.** 50°F to degrees Celsius
- 44.** 5°C to degrees Fahrenheit **45.** 68°F to degrees Celsius **46.** 15°C to degrees Fahrenheit
- 47.** 5 min to seconds **48.** 4 days to hours
- 49.** 12 weeks to days **50.** 14 hr to minutes

B

- 51.** 200°F to degrees Celsius **52.** 59°C to degrees Fahrenheit **53.** 48°F to degrees Celsius
- 54.** 8°C to degrees Fahrenheit **55.** 31°C to degrees Fahrenheit **56.** 250°F to degrees Celsius
- 57.** 14°C to degrees Fahrenheit **58.** 93°F to degrees Celsius
- 59.** 1.5 yr to hours **60.** 22 sec to minutes
- 61.** 250 min to days **62.** 365 days to weeks

C

- 63.** $(12 \text{ lb } 2 \text{ oz}) + (2 \text{ lb } 15 \text{ oz}) + (11 \text{ lb } 5 \text{ oz})$ **64.** $(16 \text{ gal } 3 \text{ qt}) + (13 \text{ gal } 3 \text{ qt}) + (17 \text{ gal } 2 \text{ qt})$
- 65.**
$$\begin{array}{r} 2 \text{ gal } 3 \text{ qt } 1 \text{ pt} \\ + 4 \text{ gal } 2 \text{ qt } 1 \text{ pt} \\ \hline \end{array}$$
- 66.**
$$\begin{array}{r} 17 \text{ lb } 3 \text{ oz} \\ - 12 \text{ lb } 14 \text{ oz} \\ \hline \end{array}$$

67. The Corner Grocery sold 20 lb 6 oz of hamburger on Wednesday, 13 lb 8 oz on Thursday, and 21 lb 9 oz on Friday. How much hamburger was sold during the 3 days?
68. Normal body temperature is considered 98.6°F. What is normal body temperature on the Celsius scale?
69. Spencer, who is a lab assistant, has 300 mL of acid that is to be divided equally among 24 students. How many milliliters will each student receive?
70. A doctor prescribes allergy medication of two tablets, 20 mg each, to be taken three times per day for a full week. How many milligrams of medication will the patient get in a week?
71. A bank time-temperature sign is stuck on 25°C. What is the Fahrenheit reading?
72. Mikal combines two cans of soup for lunch. The chicken vegetable soup is 1 lb 8 oz and the chicken noodle is 15 oz. How much soup does Mikal have?
73. Christine has a 2-liter bottle of Mountain Dew that she needs to divide evenly among 10 children at a birthday party. How much Mountain Dew does each child get?
74. The table gives the daily high temperature for Ocala, Florida, for 1 week in December. Find the average temperature for the week.

Daily High Temperatures

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
62°F	54°F	57°F	64°F	68°F	69°F	67°F

75. If a bag contains 397 g of potato chips, how many grams are contained in 4 bags?
76. An elevator has a maximum capacity of 2500 lb. A singing group of 8 men and 8 women get on. The average weight of the women is 125 lb, and the average weight of the men is 190 lb. Can they ride safely together?
77. The weight classes for Olympic wrestling (both freestyle and Greco-Roman) are given in the table. Fill in the equivalent pound measures, rounded to the nearest whole pound.
78. In the shipping industry, most weights are measured in long tons, which are defined as 2240 lb. The largest tanker in the world is the *Jahre Viking*, which weighs 662,420 (long) tons fully loaded. How much does the *Jahre Viking* weigh in pounds?

Olympic Wrestling Weight Classes

Kilograms	54	58	63	69	76	85	97	130
Pounds								

79. How much does the *Jahre Viking* weigh in kilograms? (See Exercise 78.) Round to the nearest million.
80. A rule of thumb for a person's daily protein requirement is that the number of grams of protein needed is the same as half the person's weight in pounds. Steve weighs 71 kg. How much protein should he eat per day?

- 81.** One serving of Life cereal has 3 g of protein, and putting $\frac{1}{2}$ c of 1% milk on it adds 4.5 g of protein. If Steve eats only milk and cereal for a day, how many servings does he need in order to get sufficient protein? (See Exercise 80).
- 82.** Scientists give the average surface temperature of Earth as 59°F. They estimate that the temperature increases about 1°F for every 200 ft drop in depth below the surface. What is the temperature 1 mi below the surface?

STATE YOUR UNDERSTANDING

- 83.** Can you add 4 g to 5 in.? Explain how, or explain why you cannot do it.
- 84.** If 8 in. + 10 in. = 1 ft 6 in., why isn't it true that 8 oz + 10 oz = 1 lb 6 oz?

GROUP WORK

Use the information in the list of conversions to answer Exercises 85–87. Compare your answers with those of another group. Precious metals and gems are measured in troy weight according to the following:

$$\begin{array}{ll} 1 \text{ pennyweight (dwt)} & = 24 \text{ grains} \\ 1 \text{ ounce troy (oz t)} & = 20 \text{ pennyweights} \\ 1 \text{ pound troy (lb t)} & = 12 \text{ ounce troy} \end{array}$$

- 85.** How many grains are in 1 oz t? How many grains are in 1 lb t?
- 86.** Suppose you have a silver bracelet that you want a jeweler to melt down and combine with the silver of two old rings to create a medallion that weighs 5 oz t 14 dwt. The bracelet weighs 3 oz t 18 dwt and one ring weighs 1 oz t 4 dwt. What does the second ring weigh?
- 87.** Kayla has an ingot of platinum that weighs 2 lb t 8 oz t 15 dwt. She wants to divide it equally among her six grandchildren. How much will each piece weigh?

MAINTAIN YOUR SKILLS

Do the indicated operations.

88. $54 + 20 + 97$ **89.** $451 + 88 + 309$ **90.** $3.7 + 12.99$ **91.** $34.7 + 6.8 + 0.44$

92. $4\frac{3}{8} + 2\frac{7}{8}$ **93.** $12\frac{3}{5} + 32\frac{3}{4}$ **94.** $\frac{4}{9} + \frac{3}{8} + \frac{11}{12}$ **95.** $5.2 + 148 + \frac{1}{4}$

- 96.** Convert 4 yd 2 ft 11 in. into inches.
- 97.** Convert 4.82 km into meters.

7.3 Perimeter

VOCABULARY

A **polygon** is any closed figure whose sides are line segments.

Polygons are named according to the number of sides they have. Table 7.9 lists some common polygons.

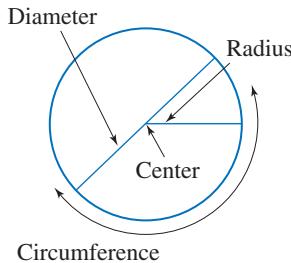
Quadrilaterals are polygons with four sides. Table 7.10 lists the characteristics of common quadrilaterals.

The **perimeter** of a polygon is the distance around the outside of the polygon.

The **circumference** of a circle is the distance around the circle.

The **radius** of a circle is the distance from the center to any point on the circle.

The **diameter** of a circle is twice the radius.



OBJECTIVES

- Find the perimeter of a polygon.
- Find the circumference of a circle.

Table 7.9 Common Polygons

Number of Sides	Name	Picture	Number of Sides	Name	Picture
3	Triangle ABC	A triangle with vertices labeled A, B, and C. Vertex A is at the top, B is at the bottom right, and C is at the bottom left.	6	Hexagon ABCDEF	A regular hexagon with vertices labeled A through F in clockwise order starting from the top vertex A.
4	Quadrilateral ABCD	A quadrilateral with vertices labeled A, B, C, and D in clockwise order starting from the top-left vertex A.	8	Octagon ABCDEFGH	An octagon with vertices labeled A through H in clockwise order starting from the top vertex A.
5	Pentagon ABCDE	A pentagon with vertices labeled A through E in clockwise order starting from the top vertex A.			

Table 7.10 Common Quadrilaterals

Trapezoid



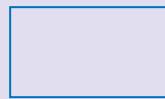
One pair of parallel sides

Parallelogram



Two pairs of equal parallel sides

Rectangle



A parallelogram with four right angles

Square



A rectangle with all sides equal

How & Why

OBJECTIVE 1

Find the perimeter of a polygon.

The perimeter of a figure can be thought of in terms of the distance traveled by walking around the outside of it or by the length of a fence around the figure. The units of measure used for perimeters are length measures, such as inches, feet, and meters. Perimeter is calculated by adding the length of all the individual sides.

For example, to calculate the perimeter of this figure, we add the lengths of the sides.

$$\begin{array}{r} \text{1 ft 6 in.} \\ \text{11 in.} \\ \hline \text{1 ft 9 in.} \end{array} \quad \begin{array}{r} \text{1 ft 9 in.} \\ \text{1 ft 2 in.} \\ \text{1 ft 6 in.} \\ + \text{11 in.} \\ \hline \text{3 ft 28 in.} \end{array}$$

$$\begin{aligned} 3 \text{ ft 28 in.} &= 3 \text{ ft} + 2 \text{ ft 4 in.} & 28 \text{ in.} &= 2 \text{ ft 4 in.} \\ &= 5 \text{ ft 4 in.} & & \end{aligned}$$

The perimeter is 5 ft 4 in.

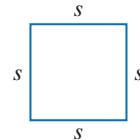
To find the perimeter of a polygon

Add the lengths of the sides.

To find the perimeter of a square

Multiply the length of one side by 4.

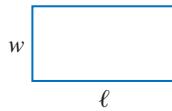
$$P = 4s$$



To find the perimeter of a rectangle

Add twice the length and twice the width.

$$P = 2\ell + 2w$$



Examples A–D

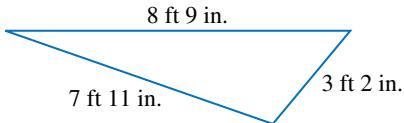


BASIC Mathematics Now™

DIRECTIONS: Find the perimeters of the given polygons.

STRATEGY: Add the lengths of the sides.

- A. Find the perimeter of the triangle.



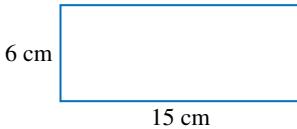
$$\begin{array}{r} 8 \text{ ft } 9 \text{ in.} \\ 7 \text{ ft } 11 \text{ in.} \\ + 3 \text{ ft } 2 \text{ in.} \\ \hline 18 \text{ ft } 22 \text{ in.} = 19 \text{ ft } 10 \text{ in.} \end{array}$$

Add the lengths.

$$22 \text{ in.} = 1 \text{ ft } 10 \text{ in.}$$

The perimeter is 19 ft 10 in.

- B. Find the perimeter of the rectangle.



$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(15 \text{ cm}) + 2(6 \text{ cm}) \\ &= 30 \text{ cm} + 12 \text{ cm} \\ &= 42 \text{ cm} \end{aligned}$$

Perimeter formula for rectangles

Substitute.

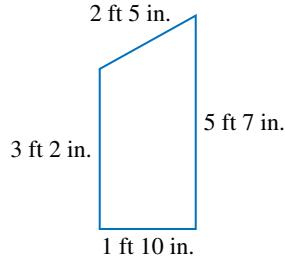
Multiply.

Add.

The perimeter of the rectangle is 42 cm.

Warm-Ups A–D

- A. Find the perimeter of the trapezoid.



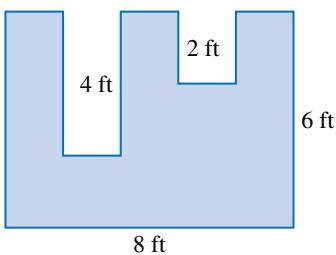
- B. Find the perimeter of the square.



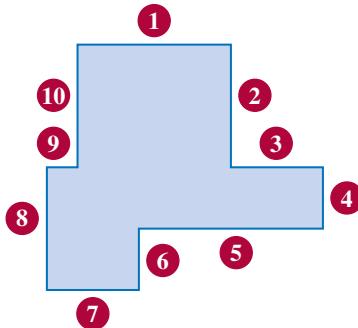
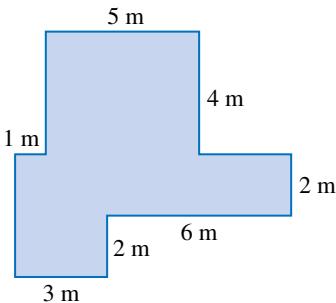
Answers to Warm-Ups

- A. 13 ft B. 52 yd

- C. Find the perimeter of the polygon.



- C. Find the perimeter of the polygon.



Side	Length
1	5 m
2	4 m
3	?
4	2 m
5	6 m
6	2 m
7	3 m
8	?
9	1 m
10	?
3	3 m
8 + 10	8 m

To find the perimeter, number the sides (there are 10) and write down their lengths.

$$\begin{aligned} \text{To find the length of side 3, notice that} \\ \text{side } 3 &= \text{side } 5 + \text{side } 7 - \text{side } 9 - \text{side } 1 \\ &= 6 \text{ m} + 3 \text{ m} - 1 \text{ m} - 5 \text{ m} \\ &= 3 \text{ m} \end{aligned}$$

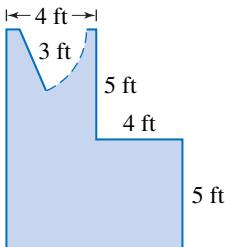
$$\begin{aligned} \text{Sides 8 and 10 are not given, but} \\ \text{side } 8 + \text{side } 10 &= \text{side } 2 + \text{side } 4 + \text{side } 6 \\ &= 4 \text{ m} + 2 \text{ m} + 2 \text{ m} \\ &= 8 \text{ m} \end{aligned}$$

So,

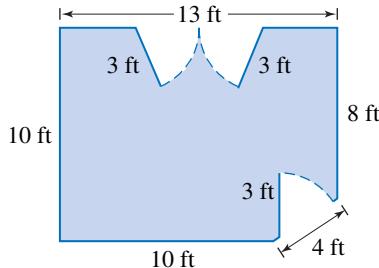
$$\begin{aligned} P &= 5 \text{ m} + 4 \text{ m} + 3 \text{ m} + 2 \text{ m} + 6 \text{ m} + 2 \text{ m} + 3 \text{ m} + 1 \text{ m} + 8 \text{ m} \\ &= 34 \text{ m} \end{aligned}$$

The perimeter of the polygon is 34 m.

- D. How much baseboard lumber is needed for the room pictured?



- D. A carpenter is replacing the baseboards in a room. The floor of the room is pictured. How many feet of baseboard are needed?



$$\begin{aligned} P &= 13 \text{ ft} + 10 \text{ ft} + 10 \text{ ft} + 4 \text{ ft} + 8 \text{ ft} \\ P &= 45 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{Baseboard} &= 45 \text{ ft} - (6 \text{ ft} + 3 \text{ ft}) \\ &= 45 \text{ ft} - 9 \text{ ft} \\ &= 36 \text{ ft} \end{aligned}$$

The carpenter needs 36 ft of baseboard.

Find the perimeter of the room, including all the doors. Subtract the combined width of the doors. Simplify.

Answers to Warm-Ups

- C. 40 ft
D. The carpenter needs 33 ft of baseboard.

How & Why

OBJECTIVE 2

Find the circumference of a circle.

Formulas for geometric figures that involve circles contain the number called pi (π). The number π is the quotient of the circumference of (distance around) the circle and its diameter. This number is the same for every circle no matter how large or small. This remarkable fact was discovered over a long period of time, and during that time a large number of approximations have been used. Today, the most commonly used approximations are

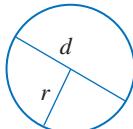
$$\pi \approx 3.14 \quad \text{or} \quad \pi \approx \frac{22}{7}$$

Calculator note: Scientific and graphing calculators have a $\boxed{\pi}$ key that generates a decimal value for π with 8, 10, or 12 decimal places depending on the calculator. Using the $\boxed{\pi}$ key instead of one of the approximations increases the accuracy of your calculations.

To find the circumference of a circle

If C is the circumference, d is the diameter, and r is the radius of a circle, then the circumference is the product of π and the diameter or the product of π and twice the radius.

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$



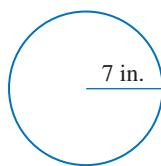
Because the radius, diameter and circumference of a circle are all lengths, they are measured in units of length.

Examples E–F

DIRECTIONS: Find the circumference of the circle.

STRATEGY: Use the formula $C = \pi d$ or $C = 2\pi r$ and substitute.

- E. Find the circumference of the circle.



$$C = 2\pi r$$

$$C \approx 2(3.14)(7 \text{ in.})$$

Formula for circumference

Substitute. Because we are using an approximation for the value of π , the value of the circumference is also an approximation. We show this using \approx .

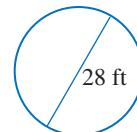
$$C \approx 43.96$$

Simplify.

The circumference is about 43.96 in.

Warm-Ups E–F

- E. Find the circumference of the circle. Let $\pi \approx 3.14$.



Answers to Warm-Ups

E. 87.92 ft

ALTERNATIVE SOLUTION: Use a calculator and the π key.

$$C = 2\pi r$$

Formula for circumference

$$C = 2\pi (7 \text{ in.})$$

Substitute. Use the π key.

$$C \approx 43.9822971503 \text{ in.}$$

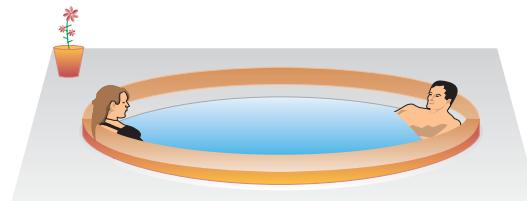
$$C \approx 43.98 \text{ in.}$$

Round to the nearest hundredth.

The circumference is about 43.98 in. Because a more accurate approximation for π was used, we conclude that this is a more accurate approximation for the circumference than the first calculation.

- F. How much duct tape is needed to tape around an irrigation pipe that has a radius of $1\frac{1}{2}$ in.? Use the π key and round to the nearest tenth.

- F. Chia-Ching wants to install a border of tile around her new in-ground spa. The spa is circular, with a diameter of 4.25 m. What is the circumference of the spa? Round to the nearest hundredth.



$$C = \pi d$$

Formula for circumference

$$C = \pi(4.25 \text{ m})$$

Substitute. Use the π key.

$$C \approx 13.3517687778 \text{ m}$$

Round.

The circumference of the spa is about 13.35 m.

Answers to Warm-Ups

- F. About 9.4 in. of duct tape are needed.

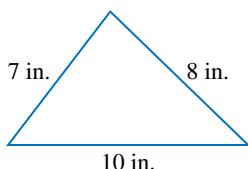
Exercises 7.3

OBJECTIVE 1

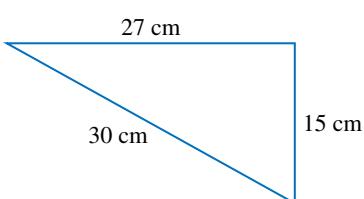
Find the perimeter of a polygon.

A Find the perimeter of the following polygons.

1.



2.



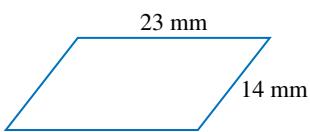
3.



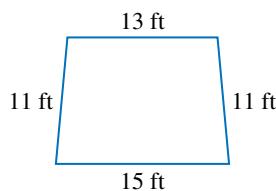
4.



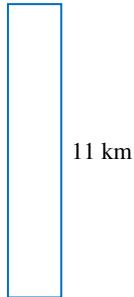
5.



6.



7.



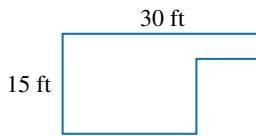
8.

**B**

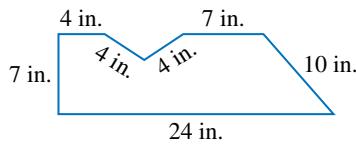
9. Find the perimeter of a triangle with sides of 16 mm, 27 mm, and 40 mm.
10. Find the perimeter of a square with sides of 230 ft.
11. Find the distance around a rectangular field with a length of 50 m and a width of 35 m.
12. Find the distance around a rectangular swimming pool with a width of 20 yd and a length of 25 yd.

Find the perimeter of the following polygons.

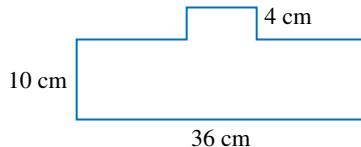
13.

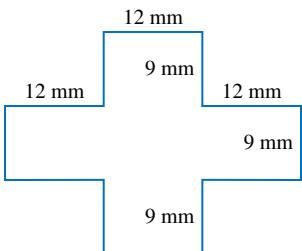
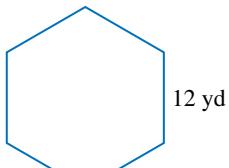
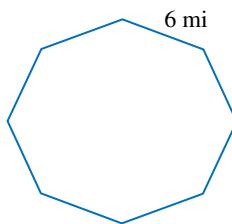
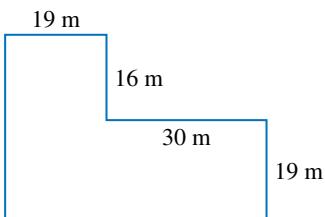
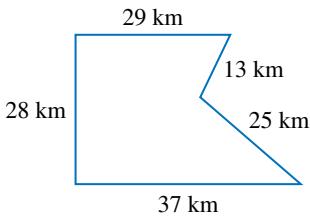


14.

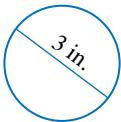
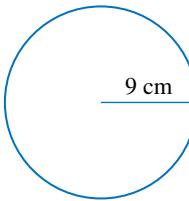
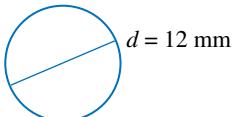
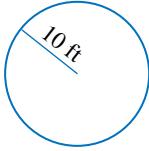
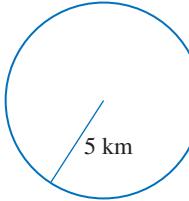
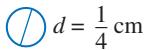
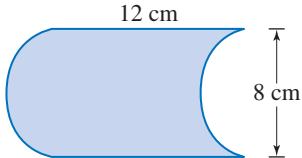
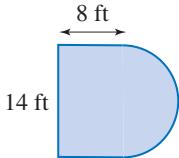
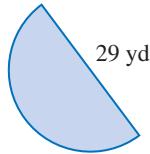


15.

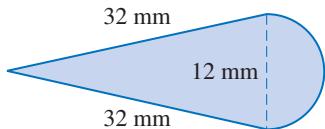


16.**17.****18.****19.****20.****OBJECTIVE 2**

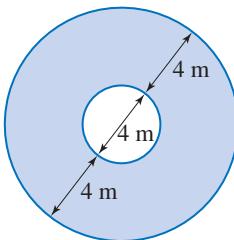
Find the circumference of a circle.

A Find the circumference of the given circles. Let $\pi \approx 3.14$.**21.****22.****23.****24.****25.****26.****B** Find the perimeter of each shaded figure. Use the π key, and round to the nearest hundredth.**27.****28.****29.**

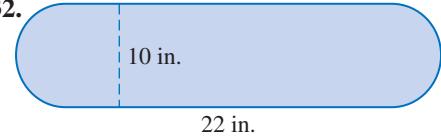
30.



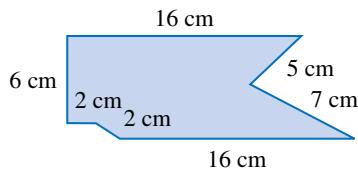
31.



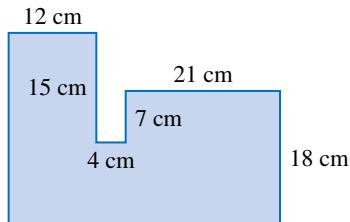
32.

**C** Find the perimeter of the shaded regions. Use the π key, and round decimals to the nearest hundredth when necessary.

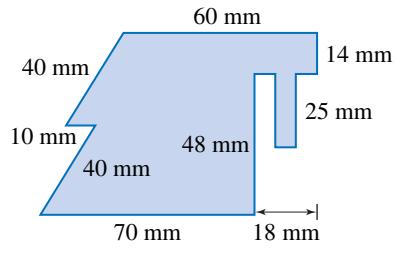
33.



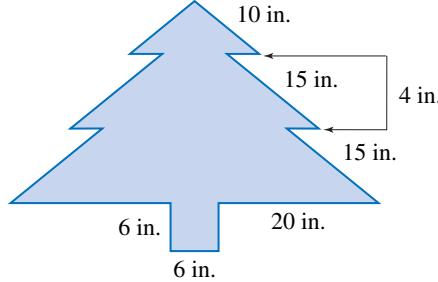
34.



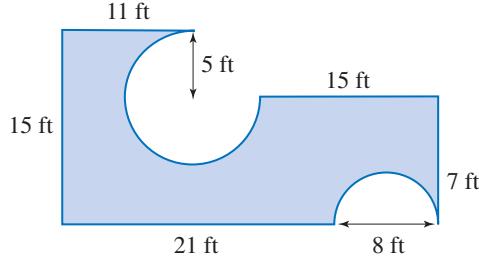
35.



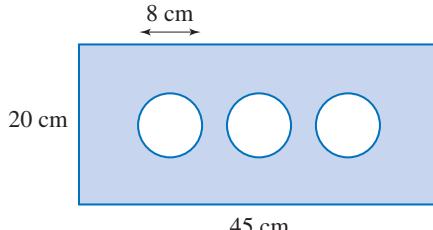
36.



37.



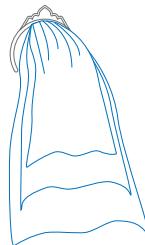
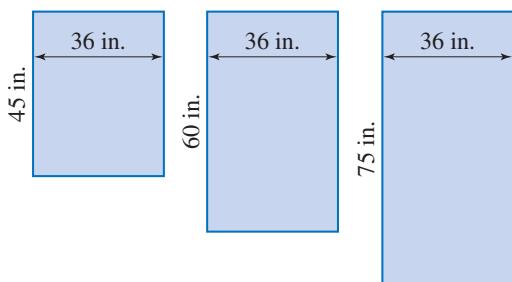
38.



39. How many feet of picture molding are needed to frame five pictures, each measuring 8 in. by 10 in.? The molding is wide enough to require an extra inch added to each dimension to allow for the corners to be mitered.
41. If Hazel needs 2 minutes to put 1 ft of binding on a rug, how long will it take her to put the binding on a rug that is 15 ft by 12 ft?

40. If fencing costs \$15 per meter, what will be the cost of fencing a rectangular lot that is 80 km long and 22 km wide?

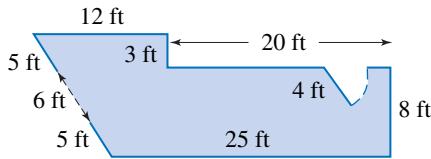
42. Annessa is making a three-tiered bridal veil. Each tier is rectangular, with one of the short ends being gathered into the headpiece. The other three sides of each tier are to be trimmed in antique lace. How many yards of lace does Annessa need?



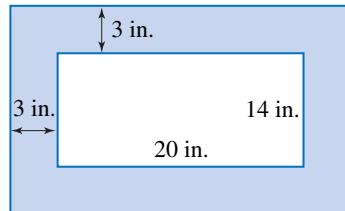
44. Jorge is lining the windows in his living room with Christmas lights. He has one picture window that is 5 ft 8 in. by 4 ft. On each side, there is a smaller window that is 2 ft 6 in. by 4 ft. What length of Christmas lights does Jorge need for the three windows?

46. As a conditioning exercise, a soccer coach has his team run around the outside of the field three times. If the field measures 60 yd \times 100 yd, how far did the team run?

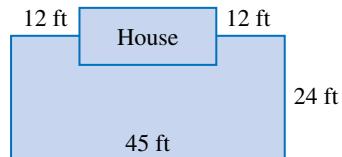
48. A carpenter is putting baseboards in the family room/dining room pictured here. How many feet of baseboard molding are needed?



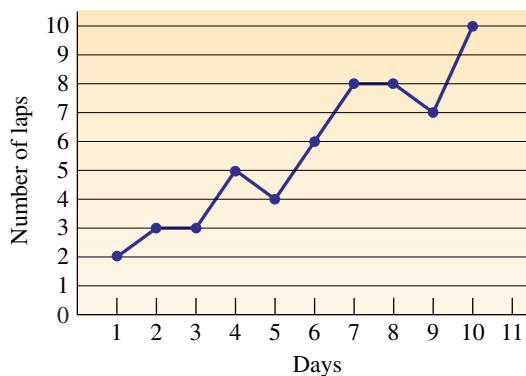
43. Holli has a watercolor picture that is 14 in. by 20 in. She puts it in a mat that is 3 in. wide on all sides. What are the inside dimensions of the frame she needs to buy?



45. Jenna and Scott just bought a puppy and need to fence their backyard. How much fence should they order?



47. A high-school football player charted the number of laps he ran around the football field during the first 10 days of practice. If the field measures 120 yd by 53 yd, how far did he run during the 10 days? Convert your answer to the nearest whole mile.

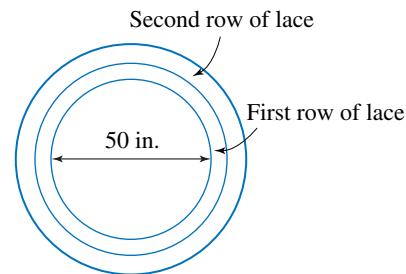


Exercises 49–51 refer to the following. Marylane is constructing raised beds in her backyard for her vegetable garden. Each bed is made from a frame of 2- \times -12-in. pressure-treated lumber. Marylane is planning three rectangular frames that measure 4 ft by 8 ft. The lumber comes in 8-ft lengths, and costs \$15.49 per board.

49. How much lumber is needed for the project?
50. How many boards are needed for the project?
51. What is the total cost of the lumber for the project?

- 52.** Yolande has lace that is 2 in. wide that she plans to sew around the edge of a circular tablecloth that is 50 in. in diameter. How much lace does she need?

- 53.** Yolande wants to sew a second row of her 2-in.-wide lace around the tablecloth in Exercise 52. How much lace does she need for the second row?



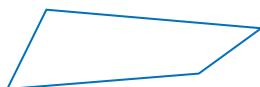
Exercises 54–57 refer to the chapter application, see page 579.

- 54.** What is the shape of the patio?
- 56.** Estimate the perimeter of the flowerbed that wraps around the right side of the patio.
- 58.** Are rectangles symmetrical around a line drawn lengthwise through the center? Are squares symmetrical?
- 60.** Draw a circle that is symmetrical and another that is not symmetrical.

- 55.** What is the perimeter of the patio?
- 57.** Draw a line down the center of the backyard. What do you notice about the two halves? Mathematicians call figures like this symmetrical, because a center line divides the figure into “mirror images.” Colonial architecture typically makes use of symmetry.
- 59.** Draw a triangle that is symmetrical, and another that is not symmetrical.

STATE YOUR UNDERSTANDING

- 61.** Explain what perimeter is and how the perimeter of the figure below is determined.
- 62.** Is a rectangle a parallelogram? Why or why not?

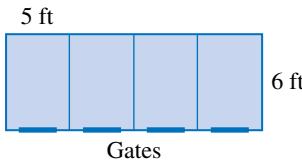


What possible units (both English and metric) would the perimeter be likely to be measured in if the figure is a national park? If the figure is a room in a house? If the figure is a scrap of paper?

- 63.** Explain the difference between a square and a rectangle.

CHALLENGE

- 64.** A farmer wants to build the goat pens pictured below. Each pen will have a gate 2 ft 6 in. wide in the end. What is the total cost of the pens if the fencing is \$3.00 per linear foot and each gate is \$15.00?

**GROUP WORK**

- 65.** FedEx will accept packages according to the formula

$$L_1 + 2L_2 + 2L_3 \leq 160 \text{ in.}$$

in which L_1 is the longest side, L_2 is the next longest side, and L_3 is the shortest side.

Make a table of dimensions of boxes that can be shipped by FedEx. Try to find the maximum sizes possible.

Longest Side	Next Longest Side	Shortest Side	Total

MAINTAIN YOUR SKILLS

- 66.** Write the place value name for nine hundred thousand, fifty.
- 68.** Round 32,571,600 to the nearest ten thousand.
- 70.** Find the product of 733 and 348.
- 72.** Find the sum of 2 lb 3 oz and 3 lb 14 oz.
- 74.** Find the average bowling score for Fred if he bowled games of 167, 182, and 146.
- 67.** Write the place value name for nine hundred fifty thousand.
- 69.** Find the difference between 733 and 348.
- 71.** Find the quotient of 153,204 and 51.
- 73.** Find the difference of 6 ft and 7 in.
- 75.** What is the total cost for a family of one mother and three children to attend a hockey game if adult tickets are \$15, student tickets are \$10, and the parking fee is \$6?

7.4 Area

VOCABULARY

Area is a measure of surface—that is, the amount of space inside a two-dimensional figure. It is measured in square units.

The **base** of a geometric figure is a side parallel to the horizon.

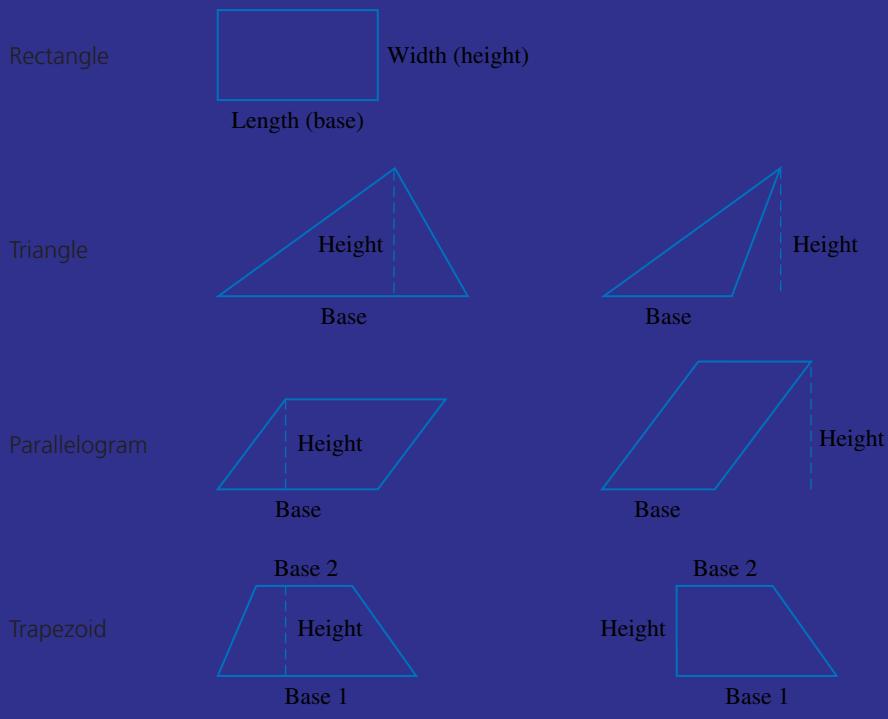
The **altitude** or **height** of a geometric figure is the perpendicular (shortest) distance from the base to the highest point of the figure.

Table 7.11 shows the base and altitude of some common geometric figures.

OBJECTIVES

- Find the area of common polygons and circles.
- Convert units of area measure.

Table 7.11 Base and Height of Common Geometric Figures



How & Why

OBJECTIVE 1

Find the area of common polygons and circles.

Suppose you wish to tile a rectangular bathroom floor that measures 5 ft by 6 ft. The tiles are 1-ft-by-1-ft squares. How many tiles do you need?

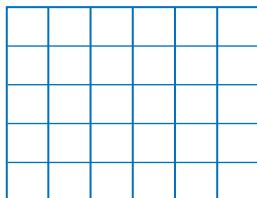


Figure 7.1

Using Figure 7.1 as a model of the tiled floor, you can count that 30 tiles are necessary.

Area is a measure of the surface—that is, the amount of space inside a two-dimensional figure. It is measured in square units. Square units are literally squares that measure one unit of length on each side. Figure 7.2 shows two examples.

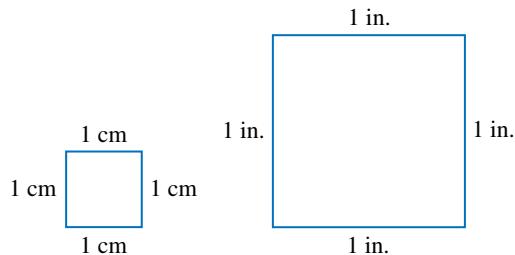


Figure 7.2

The measure of the square area on the left is 1 square centimeter, abbreviated 1 cm^2 . The measure of the square area on the right is 1 square inch, abbreviated 1 in^2 . Recall that the superscript in the abbreviation is simply part of the unit's name. It does not indicate an exponential operation.

CAUTION

$10 \text{ cm}^2 \neq 100 \text{ cm}$

In the bathroom floor example, each tile has an area of 1 ft^2 . So the number of tiles needed is the same as the area of the room, 30 ft^2 . We could have arrived at this number by multiplying the length of the room by the width. This is not a coincidence. It works for all rectangles.

To find the area of a rectangle

Multiply the length by the width.

$$A = \ell w$$



To find the area of other geometric shapes, we use the area of a rectangle as a reference.

Because a square is a special case of a rectangle, with all sides equal, the formula for the area of a square is

$$\begin{aligned} A &= \ell w \\ &= s(s) \\ &= s^2 \end{aligned}$$

To find the area of a square

Square the length of one of the sides.

$$A = s^2$$



Now let's consider the area of a triangle. We start with a right triangle that is, a triangle with one 90° angle. See Figure 7.3.

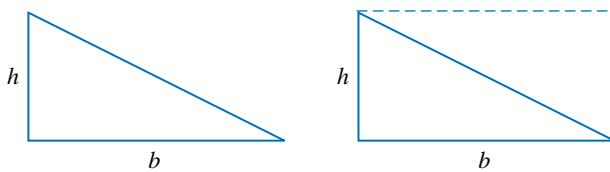


Figure 7.3

The triangle on the left has base b and height h . The figure on the right is a rectangle with length b and width h . According to the formula for rectangles, the area is $A = \ell w$ is $A = bh$. However the rectangle is made up of two triangles, both of which have a base of b and a height of h . The area of the rectangle (bh) is exactly twice the area of the triangle. So we conclude that the area of the triangle is

$$\frac{1}{2} \cdot bh \quad \text{or} \quad \frac{bh}{2} \quad \text{or} \quad bh \div 2.$$

Now let's consider a more general triangle. See Figure 7.4.

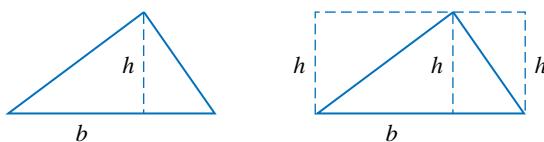


Figure 7.4

Again, the area on the left is a triangle with base b and height h . And the figure on the right is a rectangle with length b and width h . Can you see that the rectangle must be exactly twice the area of the original triangle? So again we conclude that the area of the triangle is

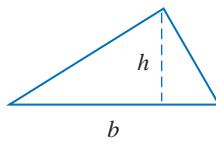
$$\frac{1}{2} \cdot bh \quad \text{or} \quad \frac{bh}{2} \quad \text{or} \quad bh \div 2.$$

Recall that height is the perpendicular distance from the base to the highest point of a figure.

To find the area of a triangle

Multiply the base times the height and divide by 2.

$$A = \frac{1}{2} \cdot bh \quad \text{or} \quad A = \frac{bh}{2} \quad \text{or} \quad A = bh \div 2$$

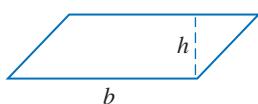


It is possible to use rectangles to find the formulas for the areas of parallelograms and trapezoids. This is left as an exercise. (See Exercise 79.)

To find the area of a parallelogram

Multiply the base times the height.

$$A = bh$$



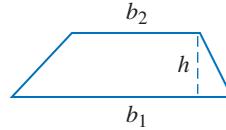
CAUTION

The base of a parallelogram is the same as its longest side, but the height of the parallelogram is *not* the same as the length of its shortest side.

To find the area of a trapezoid

Add the two bases together, multiply by the height, then divide by 2.

$$A = \frac{(b_1 + b_2)h}{2}$$



The lengths, widths, bases, sides, and heights must all be measured using the same units before the area formulas can be applied. If the units are different, convert to a common unit before calculating area.

The formula for the area of a circle is less intuitive than the formulas discussed thus far. Nevertheless, the formula for the area of a circle has been known and used for many years. Even though the sides of a circle are not line segments, we still measure the area in square units.

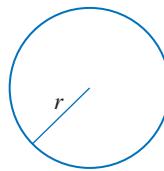
Reminder

You can use 3.14 or $\frac{22}{7}$ to approximate π or use the π key on your calculator for a more accurate approximation.

To find the area of a circle

Square the radius and multiply by π .

$$A = \pi r^2$$



Warm-Ups A–G

- A. Find the area of a square that is 12 cm on each side.

- B. A decorator found a 15 yd^2 remnant of carpet. Will it be enough to carpet a 5-yd-by-4-yd playroom?

Answers to Warm-Ups

- A. The area is 144 cm^2 .
B. The remnant will not be enough because the decorator needs 20 yd^2 .

Examples A–G



BASIC Mathematics Now™

DIRECTIONS: Find the area.

STRATEGY: Use the area formulas.

- A. Find the area of a square that is 7 in. on each side.

$$\begin{aligned} A &= s^2 && \text{Formula} \\ &= (7 \text{ in.})^2 && \text{Substitute.} \\ &= 49 \text{ in}^2 && 1 \text{ in.} \times 1 \text{ in.} = 1 \text{ in}^2. \end{aligned}$$

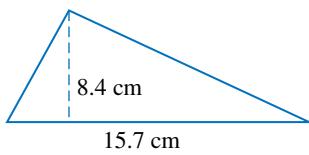
The area is 49 in^2 .

- B. A gallon of deck paint will cover 400 ft^2 . A contractor needs to paint a rectangular deck that is 26 ft long and 15 ft wide. Will 1 gal of paint be enough?

$$\begin{aligned} A &= \ell w && \text{Formula} \\ &= (26 \text{ ft})(15 \text{ ft}) && \text{Substitute.} \\ &= 390 \text{ ft}^2 && 1 \text{ ft} \times 1 \text{ ft} = 1 \text{ ft}^2. \end{aligned}$$

Because $390 \text{ ft}^2 < 400 \text{ ft}^2$, 1 gal of paint will be enough.

- C. Find the area of this triangle.



$$\begin{aligned}
 A &= \frac{bh}{2} && \text{Formula} \\
 &= \frac{(15.7 \text{ cm})(8.4 \text{ cm})}{2} && \text{Substitute.} \\
 &= \frac{131.88 \text{ cm}^2}{2} && \text{Simplify. } 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2. \\
 &= 65.94 \text{ cm}^2
 \end{aligned}$$

The area is 65.94 cm^2 .

- D. Find the area of the parallelogram with a base of 1 ft and a height of 4 in.

$$\begin{aligned}
 A &= bh && \text{Formula} \\
 &= (1 \text{ ft})(4 \text{ in.}) && \text{Substitute.} \\
 &= (12 \text{ in.})(4 \text{ in.}) && \text{Convert so that units match.} \\
 &= 48 \text{ in}^2 && \text{Simplify.}
 \end{aligned}$$

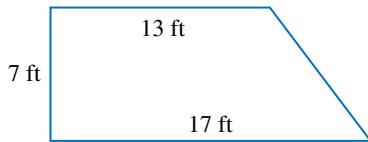
The area of the parallelogram is 48 in^2 .

- E. Find the area of the top of a coffee can that has a radius of 102 mm. Let $\pi \approx 3.14$.

$$\begin{aligned}
 A &= \pi r^2 && \text{Formula} \\
 &\approx (3.14)(102 \text{ mm})^2 && \text{Substitute.} \\
 &\approx (3.14)(10404 \text{ mm}^2) && \text{Simplify.} \\
 &\approx 32,668.56 \text{ mm}^2
 \end{aligned}$$

The area of the top of the coffee can is about $32,668.56 \text{ mm}^2$.

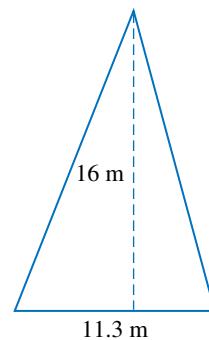
- F. Find the area of the trapezoid pictured.



$$\begin{aligned}
 A &= \frac{(b_1 + b_2)h}{2} && \text{Formula} \\
 &= \frac{(17 \text{ ft} + 13 \text{ ft})(7 \text{ ft})}{2} && \text{Substitute.} \\
 &= \frac{(30 \text{ ft})(7 \text{ ft})}{2} && \text{Simplify.} \\
 &= \frac{210 \text{ ft}^2}{2} \\
 &= 105 \text{ ft}^2
 \end{aligned}$$

The area is 105 ft^2 .

- C. Find the area of this triangle.



- D. Find the area of the parallelogram with a base of 6 yd and a height of 5 ft.

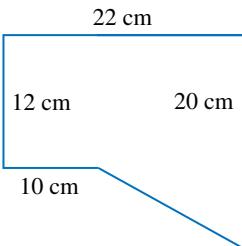
- E. What is the area of a quarter that has a radius of 13 mm? Let $\pi \approx 3.14$.

- F. Find the area of the trapezoid with bases of 51 m and 36 m and a height of 11 m.

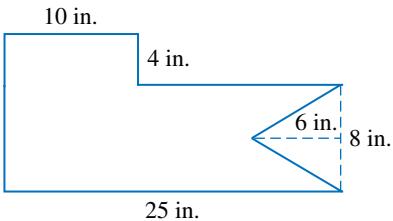
Answers to Warm-Ups

- C. The area is 90.4 m^2 .
- D. The area is 90 ft^2 .
- E. The area is about 530.66 mm^2 .
- F. The area is 478.5 m^2 .

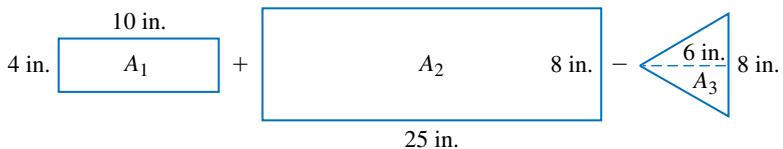
G. Find the area of the polygon.



G. Find the area of the polygon.



STRATEGY: To find the area of a polygon that is a combination of two or more common figures, first divide it into the common figure components.



$$\text{Total area} = A_1 + A_2 - A_3$$

$$A_1 = (10 \text{ in.})(4 \text{ in.}) = 40 \text{ in}^2$$

$$A_2 = (25 \text{ in.})(8 \text{ in.}) = 200 \text{ in}^2$$

$$A_3 = \frac{(6 \text{ in.})(8 \text{ in.})}{2} = \frac{48 \text{ in}^2}{2} = 24 \text{ in}^2$$

$$\begin{aligned}\text{Total area} &= 40 \text{ in}^2 + 200 \text{ in}^2 - 24 \text{ in}^2 \\ &= 216 \text{ in}^2\end{aligned}$$

Divide into components.

Compute the areas of each component.

Combine the areas.

The total area of the figure is 216 in^2 .

How & Why

OBJECTIVE 2

Convert units of area measure.

In the United States, we usually measure the dimensions of a room in feet, and so naturally, we measure its area in square feet. However, carpeting is measured in square yards. Therefore, if we want to buy carpeting for a room, we need to convert units of area measure—in this case, from square feet to square yards. We proceed as before, using unit fractions. The only difference is that we use the appropriate unit fraction twice, since we are working with square units.

To change 100 ft^2 to square yards, we write

$$\begin{aligned}100 \text{ ft}^2 &= 100(\text{ft})(\text{ft})\left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)\left(\frac{1 \text{ yd}}{3 \text{ ft}}\right) \\ &= \frac{100}{9} \text{ yd}^2 \\ &= 11\frac{1}{9} \text{ yd}^2\end{aligned}$$

Answers to Warm-Ups

G. The area is 312 cm^2 .

Examples H–I



DIRECTIONS: Convert as indicated.

STRATEGY: Use the appropriate unit fraction as a factor twice.

- H. Convert 25 mi^2 to square kilometers. Round to the nearest thousandth.

$$25 \text{ mi}^2 \approx 25(\text{mi})(\text{mi}) \left(\frac{1.6093 \text{ km}}{1 \text{ mi}} \right) \left(\frac{1.6093 \text{ km}}{1 \text{ mi}} \right) \quad 1.6093 \text{ km} \approx 1 \text{ mi}$$
$$\approx 64.746 \text{ km}^2 \qquad \text{Simplify.}$$

So $25 \text{ mi}^2 \approx 64.746 \text{ km}^2$.

- I. Convert 378 cm^2 to square meters.

$$378 \text{ cm}^2 = 378(\text{cm})(\text{cm}) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \quad 100 \text{ cm} = 1 \text{ m.}$$
$$= \frac{378}{10,000} \text{ m}^2 \qquad \text{Simplify.}$$
$$= 0.0378 \text{ m}^2$$

So $378 \text{ cm}^2 = 0.0378 \text{ m}^2$.

Warm-Ups H–I

- H. Convert 5 m^2 to square feet. Round to the nearest hundredth.

- I. Convert 3000 m^2 to square kilometers.

Answers to Warm-Ups

- H. 53.82 ft^2 I. 0.003 km^2

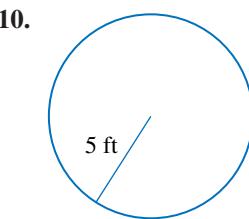
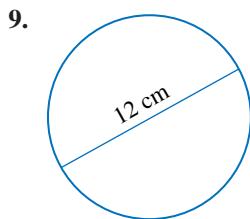
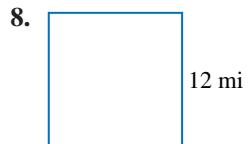
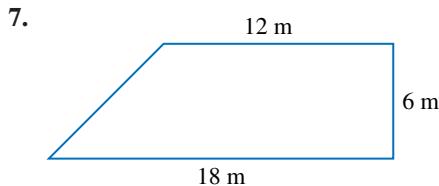
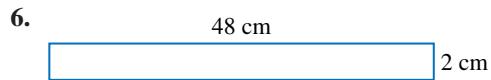
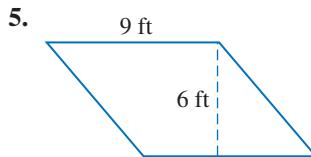
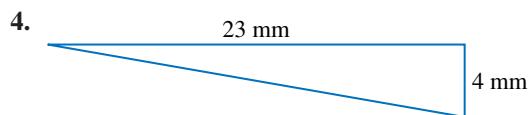
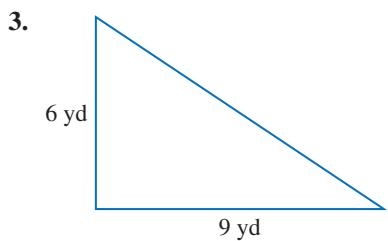
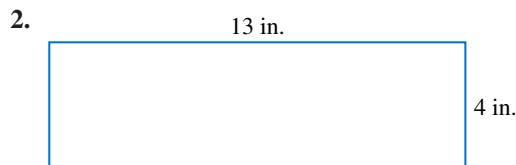
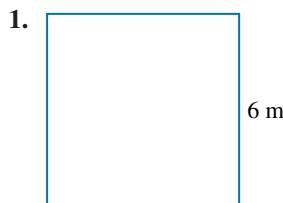
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Exercises 7.4

OBJECTIVE 1

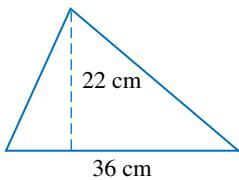
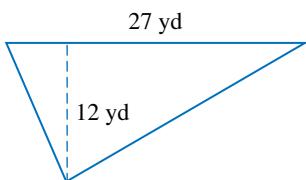
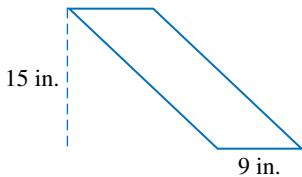
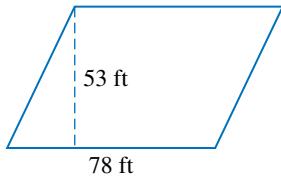
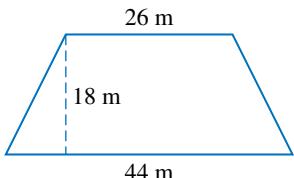
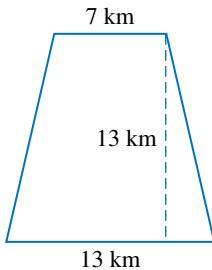
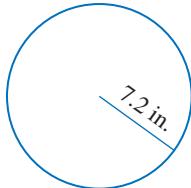
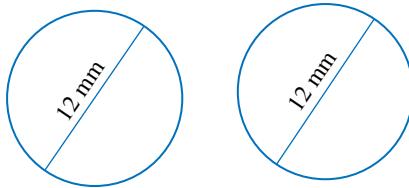
Find the area of common polygons and circles.

- A Find the area of the following figures. Use $\pi \approx \frac{22}{7}$.



B Find the areas of the figures. Use $\pi \approx 3.14$.

- 11.** Find the area of a rectangle that has a length of 14 km and a width of 10 km.
- 12.** Find the area of a square with sides of 136 cm.
- 13.** Find the area of a triangle with a base of 26 yd and a height of 98 yd.
- 14.** Find the area of a circle with a radius of 6 m.
- 15.** Find the area of a parallelogram with a base of 54 in. and a height of 30 in.
- 16.** Find the area of a circle with a diameter of 40 mm.

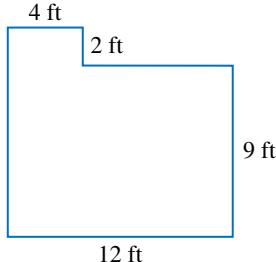
17.**18.****19.****20.****21.****22.****23.****24.**

OBJECTIVE 2

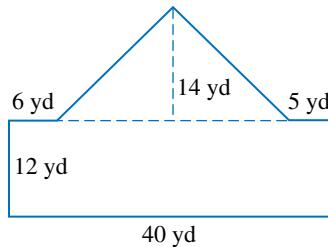
Convert units of area measure.

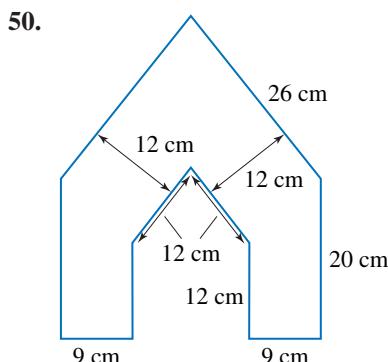
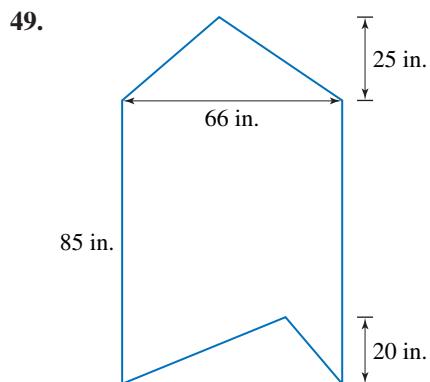
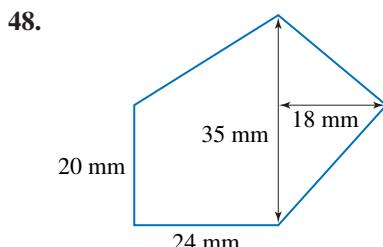
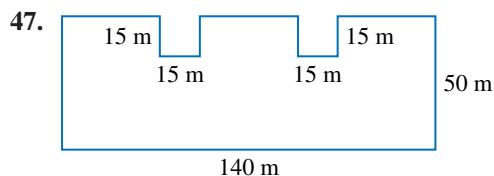
A Convert units as indicated. If necessary, round to the nearest hundredth.25. Convert 4 ft^2 to square inches.26. Convert 4 cm^2 to square millimeters.27. Convert 5 yd^2 to square feet.28. Convert 13 m^2 to square centimeters.29. Convert 1 mi^2 to square feet.30. Convert 8 km^2 to square meters.31. Convert 1296 in^2 to square feet.32. Convert $2,000,000 \text{ cm}^2$ to square meters.33. Convert 117 ft^2 to square yards.34. Convert 8700 mm^2 to square centimeters.**B**35. Convert 6.5 yd^2 to square feet.36. Convert 54 km^2 to square centimeters.37. Convert $15,000,000 \text{ ft}^2$ to square miles.38. Convert $456,789,123 \text{ cm}^2$ to square meters.39. Convert 250 mm^2 to square inches.40. Convert 100 in^2 to square centimeters.41. Convert 3 m^2 to square yards.42. Convert 80 km^2 to square miles.43. Convert 15 ft^2 to square centimeters.44. Convert 100 mi^2 to square kilometers.
Let $1 \text{ mi}^2 \approx 2.5900 \text{ km}^2$.**C** Find the areas of the figures.

45.

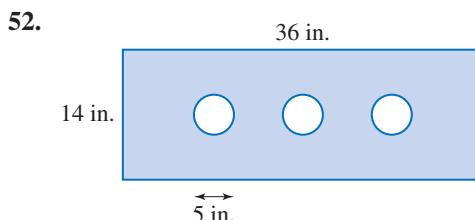
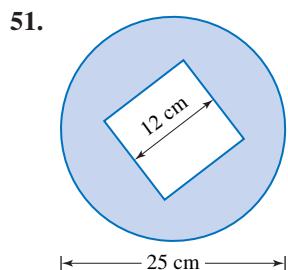


46.





Find the areas of the shaded regions. Use the π key, and round to the nearest hundredth.



53. The side of Jane's house that has no windows measures 35 ft by 22 ft. If 1 gal of stain will cover 250 ft^2 , will 2 gal of stain be enough to stain this side?

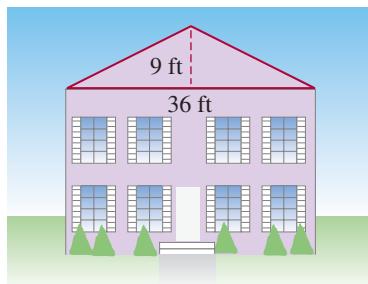
54. The south side of Jane's house measures 75 ft by 22 ft and has two windows, each 4 ft by 6 ft. Will 4 gal of stain be enough for the south side? See Exercise 53.

55. April and Larry are building an in-ground circular spa. The spa is 4 ft deep and 8 ft in diameter. They intend to tile the sides and bottom of the spa. How many square feet of tile do they need? (Hint: The sides of a cylinder form a rectangle whose length is the circumference of the circle.) Round to the nearest whole number.

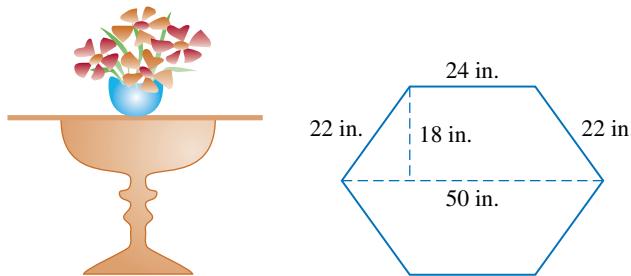
56. Ingrid plans on carpeting two rooms in her house. The floor in one room measures 30 ft by 24 ft and the floor in the other room measures 22 ft by 18 ft. If the carpet costs \$27 per square yard installed, what will it cost Ingrid to have the carpet installed?

57. If 1.25 oz of weed killer treats 1 m² of lawn, how many ounces of weed killer will Debbie need to treat a rectangular lawn that measures 15 m by 30 m?

59. How much glass is needed to replace a set of two sliding glass doors in which each pane measures 3 ft by 6 ft?
61. How many square feet of sheathing are needed for the gable end of a house that has a rise of 9 ft and a span of 36 ft? (See the drawing.)



63. How much padding is needed to make a pad for the hexagonal table pictured?



65. In Exercise 64, the material for Sheila's tablecloth is 100 in. wide. How many yards of material should she buy? Round to the nearest yard.

58. To the nearest acre, how many acres are contained in a rectangular plot of ground if the length is 1850 ft and the width is 682 ft? ($43,560 \text{ ft}^2 = 1 \text{ acre}$.) Round to the nearest whole number.

60. How many square yards of carpet are needed to cover a rectangular room that is 9 ft by 12 ft?
62. Frederica wants to construct a toolshed that is 6 ft by 9 ft around the base and 8 ft high. How many gallons of paint will be needed to cover the outside of all four walls of the shed? (Assume that 1 gal covers 250 ft².)

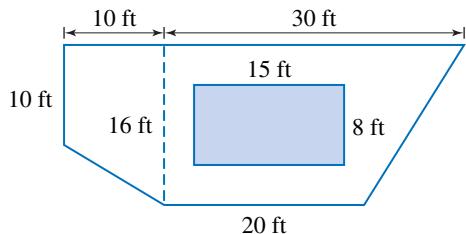
64. Sheila is making a circular tablecloth for an accent table. The top of the table is 20 in. in diameter, and the table is 3 ft tall. Sheila wants the tablecloth to touch the floor all around. Assuming a $\frac{1}{2}$ in. hem, how much material does she need? Use $\pi \approx 3.14$, and round to the nearest square inch.

66. A window manufacturer is reviewing the plans of a duplex to determine the amount of glass needed to fill the order. The number and size of the windowpanes and sliding glass doors are listed in the table.

Dimensions	Number Needed
Windows needed	3 ft by 3 ft
	3 ft by 4 ft
	3 ft by 5 ft
	4 ft by 4 ft
	5 ft by 6 ft
Sliding doors	7 ft by 3 ft

How much glass does he need to fill the order?

- 67.** One 2-lb bag of wildflower seed will cover 70 ft^2 . How many bags of seed are needed to cover the region shown here? Do not seed the shaded area.



- 69.** How many 8-ft-by-4-ft sheets of paneling are needed to panel two walls of a den that measure 14 ft by 8 ft and 10 ft by 8 ft?

- 68.** How many square yards of carpet are needed to carpet a flight of stairs if each stair is 10 in. by 40 in., the risers are 8 in., and there are 12 stairs and 13 risers in the flight? Round to the nearest square yard.

Exercises 71–72 refer to the chapter application. See page 579.

- 71.** The contractor who was hired to build the brick patio will begin by pouring a concrete slab. Then he will put the bricks on the slab. The estimate of both the number of bricks and the amount of mortar needed is based on the area of the patio and walkways. Subdivide the patio and the walkways into geometric figures; then calculate the total area to be covered in bricks.

- 70.** Joy is making a king-sized quilt that measures 72 in. by 85 in. She needs to buy fabric for the back of the quilt. How many yards should she purchase if the fabric is 45 in. wide? Round to the nearest square yard.

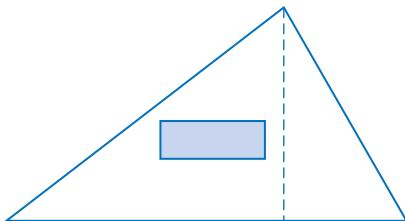
STATE YOUR UNDERSTANDING

- 73.** What kinds of units measure area? Give examples from both systems.

- 72.** The number of bricks and amount of mortar needed also depend on the thickness of the mortar between the bricks. The plans specify a joint thickness of $\frac{1}{4}$ in. According to industry standards, this will require seven bricks per square foot. Find the total number of bricks required for the patio and walkways.

- 75.** Describe how you could approximate the area of a geometric figure using 1-in. squares.

- 74.** Explain how to calculate the area of the figure below. Do not include the shaded area.



CHALLENGE

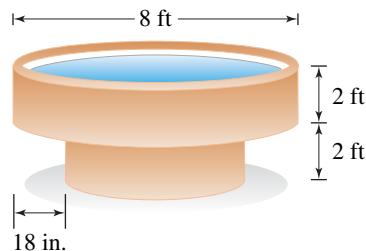
- 76.** Joe is going to tile his kitchen floor. Along the outside he will put black squares that are 6 in. on each side. The next (inside) row will be white squares that are 6 in. on each side. The remaining interior space will be a checker board pattern of alternating black and white tiles that are 1 ft on each side. How many tiles of each color will he need for the kitchen floor, which measures 12 ft by 10 ft?
- 77.** A rectangular plot of ground measuring 120 ft by 200 ft is to have a cement walkway 5 ft wide placed around the inside of the perimeter. How much of the area of the plot will be used by the walkway and how much of the area will remain for the lawn?

200 ft

120 ft



- 78.** April and Larry decide that their spa should have a built-in ledge to sit on that is 18 in. wide. The ledge will be 2 ft off the bottom. How many square feet of tile do they need for the entire inside of the spa? See Exercise 55. Round to the nearest whole square foot.

**GROUP WORK**

- 79.** Use the formula for the area of a rectangle to find the area of a parallelogram and a trapezoid. Draw pictures to illustrate your argument.
- 80.** Determine the coverage of 1 gallon of semigloss paint. How much of this paint is needed to paint your classroom, excluding chalkboards, windows, and doors? What would it cost? Compare your results with the other groups in the class. Did all groups get the same results? Give possible explanations for the differences.

MAINTAIN YOUR SKILLS

- 81.** Multiply 45.79 by 10,000.
- 82.** What percent of 568 is 122? Round to the nearest whole percent.
- 83.** Simplify: $2(46 - 28) + 50 \div 5$
- 84.** A family of six attends a weaving exhibition. Parking is \$4, adult admission is \$6, senior admission is \$4, and child admission is \$3. How much does it cost for two parents, one grandmother, and three children to attend the exhibition?

85. Simplify: $3^2 + 2^3$

86. Solve the proportion: $\frac{x}{45} = \frac{13}{16}$. Round to the nearest hundredth.

87. Find the average, median, and mode:
16, 17, 18, 19, 16, and 214

88. Find the perimeter of a rectangle that is 4 in. wide and 2 ft long.

89. The table lists calories burned per hour for various activities.

90. Convert 4.5 ft to centimeters. Round to the nearest hundredth.

Activity	Calories Burned/Hr
Sitting	50
Slow walking	125
Stacking firewood	350
Hiking	500

Make a bar graph that summarizes this information.

7.5 Volume

VOCABULARY

A **cube** is a three-dimensional geometric solid that has six sides (called **faces**), each of which is a square.

Volume is the name given to the amount of space that is contained inside a three-dimensional object.

OBJECTIVES

- Find the volume of common geometric shapes.
- Convert units of volume.

How & Why

OBJECTIVE 1

Find the volume of common geometric shapes.

Suppose you have a shoebox that measures 12 in. long by 4 in. wide by 5 in. high that you want to use to store toy blocks that are 1 in. by 1 in. by 1 in. How many blocks will fit in the box? See Figure 7.5.

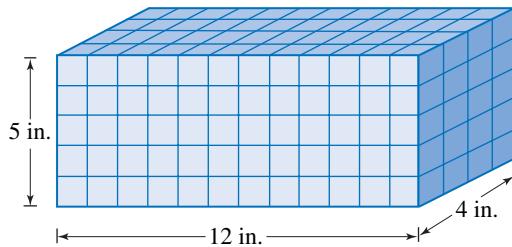


Figure 7.5

In each layer there are $12(4) = 48$ blocks and there are 5 layers. Therefore, the box holds $12(4)(5) = 240$ blocks.

Volume is a measure of the amount of space contained in a three-dimensional object. Often, volume is measured in cubic units. These units are literally cubes that measure one unit on each side. For example, Figure 7.6 shows a cubic inch (1 in^3) and a cubic centimeter (1 cm^3).

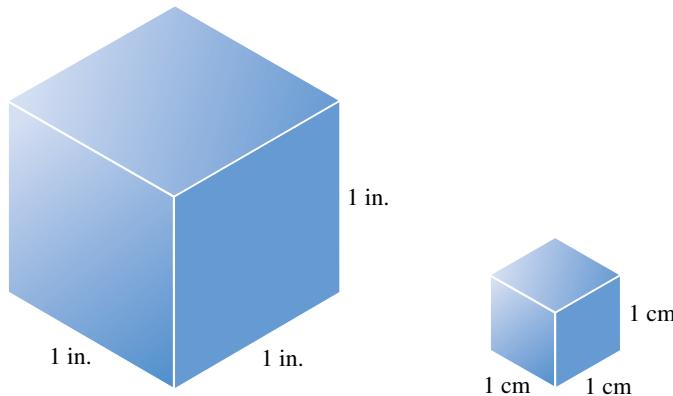


Figure 7.6

The shoebox has a volume of 240 in^3 because exactly 240 blocks, which have a volume of 1 in^3 each, can fit in the box and totally fill it up.

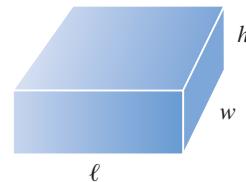
In general, volume can be thought of as the number of cubes that fill up a space. If the space is a rectangular solid, like the shoebox, it is a relatively easy matter to determine the volume by making a layer of cubes that covers the bottom and then deciding how many layers are necessary to fill the box.

Observe that the number of cubes necessary for the bottom layer is the same as the area of the base of the box, ℓw . The number of layers needed is the same as the height of the box, h . So we have the following volume formulas.

To find the volume of a rectangular solid

Multiply the length by the width by the height.

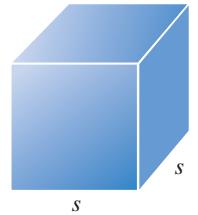
$$V = \ell wh$$



To find the volume of a cube

Cube the measure of one of the sides.

$$V = s^3$$



The length, width, and height must all be measured using the same units before the volume formulas may be applied. If the units are different, convert to a common unit before calculating the volume.

The principle used for finding the volume of a box can be extended to any solid with sides that are perpendicular to the base. The area of the base gives the number of cubes necessary to make the bottom layer, and the height gives the number of layers necessary to fill the solid. See Figure 7.7.

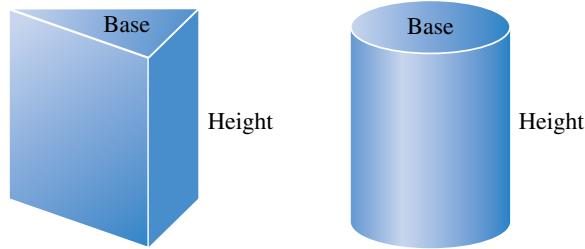


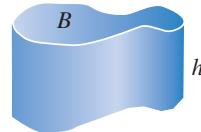
Figure 7.7

To find the volume of a solid with sides perpendicular to the base

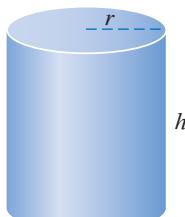
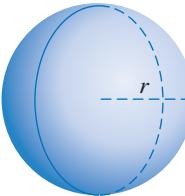
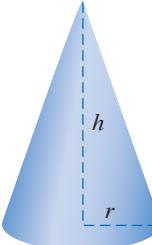
Multiply the area of the base by the height.

$$V = Bh$$

where B is the area of the base.



Because cylinders, spheres, and cones contain circles, their volume formulas also contain the number π . Even though these shapes are curved instead of straight, their volumes are measured in cubic units.

Name	Picture	Formula
Cylinder (right circular cylinder)		$V = \pi r^2 h$ r = radius of the base h = height of the cylinder
Sphere		$V = \frac{4}{3} \pi r^3$ r = radius of the sphere
Cone (right circular cone)		$V = \frac{1}{3} \pi r^2 h$ r = radius of the cone h = height of the cone

Examples A–D



BASIC
Mathematics Now™

DIRECTIONS: Find the volume.

STRATEGY: Use the volume formulas.

- A. Find the volume of a cube that is 5 mm on each side.

$$\begin{aligned} V &= s^3 && \text{Formula} \\ &= (5 \text{ mm})^3 && \text{Substitute } 5 \text{ mm for } s. \\ &= 125 \text{ mm}^3 && \text{Simplify.} \end{aligned}$$

The volume of the cube is 125 mm³.

- B. How much concrete is needed to pour a step that is 4 ft long, 3 ft wide, and 6 in. deep?

$$\begin{aligned} V &= \ell wh && \text{Formula} \\ &= (4 \text{ ft})(3 \text{ ft})(6 \text{ in.}) && \text{Substitute.} \\ &= (4 \text{ ft})(3 \text{ ft})(0.5 \text{ ft}) && \text{Convert inches to feet.} \\ &= 6 \text{ ft}^3 && \text{Simplify.} \end{aligned}$$

The step requires 6 ft³ of concrete.

ALTERNATIVE SOLUTION:

$$\begin{aligned} V &= \ell wh && \text{Formula} \\ &= (4 \text{ ft})(3 \text{ ft})(6 \text{ in.}) && \text{Substitute.} \\ &= (48 \text{ in.})(36 \text{ in.})(6 \text{ in.}) && \text{Convert feet to inches.} \\ &= 10,368 \text{ in}^3 && \end{aligned}$$

The step requires 10,368 in³ of concrete.

Warm-Ups A–D

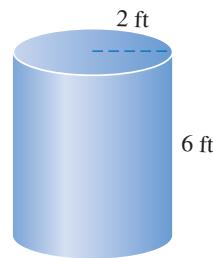
- A. Find the volume of a cube that is 8 ft on each side.

- B. How much concrete is needed for a rectangular stepping stone that is 1 ft long, 8 in. wide, and 3 in. deep?

Answers to Warm-Ups

- A. 512 ft³
B. It will take 288 in³ or $\frac{1}{6}$ ft³ of concrete.

- C. Find the volume of a cylinder with a radius of 12 cm and a height of 13 cm. Round to the nearest hundredth. Let $\pi \approx 3.14$.



$$\begin{aligned} V &= \pi r^2 h \\ &\approx (3.14)(2 \text{ ft})^2(6 \text{ ft}) \\ &\approx 75.36 \text{ ft}^3 \end{aligned}$$

Formula

Substitute.

Simplify.

The volume of the can is about 75.36 ft^3 .

ALTERNATIVE SOLUTION: Use the π key and round to the nearest hundredth.

$$\begin{aligned} V &= \pi r^2 h \\ &= (\pi)(2 \text{ ft})^2(6 \text{ ft}) \\ &\approx 75.40 \text{ ft}^3 \end{aligned}$$

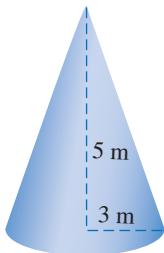
Formula

Substitute.

Simplify using the π key on your calculator.

The volume of the can is about 75.40 ft^3 .

- D. Find the volume of a right circular cone that has a base diameter of 3 ft and a height of 7 ft. Use the π key, and round to the nearest hundredth.



$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(3 \text{ m})^2(5 \text{ m}) \\ &\approx 47.12 \text{ m}^3 \end{aligned}$$

Formula

Substitute.

Simplify, using π key on your calculator. Round to the nearest hundredth.

The volume of the cone is about 47.12 m^3 .

How & Why

OBJECTIVE 2

Convert units of volume.

As Example B suggests, it is possible to measure the same volume in both cubic feet and cubic inches. We calculated that the amount of concrete needed for the step is 6 ft^3 , or $10,368 \text{ in}^3$.

To verify that $6 \text{ ft}^3 = 10,368 \text{ in}^3$, and to convert units of volume in general, we use unit fractions. Because we are converting cubic units, we use the unit fraction three times.

$$\begin{aligned} 6 \text{ ft}^3 &= 6(\text{ft})(\text{ft})(\text{ft}) \\ &= 6 \left(\frac{\text{ft}}{1 \text{ ft}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \right) \left(\frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \right) \\ &= 6 \cdot 12 \cdot 12 \cdot 12(\text{in.})(\text{in.})(\text{in.}) \\ &= 10,368 \text{ in}^3 \end{aligned}$$

Answers to Warm-Ups

- C. The volume is about 5878.08 cm^3 .
D. The volume is about 16.49 ft^3 .

When measuring the capacity of a solid to hold liquid, we sometimes use special units. Recall that in the English system, liquid capacity is measured in ounces, quarts, and gallons. In the metric system, milliliters, liters, and kiloliters are used. One cubic centimeter measures the same volume as one milliliter. That is, $1 \text{ cm}^3 = 1 \text{ mL}$. So a container with a volume of 50 cm^3 holds 50 mL of liquid.

Remember this special relationship:

$$1 \text{ cm}^3 = 1 \text{ mL}$$

Examples E–H



BASIC Mathematics Now™

DIRECTIONS: Convert units of volume.

STRATEGY: Use the appropriate unit fraction three times.

- E. Convert 25 yd^3 to cubic feet.

$$\begin{aligned} 25 \text{ yd}^3 &= 25 \text{ yd}^3 \cdot \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) && \text{Use the unit fraction } \frac{3 \text{ ft}}{1 \text{ yd}}. \\ &= 25 \cdot 3 \cdot 3 \cdot 3 \text{ ft}^3 \\ &= 675 \text{ ft}^3 \end{aligned}$$

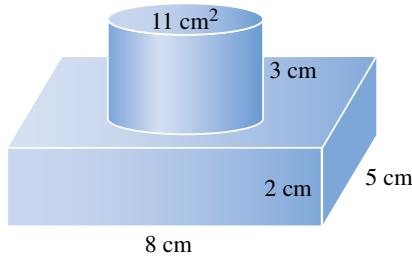
So $25 \text{ yd}^3 = 675 \text{ ft}^3$.

- F. How many cubic meters in 76 cm^3 ?

$$\begin{aligned} 76 \text{ cm}^3 &= 76 \text{ cm}^3 \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \\ &= \frac{76}{100 \cdot 100 \cdot 100} \text{ m}^3 \\ &= 0.000076 \text{ m}^3 \end{aligned}$$

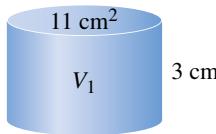
So $76 \text{ cm}^3 = 0.000076 \text{ m}^3$.

- G. How many milliliters of water does this container hold?

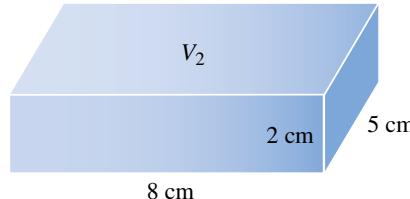


$$\text{Total volume} = V_1 + V_2$$

$$\begin{aligned} V_1 &= Bh \\ &= (11 \text{ cm}^2)(3 \text{ cm}) \\ &= 33 \text{ cm}^3 \end{aligned}$$



$$\begin{aligned} V_2 &= \ell wh \\ &= (8 \text{ cm})(5 \text{ cm})(2 \text{ cm}) \\ &= 80 \text{ cm}^3 \end{aligned}$$



$$\begin{aligned} \text{Total volume} &= 33 \text{ cm}^3 + 80 \text{ cm}^3 \\ &= 113 \text{ cm}^3 \\ &= 113 \text{ mL} \quad 1 \text{ cm}^3 = 1 \text{ mL} \end{aligned}$$

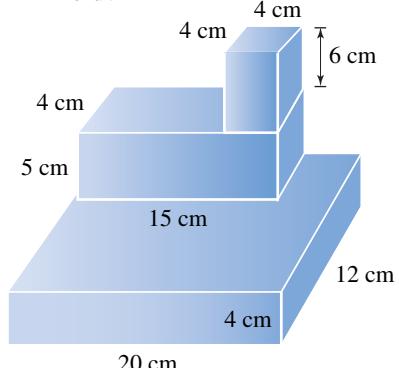
The container holds 113 mL of water.

Warm-Ups E–H

- E. Convert 10 ft^3 to cubic inches.

- F. How many cubic kilometers in $450,000 \text{ m}^3$?

- G. How many milliliters of water does this container hold?



Answers to Warm-Ups

E. $17,280 \text{ in}^3$

F. 0.00045 km^3

G. It holds 1356 mL of water.

- H.** June wants to mulch a flowerbed along her house. The bed is 25 ft by 3 ft. She wants the mulch to be 6 in. deep. How many 0.5 yd^3 bags of mulch does she need?

- H.** Dave has a rose garden that is 6 ft by 15 ft that he wants to cover in mulch 4 in. deep. Bags of mulch hold 0.5 yd^3 each. How many bags of mulch does Dave need for his garden?

$$\begin{aligned}V &= \ell wh \\&= (15 \text{ ft})(6 \text{ ft})(4 \text{ in.}) \\&= (15 \text{ ft})(6 \text{ ft})\left(\frac{1}{3} \text{ ft}\right) \\&= 30 \text{ ft}^3 \\&= 30(\text{ft})(\text{ft})(\text{ft})\left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)\left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)\left(\frac{1 \text{ yd}}{3 \text{ ft}}\right) \\&= \frac{30}{27} \text{ yd}^3 \\&\approx 1.1 \text{ yd}^3\end{aligned}$$

Formula

Substitute 15 ft for ℓ , 6 ft for w , and 4 in. for h .

Convert inches to feet.

Simplify.

Convert cubic feet to cubic yards.

Simplify.

Round to the nearest tenth of a cubic yard.

Because $1.1 \div 0.5 = 2.2$, Dave will need three bags of mulch to cover his garden to a depth of 4 in. with some mulch left over. Or, because 1.1 yd^3 is pretty close to 1 yd^3 , Dave could choose to buy only two bags of mulch and settle for the mulch being not quite 4 in. deep.

Answers to Warm-Ups

- H.** June needs three bags of mulch.

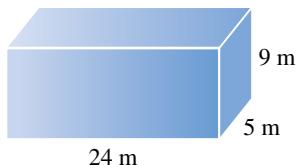
Exercises 7.5

OBJECTIVE 1

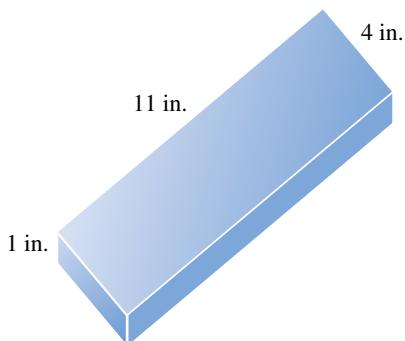
Find the volume of common geometric shapes.

A Find the volume of the figures. Let $\pi \approx 3.14$.

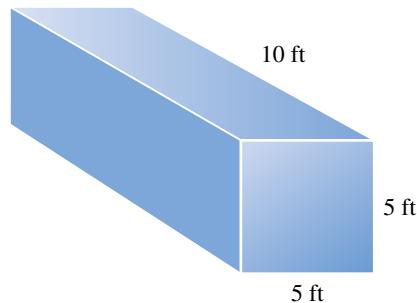
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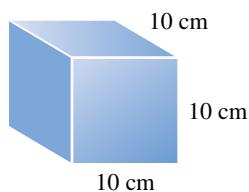
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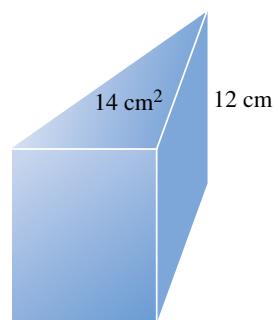
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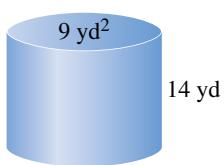
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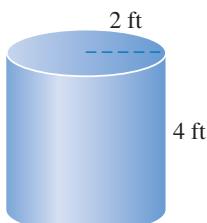
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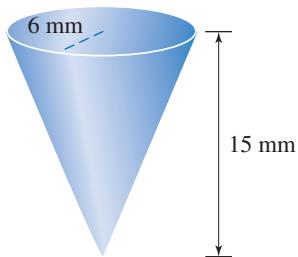
6.



7.



8.



B

9. How many mL of water will fill up a box that measures 50 cm long, 30 cm wide, and 18 cm high?
10. Find the volume of a cube that measures 22 in. on each side.
11. Find the volume of a garbage can that is 3 ft tall and has a circular base of 14 ft^2 .
12. Find the volume of two identical fuzzy dice tied to the mirror of a 1957 Chevy if one edge measures 12 cm.
13. Find the volume of a beach ball that is 18 in. in diameter. Use 3.14 for π .
14. Find the volume of an ice cream cone that is $4\frac{1}{2}$ in. tall and has a top diameter of 4 in. Use $\frac{22}{7}$ for π .

Find the volume of the figure. Use the $\boxed{\pi}$ key and if necessary round to the nearest hundredth.

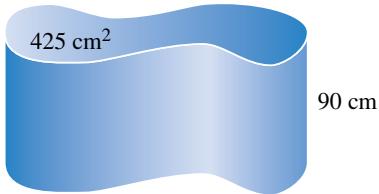
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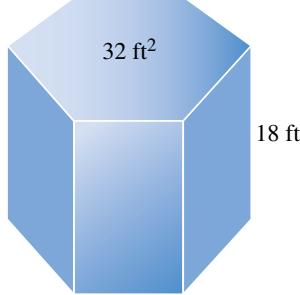
16.



17.



18.

**OBJECTIVE 2**

Convert units of volume.

A Convert units of volume.

19. Convert 1 ft^3 to cubic inches.
20. Convert 1 yd^3 to cubic feet.
21. Convert 1 m^3 to cubic centimeters.
22. Convert 1 km^3 to cubic meters.
23. Convert 1 yd^3 to cubic inches.
24. Convert 1 km^3 to cubic centimeters.
25. Convert 8640 in^3 to cubic feet.
26. Convert 270 ft^3 to cubic yards.
27. Convert $10,000 \text{ mm}^3$ to cubic centimeters.
28. Convert $7,000,000,000 \text{ m}^3$ to cubic kilometers.

B Convert. Round decimal values to the nearest hundredth.

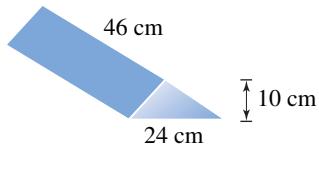
29. Convert 15 ft^3 to cubic inches.
 30. Convert 1 mi^3 to cubic feet.
 31. Convert 15 m^3 to cubic centimeters.
 32. Convert $987,654,321 \text{ m}^3$ to cubic kilometers.
 33. Convert 180 in^3 to cubic feet.
 34. Convert 4 ft^3 to cubic centimeters.
 35. Convert 1 mi^3 to cubic kilometers.
 36. Convert 375 m^3 to cubic yards.
 37. Convert 8700 cm^3 to cubic feet.
 38. Convert 30 in^3 to cubic millimeters.

C

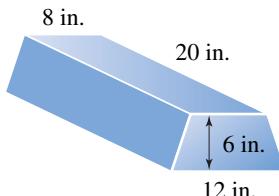
39. Find the volume of a can that has a base with an area of 265 in^2 and a height of 2 ft.
 40. How many cubic inches of concrete are needed to pour a sidewalk that is 3 ft wide, 4 in. deep, and 54 ft long? Concrete is commonly measured in cubic yards, so convert your answer to cubic yards. (*Hint:* Convert to cubic feet first, and then convert to cubic yards.)

Find the volume of the figures.

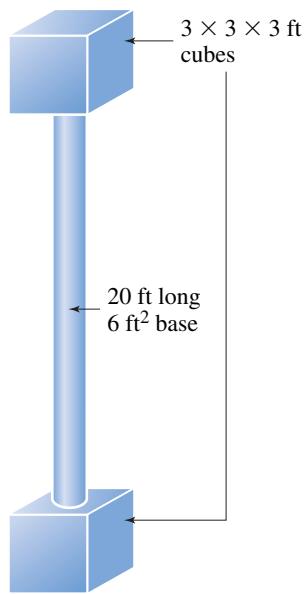
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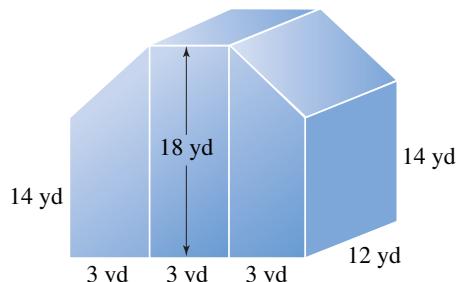
42.

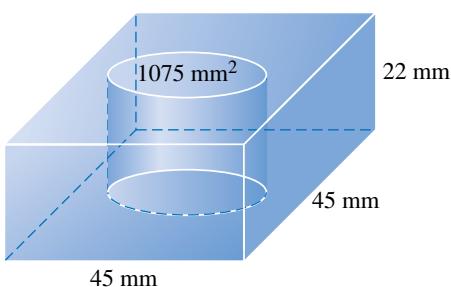
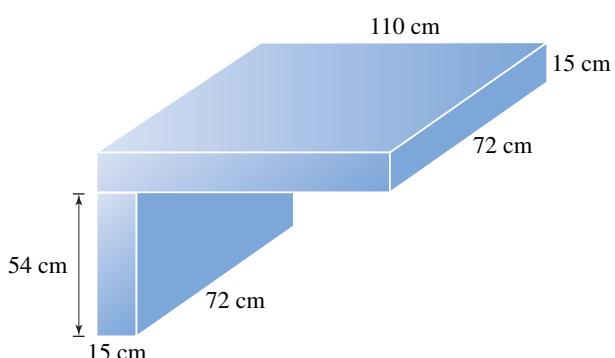


43.



44.



45.**46.**

- 47.** A bag of potting soil contains 4000 in³. How many bags are needed to fill four flower boxes, each of which measures 3 ft long, 8 in. wide, and 8 in. deep? Remember that you can buy only entire bags of potting soil.

- 49.** A farming corporation is building four new grain silos. The inside dimensions of the silos are given in the table.

	Area of Base	Height
Silo A	1800 ft ²	60 ft
Silo B	1200 ft ²	75 ft
Silo C	900 ft ²	80 ft
Silo D	600 ft ²	100 ft

Find the total volume in the four silos.

Exercises 50–54 refer to the chapter application. See page 579.

- 50.** According to industry standards, a joint thickness of $\frac{1}{4}$ in. means the bricklayer will need 9 ft³ of mortar per 1000 bricks. Find the total amount of mortar needed for the patio and walkways. Round to the nearest cubic foot. (See Exercise 72 in Section 7.4.)

- 52.** Explain why in Exercise 51 it is necessary to round up to the next whole unit, rather than using the rounding rule stated in Section 1.1.

- 48.** An excavation is being made for a basement. The hole is 24 ft wide, 36 ft long, and 7 ft deep. If the bed of a truck holds 378 ft³, how many truckloads of dirt will need to be hauled away?

- 51.** The cement subcontractor orders materials based on the total volume of the slab. The industry standard for patios and walkways is 4 to 5 in. of thickness. Because the slab will be topped with bricks, the contractor decides on a thickness of $4\frac{1}{2}$ in. Find the volume of the slab in cubic inches. Convert this to cubic feet, rounding up to the next whole cubic foot. Convert this to cubic yards. Round up to the next whole cubic yard if necessary. (See Exercise 71 in Section 7.4.)

- 53.** The cement contractor in Exercise 51 must first build a wood form that completely outlines the slab. How many linear feet of wood are needed to build the form?

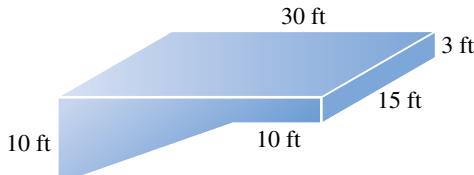
- 54.** The landscaper recommends that Barbara and Paul buy topsoil before planting the garden. They must buy enough to be able to spread the topsoil to a depth of 8 in. How many cubic inches of topsoil do they need? Because soil is usually sold in cubic yards, convert to cubic yards. Round this figure up to the next cubic yard.
- 55.** One gallon is 231 in^3 . A cylindrical trash can holds 32 gal. If it is 4 ft tall, what is the area of the base of the trash can?
- 56.** One gallon is 231 in^3 . A cylindrical trash can holds 36 gal and is 24 in. in diameter. How tall is the trash can?
- 57.** Norma is buying mushroom compost to mulch her garden. The garden is pictured here. How many cubic yards of compost does she need to mulch the entire garden 4 in. deep? (She cannot buy fractional parts of a cubic yard.)
-
- 58.** One gallon is 231 in^3 . How many cubic inches does a 20-gal fish tank hold? Find a possible length, width, and height in inches for a 20-gal fish tank.
- 59.** Malisha is packing water polo balls into a box to ship to a tournament. Each ball has a radius of 6 in. Her box is 3 ft by 2 ft by 2 ft. How many balls can she fit into the box?
- 60.** How much space in Malisha's box is empty, and how much of it is occupied by water polo balls? Use the π key, and round to the nearest cubic inch. See Exercise 59.

STATE YOUR UNDERSTANDING

- 61.** Explain what is meant by "volume." Name three occasions in the past week when the volume of an object was relevant.
- 62.** Explain how to find the volume of the following figure.
-
- 63.** Explain why the formula for the volume of a box is a special case of the formula $V = Bh$.

CHALLENGE

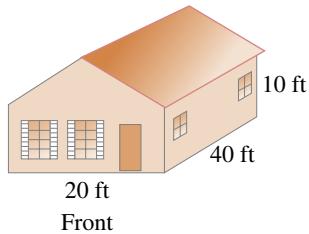
- 64.** The Bakers are constructing an in-ground pool in their backyard. The pool will be 15 ft wide and 30 ft long. It will be 3 ft deep for 10 ft at one end. It will then drop to a depth of 10 ft at the other end. How many cubic feet of water are needed to fill the pool?



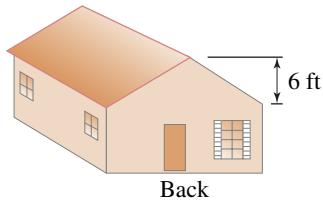
- 65.** Doug is putting a 5-m-by-20-m swimming pool in his backyard. The pool will be 2 m deep. What is the size of the hole for the pool? The hole will be lined with concrete that is 15 cm thick. How much concrete is needed to line the pool?

GROUP WORK

- 66.** The insulating ability of construction materials is measured in R-values. Industry standards for exterior walls are currently R-19. An 8-in. thickness of loose fiberglass is necessary to achieve an R-19 value. Calculate the amount of cubic feet of loose fiberglass needed to



insulate the exterior walls of the mountain cabin pictured. All four side windows measure 2×3 ft. Both doors measure 4×7 ft. The front and back windows measure 5×3 ft.

**MAINTAIN YOUR SKILLS**

- 67.** 8^3 **68.** $14^2 + 15^2$ **69.** $2^3 + 3^3 + 4^3$ **70.** $19^2 - 14^2$
- 71.** $(6^2 + 7^2)^2$ **72.** $3168 \div 24$ **73.** $(10^2)(2^2)(3^2)$ **74.** $5(36 \div 12 + 1 - 2)$
- 75.** Find the area of a rectangle that is 3 in. wide and 2 ft long.
- 76.** A warehouse store sells 5-lb bags of Good & Plenty®. Bob buys a bag and stores the candy in 20-oz jars. How many jars does he need?

7.6 Square Roots and the Pythagorean Theorem

VOCABULARY

A **perfect square** is a whole number that is the square of another whole number. For example, $16 = 4^2$, so 16 is a perfect square.

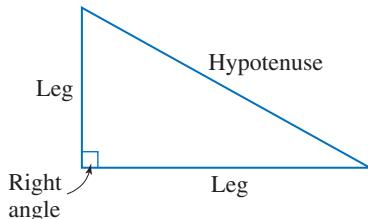
A **square root** of a positive number is a number that is squared to give the original number. The symbol $\sqrt{}$, called a radical sign, is used to name a square root of a number.

$$\sqrt{16} = 4 \text{ because } 4^2 = 4 \cdot 4 = 16$$

A **right triangle** is a triangle with one right (90°) angle.

The **hypotenuse** of a right triangle is the side that is across from the right angle.

The **legs** of a right triangle are the sides adjacent to the right angle.



OBJECTIVES

- Find the square root of a number.
- Apply the Pythagorean theorem.

How & Why

OBJECTIVE 1

Find the square root of a number.

The relationship between squares and square roots is similar to the relationship between multiples and factors. In Chapter 2, we saw that if one number is a multiple of a second number, then the second number is a factor of the first number. It is also true that if one number is the square of a second number, then the second number is a square root of the first. For example, the number 81 is the square of 9, so 9 is a square root of 81. We write this: $9^2 = 81$ so $\sqrt{81} = 9$.

To find the square root of a perfect square

Use guess and check. Square whole numbers until you find the correct value.

Examples A–C



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DIRECTIONS: Find a square root.

STRATEGY: Use guess and check.

A. $\sqrt{121}$

$$\sqrt{121} = 11 \quad \text{Because } 11 \cdot 11 = 121$$

Warm-Ups A–C

A. $\sqrt{144}$

Answers to Warm-Ups

A. 12

B. $\sqrt{729}$

B. $\sqrt{529}$

Trial number	Square	
--------------	--------	--

20	20 \cdot 20 = 400	Too small
25	25 \cdot 25 = 625	Too large
24	24 \cdot 24 = 576	Too large
23	23 \cdot 23 = 529	

So $\sqrt{529} = 23$

C. $\sqrt{\frac{9}{25}}$

C. $\sqrt{\frac{4}{49}}$

$$2 \cdot 2 = 4 \quad \text{and} \quad 7 \cdot 7 = 49$$

$$\text{So } \sqrt{\frac{4}{49}} = \frac{2}{7}.$$

When we need to find the square root of a number that is not a perfect square, the method of guess and check is possible but usually too time-consuming to be practical. To further complicate matters, square roots of nonperfect squares are nonterminating decimals. In these cases, we find an approximation of the desired square root and round to a convenient decimal place. For instance, consider $\sqrt{5}$.

Guess	So $\sqrt{5}$ is between ...
$2^2 = 4$	
$3^2 = 9$	2 and 3
$2.2^2 = 4.84$	
$2.3^2 = 5.29$	2.2 and 2.3
$2.23^2 = 4.9729$	
$2.24^2 = 5.0176$	2.23 and 2.24
$2.236^2 = 4.999696$	
$2.237^2 = 5.004169$	2.236 and 2.237

At this point, we discontinue the process and conclude that $\sqrt{5} \approx 2.24$ to the nearest hundredth.

Because all but the most basic calculators have a square root key, $\sqrt{}$, we usually find the square roots of nonperfect squares by using a calculator. The calculator displays $\sqrt{5} \approx 2.2360679775$, which is consistent with our previous calculation.

Calculator note: Some calculators require that you press the square root key, $\sqrt{}$, before entering the number, and others require that you enter the number first.

To find the square root of a number

Use the square root key, $\sqrt{}$, on a calculator. Round as necessary.

Answers to Warm-Ups

B. 27 C. $\frac{3}{5}$

Examples D–E

DIRECTIONS: Find a square root. Round decimals to the nearest hundredth if necessary.

STRATEGY: Use the square root key, $\sqrt{}$, on a calculator, then round.

D. $\sqrt{95}$

The calculator displays 9.74679434481

So $\sqrt{95} \approx 9.75$, to the nearest hundredth.

E. $\sqrt{213.16}$

The calculator displays 14.6

So $\sqrt{213.16} = 14.6$.

Warm-Ups D–E

D. $\sqrt{87}$

E. $\sqrt{696.96}$

How & Why

OBJECTIVE 2

Apply the Pythagorean theorem.

A right triangle is a triangle with one right (90°) angle. Figure 7.8 shows the right angle, legs, and hypotenuse of a right triangle.

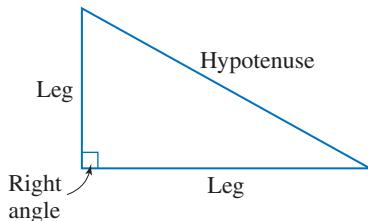


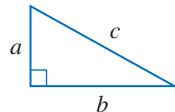
Figure 7.8

Thousands of years ago, a Greek named Pythagoras discovered that the lengths of the sides of a right triangle have a special relationship. This relationship is known as the Pythagorean theorem.

The Pythagorean theorem enables us to calculate the length of the third side of a right triangle when we know the lengths of the other two sides. Because the Pythagorean theorem involves the squares of the sides, when we want the length of a side, we need to use square roots. The following formulas are used to calculate an unknown side of a right triangle.

Pythagorean Theorem

In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse or

$$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$$


To calculate an unknown side of a right triangle

Use the appropriate formula.

$$\text{Hypotenuse} = \sqrt{(\text{leg}_1)^2 + (\text{leg}_2)^2} \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

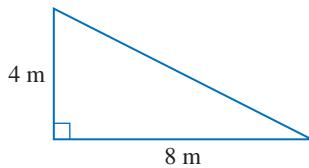
$$\text{Leg} = \sqrt{(\text{hypotenuse})^2 - (\text{known leg})^2} \quad \text{or} \quad a = \sqrt{c^2 - b^2}$$

Answers to Warm-Ups

D. 9.33 E. 26.4

Warm-Ups F–H

- F. Find the approximate length of the hypotenuse.



Examples F–H

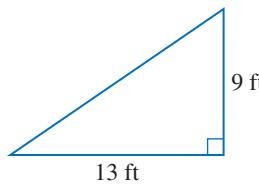


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DIRECTIONS: Find the unknown side of the given right triangle.

STRATEGY: Use the formulas. Round decimal values to the nearest hundredth.

- F. Find the approximate length of the hypotenuse.



$$\begin{aligned}
 \text{Hypotenuse} &= \sqrt{(\text{leg}_1)^2 + (\text{leg}_2)^2} && \text{Formula} \\
 &= \sqrt{9^2 + 13^2} && \text{Substitute 9 and 13 for the lengths.} \\
 &= \sqrt{81 + 169} && \text{Simplify.} \\
 &= \sqrt{250} \\
 &\approx 15.81
 \end{aligned}$$

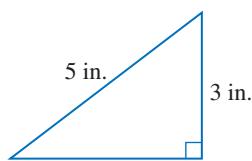
So the length of the hypotenuse is about 15.81 ft.

ALTERNATIVE SOLUTION: Use the formula $c = \sqrt{a^2 + b^2}$

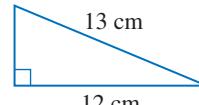
$$\begin{aligned}
 c &= \sqrt{a^2 + b^2} \\
 &= \sqrt{9^2 + 13^2} && \text{Substitute 9 for } a \text{ and 13 for } b. \\
 &= \sqrt{81 + 169} && \text{Simplify.} \\
 &= \sqrt{250} \\
 &\approx 15.81
 \end{aligned}$$

The hypotenuse is about 15.81 ft.

- G. Find the length of the unknown leg.



- G. Find the length of the unknown leg.



$$\begin{aligned}
 \text{Leg} &= \sqrt{(\text{hypotenuse})^2 - (\text{known leg})^2} && \text{Formula} \\
 &= \sqrt{13^2 - 12^2} \\
 &= \sqrt{169 - 144} \\
 &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

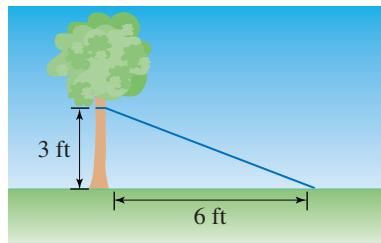
Substitute 13 for the length of the hypotenuse and 12 for the length of the known leg.

So, the length of the unknown leg is 5 cm.

Answers to Warm-Ups

F. 8.94 m G. 4 in.

- H.** Ted is staking a small tree to keep it growing upright. He plans to attach the rope 3 ft above the ground on the tree and 6 ft from the base of the tree. He needs one extra foot of rope on each end for tying. How much rope does he need?

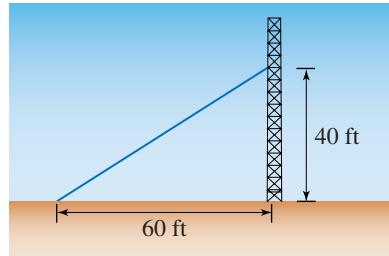


STRATEGY: The tree trunk, the ground, and the rope form a right triangle, with the rope being the hypotenuse of the triangle. We use the formula for finding the hypotenuse.

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{leg}_1)^2 + (\text{leg}_2)^2} && \text{Formula} \\ &= \sqrt{3^2 + 6^2} && \text{Substitute 3 and 6 for the lengths of leg}_1 \text{ and leg}_2. \\ &= \sqrt{9 + 36} && \text{Simplify.} \\ &= \sqrt{45} \\ &\approx 6.71\end{aligned}$$

The rope support is about 6.71 ft long, so Ted needs 8.71 ft, including the extra 2 ft for the ties.

- H.** A guy wire for a radio tower is attached 40 ft above the ground and 60 ft from the base of the tower. An additional 6 in. is needed on each end for fastening. How long is the guy wire?



Answers to Warm-Ups

- H.** The guy wire is about 73.11 ft.

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Exercises 7.6

OBJECTIVE 1

Find the square root of a number.

A Simplify.

1. $\sqrt{49}$

2. $\sqrt{25}$

3. $\sqrt{36}$

4. $\sqrt{64}$

5. $\sqrt{100}$

6. $\sqrt{169}$

7. $\sqrt{400}$

8. $\sqrt{225}$

9. $\sqrt{\frac{25}{81}}$

10. $\sqrt{\frac{64}{121}}$

11. $\sqrt{0.25}$

12. $\sqrt{0.64}$

B Simplify. Use the square root key, $\boxed{\sqrt{}}$, on your calculator. Round to the nearest hundredth.

13. $\sqrt{150}$

14. $\sqrt{888}$

15. $\sqrt{750}$

16. $\sqrt{910}$

17. $\sqrt{10.67}$

18. $\sqrt{50.41}$

19. $\sqrt{\frac{6}{13}}$

20. $\sqrt{\frac{5}{22}}$

21. $\sqrt{11,500}$

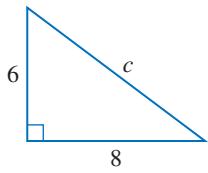
22. $\sqrt{12,000}$

OBJECTIVE 2

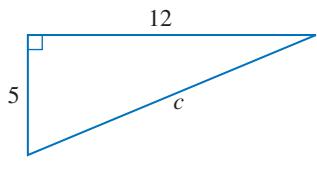
Apply the Pythagorean theorem.

A Find the unknown side of each right triangle. Round decimals to the nearest hundredth.

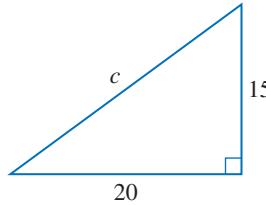
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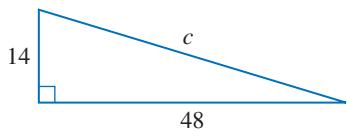
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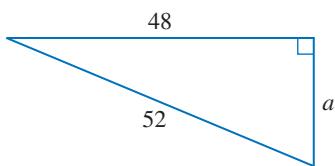
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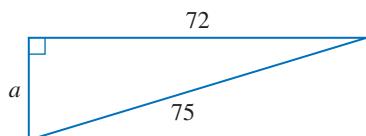
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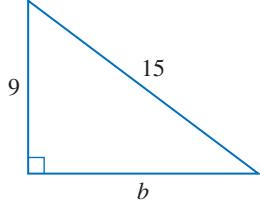
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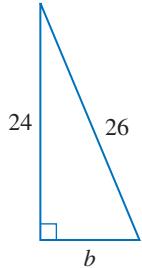
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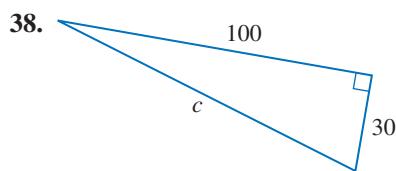
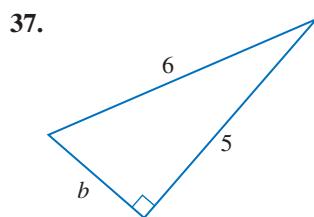
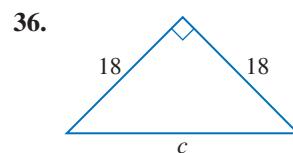
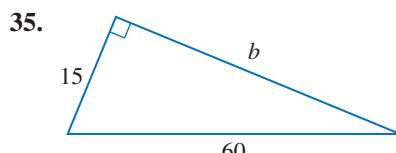
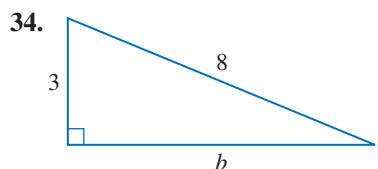
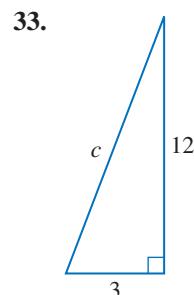
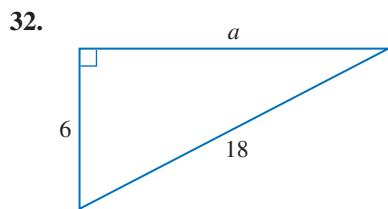
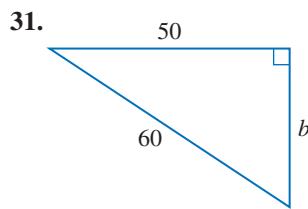


29.

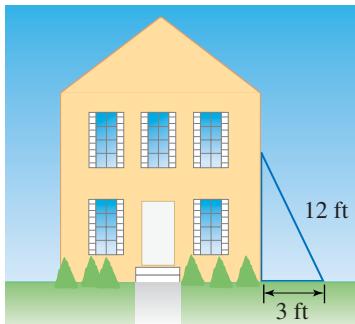


30.



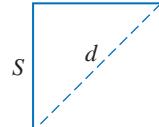
B**C**

39. Find the hypotenuse of a right triangle that has legs of 21 cm and 72 cm.
41. What is the length of the side of a square whose area is 144 ft^2 ? The formula for the length of a side of a square is $S = \sqrt{A}$, where A is the area.
43. What is the length of the side of a square whose area is 361 m^2 ? See Exercise 41.
45. Paul has a 12-ft ladder. If he sets the foot of the ladder 3 ft away from the base of his house, how far up the house is the top of the ladder?



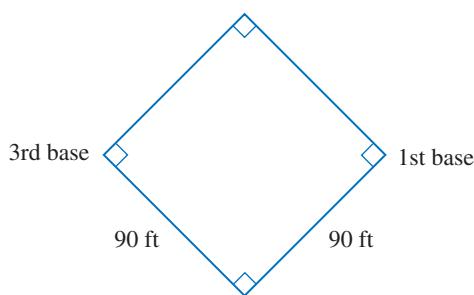
40. Find the length of one leg of a right triangle if the other leg is 24 in. and the hypotenuse is 30 in.

42. What is the length of the diagonal of a square whose area is 144 ft^2 ? See Exercise 41.

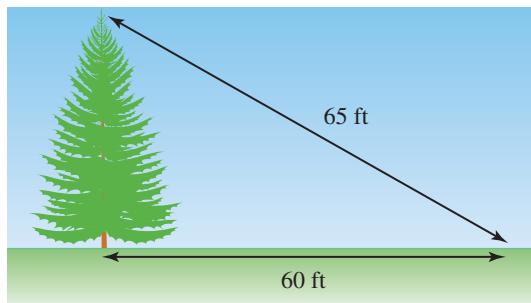


44. What is the length of the diagonal of a square whose area is 361 m^2 ?
46. Lynzie drives 35 mi due north and then 16 mi due east. How far is she from her starting point?

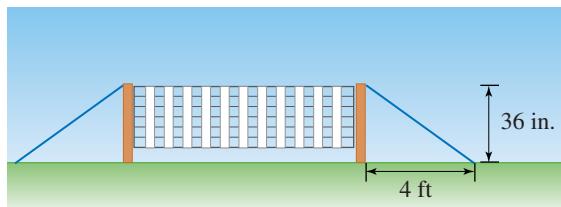
- 47.** A baseball diamond is actually a square that is 90 ft on a side. How far is it from first base across to third base?



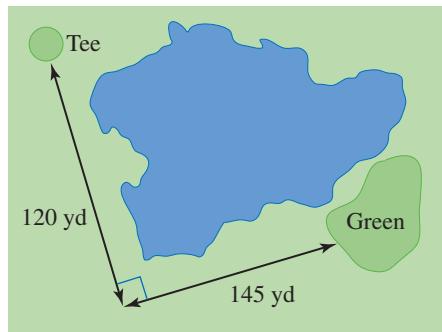
- 48.** How tall is the tree?



- 49.** Scott wants to install wire supports for the net on his pickleball court. The net is 36 in. high, and he plans to anchor the supports in the ground 4 ft from the base of the net. Find how much wire he needs for two supports, figuring an additional foot of wire for each support to fasten the ends.



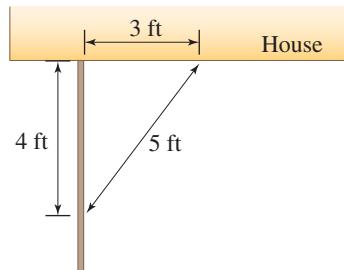
- 50.** The 7th hole on Jim's favorite golf course has a pond between the tee and the green. If Jim can hit the ball 170 yards, should he hit over the pond or go around it?



STATE YOUR UNDERSTANDING

- 51.** What is a square root? Explain how to find $\sqrt{289}$ without using a calculator.

- 52.** A carpenter is building a deck on the back of a house. He wants the joists of the deck to be perpendicular to the house, so he attaches a joist to the house, and measures 4 ft out on the joist and 3 ft over from the joint on the house. He then wiggles the joist until his two marks are 5 ft apart. At this point, the carpenter knows that the joist is perpendicular (makes a right angle) with the house. Explain how he knows.



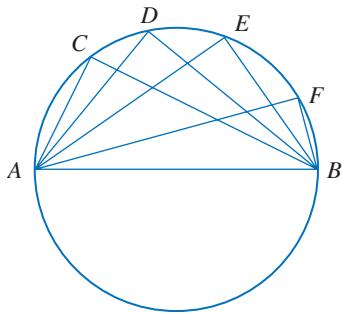
CHALLENGE

- 53.** Sam has a glass rod that is 1 in. in diameter and 35 in. long that he needs to ship across the country. He is having trouble finding a box. His choices are a box that

measures 2 ft by 2 ft by 6 in., or a box that measures 18 in. by 32 in. by 5 in. Will Sam's rod fit in either of the boxes?

GROUP WORK

- 54.** Trace a circle at least 6 inches in diameter on a piece of paper. Now fold the circle in half. Be as exact as you can. Mark the diameter of the fold. Label the ends of the diameter *A* and *B*. Pick four points on the circumference of the circle and label them *C*, *D*, *E*, and *F*. Connect *C*, *D*, *E*, and *F* to *A* and *B* to make four triangles as shown in the figure.



Use a ruler marked in millimeters to measure, as accurately as possible, the sides of all the triangles and record your results in the table.

Triangle	Side 1	Side 2	Side 3	Right Triangle?
$\triangle ACB$				
$\triangle ADB$				
$\triangle AEB$				
$\triangle AFB$				

Geometry tells us that every triangle inscribed in a semicircle is a right triangle. Do your measurements confirm this fact? If not, explain why not.

MAINTAIN YOUR SKILLS

55. What is 15% of 45,000?

56. $36.9 + 231 + 0.087$

57. $\left(4\frac{1}{5}\right)\left(3\frac{6}{7}\right)$

58. Find the prime factorization of 80.

59. $23 - 14\frac{3}{8}$

60. $4.65 \div 0.002$

- 61.** At a certain time of day, a 15-ft tree casts a shadow of 8 ft. How tall is another tree if it casts a shadow of 5 ft?

- 62.** Convert 45 ft^2 to square meters. Round to the nearest hundredth.

- 63.** Find the average and median of 3, 18, 29, 31, and 32.

- 64.** Find the volume of a can with a diameter of 8 in. and a height of 10 in. Use the π key, and round to the nearest hundredth.

Key Concepts CHAPTER 7

Section 7.1 Measuring Length

Definitions and Concepts		Examples														
English	Metric															
inch	centimeter															
foot	meter															
mile	kilometer															
To convert units, multiply by the appropriate unit fraction.		Convert 60 in. to feet.														
		$60 \text{ in.} = \frac{60 \text{ in.}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in.}}$ $= 5 \text{ ft}$														
To convert within the metric system, use the chart and move the decimal point the same number of places.		Convert 25 cm to meters.														
		<table style="margin-left: auto; margin-right: auto;"> <tr> <td>km</td> <td>hm</td> <td>dam</td> <td>m</td> <td>dm</td> <td>cm</td> <td>mm</td> </tr> <tr> <td style="text-align: center;">↑</td> <td style="text-align: center;">↑</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> Move the decimal two places to the left.	km	hm	dam	m	dm	cm	mm	↑	↑					
km	hm	dam	m	dm	cm	mm										
↑	↑															
Only like measures can be added or subtracted.		$14 \text{ cm} + 50 \text{ cm} = 64 \text{ cm}$ $8 \text{ ft} - 3 \text{ ft} = 5 \text{ ft}$														

Section 7.2 Measuring Capacity, Weight, and Temperature

Definitions and Concepts		Examples
English	Metric	
Capacity	quart	liter
Weight	ounce	gram
	pound	kilogram
Temperature	degrees Fahrenheit	degrees Celsius
To convert units, multiply by the appropriate unit fraction.		Convert 5 qt to liters.
		$5 \text{ qt} \approx \frac{5 \text{ qt}}{1} \cdot \frac{0.9464 \text{ L}}{1 \text{ qt}}$ $\approx 4.732 \text{ L}$
To convert temperatures, use the formulas.		Convert 41°F to degrees Celsius.
$C = \frac{5}{9} \cdot (F - 32)$		$C = \frac{5}{9} \cdot (41 - 32)$ $= \frac{5}{9} \cdot 9$ $= 5$
$F = \frac{9}{5} \cdot C + 32$		So 41°F is 5°C.

Section 7.3 Perimeter**Definitions and Concepts**

Perimeter is measured in linear units.

To find the perimeter of a polygon, add the measures of all of its sides.

Perimeter of a square:

$$P = 4s$$

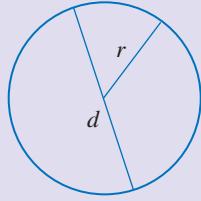
Perimeter of a rectangle:

$$P = 2\ell + 2w$$

The circumference of a circle is the distance around the circle.

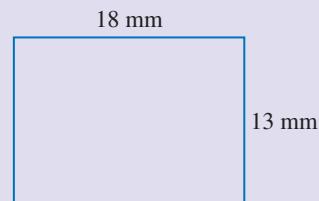
The radius of a circle is the distance from the center to any point on the circle.

The diameter of a circle is twice the radius.



The circumference of a circle:

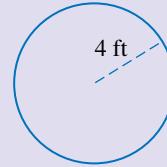
$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

Examples

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(18) + 2(13) \\ &= 36 + 26 \\ &= 62 \end{aligned}$$

The perimeter is 62 mm.

Find the circumference of



$$\begin{aligned} C &= 2\pi r \quad \pi \approx 3.14 \\ &\approx 2(3.14)(4) \\ &\approx 25.12 \end{aligned}$$

So the circumference is about 25.12 ft.

Section 7.4 Area**Definitions and Concepts**

Area is measured in square units.

Area of a square:

$$A = s^2$$

Area of a rectangle:

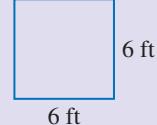
$$A = \ell w$$

Area of a triangle:

$$A = \frac{bh}{2}$$

Area of a circle:

$$A = \pi r^2$$

Examples

$$\begin{aligned} A &= s^2 \\ &= 6^2 \\ &= 36 \end{aligned} \qquad \begin{aligned} A &= \frac{bh}{2} \\ &= \frac{20 \cdot 8}{2} \\ &= 80 \end{aligned}$$

The area is 36 ft^2 .

The area is 80 cm^2 .

Section 7.5 Volume**Definitions and Concepts**

Volume is measured in cubic units.

Volume of a cube:

$$V = s^3$$

Volume of a rectangular solid:

$$V = \ellwh$$

Volume of a solid with perpendicular sides:

$$V = Bh, \text{ where } B \text{ is the area of the base}$$

The volume of a cylinder:

$$V = \pi r^2 h$$

Examples

$$\begin{aligned} V &= \ellwh \\ &= (10)(6)(3) \\ &= 180 \end{aligned}$$

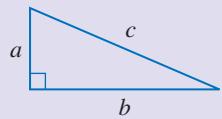
The volume is 180 in³.

Section 7.6 Square Roots and the Pythagorean Theorem**Definitions and Concepts**

A square root of a number is the number that is squared to give the original number.

To find the square root of a number, use the $\sqrt{}$ key on a calculator and round as necessary.

Pythagorean theorem: In a right triangle,



$$a^2 + b^2 = c^2$$

To find a missing side of a right triangle, use the formulas.

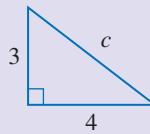
$$c = \sqrt{a^2 + b^2} \quad \text{or} \quad a = \sqrt{c^2 - b^2}$$

Examples

$$\sqrt{49} = 7 \quad \text{because} \quad 7 \cdot 7 = 49$$

$$\sqrt{436} \approx 20.88$$

Find c .



$$\begin{aligned} c &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

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Review Exercises CHAPTER 7

Section 7.1

Do the indicated operations.

1. $12 \cdot 5 \text{ cm}$

2. $330 \text{ m} \div 15$

3. $4 \text{ yd} - 22 \text{ in.}$

4. $5 \text{ ft} + 29 \text{ ft} + 8 \text{ ft}$

5. $(6 \text{ ft } 2 \text{ in.}) - (4 \text{ ft } 10 \text{ in.})$

Convert as indicated. Round decimal values to the nearest hundredth.

6. 24 cm to meters

7. 2 mi to feet

8. 5 km to miles

9. 15 in. to millimeters

10. Ted is building two plant supports out of copper tubing. Each support requires three vertical tubes that are 8 ft long and six horizontal tubes that are 2 ft long. How much tubing does he need for the two supports?

Section 7.2

11. $18 \text{ gal} \cdot 3$

12. $(322 \text{ hr}) \div 14$

13. $7 \text{ oz} + 14 \text{ oz}$

14. $34 \text{ L} - 8 \text{ L}$

15. $(6 \text{ hr } 42 \text{ min}) + (3 \text{ hr } 38 \text{ min})$

Convert as indicated. Round decimal values to the nearest hundredth.

16. 6 g to milligrams

17. 24 oz to pounds

18. 4 L to quarts

19. 59°F to degrees Celsius

20. Gail estimates that she has 45 min to work in her yard for each of the next 5 days. She is building a retaining wall that should take her 4 hr. How much additional time will she need to finish the wall?

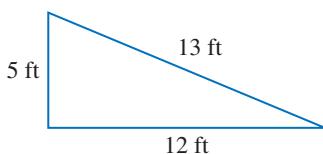
Section 7.3

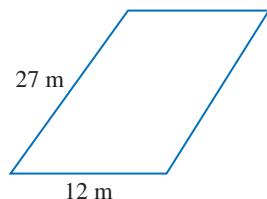
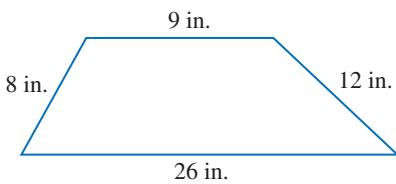
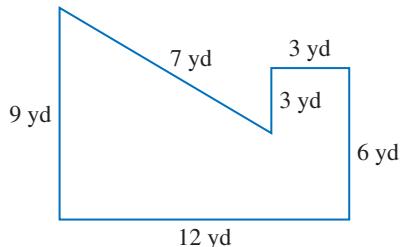
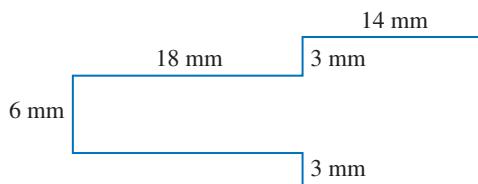
Find the perimeter of the following polygons.

21.



22.



**25.****26.**

- 27.** Find the perimeter of a square plot of land that is 75 m on each side.

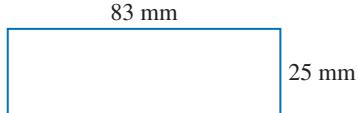
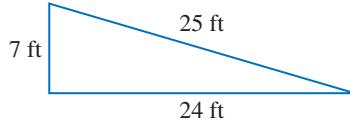
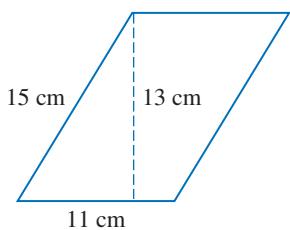
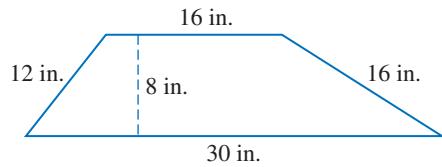
- 28.** Find the distance around a circular hot tub that measures 8 ft in diameter. Let $\pi \approx 3.14$.

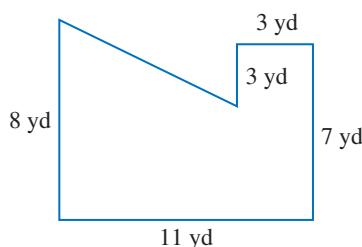
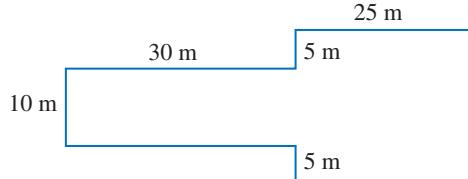
- 29.** How much fencing is needed for a rectangular field that is 55 ft by 35 ft?

- 30.** Larry is buying new baseboards for his den, which measures 9 ft by 12 ft. How much does he need if the door is 4 ft wide?

Section 7.4

Find the area of the following polygons.

31.**32.****33.****34.**

35.**36.**

- 37.** What is the area of a square that measures 600 mm on a side?

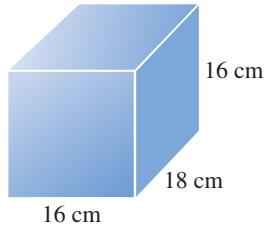
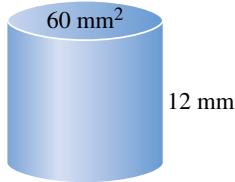
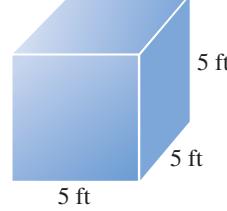
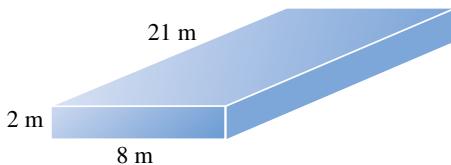
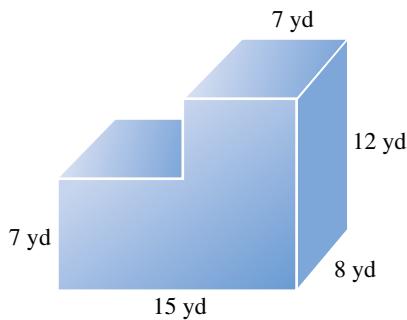
- 39.** Felicity is painting the walls of her bathroom, which measures 6 ft by 8 ft. The bathroom has an 8-ft ceiling and one 3-ft-by-6-ft door. She has 1 gal of paint that covers 350 ft^2 . Does she have enough paint for two coats?

- 38.** What is the area of a rectangular driveway that is 35 ft by 22 ft?

- 40.** How many square meters are there in the sides and bottom of a rectangular swimming pool that measures 50 m by 30 m and is 3 m deep?

Section 7.5

Find the volume of the following figures.

41.**42.****43.****44.****45.****46.**

47. What is the volume of a can of baked beans that is 8 in. tall and has a base of 13 in^2 ?
48. What is the volume of a kitchen that measures 14 ft long by 9.5 ft wide by 8 ft high?
49. How many cubic inches are there in a cubic yard?
50. Will a cubic yard of bark dust be sufficient to cover a rose garden that is 20 ft by 6 ft to a depth of 3 in.?

Section 7.6

Find the square roots. If necessary, round to the nearest hundredth.

51. $\sqrt{289}$

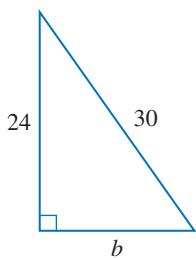
52. $\sqrt{60}$

53. $\sqrt{\frac{81}{49}}$

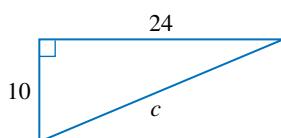
54. $\sqrt{28.09}$

Find the missing side. Round decimal values to the nearest hundredth.

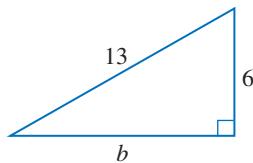
55.



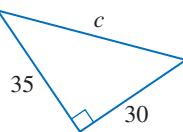
56.



57.

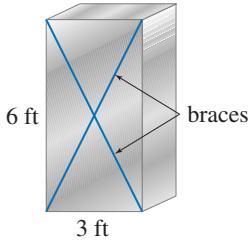


58.



59. A 18-ft ladder is leaning against a wall. If the base of the ladder is 3 ft away from the base of the wall, how high is the top of the ladder?

60. Mark wants to brace a metal storage unit by adding metal strips on the back on both diagonals. If the storage unit is 3 ft wide and 6 ft high, how long are the diagonals and how much metal stripping does he need?



True/False Concept Review CHAPTER 7

ANSWERS

Check your understanding of the language of algebra and geometry. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a true statement.

1. Metric measurements are the most commonly used in the world. 1. _____
2. Equivalent measures have the same number value, as in 7 ft and 7 yd. 2. _____
3. A liter is a measure of capacity or volume. 3. _____
4. The perimeter of a square can be found in square inches or square meters. 4. _____
5. Volume is the measure of the inside of a solid, such as a box or can. 5. _____
6. The volume of a square is $V = s^2$. 6. _____
7. The area of a circle is $A = 3.14r^2$. 7. _____
8. It is possible to find equivalent measures without remeasuring the original object. 8. _____
9. The metric system uses the base 10 place value system. 9. _____
10. In a right triangle, the longest side is the hypotenuse. 10. _____
11. $\sqrt{3} = 9$ 11. _____
12. Weight can be measured in pounds, grams, or kilograms. 12. _____
13. A trapezoid has four sides. 13. _____
14. One milliliter is equivalent to 1 square centimeter. 14. _____
15. Measurements can be added or subtracted only if they are all metric measures or all English measures. 15. _____
16. The distance around a polygon is called the perimeter. 16. _____
17. Volume is always measured in gallons or liters. 17. _____

Name _____ Class _____ Date _____

- 18.** The Pythagorean theorem states that the sum of the lengths of the legs of a right triangle is equal to the length of the hypotenuse. **18.** _____

- 19.** $1 \text{ yd}^2 = 3 \text{ ft}^2$. **19.** _____

- 20.** The prefix kilo- means 1000 **20.** _____

Test CHAPTER 7**ANSWERS**

1. $8 \text{ m} + 329 \text{ mm} = ? \text{ mm}$

1. _____

2. Find the perimeter of a rectangle that measures 24 in. by 35 in.

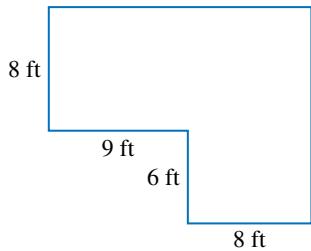
2. _____

3. Find the volume of a box that is 6 in. high, 20 in. wide, and 12 in. deep.

3. _____

4. How many square feet of tile are needed to cover the floor of the room pictured below?

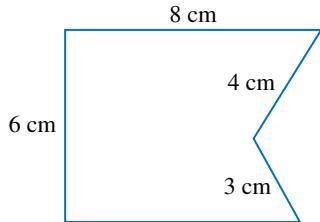
4. _____

5. Find $\sqrt{310}$ to the nearest thousandth.

5. _____

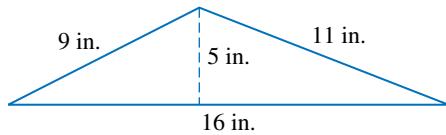
6. Find the perimeter of the figure.

6. _____



7. Find the area of the triangle.

7. _____



8. Subtract:

$$\begin{array}{r} 6 \text{ yd } 2 \text{ ft} \\ - 3 \text{ yd } 2 \text{ ft } 5 \text{ in.} \\ \hline \end{array}$$

8. _____

9. Name two units of measure in the English system for weight. Name two units of measure in the metric system for weight.

9. _____

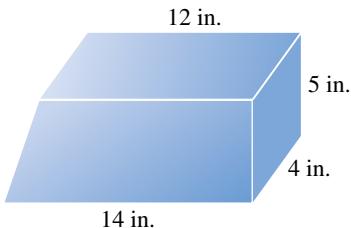
10. Change 6 gal to pints.

10. _____

11. How much molding is needed to trim a picture window 10 ft wide by 8 ft tall and two side windows each measuring 4 ft wide and 8 ft tall? Assume that the corners will be mitered and that the miters require 2 in. extra on each end of each piece of molding.

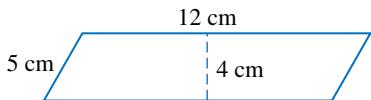
12. Find the volume of the figure.

12. _____



13. Find the area of the parallelogram.

13. _____



14. Find the volume of a hot water tank that has a circular base of 4 ft^2 and a height of 6 ft.

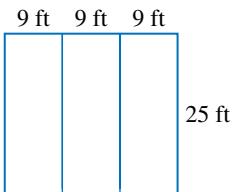
14. _____

15. Miss Katlie, a kindergarten teacher, has 192 oz of grape juice to divide equally among her 24 students. How much juice will she give each student?

15. _____

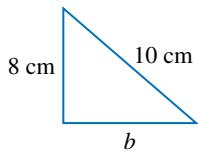
16. The cost of heavy-duty steel landscape edging is \$1.20 per linear foot. How much will it cost to line the three garden plots pictured, including the edging between the plots?

16. _____

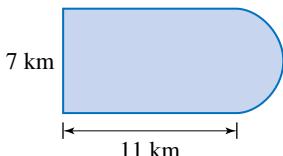


17. Find the missing side.

17. _____

18. Find the area of the figure. Round to the nearest tenth. Let $\pi \approx 3.14$.

18. _____



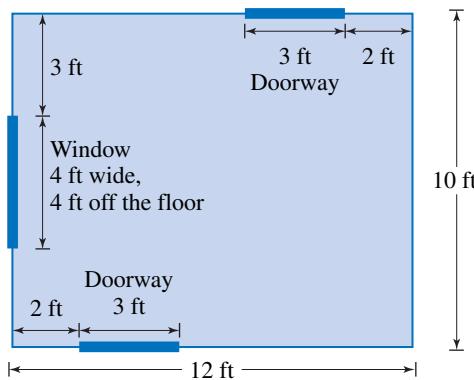
19. The Green Bay Packers condition the team by having them run around the edge of the field four times each hour. A football field measures 100 yd long by 60 yd wide. How far does each player run in a 4-hr practice?
20. Both area and volume describe interior space. Explain how they are different.
21. John's Meat Market has a big sale on steak. They sell 340 lb on Monday, 495 lb on Tuesday, 432 lb on Wednesday, 510 lb on Thursday, and 670 lb on Friday. What is the average number of pounds of steak sold each day?
22. Li is buying lace, with which she plans to edge a circular tablecloth. The tablecloth is 90 in. in diameter. How much lace does she need? If the lace is available in whole-yard lengths only, how much must she buy? Let $\pi \approx 3.14$.
23. Khallil has a 40-lb bag of dog food that is approximately 52 in. long, 16 in. wide, and 5 in. deep. He wants to transfer it to a plastic storage box that is 24 in. long, 18 in. wide, and 12 in. high. Will all of the dog food fit into the box?
24. Macy walks due north for 3.5 mi and then due east for 1.2 mi. How far is she from her starting point?
25. Neela has a gift box that is 12 in. by 14 in. by 4 in. What minimum amount of wrapping paper does she need to wrap the gift?
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____

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Group Project (2–3 WEEKS) CHAPTER 7

OPTIONAL

You are working for a kitchen design firm that has been hired to design a kitchen for the 10 ft by 12 ft room pictured here.

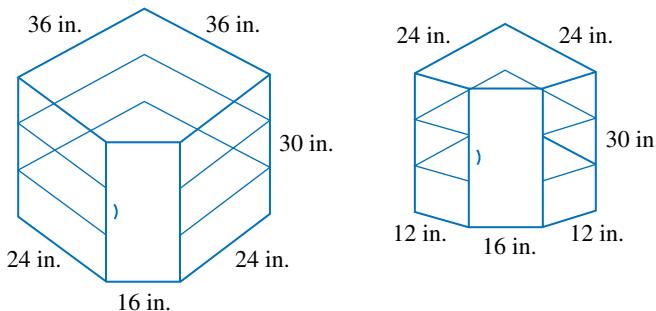


The following table lists appliances and dimensions. Some of the appliances are required and others are optional. All of the dimensions are in inches.

Appliance	High	Wide	Deep	Required
Refrigerator	68	30 or 33	30	Yes
Range/oven	30	30	26	Yes
Sink	12	36	22	Yes
Dishwasher	30	24	24	Yes
Trash compactor	30	15	24	No
Built-in microwave	24	24	24	No

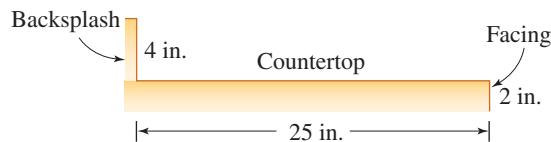
The base cabinets are all 30 in. high and 24 in. deep. The widths can be any multiple of 3 from 12 in. to 36 in. Corner units are 36 in. along the wall in each direction. The base cabinets (and the range, dishwasher, and compactor) will all be installed on 4-in. bases that are 20 in. deep.

The wall (upper) cabinets are all 30 in. high and 12 in. deep. Here, too, the widths can be any multiple of 3 from 12 in. to 36 in. Corner units are 24 in. along the wall in each direction.



1. The first step is to place the cabinets and appliances. Your client has specified that there must be at least 80 ft^3 of cabinet space. Make a scale drawing of the kitchen and indicate the placement of the cabinets and appliances. Show calculations to justify that your plan satisfies that 80 ft^3 requirement.

2. Countertops measure 25 in. deep with a 4-in. backsplash. The countertops can either be tile or Formica. If the counters are Formica, there will be a 2-in. facing of Formica. If the counters are tile, the facing will be wood that matches the cabinets. (See the figure.) Calculate the amount of tile and wood needed for the counters and the amount of Formica needed for the counters.



3. The bases under the base cabinets will be covered with a rubber kickplate that is 4 in. high and comes in 8-ft lengths. Calculate the total length of kickplate material needed and the number of lengths of kickplate material necessary to complete the kitchen.
4. Take your plan to a store that sells kitchen appliances, cabinets, and counters. Your goal is to get the best-quality materials for the least amount of money. Prepare at least two cost estimates for the client. Do not include labor in your estimates, but do include the cost of the appliances. Prepare a rationale for each estimate, explaining the choices you made. Which plan will you recommend to the client, and why?

Cumulative Review CHAPTERS 1–7

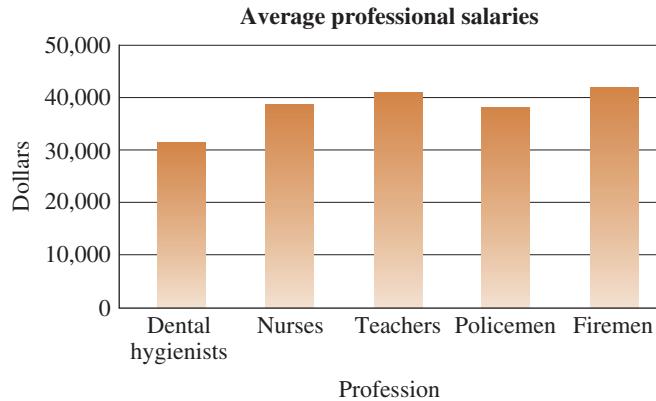
1. Write the word name for 7048.
2. Round to the nearest hundred: 82,549
3. List from smallest to largest:
381, 390, 375, 384, 309
4. Add:

$$\begin{array}{r}
 491 \\
 2367 \\
 \quad 59 \\
 \quad 2301 \\
 + \underline{4120}
 \end{array}$$
5. Subtract: $642 - 467$
6. Multiply:

$$\begin{array}{r}
 340 \\
 \times \underline{212}
 \end{array}$$
7. Multiply: $(62)(21)(16)$
8. Multiply: $452(10,000)$
9. Divide: $610,000 \div 1000$
10. Divide: $854 \overline{)53,802}$
11. Find the average, median, and mode of the set of numbers: 32, 56, 41, 47, 56, 39, 44
12. Perform the indicated operations.

$$8 \cdot 4 + 2 - 5(9 - 4)$$

Exercises 13–14. The graph gives the average annual salary for five professions.



13. Which profession has the lowest average salary?
14. What is the difference in salary between dental hygienists and nurses?
15. During the United Way campaign, Sylvia collected the following pledge amounts: \$250, \$310, \$500, \$225, \$150, \$175, and \$245. What is the total amount of the pledges Sylvia collected?
16. Jim works for Esso at an annual salary of \$52,800. Wilson Inc. has offered him a job that pays \$72,400. How much more will his annual salary be if he takes the job at Wilson Inc.?

- 17.** Millie worked 14 hours at a job paying \$14 per hour and 23 hours at a job that pays \$19 per hour. How much did Millie earn at the two jobs?
- 18.** The 13 starting linemen for Westside University weighed in as follows: 3 at 210 lb, 2 at 234 lb, 4 at 256 lb, 2 at 285 lb, 1 at 310 lb, and 1 at 326 lb. What is the average weight of the starting linemen?
- 19.** List all the factors of 204.
- 20.** Write 235 as a product of two factors in as many ways as possible.
- 21.** Write the first five multiples of 31.
- 22.** Find the least common multiple (LCM) of 36, 80, and 84.
- 23.** Is 3435 divisible by 2, 3, or 5?
- 24.** Is 2011 prime or composite?
- 25.** Write the prime factorization of 840.
- 26.** Is $\frac{25}{17}$ an improper or proper fraction?
- 27.** Simplify: $\frac{90}{210}$
- 28.** Multiply: $\frac{5}{9} \cdot \frac{20}{28} \cdot \frac{3}{25}$
- 29.** Multiply: $3\frac{4}{5} \cdot 5\frac{3}{4}$
- 30.** Divide: $\frac{8}{15} \div \frac{16}{45}$
- 31.** Divide: $11\frac{1}{5} \div 12\frac{3}{5}$
- 32.** List the numbers from smallest to largest:
 $\frac{5}{12}, \frac{4}{9}, \frac{7}{15}, \frac{2}{5}, \frac{13}{30}$
- 33.** Add: $\frac{3}{70} + \frac{5}{14} + \frac{7}{10} + \frac{3}{5}$
- 34.** Add:

$$\begin{array}{r} 41\frac{13}{15} \\ + 28\frac{9}{20} \\ \hline \end{array}$$
- 35.** Subtract: $\frac{17}{28} - \frac{12}{35}$
- 36.** Subtract:

$$\begin{array}{r} 66 \\ - 23\frac{7}{15} \\ \hline \end{array}$$
- 37.** Perform the indicated operations:

$$\left(\frac{2}{3}\right)\left(\frac{3}{8}\right) + \left(2\frac{1}{4} - 1\frac{7}{8}\right) \div \frac{1}{4}$$
- 38.** On January 1, 2005, Huong measured $57\frac{2}{3}$ in. tall. On January 1, 2006, he measured $59\frac{3}{8}$ in. tall. How much did Huong grow during the year?
- 39.** In a class of 424 students, three-fourths of them passed the state competency exam in mathematics. How many students passed the competency exam?
- 40.** Mr. Doty has $107\frac{2}{3}$ oz of candy to divide among the 34 children in his day care program. How much candy does each child get?
- 41.** Write the word name for 11.063.
- 42.** Round 78.3948 to the nearest ten, tenth, hundredth, and thousandth.

- 43.** List the decimals from smallest to largest:
1.35, 1.046, 1.4005, 1.307, 1.3491

45. Subtract: $693.22 - 413.98$

47. Write in scientific notation: 0.0000345

49. Divide: $709.571 \div 45.34$

51. Write as a decimal: $\frac{11}{16}$

53. Perform the indicated operations.
 $(6.2)^2 - 8(1.16) - 3(5.94 - 3.66)$

44. Add: $3.78 + 11.9 + 0.036 + 15.17 + 110$

46. Multiply: 3.456×10^5

48. Multiply: $(23.55)(6.13)$

50. Divide; round to the nearest hundredth: $33.7 \overline{)213.8}$

- 52.** Find the average and median of 6.37, 8.91, 7.42, and 5.7.

- 54.** Marilyn needs to buy four textbooks for this semester. She goes to the bookstore to find out how much they cost:

Text	Cost
Algebra	\$89.75
Chemistry	\$108.50
Psychology	\$66.00
American History	\$72.75

What is the total cost of the books that she needs?

What is the average cost of the books?

- 55.** Bill is making a tabletop. The two ends of the piece of wood he is using need to be trimmed and sanded. The piece of wood is 53.25 in. long. He requires a length of 48.5 in. The saw removes $\frac{3}{16}$ in. during each cut, and he wants to allow $\frac{1}{16}$ in. for finishing sanding on each end. If he cuts an equal amount off of each end, how much should he cut from each end?

- 57.** In Ms. Adam's algebra class there are 22 females and 18 males. Write a ratio that represents the female population in the class.

59. Solve: $\frac{4}{9} = \frac{48}{A}$

- 61.** A flagpole that is 36 ft tall casts a shadow of 18.5 ft. What is the length of a shadow cast by a tree that is 50 ft tall? Round to the nearest tenth of a foot.

- 56.** Mr. Hardgrave bought one thousand shares of Microsoft at \$26.34 a share and sold them for \$29.43 a share. If his broker fees totaled \$134.50 for both trades, what was Mr. Hardgrave's profit?

58. Solve: $\frac{5}{9} = \frac{B}{12}$

- 60.** Safeway advertises bananas at 3 lb for \$2.16. At this rate, what will 14 lb of bananas cost?

- 62.** The property tax on a home valued at \$215,000 is \$1850. At the same tax rate, what will be the tax on a home valued at \$385,000? Round to the nearest dollar.

Change to a decimal.

63. 7.9%

64. 418%

Change to a percent.

65. 0.421

66. 0.0092

Change to a percent.

67. $\frac{11}{32}$

68. $\frac{13}{35}$ (Round to the nearest tenth of a percent.)

Change to a fraction.

69. 71.5%

70. 1.5%

71. What is 27.4% of 85?

72. What percent of 215 is 86.86?

73. 20.37 is 42% of what number?

74. Of the 724 employees of Compugo, 76.4% contributed to the tsunami relief fund of 2005. How many employees contributed to the fund? Find to the nearest employee.

75. Helga earns \$150 per week plus an 8% commission on her total sales. Last week, Helga had sales of \$12,564. How much did Helga earn last week?

76. Pedro bought a new inboard power ski boat. The price of the boat was \$16,750. His total bill, including sales tax, was \$17,721.50. What was the sales tax rate?

Add.

77.
$$\begin{array}{r} 3 \text{ yd } 2 \text{ ft } 10 \text{ in.} \\ + 4 \text{ yd } 2 \text{ ft } 8 \text{ in.} \\ \hline \end{array}$$

78.
$$\begin{array}{r} 3 \text{ qt } 1 \text{ pt } 1 \text{ c} \\ + 1 \text{ qt } 1 \text{ pt } 1 \text{ c} \\ \hline \end{array}$$

79. How many milliliters are there in 0.82 L?

Convert the units of measure.

80. $108 \text{ min} = ? \text{ hr}$

81. $19,536 \text{ ft} = ? \text{ mi}$

82. $34.29 \text{ cm} = ? \text{ in.}$ (Round to the nearest tenth.)

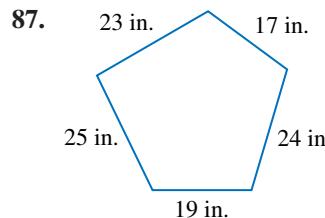
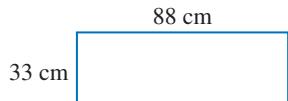
83. $3.5 \text{ lb} = ? \text{ kg}$ (Round to the nearest hundredth.)

84. $74.1 \text{ m}^2 = ? \text{ ft}^2$ (Round to the nearest tenth.)

85. The record ground temperature in Death Valley, 201°F , was recorded at Furnace Creek on July 15, 1972. What is this record high measured in Celsius? Round to the nearest tenth.

Find the perimeter of the following figures.

86.

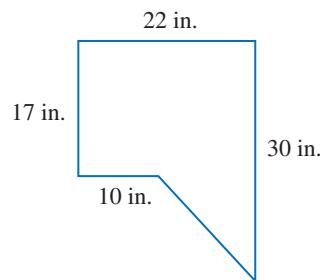


88. Find the circumference of a circle that has a diameter of 72.5 cm. Let $\pi \approx 3.14$.

90. Find the area of a circle that has a diameter of 45 cm. Let $\pi \approx 3.14$.

89. Find the area of a triangle that has a base of 11 ft and a height of 9 ft.

91. Find the area of the polygon.



92. Find the volume of a cube that is 13 m on a side.

93. Find the volume of a cylinder with a circular base with radius of 6 cm and a height of 9 cm. Let $\pi \approx 3.14$.

94. How many cubic inches of concrete are needed to pour a sidewalk that is 4 ft wide, 5 in. deep, and 70 ft long? Concrete is commonly measured in cubic yards, so convert your answer to cubic yards.

95. Trudy is carpeting three rooms in her house. The rooms measure 18 ft by 15 ft, 22 ft by 20 ft, and 21 ft by 25 ft. She picks out a carpet that sells for \$45 a square yard. Assuming that she must buy a whole number of square yards, what is the cost of the carpeting?

Find the square root. If necessary, round to the nearest hundredth.

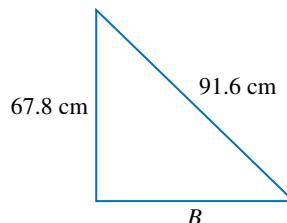
96. $\sqrt{12,321}$

97. $\sqrt{85.49}$

98. $\sqrt{0.03672}$

99. Find the length of the hypotenuse of a right triangle with legs measuring 36 ft and 27 ft.

100. Find the missing side of the following right triangle. Round to the nearest tenth of a centimeter.



Evaluating Your Performance

Pick up your graded test as soon as possible, while the test is still fresh in your mind. No matter what your score, the time you spend reviewing errors you may have made can help improve future test scores. Don't skip the important step of "evaluating your performance." Begin by categorizing your errors:

- 1. Careless errors.** These occur when you know how to do the problem correctly but don't. Careless errors happen when you read the directions incorrectly, make computational errors, or forget to do the problem.
- 2. Concept errors.** These occur because you don't grasp the concept fully or accurately. Even if you did the problem again, you would probably make the same error.
- 3. Study errors.** These occur when you do not spend enough time studying the material most pertinent to the test.
- 4. Application errors.** These usually occur when you are not sure what concept to use.

If you made a careless error, ask yourself if you followed all the suggestions for better test taking. If you didn't, vow to do so on the next test. List what you will do differently next time so that you can minimize these careless errors in the future.

Concept errors need more time to correct. You must review what you didn't understand or you will repeat your mistakes. It is important to grasp the concepts because you will use them later. You may need to seek help from your instructor or a tutor for these kinds of errors.

Avoid study errors by asking the teacher before the test what concepts are most important. Also, pay careful attention to the section objectives because these clarify what you are expected to know.

You can avoid application errors by doing as many of the word problems as you can. Reading the strategies for word problems can help you think about how to start a problem. It also helps to mix up the problems between sections and to do even-numbered problems, for which you do not have answers. It is especially important to try to estimate a reasonable answer before you start calculations.

As a final step in evaluating your performance, take time to think about what you said to yourself during the test. Was this self-talk positive or negative? If the talk was helpful, remember it. Use it again. If not, here are a few suggestions. Be certain that you are keeping a detailed record of your reactions to anxiety. Separate your thoughts from your feelings. Use cue words as signals to begin building coping statements. Your coping statements must challenge your negative belief patterns and they must be believable, not merely pep talk such as "I can do well, I won't worry about it." Make the statements brief and state them in the present tense.

Learn from your mistakes so you can use what you have learned on the next test. If you are satisfied with the results of the test overall, congratulate yourself. You have earned it.

8

ALGEBRA PREVIEW; SIGNED NUMBERS



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SECTIONS

- 8.1** Opposites and Absolute Value
- 8.2** Adding Signed Numbers
- 8.3** Subtracting Signed Numbers
- 8.4** Multiplying Signed Numbers
- 8.5** Dividing Signed Numbers
- 8.6** Order of Operations: A Review
- 8.7** Solving Equations

BASIC
Mathematics Now™

Table 8.1 Continental Elevations

Continent	Highest Point	Feet above Sea Level	Lowest Point	Feet below Sea Level
Africa	Kilimanjaro, Tanzania	19,340	Lake Assal	512
Antarctica	Vinson Massif	16,864	Bentley, Sub-glacial Trench	8327
Asia	Mount Everest, Nepal-Tibet	29,028	Dead Sea, Israel-Jordan	1312
Australia	Mount Kosciusko, New South Wales	7310	Lake Eyre, South Australia	52
Europe	Mount El'brus, Russia	18,510	Caspian Sea, Russia-Azerbaijan	92
North America	Mount McKinley, Alaska	20,320	Death Valley, California	282
South America	Mount Aconcagua, Argentina	22,834	Valdes Peninsula, Argentina	131

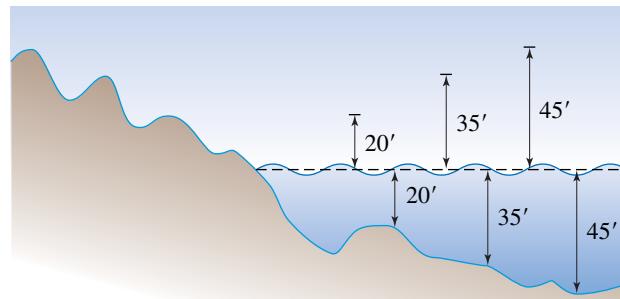


Figure 8.1

What location has the highest continental altitude on Earth? What location has the lowest continental altitude? Is there a location on Earth with a lower altitude? Explain.

8.1 Opposites and Absolute Value

OBJECTIVES

- Find the opposite of a signed number.
- Find the absolute value of a signed number.

VOCABULARY

Positive numbers are the numbers of arithmetic and are greater than zero.

Negative numbers are numbers less than zero. Zero is neither positive nor negative. Positive numbers, zero, and negative numbers are called **signed numbers**.

The **opposite** or **additive inverse** of a signed number is the number on the number line that is the same distance from zero but on the opposite side of it. Zero is its own opposite. The opposite of 5 is written -5 . This can be read “the opposite of 5” or “negative 5,” since they both name the same number.

The **absolute value** of a signed number is the number of units between the number and zero. The expression $|6|$ is read “the absolute value of 6.”

How & Why

OBJECTIVE 1

Find the opposite of a signed number.

Expressions such as

$$6 - 8 \quad 9 - 24 \quad 11 - 12 \quad \text{and} \quad 4 - 134$$

do not have answers in the numbers of arithmetic. The answer to each is a signed number. Signed numbers (which include both numbers to the right of zero and to the left of zero) are used to represent quantities with opposite characteristics. For instance,

right and left
up and down
above zero and below zero
gain and loss

A few signed numbers are shown on the number line in Figure 8.2.



Figure 8.2

The negative numbers are to the left of zero. The negative numbers have a dash, or negative sign, in front of them. The numbers to the right of zero are called positive (and may be written with a plus sign). Zero is neither positive nor negative. Here are some signed numbers.

- | | |
|----------------|---------------------------------------|
| 7 | Seven, or positive seven |
| -3 | Negative three |
| -0.12 | Negative twelve hundredths |
| 0 | Zero is neither positive nor negative |
| $+\frac{1}{2}$ | One half, or positive one half |

Positive and negative numbers are used many ways in the physical world here are some examples:

Positive	Negative
Temperatures above zero (83°)	Temperatures below zero (−11°)
Feet above sea level (6000 ft)	Feet below sea level (−150 ft)
Profit (\$94)	Loss (\$51)
Right (15)	Left (−12)

In any situation in which quantities can be measured in opposite directions, positive and negative numbers can be used to show direction.

The dash in front of a number is read in two different ways:

−23 The opposite of 23

−23 Negative 23

The opposite of a number is the number on the number line that is the same distance from zero but on the opposite side. To find the opposite of a number, we refer to a number line. See Figure 8.3.

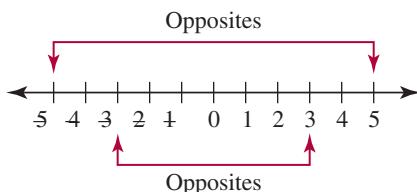


Figure 8.3

$-(5) = -5$ The opposite of positive 5 is negative 5.

$-(-3) = 3$ The opposite of negative 3 is positive 3.

$-0 = 0$ The opposite of 0 is 0.

To find the opposite of a positive number

1. Locate the number on the number line.
2. Count the number of units it is from zero.
3. Count this many units to the left of zero. Where you stop is the opposite of the positive number.

To find the opposite of a negative number

1. Locate the number on the number line.
2. Count the number of units it is from zero.
3. Count this many units to the right of zero. Where you stop is the opposite of the negative number.

The opposite of a positive number is negative and the opposite of a negative number is positive. The opposite of 0 is 0.

Warm-Ups A–E

Examples A–E BASIC Mathematics = Now™

DIRECTIONS: Find the opposite.

STRATEGY: The opposite of a number is found on the number line the same number of units from zero but on the opposite side of zero.

- A. Find the opposite of 26.

STRATEGY: With no sign in front, the number is positive; 14 is read “fourteen” or “positive fourteen.”

$$-(14) = ? \quad \text{Because 14 is 14 units to the right of zero, the opposite of 14 is 14 units to the left of zero.}$$

$$-(14) = -14$$



The opposite of 14 is -14 .

- B. Find the opposite of -19 .

STRATEGY: The number -28 can be thought of in two ways: as a negative number that is 28 units to the left of zero, or as a number that is 28 units on the opposite side of zero from 28; -28 is read “negative 28” or “the opposite of 28.”

$$-(-28) = ? \quad \text{The opposite of negative 28 is written } -(-28) \text{ and is found 28 units on the opposite side of zero from } -28.$$

The opposite of -28 is 28 .

- C. Find the opposite of $-(-13)$.

CAUTION

The dash in front of the parentheses is *not* read “negative.” Instead, it is read “the opposite of.” The dash directly in front of 11 is read “negative” or “the opposite of.”

STRATEGY: First find $-(-11)$. Then find the opposite of that value.

$-(-11)$ is “the opposite of negative 11” $-(-11)$ can also be read “the opposite of the opposite of 11.”

$-(-11) = ?$ The opposite of -11 is 11 units to the right of zero, which is 11.

So the opposite of -11 is 11.

Continuing, we find the opposite of $-(-11)$, which is written $-[-(-11)]$.

$$\begin{aligned} -[-(-11)] &= -[11] \\ &= -11 \end{aligned}$$

The opposite of $-(-11) = -11$.

Answers to Warm-Ups

- A. -26 B. 19 C. -13

CALCULATOR EXAMPLE



- D. Find the opposite of 19, -2.3 , and 4.7 .

STRATEGY: The $[+/-]$ or the $[(-)$ key on the calculator will give the opposite of the number.

Calculator note: Some calculators require that you enter the number and then press the $[+/-]$ key. Others require that you press the $[(-)$ key and then enter the number.

The opposites are -19 , 2.3 , and -4.7 .

- E. If 10% of Americans purchased products with no plastic packaging just 10% of the time, approximately 144,000 lb of plastic would be eliminated (taken out of or decreased) from our landfills.

1. Write this decrease as a signed number.
2. Write the opposite of eliminating (decreasing) 144,000 lb of plastic from our landfills as a signed number.
1. Decreases are often represented by negative numbers. Therefore, a decrease of 144,000 lb is $-144,000$ lb.
2. The opposite of a decrease is an increase, so $-(-144,000 \text{ lb})$ is $144,000$ lb.

- D. Find the opposite of 13, -7.8 , and 7.1 .

- E. The energy used to produce a pound of rubber is 15,700 BTU. Recycled rubber requires 4100 BTU less.

1. Express this decrease as a signed number.
2. Express the opposite of a decrease of 4100 BTU as a signed number.

How & Why

OBJECTIVE 2

Find the absolute value of a signed number.

The absolute value of a signed number is the number of units between the number and zero on the number line. Absolute value is defined as the number of units only; direction is not involved. Therefore, the absolute value is never negative.

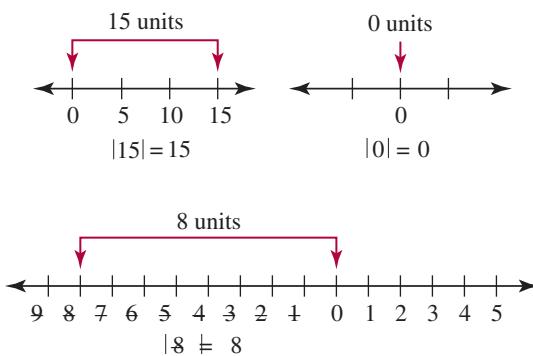


Figure 8.4

From Figure 8.4, we can see that the absolute value of a number and the absolute value of the number's opposite are equal. For example

$$|45| = |-45| \quad \text{Because both equal } 45.$$

The absolute value of a positive number is the number itself. The absolute value of a negative number is its opposite. The absolute value of zero is zero.

To find the absolute value of a signed number

The value is

1. zero if the number is zero.
2. the number itself if the number is positive.
3. the opposite of the number if the number is negative.

Answers to Warm-Ups

- D. $-13, 7.8, -7.1$
E. As a signed number, the decrease is -4100 BTU. As a signed number, the opposite of a decrease of 4100 BTU is 4100 BTU.

Warm-Ups F–I

Examples F–I



BASIC
Mathematics = Now™

F. Find the absolute value of 29.

G. Find the absolute value of -90 .

H. Find the absolute value of -1 .

I. Find the absolute value of $-\frac{3}{7}$.

DIRECTIONS: Find the absolute value of the number.

STRATEGY: If the number is positive or 0, write the number. If the number is negative, write its opposite.

F. Find the absolute value of 17.

$$|17| = 17 \quad \text{The absolute value of a positive number is the number itself.}$$

G. Find the absolute value of 41.

$$\begin{aligned} |-41| &= -(-41) && \text{The absolute value of a negative number is its opposite.} \\ &= 41 \end{aligned}$$

H. Find the absolute value of 0.

$$|0| = 0 \quad \text{The absolute value of zero is zero.}$$

I. Find the absolute value of $-\frac{5}{9}$.

$$\begin{aligned} \left| -\frac{5}{9} \right| &= -\left(-\frac{5}{9} \right) && \text{The absolute value of a negative number is its opposite.} \\ &= \frac{5}{9} \end{aligned}$$

Answers to Warm-Ups

F. 29 G. 90 H. 1 I. $\frac{3}{7}$

Exercises 8.1

OBJECTIVE 1

Find the opposite of a signed number.

A Find the opposite of the signed number.

1. -4

2. -8

3. 5

4. 13

5. -2.3

6. 3.3

7. $\frac{3}{5}$

8. $\frac{3}{4}$

9. $-\frac{6}{7}$

10. $-\frac{3}{8}$

11. The opposite of _____ is 61.

12. The opposite of -23 is _____.**B** Find the opposite of the signed number.

13. -42

14. -57

15. -3.78

16. -6.27

17. $\frac{17}{5}$

18. $\frac{23}{7}$

19. 0.55

20. 0.732

21. 113.8

22. 243.7

23. -0.0123

24. -0.78

OBJECTIVE 2

Find the absolute value of a signed number.

A Find the absolute value of the signed number.

25. $|-4|$

26. $|-11|$

27. $|24|$

28. $|61|$

29. $|-3.17|$

30. $|-4.6|$

31. $\left| \frac{5}{11} \right|$

32. $\left| \frac{7}{13} \right|$

33. $\left| -\frac{3}{11} \right|$

34. $\left| -\frac{7}{12} \right|$

35. The absolute value of _____ is 32.

36. The absolute value of _____ is 17.

B Find the absolute value of a signed number.

37. $|0.0065|$

38. $|0.0021|$

39. $|-355|$

40. $|-922|$

41. $\left| -\frac{11}{3} \right|$

42. $\left| -\frac{17}{6} \right|$

43. $\left| -\frac{33}{7} \right|$

44. $\left| -\frac{18}{5} \right|$

45. $|0|$

46. $|-100|$

47. $|-0.341|$

48. $|-4.96|$

C Find the value of the following.

49. Opposite of $\left| -\frac{5}{17} \right|$

50. Opposite of $\left| \frac{9}{8} \right|$

51. Opposite of $|78|$

52. Opposite of $|-91|$

- 53.** At the New York Stock Exchange, positive and negative numbers are used to record changes in stock prices on the board. What is the opposite of a gain of 1.54?
- 55.** On a thermometer, temperatures above zero are listed as positive and those below zero as negative. What is the opposite of a reading of 14°C ?
- 57.** The modern calendar counts the years after the birth of Christ as positive numbers (A.D. 2001 or $+2001$). Years before Christ are listed using negative numbers (2011 B.C. or -2011). What is the opposite of 1915 B.C., or 4915?
- 59.** A cyclist travels up a mountain 1415 ft, then turns around and travels down the mountain 1143 ft. Represent each trip as a signed number.
- 61.** If 80 miles north is represented by $+80$, how would you represent 80 miles south?
- 54.** On the NASDAQ, a stock is shown to have taken a loss of three-eighths of a point (-0.375). What is the opposite of this loss?
- 56.** On a thermometer such as the one in Exercise 55, what is the opposite of a reading of 46°C ?
- 58.** The empty-weight center of gravity of an airplane is determined. A generator is installed at a moment of -278 . At what moment could a weight be placed so that the center of gravity remains the same? (Moment is the product of a quantity such as weight and its distance from a fixed point. In this application the moments must be opposites to keep the same center of gravity.)
- 60.** An energy audit indicates that the Gates family could reduce their average electric bill by \$17.65 per month by doing some minor repairs, insulating their attic and crawl space, and caulking around the windows and other cracks in the siding.
- Express this savings as a signed number.
 - Express the opposite of the savings as a signed number.

Exercises 62–64 refer to the chapter application. See page 673.

- 62.** Rewrite the continental altitudes in the application using signed numbers.
- 63.** The U.S. Department of Defense has extensive maps of the ocean floors because they are vital information for the country's submarine fleet. The table lists the deepest part and the average depth of the world's major oceans.

Ocean	Deepest Part (ft)	Average Depth (ft)
Pacific	Mariana Trench, 35,840	12,925
Atlantic	Puerto Rico Trench, 28,232	11,730
Indian	Java Trench, 23,376	12,598
Arctic	Eurasia Basin, 17,881	3,407
Mediterranean	Ionian Basin, 16,896	4,926

Rewrite the table using signed numbers.

- 64.** Is the highest point on Earth farther away from sea level than the deepest point in the ocean? Explain. What mathematical concept allows you to answer this question? See Exercise 63.

Simplify.

65. $-(-24)$

66. $-(81)$

67. $-(-(-14))$

68. $-(-(33))$

- 69.** The Buffalo Bills are playing a football game against the Seattle Seahawks. On the first play, the Seahawks lose 8 yd. Represent this as a signed number. What is the opposite of a loss of 8 yd? Represent this as a signed number.

- 70.** The Dallas Cowboys and the New York Giants are having an exhibition game in London, England. The Cowboy offensive team runs a gain of 6 yd, a loss of 8 yd, a gain of 21 yd, and a loss of 15 yd. Represent these yardages as signed numbers.

- 71.** The Golden family is on a vacation in the southwestern United States. Consider north and east as positive directions and south and west as negative directions. On 1 day they drive north 137 mi then east 212 mi. The next day they drive west 98 mi then 38 mi south. Represent each of these distances as signed numbers.

STATE YOUR UNDERSTANDING

- 72.** Is zero the only number that is its own opposite?
Justify your answer.
- 73.** Is there a set of numbers for which the absolute value of each number is the number itself? If yes, identify that set and tell why this is true.
- 74.** Explain -4 . Draw it on a number line. On the number line, use the concepts of opposites and absolute value as they relate to -4 . Give an instance in the world when -4 is useful.

CHALLENGE*Simplify.*

75. $|16 - 10| - |14 - 9| + 6$

76. $8 - |12 - 8| - |10 - 8| + 2$

77. If n is a negative number, what kind a number is $-n$?78. For what numbers is $|-n| = n$ always true?**GROUP WORK**

79. Develop a rule (procedure) for adding a positive and a negative number. (*Hint:* You might want to visualize that walking forward represents a positive number and walk-

ing backward represents a negative number.) See if you can find answers to:

$$6 + (-4) = ? \quad 7 + (-10) = ? \quad \text{and} \quad -6 + 11 = ?$$

MAINTAIN YOUR SKILLS*Add.*

80. $7 + 11 + 32 + 9$

81. $5 + 12 + 21 + 14$

Subtract.

82. $17 - 12$

83. $45 - 23$

Add.

$$\begin{array}{r} 6 \text{ m } 250 \text{ cm} \\ + 7 \text{ m } 460 \text{ cm} \\ \hline = ? \text{ m} \end{array}$$

$$\begin{array}{r} 5 \text{ L } 78 \text{ mL} \\ + 2 \text{ L } 95 \text{ mL} \\ \hline = ? \text{ mL} \end{array}$$

Subtract.

$$\begin{array}{r} 3 \text{ yd } 2 \text{ ft } 8 \text{ in.} \\ - 1 \text{ yd } 2 \text{ ft } 11 \text{ in.} \\ \hline = ? \text{ in.} \end{array}$$

$$\begin{array}{r} 7 \text{ gal } 1 \text{ qt } 1 \text{ pt} \\ - 4 \text{ gal } 3 \text{ qt } 1 \text{ pt} \\ \hline = ? \text{ pt} \end{array}$$

88. A 12-m coil of wire is to be divided into eight equal parts. How many meters are in each part?

87. Some drug doses are measured in grains. For example, a common aspirin tablet contains 5 grains. If there are 15.43 g in a grain, how many grams do two aspirin tablets contain?

8.2 Adding Signed Numbers

How & Why

OBJECTIVE

Add signed numbers.

OBJECTIVE

Add signed numbers.

Positive and negative numbers are used to show opposite quantities:

- +482 lb may show 482 lb loaded
- 577 lb may show 577 lb unloaded
- +27 dollars may show 27 dollars earned
- 19 dollars may show 19 dollars spent

We can use this idea to find the sum of signed numbers. We already know how to add two positive numbers, so we concentrate on adding positive and negative numbers.

Think of 27 dollars earned (positive) and 19 dollars spent (negative). The result is 8 dollars left in your pocket (positive). So

$$27 + (-19) = 8$$

To get this sum, we subtract the absolute value of -19(19) from the absolute value of 27(27). The sum is positive.

Think of 23 dollars spent (negative) and 15 dollars earned (positive). The result is that you still owe 8 dollars (negative). So

$$-23 + 15 = -8$$

To get this sum, subtract the absolute value of 15(15) from the absolute value of -23(23). The sum is negative.

Think of 5 dollars spent (negative) and another 2 dollars spent (negative). The result is 7 dollars spent (negative). So

$$-5 + (-2) = -7$$

To get this sum we add 5 to 2 (the absolute value of each). The sum is negative.

The results of the examples lead us to the procedure for adding signed numbers.

To add signed numbers

1. If the signs are alike, add their absolute values and use the common sign for the sum.
2. If the signs are not alike, subtract the smaller absolute value from the larger absolute value. The sum will have the sign of the number with the larger absolute value.

As a result of this definition, the sum of a number and its opposite is zero. Thus, $5 + (-5) = 0$, $8 + (-8) = 0$, $-12 + 12 = 0$, and so on.

Warm-Ups A–J

Examples A–J



BASIC
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A. Add: $56 + (-30)$

DIRECTIONS: Add.

STRATEGY: If the signs are alike, add the absolute values and use the common sign for the sum. If the signs are unlike, subtract their absolute values and use the sign of the number with larger absolute value.

B. Find the sum: $-63 + 44$

A. Add: $45 + (-23)$

STRATEGY: Since the signs are unlike, subtract their absolute values.

$$\begin{aligned}|45| - |-23| &= 45 - 23 \\&= 22\end{aligned}$$

Because the positive number has the larger absolute value, the sum is positive.

$$\text{So } 45 + (-23) = 22$$

B. Find the sum: $-72 + 31$

$$\begin{aligned}|-72| - |31| &= 72 - 31 \\&= 41\end{aligned}$$

$-72 + 31 = -41$ The sum is negative, because -72 has the larger absolute value.

C. Add: $42 + (-81)$

C. Add: $37 + (-53)$

$$\begin{aligned}|-53| - |37| &= 53 - 37 \\&= 16\end{aligned}$$

$37 + (-53) = -16$ The sum is negative, because -53 has the larger absolute value.

D. Add: $-5.6 + (-2.1)$

D. Add: $-0.12 + (-2.54)$

$$\begin{aligned}|-0.12| + |-2.54| &= 0.12 + 2.54 \\&= 2.66\end{aligned}$$

$-0.12 + (-2.54) = -2.66$ The numbers are negative, therefore the sum is negative.

E. Find the sum: $\frac{2}{3} + \left(-\frac{5}{6}\right)$

E. Find the sum: $\frac{5}{8} + \left(-\frac{3}{4}\right)$

$$\begin{aligned}\left|-\frac{3}{4}\right| - \left|\frac{5}{8}\right| &= \frac{6}{8} - \frac{5}{8} \\&= \frac{1}{8}\end{aligned}$$

$$\frac{5}{8} + \left(-\frac{3}{4}\right) = -\frac{1}{8}$$

Because the negative number has the larger absolute value, the sum is negative.

Answers to Warm-Ups

A. 26 B. -19 C. -39

D. -7.7 E. $-\frac{1}{6}$

- F. Find the sum of -16 , 41 , -33 , and -14 .

STRATEGY: Where there are more than two numbers to add, it may be easier to add the numbers with the same sign first.

$$\begin{array}{r} -16 \\ -33 \\ -14 \\ \hline -63 \end{array}$$

Add the negative numbers.

Now add their sum to 41 .

$$|-63| - |41| = 63 - 41 \\ = 22$$

Because the signs are different, find the difference in their absolute values.

$$-16 + 41 + (-33) + (-14) = -22$$

The sum is negative, because -63 has the larger absolute value.

- G. Add: $-0.4 + 0.7 + (-5.4) + 1.3$

$$\begin{array}{r} -0.4 \quad 0.7 \\ -5.4 \quad 1.3 \\ \hline -5.8 \quad 2.0 \end{array}$$

Now add the sums.

$$-5.8 + 2.0 \quad \text{The signs are different so subtract their absolute values.}$$

$$|-5.8| - |2.0| = 5.8 - 2.0 \\ = 3.8$$

$$-0.4 + 0.7 + (-5.4) + 1.3 = -3.8 \quad \text{Because } -5.8 \text{ has the larger absolute value, the sum is negative.}$$

- H. Add: $-\frac{5}{6} + \frac{4}{9} + \left(-\frac{5}{18}\right) + \left(-\frac{7}{18}\right)$

- F. Find the sum of -40 , 62 , -85 , and 57 .
- G. Add: $0.42 + (-5.21) + (-0.94) + 3.65$

- H. $-\frac{4}{5} + \frac{1}{2} + \left(-\frac{3}{4}\right) + \left(-\frac{2}{5}\right)$

STRATEGY: First add the negative numbers.

$$\begin{aligned} -\frac{5}{6} + \left(-\frac{5}{18}\right) + \left(-\frac{7}{18}\right) &= -\frac{15}{18} + \left(-\frac{5}{18}\right) + \left(-\frac{7}{18}\right) \\ &= -\frac{27}{18} \end{aligned}$$

Add the sum of the negative numbers and the positive number. To do this, find the difference of their absolute values.

$$\begin{aligned} \left|-\frac{27}{18}\right| - \left|\frac{4}{9}\right| &= \frac{27}{18} - \frac{8}{18} \\ &= \frac{19}{18} \end{aligned}$$

$$-\frac{5}{6} + \frac{4}{9} + \left(-\frac{5}{18}\right) + \left(-\frac{7}{18}\right) = -\frac{19}{18}$$

The sum is negative because the negative number has the larger absolute value.

Answers to Warm-Ups

F. -6 G. -2.08

H. $-\frac{29}{20}$

CALCULATOR EXAMPLE

- I. Find the sum: $-88 + 97 + (-117) + (-65)$

- J. A second stock that John owns has the following changes for a week:
Monday, gains \$1.34;
Tuesday, gains \$0.84;
Wednesday, loses \$4.24;
Thursday, loses \$2.21;
Friday, gains \$0.36. What is the net change in the price of the stock for the week?

- I. Find the sum: $-89 + 77 + (-104) + (-93)$

Use the [+/-] or [(-) key on the calculator to indicate negative numbers.

The sum is -209 .

- J. John owns stock that is traded on the NASDAQ. On Monday the stock gains \$2.04, on Tuesday it loses \$3.15, on Wednesday it loses \$1.96, on Thursday it gains \$2.21, and on Friday it gains \$1.55. What is the net price change in the price of the stock for the week?

STRATEGY: To find the net change in the price of the stock, write the daily changes as signed numbers and find the sum of these numbers.

Monday	gains \$2.04	2.04
Tuesday	loses \$3.15	-3.15
Wednesday	loses \$1.96	-1.96
Thursday	gains \$2.21	2.21
Friday	gains \$1.55	1.55

$$2.04 + (-3.15) + (-1.96) + 2.21 + 1.55 = 5.8 + (-5.11) \\ = 0.69$$

The stock gains \$0.69 during the week.

Add the positive numbers and add the negative numbers.

Answers to Warm-Ups

- I. -173
J. The stock lost \$3.91 (-3.91) for the week.

Exercises 8.2

OBJECTIVE

Add signed numbers.

A Add.

1. $-7 + 9$

2. $-7 + 3$

3. $8 + (-6)$

4. $10 + (-6)$

5. $-9 + (-6)$

6. $-7 + (-8)$

7. $-11 + (-3)$

8. $-15 + (-9)$

9. $-11 + 11$

10. $16 + (-16)$

11. $0 + (-12)$

12. $-32 + 0$

13. $-23 + (-23)$

14. $-13 + (-13)$

15. $3 + (-17)$

16. $12 + (-15)$

17. $-19 + (-7)$

18. $-11 + (-22)$

19. $24 + (-17)$

20. $-18 + 31$

B

21. $-5 + (-7) + 6$

22. $-5 + (-8) + (-10)$

23. $-65 + (-43)$

24. $-56 + (-44)$

25. $-98 + 98$

26. $108 + (-108)$

27. $-45 + 72$

28. $53 + (-38)$

29. $-36 + 43 + (-17)$

30. $-37 + 21 + (-9)$

31. $62 + (-56) + (-13)$

32. $-34 + (-18) + 29$

33. $-4.6 + (-3.7)$

34. $-9.7 + (-5.2)$

35. $10.6 + (-7.8)$

36. $-13.6 + 8.4$

37. $-\frac{5}{6} + \frac{1}{4}$

38. $\frac{3}{8} + \left(-\frac{1}{2}\right)$

39. $-\frac{7}{10} + \left(-\frac{7}{15}\right)$

40. $-\frac{5}{12} + \left(-\frac{2}{9}\right)$

C Simplify.

41. $135 + (-256)$

42. $233 + (-332)$

43. $-81 + (-32) + (-76)$

44. $-75 + (-82) + (-71)$

45. $-31 + 28 + (-63) + 36$

46. $-44 + 37 + (-59) + 45$

47. $49 + (-67) + 27 + 72$

48. $81 + (-72) + 33 + 49$

49. $356 + (-762) + (-892) + 541$

50. $-923 + 672 + (-823) + (-247)$

51. Find the sum of 542, -481, and -175.

52. Find the sum of 293, -122, and -211.

Exercises 53–56. The table gives the temperatures recorded by Rover for a 5-day period at one location on the surface of Mars.

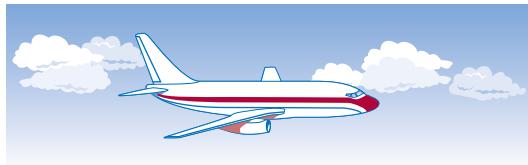
	Day 1	Day 2	Day 3	Day 4	Day 5
5:00 A.M.	−92°C	−88°C	−115°C	−103°C	−74°C
9:00 A.M.	−57°C	−49°C	−86°C	−93°C	−64°C
1:00 P.M.	−52°C	−33°C	−46°C	−48°C	−10°C
6:00 P.M.	−45°C	−90°C	−102°C	−36°C	−42°C
11:00 P.M.	−107°C	−105°C	−105°C	−98°C	−90°C

- 53.** What is the sum of the temperatures recorded at 9:00 A.M.?
- 54.** What is the sum of the temperatures recorded on day 1?
- 55.** What is the sum of the temperatures recorded at 11:00 P.M.?
- 56.** What is the sum of the temperatures recorded on day 4?
- 57.** An airplane is being reloaded; 963 lb of baggage and mail are removed (−963 lb) and 855 lb of baggage and mail are loaded (+855 lb). What net change in weight should the cargo master report?
- 58.** At another stop, the plane in Exercise 57 unloads 2341 lb of baggage and mail and takes on 2567 lb. What net change should the cargo master report?
- 59.** The change in altitude of a plane in flight is measured every 10 min. The figures between 3:00 P.M. and 4:00 P.M. are as follows:

3:00	30,000 ft initially	(+30,000)
3:10	increase of 220 ft	(+220 ft)
3:20	decrease of 200 ft	(−200 ft)
3:30	increase of 55 ft	(+55 ft)
3:40	decrease of 110 ft	(−110 ft)
3:50	decrease of 55 ft	(−55 ft)
4:00	decrease of 40 ft	(−40 ft)

- 60.** What is the final altitude of the airplane in Exercise 59 if it is initially flying at 23,000 ft with the following changes in altitude?

3:00	23,000 ft initially	(+23,000)
3:10	increase of 315 ft	(+315 ft)
3:20	decrease of 825 ft	(−825 ft)
3:30	increase of 75 ft	(+75 ft)
3:40	decrease of 250 ft	(−250 ft)
3:50	decrease of 85 ft	(−85 ft)
4:00	decrease of 70 ft	(−70 ft)



What is the altitude of the plane at 4 P.M.? (*Hint:* Find the sum of the initial altitude and the six measured changes between 3 and 4 P.M.)

- 61.** The Pacific Northwest Book Depository handles most textbooks for the local schools. On September 1, the inventory is 34,945 volumes. During the month, the company makes the following transactions (positive numbers represent volumes received, negative numbers represent shipments): 3456, −2024, −3854, 612, and −2765. What is the inventory at the end of the month?
- 62.** The Pacific Northwest Book Depository has 18,615 volumes on November 1. During the month, the depository has the following transactions: −4675, 912, −6764, 1612, and −950. What is the inventory at the end of the month?

- 63.** The Buffalo Bills made the following consecutive plays during a recent Monday night football game: 8-*yd* loss, 10-*yd* gain, and 7-*yd* gain. A first down requires a gain of 10 *yd*. Did they get a first down?
- 65.** Nordstrom stock has the following changes in 1 week: up 0.61, down 0.45, down 1.13, up 2.61, and up 0.12.
- What is the net change for the week?
 - If the stock starts at 72.83 at the beginning of the week, what is the closing price?
- 67.** A new company has the following weekly balances after the first month of business: a loss of \$2376, a gain of \$5230, a loss of \$1055, and a gain of \$3278. What is their net gain or loss for the month?
- 64.** The Seattle Seahawks have these consecutive plays one Sunday: 12-*yd* loss, 19-*yd* gain, and 4-*yd* gain. Do they get a first down?
- 66.** On a January morning in a small town in upstate New York, the lowest temperature is recorded as 17 degrees below zero. During the following week the daily lowest temperature readings are up 3 degrees, up 7 degrees, down 5 degrees, up 1 degree, no change, and down 3 degrees. What is the low temperature reading for the last day?
- 68.** Marie decided to play the state lottery for 1 month. This meant that she played every Wednesday and Saturday for a total of nine times. This is her record: lost \$10, won \$45, lost \$15, lost \$12, won \$65, lost \$12, lost \$16, won \$10, and lost \$6. What is the net result of her playing?

STATE YOUR UNDERSTANDING

- 69.** When adding a positive and a negative number, explain how to determine whether the sum is positive or negative.

CHALLENGE

- 70.** What is the sum of 46 and (-52) increased by 23?
- 72.** What number added to 47 equals 28?
- 74.** What number added to (-123) equals (-163) ?
- 71.** What is the sum of (-73) and (-32) added to (-19) ?
- 73.** What number added to (-75) equals (-43) ?

GROUP WORK

- 75.** Develop a rule for subtracting two signed numbers.
(Hint: Subtraction is the inverse of addition; that is, it undoes addition. So if adding $+6$ can be thought of as walking forward six paces, then subtracting $+6$ may be

thought of as walking backward 6 paces.) See if you can find answers to

$$4 - (+6) = ? \quad -5 - (+6) = ? \quad \text{and} \quad 8 - (-4) = ?$$

MAINTAIN YOUR SKILLS*Subtract.*

76. $145 - 67$

77. $356 - 89$

78.
$$\begin{array}{r} 178 \\ - 95 \\ \hline \end{array}$$

79.
$$\begin{array}{r} 897 \\ - 698 \\ \hline \end{array}$$

- 80.** Find the perimeter of a rectangle that is 42 m long and 29 m wide.
- 81.** Find the circumference of a circle with a radius of 17 cm. The formula is $C = \pi d$, where d is the diameter of the circle. Let $\pi \approx 3.14$.
- 82.** Find the perimeter of a square that is 31 in. on each side.
- 83.** Find the perimeter of a triangle with sides of 65 cm, 78 cm, and 52 cm.
- 84.** Bananas are put on sale for 3 lb for \$2.16. What is the cost of 1 lb?
- 85.** The cost of pouring a 3-ft-wide cement sidewalk is estimated to be \$21 per square foot. If a walk is to be placed around a rectangular plot of ground that is 42 ft along the width and 50 ft along the length, what is the cost of pouring the walk?

8.3 Subtracting Signed Numbers

How & Why

OBJECTIVE

Subtract signed numbers.

OBJECTIVE

Subtract signed numbers.

The expression $11 - 7 = ?$ can be restated using addition: $7 + ? = 11$. We know that $7 + 4 = 11$ and so $11 - 7 = 4$. We use the fact that every subtraction fact can be restated as addition to discover how to subtract signed numbers. Consider $-4 - 8 = ?$. Restating, we have $8 + ? = -4$. We know that $8 + (-12) = -4$ so we conclude that $-4 - 8 = -12$. The expression $-5 - (-3) = ?$ can be restated as $-3 + ? = -5$. Because $-3 + (-2) = -5$ we conclude that $-5 - (-3) = -2$.

To discover the rule for subtraction of signed numbers, compare:

$11 - 7$	and	$11 + (-7)$	Both equal 4
$-4 - 8$	and	$-4 + (-8)$	Both equal -12
$-5 - (-3)$	and	$-5 + 3$	Both equal -2

Every subtraction problem can be worked by asking what number added to the subtrahend will yield the minuend. However, when we look at the second column we see an addition problem that gives the same answer as the original subtraction problem. In each case the opposite (additive inverse) of the number being subtracted is added. Let's look at three more examples.

Answer Obtained by Adding to the Subtrahend

$$19 - 7 = 12$$

$$-32 - 45 = -77$$

$$-21 - (-12) = -9$$

Answer Obtained by Adding the Opposite

$$19 + (-7) = 12$$

$$-32 + (-45) = -77$$

$$-21 + 12 = -9$$

-7 is the opposite of 7 , the number to be subtracted.

-45 is the opposite of 45 , the number to be subtracted.

12 is the opposite of -12 , the number to be subtracted.

This leads us to the rule for subtracting signed numbers.

To subtract signed numbers

1. Rewrite as an addition problem by adding the opposite of the number to be subtracted.
2. Find the sum.

Examples A–J



BASIC
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DIRECTIONS: Subtract.

STRATEGY: Add the opposite of the number to be subtracted.

A. Subtract: $42 - 28$

STRATEGY: Rewrite as an addition problem by adding -28 , which is the opposite of 28 .

$$\begin{aligned}42 - 28 &= 42 + (-28) \\&= 14\end{aligned}$$

Rewrite as addition.

Add. Because the signs are different, subtract their absolute values and use the sign of the number with the larger absolute value, which is 42 .

Warm-Ups A–J

A. Subtract: $75 - 42$

Answers to Warm-Ups

Because both numbers are positive we can also do the subtraction in the usual manner: $42 - 28 = 14$.

B. Subtract: $-56 - 70$

B. Subtract: $-38 - 61$

$$\begin{aligned}-38 - 61 &= -38 + (-61) && \text{Rewrite as addition.} \\ &= -99\end{aligned}$$

Add. Because both numbers are negative, add their absolute values and keep the common sign.

C. Find the difference of -34 and -40 .

C. Find the difference of -54 and -12 .

$$\begin{aligned}-54 - (-12) &= -54 + 12 && \text{Rewrite as addition.} \\ &= -42 && \text{Add.}\end{aligned}$$

D. Subtract: $32 - 51$

D. Subtract: $63 - 91$

$$\begin{aligned}63 - 91 &= 63 + (-91) && \text{Rewrite as addition.} \\ &= -28\end{aligned}$$

E. Find the difference:

$$\frac{1}{4} - \left(-\frac{3}{5}\right)$$

E. Find the difference: $\frac{5}{8} - \left(-\frac{2}{3}\right)$

$$\begin{aligned}\frac{5}{8} - \left(-\frac{2}{3}\right) &= \frac{5}{8} + \frac{2}{3} && \text{Rewrite as addition.} \\ &= \frac{15}{24} + \frac{16}{24} && \text{Write each fraction with a common denominator and add.} \\ &= \frac{31}{24}\end{aligned}$$

F. Subtract: $45 - (-32) - 24$

F. Subtract: $23 - (-33) - 19$

STRATEGY: Change both subtractions to add the opposite.

$$\begin{aligned}23 - (-33) - 19 &= 23 + 33 + (-19) && \text{Rewrite as addition.} \\ &= 56 + (-19) && \text{Add 23 and 33.} \\ &= 37 && \text{Add.}\end{aligned}$$

G. Subtract: $-0.74 - 1.16 - (-0.93) - (-2.46)$

G. Subtract: $-0.34 - (-1.04) - (-3.02) - 0.81$

STRATEGY: Change all subtractions to add the opposite and add.

$$\begin{aligned}-0.34 - (-1.04) - (-3.02) - 0.81 &= -0.34 + 1.04 + 3.02 + (-0.81) \\ &= 2.91\end{aligned}$$

H. Subtract:

$$-\frac{3}{5} - \left(-\frac{3}{8}\right) - \left(-\frac{7}{10}\right) - \frac{7}{20}$$

H. Subtract: $-\frac{3}{4} - \frac{7}{8} - \left(-\frac{1}{2}\right) - \left(-\frac{1}{8}\right)$

$$\begin{aligned}-\frac{3}{4} - \frac{7}{8} - \left(-\frac{1}{2}\right) - \left(-\frac{1}{8}\right) &= -\frac{3}{4} + \left(-\frac{7}{8}\right) + \frac{1}{2} + \frac{1}{8} \\ &= -\frac{6}{8} + \left(-\frac{7}{8}\right) + \frac{4}{8} + \frac{1}{8} \\ &= -\frac{8}{8} = -1\end{aligned}$$

Answers to Warm-Ups

B. ± 26 C. 6 D. 49

E. $\frac{17}{20}$ F. 53 G. 1.49

H. $\frac{1}{8}$

CALCULATOR EXAMPLE

- I. Subtract: $-784.63 - (-532.78)$

STRATEGY: The calculator does not require you to change the subtraction to add the opposite.

The difference is -251.85 .

- J. The highest point in North America is Mount McKinley, a peak in central Alaska, which is approximately 20,320 ft above sea level. The lowest point in North America is Death Valley, a deep basin in southeastern California, which is approximately 282 ft below sea level. What is the difference in height between Mount McKinley and Death Valley? (Above sea level is positive and below sea level is negative.)

STRATEGY: To find the difference in height, write each height as a signed number and subtract the lower height from the higher height.

Mount McKinley	20,320 above	20,320
Death Valley	282 below	-282

$$20,320 - (-282) = 20,320 + 282 \quad \text{Change subtraction to add the opposite and add.}$$
$$= 20,602$$

The difference in height is approximately 20,602 ft.

- I. Subtract:
 $-483.34 - (-312.43)$

- J. One night last winter, the temperature dropped from 24°F to -11°F . What was the difference between the high and the low temperatures?

Answers to Warm-Ups

- I. $+70.91$
J. The difference in temperatures was 35°F .

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Exercises 8.3

OBJECTIVE

Subtract signed numbers.

A Subtract.

1. $7 - 3$

2. $9 - 5$

3. $-7 - 5$

4. $-8 - 7$

5. $-8 - (-3)$

6. $-7 - (-2)$

7. $11 - 8$

8. $13 - 9$

9. $-12 - 8$

10. $-13 - 6$

11. $-23 - (-14)$

12. $-17 - (-8)$

13. $19 - (-13)$

14. $26 - (-9)$

15. $-23 - 11$

16. $-24 - 12$

17. $-15 - 13$

18. $-33 - 20$

19. $-13 - (-14)$

20. $-34 - (-35)$

21. $-16 - (-16)$

22. $-23 - (-23)$

23. $-9 - 9$

24. $-31 - 31$

25. $-40 - 40$

26. $-14 - 14$

B

27. $72 - (-46)$

28. $54 - (-61)$

29. $-57 - 62$

30. $-82 - 91$

31. $-48 - (-59)$

32. $-66 - (-81)$

33. $-91 - 91$

34. $-43 - 43$

35. $102 - (-102)$

36. $78 - (-78)$

37. $-69 - (-69)$

38. $-83 - (-83)$

39. $134 - (-10)$

40. $164 - (-20)$

41. $132 - (-41)$

42. $173 - (45)$

43. $-6.74 - 3.24$

44. $-13.34 - 9.81$

45. $-4.65 - (-3.21)$

46. $-7.54 - (-8.12)$

47. $-23.43 - 32.71$

48. $-18.63 - (-13.74)$

- 49.** Find the difference between 43 and -73 .
- 50.** Find the difference between -88 and -97 .
- 51.** Subtract 338 from -349 .
- 52.** Subtract 145 from -251 .
- 53.** Rover records high and low temperatures of -24°C and -109°C for 1 day on the surface of Mars. What is the change in temperature for that day?

- 55.** At the beginning of the month, Joe's bank account had a balance of \$782.45. At the end of the month, the account was overdrawn by \$13.87 ($-\13.87). If there were no deposits during the month, what was the total amount of checks Joe wrote? (Hint: Subtract the ending balance from the original balance.)
- 56.** At the beginning of the month, Jack's bank account had a balance of \$512.91. At the end of the month, the balance was $-\$67.11$. If there were no deposits, find the amount of checks Jack wrote. (Refer to Exercise 55.)

Exercises 57–59 refer to the chapter application. See page 673.

- 57.** The range of a set of numbers is defined as the difference between the largest and the smallest numbers in the set. Calculate the range of altitude for each continent. Which continent has the smallest range, and what does this mean in physical terms?
- 58.** What is the difference between the lowest point in the Mediterranean and the lowest point in the Atlantic? See Exercise 63 in Section 8.1.
- 59.** Some people consider Mauna Kea, Hawaii, to be the tallest mountain in the world. It rises 33,476 ft from the ocean floor, but is only 13,796 ft above sea level. What is the depth of the ocean floor at this location?

Exercises 60–63 refer to the table below, which shows temperature recordings by a Martian probe for a 5-day period at one location on the surface of Mars.

Temperatures on the Surface of Mars

	Day 1	Day 2	Day 3	Day 4	Day 5
5:00 A.M.	-92°C	-88°C	-115°C	-103°C	-74°C
9:00 A.M.	-57°C	-49°C	-86°C	-93°C	-64°C
1:00 P.M.	-52°C	-33°C	-46°C	-48°C	-10°C
6:00 P.M.	-45°C	-90°C	-102°C	-36°C	-42°C
11:00 P.M.	-107°C	-105°C	-105°C	-98°C	-90°C

- 60.** What is the difference between the high and low temperatures recorded on day 3?
- 61.** What is the difference between the temperatures recorded at 11:00 P.M. on day 3 and day 5?

62. What is the difference between the temperatures recorded at 5:00 A.M. on day 2 and 6:00 P.M. on day 4?
64. Al's bank account had a balance of \$318. He writes a check for \$412.75. What is his account balance now?
66. Carol started school owing her mother \$18; by school's end she borrowed \$123 more from her mother. How does her account with her mother stand now?
68. What is the difference in altitude between the highest point and the lowest point in California?
Highest point: Mount Whitney is 14,494 ft above sea level (+14,494)
Lowest point: Death Valley is 282 ft below sea level (-282)
70. The New York Jets started on their 40-yd line. After three plays, they were on their 12-yd line. Did they gain or lose yards? Represent their gain or loss as a signed number.
63. What is the difference between the highest and lowest temperatures recorded during the 5 days?
65. Thomas started with \$210.34 in his account. He writes a check for \$216.75. What is his account balance now?
67. At the beginning of the month, Janna's bank account had a balance of \$467.82. At the end of the month, the account was overdrawn by \$9.32. If there were no deposits during the month, what was the total amount of checks Janna wrote?
69. On the first sale of the day, Gina makes a profit of \$145.67. However, on the next sale, Gina loses \$310.12 because of another employee's misquote on the price of an item. After these two sales, what is the status of Gina's sales?
71. During 1980, the net import of coal was -2.39 quadrillion BTU. In 1990, the net import of coal was -2.70 quadrillion BTU. In 2004, the net import of coal was -0.46 quadrillion BTU. Find the difference in coal imports for each pair of years.

STATE YOUR UNDERSTANDING

72. Explain the difference between adding and subtracting signed numbers.
73. How would you explain to a 10-year-old how to subtract -8 from 12 ?
74. Explain why the order in which two numbers are subtracted is important, but the order in which they are added is not.
75. Explain the difference between the problems $5 + (-8)$ and $5 - 8$.

CHALLENGE

76. $-13 - (-54) - |-21|$

77. $-9.46 - [-(-3.22)]$

78. $-76 - (-37) - | -(-55)|$

79. $57 - |-67 - (-51)| - 82$

80. $|17.5 - 13.5| - |21.7 - (-19.6)|$

GROUP WORK

81. Determine the normal daily mean (average) temperature for your city for each month of the year. Find the difference from month to month. Chart this result.

MAINTAIN YOUR SKILLS

Multiply.

82. $16(7)$

83. $42(13)$

84.
$$\begin{array}{r} 156 \\ \times 37 \\ \hline \end{array}$$

85.
$$\begin{array}{r} 4782 \\ \times 362 \\ \hline \end{array}$$

86. Find the area of a circle that has a radius of 14 in. (Let $\pi \approx 3.14$.)

87. Find the area of a square that is 16.7 m on each side.

88. Find the area of a rectangle that is 13.6 cm long and 9.4 cm wide.

89. Find the area of a triangle that has a base of 14.4 m and a height of 7.8 m.

90. How many square yards of wall-to-wall carpeting are needed to carpet a rectangular flor that measures 22.5 ft by 30.5 ft?

91. How many square tiles, which are 9 in. on a side, are needed to cover a flor that is 21 ft by 24 ft?

8.4 Multiplying Signed Numbers

How & Why

OBJECTIVE 1

Multiply a positive number and a negative number.

Consider the following multiplications:

$$\begin{aligned}3(4) &= 12 \\3(3) &= 9 \\3(2) &= 6 \\3(1) &= 3 \\3(0) &= 0 \\3(-1) &= ? \\3(-2) &= ?\end{aligned}$$

Each product is 3 smaller than the one before it. Continuing this pattern,

$$3(-1) = -3 \quad \text{Because } 0 - 3 = -3$$

and

$$3(-2) = -6 \quad \text{Because } -3 - 3 = -6$$

The pattern indicates that the product of a positive and a negative number is negative; that is, the opposite of the product of their absolute values.

OBJECTIVES

1. Multiply a positive number and a negative number.
2. Multiply two negative numbers.

To find the product of a positive and a negative number

1. Find the product of the absolute values.
2. Make this product negative.

This is sometimes stated, “The product of two unlike signs is negative.”

The commutative property of multiplication dictates that no matter the order in which the positive and negative numbers appear, their product is always negative. Thus,

$$3(-4) = -12 \quad \text{and} \quad -4(3) = -12$$

Examples A–F



BASIC Mathematics Now™

DIRECTIONS: Multiply.

STRATEGY: Multiply the absolute values and write the opposite of that product.

- A. Find the product: $-7(6)$

$$-7(6) = -42 \quad \text{The product of a positive and a negative number is negative.}$$

- B. Find the product: $18(-9)$

$$18(-9) = -162 \quad \text{The product of two factors with unlike signs is negative.}$$

- C. Find the product: $5(-3.3)$

$$5(-3.3) = -16.5 \quad \text{The product of two factors with unlike signs is negative.}$$

Warm-Ups A–F

- A. Find the product: $-12(5)$

- B. Find the product: $25(-7)$

- C. Find the product: $6(-1.9)$

Answers to Warm-Ups

A. 60 B. -175 C. -11.4

D. Multiply: $\left(-\frac{5}{9}\right)\left(\frac{3}{5}\right)$

D. Multiply: $\left(-\frac{4}{5}\right)\left(\frac{3}{4}\right)$

$$\left(-\frac{4}{5}\right)\left(\frac{3}{4}\right) = -\frac{3}{5}$$

Simplify. Find the product of the numerators and the denominators. The product is negative.

E. Multiply: $9(-4)(3)$

$$8(-7)(4) = -56(4)$$
$$= -224$$

Multiply the first two factors.

Multiply again.

- F. James wants to lose weight. His goal is to lose an average of 4.5 lb (-4.5 lb) per week. At this rate, how much will he lose in 8 weeks? Express this weight loss as a signed number.

- F. In order to attract business, the Family grocery store ran a “loss leader” sale last week. The store sold eggs at a loss of \$0.23 ($-\0.23) per dozen. If 238 dozen eggs were sold last weekend, what was the total loss from the sale of eggs? Express this loss as a signed number.

$$238(-0.23) = -54.74$$

To find the total loss, multiply the loss per dozen by the number of dozens sold.

Therefore, the loss, written as a signed number, is $-\$54.74$.

How & Why

OBJECTIVE 2

Multiply two negative numbers.

We use the product of a positive and a negative number to develop a pattern for multiplying two negative numbers.

$$\begin{aligned}-3(4) &= -12 \\-3(3) &= -9 \\-3(2) &= -6 \\-3(1) &= -3 \\-3(0) &= 0 \\-3(-1) &= ? \\-3(-2) &= ?\end{aligned}$$

Each product is three larger than the one before it. Continuing this pattern,

$$\begin{aligned}-3(-1) &= 3 && \text{Because } 0 + 3 = 3. \\-3(-2) &= 6 && \text{Because } 3 + 3 = 6.\end{aligned}$$

In each case the product is positive.

To multiply two negative numbers

- Find the product of the absolute values.
- Make this product positive.

The product of two like signs is positive. When multiplying more than two signed numbers, if there is an even number of negative factors, the product is positive.

Answers to Warm-Ups

D. $-\frac{1}{3}$ E. -108

F. James's weight loss will be -36 lb.

Examples G–M



BASIC Mathematics Now™

DIRECTIONS: Multiply.

STRATEGY: Multiply the absolute values; make the product positive.

G. Multiply: $-11(-7)$

$$-11(-7) = 77 \quad \text{The product of two negative numbers is positive.}$$

H. Multiply: $-5.2(-0.32)$

$$-5.2(-0.32) = 1.664 \quad \text{The product of two numbers with like signs is positive.}$$

I. Find the product: $-98(-7)$

$$-98(-7) = 686$$

J. Find the product of $-\frac{2}{7}$ and $-\frac{3}{8}$.

$$\left(-\frac{2}{7}\right)\left(-\frac{3}{8}\right) = \frac{3}{28} \quad \text{Negative times negative is positive.}$$

So the product is $\frac{3}{28}$.

K. Multiply: $-16(-3)(-5)$

$$\begin{aligned} -16(-3)(-5) &= 48(-5) && \text{Multiply the first two factors.} \\ &= -240 && \text{Multiply again.} \end{aligned}$$

L. Find the product of $-3, 12, 3, -1$, and -5 .

STRATEGY: There is an odd number of negative factors, therefore the product is negative.

$$(-3)(12)(3)(-1)(-5) = -540$$

The product is -540 .

CALCULATOR EXAMPLE



M. Multiply: $-82(-9.6)(-12.9)$

The product is $-10,154.88$.

Warm-Ups G–M

G. Multiply: $-12(-8)$

H. Multiply: $-3.6(-2.7)$

I. Find the product: $-111(-6)$

J. Find the product of $-\frac{7}{12}$ and $-\frac{8}{25}$.

K. Multiply: $-13(-3)(-4)$

L. Find the product of $-2, 11, 4, -2$, and -3 .

M. Multiply:
 $-63(-4.9)(-13.5)$

Answers to Warm-Ups

- G. 96 H. 9.72 I. 666
J. $\frac{14}{75}$ K. 456 L. 528
M. -4167.45

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Exercises 8.4

OBJECTIVE 1

Multiply a positive number and a negative number.

A *Multiply.*

1. $-2(4)$

2. $4(-5)$

3. $-5(2)$

4. $(-6)(8)$

5. $10(-8)$

6. $-13(5)$

7. $-11(7)$

8. $12(-3)$

9. The product of -5 and _____ is -55 .

10. The product of 8 and _____ is -72 .

B

11. $-9(34)$

12. $11(-23)$

13. $-17(15)$

14. $23(-18)$

15. $2.5(-3.6)$

16. $-3.4(2.7)$

17. $-0.35(1000)$

18. $2.57(-10,000)$

19. $-\frac{2}{3} \cdot \frac{3}{8}$

20. $\left(\frac{3}{8}\right)\left(-\frac{4}{5}\right)$

OBJECTIVE 2

Multiply two negative numbers.

A *Multiply.*

21. $(-1)(-3)$

22. $(-2)(-4)$

23. $-7(-4)$

24. $-6(-5)$

25. $-11(-9)$

26. $(-4)(-3)$

27. $-12(-5)$

28. $-6(-16)$

29. The product of -6 and _____ is 66 .

30. The product of -13 and _____ is 39 .

B

31. $-14(-15)$

32. $-23(-17)$

33. $-1.2(-4.5)$

34. $-0.9(-0.72)$

35. $(-5.5)(-4.4)$

36. $(-6.3)(-2.3)$

37. $\left(-\frac{3}{14}\right)\left(-\frac{7}{9}\right)$

38. $\left(-\frac{8}{15}\right)\left(-\frac{5}{24}\right)$

39. $(-0.35)(-4.7)$

40. $(-7.2)(-2.1)$

C

41. $(-56)(45)$

42. $(16)(-32)$

43. $(15)(31)$

44. $(-23)(71)$

45. $(-1.4)(-5.1)$

46. $(-2.4)(6.1)$

47. $\left(-\frac{9}{16}\right)\left(\frac{8}{15}\right)$

48. $\left(-\frac{8}{21}\right)\left(-\frac{7}{16}\right)$

49. $(-4.01)(3.5)$

50. $(-6.7)(-0.45)$

51. $(-3.19)(-1.7)(0.1)$

52. $-1.3(4.6)(-0.2)$

53. $-2(4)(-1)(0)(-5)$

54. $(-4)(-7)(3)(-8)(0)$

55. $-0.07(0.3)(-10)(100)$

56. $(0.3)(-0.05)(-10)(-10)$

57. $-2(-5)(-6)(-4)(-1)$

58. $9(-2)(-3)(-5)(-4)$

59. $\left(-\frac{2}{3}\right)\left(-\frac{3}{4}\right)\left(-\frac{4}{5}\right)\left(-\frac{5}{6}\right)$

60. $\left(-\frac{5}{12}\right)\left(\frac{7}{8}\right)\left(\frac{3}{14}\right)\left(-\frac{8}{15}\right)$

61. The formula for converting a temperature measurement from Fahrenheit to Celsius is $C = \frac{5}{9}(F - 32)$. What Celsius measure is equal to 5°F?

63. While on a diet for 8 consecutive weeks, Ms. Riles averages a weight loss of 2.6 lb each week. If each loss is represented by -2.6 lb, what is her total loss for the 8 weeks, expressed as a signed number?

65. The Dow Jones Industrial Average sustains 12 straight days of a 1.74 decline. What is the total decline during the 12-day period, expressed as a signed number?

Simplify.

67. $(15 - 8)(5 - 12)$

68. $(17 - 20)(5 - 9)$

69. $(15 - 21)(13 - 6)$

70. $(25 - 36)(5 - 9)$

71. $(-12 + 30)(-4 - 10)$

72. $(11 - 18)(-13 + 5)$

73. Safeway Inc. offers as a loss leader 10 lb of sugar at a loss of 17¢ per bag (-17 ¢). If 386 bags are sold during the sale, what is the total loss, expressed as a signed number?

75. Winn Dixie's loss leader is a soft drink on which the store loses 6¢ per six-pack. They sell 523 of these six-packs. What is Winn Dixie's total loss expressed as a signed number?

62. Use the formula in Exercise 61 to find the Celsius measure that is equal to 86°F.

64. Mr. Riles goes on a diet for 8 consecutive weeks. He averages a loss of 3.2 lb per week. If each loss is represented by -3.2 lb, what is his total loss expressed as a signed number?

66. The Dow Jones Industrial Average sustains 7 straight days of a 2.33 decline. What is the total decline during the 7-day period, expressed as a signed number?

74. Albertsons offers a loss leader of coffee at a loss of 23¢ per 3-lb tin (-23 ¢). If they sell 412 tins of coffee, find the total loss expressed as a signed number.

76. Kroger's loss leader is soap powder on which the store loses 28¢ per carton. They sell 264 cartons. What is Kroger's total loss expressed as a signed number?

- 77.** Safeway's loss leader is 1 dozen eggs on which the store loses 31¢ per dozen. The store sells 809 dozen eggs that week. Express Safeway's total loss as a signed number.
- 79.** A certain junk bond trader purchased 670 shares of stock at 9.34. When she sold her shares, the stock sold for 6.45. What did she pay for the stock? How much money did she receive when she sold this stock? How much did she lose or gain? Represent the loss or gain as a signed number.
- 78.** A scientist is studying movement of a certain spider within its web. Any movement up is considered to be positive, whereas any movement down is negative. Determine the net movement of a spider that goes up 2 cm five times and down 3 cm twice.
- 80.** A company bought 450 items at \$1.23 each. They tried to sell them for \$2.35 and sold only 42 of them. They lowered the price to \$2.10 and sold 105 more. The price was lowered a second time to \$1.35 and 85 were sold. Finally they advertised a close-out price of \$0.95 and sold the remaining items. Determine the net profit or loss for each price. Did they make a profit or lose money on this item overall?

Exercises 81–82 refer to the chapter application. See page 673.

- 81.** Which continent has a low point that is approximately 10 times the low point of South America?
- 82.** Which continent has a high point that is approximately twice the absolute value of the lowest point?

STATE YOUR UNDERSTANDING

- 83.** Explain why the product of an even number of negative numbers is positive.
- 84.** Explain the procedure for multiplying two signed numbers.

CHALLENGE

Simplify.

- 85.** $| -(-5)| (-9 - [-(-5)])$
- 86.** $| -(-8)| (-8 - [-(-9)])$
- 87.** Find the product of -8 and the opposite of 7 .
- 88.** Find the product of the opposite of 12 and the absolute value of -9 .

GROUP WORK

- 89.** Throughout the years, mathematicians have used a variety of examples to show students that the product of two negative numbers is positive. Talk to science and math instructors and record their favorite explanation. Discuss your findings in class.

MAINTAIN YOUR SKILLS*Divide.*

90. $66 \div 11$

91. $816 \div 12$

92. $\frac{1005}{15}$

93. $54 \overline{)14,472}$

94. $34 \overline{)4876}$, round to the nearest hundredth

95. What is the equivalent piecework wage (dollars per piece) if the hourly wage is \$15.86 and the average number of articles completed in 1 hr is 6.1?
97. How far does the tip of the hour hand of a clock travel in 6 hr if the length of the hand is 3 in.? Let $\pi \approx 3.14$.
99. A mini-storage complex has one unit that is 40 ft by 80 ft and rents for \$1800 per year. What is the cost of a square foot of storage for a year?

96. If carpeting costs \$34.75 per square yard installed, what is the cost of wall-to-wall carpeting needed to cover the floor in a rectangular room that is 24 ft wide by 27 ft long?
98. How many square feet of sheet metal are needed to make a box without a top that has measurements of 5 ft 6 in. by 4 ft 6 in. by 9 in.?

8.5 Dividing Signed Numbers

How & Why

OBJECTIVE 1

Divide a positive number and a negative number.

To divide two signed numbers, we find the number that when multiplied times the divisor equals the dividend. The expression $-9 \div 3 = ?$ asks $3(?) = -9$; we know $3(-3) = -9$, so $-9 \div 3 = -3$. The expression $24 \div (-6) = ?$ asks $-6(?) = 24$; we know $-6(-4) = 24$, so $24 \div (-6) = -4$.

When dividing unlike signs, we see that the quotient is negative. We use these examples to state how to divide a negative number and a positive number.

OBJECTIVES

1. Divide a positive number and a negative number.
2. Divide two negative numbers.

To divide a positive and a negative number

1. Find the quotient of the absolute values.
2. Make the quotient negative.

Examples A–D



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DIRECTIONS: Divide.

STRATEGY: Divide the absolute values and make the quotient negative.

A. Divide: $32 \div (-8)$

$32 \div (-8) = -4$ The quotient of two numbers with unlike signs is negative.

B. Divide: $(-4.8) \div 3.2$

$(-4.8) \div 3.2 = -1.5$ When dividing unlike signs, the quotient is negative.

C. Divide: $\left(\frac{4}{35}\right) \div \left(-\frac{4}{5}\right)$

$$\left(\frac{4}{35}\right) \div \left(-\frac{4}{5}\right) = \left(\frac{\cancel{4}}{\cancel{35}}\right) \cdot \left(-\frac{\cancel{4}}{\cancel{5}}\right) \quad \text{Multiply by the reciprocal.}$$
$$= -\frac{1}{7} \quad \text{The product is negative.}$$

D. Over a period of 18 weeks, Mr. Rich loses a total of \$4230 ($-\4230) in his stock market account. What is his average loss per week, expressed as a signed number?

STRATEGY: To find the average loss per week, divide the total loss by the number of weeks.

$$-4230 \div 18 = -235$$

Mr. Rich has an average loss of \$235 ($-\235) per week.

Warm-Ups A–D

A. Divide: $72 \div (-4)$

B. Divide: $(-4.9) \div 1.4$

C. Divide: $\left(\frac{3}{8}\right) \div \left(-\frac{3}{4}\right)$

D. Ms. Rich loses \$6225 in her stock market account over a period of 15 weeks. What is her average loss per week, expressed as a signed number?

Answers to Warm-Ups

- A. -18 B. -3.5 C. $-\frac{1}{2}$
D. Ms. Rich lost \$415 ($-\415) per week.

How & Why

OBJECTIVE 2

Divide two negative numbers.

To determine how to divide two negative numbers, we again use the relationship to multiplication.

The expression $-21 \div (-7) = ?$ asks $(-7)(?) = -21$; we know that $-7(3) = -21$, so $-21 \div (-7) = 3$. The expression $-30 \div (-6) = ?$ asks $(-6)(?) = -30$; we know that $-6(5) = -30$, so $-30 \div (-6) = 5$. We see that in each case, when dividing two negative numbers, the quotient is positive. These examples lead us to the following rule.

To divide two negative numbers

1. Find the quotient of the absolute values.
2. Make the quotient positive.

Warm-Ups E–G

E. Find the quotient:
 $-48 \div (-6)$

F. Find the quotient:
 $-22.75 \div (-2.6)$

G. Divide:
 $\left(-\frac{9}{30}\right)$ by $\left(-\frac{18}{25}\right)$

Examples E–G



BASIC Mathematics Now™

DIRECTIONS: Divide.

STRATEGY: Find the quotient of the absolute values.

E. Find the quotient: $-44 \div (-11)$
 $-44 \div (-11) = 4$ The quotient of two negative numbers is positive.

CALCULATOR EXAMPLE



F. Find the quotient: $-7.74 \div (-3.6)$
 $-7.74 \div (-3.6) = 2.15$

G. Divide: $\left(-\frac{11}{9}\right)$ by $\left(-\frac{22}{27}\right)$

$$\left(-\frac{11}{9}\right) \div \left(-\frac{22}{27}\right) = \left(-\frac{11}{9}\right) \left(\frac{27}{22}\right)$$

$\frac{1}{9}$ $\frac{3}{2}$ Invert and multiply.
 $\frac{1}{2}$

$$= \frac{3}{2}$$

Simplify.

Answers to Warm-Ups

E. 8 F. 8.75 G. $\frac{5}{12}$

Exercises 8.5

OBJECTIVE 1

Divide a positive number and a negative number.

A Divide.

1. $-10 \div 5$ 2. $10 \div (-2)$ 3. $-16 \div 4$ 4. $15 \div (-3)$
 5. $18 \div (-6)$ 6. $-18 \div 3$ 7. $24 \div (-3)$ 8. $-33 \div 11$
 9. The quotient of -48 and _____ is -6 . 10. The quotient of 70 and _____ is -14 .

B

11. $72 \div (-12)$ 12. $84 \div (-12)$ 13. $6.06 \div (-3)$
 14. $3.05 \div (-5)$ 15. $-210 \div 6$ 16. $-315 \div 9$
 17. $\left(-\frac{6}{7}\right) \div \frac{2}{7}$ 18. $\left(-\frac{4}{3}\right) \div \frac{8}{3}$ 19. $0.75 \div (-0.625)$
 20. $0.125 \div (-0.625)$

OBJECTIVE 2

Divide two negative numbers.

A Divide.

21. $-10 \div (-5)$ 22. $-10 \div (-2)$ 23. $-12 \div (-4)$ 24. $-14 \div (-2)$
 25. $-28 \div (-4)$ 26. $-32 \div (-4)$ 27. $-54 \div (-9)$ 28. $-63 \div (-7)$
 29. The quotient of -105 and _____ is 21 . 30. The quotient of -75 and _____ is 15 .

B

31. $-98 \div (-14)$ 32. $-88 \div (-11)$ 33. $-96 \div (-12)$
 34. $-210 \div (-10)$ 35. $-12.12 \div (-3)$ 36. $-18.16 \div (-4)$
 37. $\left(-\frac{3}{8}\right) \div \left(-\frac{3}{4}\right)$ 38. $\left(-\frac{1}{2}\right) \div \left(-\frac{5}{8}\right)$ 39. $-0.65 \div (-0.13)$
 40. $-0.056 \div (-0.4)$

C

41. $-540 \div 12$ 42. $-1071 \div 17$ 43. $-3364 \div (-29)$
 44. $-4872 \div (-48)$ 45. $3.735 \div (-0.83)$ 46. $-2.352 \div (-0.42)$

47. $0 \div (-35)$

48. $-85 \div 0$

49. $-0.26 \div 100$

50. $-0.56 \div (-100)$

51. $\frac{-16,272}{36}$

52. $\frac{-34,083}{-63}$

53. Find the quotient of -384 and -24 .54. Find the quotient of -357 and 21 .

55. The membership of the Burlap Baggers Investment Club takes a loss of $\$753.90$ ($-\$753.90$) on the sale of stock. If there are six co-equal members in the club, what is each member's share of the loss, expressed as a signed number?

56. The temperature in Nome, Alaska, drops from 10° above zero ($+10^\circ$) to 22° below zero (-22°) in an 8-hour period. What was the average drop in temperature per hour, expressed as a signed number?

57. Mr. Harkness loses a total of 115 lb in 25 weeks. Express the average weekly loss as a signed number.

58. Ms. Harkness loses a total of 65 lb in 25 weeks. Express the average weekly loss as a signed number.

59. A certain stock loses 45.84 points in 12 days. Express the average daily loss as a signed number.

60. A certain stock loses 31.59 points in 9 days. Express the average daily loss as a signed number.

61. Determine the population of Los Angeles in 1995, 2000, and 2005. Determine the population of your city in 1995, 2000, and 2005. Find the average yearly loss or gain for each 5 years and also for the 10-year period for each city (written as a signed number). List the possible reasons for these changes.

62. Central Electronics lost $\$967,140$ during one 20-month period. Determine the average monthly loss (written as a signed number). If there are 30 stockholders in this company, determine the total loss per stockholder (written as a signed number).

Exercises 63–64 refer to the chapter application. See page 673.

63. Which continent has a high point that is approximately one-fourth of the height of Mt. Everest?

64. Which continent has a low point that is approximately one-fourth the low point of Africa?

STATE YOUR UNDERSTANDING

65. The sign rules for multiplication and division of signed numbers may be summarized as follows:

If the numbers have the same sign, the answer is positive.

If the numbers have different signs, the answer is negative.

Explain why the rules for division are the same as the rules for multiplication.

66. When dividing signed numbers, care must be taken not to divide by zero. Why?

CHALLENGE*Simplify.*

67. $[-|-10|(6 - 11)] \div [(8 - 13)(11 - 10)]$

68. $[(14 - 20)(-5 - 9)] \div [-(-12)(-8 + 7)]$

69. $\left(-\frac{5}{6} - \frac{1}{2}\right)\left(-\frac{2}{3} + \frac{1}{6}\right) \div \left(\frac{1}{3} - \frac{3}{4}\right)$

70. $\left(-\frac{1}{3} - \frac{1}{4}\right)\left(-\frac{1}{3} + \frac{1}{6}\right) \div \left(\frac{1}{3} - \frac{3}{4}\right)$

71. $(-0.82 - 1.28)(1.84 - 2.12) \div [3.14 + (-3.56)]$

GROUP WORK

72. Determine the temperature on the first day of each month over a 12-month period in one city in Alaska, one city in Canada, and one city in Hawaii. Find the average monthly changes in temperature for each city.

Find the averages for the temperatures on the first day of the month for each city. Make a chart or graph from these data. Be sure no two groups choose the same cities. Compare the results with your classmates.

MAINTAIN YOUR SKILLS*Simplify.*

73. $16 \div 4 + 8 - 5$

74. $75 \div 3 \cdot 5 + 3 - (5 - 2)$

75. $14 - 3^2 + 8 - 3 \cdot 2 + 11$

76. $(17 - 3 \cdot 5)^3 + [16 - (19 - 2 \cdot 8)]$

77. Find the volume of a cylinder that has a radius of 8 in. and a height of 24 in. (Let $\pi \approx 3.14$.)

78. Find the volume of a cone that has a radius of 12 in. and a height of 9 in. (Let $\pi \approx 3.14$.)

79. An underground gasoline storage tank is a cylinder that is 72 in. in diameter and 18 ft long. If there are 231 in³ in a gallon, how many gallons of gasoline will the tank hold? (Let $\pi \approx 3.14$.) Round the answer to the nearest gallon.
81. To remove the dirt from the swimming pool in Exercise 80, trucks that can haul 8 yd³ per load are used. How many truckloads will there be?

80. A swimming pool is to be dug and the dirt hauled away. The pool is to be 27 ft long, 16 ft wide, and 6 ft deep. How many cubic yards of dirt must be removed?

82. A real-estate broker sells a lot that measures 88.75 ft by 180 ft. The sale price is \$2 per square foot. If the broker's commission is 8%, how much does she make?

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8.6 Order of Operations: A Review

How & Why

OBJECTIVE

Do any combination of operations with signed numbers.

The order of operations for signed numbers is the same as that for whole numbers, fractions, and decimals.

To evaluate expressions with more than one operation

- Step 1. Parentheses**—Do the operations within grouping symbols first (parentheses, fraction bar, etc.), in the order given in steps 2, 3, and 4.
- Step 2. Exponents**—Do the operations indicated by exponents.
- Step 3. Multiply and Divide**—Do multiplication and division as they appear from left to right.
- Step 4. Add and Subtract**—Do addition and subtraction as they appear from left to right.

Examples A–G



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DIRECTIONS: Perform the indicated operations.

STRATEGY: Follow the order of operations.

A. Perform the indicated operations: $-63 + (-21) \div 7$

$$-63 + (-21) \div 7 = -63 + (-3) \quad \text{Divide first.}$$
$$= -66$$

B. Perform the indicated operations: $(-15)(-3) - 44 \div (-11)$

$$(-15)(-3) - 44 \div (-11) = 45 - (-4) \quad \text{Multiply and divide first.}$$
$$= 45 + 4 \quad \text{Add the opposite of } -4.$$
$$= 49$$

C. Perform the indicated operations: $12 \div (-0.16) + 3(-1.45)$

$$12 \div (-0.16) + 3(-1.45) = -75 + (-4.35) \quad \text{Multiply and divide first.}$$
$$= -79.35$$

D. Perform the indicated operations: $10 - \left(\frac{3}{5}\right)(-20)$

$$10 - \left(\frac{3}{5}\right)(-20) = 10 - (-12) \quad \text{Add the opposite of } -12.$$
$$= 10 + 12$$
$$= 22$$

E. Perform the indicated operations: $3(-5)^2 - 3^2 + 7(-4)^2$

$$3(-5)^2 - 3^2 + 7(-4)^2 = 3(25) - 9 + 7(16) \quad \text{Do exponents first.}$$
$$= 75 - 9 + 112$$
$$= 178$$

OBJECTIVE

Do any combination of operations with signed numbers.

Warm-Ups A–G

A. Perform the indicated operations:
 $-56 + (-32) \div 4$

B. Perform the indicated operations:
 $(-8)(5) - 72 \div (-12)$

C. Perform the indicated operations:
 $9 \div (-0.15) + 6(-2.15)$

D. Perform the indicated operations:
 $15 - \left(\frac{7}{12}\right)(-36)$

E. Perform the indicated operations:
 $(-6)(-3)^2 + 43 - (-5)^2$

Answers to Warm-Ups

- A. 64 B. 34 C. 72.9
D. 36 E. 36

CALCULATOR EXAMPLE

- F. Perform the indicated operations:
 $(-28)(14) - 225 \div (-3)$
- G. How many degrees Celsius is -4°F ?

- F. Perform the indicated operations: $(-18)(23) - (-84) \div (-7)$

The result is -426 .

- G. Hilda keeps the thermostat on her furnace set at 68°F . Her pen pal in Germany says that her thermostat is set at 20°C . They wonder whether the two temperatures are equal.

STRATEGY: To find out whether $68^{\circ}\text{F} = 20^{\circ}\text{C}$, substitute 68 for F in the formula.

$$C = \frac{5}{9}(F - 32) \quad \text{Formula}$$

$$C = \frac{5}{9}(68 - 32)$$

$$C = \frac{5}{9}(36)$$

$$C = 20$$

Therefore, 68°F equals 20°C .

Answers to Warm-Ups

F. -317

G. The Celsius temperature is -20°C .

Exercises 8.6

OBJECTIVE

Do any combination of operations with signed numbers.

A Perform the indicated operations.

- 1.** $2(-7) - 10$ **2.** $13 + 4(-5)$ **3.** $(-2)(-4) + 11$
4. $15 + (-3)(-5)$ **5.** $3(-6) + 12$ **6.** $(-5)(6) + 19$
7. $-7 + 3(-3)$ **8.** $-14 + (-6)2$ **9.** $(-3)8 \div 4$
10. $(-8)6 \div 3$ **11.** $(-4)8 \div (-4)$ **12.** $(-7)12 \div (-6)$
13. $(-8) \div 4(-2)$ **14.** $(-18) \div 3(2)$ **15.** $(-3)^2 + (-2)^2$
16. $6^2 - 4^2$ **17.** $(11 - 3) + (9 - 6)$ **18.** $(4 - 9) + (5 - 7)$
19. $(3 - 5)(6 - 10)$ **20.** $(8 - 5)(11 - 15)$ **21.** $(-3)^2 + 4(-2)$
22. $(-2)^2 - 4(-2)$ **23.** $-5 + (6 - 8) - 5(3)$ **24.** $-7 + (5 - 11) - 4(2)$

B

- 25.** $(-13)(-2) + (-16)2$ **26.** $(-16)(-5) + (-14)5$
27. $(9 - 7)(-2 - 5) + (15 - 9)(2 + 7)$ **28.** $(10 - 15)(-4 - 3) + (12 - 7)(3 + 2)$
29. $7(-11 + 5) - 44 \div (-11)$ **30.** $8(-7 + 4) - 54 \div (-9)$
31. $18(-2) \div (-6) - 14$ **32.** $(-4)(-9) \div (-12) + 11$
33. $-120 \div (-20) - (9 - 11)$ **34.** $-135 \div (-15) - (12 - 17)$

- 35.** $-2^3 - (-2)^3$ **36.** $-4^3 - 4^3$
37. $-35 \div 7(-5) - 7^2$ **38.** $-28 \div 7(-4) - 7^2$
39. $2^2(5 - 4)(7 - 3)^2$ **40.** $3^2(8 - 6)(6 - 8)^2$
41. $(9 - 13) + (-5)(-2) - (-2)5 - 3^3$ **42.** $(8 - 11) - (7)(-3) + (-4)(3) - 2^2$

- 43.** $(-3)(-2)(-3) - (-4)(-3) - (3)(-5)$ **44.** $(-5)(-6)(-1) - (-3)(-6) - (-5)(2)$
45. $(-1)(-6)^2(-2) - (-3)^2(-2)^3$ **46.** $(-1)(-3)^3(-4) - (-4)^2(-2)^2$

C

- 47.** Find the sum of the product of 12 and -4 and the product of -3 and -12 .
48. Find the difference of the product of 3 and 9 and the product of -8 and 3.

Exercises 49–52 refer to the following table, which shows temperatures a satellite recorded during a 5-day period at one location on the surface of Mars.

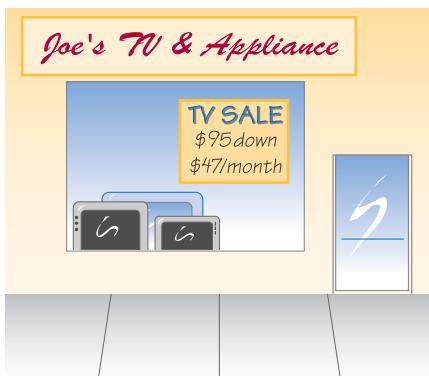
Temperatures on the Surface of Mars

	Day 1	Day 2	Day 3	Day 4	Day 5
5:00 A.M.	-92°C	-88°C	-115°C	-103°C	-74°C
9:00 A.M.	-57°C	-49°C	-86°C	-93°C	-64°C
1:00 P.M.	-52°C	-33°C	-46°C	-48°C	-10°C
6:00 P.M.	-45°C	-90°C	-102°C	-36°C	-42°C
11:00 P.M.	-107°C	-105°C	-105°C	-98°C	-90°C

49. What is the average temperature recorded during day 5? 50. What is the average temperature recorded at 6:00 P.M.?
51. What is the average high temperature recorded for the 5 days? 52. What is the average low temperature recorded for the 5 days?

Simplify.

53. $[-3 + (-6)]^2 - [-8 - 2(-3)]^2$ 54. $[-5(-9) - (-6)^2]^2 + [(-8)(-1)^3 + 2]^2$
55. $[46 - 3(-4)^2]^3 - [-7(1)^3 + (-5)(-8)]$ 56. $[30 - (-5)^2]^2 - [-8(-2) - (-2)(-4)]^2$
57. $-15 - \frac{8^2 - (-4)}{3^2 + 3}$ 58. $-22 + \frac{9^2 - 6}{6^2 - 11}$
59. $\frac{12(8 - 24)}{5^2 - 3^2} \div (-12)$ 60. $\frac{15(12 - 45)}{6^2 - 5^2} \div (-9)$
61. $-8|125 - 321| - 21^2 + 8(-7)$ 62. $-9|482 - 632| - 17^2 + 9(-9)$
63. $-6(8^2 - 9^2)^2 - (-7)20$ 64. $-5(6^2 - 7^2)^2 - (-8)19$
65. Find the difference of the quotient of 28 and -7 and the product of -4 and -3 .
66. Find the sum of the product of -3 and 7 and the quotient of -15 and -5 .
67. Keshia buys a TV for \$95 down and \$47 per month for 15 months. What is the total price she pays for the TV? (*Hint:* When a deferred payment plan is used, the total cost of the article is the down payment plus the total of the monthly payments.)
68. The E-Z Chair Company advertises recliners for \$40 down and \$17 per month for 24 months. What is the total cost of the recliner?



- 69.** During an “early bird” special, K-Mart sold 24 fishing poles at a loss of \$3 per pole. During the remainder of the day, they sold 9 poles at a profit of \$7 per pole. Express the profit or loss on the sale of fishing poles as a signed number.

Exercises 71–74 refer to the chapter application. See page 673.

- 71.** For each continent, calculate the average of the highest and lowest points. Which continent has the largest average, and which has the smallest average?

- 70.** Fly America sells 40 seats on Flight 402 at a loss of \$52 per seat ($-\52). Fly America also sells 67 seats at a profit of \$78 per seat. Express the profit or loss on the sale of the seats as a signed number.

The following table gives the altitudes of selected cities around the world.

Altitudes of Cities

City	Altitude (ft)	City	Altitude (ft)
Athens, Greece	300	Mexico City, Mexico	7347
Bangkok, Thailand	0	New Delhi, India	770
Berlin, Germany	110	Quito, Ecuador	9222
Bogota, Columbia	8660	Rome, Italy	95
Jakarta, Indonesia	26	Tehran, Iran	5937
Jerusalem, Israel	2500	Tokyo, Japan	30

- 72.** Find a group of five cities with an average altitude of less than 100 ft.
- 74.** Find a group of four cities with an average altitude of approximately 7000 ft.

- 73.** Find a group of three cities with an average altitude of approximately 350 ft.

- 75.** The treasurer of a local club records the following transactions during one month:

Opening Balance	\$4756
Deposit	\$345
Check #34	\$212
Check #35	\$1218
New check cost	\$15
Deposit	\$98
National dues paid	\$450
Electric bill	\$78

What is the balance at the end of the month?

- 76.** The Chicago Bears made the following plays during a quarter of a game:
- 3 plays lost 8 yd each
 - 8 plays lost 5 yd each
 - 1 quarterback sack lost 23 yd
 - 1 pass for 85 yd
 - 5 plays gained 3 yd each
 - 2 plays gained 12 yd each
 - 1 fumble lost 7 yd
 - 2 passes for 10 yd each

Determine the average movement per play during this quarter. Round to the nearest tenth.

- 77.** Consider traveling north and east as positive values and traveling south and west as negative values. A certain trip requires going 81 miles north, followed by 67 miles west. The next day the trip requires traveling 213 miles south, followed by 107 miles west. The last day of the trip requires traveling 210 miles north and 83 miles east. Determine your position at the end of your trip in relation to your starting point.

- 78.** Try this game on your friends. Have them pick a number. Tell them to double it, then add 20 to that number, divide the sum by 4, subtract 5 from that quotient,

square the difference, and multiply the square by 4. They should now have the square of the original number. Write a mathematical representation of this riddle.

STATE YOUR UNDERSTANDING

Locate the error in Exercises 79 and 80. Indicate why each is not correct. Determine the correct answer.

$$\begin{aligned} 79. \quad 2[3 + 5(-4)] &= 2[8(-4)] \\ &= 2[-32] \\ &= -64 \end{aligned}$$

$$\begin{aligned} 80. \quad 3 - [5 - 2(6 - 4^2)^3] &= 3 - [5 - 2(6 - 16)^3] \\ &= 3 - [5 - 2(-10)^3] \\ &= 3 - [5 - (-20)^3] \\ &= 3 - [5 - (-8000)] \\ &= 3 - [5 + 8000] \\ &= 3 - 8005 \\ &= -8002 \end{aligned}$$

- 81.** Is there ever a case when exponents are not computed first? If so give an example.

CHALLENGE

Simplify.

$$82. \frac{3^2 - 5(-2)^2 + 8 + [4 - 3(-3)]}{4 - 3(-2)^3 - 18}$$

$$83. \frac{(5 - 9)^2 + (-6 + 8)^2 - (14 - 6)^2}{[3 - 4(7) + 3^3]^2}$$

$$84. \frac{3(4 - 7)^2 + 2(5 - 8)^3 - 18}{(6 - 9)^2 + 6}$$

GROUP WORK

- 85.** Engage the entire class in a game of KRYPTO. This card game consists of 41 cards numbered from -20 to 20. Each group gets four cards. A card is chosen at random from the remaining cards and the number is put

on the board. Each group must find a way using addition, subtraction, multiplication, and/or division to combine their given cards to equal the number on the board. Operations may be used more than once.

MAINTAIN YOUR SKILLS*Add.*

86. $(-17.2) + (-18.6) + (-2.7) + 9.1$

87. $(28.31) + (-8.14) + (-21.26) + (-16)$

Subtract.

88. $48 - (-136)$

89. $-62.7 - (-78.8)$

Multiply.

90. $(-36)(84)(-21)$

91. $(-62)(-22)(-30)$

Divide.

92. $(-800) \div (-32)$

93. $(-25,781) \div 3.5$

- 94.** The four Zapple brothers form a company. The first year, the company loses \$5832 ($-\5832). The brothers share equally in the loss. Represent each brother's loss as a signed number.

- 95.** AVI Biopharma stock recorded the following gains and losses for the week:

Monday	loss 0.34
Tuesday	loss 0.54
Wednesday	gain 1.32
Thursday	gain 0.67
Friday	loss 0.672

Use signed numbers to find out whether the stock gains or loses for the week.

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8.7 Solving Equations

VOCABULARY

Recall that the **coefficient** of the variable is the number that is multiplied times the variable.

OBJECTIVE

Solve equations of the form $ax + b = c$ or $ax - b = c$, where a , b , and c are signed numbers.

How & Why

OBJECTIVE

Solve equations of the form $ax + b = c$ or $ax - b = c$, where a , b , and c are signed numbers.

Note: Before starting this section, you may want to review Getting Ready for Algebra sections in earlier chapters.

The process of solving equations that are of the form $ax + b = c$ and $ax - b = c$, using signed numbers, involves two operations to isolate the variable. To isolate the variable is to get an equation in which the variable is the only symbol on a particular side of the equation.

To find the solution of an equation of the form $ax + b = c$ or $ax - b = c$

1. Add (subtract) the constant to (from) each side of the equation.
2. Divide both sides by the coefficient of the variable.

Examples A–C



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DIRECTIONS: Solve.

STRATEGY: First, add or subtract the constant to or from both sides of the equation. Second, divide both sides of the equation by the coefficient of the variable.

A. Solve: $-6x + 23 = -7$

$$\begin{aligned} -6x + 23 &= -7 && \text{Original equation} \\ -6x + 23 - 23 &= -7 - 23 && \text{Subtract.} \\ -6x &= -30 \\ \frac{-6x}{-6} &= \frac{-30}{-6} && \text{Divide.} \\ x &= 5 \end{aligned}$$

CHECK: Substitute 5 for x in the original equation.

$$\begin{aligned} -6(5) + 23 &= -7 \\ -30 + 23 &= -7 \\ -7 &= -7 \end{aligned}$$

The solution is $x = 5$.

Warm-Ups A–C

A. Solve: $-7x + 12 = -23$

Answers to Warm-Ups

A. $x = 5$

B. Solve: $37 = -13x - 15$

B. Solve: $-43 = -14x - 71$

$$\begin{array}{rcl} -43 & = & -14x - 71 & \text{Original equation} \\ -43 + 71 & = & -14x - 71 + 71 & \text{Add.} \\ 28 & = & -14x \\ \frac{28}{-14} & = & \frac{-14x}{-14} & \text{Divide.} \\ -2 & = & x \end{array}$$

CHECK: Substitute -2 in the original equation.

$$\begin{aligned} -43 &= -14(-2) - 71 \\ -43 &= 28 - 71 \\ -43 &= 28 + (-71) \\ -43 &= -43 \end{aligned}$$

The solution is $x = -2$.

C. $6x - 45 = -87$

C. Solve: $7x - 32 = -88$

$$\begin{array}{rcl} 7x - 32 & = & -88 & \text{Original equation} \\ + 32 & & + 32 & \text{Add 32 to both sides.} \\ \hline 7x & = & -56 \\ \frac{7x}{7} & = & \frac{-56}{7} & \text{Divide both sides by 7.} \\ x & = & -8 & \text{The check is left for the student.} \end{array}$$

The solution is $x = -8$.

Answers to Warm-Ups

B. $x = -4$ C. $x = -7$

Exercises 8.7

OBJECTIVE

Solve equations of the form $ax + b = c$ or $ax - b = c$, where a , b , and c are signed numbers.

A Solve.

- | | | |
|------------------------------|---------------------------|----------------------------|
| 1. $-3x + 25 = 4$ | 2. $-4y + 11 = -9$ | 3. $-6 + 3x = 9$ |
| 4. $-11 + 5y = 14$ | 5. $4y - 9 = -29$ | 6. $3x - 13 = -43$ |
| 7. $2a - 11 = 3$ | 8. $5a + 17 = 17$ | 9. $-5x + 12 = -23$ |
| 10. $-11y - 32 = -65$ | 11. $4x - 12 = 28$ | 12. $9y - 14 = 4$ |

B

- | | | |
|-----------------------------|---------------------------|-----------------------------|
| 13. $-14 = 2x - 8$ | 14. $26 = 3x - 4$ | 15. $-40 = 5x - 10$ |
| 16. $-30 = -5x - 10$ | 17. $-6 = -8x - 6$ | 18. $9 = -5x + 9$ |
| 19. $-10 = -4x + 2$ | 20. $20 = -8x + 4$ | 21. $-14y - 1 = -99$ |
| 22. $-16x + 5 = -27$ | 23. $-3 = -8a - 3$ | 24. $-12 = 5b + 18$ |

C

- | | |
|---|--|
| 25. $-0.6x - 0.15 = 0.15$ | 26. $-1.05y + 5.08 = 1.72$ |
| 27. $0.03x + 2.3 = 1.55$ | 28. $0.02x - 2.4 = 1.22$ |
| 29. $-135x - 674 = 1486$ | 30. $94y + 307 = -257$ |
| 31. $-102y + 6 = 414$ | 32. $-63c + 22 = 400$ |
| 33. $\frac{1}{2}a - \frac{3}{8} = \frac{1}{40}$ | 34. $-\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$ |
| 35. If 98 is added to 6 times some number, the sum is 266.
What is the number? | |
| 36. If 73 is added to 11 times a number, the sum is -158 .
What is the number? | |
| 37. The difference of 15 times a number and 181 is -61 .
What is the number? | |
| 38. The difference of 24 times a number and 32 is -248 .
What is the number? | |
| 39. A formula for distance traveled is $2d = t^2a + 2v$,
where d represents distance, v represents initial velocity,
t represents time, and a represents acceleration.
Find a if $d = 240$, $v = -20$, and $t = 4$. | |
| 40. Use the formula in Exercise 39 to find a if $d = 240$,
$v = -35$, and $t = 5$. | |

- 41.** The formula for the balance of a loan (D) is $D = B - NP$, where P represents the monthly payment, N represents the number of payments, and B represents the money borrowed. Find N when $D = \$575$, $B = \$925$, and $P = \$25$.

- 42.** Use the formula in Exercise 41 to determine the monthly payment (P) if $D = \$820$, $B = \$1020$, and $N = 5$.

Exercises 43–45 refer to the chapter application. See page 673. Use negative numbers to represent feet below sea level.

- 43.** The high point of Australia is 12,558 ft more than 4 times the lowest point of one of the continents. Write an algebraic equation that describes this relationship. Which continent's lowest point fits the description?

- 44.** The lowest point of Antarctica is 2183 ft less than 12 times the lowest point of one of the continents. Write an algebraic equation that describes this relationship. Which continent's lowest point fits this description?

- 45.** The Mariana Trench in the Pacific Ocean is about 2000 ft deeper than twice one of the other ocean's deepest parts. Write an algebraic equation that describes this relationship. Which ocean's deepest part fits this description? See Exercise 63, Section 8.1.

Solve.

46. $5x + 12 + (-9) = 18$

47. $8z - 12 + (-6) = 38$

48. $-3b - 12 + (-4) = 11 + (-6)$

49. $-5z - 15 + 6 = -21 - 18$

STATE YOUR UNDERSTANDING

- 50.** Explain what it means to solve an equation.

- 51.** Explain how to solve the equation $-3x + 10 = 4$.

CHALLENGE

Solve.

52. $8x - 9 = 3x + 6$

53. $7x + 14 = 3x - 2$

54. $9x + 16 = 7x - 12$

55. $10x + 16 = 5x + 6$

GROUP WORK

- 56.** Bring in your last two electricity bills. Develop a formula for determining how your bills are computed. Share your group's synopsis with the class. Try to predict your next month's bill.

Key Concepts CHAPTER 8

Section 8.1 Opposites and Absolute Value

Definitions and Concepts	Examples
Positive numbers are greater than zero.	Positive numbers: $4, 7.31, \frac{5}{9}$
Negative numbers are written with a dash (-) and are less than zero.	Negative numbers: $-4, -7.31, -\frac{5}{9}$
The opposite of a signed number is the number that is the same distance from zero but has the opposite sign.	Opposites: 4 and $-4, 7.31$ and $-7.31, \frac{5}{9}$ and $-\frac{5}{9}$
The absolute value of a signed number is its distance from zero on a number line.	$ -5 = 5$ $ 5 = 5$

Section 8.2 Adding Signed Numbers

Definitions and Concepts	Examples
To add signed numbers: <ul style="list-style-type: none">• If the signs are alike, add their absolute values and use the common sign.• If the signs are unlike, subtract the smaller absolute value from the larger absolute value. The sum has the sign of the number with the larger absolute value.	$4 + 9 = 13$ $(-4) + (-9) = -13$ $-4 + 9 = 9 - 4 = 5$ $4 + (-9) = -(9 - 4) = -5$

Section 8.3 Subtracting Signed Numbers

Definitions and Concepts	Examples
To subtract signed numbers: <ul style="list-style-type: none">• Rewrite as an addition problem by adding the opposite of the number to be subtracted.• Find the sum.	$5 - 8 = 5 + (-8) = -3$ $-5 - 8 = -5 + (-8) = -13$ $5 - (-8) = 5 + 8 = 13$

Section 8.4 Multiplying Signed Numbers

Definitions and Concepts	Examples
To multiply signed numbers: <ul style="list-style-type: none"> Find the product of the absolute values. If there is an even number of negative factors, the product is positive. If there is an odd number of negative factors, the product is negative. 	$3(-9) = -27$ $(-3)(-9) = 27$ $(-2)(-2)(-2)(-2) = 16$ $(-4)(-3)(-2) = -24$

Section 8.5 Dividing Signed Numbers

Definitions and Concepts	Examples
To divide signed numbers: <ul style="list-style-type: none"> Find the quotient of the absolute values. If the signs are alike, the quotient is positive. If the signs are unlike, the quotient is negative. 	$-12 \div 6 = -2$ $(-12) \div (-6) = 2$ $12 \div (-6) = -2$

Section 8.6 Order of Operations: A Review

Definitions and Concepts	Examples
The order of operations for signed numbers is the same as that for whole numbers: <ul style="list-style-type: none"> Parentheses Exponents Multiplication/Division Addition/Subtraction 	$(-5)(-2)^3 + (-24) \div (6 - 8)$ $(-5)(-2)^3 + (-24) \div (-2)$ $(-5)(-8) + (-24) \div (-2)$ $40 + 12$ 52

Section 8.7 Solving Equations

Definitions and Concepts	Examples
To solve equations of the form $ax + b = c$ or $ax - b = c$: <ul style="list-style-type: none"> Add (or subtract) the constant to (from) both sides. Divide both sides by the coefficient of the variable. 	$\begin{aligned} 3x + 8 &= -6 \\ 3x + 8 - 8 &= -6 - 8 \\ 3x &= -14 \\ \frac{3x}{3} &= -\frac{14}{3} \\ x &= -\frac{14}{3} \end{aligned}$

Review Exercises

CHAPTER 8

Section 8.1

Find the opposite of the signed number.

1. 39

2. 57

3. -0.91

4. -0.134

Find the absolute value of the signed number.

5. $|46.5|$

6. $|-386|$

7. $|71|$

8. $|3.03|$

9. Find the opposite of $|-6.4|$.

10. If 93 miles north is represented by $+93$ miles, how would you represent 93 miles south?

Section 8.2

Add.

11. $-75 + (-23)$

12. $-75 + 23$

13. $75 + (-23)$

14. $65 + (-45) + (-82)$

15. $-7.8 + (-5.3) + 9.9$

16. $-24 + 65 + (-17) + 31$

17. $-6.8 + (-4.3) + 7.12 + 3.45$

18. $-\frac{7}{12} + \frac{7}{15} + \left(-\frac{7}{10}\right)$

19. The Chicago Bears made the following consecutive plays during a recent football game: 9-yd gain, 5-yd loss, 6-yd loss, and 12-yd gain. Did they get a first down?

20. Intel stock has the following changes in 1 week: up 0.78, down 1.34, down 2.78, up 3.12, and down 0.15. What is the net change for the week?

Section 8.3

Subtract.

21. $19 - (-3)$

22. $-45 - 81$

23. $16 - (-75)$

24. $-134 - (-134)$

25. $-4.56 - 3.25$

26. $-4.56 - (-3.25)$

27. Find the difference between -127 and -156 .

28. Subtract -56 from -45 .

29. At the beginning of the month, Maria's bank account had a balance of \$562.75. At the end of the month, the account was overdrawn by \$123.15 ($-\123.15). If there were no deposits during the month, what was the total amount of the checks Maria wrote?

30. Microsoft stock opened the day at 25.62 and closed the day at 24.82. Did the stock gain or lose for the day? Express the gain or loss as a signed number.

Section 8.4*Multiply.*

31. $-6(11)$

32. $5(-28)$

33. $-1.2(3.4)$

34. $7.4(-5.1)$

35. $-3(-17)$

36. $-7(-21)$

37. $-4.03(-2.1)$

38. $(-1)(-4)(-6)(5)(-2)$

39. Kroger's promotes a gallon of milk as a loss leader. If Kroger loses 45¢ per gallon, what will be the total loss if they sell 632 gallons? Express the loss as a signed number.

40. Pedro owns 723.5 shares of Pfizer. If the stock loses \$0.32 ($-\0.32) a share, what is Pedro's loss expressed as a signed number?

Section 8.5*Divide.*

41. $-18 \div 6$

42. $153 \div (-3)$

43. $-45 \div (-9)$

44. $-4.14 \div (-1.2)$

45. $-2448 \div 153$

46. $-8342 \div (-97)$

47. Find the quotient of -84.3 and 1.5 .

48. Divide -712 by -32 .

49. A share of UPS stock loses 15.5 points in 4 days. Express the average daily loss as a signed number.

50. Albertsons grocery store lost \$240.50 on the sale of Wheaties as a loss leader. If the store sold 650 boxes of the cereal during the sale, what is the loss per box, expressed as a signed number?

Section 8.6*Perform the indicated operations.*

51. $5(-9) - 11$

52. $-3(17) + 45$

53. $-18 + (-4)(5) - 12$

54. $-84 \div 4(7)$

55. $-72 \div (-12)(3) + 6(-7)$

56. $(7 - 4)(4)(-3) + 6(7 - 9)$

57. $(-4)^2(-1)(-1) + (-6)(-3) - 17$

58. $(-1)(3^2)(-4) + 4(-5) - (-18 - 5)$

59. Find the difference of the quotient of 71 and -2.5 and the product of 3.2 and -2.4 .

60. A local airline sells 65 seats for $\$324$ each and 81 seats for $\$211$ each. If the break-even point for the airline is $\$256$ a seat, express the profit or loss for the airline for this flight as a signed number.

Section 8.7*Solve.*

61. $7x + 25 = -10$

62. $-6x + 21 = -21$

63. $71 - 5x = -54$

64. $-55 = 3x + 41$

65. $-43 = 7x - 43$

66. $12y - 9 = -45$

67. $78a + 124 = -890$

68. $-55b + 241 = -144$

- 69.** A formula for relating degrees Fahrenheit (F) and degrees Celsius (C) is $9C = 5F - 160$. Find the degrees Fahrenheit that is equal to -22°C .

- 70.** Using the formula in Exercise 69, find the degrees Fahrenheit that is equal to -8°C .

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True/False Concept Review CHAPTER 8

ANSWERS

Check your understanding of the language of basic mathematics. Tell whether each of the following statements is true (always true) or false (not always true). For each statement you judge to be false, revise it to make a statement that is true.

1. Negative numbers are found to the left of zero on the number line. 1. _____
2. The opposite of a signed number is always positive. 2. _____
3. The absolute value of a number is always positive. 3. _____
4. The opposite of a signed number is the same distance from zero as the number on the number line but in the opposite direction. 4. _____
5. The sum of two signed numbers is always positive or negative. 5. _____
6. The sum of a positive signed number and a negative signed number is always positive. 6. _____
7. To find the sum of a positive signed number and a negative signed number, subtract their absolute values and use the sign of the number with the larger absolute value. 7. _____
8. To subtract two signed numbers, add their absolute values. 8. _____
9. If a negative number is subtracted from a positive number, the difference is always positive. 9. _____
10. The product of two negative numbers is never negative. 10. _____
11. The sign of the product of a positive number and a negative number depends on which number has the larger absolute value. 11. _____
12. The sign of the quotient, when dividing two signed numbers, is the same as the sign obtained when multiplying the two numbers. 12. _____
13. The order of operations for signed numbers is the same as the order of operations for positive numbers. 13. _____
14. Subtracting a number from both sides of an equation results in an equation that has the same solution as the original equation. 14. _____

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Test CHAPTER 8**ANSWERS***Perform the indicated operations.*

1. $-32 + (-19) + 39 + (-21)$

1. _____

2. $(45 - 52)(-16 + 21)$

2. _____

3. $\left(-\frac{3}{8}\right) \div \left(\frac{3}{10}\right)$

3. _____

4. $\left(-\frac{7}{15}\right) - \left(-\frac{3}{5}\right)$

4. _____

5. $(-11 - 5) - (5 - 22) + (-6)$

5. _____

6. $-5.78 + 6.93$

6. _____

7. a.
- $-(-17)$
-
- b.
- $|-33|$

7. _____

8. $-65 - (-32)$

8. _____

9. $(-18 + 6) \div 3 \cdot 4 - (-7)(-2)(-1)$

9. _____

10. $-110 \div (-55)$

10. _____

11. $(-6)(-8)(2)$

11. _____

12. $(|-7|)(-3)(-1)(-1)$

12. _____

13. $-63.2 - 45.7$

13. _____

14. $(-2)^2(-2)^2 + 4^2 \div (2)(3)$

14. _____

15. $21.84 \div (-0.7)$

15. _____

16. $\left(-\frac{1}{3}\right) + \frac{5}{6} + \left(-\frac{1}{2}\right) + \left(-\frac{1}{6}\right)$

16. _____

17. $-112 \div (-8)$

17. _____

18. $-56 - 24$

18. _____

19. $(-7)(4 - 13)(-2) - 5(-2 - 6)$

19. _____

20. $\left(-\frac{3}{8}\right)\left(\frac{12}{15}\right)$

20. _____

21. $-45 + (-23)$

21. _____

22. $6(-7) + 37$

22. _____

Solve.

23. $-16 = 5x + 14$

23. _____

24. $7x - 32 = 17$

24. _____

25. $24 - 6a = 45$

25. _____

26. Ms. Rosier lost an average of 1.05 lb per week (-1.05 lb) during her 16-week diet. Express Ms. Rosier's total weight loss during the 16 weeks as a signed number.

26. _____

27. The temperature in Chicago ranges from a high of 12°F to a low of -9°F within a 24-hr period. What is the drop in temperature, expressed as a signed number?

27. _____

28. A stock on the New York Stock Exchange opens at 17.65 on Monday. It records the following changes during the week: Monday, $+0.37$; Tuesday, -0.67 ; Wednesday, $+1.23$; Thursday, -0.87 ; Friday, $+0.26$. What is its closing price on Friday?

28. _____

29. What Fahrenheit temperature is equal to a reading of -10°C ? Use the formula

29. _____

$$F = \frac{9}{5}C + 32.$$

30. Find the average of $-11, -15, 23, -19, 10$, and -12 .

30. _____

Group Project (4 WEEKS) CHAPTER 8

OPTIONAL

On three consecutive Mondays, locate the final scores for each of the three major professional golf tours in the United States: the Professional Golf Association, PGA; the Ladies Professional Golf Association, LPGA; and the Champions Tour. These scores can usually be found on the summary page in the sports section of the daily newspaper.

1. Record the scores, against par, for the 30 top finishers and ties on each tour. Display the data using bar graphs for week 1, line graphs for week 2, and bar graphs for week 3. Which type of graph best displays the data?
2. Calculate the average score, against par, for each tour for each week. When finding the average, if there is a remainder and it is half of or more than the divisor, round up, otherwise round down. Now average the average scores for each tour. Which tour scored the best? Why?
3. What is the difference between the best and worst scores on each tour for each week?
4. What is the average amount of money earned by the player whose scores were recorded on each tour for each week? Which tour pays the best?
5. How much did the winner on each tour earn per stroke under par in the second week of your data? Compare the results. Is this a good way to compare the earnings on the tour? If not, why not?

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APPENDIX A

CALCULATORS

The wide availability and economical price of current hand-held calculators make them ideal for doing time-consuming arithmetic operations. Even people who are very good at math use calculators under certain circumstances (for instance, when balancing their checkbooks). You are encouraged to use a calculator as you work through this text. Learning the proper and appropriate use of a calculator is a vital skill for today's math students.

As with all new skills, your instructor will give you guidance as to where and when to use it. Calculators are especially useful in the following instances:

- For doing the fundamental operations of arithmetic (addition, subtraction, multiplication, and division)
- For finding powers or square roots of numbers
- For evaluating complicated arithmetic expressions
- For checking solutions to equations

Several different kinds of calculators are available.

- A basic 4-function calculator will add, subtract, multiply and divide. Sometimes these calculators also have a square root key. These calculators are not powerful enough to do all of the math in this text, and they are not recommended for math students at this level.
- A scientific calculator generally has about eight rows of keys on it and is usually labeled "scientific." Look for keys labeled "sin," "tan," and "log." Scientific calculators also have power keys and parenthesis keys, and the order of operations is built into them. These calculators are recommended for math students at this level.
- A graphing calculator also has about eight rows of keys, but it has a large, nearly square display screen. These calculators are very powerful, and you may be required to purchase them in later math courses. However, you will not need all that power to be successful in this course, and they are significantly more expensive than scientific calculators.

We will assume that you are operating a scientific calculator. (Some of the keystrokes are different on graphing calculators, so if you are using one of these calculators, please consult your owner's manual.) Study the following table to discover how the basic keys are used.

Expression	Key Strokes	Display
$144 \div 3 - 7$	[144] [÷] [3] [−] [7] [=]	41.
$3(2) + 4(5)$	[3] [×] [2] [+] [4] [×] [5] [=]	26.
$13^2 - 2(12 + 10)$	[13] [x^2] [−] [2] [×] [(] [12] [+] [10] [)] [=]	125.
$\frac{28 + 42}{10}$	[(] [28] [+] [42] [)] [÷] [10] [=]	7. or [28] [+] [42] [=] [÷] [10] [=]
$\frac{288}{6 + 12}$	[288] [÷] [(] [6] [+] [12] [)] [=]	16.
$19^2 - 3^5$	[19] [x^2] [−] [3] [x^5] [=]	118.
$2\frac{1}{3} + \frac{5}{6}$	[2] [a^b/c] [1] [a^b/c] [3] [+] [5] [a^b/c] [6] [=]	$3\frac{1}{2} 1\frac{1}{6}$

Notice that the calculator does calculations when you hit the $=$ or ENTER key. The calculator automatically uses the order of operations when you enter more than one operation before hitting $=$ or ENTER. Notice that if you begin a sequence with an operation sign, the calculator automatically uses the number currently displayed as part of the calculation. There are three operations that require only one number: squaring a number, square rooting a number, and taking the opposite of a number. In each case, enter the number first and then hit the appropriate operation key. Be especially careful with fractions. Remember that when there is addition or subtraction inside a fraction, the fraction bar acts as a grouping symbol. But the only way to convey this to your calculator is by using the grouping symbols $($ and $)$. Notice that the fraction key is used between the numerator and denominator of a fraction and also between the whole number and fractional part of a mixed number. It automatically calculates the common denominator when necessary.

Model Problem Solving

Practice the following problems until you can get the results shown.

Answers

- | | |
|----------------------------------|-----------------|
| a. $47 + \frac{525}{105}$ | 52 |
| b. $\frac{45 + 525}{38}$ | 15 |
| c. $\frac{648}{17 + 15}$ | 20.25 |
| d. $\frac{140 - 5(6)}{11}$ | 10 |
| e. $\frac{3870}{9(7) + 23}$ | 45 |
| f. $\frac{5(73) + 130}{33}$ | 15 |
| g. $100 - 2^5$ | 68 |
| h. $100 - (-2)^5$ | 132 |
| i. $4\frac{2}{7} - 3\frac{3}{5}$ | $\frac{24}{35}$ |

APPENDIX B

PRIME FACTORS OF NUMBERS 1 THROUGH 100

	Prime Factors		Prime Factors		Prime Factors		Prime Factors
1	none	26	$2 \cdot 13$	51	$3 \cdot 17$	76	$2^2 \cdot 19$
2	2	27	3^3	52	$2^2 \cdot 13$	77	$7 \cdot 11$
3	3	28	$2^2 \cdot 7$	53	53	78	$2 \cdot 3 \cdot 13$
4	2^2	29	29	54	$2 \cdot 3^3$	79	79
5	5	30	$2 \cdot 3 \cdot 5$	55	$5 \cdot 11$	80	$2^4 \cdot 5$
6	$2 \cdot 3$	31	31	56	$2^3 \cdot 7$	81	3^4
7	7	32	2^5	57	$3 \cdot 19$	82	$2 \cdot 41$
8	2^3	33	$3 \cdot 11$	58	$2 \cdot 29$	83	83
9	3^2	34	$2 \cdot 17$	59	59	84	$2^2 \cdot 3 \cdot 7$
10	$2 \cdot 5$	35	$5 \cdot 7$	60	$2^2 \cdot 3 \cdot 5$	85	$5 \cdot 17$
11	11	36	$2^2 \cdot 3^2$	61	61	86	$2 \cdot 43$
12	$2^2 \cdot 3$	37	37	62	$2 \cdot 31$	87	$3 \cdot 29$
13	13	38	$2 \cdot 19$	63	$3^2 \cdot 7$	88	$2^3 \cdot 11$
14	$2 \cdot 7$	39	$3 \cdot 13$	64	2^6	89	89
15	$3 \cdot 5$	40	$2^3 \cdot 5$	65	$5 \cdot 13$	90	$2 \cdot 3^2 \cdot 5$
16	2^4	41	41	66	$2 \cdot 3 \cdot 11$	91	$7 \cdot 13$
17	17	42	$2 \cdot 3 \cdot 7$	67	67	92	$2^2 \cdot 23$
18	$2 \cdot 3^2$	43	43	68	$2^2 \cdot 17$	93	$3 \cdot 31$
19	19	44	$2^2 \cdot 11$	69	$3 \cdot 23$	94	$2 \cdot 47$
20	$2^2 \cdot 5$	45	$3^2 \cdot 5$	70	$2 \cdot 5 \cdot 7$	95	$5 \cdot 19$
21	$3 \cdot 7$	46	$2 \cdot 23$	71	71	96	$2^5 \cdot 3$
22	$2 \cdot 11$	47	47	72	$2^3 \cdot 3^2$	97	97
23	23	48	$2^4 \cdot 3$	73	73	98	$2 \cdot 7^2$
24	$2^3 \cdot 3$	49	7^2	74	$2 \cdot 37$	99	$3^2 \cdot 11$
25	5^2	50	$2 \cdot 5^2$	75	$3 \cdot 5^2$	100	$2^2 \cdot 5^2$

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APPENDIX C

SQUARES AND SQUARE ROOTS (0 TO 199)

n	n^2	\sqrt{n}	n	n^2	\sqrt{n}	n	n^2	\sqrt{n}	n	n^2	\sqrt{n}
0	0	0.000	50	2,500	7.071	100	10,000	10.000	150	22,500	12.247
1	1	1.000	51	2,601	7.141	101	10,201	10.050	151	22,801	12.288
2	4	1.414	52	2,704	7.211	102	10,404	10.100	152	23,104	12.329
3	9	1.732	53	2,809	7.280	103	10,609	10.149	153	23,409	12.369
4	16	2.000	54	2,916	7.348	104	10,816	10.198	154	23,716	12.410
5	25	2.236	55	3,025	7.416	105	11,025	10.247	155	24,025	12.450
6	36	2.449	56	3,136	7.483	106	11,236	10.296	156	24,336	12.490
7	49	2.646	57	3,249	7.550	107	11,449	10.344	157	24,649	12.530
8	64	2.828	58	3,346	7.616	108	11,664	10.392	158	24,964	12.570
9	81	3.000	59	3,481	7.681	109	11,881	10.440	159	25,281	12.610
10	100	3.162	60	3,600	7.746	110	12,100	10.488	160	25,600	12.649
11	121	3.317	61	3,721	7.810	111	12,321	10.536	161	25,921	12.689
12	144	3.464	62	3,844	7.874	112	12,544	10.583	162	26,244	12.728
13	169	3.606	63	3,969	7.937	113	12,769	10.630	163	26,569	12.767
14	196	3.742	64	4,096	8.000	114	12,996	10.677	164	26,896	12.806
15	225	3.873	65	4,225	8.062	115	13,225	10.724	165	27,225	12.845
16	256	4.000	66	4,356	8.124	116	13,456	10.770	166	27,556	12.884
17	289	4.123	67	4,489	8.185	117	13,689	10.817	167	27,889	12.923
18	324	4.243	68	4,624	8.246	118	13,924	10.863	168	28,224	12.961
19	361	4.359	69	4,761	8.307	119	14,161	10.909	169	28,561	13.000
20	400	4.472	70	4,900	8.367	120	14,400	10.954	170	28,900	13.038
21	441	4.583	71	5,041	8.426	121	14,641	11.000	171	29,241	13.077
22	484	4.690	72	5,184	8.485	122	14,884	11.045	172	29,584	13.115
23	529	4.796	73	5,329	8.544	123	15,129	11.091	173	29,929	13.153
24	576	4.899	74	5,476	8.602	124	15,376	11.136	174	30,276	13.191
25	625	5.000	75	5,625	8.660	125	15,625	11.180	175	30,625	13.229
26	676	5.099	76	5,776	8.718	126	15,876	11.225	176	30,976	13.266
27	729	5.196	77	5,929	8.775	127	16,129	11.269	177	31,329	13.304
28	784	5.292	78	6,084	8.832	128	16,384	11.314	178	31,684	13.342
29	841	5.385	79	6,241	8.888	129	16,641	11.358	179	32,041	13.379
30	900	5.477	80	6,400	8.944	130	16,900	11.402	180	32,400	13.416
31	961	5.568	81	6,561	9.000	131	17,161	11.446	181	32,761	13.454
32	1,024	5.657	82	6,724	9.055	132	17,424	11.489	182	33,124	13.491
33	1,089	5.745	83	6,889	9.110	133	17,689	11.533	183	33,489	13.528
34	1,156	5.831	84	7,056	9.165	134	17,956	11.576	184	33,856	13.565
35	1,225	5.916	85	7,225	9.220	135	18,225	11.619	185	34,225	13.601
36	1,296	6.000	86	7,396	9.274	136	18,496	11.662	186	34,596	13.638
37	1,369	6.083	87	7,569	9.327	137	18,769	11.705	187	34,969	13.675
38	1,444	6.164	88	7,744	9.381	138	19,044	11.747	188	35,344	13.711
39	1,521	6.245	89	7,921	9.434	139	19,321	11.790	189	35,721	13.748
40	1,600	6.325	90	8,100	9.487	140	19,600	11.832	190	36,100	13.784
41	1,681	6.403	91	8,281	9.539	141	19,881	11.874	191	36,481	13.820
42	1,764	6.481	92	8,464	9.592	142	20,164	11.916	192	36,864	13.856
43	1,849	6.557	93	8,649	9.644	143	20,449	11.958	193	37,249	13.892
44	1,936	6.633	94	8,836	9.695	144	20,736	12.000	194	37,636	13.928
45	2,025	6.708	95	9,025	9.747	145	21,025	12.042	195	38,025	13.964
46	2,116	6.782	96	9,216	9.798	146	21,316	12.083	196	38,416	14.000
47	2,209	6.856	97	9,409	9.849	147	21,609	12.124	197	38,809	14.036
48	2,304	6.928	98	9,604	9.899	148	21,904	12.166	198	39,204	14.071
49	2,401	7.000	99	9,801	9.950	149	22,201	12.207	199	39,601	14.107

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APPENDIX D

COMPOUND INTEREST TABLE (FACTORS)

	Years					
	1	5	10	15	20	25
2%						
Quarterly	1.0202	1.1049	1.2208	1.3489	1.4903	1.6467
Monthly	1.0202	1.1051	1.2212	1.3495	1.4913	1.6480
Daily	1.0202	1.1052	1.2214	1.3498	1.4918	1.6487
3%						
Quarterly	1.0303	1.1612	1.3483	1.5657	1.8180	2.1111
Monthly	1.0304	1.1616	1.3494	1.5674	1.8208	2.1150
Daily	1.0305	1.1618	1.3498	1.5683	1.8221	2.1169
4%						
Quarterly	1.0406	1.2202	1.4889	1.8167	2.2167	2.7048
Monthly	1.0407	1.2210	1.4908	1.8203	2.2226	2.7138
Daily	1.0408	1.2214	1.4918	1.8221	2.2254	2.7181
5%						
Quarterly	1.0509	1.2820	1.6436	2.1072	2.7015	3.4634
Monthly	1.0512	1.2834	1.6470	2.1137	2.7126	3.4813
Daily	1.0513	1.2840	1.6487	2.1169	2.7181	3.4900
6%						
Quarterly	1.0614	1.3469	1.8140	2.4432	3.2907	4.4320
Monthly	1.0617	1.3489	1.8194	2.4541	3.3102	4.4650
Daily	1.0618	1.3498	1.8220	2.4594	3.3198	4.4811
7%						
Quarterly	1.0719	1.4148	2.0016	2.8318	4.0064	5.6682
Monthly	1.0723	1.4176	2.0097	2.8489	4.0387	5.7254
Daily	1.0725	1.4190	2.0136	2.8574	4.0547	5.7536

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Midterm Examination

CHAPTERS 1–4

ANSWERS

1. Write the place value of the digit 7 in 247,598.

1. _____

2. Add:

$$\begin{array}{r}
 289 \\
 4675 \\
 52 \\
 78,612 \\
 + 555 \\
 \hline
 \end{array}$$

2. _____

3. Write the word name for 67,509.

3. _____

4. Subtract:

$$\begin{array}{r}
 6123 \\
 -3782 \\
 \hline
 \end{array}$$

4. _____

5. Add: $703 + 25,772 + 1098 + 32$

5. _____

6. Multiply: $(715)(64)$

6. _____

7. Estimate the product and multiply:

$$\begin{array}{r}
 803 \\
 \times 906 \\
 \hline
 \end{array}$$

7. _____

8. Divide: $347 \overline{)73,911}$

8. _____

9. Divide: $48 \overline{)76,446}$

9. _____

10. Perform the indicated operations: $19 - 3 \cdot 4 + 18 \div 3$

10. _____

11. Find the sum of the quotient of 72 and 9 and the product of 33 and 2.

11. _____

12. Find the average of 136, 213, 157, and 186.

12. _____

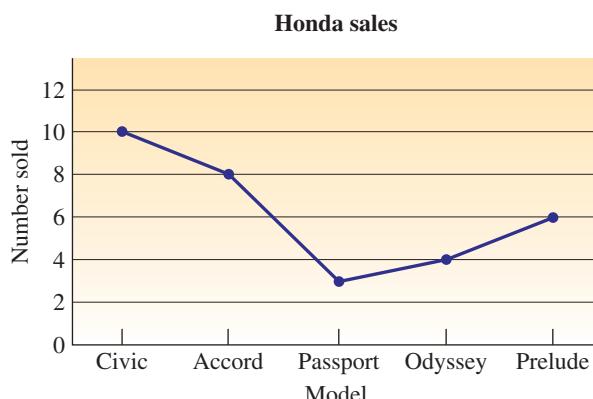
13. Find the average, median, and mode of 52, 64, 64, 97, 128, 97, 82, and 64.

13. _____

14. The graph shows the number of Honda vehicles sold at a local dealership by model.

- a. Which model has the highest sales?
- b. How many more Accords are sold than Passports?

14. _____



Name _____ Class _____ Date _____

15. Write the least common multiple (LCM) of 20, 15, and 24.

15. _____

16. Is 1785 divisible by 2, 3, or 5?

16. _____

17. Is 107 a prime number or a composite number?

17. _____

18. Is 7263 a multiple of 9?

18. _____

19. List the first five multiples of 41.

19. _____

20. List all the factors of 304.

20. _____

21. Write the prime factorization of 540.

21. _____

22. Change to a mixed number: $\frac{83}{7}$

22. _____

23. Change to an improper fraction: $15\frac{3}{8}$

23. _____

24. Which of these fractions are proper? $\frac{3}{4}, \frac{7}{6}, \frac{8}{8}, \frac{7}{8}, \frac{9}{8}, \frac{5}{4}, \frac{4}{4}$

24. _____

25. List these fractions from the smallest to the largest: $\frac{5}{9}, \frac{5}{8}, \frac{7}{12}, \frac{2}{3}$

25. _____

26. Simplify: $\frac{180}{225}$

26. _____

27. Multiply and simplify: $\frac{3}{16} \cdot \frac{4}{9} \cdot \frac{5}{6}$

27. _____

28. Multiply. Write the answer as a mixed number. $\left(3\frac{3}{5}\right)\left(12\frac{6}{7}\right)$

28. _____

29. Divide and simplify: $\frac{45}{70} \div \frac{9}{14}$

29. _____

30. What is the reciprocal of $2\frac{3}{8}$?

30. _____

31. Add: $\frac{7}{15} + \frac{11}{18}$

31. _____

32. Add:
$$\begin{array}{r} 6\frac{2}{3} \\ + 11\frac{7}{8} \\ \hline \end{array}$$

32. _____

Name _____ Class _____ Date _____

33. Subtract: $15 - 9\frac{5}{8}$

33. _____

34. Subtract: $54\frac{1}{9} - 36\frac{4}{7}$

34. _____

35. Find the average of $4\frac{5}{6}$, $6\frac{2}{3}$, and $11\frac{3}{4}$.

35. _____

36. Perform the indicated operations: $\frac{3}{5} - \frac{1}{3} \cdot \frac{5}{6} \div \frac{5}{6}$

36. _____

37. Write the place value name for eighty thousand two hundred forty and one hundred twenty-two thousandths.

37. _____

38. Round 6.73582 to the nearest tenth, hundredth, and thousandth.

38. _____

39. Write 0.26 as a fraction.

39. _____

40. List the following numbers from smallest to largest: 1.034, 1.109, 1.044, 1.094, 1.02, 1.07

40. _____

41. Add: $134.76 + 7.113 + 0.094 + 5.923 + 25.87$

41. _____

42. Subtract: $8.3 - 5.763$

42. _____

43. Multiply: 23.075×7.12

43. _____

44. Multiply: $0.046 \times 100,000$

44. _____

45. Divide: $0.902 \div 1,000$

45. _____

46. Write 0.00058 in scientific notation.

46. _____

47. Divide: $7.85 \overline{)445.88}$

47. _____

48. Change $\frac{9}{17}$ to a decimal rounded to the nearest thousandth.

48. _____

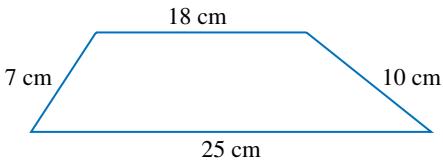
49. Find the average and median of 4.5, 6.9, 8.3, 9.5, 3.1, 10.3, and 2.2.

49. _____

50. Perform the indicated operations. $(5.5)^2 - 2.3(4.1) + 11.8 - 9.3 \div 0.3$

50. _____

51. Find the perimeter of the trapezoid.



51. _____

52. Find the area of a rectangle that is $6\frac{3}{5}$ ft wide and $9\frac{2}{3}$ ft long.

52. _____

53. On Thursday, October 4, the following counts of chinook salmon going over the dams were recorded: Bonneville, 1577; The Dalles, 1589; John Day, 1854; McNary, 1361; Ice Harbor, 295; and Little Goose, 274. How many chinook salmon were counted?

53. _____

54. Recently General Motors declared a stock dividend of 14¢ a share. Maria owns 765.72 shares of General Motors. To the nearest cent, what was Maria's dividend?

54. _____

55. Houn paid \$13.69 for 18.5 lb of bananas. What was the price per pound of the bananas?

55. _____

Final Examination *CHAPTERS 1–8***ANSWERS**

- 1.** Add: $\frac{5}{8} + \frac{7}{16}$
- 2.** Write the LCM (least common multiple) of 15, 18, and 35.
- 3.** Subtract:
$$\begin{array}{r} 93.42 \\ -57.69 \\ \hline \end{array}$$
- 4.** Add: $53.82 + 0.458 + 0.085$
- 5.** Divide: $\frac{15}{16} \div \frac{5}{8}$
- 6.** Multiply: $3\frac{5}{7} \cdot 35$
- 7.** Which of these numbers is a prime number? 99, 199, 299, 699
- 8.** Divide. Round the answer to the nearest hundredth: $0.62 \overline{)23.764}$
- 9.** Subtract: $17 - 7\frac{11}{12}$
- 10.** Multiply: $(0.0945)(10,000)$
- 11.** Round to the nearest hundredth: 246.7089
- 12.** Multiply: $(11.6)(4.07)$
- 13.** Write as a fraction and simplify: 64%
- 14.** Add:
$$\begin{array}{r} 9\frac{11}{15} \\ + \frac{5}{6} \\ \hline \end{array}$$
- 15.** Solve the proportion: $\frac{11}{15} = \frac{x}{10}$
- 16.** What is the place value of the 6 in 23.11567?
- 17.** List these fractions from the smallest to the largest: $\frac{3}{4}, \frac{4}{5}, \frac{7}{10}$
- 18.** Change to percent: $\frac{23}{25}$
- 19.** Divide. Round to the nearest hundredth. $39 \overline{)672}$
- 1.** _____
- 2.** _____
- 3.** _____
- 4.** _____
- 5.** _____
- 6.** _____
- 7.** _____
- 8.** _____
- 9.** _____
- 10.** _____
- 11.** _____
- 12.** _____
- 13.** _____
- 14.** _____
- 15.** _____
- 16.** _____
- 17.** _____
- 18.** _____
- 19.** _____

Name _____ Class _____ Date _____

20. Write as an approximate decimal to the nearest thousandth: $\frac{17}{33}$ 20. _____
21. Write the place value name for six thousand and fifteen thousandths. 21. _____
22. Divide: $0.043 \overline{)0.34787}$ 22. _____
23. Write as a decimal: $57\frac{5}{8}\%$ 23. _____
24. Thirty-nine percent of what number is 19.5? 24. _____
25. Write as a percent: 4.52 25. _____
26. Write as a decimal: $\frac{71}{250}$ 26. _____
27. Seventy-three percent of 82 is what number? 27. _____
28. Multiply: $(0.26)(4.5)(0.55)$ 28. _____
29. If 6 lb of strawberries cost \$11.10, how much would 25 lb cost? 29. _____
30. An iPod is priced at \$345. It is on sale for \$280. What is the percent of discount based on the original price? Round to the nearest tenth of a percent. 30. _____
31. List the first five multiples of 51. 31. _____
32. Write the word name for 8037.037. 32. _____
33. Is the following proportion true or false?
$$\frac{1.9}{22} = \frac{5.8}{59}$$
 33. _____
34. Write the prime factorization of 680. 34. _____
35. Simplify: $\frac{315}{450}$ 35. _____
36. Change to a fraction and simplify: 0.945 36. _____
37. Change to a mixed number: $\frac{341}{8}$ 37. _____
38. Multiply and simplify: $\frac{6}{35} \cdot \frac{42}{54}$ 38. _____
39. Change to a fraction: 0.084 39. _____
40. A survey at a McDonald's showed that 27 of 50 customers asked for a Big Mac. What percent of the customers wanted a Big Mac? 40. _____

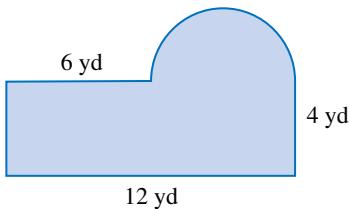
Name _____ Class _____ Date _____

41. Is 2546 a multiple of 3?
42. Subtract: $14\frac{3}{5} - \frac{11}{15}$
43. Change to an improper fraction: $21\frac{8}{11}$
44. List the following decimals from smallest to largest: 2.32, 2.332, 2.299, 2.322
45. List all the divisors of 408.
46. Divide: $45.893 \div 10^5$
47. Divide: $4\frac{7}{12} \div 1\frac{9}{16}$
48. Write a ratio in fraction form to compare 85¢ to \$5 (using common units) and simplify.
49. Mildred calculates that she pays \$0.67 for gas and oil to drive 5 miles. In addition, she pays 30¢ for maintenance for each 5 miles she travels. How much will it cost her to drive 7500 miles?
50. Billy works for a large furniture manufacturer. He earns a fixed salary of \$1500 per month plus a 3.5% commission on all sales. What does Billy earn in a month in which his sales are \$467,800?
51. The sales tax on a \$55 purchase is \$4.75. What is the sales tax rate, to the nearest tenth of a percent?
52. Melissa buys a new vacuum cleaner that is on sale at 30% off the original price. If the original price is \$245.50 and there is a 6.5% sales tax, what is the final cost of the vacuum cleaner?
53. Perform the indicated operations: $65 - 5 \cdot 7 + 99 \div 11$
54. Perform the indicated operations: $79.15 - 5.1(8.3) \div 3.4$
55. Add:
$$\begin{array}{r} 5 \text{ hr } 47 \text{ min } 32 \text{ sec} \\ + 2 \text{ hr } 36 \text{ min } 48 \text{ sec} \\ \hline \end{array}$$
56. Subtract:
$$\begin{array}{r} 6 \text{ m } 35 \text{ cm} \\ - 4 \text{ m } 52 \text{ cm} \\ \hline \end{array}$$
57. Convert 5¢ per gram to dollars per kilogram.
58. Find the perimeter of a trapezoid with bases of 63.7 ft and 74.2 ft and sides of 21.5 ft and 23.6 ft.
41. _____
42. _____
43. _____
44. _____
45. _____
46. _____
47. _____
48. _____
49. _____
50. _____
51. _____
52. _____
53. _____
54. _____
55. _____
56. _____
57. _____
58. _____

59. Find the area of a triangle with a base of 8.4 m and a height of 6.3 m.

59. _____

60. Find the area of the following geometric figure (let $\pi \approx 3.14$):



60. _____

61. Find the volume of a box with length of 4.6 ft, width of 7 in., and height of 5 in. (in cubic inches).

61. _____

62. Find the square root of 456.8 to the nearest hundredth.

62. _____

63. Find the hypotenuse of a right triangle with legs of 23 cm and 27 cm. Find to the nearest tenth of a centimeter.

63. _____

64. Add: $(-34) + (-23) + 41$

64. _____

65. Subtract: $(-62) - (-84)$

65. _____

66. Multiply: $(5)(-7)(-1)(-3)$

66. _____

67. Divide: $(-46.5) \div (-15)$

67. _____

68. Perform the indicated operations: $(-6 - 4)(-4) \div (-5) - (-10)$

68. _____

69. Solve: $12a + 98 = 2$

69. _____

70. Find the Fahrenheit temperature that is equivalent to -15°C . Use the formula, $9C = 5F - 160$.

70. _____

ANSWERS

CHAPTER 1

Section 1.1

1. five hundred seventy-four 3. eight hundred ninety
 5. seven thousand, twenty 7. 57 9. 9500
 11. 100,000,000 13. twenty-seven thousand, six hundred
 ninety 15. two hundred seven thousand, six hundred
 ninety 17. forty-five million 19. 359,800
 21. 22,570 23. 76,000,000,000 25. <
 27. > 29. < 31. < 33. 840 35. 1500

Number	Ten	Hundred	Thousand	Ten	Thou-	sand
37.	607,546	607,550	607,500	608,000	610,000	
39.	7,635,753	7,635,750	7,635,800	7,636,000	7,640,000	

41. The percent who exercise regularly is 40%.
 43. The under \$15,000 category has the highest percentage of nonexercisers.
 45. As income level goes up, so does the percentage of regular exercisers.
 47. Families with children, single women, and unaccompanied youth increased over the 10-year period. Single men, severely mentally ill, substance abusers, the employed, and veterans all decreased.
 49. In 1994, 59% of the homeless were single men or women. By 2004, this figure decreased to 55% of the homeless. 51. 656,732,410
 53. thirteen thousand, one hundred forty-eight dollars
 55. > 57. 1000 59. 81,600,000
 61. 63,700; 63,800: Rounding the second time changes the tens digit to 5, so rounding to the hundreds place from 63,750 results in a different answer. If told to round to the nearest hundred, the first method is correct.
 63. Kimo wrote eleven thousand, four hundred seventy-five on the check.
 65. It is estimated that two hundred seventy-six thousand, four hundred mallard ducks stayed in Wisconsin to breed.
 67. The place value name for the bid is \$36,407.
 69. To the nearest thousand dollars, the value of the Income Fund of America shares is \$185,000.
 71. There were 15,429,000 short tons of emission of volatile organic compounds in 2003.

73. The per capita personal income in Maine is twenty-eight thousand, eight hundred thirty-one dollars.

75. Rhode Island has the smallest per capita personal income.

77. To the nearest million miles, the distance from Earth to the sun is 93,000,000 miles.

79.

Month	Number of Marriages	Month	Number of Marriages
June	242,000	December	184,000
May	241,000	April	172,000
August	239,000	November	171,000
July	235,000	February	155,000
October	231,000	March	118,000
September	225,000	January	110,000

81. The rivers in increasing length are Yukon, Colorado, Arkansas, Rio Grande, Missouri, Mississippi.

83. Yes, the motor vehicle department's estimate was correct.

85. Jif has fewer of the following nutrients: fat, saturated fat, sodium, carbohydrates, and sugars.

87. *Shrek 2* took in four hundred thirty-six million, four hundred seventy-one thousand, thirty-six dollars.

89. *The Incredibles* took in about \$259,000,000.

91. Base ten's a good name for our number system because each place value in the system is 10 times the previous place and one-tenth the succeeding place.

93. Rounding a number is a method of calculating an approximation of that number. The purpose is to get an idea of the value of the number without listing digits that do not add to our understanding. The number 87,452 rounds to 87,000, to the nearest thousand, because 452 is less than halfway between 0 and 1000. Rounded to the nearest hundred it is 87,500 because 52 is more than halfway between 0 and 100.

95. five trillion, three hundred twenty-six billion, nine hundred one million, five hundred seventy thousand

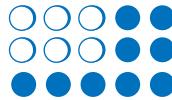
97. 7 99. 0

Section 1.2

1. 113 3. 979 5. 707 7. 1 9. 5737
 11. 5789 13. 17,500 15. 403 17. 334
 19. 531 21. 10 23. 474 25. 136 27. 4700

- 29.** 1100 **31.** 10,000 **33.** 300 **35.** 4800
37. 19,000 **39.** 161,000 **41.** 5000
43. 34,000 **45.** 44 cm **47.** 221 in. **49.** 56 ft
51. 134 in. **53.** The total number of Fords, Toyotas, and Lexuses sold is 3561.

- 55.** 163 more Hondas than Fords are sold.
57. The total of the three best-selling cars sold is 5320.
59. For the week, 5487 salmon went through the ladder. Tuesday's count was 332 more than Saturday's count.
61. The estimated answer is 2000 so Ralph's answer is reasonable.
63. The estimated cost of the items is \$1300.
65. The total number of property crimes is 51,343.
67. Sasha consumes 520 calories.
69. Sasha could have eaten 1180 calories for breakfast and lunch.
71. A total of 22,935 trees can be harvested.
73. Fong's Grocery still owes \$14,800.
75. The median family income was probably rounded to the nearest hundred. The San Francisco median income was \$23,100 higher than Seattle's median income.
77. On average, 530 more husbands killed their wives than wives killed their husbands.
79. On average, there were 209 people who killed a sibling.
81. There were 600,178 more F-Series trucks sold than Explorers.
83. There were 143,691 more F-Series trucks sold than the next three top sellers combined.
85. There are 3 million children with HIV.
87. The perimeter of the house is 200 ft.
89. Blanche needs 224 in. of lace.
91. Explanations for subtracting that are aimed at 6-year-olds are usually based on physical objects. So $15 - 9 = 6$ because when 9 circles (or pencils or apples or whatever) are removed from 15 circles, 6 circles remain.



Count the circles that are empty.

- 93.** A sum is the result of adding numbers. The sum of 8, 8, and 2 is 18. Mathematically, we write $8 + 8 + 2 = 18$.
95. seven hundred thousand, nine hundred
97. The dollar sales for the nine cars is \$184,000. The Accords' sales were \$29,476 more than the Civics' sales.
99. A = 7, B = 2, C = 3, D = 1

Getting Ready for Algebra

- 1.** $x = 12$ **3.** $x = 23$ **5.** $z = 14$ **7.** $c = 39$
9. $a = 151$ **11.** $x = 14$ **13.** $y = 58$
15. $k = 168$ **17.** $x = 11$ **19.** $w = 116$
21. The markup is \$348.
23. The length of the garage is 9 meters.
25. Let S represent the EPA highway rating of the Saturn and I represent the EPA highway rating of the Impreza. $I + 5 = S$; the Impreza has a highway rating of 30 mpg.

- 27.** Let B represent the total dollars budgeted in a category, S represent the dollars spent in a category, and R represent the dollars not yet spent in a category. $S + R = B$

Section 1.3

- 1.** 581 **3.** 231 **5.** 304 **7.** 651 **9.** 432
11. 0 **13.** 4400 **15.** thousands **17.** 3712
19. 5628 **21.** 3551 **23.** 3478 **25.** 126,000
27. 50,464 **29.** 35,856 **31.** 33,700 **33.** 15,600
35. 38,471 **37.** 18,525 **39.** 328,396 **41.** 3200
43. 36,000 **45.** 250,000 **47.** 240,000
49. 800,000 **51.** 18,000,000 **53.** 243 square yd
55. 529 square ft **57.** 1944 square cm
59. 102 square ft **61.** 52,224 square cm
63. 336 square m **65.** 294 square ft
67. 198 square ft **69.** 510,656 **71.** 2,376,000
73. The estimated product is 30,000, which is close to 28,438, so Maria's answer is reasonable.
75. The Rotary Club estimates it will sell 5115 dozen roses.
77. The gross receipts from the sale of Grand Caravans are \$838,371.
79. The gross receipts are \$2,250,000, rounded to the nearest thousand.
81. Washington County grew by 22,488 people during 2004.
83. The comptroller realized \$44,275 from the sale of the shares.
85. The estimated cost of the 12 blouses is \$400, so Carmella should have enough money in the budget.
87. There are 6 feet in a fathom.
89. There are about 18,228 ft in a league.
91. It is not possible to be 20,000 leagues under the sea. The author was taking literary license.
93. Sirius is about 47,040,000,000,000 miles from Earth.
95. There were 136 pages printed in 17 minutes.
97. A RAM of 256 KB has 262,144 bytes.
99. A tablespoon of olive oil has 126 calories from fat.
101. To the nearest thousand gallons, in a 31-day month the water usage is 16,574,000 gallons.
103. The tires cost Ms. Perta \$22,272. The gross income from the sale of the tires is \$49,184. The profit from the sale of the tires is \$26,912.
105. Yes, the Harry Potter movie would have made \$498,717,454 if it had doubled its gross earnings.
107. Explanations for multiplication that are aimed at 8-year-olds are usually based on physical objects. For example, $3(8) = 24$ because 3 groups, each of which has 8 circles (or pencils or apples or whatever), contain a total of 24 circles.



Count the total number of circles.

- 109.** A product is the result of multiplying numbers. The product of 11 and 8 is 88. Mathematically, we write $11 \times 8 = 88$.
111. To the nearest thousand, the crop is worth \$4,449,000.
113. A = 3, B = 2, C = 4, D = 5, E = 9

Section 1.4

1. 9 3. 9 5. 87 7. 91 9. 17 11. 40
13. 82 15. 4 R 4 17. 4 R 13 19. divisor
21. 4062 23. 32 25. 54 27. 87 29. 67
31. 391 33. 239 R 17 35. 17 R 30 37. 58
39. 12,802 R 18 41. 1160 43. 3400
45. The taxes paid per return in week 1 are \$5476.
47. The taxes paid per return in week 3 are \$4100, rounded to the nearest hundred.
49. The survey finds that 135 trees per acre are ready to harvest.
51. Ms. Munos will pay \$2016 for the radios.
53. You would need to spend \$1,400,000 per day.
55. The population density of China was about 138 people per square kilometer.
57. The population density of the United States was about 31 people per square kilometer.
59. Approximately 38,800,000 households own cats.
61. Each California representative represents about 625,380 people.
63. The gross state product per person in Florida was about \$32,856.
65. There are about 45 calories per serving.
67. Juan can take 8 capsules per day.
69. *Spider-Man 2* took in about twice as much as *The Day After Tomorrow*.
71. The average salary for the Ravens was approximately \$1,269,000.
73. The remainder, after division, is the amount left over after all possible groups of the appropriate size have been formed. When 46 objects are separated into groups of 9 (you can make 5 groups of 9) there is 1 object remaining. So $46 \div 9 = 5$ R 1.
75. The company will get the greater gross return in France. The difference in gross receipts is \$7125.
77. A = 2, B = 7, C = 9

Getting Ready for Algebra

1. $x = 5$ 3. $c = 18$ 5. $x = 4$ 7. $b = 46$
9. $x = 12$ 11. $y = 312$ 13. $x = 24$
15. $b = 24,408$ 17. $x = 61$ 19. $w = 782$
21. The width of the garden plot is 17 feet.
23. He sells 2340 lb of crab.
25. The wholesale cost of one set is \$310.
27. Let L represent the low temperature in July and H represent the high temperature in January. $2H = L$; the high temperature is 30°F.

Section 1.5

1. 12^7 3. 49 5. 8 7. 15
9. base; exponent; power or value 11. 216
13. 441 15. 1000 17. 512 19. 6561
21. 7200 23. 2,100,000 25. 23 27. 3700
29. exponent 31. 499,000,000 33. 8710
35. 67,340,000 37. 612 39. 562,000,000,000
41. 4500 43. 10^{11} 45. 161,051

47. 134,217,728 49. 47,160,000,000,000
51. 7900 53. The size of Salvador's lot is 10,800 ft².
55. The city parks operating budget is approximately \$84,000,000.
57. It is approximately 25,500,000,000,000 miles from Earth to Alpha Centauri.
59. The distance, 6 trillion miles, can also be written as 6×10^{12} and as 6,000,000,000,000.
61. There are 3^5 or 243 bacteria after 4 hours.
63. The number of bacteria will exceed 1000 during the sixth hour.
65. The surface area of the Pacific Ocean is about 64,200,000 square miles.
67. A gigabyte is 10^9 bytes.
69. *Meet the Fockers* earned about \$273,000,000, or 273×10^6 .
71. 50^5 is larger than the gross earnings of *The Incredibles*.
73. The expression 4^{10} represents the product of 10 fours, that is, $4 \times 4 \times 4$.
75. Mitchell's grandparents will deposit \$1,048,576 on his 10th birthday. They will have deposited a total of \$1,398,100. 77. 8962

Section 1.6

1. 44 3. 0 5. 59 7. 28 9. 48
11. 49 13. 30 15. 53 17. 48
19. 16 21. 42 23. 106 25. 35
27. 71 29. 381 31. 2757 33. 45
35. There were 595 more mallards and canvasbacks than teals and wood ducks.
37. Four times the number of wood ducks added to the number of mallards would be 165 more than the number of teals and canvasbacks. 39. 44 41. 521
43. Elmo's Janitorial Service used supplies costing \$1570 for the month.
45. The trucker's average weekly income is \$1291. His yearly income is \$64,550.
47. Marla consumes 1150 calories for breakfast.
49. Sally's total charge is \$190.
51. Clay and Connie have \$38 left on their certificate.
53. June's target training rate is 110 bpm.
55. Your target training rate decreases as you age because the MHR decreases.
57. The result is \$517,700,480.
59. Using the order of operations, division takes precedence over subtraction, so $20 - 10 \div 2 = 20 - 5$, or 15.
61. 196

Getting Ready for Algebra

1. $x = 7$ 3. $y = 33$ 5. $x = 6$ 7. $c = 32$
9. $x = 2$ 11. $c = 12$ 13. $a = 196$
15. $b = 21$ 17. Remy bought seven tickets.
19. Rana made six arrangements.
21. $8v = C$; Jessica can purchase nine visits at B-Fit.
23. $4v + 32 = C$; Jessica can purchase 10 visits from Gym Rats.

Section 1.7

1. 10 3. 15 5. 14 7. 11 9. 7 11. 9
 13. 14 15. 34 17. 46 19. 37 21. 138
 23. 156 25. 137 27. 59 29. 56 31. 26
 33. 40 35. 107 37. 126 39. 86 41. 28
 43. 3 45. no mode 47. 40 49. 45 and 60

51. no mode

53. The average number of points is 101 and the median number of points is 100.

55. The average score is 133; the median score is 134.

57. The average gas mileage for the cars is 32 mpg, the median mileage is 32 mpg, and the mode mileage is 34 mpg.

59. The average weight of the players on the wrestling team is 136 lb.

61. The average cost per mile to build the new lines is \$63 million.

63. The average price of the coffee makers is \$117. Two of the models cost less than the average price.

65. The average population was 1,720,059 over the 13-year period.

67. Population by Year in Nevada

Year	Population	Year	Population
1990	1,198,954	1997	1,760,144
1991	1,279,124	1998	1,840,314
1992	1,359,294	1999	1,920,484
1993	1,439,464	2000	2,000,654
1994	1,519,634	2001	2,080,824
1995	1,599,804	2002	2,160,994
1996	1,679,974	2003	2,241,164

69. The average yearly attendance for the 4 years was about 12,800,000.

71. The average assets for the top three banks were \$648,486 million.

73. The average assets for the top five banks were \$531,498 million.

75. The mean number of Internet users over the 8 years is about 127 million.

77. The median number of Internet users over the 8-year period is 130 million.

79. The median number of users for the last 5 years is 158 million.

81. Half of the houses on Jupiter Island cost more than \$4 million and half cost less than \$4 million.

83. In 1950, about half of all men getting married for the first time were 23 years old or younger. In 2002, about half of all men getting married were 27 years old or younger.

This means that men are waiting longer to get married. The 23-, 24-, 25-, and 26-year-old men are now below the average of those getting married for the first time.

85. The average earnings were about \$342,509,984.

87. The average earnings of all 10 movies were about \$265 million.

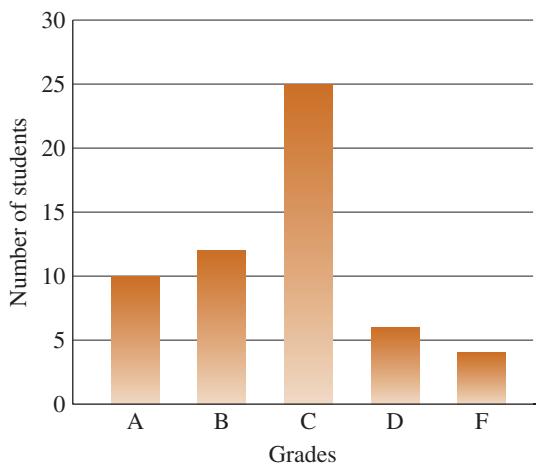
89. The average, or mean, of 2, 4, 5, 5, and 9 is their sum, 25, divided by 5, the number of numbers. So, the mean is 5. The average gives one possible measure of the center of the group.

Section 1.8

1. Delta 3. 10,000 5. about 124,000
 7. 40 9. Full-size
 11. The number of vehicles in for repair is 400.
 13. Australia 15. 575 17. 450
 19. The total paid for paint and lumber is \$45,000.
 21. Steel casting costs \$15,000 less than plastic.
 23. The company will pay \$40,000 for steel casting to double production.

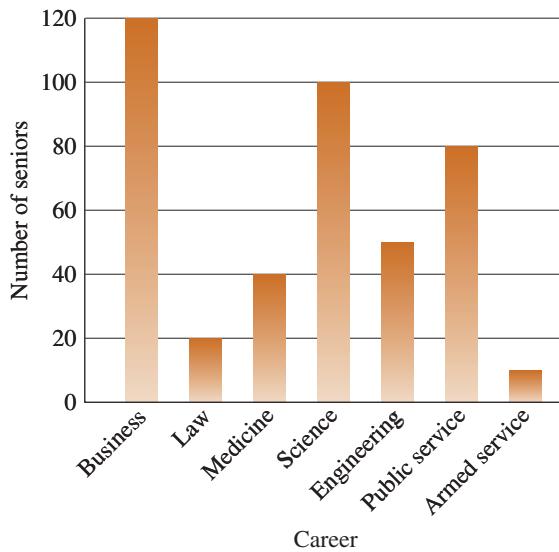
25.

History class grades

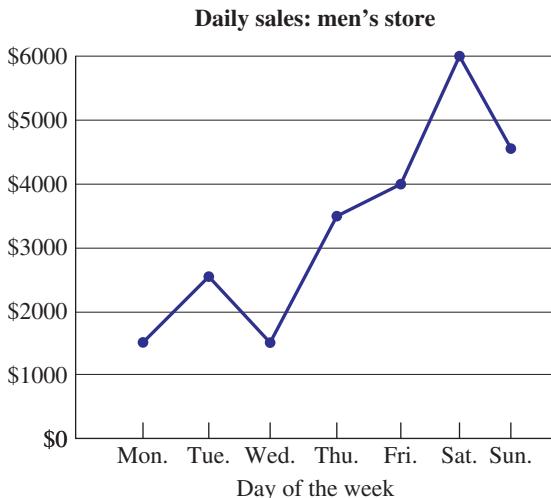


27.

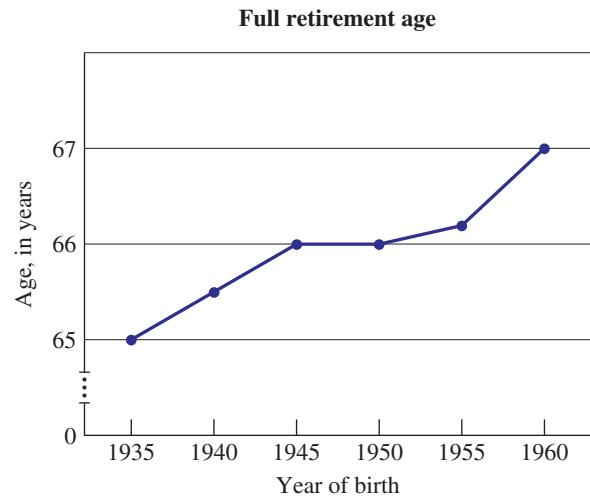
Career preference



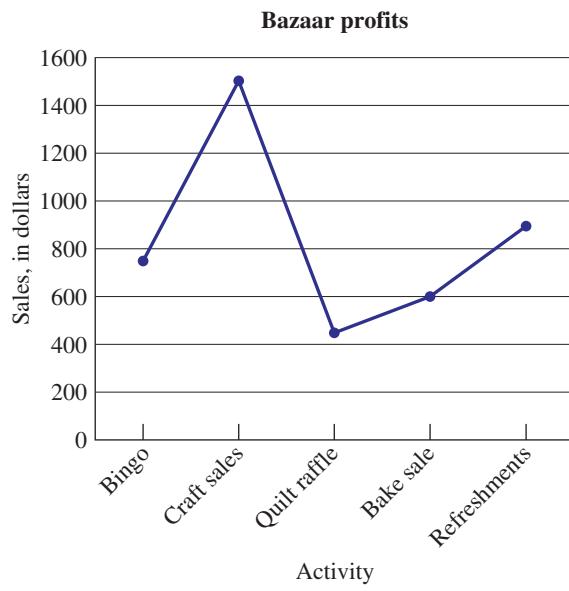
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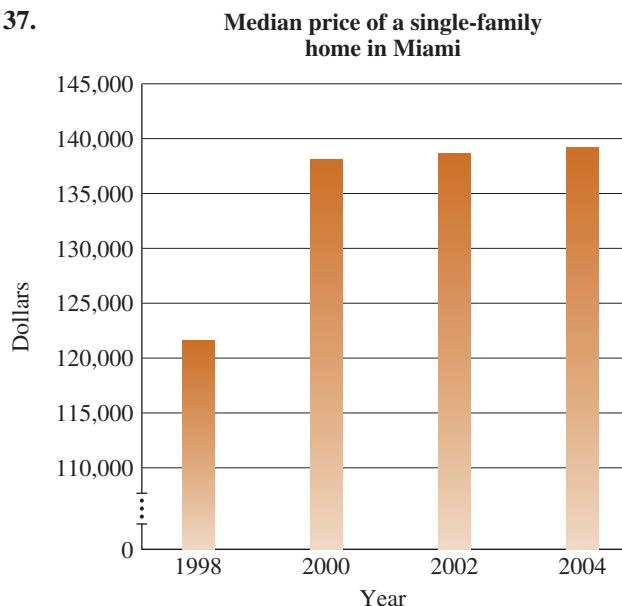
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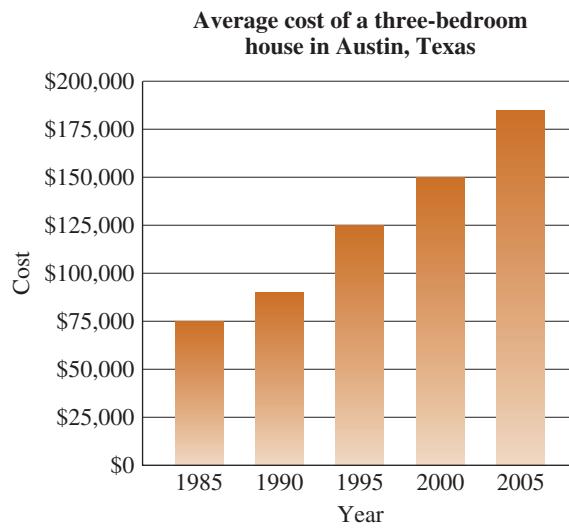
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37.



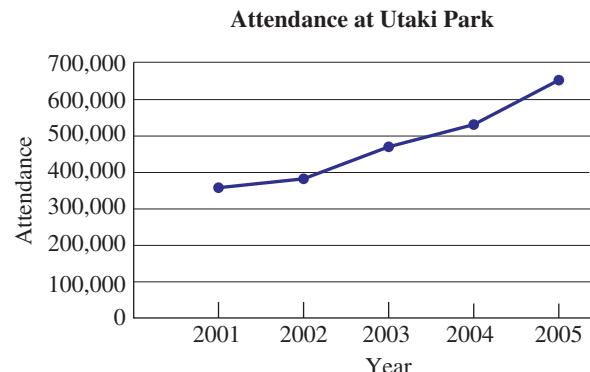
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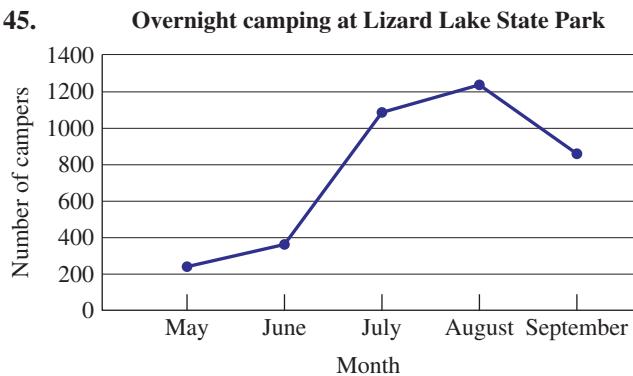
39. Tokyo had the largest population in 2000.

41. Mexico City is expected to grow the most, 3 million people, during the 15-year period.

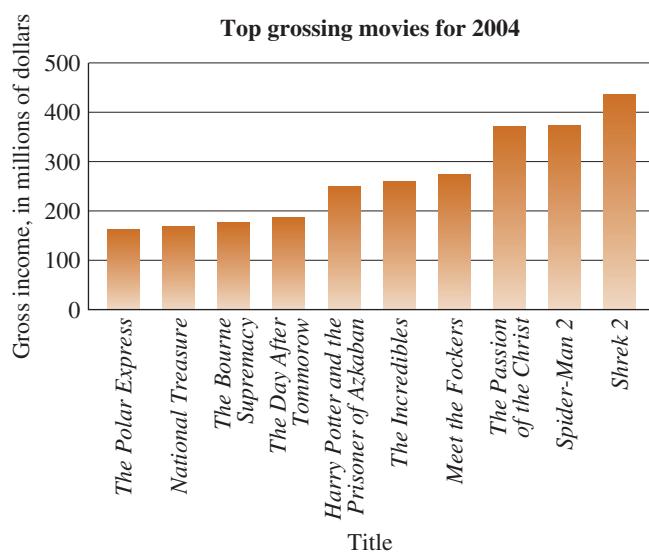
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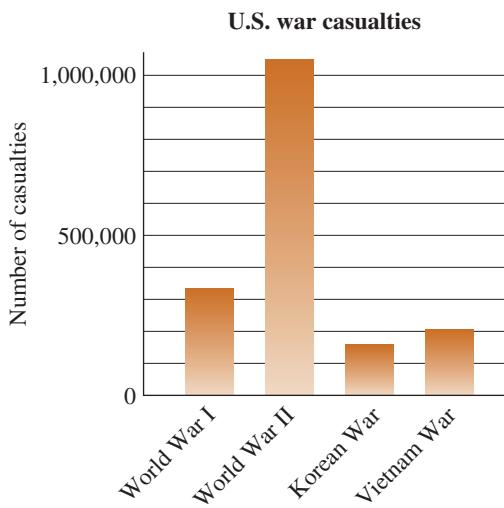
45.



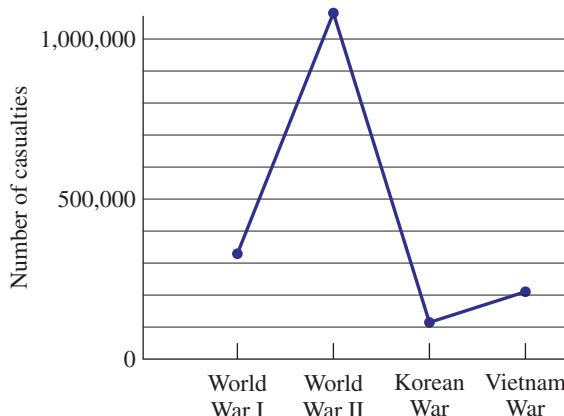
47.



49.



U.S. war casualties



Chapter 1 Review Exercises

Section 1.1

1. six hundred seven thousand, three hundred twenty-one
3. 62,337 5. < 7. <
9. 183,660, 183,700, 184,000, and 180,000
11. There are 74 more people in the under-15 group.
13. There are 383 more people in the under-15 group.

Section 1.2

15. 917 17. 1764 19. 221 21. 1332
23. 102 in. 25. 4000, 3200

Section 1.3

27. 11,774 29. 76,779 31. 3,000,000

Section 1.4

33. 15 35. 86 R 5 37. 5300

Section 1.5

39. 1331 41. 23,000 43. 712,000,000
45. \$3,400,000,000

Section 1.6

47. 65 49. 23

Section 1.7

51. 57, 64, no mode 53. 62, 63, 63
55. The average salary of the siblings, to the nearest hundred, is \$87,600.

Section 1.8

57. The difference in the temperatures is 15°.

Chapter 1 True/False Concept Review

1. false; to write one billion takes 10 digits. 2. true
3. true 4. false; seven is less than twenty-three.
5. false; $2567 > 2566$ 6. true 7. true 8. true
9. false; the sum is 87. 10. true 11. true

12. false; the product is 45. 13. true 14. true
 15. true 16. false; a number multiplied by 0 is 0.
 17. true 18. false; the quotient is 26. 19. true
 20. true 21. false; division by zero is undefined.
 22. false; the value is 49. 23. true 24. true
 25. false; the product is 450,000. 26. true
 27. false; in $(3 + 1)^2$, the addition is done first.
 28. false; in $(11 - 7) \cdot 9$, subtraction is done first.
 29. true 30. true 31. true 32. true
 33. false; the median of 34, 54, 14, 44, 67, 81, and 90 is 54.
 34. true

Chapter 1 Test

1. 212 2. 3266 3. 28 4. 15,836 5. <
 6. 55,000,000 7. 238,336 8. 730,061 9. 1991
 10. 372,600 11. 39,000 12. 230,000
 13. 729 14. 8687 15. 120 ft 16. 32,000
 17. 6158 18. four thousand, five 19. 86
 20. 259,656 21. 7730 22. 676,000,000
 23. 1160 R 32 24. 43 25. 345 cm²
 26. 514 27. average, 697; median, 743; mode, 795
 28. It will take the secretary 168 minutes or 2 hours and 48 minutes to type 18 pages.
 29. Each person will win \$13,862,000. Each person will receive \$693,100 per year.
 30. \$120,001–\$250,000 31. 40 32. 5
 33. Division B has the most employees.
 34. There are 120 more employees in division A.
 35. There is a total of 1020 employees.

CHAPTER 2

Section 2.1

1. no 3. yes 5. yes 7. no 9. yes
 11. no 13. yes 15. yes 17. no 19. 2, 3, 5
 21. 2 23. 3, 5 25. 5 27. 3, 5 29. yes
 31. no 33. yes 35. yes 37. no 39. yes
 41. no 43. no 45. yes 47. 6, 10 49. 9
 51. none 53. 6, 10 55. 6, 9, 10
 57. Yes, 3231 is divisible by 3.
 59. The band can march in rows of 5, but not rows of 3 or 10 with the same number in each row.
 61. Lucia can divide the class into groups of 5 or 6 because 120 is divisible by 5 and 6. Groups of 9 are not possible because 120 is not divisible by 9.
 63. Yes, each will get 104 comic books.
 65. There are 15 animals in the pen.
 67. There is a total of 24 elephants and riders in the act.
 69. A number is divisible by 5 when the remainder after dividing by 5 is 0. For example, 115 is divisible by 5 because $115 \div 5 = 23$ R 0. However, 116 is not divisible by 5 because $116 \div 5 = 123$ R 1.
 71. In the divisibility test for 2, we look at the last digit. If the last digit is even, then the number is divisible by 2. In the divisibility test for 3, we look at the *sum* of the digits. If the sum is divisible by 3, then the number is divisible by 3.

73. yes 75. Yes. A number divisible by both 3 and 10 is divisible by 30. 81. 5,060,000; 5,100,000
 83. 21 85. 196 cm² 87. 13, 26, 39, 52, 65, and 78
 89. 1231, 2462, 3693, 4924, 6155, and 7386

Section 2.2

1. 3, 6, 9, 12, 15 3. 17, 34, 51, 68, 85
 5. 21, 42, 63, 84, 105 7. 30, 60, 90, 120, 150
 9. 50, 100, 150, 200, 250 11. 54, 108, 162, 216, 270
 13. 64, 128, 192, 256, 320 15. 85, 170, 255, 340, 425
 17. 157, 314, 471, 628, 785
 19. 361, 722, 1083, 1444, 1805 21. yes 23. yes
 25. no 27. no 29. no 31. yes 33. yes
 35. yes 37. yes 39. multiple of 6 and 9
 41. multiple of 6 and 15 43. multiple of 6 and 15
 45. multiple of 13 47. multiple of 13 and 19
 49. No, Jean's car will not be selected because 14 is not a multiple of 4.
 51. The students should work problems 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, and 52.
 53. Because goats have 4 feet, the number of goat feet in the pen must be a multiple of 4. But 30 is not a multiple of 4, so Katy counted incorrectly.
 55. The team should check bottles with the numbers 510, 525, 540, 555, 570, 585, 600, 615, 630, 645, 660, 675, and 690.
 57. Yes, the gear is in its original position because 240 is a multiple of 20.
 59. Yes, the number of in-line skating injuries is a multiple of the number of skateboarding injuries. There were approximately three times as many injuries from in-line skating as from skateboarding.
 61. Using 2, 3, 4, and 10 with the Pythagorean triple 3, 4, 5 we have 6, 8, 10; 9, 12, 15; 12, 16, 20; and 30, 40, 50. Verification: $6^2 + 8^2 = 10^2$ ($36 + 64 = 100$); $9^2 + 12^2 = 15^2$ ($81 + 144 = 225$); $12^2 + 16^2 = 20^2$ ($144 + 256 = 400$); and $30^2 + 40^2 = 50^2$ ($900 + 1600 = 2500$).
 63. Using 2, 3, and 10 with the Pythagorean triple 5, 12, 13, we have 10, 24, 26; 15, 36, 39; and 50, 120, 130. Verification: $10^2 + 24^2 = 26^2$ ($100 + 576 = 676$); $15^2 + 36^2 = 39^2$ ($225 + 1296 = 1521$); $50^2 + 120^2 = 130^2$ ($2500 + 14,400 = 16,900$).
 65. The different words help us to describe the process we are using. Saying 9 is a factor of 135 means that 9 times some whole number is 135. The process is multiplication. The statement 9 is a divisor of 135 means that 9 divides 135 evenly. The process is division.
 67. yes 69. 5814 73. 1, 2, 3, 4, 6, 12
 75. 1, 3, 5, 15 77. 1, 17 79. 6 81. 17
 83. No, 12 is divisible by both 3 and 6 but not by 18.

Section 2.3

1. $1 \cdot 16, 2 \cdot 8, 4 \cdot 4$ 3. $1 \cdot 23$ 5. $1 \cdot 33, 3 \cdot 11$
 7. $1 \cdot 46, 2 \cdot 23$ 9. $1 \cdot 49, 7 \cdot 7$
 11. $1 \cdot 72, 2 \cdot 36, 3 \cdot 24, 4 \cdot 18, 6 \cdot 12, 8 \cdot 9$
 13. $1 \cdot 80, 2 \cdot 40, 4 \cdot 20, 5 \cdot 16, 8 \cdot 10$
 15. $1 \cdot 95, 5 \cdot 19$

- 17.** $1 \cdot 100$, $2 \cdot 50$, $4 \cdot 25$, $5 \cdot 20$, $10 \cdot 10$
19. $1 \cdot 105$, $3 \cdot 35$, $5 \cdot 21$, $7 \cdot 15$
21. $1 \cdot 112$, $2 \cdot 56$, $4 \cdot 28$, $7 \cdot 16$, $8 \cdot 14$
23. $1 \cdot 116$, $2 \cdot 58$, $4 \cdot 29$
25. $1 \cdot 333$, $3 \cdot 111$, $9 \cdot 37$
27. $1 \cdot 339$, $3 \cdot 113$ **29.** $1, 2, 4, 8, 16$
31. $1, 29$ **33.** $1, 3, 11, 33$ **35.** $1, 2, 23, 46$
37. $1, 2, 4, 13, 26, 52$
39. $1, 5, 13, 65$ **41.** $1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$
43. $1, 2, 3, 6, 13, 26, 39, 78$
45. $1, 2, 4, 5, 10, 20, 25, 50, 100$ **47.** $1, 2, 61, 122$
49. $1, 2, 3, 6, 9, 18, 27, 54, 81, 162$
51. $1 \cdot 444$, $2 \cdot 222$, $3 \cdot 148$, $4 \cdot 111$, $6 \cdot 74$, $12 \cdot 37$
53. $1 \cdot 652$, $2 \cdot 326$, $4 \cdot 163$ **55.** $1 \cdot 680$, $2 \cdot 340$,
 $4 \cdot 170$, $5 \cdot 136$, $8 \cdot 85$, $10 \cdot 68$, $17 \cdot 40$, $20 \cdot 34$

57. Number of Programs	Length of Each
1	120 min
2	60 min
3	40 min
4	30 min
5	24 min
6	20 min
8	15 min
10	12 min
12	10 min
15	8 min
20	6 min
24	5 min
30	4 min
40	3 min
60	2 min
120	1 min

- 59.** To provide 1 adult for every 4 infants, they need 6 adults. To provide 1 adult for every 3 infants, they need 8 adults.
61. The band can make rectangles of 2 by 25 or 10 by 5.
63. We know that the number of band members is not a multiple of 4 or 5. In fact, the number of members is 1 more than a multiple of 4 and also 1 more than a multiple of 5. The multiples of $4 + 1$ are in the list: 5, 9, 13, 17, 21, 25, 29, 33, The multiples of $5 + 1$ are in the list: 6, 11, 16, 21, 26, 31, The smallest number in both lists is 21.
65. The divisors of 6 are 1, 2, 3, and 6. The sum $1 + 2 + 3 = 6$. Therefore, 6 is a perfect number.
67. The divisors of 28 are 1, 2, 4, 7, 14, and 28. Because $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number.
69. The difference between a factor and a divisor is the operation we use. If we are multiplying, we generally use the word *factor*. If we are dividing, we generally use the word *divisor*.

- 71.** The largest factor is 991. **75.** 2499 **77.** 38,016
79. 33 **81.** 27 R 3
83. There is enough wire for 41 speakers, with 16 feet left over.

Section 2.4

- 1.** composite **3.** composite **5.** composite
7. prime **9.** composite **11.** prime
13. composite **15.** prime **17.** composite
19. composite **21.** composite **23.** composite
25. composite **27.** composite **29.** prime
31. prime **33.** composite **35.** prime
37. composite **39.** prime **41.** composite
43. composite **45.** composite **47.** prime
49. prime **51.** composite **53.** composite
55. The next year that is a prime number is 2011.
57. No, 1957 is not a prime number.
59. The number of impulses was 59.
61. Only one arrangement is possible because 19 is a prime number.

- 63.** $M_3 = 2^3 - 1 = 7$; $M_5 = 2^5 - 1 = 31$;
 and $M_7 = 2^7 - 1 = 127$
65. $M_3 \times (2^{3-1}) = 7 \times (2^2) = 7 \times 4 = 28$
67. A prime number has only two factors, itself and 1. The number 7 is prime. A composite number has three or more factors, itself, 1, and at least one other factor. The number 9 is composite because it has three factors, 1, 3, and 9. A composite number is composed of numbers other than itself and 1.
69. The number 6 has exactly four factors. They are 1, 2, 3, and 6. So does the number 15 with factors 1, 3, 5, and 15. Any number that is the product of exactly two prime numbers has four factors.
71. composite **75.** Yes. The final quotient is 5.
77. Yes. The final quotient is 29.
79. No. The quotient of 1029 and 3 is 343, which is not divisible by 3. The final quotient is 343.
81. No. The quotient of 2880 and 5 is 576, which is not divisible by 5. The final quotient is 576.
83. Yes. The final quotient is 11.

Section 2.5

- 1.** $2^2 \cdot 3$ **3.** $3 \cdot 5$ **5.** $3 \cdot 7$ **7.** $2^3 \cdot 3$ **9.** $2^2 \cdot 7$
11. $2 \cdot 17$ **13.** $3 \cdot 13$ **15.** $3^2 \cdot 5$ **17.** $2^2 \cdot 19$
19. $5 \cdot 17$ **21.** $2 \cdot 3^2 \cdot 5$ **23.** $2^2 \cdot 23$ **25.** $3 \cdot 5 \cdot 7$
27. prime **29.** $2^2 \cdot 3 \cdot 13$ **31.** $7 \cdot 23$ **33.** $2^2 \cdot 3^2 \cdot 5$
35. $2^3 \cdot 5^2$ **37.** $2 \cdot 5 \cdot 31$ **39.** $3 \cdot 5 \cdot 23$
41. $2 \cdot 3^2 \cdot 5^2$ **43.** $17 \cdot 19$ **45.** $3^3 \cdot 17$
47. $3 \cdot 5 \cdot 31$ **49.** 5^4 **51.** $2 \cdot 5 \cdot 7 \cdot 17$
53. Answers will vary. **55.** Answers will vary.
57. The numbers 119 and 143 are relatively prime because they have no common factors, as we can see from the prime factorization of each: $119 = 7 \cdot 17$ and $143 = 11 \cdot 13$.
59. The number 97 is relatively prime to 180.
61. A number is written in prime-factored form when (1) it is a prime number or (2) it is written as a product and every factor in the product is a prime number. For example, $18 = 2 \cdot 3^2$ is in prime-factored form, whereas $18 = 3 \cdot 6$ and $18 = 2 \cdot 9$ are not. **63.** $2 \cdot 3 \cdot 5 \cdot 73$ **65.** $2^8 \cdot 7$
67. The perimeter is 36 in. **69.** yes **71.** no
73. yes **75.** Answers will vary, but may include 15, 30, 45, and 60.

Section 2.6

1. 12 3. 21 5. 30 7. 16 9. 12 11. 36
13. 12 15. 18 17. 30 19. 10 21. 36
23. 48 25. 60 27. 48 29. 72 31. 48
33. 180 35. 24 37. 48 39. 168 41. 224
43. 204 45. 1400 47. 24 49. 240

51. The two gears will return to their original positions in one turn of the 36-tooth gear because 12 is a divisor of 36.
53. The two gears will return to their original positions in two turns of the 36-tooth gear.
55. The least price they could pay is \$100, which is the LCM of 20, 50, and 5.

57. To find the LCM of 20, 24, and 45, first find the prime factorization of each number. Then build the LCM by using each different prime factor to the highest power that occurs in any single number.

59. 720 61. 1296

65. 1, 2, 4, 5, 10, 20, 23, 46, 92, 115, 230, 460
67. 1, 2, 4, 5, 7, 8, 10, 14, 16, 20, 28, 35, 40, 56, 70, 80,
112, 140, 280, 560 69. yes 71. no
73. The average attendance was 1350 people.

Chapter 2 Review Exercises

Section 2.1

1. 6, 36, 636 3. 15, 255, 525 5. 36, 63
7. 444, 666, 888 9. 645 11. 7, 9 13. 6, 7, 9
15. 4, 5, 10 17. 6, 15
19. Yes, the profit can be evenly divided because 2060 is divisible by 5.

Section 2.2

21. 8, 16, 24, 32, 40 23. 51, 102, 153, 204, 255
25. 85, 170, 255, 340, 425 27. 122, 244, 366, 488, 610
29. 141, 282, 423, 564, 705 31. multiple of 6 and 9
33. multiple of 6 and 15 35. multiple of 15
37. multiple of 12 and 16 39. Yes, the prediction is accurate because 605 is the 11th multiple of 55.

Section 2.3

41. $1 \cdot 15, 3 \cdot 5$ 43. $1 \cdot 38, 2 \cdot 19$
45. $1 \cdot 236, 2 \cdot 118, 4 \cdot 59$ 47. $1 \cdot 338, 2 \cdot 169, 13 \cdot 26$
49. $1 \cdot 343, 7 \cdot 49$ 51. 1, 3, 13, 39
53. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
55. 1, 97 57. 1, 2, 3, 6, 17, 34, 51, 102
59. The possible energy costs are \$1590 for 1 person, \$795 for 2 people, \$530 for 3 people, \$318 for 5 people, \$265 for 6 people, and \$159 for 10 people.

Section 2.4

61. prime 63. prime 65. composite
67. prime 69. composite 71. prime
73. prime 75. composite 77. prime
79. The prime number year after 2003 is 2011.

Section 2.5

81. 3^3 83. $2 \cdot 19$ 85. $2^2 \cdot 13$ 87. $2 \cdot 3 \cdot 37$
89. $2^2 \cdot 3^2 \cdot 7$ 91. $2 \cdot 3 \cdot 43$ 93. $2^2 \cdot 5 \cdot 13$

95. prime 97. $5 \cdot 53$ 99. $2006 = 2 \cdot 17 \cdot 59$;
 $2007 = 3^2 \cdot 223$; $2008 = 2^3 \cdot 251$; $2009 = 7^2 \cdot 41$;
and so forth.

Section 2.6

101. 24 103. 60 105. 144 107. 100
109. 210 111. 24 113. 72 115. 180
117. 840

119. Possible answers include 2 and 49; 14 and 49; 2 and 98; and 14 and 98. Other answers are possible.

Chapter 2 True/False Concept Review

1. false; not all multiples of 3 end with the digit 3. For example, 15 is a multiple of 3.
2. true 3. true 4. true 5. true 6. true
7. false; only one multiple of 300 is also a factor of 300—itself.
8. false; the square of 25 is 625. Twice 25 is 50.
9. false; not all natural numbers ending in 6 are divisible by 6. For example, 16 is not divisible by 6. 10. true
11. false; not all natural numbers ending in 9 are divisible by 3. For example, 29 is not divisible by 3. 12. true
13. false; the sum of the digits $7 + 7 + 7 + 7 + 3 = 31$, which is not divisible by 3. 14. true 15. true
16. false; all prime numbers except 2 are odd. 17. true
18. false; every prime number has exactly two factors.
19. true 20. true
21. false; the least common multiple (LCM) of three different prime numbers is the product of the three numbers.
22. true
23. false; the largest divisor of the least common multiple (LCM) of three numbers is not necessarily the largest of the three numbers. For example, the LCM of 2, 4, and 6 is 12. The largest divisor of 12 is 12, not 6. 24. true

Chapter 2 Test

1. yes 2. 1, 2, 4, 7, 8, 14, 16, 28, 56, 112 3. yes
4. no 5. 96 6. $1 \cdot 75, 3 \cdot 25, 5 \cdot 15$ 7. $2^3 \cdot 5 \cdot 7$
8. 252 9. yes 10. 208, 221, 234, 247 11. no
12. prime 13. composite 14. $5 \cdot 11^2$ 15. 504
16. 2 17. 299 18. 360
19. No, a set of prime factors has only one product.
20. Any two of the following sets of numbers: 2, 3, 7; or 2, 3, 14; or 2, 3, 21; or 2, 3, 42; or 2, 7, 42; or 3, 7, 14; or 6, 7, 42; or 14, 21, 42.

CHAPTER 3

Section 3.1

1. $\frac{4}{7}$ 3. $\frac{2}{5}$ 5. $\frac{4}{5}$ 7. $\frac{4}{3}$ 9. $\frac{11}{8}$ 11. $\frac{13}{9}$
13. Proper fractions: $\frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}$; Improper fractions:
 $\frac{11}{11}, \frac{12}{11}, \frac{13}{11}$

15. Proper fractions: none; Improper fractions:

$$\frac{17}{16}, \frac{18}{17}, \frac{29}{19}, \frac{30}{21}, \frac{23}{23}$$

17. Proper fractions: $\frac{6}{11}, \frac{10}{12}$; Improper fractions: $\frac{10}{8}, \frac{11}{6}, \frac{9}{9}$

19. Proper fractions: $\frac{6}{10}, \frac{18}{20}, \frac{24}{25}$; Improper fractions:

$$\frac{4}{4}, \frac{5}{4}, \frac{12}{12}, \frac{15}{13}, \frac{17}{17}, \frac{23}{20} \quad \text{21. } 4\frac{3}{4} \quad \text{23. } 12\frac{5}{7} \quad \text{25. } 12\frac{4}{9}$$

$$27. \frac{6\frac{11}{13}}{13} \quad \text{29. } 14\frac{21}{22} \quad \text{31. } 22\frac{1}{15} \quad \text{33. } \frac{41}{9}$$

$$35. \frac{15}{1} \quad \text{37. } \frac{31}{4} \quad \text{39. } \frac{109}{4} \quad \text{41. } \frac{202}{5}$$

43. $\frac{275}{6}$ **45.** The fraction $\frac{0}{1}$ is a proper fraction. Any whole number except 0 can be put in the denominator.

47. The error occurred because both numerator and

denominator were multiplied by 4: $4\frac{2}{3} = \frac{14}{3}$.

$$\text{49. } \frac{7}{10}$$

$$\text{51. } \frac{11}{8} \quad \text{53. } \begin{array}{|c|c|c|c|c|c|c|} \hline \text{blue} & \text{blue} & \text{white} & \text{white} & \text{white} & \text{white} & \text{white} \\ \hline \end{array}$$

55. Of the class, $\frac{17}{31}$ are women.

57. The mixed numbers $11\frac{1}{2}$ and $6\frac{1}{8}$ can be written as improper fractions because they represent numbers larger than 1. The dimensions of nonstandard mail are longer than $\frac{23}{2}$ in., taller than $\frac{49}{8}$ in., and/or thicker than $\frac{1}{4}$ in.

59. The tank is $\frac{3}{8}$ full.

61. The number 6 is closest to the mark.

63. Beverly needs 99 tiles in each row.

65. Seven inches is $\frac{7}{12}$ of a foot, so $5'7"$ is $5\frac{7}{12}$ ft.

67. The cup contains $\frac{5}{8}$ cup of oil.

69. This is an improper fraction because the numerator is larger than the denominator.

71. To change $\frac{34}{5}$ to a mixed number, divide 34 by 5.

Write the remainder, 4, over the divisor, 5, to form the

fraction part. We have $\frac{34}{5} = 6\frac{4}{5}$. To change $7\frac{3}{8}$ to an

improper fraction multiply the denominator, 8, times the whole number, 7. Add the product to 3 to get 59. Write this

sum over the denominator. So $7\frac{3}{8} = \frac{59}{8}$.

73. A mixed number has a value of 1 or more. A proper fraction has a value less than 1.

$$\text{75. } \frac{144}{9}, \frac{2304}{144}$$

77. There will be 87 pieces of rope, 5 pieces for each child. There will be 12 pieces left over.

$$\text{83. } 39 \quad \text{85. } 15 \quad \text{87. composite} \quad \text{89. } 3^2 \cdot 31$$

91. Bonnie averaged 201 miles per tank and about 50 miles per gallon.

Section 3.2

$$\text{1. } \frac{2}{3} \quad \text{3. } \frac{2}{3} \quad \text{5. } \frac{2}{5} \quad \text{7. } \frac{3}{5} \quad \text{9. } \frac{3}{4} \quad \text{11. } \frac{2}{5}$$

$$\text{13. } \frac{7}{8} \quad \text{15. } \frac{15}{8} \quad \text{17. } \frac{7}{9} \quad \text{19. } \frac{3}{5} \quad \text{21. } 9$$

$$\text{23. } \frac{7}{3} \quad \text{25. } \frac{1}{3} \quad \text{27. } \frac{1}{3} \quad \text{29. } \frac{3}{4} \quad \text{31. } \frac{31}{37}$$

$$\text{33. } \frac{2}{3} \quad \text{35. } \frac{3}{5} \quad \text{37. } \frac{1}{3} \quad \text{39. } \frac{9}{16} \quad \text{41. } \frac{3}{4}$$

$$\text{43. } 6 \quad \text{45. } \frac{17}{21} \quad \text{47. } \frac{16}{21} \quad \text{49. } \frac{3}{4} \quad \text{51. } \frac{21}{34}$$

$$\text{53. } \frac{7}{12} \quad \text{55. } \frac{14}{15} \quad \text{57. } \frac{3}{5}$$

59. Four inches is $\frac{1}{3}$ of a foot, so $5'4"$ is $5\frac{1}{3}$ feet.

61. Martin Brodeur saved $\frac{15}{16}$ of the shots on goal. The opposing team scored two points on him.

63. The fraction of fouled plugs is $\frac{1}{3}$.

65. She has $\frac{1}{8}$ of her shift left.

67. The DA prosecuted $\frac{11}{16}$ of his cases successfully.

69. Of the elk, $\frac{3}{4}$ are cows.

$$\text{71. } 270 \text{ in.} = \frac{270}{12} \text{ ft} = \frac{45}{2} \text{ ft} = 22\frac{1}{2} \text{ ft}$$

73. Find the prime factorization of both numerator and denominator. Then eliminate the common factors. The product of the remaining factors is the simplified fraction.

$$\frac{3 \cdot 5 \cdot 5 \cdot 7}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 5} = \frac{7}{15}$$

75. Yes, all simplify to $\frac{5}{12}$. **79.** 616 **81.** 990

83. 3552 **85.** 18 **87.** 16

Section 3.3

$$\text{1. } \frac{2}{25} \quad \text{3. } \frac{15}{28} \quad \text{5. } \frac{7}{8} \quad \text{7. } \frac{1}{5} \quad \text{9. } \frac{16}{5} \text{ or } 3\frac{1}{5}$$

$$\text{11. } \frac{2}{15} \quad \text{13. } \frac{3}{2} \text{ or } 1\frac{1}{2} \quad \text{15. } 1 \quad \text{17. } \frac{7}{12} \quad \text{19. } 0$$

$$\text{21. } \frac{10}{7} \quad \text{23. } \frac{1}{3} \quad \text{25. none} \quad \text{27. } \frac{14}{51}$$

$$\text{29. } 17 \quad \text{31. } \frac{3}{8} \quad \text{33. } \frac{28}{27} \text{ or } 1\frac{1}{27} \quad \text{35. } 3$$

37. $\frac{9}{25}$ 39. $\frac{5}{2}$ or $2\frac{1}{2}$ 41. $\frac{49}{30}$ or $1\frac{19}{30}$ 43. $\frac{3}{4}$

45. $\frac{10}{3}$ or $3\frac{1}{3}$ 47. $\frac{10}{11}$ 49. 1 51. $\frac{8}{45}$

53. $\frac{3}{10}$ 55. $\frac{41}{4}$ or $10\frac{1}{4}$ 57. $\frac{8}{3}$ or $2\frac{2}{3}$

59. $\frac{3}{4} \cdot \frac{7}{6} = \frac{7}{8}$ because $\frac{3}{4} \cdot \frac{7}{6}$ can be simplified to $\frac{1}{4} \cdot \frac{7}{2}$.

then we multiply 1 times 7 to get 7 and 4 times 2 to get 8.

61. The error comes from inverting $\frac{4}{5}$ before multiplying.

We invert only in division: $\frac{3}{8} \cdot \frac{4}{5} = \frac{12}{40}$ or $\frac{3}{10}$.

63. There are 8 combinations of four children with a boy as firstborn, and 8 combinations with a girl as firstborn.

65. One combination has a firstborn boy followed by three girls.

67. When it is $\frac{3}{4}$ full, the container holds $\frac{21}{20}$ or $1\frac{1}{20}$ gallons.

69. Becky might save 41 gallons per year.

71. About 816,714 people will survive 5 years after their diagnosis.

73. About 28,339,200 Mexicans did not have access to safe drinking water.

75. Multiply $\frac{35}{24}$ by $\frac{14}{40}$ (the reciprocal of $\frac{40}{14}$). Then simplify:

$$\frac{35}{24} \div \frac{40}{14} = \frac{49}{96}. \quad 77. \frac{9}{10} \quad 83. 3\frac{3}{5} \quad 85. 1\frac{13}{16}$$

$$87. \frac{9}{2} \quad 89. \frac{16}{5}$$

91. Kevin can haul 15 scoops.

Section 3.4

1. $1\frac{5}{16}$ 3. $4\frac{3}{8}$ 5. $7\frac{1}{3}$ 7. $22\frac{1}{2}$ 9. 24

11. $3\frac{1}{3}$ 13. $18\frac{3}{4}$ 15. 0 17. 55 19. $16\frac{1}{2}$

21. $32\frac{1}{15}$ 23. 105 25. $3\frac{1}{5}$ 27. $\frac{3}{4}$ 29. $4\frac{1}{10}$

31. $1\frac{5}{13}$ 33. $1\frac{1}{3}$ 35. $\frac{16}{25}$ 37. $\frac{5}{26}$ 39. $\frac{11}{20}$

41. $14\frac{1}{2}$ 43. $14\frac{2}{3}$ 45. $17\frac{7}{16}$ 47. $4\frac{2}{3}$

49. $70\frac{2}{5}$ 51. $\frac{1}{2}$

53. Mixed numbers cannot be multiplied by multiplying the whole number parts and the fraction parts separately.

Change them to fractions first: $\frac{5}{3} \cdot \frac{3}{2} = \frac{5}{2} = 2\frac{1}{2}$.

55. The interior of the shed has $6253\frac{1}{2}$ in² of space.

57. The first ring is \$1400 per carat and the second ring is \$2000 per carat.

59. The iron content is 22 parts per million.

61. Mike can store 28 DVDs on the shelf.

63. The less-efficient car emits 3000 lb of CO₂.

65. Shane will need three 2 × 4s.

67. There are exactly $77\frac{5}{7}$ boards in 272 in. Shane will need 78 boards.

69. When we divide by $1\frac{1}{2}$ we are dividing by $\frac{3}{2}$. Dividing by $\frac{3}{2}$ is the same as multiplying by $\frac{2}{3}$. Because $\frac{2}{3}$ is less than 1, the multiplication gives us a smaller number.

71. 1230 75. $8\frac{1}{8}$ 77. $1\frac{2}{15}$ 79. $13\frac{1}{5}$ 81. 88

83. No, because 234,572 is not a multiple of 3.

Getting Ready for Algebra

1. $x = \frac{3}{4}$ 3. $y = \frac{10}{9}$ or $y = 1\frac{1}{9}$ 5. $z = \frac{5}{16}$

7. $x = \frac{17}{8}$ or $x = 2\frac{1}{8}$ 9. $a = \frac{10}{7}$ or $a = 1\frac{3}{7}$

11. $b = \frac{3}{2}$ or $b = 1\frac{1}{2}$ 13. $z = \frac{1}{2}$ 15. $a = \frac{11}{30}$

17. The distance is $\frac{3}{4}$ mi.

19. 180 lb of tin were recycled.

Section 3.5

1. $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}$ 3. $\frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \frac{5}{30}$

5. $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}, \frac{20}{45}$ 7. $\frac{22}{26}, \frac{33}{39}, \frac{44}{52}, \frac{55}{65}$

9. $\frac{14}{6}, \frac{21}{9}, \frac{28}{12}, \frac{35}{15}$ 11. 7 13. 24 15. 28

17. 3 19. 15 21. 4 23. 38 25. 16

27. 38 29. 104 31. $\frac{2}{13}, \frac{4}{13}, \frac{6}{13}$ 33. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$

35. $\frac{1}{3}, \frac{3}{8}, \frac{1}{2}$ 37. true 39. false 41. true

43. $\frac{3}{7}, \frac{3}{5}, \frac{2}{3}$ 45. $\frac{4}{15}, \frac{5}{6}, \frac{13}{15}, \frac{9}{10}$ 47. $\frac{11}{24}, \frac{17}{36}, \frac{35}{72}$

49. $\frac{3}{7}, \frac{13}{28}, \frac{17}{35}$ 51. $1\frac{9}{16}, 1\frac{5}{8}, 1\frac{13}{20}$ 53. true

55. true 57. true 59. LCM = 24; $\frac{12}{24}, \frac{16}{24}, \frac{4}{24}, \frac{15}{24}$

61. From smallest to largest, the strengths are

$\frac{3}{32}, \frac{1}{8}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{1}{2}$, and $\frac{9}{16}$.

63. More of the population is from the ethnic group, Fula.

65. African Americans make up a larger share of the total U.S. population.

67. Chang's measurement is heaviest.

69. Moe owns the largest part and Curly owns the smallest.

71. To simplify a fraction is to find a fraction with a smaller numerator and a smaller denominator that is equivalent to the original fraction. To build a fraction is to find a fraction with a larger numerator and a larger denominator that is equivalent to the original.

73. $\frac{50}{70}, \frac{65}{91}, \frac{115}{161}, \frac{560}{784}, \frac{2905}{4067}$ **77.** 8 **79.** 30

81. 32 **83.** $2^3 \cdot 3^2$

85. The pharmacist should give her 56 capsules.

Section 3.6

1. $\frac{9}{11}$ **3.** $\frac{2}{3}$ **5.** 2 **7.** $\frac{5}{6}$ **9.** $\frac{10}{13}$ **11.** $1\frac{1}{4}$

13. $\frac{1}{2}$ **15.** $\frac{5}{16}$ **17.** $\frac{7}{10}$ **19.** $\frac{3}{4}$ **21.** $\frac{13}{24}$

23. $\frac{5}{12}$ **25.** $\frac{13}{16}$ **27.** $\frac{3}{5}$ **29.** $\frac{4}{5}$ **31.** $\frac{26}{35}$

33. $1\frac{7}{60}$ **35.** $1\frac{2}{5}$ **37.** $\frac{23}{32}$ **39.** $2\frac{5}{8}$ **41.** $2\frac{7}{27}$

43. $2\frac{19}{90}$ **45.** $\frac{2}{3}$ **47.** $1\frac{29}{100}$ **49.** $1\frac{13}{144}$

51. Contributions and tuition and fees were $\frac{149}{200}$ of the total revenues for the school.

53. Asia and Africa contain more than $\frac{19}{20}$ of the world's Muslims.

55. $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$. When adding fractions, the parts of the unit must be the same size. So we must keep the common denominator in the result and not add the denominators.

57. The recipe makes $3\frac{3}{4}$ gallons of punch.

59. Jonnie needs a bolt that is $1\frac{13}{16}$ in. long.

61. The length of the pin is $\frac{3}{4}$ in.

63. The length of the rod is $3\frac{3}{8}$ in.

65. Shane has $\frac{5}{6}$ of the deck boards installed.

67. Each denominator indicates into how many parts a unit has been divided. Unless the units are the same size it would be like adding apples and oranges. You cannot add units of different sizes.

69. $\frac{4}{7}$ **71.** Jim consumes 4 g of fat, which is $\frac{6}{55}$ of the number of calories.

73. $10\frac{5}{12}$ **75.** $19\frac{1}{15}$ **77.** $6\frac{11}{16}$ **79.** 78

81. The average class size is 40.

Section 3.7

1. $4\frac{6}{7}$ **3.** $11\frac{3}{5}$ **5.** $5\frac{5}{6}$ **7.** $6\frac{2}{9}$ **9.** $7\frac{5}{14}$

11. $10\frac{2}{15}$ **13.** $15\frac{1}{16}$ **15.** $11\frac{2}{3}$ **17.** $26\frac{5}{8}$

19. $11\frac{5}{24}$ **21.** $10\frac{1}{6}$ **23.** $20\frac{7}{12}$ **25.** $49\frac{5}{6}$

27. $337\frac{11}{36}$ **29.** $67\frac{7}{30}$ **31.** $55\frac{11}{12}$ **33.** $94\frac{2}{45}$

35. $103\frac{11}{70}$ **37.** $50\frac{3}{5}$ **39.** $14\frac{8}{15}$ **41.** $64\frac{3}{8}$

43. Nancy needs $98\frac{1}{2}$ in. of molding for the frame.

45. Elizabeth needs $10\frac{5}{8}$ yd of fabric for the dress and jacket.

47. Scott needs $11\frac{1}{4}$ lb of seafood for his paella.

49. The perimeter is $42\frac{1}{6}$ in.

51. The total rainfall for the three cities is $92\frac{5}{24}$ in.

53. There are $61\frac{27}{80}$ mi of road that need to be resurfaced. It will cost about \$920,000.

55. It is $17\frac{1}{2}$ in. from point A to point D.

57. I prefer the methods of this section rather than changing to improper fractions first because (a) it avoids fractions with large numerators and (b) requires fewer steps.

59. Yes, the statement is true.

63. 84 **65.** 424 **67.** 15 **69.** $\frac{6}{11}$

71. It will take 186 lb of fresh prunes.

Section 3.8

1. $\frac{5}{9}$ **3.** $\frac{1}{3}$ **5.** $\frac{1}{3}$ **7.** $\frac{1}{2}$ **9.** $\frac{7}{16}$ **11.** $\frac{7}{45}$

13. $\frac{1}{6}$ **15.** $\frac{1}{18}$ **17.** $\frac{13}{20}$ **19.** $\frac{9}{20}$ **21.** $\frac{1}{24}$

23. $\frac{17}{48}$ **25.** $\frac{4}{21}$ **27.** $\frac{19}{48}$ **29.** $\frac{1}{18}$ **31.** $\frac{13}{24}$

33. $\frac{9}{20}$ **35.** $\frac{5}{24}$ **37.** $\frac{19}{75}$ **39.** $\frac{17}{48}$ **41.** $\frac{5}{36}$

43. $\frac{3}{200}$ **45.** $\frac{1}{72}$ **47.** The board will be $1\frac{3}{40}$ in. thick.

49. Ben still has $\frac{3}{10}$ of his refund, or \$117, left.

51. In that year, $\frac{9}{25}$ of the population did not hold stock.

53. About $\frac{1}{10}$ of the total sales was neither CDs nor cassettes.

55. For Americans 65 or older, $\frac{21}{50}$ of the population are male.

57. The difference in diameters is $\frac{11}{16}$ in.

59. The interest payments were $\frac{2}{25}$ of the total budget.

61. They are not equal because the denominators (parts of the unit), 4 and 2, are not the same. We cannot get the result by subtracting the numerators and the denominators. We must first find the common denominator. So,

$$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}. \quad \text{63. } \frac{218}{625}$$

65. The project will require 101 bricks, but 3 of them will be cut to fit. One brick, $3\frac{3}{4}$ in. wide, plus the mortar joint on one end, $\frac{3}{8}$ in., has a width of $4\frac{1}{8}$ in. The length of the garden is 10 ft, or 120 in. We must add 16 in. to the length to account for the border on the ends. The width of the garden is 72 in. We do not need to add on to the width. Each length requires

$136 \div 4\frac{1}{8}$ bricks, or about 33 bricks. Each width requires

$72 \div 4\frac{1}{8}$ bricks, or about 18 bricks.

67. 37,726 69. $4\frac{1}{4}$ 71. $5\frac{5}{18}$ 73. $9\frac{1}{3}$

75. They can lay 11,925 bricks.

Section 3.9

1. $7\frac{2}{7}$ 3. $2\frac{1}{5}$ 5. $5\frac{1}{8}$ 7. $2\frac{3}{7}$ 9. $78\frac{5}{18}$

11. $2\frac{1}{2}$ 13. $14\frac{1}{5}$ 15. $172\frac{7}{9}$ 17. $6\frac{11}{12}$ 19. $9\frac{1}{2}$

21. $56\frac{1}{3}$ 23. $15\frac{23}{60}$ 25. $21\frac{3}{4}$ 27. $15\frac{47}{48}$

29. $21\frac{1}{4}$ 31. $2\frac{7}{32}$ 33. $9\frac{29}{36}$ 35. $74\frac{7}{8}$

37. $27\frac{31}{120}$ 39. $27\frac{31}{36}$ 41. $16\frac{19}{78}$ 43. $\frac{5}{6}$

45. If 1 is borrowed from 16 in order to subtract the fraction part, $\frac{1}{4}$, then the whole number part is $15 - 13$, or 2. So

$$16 - 13\frac{1}{4} = \left(15 + \frac{4}{4}\right) - \left(13 + \frac{1}{4}\right) = 2\frac{3}{4}.$$

47. She trims $2\frac{3}{8}$ lb. 49. He has $12\frac{1}{20}$ tons left.

51. The leftover piece is $4\frac{2}{3}$ ft long and 10 ft wide.

53. Amber is charged for $12\frac{7}{8}$ extra pounds.

55. They have $19\frac{3}{10}$ mi to go.

57. The Rodrigases recycle $315\frac{9}{10}$ more pounds per year.

59. Haja can buy a bolt that is $\frac{3}{8}$ in. in diameter.

61. First change the fractions to equivalent fractions with a common denominator. Thus, $\frac{1}{3} = \frac{8}{24}$ and $\frac{5}{8} = \frac{15}{24}$. Because $\frac{15}{24}$ is the larger fraction, we must borrow 1 from the 4 and add it to $\frac{8}{24}$. We have $4\frac{8}{24} = 3 + 1\frac{8}{24} = 3\frac{32}{24}$. Now we can subtract: $3\frac{32}{24} - 2\frac{15}{24} = 1\frac{17}{24}$.

63. No, they are not equal.

65. The snail's net distance is $3\frac{2}{3}$ ft. It will take 6 days to gain over 20 ft.

67. 80 69. 28 71. 12 73. $\frac{8}{15}$

75. Karla averaged 28 miles per gallon.

Getting Ready for Algebra

1. $a = \frac{1}{2}$ 3. $c = \frac{5}{8}$ 5. $x = \frac{11}{72}$ 7. $y = 1\frac{38}{63}$

9. $a = 1\frac{11}{40}$ 11. $c = 3\frac{11}{24}$ 13. $x = 1\frac{1}{36}$

15. $3\frac{1}{6} = w$ 17. $a = 36\frac{4}{9}$ 19. $c = 21\frac{2}{21}$

21. The height 10 years ago was $43\frac{9}{16}$ ft.

23. She bought 46 lb of nails.

Section 3.10

1. $\frac{7}{13}$ 3. $\frac{2}{17}$ 5. $\frac{1}{2}$ 7. 1 9. $\frac{1}{40}$ 11. $\frac{5}{12}$

13. $\frac{19}{48}$ 15. $\frac{5}{8}$ 17. $\frac{5}{12}$ 19. $\frac{1}{4}$ 21. $\frac{5}{6}$

23. 0 25. $\frac{29}{64}$ 27. $\frac{1}{6}$ 29. $\frac{4}{9}$ 31. $\frac{11}{21}$

33. $\frac{4}{11}$ 35. $\frac{16}{45}$ 37. $2\frac{1}{3}$ 39. $\frac{15}{32}$ 41. $\frac{19}{30}$

43. $3\frac{2}{9}$ 45. $7\frac{1}{5}$ 47. $\frac{5}{12}$ 49. $2\frac{2}{5}$ 51. $3\frac{37}{54}$

53. $3\frac{1}{16}$ 55. The average length is $33\frac{2}{3}$ in.

57. The mothers need 55 cups of cereal, $1\frac{1}{4}$ cups of butter, $3\frac{3}{4}$ cups of chocolate chips, $27\frac{1}{2}$ cups of marshmallows, 40 cups of pretzels, and 10 cups of raisins.

59. The average winning throw was $69 \text{ ft } 2\frac{19}{40}$ in.

61. There are $135\frac{1}{4}$ oz of seafood. The average cost is 61¢ per ounce.

63. Shane will need three 2×12 s, although there will be 6 ft left over.

65. 1. Do operations in parentheses first, following steps 2, 3, and 4. 2. Do exponents next. 3. Do multiplication and division as they occur. 4. Do addition and subtraction as they occur. The order is the same as for whole numbers.

67. $2\frac{57}{1000}$ **69.** The total amount paid is \$19,444.

71. $6\frac{1}{9}$ **73.** $4\frac{4}{27}$ **75.** $\frac{1}{7}$

77. $2 \cdot 5^2 \cdot 13$ **79.** The tabletop is $1\frac{1}{4}$ in. thick.

Chapter 3 Review Exercises

Section 3.1

1. $\frac{4}{5}$ **3.** $\frac{9}{10}$ **5.** Proper fractions: $\frac{11}{12}, \frac{3}{20}$

7. $6\frac{1}{12}$ **9.** $114\frac{2}{3}$ **11.** $\frac{77}{12}$ **13.** $\frac{77}{6}$ **15.** $\frac{17}{1}$

17. The wholesaler can pack $2684\frac{19}{24}$ cases of beans.

Section 3.2

19. $\frac{5}{7}$ **21.** $\frac{4}{7}$ **23.** $\frac{3}{5}$ **25.** 17 **27.** $\frac{1}{3}$

29. $\frac{22}{23}$ **31.** $\frac{1}{5}$ **33.** $\frac{2}{3}$

35. He has $\frac{3}{5}$ of his shift left.

Section 3.3

37. $\frac{2}{25}$ **39.** $\frac{18}{55}$ **41.** 1 **43.** $\frac{8}{3}$ **45.** $\frac{3}{8}$

47. $\frac{3}{4}$ **49.** $\frac{21}{50}$

51. Rent accounts for $\frac{1}{7}$ of their income.

Section 3.4

53. $1\frac{5}{16}$ **55.** 12 **57.** $3\frac{1}{3}$ **59.** $18\frac{3}{4}$ **61.** 2

63. $1\frac{1}{8}$ **65.** $3\frac{3}{5}$ **67.** $28\frac{1}{5}$

69. She harvests $8362\frac{1}{2}$ bushels.

Section 3.5

71. $\frac{4}{6}, \frac{6}{9}, \frac{10}{15}, \frac{16}{24}$ **73.** $\frac{6}{28}, \frac{9}{42}, \frac{15}{70}, \frac{24}{112}$ **75.** 18

77. 120 **79.** $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$ **81.** $\frac{1}{5}, \frac{2}{9}, \frac{3}{11}$

83. $\frac{8}{25}, \frac{59}{100}, \frac{3}{5}, \frac{31}{50}$ **85.** true **87.** false

89. The smallest is $\frac{7}{16}$ ton and the largest is $\frac{3}{4}$ ton.

Section 3.6

91. $\frac{7}{11}$ **93.** $\frac{2}{3}$ **95.** $\frac{5}{8}$ **97.** $\frac{3}{5}$ **99.** $\frac{7}{15}$

101. $1\frac{5}{8}$ **103.** The bamboo grew $1\frac{1}{8}$ in.

Section 3.7

105. $6\frac{5}{14}$ **107.** $15\frac{13}{15}$ **109.** $13\frac{5}{24}$

111. $25\frac{43}{80}$ **113.** 80 **115.** $60\frac{7}{12}$

117. Russ's mileage was $11\frac{13}{24}$ mi.

Section 3.8

119. $\frac{1}{2}$ **121.** $\frac{1}{2}$ **123.** $\frac{13}{30}$ **125.** $\frac{1}{30}$

127. $\frac{19}{75}$ **129.** $\frac{3}{40}$ **131.** She has $\frac{5}{12}$ oz left.

Section 3.9

133. $118\frac{1}{6}$ **135.** $15\frac{1}{5}$ **137.** $1\frac{5}{8}$ **139.** $6\frac{7}{48}$

141. $6\frac{35}{48}$ **143.** $5\frac{25}{32}$

145. a. During the year, $3\frac{5}{8}$ in. more rain falls in Westport.

b. In a 10-yr period, $277\frac{1}{12}$ in. more rain falls in Salem.

Section 3.10

147. 1 **149.** $1\frac{7}{12}$ **151.** $\frac{1}{2}$ **153.** $1\frac{1}{64}$

155. $\frac{15}{32}$ **157.** $1\frac{13}{30}$

159. The class average is $\frac{7}{10}$ of the problems correct.

Chapter 3 True/False Concept Review

1. true **2.** false; written as a mixed number, $\frac{7}{8} = 0\frac{7}{8}$.

3. false; the numerator always equals the denominator, so the fraction is improper. **4.** true

5. false; when a fraction is completely simplified, its value remains the same. **6.** true **7.** true **8.** true

9. false; the reciprocal of an improper fraction is less than 1. **10.** true **11.** true **12.** true

13. false; like fractions have the same denominators.

14. false; to add mixed numbers, add the whole numbers and add the fractions.

15. true **16.** true **17.** true **18.** true

Chapter 3 Test

1. $20\frac{1}{3}$ 2. $1\frac{7}{24}$ 3. $\frac{79}{9}$ 4. $\frac{3}{8}, \frac{2}{5}, \frac{3}{7}$ 5. $\frac{11}{1}$

6. 27 7. $9\frac{2}{15}$ 8. $18\frac{20}{27}$ 9. 0 10. $\frac{2}{3}$

11. $6\frac{4}{5}$ 12. $\frac{1}{2}$ 13. $\frac{2}{9}$ 14. $\frac{1}{3}$ 15. $\frac{12}{35}$

16. $6\frac{13}{20}$ 17. $\frac{5}{8}$ 18. $\frac{11}{14}$ 19. $\frac{5}{16}$ 20. $2\frac{5}{8}$

21. $\frac{7}{8}, \frac{7}{9}, \frac{8}{9}$ 22. $2\frac{5}{8}$ 23. $8\frac{13}{40}$ 24. $\frac{7}{12}$

25. $7\frac{6}{11}$ 26. $\frac{4}{5}$ 27. $1\frac{7}{8}$ 28. $\frac{9}{50}$

29. true 30. $\frac{5}{5}, \frac{6}{6}, \frac{7}{7}$

31. There are 22 truckloads of hay in the rail car.

32. She needs to make 25 lb of candy.

Chapters 1–3 Cumulative Review

1. six thousand, ninety-one 3. 1,000,310

5. 654,800 7. $>$ 9. 134,199

11. 1593 13. 154 cm 15. 24,893 17. 257

19. 7^8 21. 321,000 23. 32 25. 45,406; 39,445

27. Phil Mickelson has the lowest score at 64.

29. Week 4 had the highest sales.

31. Holli spent \$3404 on redecorating.

33. The Portland Symphony generated \$262,124.

35. 2 37. 2, 3 39. 140, 280, 420, 560, 700

41. no 43. $1 \cdot 495, 3 \cdot 165, 5 \cdot 99, 9 \cdot 55, 11 \cdot 45, 15 \cdot 33$

45. 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

47. 1, 3, 61, 183 49. prime 51. $2^2 \cdot 3 \cdot 5 \cdot 7$

53. 210 55. $\frac{9}{5}$ 57. $3\frac{3}{8}$ 59. $\frac{43}{6}$ 61. $\frac{11}{17}$

63. $\frac{3}{5}$ 65. $\frac{25}{24}$ 67. $18\frac{17}{18}$ 69. $1\frac{15}{28}$

71. $\frac{12}{14}, \frac{18}{21}, \frac{24}{28}, \frac{30}{35}$ 73. 27 75. false 77. $\frac{10}{11}$

79. $\frac{11}{105}$ 81. $4\frac{41}{60}$ 83. $\frac{31}{72}$

85. average, $\frac{55}{108}$; median, $\frac{35}{72}$ 87. $16\frac{1}{32}$ square in.

89. The Perez family spends $\frac{7}{12}$ of their income on these items.

CHAPTER 4

Section 4.1

1. twenty-six hundredths 3. two hundred sixty-seven thousandths 5. three and seven thousandths

7. eleven and ninety-two hundredths 9. 0.42

11. 0.409 13. 7.033 15. 0.0105

17. eight hundred five thousandths 19. eighty and five hundredths

21. thirty-seven and seven hundred five ten-thousandths 23. ninety and three thousandths

25. 0.0035 27. 1800.028 29. 505.005

31. 67.83

Unit	Tenth	Hundredth
------	-------	-----------

33. 35.777 36. 35.8 35.78

35. 819.735 820. 819.7 819.74

37. 0.7359 1. 0.7 0.74

39. \$67.49 41. \$548.72

Ten	Hundredth	Thousandth
-----	-----------	------------

43. 35.7834 40. 35.78 35.783

45. 86.3278 90. 86.33 86.328

47. 0.9165 0. 0.92 0.917

49. \$72 51. \$5647

53. The word name is fourteen and eight thousand eight hundred ninety-nine ten-thousandths. 55. To the nearest hundredth, the millage is 16.94. 57. Ngo writes two hundred seventy-five and eighty-five hundredths dollars or two hundred seventy-five and 85/100 dollars.

59. The position of the arrow to the nearest hundredth is 3.74.

61. The position of the arrow to the nearest tenth is 3.7.

63. five hundred sixty-seven and nine thousand twenty-three ten-thousandths 65. The account value to the nearest cent is \$3478.59. 67. 213.1101 69. The population density of Kuwait is about 118 people per square kilometer.

71. The number appears to have been rounded to the nearest hundredth.

Thousand Hundredth Ten-thousandth

73. 7564.35926 8000. 7564.36 7564.3593

75. 78,042.38875 78,000. 78,042.39 78,042.3888

77. The speed was rounded to the thousandths place.

79. The speed is 763 mph to the nearest mile per hour and 763.0 to the nearest tenth.

81. In 43.29, the numeral 4 is in the tens place, which is two places to the left of the decimal point. This means there are four tens in the number. In 18.64, the 4 is in the hundredths place, which is two places to the right of the decimal point. This means that there are four hundredths in the number.

83. The rounded value, 8,283, is greater than the original value. $8.283 > 8.28282828$ or $8.28282828 < 8.283$

85. The place value name is 0.00000522. 89. 77

91. 475 93. true 95. 6209, 6215, 6218, 6223, 6227

97. Kobe must hit 34 field goals.

Section 4.2

1. $\frac{91}{100}$ 3. $\frac{13}{20}$ 5. $\frac{429}{1000}$ 7. $\frac{39}{50}$ 9. $\frac{12}{25}$

11. $7\frac{23}{100}$ 13. $\frac{1}{8}$ 15. $9\frac{4}{25}$ 17. $11\frac{43}{125}$

19. $\frac{4}{5}$ 21. 0.2, 0.5, 0.8 23. 0.19, 0.27, 0.38

25. 3.179, 3.185, 3.26 27. false 29. false
 31. 0.046, 0.047, 0.047007, 0.047015, 0.0477
 33. 0.88579, 0.88799, 0.888, 0.8881
 35. 25.005, 25.0059, 25.051, 25.055 37. false
 39. true
 41. As a reduced fraction, the probability is $\frac{1}{16}$.
 43. The best (lowest) bid for the school district is \$2.6351 made by Tillamook Dairy.
 45. As a simplified fraction, the highest percentage of free throws made in a season is $\frac{104}{125}$.
 47. As a simplified fraction, the lowest percentage of free throws made by both teams in a game is $\frac{41}{100}$.
 49. $\frac{31}{80}$ 51. $65\frac{3}{40}$ 53. Hoa needs less soap.
 55. 5.00088, 5.0009, 5.00091, 5.001, 5.00101
 57. 37.59, 37.79, 37.90, 37.95, 37.96, 38.01, 38.15, 38.25
 59. Maria should choose the 0.725 yd.
 61. The West Bank has fewer people per square kilometer.
 63. The fat grams are $7\frac{19}{20}$, $8\frac{9}{10}$, and $10\frac{17}{20}$, respectively. The least amount of fat is in the frozen plain hash browns.
 65. O'Neal made $\frac{73}{125}$ of his field goal attempts. He missed $\frac{52}{125}$ of his attempts.
 67. The table sorted by averages from highest to lowest is

National League

Year	Name	Team	Average
2000	Todd Hetton	Colorado	0.372
2002	Barry Bonds	San Francisco	0.370
2004	Barry Bonds	San Francisco	0.362
2003	Albert Pujols	St. Louis	0.359
2001	Larry Walker	Colorado	0.350

American League

Year	Name	Team	Average
2000	Nomar Garciaparra	Boston	0.372
2004	Ichiro Suzuki	Seattle	0.372
2001	Ichiro Suzuki	Seattle	0.350
2002	Manny Ramirez	Boston	0.349
2003	Bill Mueller	Boston	0.326

69. $\frac{11}{25}, \frac{101}{250}, \frac{1011}{25,000}$ 73. 11,509 75. 6785
 77. $\frac{11}{8}$ or $1\frac{3}{8}$ 79. $\frac{1}{64}$

81. The attendance at the theater was 4593.

Section 4.3

1. 1.3 3. 5.9 5. 13.2 7. 40.09 9. 4
 11. 55.867 13. 34.32 15. 7.065 17. 9.0797
 19. 118.852 21. 0.1714 23. 1.2635
 25. 120.3876 27. 795.342 29. 28.596 31. 0.3
 33. 4.3 35. 4.1 37. 3.07 39. 20.87
 41. 0.266 43. 1.564 45. 0.369 47. 3.999
 49. 81.907 51. 0.04019 53. 7.2189
 55. 0.0727 57. 4.24 59. 0.175 61. 0.2719
 63. 1.8559
 65. Manuel bought 85.2 gallons of gas on his trip.
 67. 1689.4
 69. The total sales for the six business sectors were \$7.28 billion.
 71. \$3.41 billion was spent in the nonfood sectors.
 73. Jack has \$26.29 remaining.
 75. The total cost of the groceries is \$30.85.
 77. The total length of the Seikan and the Dai-shimizu is 76.44 km.
 79. The period from 2005 to 2010 is projected as the largest increase, 3.7 million.
 81. The top of the tree is 83 ft from ground level.
 83. The distance is 6.59375 in.
 85. Sera is 0.778 second faster than Muthoni.
 87. The relay race was completed in 38.95 sec.
 89. The fourth sprinter must have a time of 13.16 sec or less.
 91. When subtracting fractions, we must first get common denominators and then subtract the numerators. When subtracting decimals, we align decimal points. This results in columns of like place value, which are then subtracted. The numbers in each column have the same place value, which means if they were written as fractions they would have common denominators. Thus, aligning the decimal points results in subtracting numbers with common denominators.
 93. A total of 26 5.83s must be added.
 95. The missing number is 0.1877.
 97. The difference is 0.4475.
 101. 391,571 103. 42,579 105. $\frac{2}{3}$ 107. $25\frac{1}{5}$
 109. Harry and David will need 8280 pears to fill the order.

Getting Ready for Algebra

1. $x = 11.1$ 3. $y = 13.83$ 5. $t = 0.484$
 7. $x = 12.51$ 9. $w = 7.2$ 11. $t = 8.45$
 13. $a = 3.84$ 15. $x = 16.17$ 17. $a = 17.7$
 19. $s = 7.273$ 21. $c = 596.36$
 23. Two years ago the price of the water heater was \$427.73.
 25. The markup is \$158.66.
 27. Let C represent the cost of the groceries, then $$24 + C = \61 . The shopper can spend \$37 on groceries.

Section 4.4

1. 4.8 3. 9.5 5. 0.42 7. 0.64 9. 0.032
 11. 0.126 13. four 15. 0.06176 17. 15.093
 19. 2.8728 21. 3.9508 23. 22.631

- 25.** 0.023852 **27.** 0.167232 **29.** 0.004455
31. 32.2812 **33.** 2.0776 **35.** 27.1 **37.** 1776.7
39. 532.79 **41.** 108.40
43. Grant purchased a total of 99 gallons of gas.
45. To the nearest cent, Grant paid \$48.37 for the fifth fill-up.
47. Grant paid the least for his fill-up at \$2.399 per gallon.
49. 413.7936 **51.** 7980.21 **53.** 6564.620
55. 3086.16 **57.** Mary paid \$18.61 for the steak.
59. It costs \$227.59 to rent the full-size car.
61. It costs \$1.07 less to rent the full-size car.
63. Store 1 is selling the freezer-refrigerator for the least total cost.
65. Steve burns 2384.25 calories per week.
67. The total linear feet of steel is 744.975 ft.
69. The total weight of red meat consumed is 528 lb in 1970, 449.2 lb in 1990, and 447.6 lb in 2003. Greater awareness of potential heart problems associated with the consumption of large quantities of red meat probably accounts for the decline.
71. The number of gallons of water used by each type of toilet is as follows: pre-1970, 1,149,005 gallons; 1970s, 731,185 gallons; low-flow, 323,810.5 gallons. A total of 825,194.5 gallons of water is saved by using the low-flow model.
73. The Gregory estate owes \$11,046 in taxes.
75. The dive was awarded 66.99 points.
77. The number of decimal places in the product is the sum of the number of decimal places in the two factors. So the product of 5.73 and 4.2 will have three decimal places.
79. The smallest whole number is 111.
81. The missing number is 0.0252.
85. 820,000 **87.** 2200 **89.** 692,000 **91.** 476
93. 4210

Section 4.5

- 1.** 0.495 **3.** 9260 **5.** 1355.7 **7.** 3.458
9. 83.25 **11.** 1070 **13.** right **15.** 78,324
17. 4.26 **19.** 57,900 **21.** 0.90775
23. 1,755,000 **25.** 60.5632 **27.** 0.0003276
29. 7.5×10^5 **31.** 9.1×10^{-5}
33. 4.1953×10^3 **35.** 80,000 **37.** 0.004
39. 943,000 **41.** 4.37×10^4 **43.** 1.41×10^{-6}
45. 6.84×10^{-11} **47.** 9.75002×10^2
49. 0.00007341 **51.** 1,770,000,000
53. 0.0000000311 **55.** 2775.4
57. The total cost of the tires is \$3968.
59. The total cost of Ms. James's land is \$310,000.
61. The land area of Earth is 5.2×10^7 square miles.
63. The length of a red light ray is 7.2×10^{-8} cm.
65. The speed of light is approximately 11,160,000 miles per minute.
67. The shortest wavelength of visible light is 0.00004 cm.
69. The total energy use of the four families is 1,232,300,000 BTUs.
71. The per capita consumption of milk in Cabot Cove in 2005 was 22.9 gallons.
73. They are multiplying the actual number by 1000.
75. The mathematically calculated average is 0.424.

- 77.** The approximate length of one parsec is 1.92×10^{13} miles.
79. 6.5×10^6 **83.** 312 **85.** 62 R 1 **87.** 2596
89. 9200
91. The width of the rectangle is 21 in. or 1.75 ft.

Section 4.6

- 1.** 0.8 **3.** 4.9 **5.** 326.7 **7.** 3030 **9.** 5.53
11. 0.7875 **13.** whole number **15.** 1.2 **17.** 9.9
19. 0.10 **21.** 8.21 **23.** 1.92 **25.** 449.74
27. 15.667 **29.** 0.016 **31.** 14.444
33. 5.6 **35.** 7.95 **37.** 5.7 **39.** 12.4
41. 8.275 **43.** 6.62, 6.75, no mode
45. 18.304, 21.5, 21.5 **47.** 15.26, 15.4, no mode
49. 0.668, 0.6685, no mode **51.** 0.7128, 0.8674, 0.8674
53. The average closing price of Microsoft was \$23.99.
55. 91.74
57. To the nearest tenth of a cent, the unit price of apples is \$0.973, or 97.3¢, per pound.
59. To the nearest tenth of a cent, the unit price of rib steak is \$5.971, or 597.1¢, per pound.
61. To the nearest cent, the cost of a 21.8-lb turkey is \$19.40.
63. To the nearest cent, the average donation was \$959.41.
65. Orlando has an average daily temperature of 82°F. The median of the average daily temperatures is 84°F and the modes of the average daily temperatures are 73°F, 78°F, 84°F, and 92°F.
67. The average price per gallon is 2.502. The median price per gallon is 2.489. The mode prices per gallon are 2.489 and 2.599.
69. The average low temperature for the cities was 25.3°F.
71. The mode for the high temperatures was 33°F.
73. To the nearest mile, June got 49 miles per gallon.
75. To the nearest foot, there are 315 ft of cable on the spool.
77. To the nearest tenth, the length of the beam is 19.3 ft.
79. Denmark has the smallest area and Norway has the smallest population.
81. Denmark, the country with the largest population density, is almost 10 times as crowded as Norway. Finland and Norway are about equal in density.
83. To the nearest hundredth, the ERA is 1.64.
85. The fraction is $\frac{7}{10}$; it means that the runner is successful 7 out of 10 times.
87. The decimal point in a quotient is positioned directly above the decimal point in the dividend when dividing by a whole number. When dividing by a decimal, move the decimal point in the divisor to make it a whole number. Move the decimal point in the dividend the same number of places. Now the decimal point in the quotient is positioned above the relocated decimal in the dividend. This works because a division problem may be thought of as a fraction $\left(\frac{\text{divided}}{\text{divisor}} \right)$ and moving the decimals in both the same number of places to the right is the same as multiplying a fraction by 1 in the form of some power of 10 over itself.

89. Division Procedure

Operation on Decimals	Procedure	Example
Division	If the divisor is not a whole number, move the decimal point to the right until it is a whole number. Move the decimal point in the dividend the same number of places to the right. Now place the decimal point in the quotient directly above the decimal point in the dividend. Now divide.	$0.25 \overline{)3.5675}$ 14.27 $\underline{25)356.75}$ $\underline{25}$ 106 $\underline{100}$ 67 $\underline{50}$ 175 $\underline{175}$ 0
93.	$\frac{5}{6}$	93.
95.	$\frac{52}{11}$	95.
97.	$17\frac{11}{12}$	97.
99.	68	99.
101.	$\frac{3}{5}$	101.

Getting Ready for Algebra

1. $x = 7$ 3. $y = 308.5$ 5. $t = 0.23$ 7. $m = 0.06$
9. $q = 562.5$ 11. $h = 28.125$ 13. $y = 2.66$
15. $c = 0.2028$ 17. $x = 1.04$ 19. $s = 0.02415$
21. $z = 85.7616$ 23. There are 23 servings.
25. The current is 9.5 amps.
27. The length of the rectangle is 18.4 ft.
29. Let S represent the number of students, then $20S = 3500$. The instructor has 175 students in her classes.

Section 4.7

- 1.** 0.75 **3.** 0.125 **5.** 0.8125 **7.** 3.35
9. 56.584

	Tenth	Hundredth
11. $\frac{3}{7}$	0.4	0.43
13. $\frac{9}{11}$	0.8	0.82
15. $\frac{9}{13}$	0.7	0.69
17. $\frac{2}{15}$	0.1	0.13
19. $7\frac{7}{18}$	7.4	7.39
21. $0.\overline{81}$	23. $0.41\overline{6}$	

	Hundredth	Thousands
25.	$\frac{28}{65}$	0.43
27.	$\frac{59}{72}$	0.82
29.	<u>0.384615</u>	31. <u>0.857142</u>
33.	The micrometer reading will be 2.375 in.	

- 35.** The measurements are 0.375 in., 1.25 in., and 0.5 in.

Hundredth	Thousandth	Ten-thousandth
37. $\frac{33}{57}$	0.58	0.579
39. $33\frac{79}{165}$	33.48	33.479
41. The cost of the fabric is $\$4\frac{2}{25}$, or \$4.08, per yard.		
Decimals are easier to use because we write money in decimal form.	43. Texas has the winning jump.	
45. On May 1, there were 14.45 hr of daylight in Portland.		
47. Radcliffe's world's record time was $2\frac{37}{144}$ hr, or about 2.26 hr.	49. The larger is $\frac{7}{625}$.	
51. The fraction is less than 0.1. Yes, the decimal equivalent is 0.015375.	55. 9	57. 13
59. 80		
61. 210,000		
63. The average sale price of the houses was \$257,682.		
Section 4.8		
1. 0.5	3. 0.02	5. 0.3
7. 15.4	9. 0.3	
11. 10.07	13. 22.6	15. 130
17. 14.41		
19. 26.264	21. 0.19	23. 0.4
25. 9		
27. 1	29. 0.0024	31. 0.016
33. hundred	35. hundred-thousandth	
37. The estimate is 0.19. The answer is reasonable.		
39. The estimate is 0.0003. The answer is reasonable.		
41. Elmer spends \$15.55.	43. d	45. 52.89
47. 4.081	49. 27.8825	
51. The estimate is 0.0007, so Alex's answer is reasonable.		
53. The estimate is \$120, but since we rounded the price down, she will not have enough money for the shirts.		
55. The perimeter is about 78 ft.		
57. The estimated cost is \$22, so Jane cannot afford all of them.		
59. Matthew paid \$116.60 for the items.		
61. a. The nine golfers won a total of \$3,555,700.		
b. The 63 golfers earned an average of about \$35,624.		
63. The Red Sox averaged 6 runs per game, and the Cardinals averaged 3 runs per game.		
65. In the problem $0.3(5.1)^2 + 8.3 \div 5$, we divide 8.3 by 5 and add the result to $0.3(5.1)^2$. We do this because the order of operations requires that we divide before adding. In the problem $[0.3(5.1)^2 + 8.3] \div 5$, the entire quantity $[0.3(5.1)^2 + 8.3]$ is divided by 5. The insertion of the brackets has changed the order of operations so that we add before dividing because the addition is inside grouping symbols.		
67. $3.62 \div (0.02 + 72.3 \cdot 0.2) = 0.25$		
69. $(1.4^2 - 0.8)^2 = 1.3456$	73. 0.84375	75. 2.3
77. $\frac{51}{125}$	79. $6\frac{21}{25}$	
81. The clerk should price the TV at \$490.38.		

Getting Ready for Algebra

1. $x = 8.16$ 3. $x = 3$ 5. $0.1 = t$ 7. $x = 741$
9. $x = 0.32$ 11. $m = 63.1$ 13. $y = 23.56$
15. $p = 4.375$ 17. $24.3 = x$ 19. $h = 26.8$
21. $c = 6.275$ 23. The Celsius temperature is 120°C.
25. Gina has made 15 payments.
27. Let H represent the hours of labor, then $36H + 137.50 = 749.50$. She put in 17 hours of labor on the repair.

Chapter 4 Review Exercises

Section 4.1

1. six and twelve hundredths 3. fifteen and fifty-eight thousandths 5. 21.05 7. 400.04

	Tenth	Hundredth	Thousandth
9.	34.7648	34.8	34.76
11.	0.467215	0.5	0.47

Section 4.2

13. $\frac{19}{25}$ 15. $\frac{8}{3125}$ 17. 0.89, 0.95, 1.01
19. 0.717, 7.017, 7.022, 7.108 21. true

Section 4.3

23. 34.708 25. 13.7187 27. 48.1149
29. 32.6 in.
31. Hilda's take-home pay is \$3464.69.

Section 4.4

33. 28.245 35. 0.0243402 37. 0.148
39. 0.0019 41. Millie pays \$1243.31 for 23.75 yd.

Section 4.5

43. 0.013765 45. $73,210$ 47. 7.8×10^{-3}
49. 1.43×10^{-5} 51. 70,000,000
53. 0.0641 55. The average price of a bat is \$37.35.

Section 4.6

57. 0.037 59. 177.1875 61. 0.84 63. 70.77
65. To the nearest cent, the average donation was \$65.45.
67. 7.912, 8.09 69. 0.519, 0.56135
71. The unit price is \$1.563 per ounce.

Section 4.7

73. 0.5625 75. 17.376 77. 0.61 79. $0.\overline{692307}$
81. The value of the Microsoft share is 24.28.

Section 4.8

83. 5.27 85. 31.998 87. 33.982
89. The estimate is 9.8, so Jose's answer is reasonable.
91. The estimate is 0.024, so Louise's answer is reasonable.
93. The estimate is \$270, so Ron should have enough money in the budget.

Chapter 4 True/False Concept Review

1. false; the word name is "seven hundred nine thousandths." 2. true
3. false; the expanded form is $\frac{8}{10} + \frac{5}{100}$. 4. true
5. false; the number on the left is smaller because 732,687 is less than 740,000. 6. true 7. true 8. true
9. false; the numeral in the tenths place is 7 followed by a 4 so it rounds to 356.7. 10. true 11. false; $9.7 - 0.2 = 9.5$ as we can see when the decimal points are aligned.
12. false; if we leave out the extra zeros in a product such as $0.5(3.02) = 1.51$, the answer has fewer decimal places.
13. true 14. false; the decimal moves to the left, resulting in a smaller number. 15. false; move the decimal left when the exponent is negative. 16. true
17. false; most fractions have no exact terminating decimal form. For example, the fractions $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{9}$ have no exact terminating decimal equivalents.
18. false; $\frac{4}{11} = 0.\overline{36}$ 19. true 20. true

Chapter 4 Test

1. 5.453 2. 0.6699, 0.6707, 0.678, 0.6789, 0.682
3. seventy-five and thirty-two thousandths 4. 33.7212
5. 0.184 6. 57.90 7. 72.163 8. $18\frac{29}{40}$
9. 7.23×10^{-7} 10. 0.739 11. 73,000
12. 21.82 13. 97.115 14. 0.0000594
15. 9045.065 16. 91.7 17. 3.0972×10^5
18. 46.737 19. 0.05676 20. 0.00037
21. The average monthly offering was \$126,774.33.
22. 7895.308 23. Grant pays \$67.86 for the plants.
24. Allen Iverson had played in 72 games.
25. The player's slugging percentage is 686.
26. Jerry lost 0.03 lb more than Harold.

CHAPTER 5

Section 5.1

1. $\frac{1}{4}$ 3. $\frac{1}{3}$ 5. $\frac{4}{5}$ 7. $\frac{1}{2}$ 9. $\frac{9}{10}$ 11. $\frac{5}{12}$
13. $\frac{2 \text{ cars}}{3 \text{ families}}$ 15. $\frac{55 \text{ mi}}{1 \text{ hr}}$ 17. $\frac{23 \text{ mi}}{1 \text{ gal}}$
19. $\frac{23 \text{ trees}}{2 \text{ rows}}$ 21. $\frac{2 \text{ trees}}{7 \text{ ft}}$ 23. $\frac{2 \text{ books}}{5 \text{ students}}$
25. $\frac{3 \text{ students}}{2 \text{ rooms}}$ 27. $\frac{15 \text{ pies}}{2 \text{ sales}}$
29. 20 mpg (miles per gallon) 31. 4 ft per sec
33. 23¢ per lb 35. 0.008 qt per mi
37. 2.4 children per family 39. 83.3 ft per sec
41. 292.2 lb per mi^2 43. 741.7 gal per hr

- 45. a.** The price per rose of the special is \$9.98.
b. This is a savings of \$2.90 over buying three separate roses.
- 47.** The 14-oz can is 6.36¢ per ounce and the 16-oz can is 6.25¢ per ounce, so the 16-oz can is the better buy.
- 49.** The 5-lb sale bag is the better buy, at a cost of 99¢ per pound.
- 51. a.** The salsa costs 22¢ per ounce. **b.** The sale price of the salsa is 16¢ per ounce. **c.** Jerry can save \$3.84.
- 53.** Answers will vary. For example, meat, fish, fruit, vegetables, and bulk food are usually priced by the pound, so the unit price is the same for any amount purchased. Boxed items such as cereal, dried pasta, cookies, laundry detergent, and pet food are usually cheaper in larger quantities. Occasionally, meat and other perishables may be discounted because they are near their expiration date, so the unit price is lower.
- 55. a.** The ratio of compact spaces to larger spaces is $\frac{3}{4}$.
b. The ratio of compact spaces to total spaces is $\frac{3}{7}$.
57. The ratio of sale price to regular price is $\frac{3}{4}$.
- 59.** The population density of Dryton City is about 97.6 people per square mile.
- 61.** Answers will vary.
- 63. a.** The ratio of laundry use to toilet use is $\frac{7}{20}$.
b. The ratio of bath/shower use to dishwashing use is $\frac{16}{3}$.
- 65.** A total of 1.25 mg of lead is enough to pollute 25 L of water.
- 67.** The retrieval rate is $\frac{105 \text{ in.}}{5 \text{ turns}} = 21 \text{ in. per turn}$.
- 69.** The average number of credits per student was $\frac{4020 \text{ credits}}{645 \text{ students}} \approx 6$ credits per student. The FTE for the term was $\frac{4020}{15 \text{ credits}} = 268$ FTE.
- 71.** This is a ratio, because no units are given. It means that 1 inch on the map represents 150,000 inches (about 2.37 miles) in the real world.
- 73.** A ratio compares two measurements. The ratio of \$5 to \$1000 is $\frac{5}{1000}$, or $\frac{1}{200}$. A rate compares unlike measurements. The rate of 350 cents per 20 oz is $\frac{350 \text{ cents}}{20 \text{ oz}}$, or $\frac{35 \text{ cents}}{2 \text{ oz}}$. A unit rate compares unlike measurements, where the second measurement is 1 unit. The unit rate of 350 cents to 20 oz is 17.5¢ per ounce.
- 75.** Examples of 2-to-1 ratios are eyes to people, hands to people, and legs to people. Examples of 3-to-1 ratios are triplets to mothers, petals to trillium flowers, and leaves to poison ivy stems.
- 79.** $\frac{1}{14}$ **81.** 0.195 **83.** 0.525 **85.** $4\frac{8}{15}$ **87.** $\frac{94}{125}$

Section 5.2

- 1.** true **3.** false **5.** false **7.** true **9.** false
11. false **13.** true **15.** $a = 14$ **17.** $c = 6$
19. $y = 10$ **21.** $c = 10$ **23.** $x = \frac{9}{4}$ **25.** $p = \frac{1}{2}$
27. $x = 11.5$ **29.** $z = 4.4$ **31.** $w = 4\frac{1}{3}$
33. $a = 12.8$ **35.** $c = 0.6$ **37.** $b = \frac{3}{8}$
39. $x = 0.45$ **41.** $w = 0.1$ **43.** $y = 128$
45. $t = 8$ **47.** $w \approx 1.4$ **49.** $y \approx 31.3$
51. $a \approx 0.33$ **53.** $c \approx 0.95$

55. The value in the box must be 10 so that the cross products are the same.

57. The error is multiplying straight across instead of finding cross products. Multiplying straight across goes with fraction multiplication, but cross multiplication goes with proportions.

59. Fran's minimum allowable level of HDL is about 48.

61. Eastlake should have about 32 teachers.

63. a. The number of migraine sufferers is 245. **b.** The number of men in the group who suffer from migraines is 102. **c.** Of 350 migraine sufferers, 70 headaches may be related to diet.

65. The proportion $\frac{1}{196,416} = \frac{1}{200,000}$ is not mathematically true, but it is close to being true. The atlas probably used 3.1 miles as a rounded number.

67. $3.5(y) = \left(\frac{1}{4}\right)7$ To solve the proportion, first cross multiply.
 $3.5y = \frac{7}{4}$
 $y = \frac{7}{4} \div 3.5$ Rewrite as division. We can simplify using decimals or fractions.

If we choose fractions, we get $y = \frac{7}{4} \div \frac{7}{2}$, or $y = \frac{1}{2}$.
If we choose decimals, we get $y = 1.75 \div 3.5$, or $y = 0.5$.

69. $a = 5$ **71.** $w \approx 8.809$ **75.** 6.8265
77. 48,350 **79.** 25.375 **81.** 2.001
83. Quan pays \$31.86.

Section 5.3

1. 6 **2.** 4 **3.** 15 **4.** h **5.** $\frac{6}{4} = \frac{15}{h}$ **6.** 10 in.
7. 30 **9.** h **11.** $\frac{30}{18} = \frac{h}{48}$ **13.** 6 **14.** 30
15. T **16.** 265 **17.** $\frac{6}{30} = \frac{T}{265}$
18. It will take 53 hr. **19.** T **21.** $\frac{3}{65} = \frac{T}{910}$

23. The school will need 4 additional teachers.

24.

	Case I	Case II
Days	1.5	14
Garbage (lb)	36	x

$$25. \frac{1.5}{36} = \frac{14}{x}$$

26. The restaurant has 336 lb of garbage at the end of 2 weeks. 27. Idamae must knit 100 rows.

29. In Bahrain; in 2004, there were an estimated 413,510 males.

31. The family would spend \$9000 on food.

33. Fifty pounds of fertilizer will cover 2500 ft².

$$35. \frac{11}{15} = \frac{x}{30} \quad 37. \text{ Dawn will earn } \$378.$$

39. Mike's onions cost \$1.48.

41. It will take 17 hr to go 935 mi.

43. The second room will cost \$350.90.

45. Ida needs 0.4 cm³ (0.4 cc) of the drug.

47. Nora will receive \$5525.

49. Betty needs 12 lb of cashews and 28 lb of peanuts.

51. Debra needs 42 quarts of blue paint.

53. Lucia needs 65 lb of ground round for the meatballs.

55. The computer costs £650.

57. The comparable price for a 60-month battery is \$59.75.

59. 12 m³ of air must have less than 2820 mg of ozone.

61. The store must reduce the price of each box by 10¢ in order to beat the warehouse outlet's price.

63. The actual bridges are about 3.09 mi apart.

65. A proportion is composed of two equal ratios or rates. The speed of a vehicle can be measured in different units:

$\frac{60 \text{ mi}}{1 \text{ hr}} = \frac{88 \text{ ft}}{1 \text{ sec}}$. Food recipes can be cut or expanded by keeping the ingredient measures in proportion. Prices for many items, especially food and clothing, are proportional to the number purchased.

67. From a consumer's viewpoint, it is better if there is a price break when buying larger quantities. Many products have a lower unit price when bought in bulk or larger packages.

69. The drive shaft speed is 1120 revolutions per minute.

73. 37.41; 37.415 75. 0.01399 > 0.011

77. The gas costs \$28.55. 79. $\frac{173}{200}$ 81. 0.559

Chapter 5 Review Exercises

Section 5.1

$$1. \frac{1}{5} \quad 3. \frac{6}{5} \quad 5. \frac{4}{5} \quad 7. \frac{2}{3} \quad 9. \frac{9 \text{ people}}{10 \text{ chairs}}$$

$$11. \frac{6 \text{ km}}{1 \text{ person}} \quad 13. \frac{8 \text{ lb}}{3 \text{ ft}} \quad 15. \frac{85 \text{ people}}{3 \text{ committees}}$$

17. 25 mph 19. 9¢ per pound of potatoes

21. 37.5 mpg 23. 16.1 gal per min

25. No, the first part of the country has a rate of 3.5 TVs per household and the second part of the country has a rate of about 3.3 TVs per household.

Section 5.2

27. true 29. false 31. false 33. $r = 9$

$$35. t = 4 \quad 37. f = \frac{1}{2} \quad 39. r = 10\frac{2}{3}$$

$$41. t = 5\frac{1}{3} \quad 43. a \approx 4.2 \quad 45. c \approx 27.4$$

47. The store brand must be cheaper than \$3.03 for a 25-load box.

Section 5.3

49. Merle will study 37.5 hr per week.

51. Juan must work 125 hr to pay for his tuition.

53. Larry makes \$43.75.

Chapter 5 True/False Concept Review

1. true 2. true 3. false;

$$\frac{18 \text{ miles}}{1 \text{ gallon}} = \frac{54 \text{ miles}}{3 \text{ gallons}}, \text{ or } \frac{18 \text{ miles}}{1 \text{ hour}} = \frac{54 \text{ miles}}{3 \text{ hours}}$$

4. false; to solve a proportion, we must know the values of three of the four numbers.

$$5. \text{ false; if } \frac{8}{5} = \frac{t}{2}, \text{ then } t = 3\frac{1}{5}.$$

6. false; in a proportion, two ratios are *stated* to be equal but they may not be equal. 7. true 8. true

9. true 10. false; the table should look like this:

	First Tree	Second Tree
Height	18	x
Shadow	17	25

Chapter 5 Test

$$1. \frac{4}{5} \quad 2. \text{ Ken would answer 45 questions correctly.}$$

$$3. w = 0.9 \quad 4. \text{ false} \quad 5. y = 6.5 \quad 6. \text{ true}$$

$$7. \text{ Mary should be paid } \$84.24. \quad 8. \frac{1}{9}$$

9. The price of 10 cans of peas is \$8.20.

10. One would expect 17.5 lb of bones.

11. $x = 0.625$ 12. They will catch 24 fish.

13. Jennie will need 10 gal of gas. 14. $a = 5.3$

15. yes 16. The tree will cast a 10.5-ft shadow.

17. The population density is 37.2 people per square mile.

18. $y \approx 1.65$ 19. The crew could do 26 jobs.

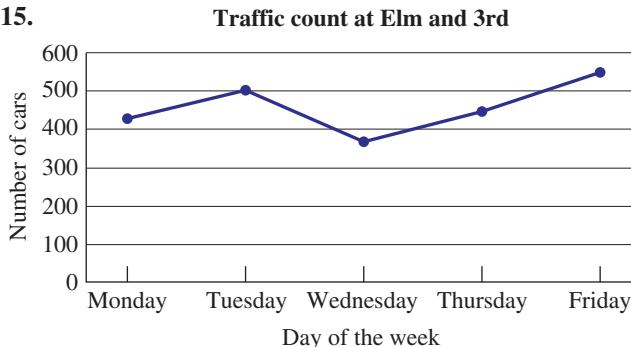
20. There are 30 females in the class.

Chapters 1–5 Cumulative Review

1. 2045 3. 3134 5. 18,630 7. 19,035

9. 128 11. 93 13. The sophomore class had the most students taking algebra.

15.



17. It is divisible by 6, 9, and 10. 19. Yes, 504 is a multiple of 24. 21. The factors of 210 are 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, and 210. 23. The number 303 is composite.

25. The LCM is 1008. 27. $\frac{88}{7}$ 29. $\frac{16}{19}$ 31. $\frac{3}{5}$

33. $\frac{9}{16}$ 35. $38\frac{2}{5}$ 37. $\frac{86}{75}$ or $1\frac{11}{75}$
 39. $\frac{43}{90}$ 41. $12\frac{9}{20}$ 43. $\frac{29}{56}$ 45. $\frac{3}{4}$

47. Jamie must get 35 problems correct.

49. There will be $\frac{5}{32}$ in. left over.

51. four hundred nineteen millionths

53. 900,000,0009

55. $\frac{13}{200}$ 57. 0.019, 0.052, 0.101, 0.21, 0.63

59. 75.745 61. 155.708 63. Jose has \$2.10 left.

65. 0.024566 67. 198.3 69. 0.082784

71. 4.6007×10^3 73. The cleaning solvent will last about 88 days. 75. 7.48 77. 1.224

79. average, 17.22; median, 16.11; mode, 16.11

81. Andy spends \$96.80 on supplies.

83. $\frac{3}{8}$ 85. \$0.195 per oz 87. false 89. $x = 63$

91. $c \approx 46.2$ 93. 25 ft 95. 15 ft

97. The second flagpole is 36 ft tall.

99. Greg will earn \$156.60 if he sells \$2610 worth of clothes.

43. The luxury tax is 11%.

45. The annual interest rate is 3.75%.

47. The tax rate is 1.48%.

49. In 2002, 1.51% of the juvenile population was arrested for property crimes.

51. In 2002, 56% of bachelor's degree recipients were women.

53. Carol spends 82% of her money on the outfit.

55. The percent paid in interest was 20%.

57. Percent is the amount per hundred. It is the numerator of a fraction with a denominator of 100. It is the number of hundredths in a decimal.

59. $\frac{109}{500}$; 21.8% 63. 4733.5 65. 207,800

67. 0.0000672 69. 0.009003

71. Ms. Henderson earned \$3172.05 during the month.

Section 6.2

1. 47% 3. 753% 5. 8% 7. 139%

9. 1900% 11. 0.65% 13. 95.2% 15. 51.7%

17. 7.31% 19. 7000% 21. 1781% 23. 0.029%

25. 710% 27. 39.54% 29. $81.\bar{1}\bar{6}\%$, or $81\frac{1}{6}\%$

31. 70.45% 33. 0.37 35. 0.73

37. 0.0641 39. 9.08 41. 3.278 43. 0.000062

45. 0.0019 47. 39.4 49. 0.00037 51. 1

53. 5.35 55. 0.005 57. 0.00875 59. 0.7375

61. 0.00375 63. 4.13773

65. The tax on income in Colorado was 4.63%.

67. The Girl Scout sold 36% of her quota of cookies on the first day.

69. The employees will get a raise of 0.0315.

71. The Credit Union will use 0.0634 to compute the interest.

73. 0.005 75. 0.577 77. The river pollution due to agricultural runoff is 0.6 of the total pollution.

79. The golf team won 87.5% of their matches.

81. As a percent, Barry Bonds's home runs are about 93.1% of Hank Aaron's.

83. In 2006, the number of physical therapy jobs will be 170.4% of the number of jobs in 1996.

85. Answers will vary.

87. The Moscow subway system has 1.153 times the number of riders of the Tokyo system.

89. The diameter of Pluto is about 0.274 times the diameter of Earth.

91. The percent of income paid for both Social Security and Medicare is 9.1%.

93. The sale price is 89% of the original price. The store will advertise "1% off."

95. The new box is 1.25 times the size of the old box.

97. The decimal will be greater than 1 when the % is greater than 100%. For example, 235% written as a decimal is 2.35 and 3.4% written as a decimal is 0.034.

99. 1,800,000% 101. 42.5% or $42\frac{1}{2}\%$; $42\frac{5}{9}\%$

103. 0.6, 0.569 107. 0.140625 109. 1.8

CHAPTER 6

Section 6.1

1. 29% 3. 72% 5. 140% 7. 62% 9. 17%

11. 56% 13. 44% 15. 55% 17. 190%

19. 100% 21. 175% 23. 42.5% 25. 60%

27. 87.5% 29. $83\frac{1}{3}\%$ 31. $66\frac{2}{3}\%$ 33. 12

35. The percent of eligible voters who exercised their right to vote was 82%.

37. $31\frac{1}{4}\%$ 39. 660% 41. $53\frac{1}{6}\%$

111. $\frac{41}{400}$ **113.** 350,000

115. A total of \$22.44 is saved on the trip.

Section 6.3

- 1.** 67% **3.** 74% **5.** 85% **7.** 50% **9.** 44%
- 11.** 130% **13.** 137.5% **15.** 3.9% **17.** 460%
- 19.** $66\frac{2}{3}\%$ **21.** $483\frac{1}{3}\%$ **23.** $741\frac{2}{3}\%$ **25.** 88.2%
- 27.** 55.6% **29.** 53.8% **31.** 3237.9% **33.** $\frac{3}{25}$
- 35.** $\frac{13}{20}$ **37.** $\frac{13}{10}$ or $1\frac{3}{10}$ **39.** 7 **41.** $\frac{21}{25}$
- 43.** $\frac{3}{4}$ **45.** $\frac{9}{20}$ **47.** $\frac{3}{2}$ or $1\frac{1}{2}$ **49.** $\frac{91}{200}$
- 51.** $\frac{17}{250}$ **53.** $\frac{121}{200}$ **55.** $\frac{1}{400}$ **57.** $\frac{3}{40}$
- 59.** $\frac{1}{225}$ **61.** $\frac{21}{40}$ **63.** $\frac{199}{60}$ or $3\frac{19}{60}$

- 65.** Kobe Bryant made 85% of his free throws.
- 67.** President George W. Bush received 50.7% of the votes.
- 69.** George and Ethel paid $\frac{9}{50}$ of their income in taxes.
- 71.** Carmelo Anthony made $\frac{21}{25}$ of his shots.
- 73.** 18.61% **75.** 844.19% **77.** $\frac{2}{45}$ **79.** $\frac{7}{800}$
- 81.** Miguel is taking 162.5% of the recommended allowance.
- 83.** It represents $\frac{28}{25}$, or $1\frac{3}{25}$, of last year's enrollment.
- 85.** The fraction of residents between the ages of 25 and 40 is $\frac{7}{40}$.

- 87.** St. Croix is $\frac{3}{5}$, or 60%, of the area of the Virgin Islands.
- 89.** In 2000, 57.8% of the days were smoggy. In 2005, 45.8% of the days were smoggy. Possible reasons include better emission controls on cars and trucks and more pollution controls on factories.
- 91.** During the 10 years, $\frac{29}{50}$ of the salmon run was lost.

- 93.** In South Africa, $\frac{479}{5000}$ of the population is white, meaning that 479 out of every 5000 people in South Africa are white.

- 95.** The percent taken off the original price is 48%.

- 97.** The grill was 33% off.

- 99.** Sales can be described either by fractions or percents. For example, you might hear about a sale of $\frac{1}{3}$ off or of 33% off.

Common statistics are also given in either form. You might hear that $\frac{1}{5}$ of the residents of a town are over 65 years old or

that 20% of the residents are over 65 years old. It is important to note that in such circumstances, fractions are generally approximations of the actual statistics.

Percent forms of statistics may be approximations, but they are generally considered more accurate than fraction forms.

- 101.** 2.3% **103.** $\frac{1}{4000}$ **105.** $1\frac{2001}{2500}$ **109.** $\frac{21}{25}$
- 111.** $4\frac{13}{200}$ **113.** 0.825 **115.** 56.7%
- 117.** 0.0813

Section 6.4

Fraction	Decimal	Percent
1. $\frac{1}{10}$	0.1	10%
3. $\frac{3}{4}$	0.75	75%
5. $1\frac{9}{20}$	1.45	145%
7. $\frac{1}{1000}$	0.001	0.1% or $\frac{1}{10}\%$
9. $2\frac{1}{4}$	2.25	225%
11. $\frac{11}{200}$	0.055	$5\frac{1}{2}\%$
13. $\frac{1}{200}$	0.005	0.5% or $\frac{1}{2}\%$
15. $\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
17. $\frac{43}{50}$	0.86	86%
19. $\frac{2}{25}$	0.08	8%
21. $\frac{1}{4}$	0.25	25%
23. $\frac{2}{5}$	0.4	40%
25. $\frac{1}{8}$	0.125	12.5% or $12\frac{1}{2}\%$
27. The swim suit is 25% off the regular price.		
29. Hank's Super Market offers the better deal.		
31. Vender B offers the better deal and falls within the boss's authorized amount.		
33. Stephanie had the best batting average.		
35. Peru has the higher literacy rate.		
37. The TV sells for $\frac{2}{5}$ off the regular price.		
39. Melinda gets the best deal from Machines Etc.		
41. Fractions are commonly used to describe units with subdivisions that are not powers of 10. For example, inches are subdivided into fourths and eighths, so it is common to have measures of $14\frac{3}{8}$ inches. Rates are usually stated using		

decimal forms. For example, unit pricing is generally given in tenths of a cent, as in Cheerios costing 19.4¢ per ounce. Statistics are most often given using percent forms. For example, 52% of total orchestra expenses are salaries of the musicians.

45. $y = 41.4$ **47.** $x = 3.735$ **49.** $B = 200$

51. $A \approx 9.5$ **53.** The Bacons should collect \$62,800 in insurance money.

Section 6.5

- | | | | | |
|--------------------|------------------|------------------------------|------------------------------|----------------|
| 1. 54 | 3. 46 | 5. 200% | 7. 50% | 9. 40 |
| 11. 27.3 | 13. 80% | 15. 100 | 17. 2 | 19. 117 |
| 21. 12.25 | 23. 2.5% | 25. 345 | 27. 65.76 | |
| 29. 155 | 31. 265 | 33. 76.8% | 35. 21.49 | |
| 37. 62.5% | 39. 205 | 41. 57.1% | 43. 609.59 | |
| 45. 227.265 | 47. 23.7% | 49. $\frac{171}{200}$ | 51. $114\frac{3}{10}$ | |
| 53. 132.0% | | | | |

55. The problem is in equating a 40¢ profit with a 40% profit. A 40¢ profit means that the company makes 40¢ on each part sold. A 40% profit means that the company makes 40% of the cost on each part. But 40% of 30¢ is only 12¢. So a 40% profit would mean selling the parts for 42¢. The company is making a much higher profit than that. The 40¢ profit is actually 133% of the cost.

57. $\frac{17}{75}$ **61.** $x = 45$ **63.** $a = 0.25$, or $a = \frac{1}{4}$

65. $w = 20.2$ **67.** The number of miles is 145.

69. The number of inches needed is $2\frac{5}{8}$.

Section 6.6

- There are about 838 undergraduate students from low-income families.
- The Hispanic or Latino population of Los Angeles County was about 4,600,120.
- John's score was 76%.
- The tip was 18% of the check.
- Gravel is 50% of the mixture.
- Delplanche Farms has about 31% of its acreage in soybeans.
- The General Fund Budget is about 42.6% of the total budget.
- Mickey Mantle got a hit 29.8% of the time.
- The number of new cars sold in Maryland was about 427,582.
- The fruit stand had 24 boxes of bananas in stock.
- Maria sold \$3720 worth of candy.
- The diesel engine is down 1% less time than the gasoline engine.
- The recommended daily values are as follows: fat, 63 g; sodium, 2444 mg; potassium, 3600 mg; and dietary fiber, 21 g.
- In 1900, the U.S. population was about 75.7 million people.
- In 2000, there were about 40,052 households in Cleveland with incomes less than \$10,000.

Amount	New Amount	Increase or Decrease	Percent of Increase or Decrease
31. 345	415	Increase of 70	20.3%
33. 764	888	Increase of 124	16.2%
35. 2900	3335	Increase of 435	Increase of 15%

37. The Portland Trail Blazers' average home attendance decreased by 14.6%.

39. The price of a gallon of gas rose about 9% during the 3 weeks.

41. The percent of increase in Jose's IRA was 16.3%.

43. The percent of decrease in the population was about 3.2%.

45. The percent of decrease in production was about 6.0%.

47. The percent of increased mileage is 7.6%.

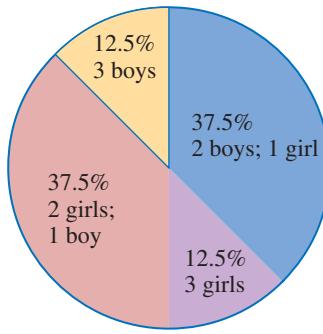
49. The number of foreign-born residents in the United States has increased about 150% over the past century.

51. African Americans are the smallest identifiable population in California.

53. The second largest ethnic group in California is Hispanic.

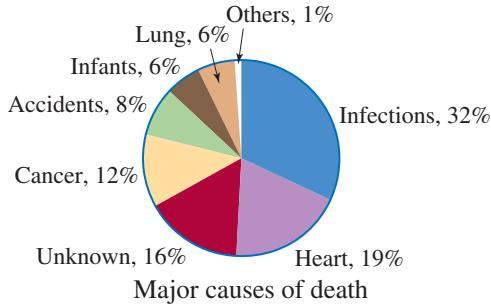
55. DaimlerChrysler sold more cars than Nissan.

57.



Possibilities of boy-girl combinations

59.



61. The percent of increase is 100%.

63. Carol's baby's weight increased by 215% during the first year.

65. Nonfood items accounted for 30.4% of the cost. Meat products accounted for 23.5%.

67. Sally receives \$7560 and Rita receives \$5040.

69. Joe pays a total of \$17,508 for the car.

71. The engine develops 113.75 horsepower.

73. Peter attended 90% of the classes.

75. The score on the algebra test is 85%.

Section 6.7

Marked Price	Sales Tax Rate	Amount of Tax	Total Cost
1. \$238	5.8%	\$13.80	\$251.80
3. \$90.10	3.8%	\$3.42	\$93.52
5. \$821	8.5%	\$69.79	\$890.79
7. \$1025	6.4%	\$65.60	\$1090.60

9. The sales tax is \$60.92.
11. The suit costs George \$506.47.
13. The sales tax rate is 6.3%.
15. Jim pays \$2634.72 for the TV set.
17. The sales tax rate is about 6.0%.
19. The sales tax rate is about 7.5%.
-
- | Original Price | Rate of Discount | Amount of Discount | Sale Price |
|----------------|------------------|--------------------|------------|
| 21. \$75.82 | 18% | \$13.65 | \$62.17 |
| 23. \$320 | 30% | \$96 | \$224 |
| 25. \$145.65 | 30% | \$43.70 | \$101.95 |
| 27. \$350 | 33% | \$115.50 | \$234.50 |
29. Joan pays \$161.46 for the painting.
31. Larry pays \$48,129.90 for the SUV.
33. The discount is \$274.40. Melvin pays \$705.59 for the generator.
35. The discount price is \$3439.99 and the sales tax rate is 5.5%.
37. Catherine will save \$36.60 off the original price.
39. The TV is on sale for \$402.71.
41. The rebate on the canopy is \$26.74.
43. The sale price of the overalls is \$32.06.
45. The claim is accurate, as the bag contains 20.8% more candy.
47. The mother pays \$77.48 for the two pairs of shoes. This is a 21.7% savings off the original price.
49. The price of the clubs will be \$928.13.

Rate of

Sales	Commission	Commission
51. \$4890	9%	\$440.10
53. \$67,320	8.5%	\$5722.20
55. \$1100	4%	\$44
57. \$346,800	9%	\$31,212

59. Grant's commission for the week was \$392.93.
61. The commission on the sale of the tickets is \$25.48.
63. He earned \$741.07 last week.
65. Juanita earned \$655.65 last week.
67. Matthew's sales for the week totaled \$5187.50.
69. Ms. James earned commissions of \$1433.50 for the week.
71. A sales tax is a percent increase in the price paid by the consumer, because it is an additional amount of money paid.
73. In July Marlene earned \$3183 and in August she earned \$5175.

75. $\frac{59}{72}$ 77. $\frac{23}{48}$ 79. $\frac{1}{6}$ 81. $\frac{13}{9}$, or $1\frac{4}{9}$

83. The perimeter is $18\frac{19}{20}$ in.

Section 6.8

Principal	Rate	Time	Interest	Total Amount Due
1. \$10,000	5%	1 year	\$500	\$10,500
3. \$5962	8%	1 year	\$476.96	\$6438.96
5. \$24,000	7.5%	4 years	\$7200	\$31,200
7. \$4500	5.5%	9 months	\$185.63	\$4685.63

9. Maria earns \$112.50 in interest.
11. Nancy has earned \$1402.50 in interest over the 3-year period.
13. Janelle earns \$135 in interest.
15. Janna pays \$2424.58 to pay off the loan.
17. a. The interest paid was \$5038. b. The interest rate was 11%.
19. The ending balance is \$29,710.80.
21. The ending balance is \$198,612.
23. The amount of interest earned is \$10,850.56.
25. The amount of interest earned is \$138,643.
27. Juanita's account will earn \$91.80 more than Jose's account.
29. Carl's account earns \$177 more interest.
31. a. Mark pays \$57.85 in interest. b. \$22.15 goes to pay off the balance. c. The unpaid balance at the end of June is \$3944.95.
33. a. Debbi pays \$157.04 in interest. b. \$17.96 goes to pay off the balance. c. The unpaid balance at the end of April is \$8981.79.
35. a. Belle pays \$88.94 in interest. b. \$25.06 goes to pay off the balance. c. The unpaid balance at the end of June is \$5677.11.
37. The minimum monthly payments are primarily interest owed on the balance, and very little goes to reducing the amount of the balance. A late fee or additional charges could easily be more than the reduction of the outstanding balance, so the balance actually increases for the month.
39. It will take 14 months to pay off the balance and the 14th payment will be for \$82.32. Linn will have paid \$146.72 in interest. 41. 315.028 43. 36.325
45. 1.30456 47. 57.015 49. Each of Mary's nieces and nephews received \$197,600.

Chapter 6 Review Exercises

Section 6.1

1. 75% 3. 78% 5. $57\frac{1}{7}\%$

Section 6.2

7. 65.2% 9. 0.017% 11. The Phoenix Suns won 75.6% of their games. 13. 0.48 15. 0.000625

- 17.** The decimal number used to compute the interest is 0.1845.

Section 6.3

- 19.** 68.75% **21.** 48.1% **23.** The Classic League basketball team won about 80.43% of their games.

25. $1\frac{13}{20}$ or $\frac{33}{20}$ **27.** $\frac{13}{40}$

- 29.** The Bears defense was on the field $\frac{29}{50}$ of the game.

Section 6.4

31–38.

Fraction	Decimal	Percent
$\frac{17}{25}$	0.68	68%
$\frac{37}{50}$	0.74	74%
$\frac{3}{200}$	0.015	1.5%
$3\frac{11}{40}$	3.275	327.5%

Section 6.5

- 39.** 100.1 **41.** 21.25% **43.** 12.58 **45.** 18.2%

Section 6.6

- 47.** Melinda's yearly salary was \$24,035.
49. Omni Plastics currently employs 429 people.
51. The Hummer H2 decreased in value by 12%.
53. Most students received a B grade.

Section 6.7

- 55.** The clubs cost Mary \$495.33.
57. The cost of the suit to the consumer is \$405.54.
59. The New Balance shoes are sold at about 28% off.

Section 6.8

- 61.** Wanda owes her uncle \$6215.
63. Minh's credit card balance will be \$1311.46.

Chapter 6 True/False Concept Review

- 1.** true **2.** true
- 3.** false; rewrite the fraction with a denominator of 100 and then write the numerator and write the percent symbol.
- 4.** false; move the decimal point two places to the right and write the percent symbol.
- 5.** true **6.** false; the base unit is always 100.
- 7.** true **8.** true **9.** false; the proportion is $\frac{A}{B} = \frac{X}{100}$.
- 10.** true **11.** true **12.** false; $4\frac{3}{4} = 475\%$
- 13.** false; $0.009\% = 0.00009$ **14.** false; $B = 43,000$
- 15.** true **16.** true

- 17.** false; it is the same as a decrease of 27.75%.

- 18.** false; her salary is 99% of what it was Monday.

- 19.** true **20.** true **21.** true

- 22.** false; $\frac{1}{2}\% = 0.005$

- 23.** true **24.** false; the sales tax rate is 6.5%

Chapter 6 Test

- 1.** The percent of discount on the computer is 27.8%.

- 2.** 3.542% **3.** The population is 56% female.

- 4.** 74.6% **5.** 84.375% **6.** 45 **7.** $\frac{54}{325}$

- 8.** 0.78% **9.** 11.7 **10.** 681.8% **11.** $\frac{68}{25}$ or $2\frac{18}{25}$

- 12.** 0.0789

- 13.** The Adams family spends \$1120 per month on rent.

14–19.

Fraction	Decimal	Percent
$\frac{13}{16}$	0.8125	81.25%
$\frac{78}{125}$	0.624	62.4%
$\frac{37}{200}$	0.185	18.5%

- 20.** 75.6% **21.** 0.04765 **22.** The total price of the tires is \$380.39.

- 23.** The sale price of the Hilfiger jacket is \$149.07.

- 24.** The percent of increase in population is 20.6%.

- 25.** The percent of B grades is 23.4%.

- 26.** Jerry earned \$5151 last month.

- 27.** Ukiah will owe his aunt \$6955 at the end of 1 year.

- 28.** The percent of calories from fat is 47.4%.

- 29.** Greg's investment is worth \$14,823.

- 30.** Loraine's credit card balance at the end of May is \$6935.79.

CHAPTER 7

Section 7.1

- 1.** feet or meters **3.** inches or millimeters

- 5.** inches or centimeters **7.** feet or meters

- 9.** feet or meters **11.** inches or centimeters

- 13.** feet or meters **15.** yards, feet, or meters

- 17.** 72 in. **19.** 41,000 m **21.** 36 ft

- 23.** 0.723 km **25.** 600,000 cm **27.** 0.57 mi

- 29.** 2.25 ft **31.** 398.78 cm **33.** 74.57 mi

- 35.** 4.79 ft **37.** 84 ft **39.** 15 in. **41.** 200 mm

- 43.** 74 ft **45.** 2600 mi **47.** 9 ft 4 in.

- 49.** 3.25 mi **51.** 24,075 mm **53.** 6 ft 7 in.

- 55.** 9792 mi **57.** 19 ft 7 in. **59.** 8 yd 3 in.

- 61.** Martha should buy 18 ft 8 in. of molding.

- 63.** Rosa can make 6 napkins and she will have 20 cm of fabric left over. **65.** Answers will vary.

- 67.** Nine lanes will require 8 lengths of dividers, which is a total of 400 m.

69. Figures for the Amazon and the Congo Rivers appear to be rounded, because of the two and three terminal zeros. This implies they are probably estimates.

71. The Amazon River is just about twice as long as the São Francisco River.

73. The yard is 20 ft wide and 28 ft long.

75. The main walkway is 4 ft wide, and the others are 2 ft wide.

77. Only like measures may be added or subtracted. Add (subtract) the numbers and keep the common unit of measure.

81. 0.0000478 **83.** $\frac{9}{35}$ **85.** $17\frac{21}{40}$ **87.** 24.96

89. 95

Section 7.2

- 1.** 41 lb **3.** 25 mL **5.** 4 gal **7.** 20 c
9. 84 qt **11.** 506 mL **13.** 82 oz **15.** 0 qt
17. 88 c **19.** 132.49 L **21.** 20 g **23.** 80 lb
25. 45 kg **27.** 8000 g **29.** 8 g **31.** 32 lb
33. 4728 oz **35.** 223 g **37.** 0.036 g
39. 4.41 oz **41.** 0°C **43.** 10°C **45.** 20°C
47. 300 sec **49.** 84 days **51.** 93.3°C **53.** 8.9°C
55. 87.8°F **57.** 57.2°F **59.** 13,140 hr
61. 0.2 day **63.** 26 lb 6 oz **65.** 7 gal 2 qt
67. The grocery sold 55 lb 7 oz of hamburger.
69. Each student receives 12.5 mL of acid. **71.** The temperature is 77°F.
73. Each child gets about 0.2 L, or 200 mL, of Mountain Dew.
75. Four bags contain 1588 g of potato chips.

Kilograms	54	58	63	69	76	85	97	130
Pounds	119	128	139	152	168	187	214	287

79. The *Jahre Viking* weighs about 673 million kg fully loaded.

81. He needs 11 servings of cereal and milk to get sufficient protein.

83. Because grams measure weight and inches measure length, it is not possible to add these measures.

89. 848 **91.** 41.94 **93.** $45\frac{7}{20}$ **95.** 153.45

97. 4820 m

Section 7.3

- 1.** 25 in. **3.** 28 m **5.** 74 mm **7.** 26 km
9. 83 mm **11.** 170 m **13.** 90 ft **15.** 100 cm
17. 72 yd **19.** 168 m **21.** 9.42 in.
23. 37.68 mm **25.** 31.4 km **27.** 49.13 cm
29. 74.55 yd **31.** 50.27 m **33.** 54 cm
35. 350 mm **37.** 105.13 ft
39. The five frames require 16 ft 8 in. of molding.
41. It will take Hazel 1 hr 48 min to bind the rug.
43. The inside dimensions of the frame are 20 in. by 26 in.
45. They need 117 ft of fence.

47. The football player ran 19,376 yd, which is about 11 mi.

49. Marylane needs 72 ft of lumber for the project.

51. The total cost of the lumber is \$139.41.

53. Yolande needs about 170 in. of lace for the second row.

55. The perimeter of the patio is 60 ft. **57.** The center line divides the yard into identical but mirror-image halves.

59.



Symmetrical



Not symmetrical

61. Perimeter is the distance around a flat surface. To find the perimeter of the figure, add the lengths of the four sides. If the figure were a park, the sides could be measured in miles or kilometers. If the figure were a room, the sides could be measured in feet or meters. If it were a piece of paper, the sides could be measured in inches or centimeters.

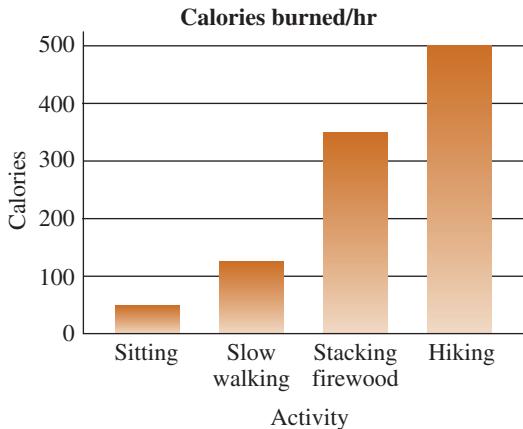
63. A square has all four sides equal. This doesn't have to be true for a rectangle. **67.** 950,000 **69.** 385

71. 3004 **73.** 5 ft 5 in. **75.** \$51

Section 7.4

- 1.** 36 m² **3.** 27 yd² **5.** 54 ft² **7.** 90 m²
9. $113\frac{1}{7}$ cm² **11.** 140 km² **13.** 1274 yd²
15. 1620 in² **17.** 396 cm² **19.** 135 in²
21. 630 m² **23.** 162.7776 in² **25.** 576 in²
27. 45 ft² **29.** 27,878,400 ft² **31.** 9 ft²
33. 13 yd² **35.** 58.5 ft² **37.** 0.54 mi²
39. 0.39 in² **41.** 3.59 yd² **43.** 13,935.46 cm²
45. 116 ft² **47.** 6550 m² **49.** 5775 in²
51. 346.87 cm² **53.** No, she needs more than 3 gal.
55. April and Larry need about 151 ft² of tile.
57. Debbie needs 562.5 oz of weed killer.
59. The doors require 36 ft² of glass.
61. The gable requires 162 ft² of sheathing.
63. The table can be covered with 1332 in² of padding.
65. Sheila should buy about 3 yd of material.
67. The region can be covered with six bags of seed.
69. The den can be paneled with six sheets of paneling.
71. The patio and walkways have an area of 326 ft².
73. Area is usually measured in square units. For example, square feet and square meters can be used. Acres are also used to measure land area.
75. Cover the area with as many whole squares as possible. For the leftover area, estimate half or quarter squares. Then find the total of the measures of all of the squares.
77. 3100 ft² will be walkway and the remaining 20,900 ft² will be lawn. **81.** 457,900 **83.** 46 **85.** 17
87. average: 50; median: 17.5; mode: 16

89.



Section 7.5

1. 1080 m^3
3. 250 ft^3
5. 168 cm^3
7. 50.24 ft^3
9. $27,000 \text{ mL}$
11. 42 ft^3
13. 3052.08 in^3
15. 4725 in^3
17. $38,250 \text{ cm}^3$
19. 1728 in^3
21. $1,000,000 \text{ cm}^3$
23. $46,656 \text{ in}^3$
25. 5 ft^3
27. 10 cm^3
29. $25,920 \text{ in}^3$
31. $15,000,000 \text{ cm}^3$
33. 0.10 ft^3
35. 4.17 km^3
37. 0.31 ft^3
39. 6360 in^3
41. 5520 cm^3
43. 174 ft^3
45. $20,900 \text{ mm}^3$
47. Three bags of potting soil are needed.
49. The total volume of the silos is $330,000 \text{ ft}^3$.
51. The slab is $211,248 \text{ in}^3 \approx 123 \text{ ft}^3 \approx 5 \text{ yd}^3$ of concrete.
53. The cement contractor needs 127 ft of lumber for the form.
55. The trash can has a 154-in^2 base.
57. Norma needs 10 yd^3 of mushroom compost.
59. Malisha can fit 12 balls into the box.
61. The volume of an object is the amount of space it will hold or displace. Volume is a consideration when trying to fit a birthday gift into a box, deciding how many servings of orange juice are in a container, or filling up the gas tank of your car.
63. In the formula $V = \ell wh$, ℓw represents the area of the base. Therefore, we can rewrite the formula as $V = \ell wh = Bh$.
65. The hole for the pool is 200 m^3 in volume. Lining the pool requires about 28.7 m^3 of concrete.
67. 512
69. 99
71. 7225
73. 3600
75. 72 in^2 or 0.5 ft^2

Section 7.6

1. 7
3. 6
5. 10
7. 20
9. $\frac{5}{9}$
11. 0.5
13. 12.25
15. 27.39
17. 3.27
19. 0.68
21. 107.24
23. $c = 10$
25. $c = 25$
27. $a = 20$
29. $b = 12$
31. $b \approx 33.17$
33. $c \approx 12.37$
35. $b \approx 58.09$
37. $b \approx 3.32$
39. The hypotenuse measures 75 cm.
41. The side of the square measures 12 ft.
43. The side of the square measures 19 m.
45. The top of the ladder is about 11.62 ft up the house.
47. It is about 127.28 ft from first base to third base.

49. Scott needs 12 ft of wire for the supports.

51. A square root of a number is a number that, when squared, gives back the original number. To find $\sqrt{289}$, we look for the number whose square is 289. Because $20^2 = 400$, we know that the number is less than 20. Working backward, we have $18^2 = 324$, which is too large but close. The number $17^2 = 289$. Therefore, $\sqrt{289} = 17$.

53. The rod will fit in the second box but not in the first one.

55. 6750 57. $16\frac{1}{5}$ 59. $8\frac{5}{8}$

61. The second tree is 9.375 ft tall.

63. The average is 22.6 and the median is 29.

Chapter 7 Review Exercises

Section 7.1

1. 60 cm
3. 3 yd 14 in.
5. 1 ft 4 in.
7. 10,560 ft
9. 381 mm

Section 7.2

11. 54 gal
13. 21 oz
15. 10 hr 20 min
17. 1.5 lb
19. 15°C

Section 7.3

21. $P = 98 \text{ cm}$
23. $P = 55 \text{ in.}$
25. $P = 40 \text{ yd}$
27. The perimeter is 300 m.
29. The field requires 180 ft of fencing.

Section 7.4

31. $A = 2075 \text{ mm}^2$
33. $A = 143 \text{ cm}^2$
35. $A = 69 \text{ yd}^2$
37. The area of the square is $360,000 \text{ mm}^2$.
39. One coat must cover 206 ft^2 , so two coats must cover 412 ft^2 ; therefore, she does not have enough paint.

Section 7.5

41. $V = 4608 \text{ cm}^3$
43. $V = 720 \text{ mm}^3$
45. $V = 336 \text{ m}^3$
47. The volume is 104 in^3 .
49. There are $46,656 \text{ in}^3$ in a cubic yard.

Section 7.6

51. 17
53. $\frac{9}{7}$, or 1.29
55. $b = 18$
57. $b \approx 11.53$
59. The top of the ladder is about 17.75 ft high.

Chapter 7 True/False Concept Review

1. true
2. false; equivalent measures have different units of measurement, as in 6 ft and 2 yd.
3. true
4. false; the perimeter of a square can be found in inches or meters or other measures of length.
5. true
6. false; the volume of a cube is $V = s^3$, or the area of a square is $A = s^2$, or a square has no volume.
7. false; the area of a circle is $A = \pi r^2$, or the area of a circle is $A \approx 3.14 r^2$.
8. true
9. true
10. true
11. false; $3^2 = 9$ or $\sqrt{9} = 3$
12. true
13. true

14. false; one milliliter is equivalent to 1 cubic centimeter.
 15. false; measurements can be added or subtracted only if they are expressed with the same unit of measure.
 16. true
 17. false; volume can also be measured in cubic units.
 18. false; the Pythagorean theorem states that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.
 19. false; $1 \text{ yd}^2 = 9 \text{ ft}^2$ 20. true

Chapter 7 Test

1. 8329 mm 2. 118 in. 3. 1440 in^3
 4. The room can be covered by 184 ft^2 of tile.
 5. 17.607 6. 29 cm 7. 40 in^2 8. 2 yd 2 ft 7 in.
 9. In the English system, there are pounds, ounces, and tons. In the metric system, there are grams, milligrams, and kilograms. 10. 48 pt
 11. The windows require 88 ft of molding.
 12. 260 in^3 13. 48 cm^2 14. The volume is 24 ft^3 .
 15. Each student will receive 8 oz of grape juice.
 16. Lining the garden plots will cost \$184.80. 17. 6 cm
 18. 96.2 km^2 19. Each player runs 5120 yd.
 20. Area is a measure of two-dimensional space, whereas volume is a measure of three-dimensional space.
 21. John's Meat Market averaged sales of 489.4 lb of steak each day.
 22. Li needs about 282.6 in. of lace. She must buy 8 yd.
 23. The box has a larger volume than the bag, so all the dog food will fit into the box.
 24. Macy is 3.7 mi from her starting point.
 25. Neela needs more than 544 in^2 of wrapping paper.

Chapters 1–7 Cumulative Review

1. seven thousand forty-eight 3. 309, 375, 381, 384,
 390 5. 175 7. 20,832 9. 610
 11. average, 45; median, 44; mode, 56
 13. Dental hygienists have the lowest average salary.
 15. Sylvia collected pledges totaling \$1855.
 17. Millie earned \$633.
 19. 1, 2, 3, 4, 6, 12, 17, 34, 51, 68, 102, 204
 21. 31, 62, 93, 124, 155 23. It is divisible by 3 and 5.
 25. $2^3 \cdot 3 \cdot 5 \cdot 7$ 27. $\frac{3}{7}$ 29. $21\frac{17}{20}$ 31. $\frac{8}{9}$
 33. $\frac{17}{10}$ or $1\frac{7}{10}$ 35. $\frac{37}{140}$ 37. $1\frac{3}{4}$
 39. The number of students who passed the mathematics competency exam is 318.
 41. eleven and sixty-three thousandths
 43. 1.046, 1.307, 1.3491, 1.35, 1.4005 45. 279.24
 47. 3.45×10^{-5} 49. 15.65 51. 0.6875
 53. 22.32
 55. Bill should cut 2.125 in. from each end of the board.
 57. The ratio of the females to the total class is $\frac{11}{20}$.
 59. $A = 108$
 61. The tree will cast a shadow of about 25.7 ft.

63. 0.079 65. 42.1% 67. 34.375%
 69. $\frac{143}{200}$ 71. 23.29 73. 48.5
 75. Helga earned \$1155.12 last week.
 77. 8 yd 2 ft 6 in. 79. 820 mL 81. 3.7 mi
 83. 1.59 kg
 85. The temperature measured in Celsius is about 93.9°C.
 87. 108 in. 89. The area of the triangle is 49.5 ft^2 .
 91. 452 in^2 93. The volume is about 1017.36 cm^3 .
 95. The carpet will cost Trudy \$6175. 97. 9.25
 99. The hypotenuse is 45 ft in length.

CHAPTER 8

Section 8.1

1. 4 3. -5 5. 2.3 7. $-\frac{3}{5}$ 9. $\frac{6}{7}$
 11. -61 13. 42 15. 3.78 17. $-\frac{17}{5}$
 19. -0.55 21. -113.8 23. 0.0123
 25. 4 27. 24 29. 3.17 31. $\frac{5}{11}$ 33. $\frac{3}{11}$
 35. 32 or -32 37. 0.0065 39. 355 41. $\frac{11}{3}$
 43. $\frac{33}{7}$ 45. 0 47. 0.341 49. $-\frac{5}{17}$
 51. -78
 53. The opposite of a gain of 1.54 is -1.54.
 55. The opposite of a reading of 14°C is -14°C.
 57. The opposite of 1915 B.C. is A.D. 1915.
 59. As signed numbers, the distance up the mountain is +1415 ft and the distance down the mountain is -1143 ft.
 61. Eighty miles south would be represented by -80 miles.
 63.

Ocean	Deepest Part (ft)	Average Depth (ft)
Pacific	Mariana Trench, -35,840	-12,925
Atlantic	Puerto Rico Trench, -28,232	-11,730
Indian	Java Trench, -23,376	-12,598
Arctic	Eurasia Basin, -17,881	-3,407
Mediterranean	Ionian Basin, -16,896	-4,926
65. 24 67. -14		
69. On the first play, the Seahawks made -8 yd. The opposite of the loss of eight yards is +8 yd.		
71. The distances driven by the Golden family expressed as signed numbers are +137 mi, +212 mi, -98 mi, and -38 mi.		
73. The positive real numbers and zero each have an absolute value that is the number itself. This is because the absolute value of a number is its distance (distance is positive) from zero and the positive numbers are all a positive distance from zero.		
75. 7 77. positive 81. 52 83. 22		
85. 7173 mL 87. 20 pt		
89. Two aspirin tablets contain 154.3 g.		

Section 8.2

1. 2 3. 2 5. -15 7. -14 9. 0
11. -12 13. -46 15. -14 17. -26
19. 7 21. -6 23. -108 25. 0 27. 27
29. -10 31. -7 33. -8.3 35. 2.8
37. $-\frac{7}{12}$ 39. $-\frac{7}{6}$ 41. -121 43. -189
45. -30 47. 81 49. -757 51. -114
53. The sum of the temperatures recorded at 9:00 A.M. is -349°C.
55. The sum of the temperatures recorded at 11:00 P.M. is -505°C.
57. The net change in weight the cargo master should report is -108 lb.
59. The altitude of the airplane at 4 P.M. is 29,870 ft.
61. The inventory at the end of the month is 30,370 volumes.
63. No, the Bills were 1 yd short of a first down.
65. a. The net change in the Nordstrom stock for the week is up 1.76. b. The closing price of the Nordstrom stock is 74.59.
67. The net gain for the month is \$5077.
69. Adding signed numbers can be thought of as moving along a number line according to the absolute value and sign of the numbers. Starting at zero, move as many spaces as the absolute value of the first number. Move right if the number is positive and left if it is negative. Then move according to the second number. The sign of the sum is determined by which side of zero you end up on. So if you have moved right for more spaces than you have moved left, the sum will be positive. And if you have moved left more spaces than you have moved right, the sum will be negative. Symbolically, compare the absolute values of the numbers being added. The sign of the one with the largest absolute value (corresponding to the most spaces moved in a particular direction) is the sign of the answer.
71. -124 73. 32 77. 267 79. 199
81. The circumference is 106.76 cm.
83. The perimeter is 195 cm.
85. The cost of pouring the walk is \$12,348.

Section 8.3

1. 4 3. -12 5. -5 7. 3 9. -20
11. -9 13. 32 15. -34 17. -28 19. 1
21. 0 23. -18 25. -80 27. 118
29. -119 31. 11 33. -182 35. 204
37. 0 39. 144 41. 173 43. -9.98
45. -1.44 47. -56.14 49. 116 51. -687
53. The change in temperature is 85°C.
55. Joe wrote checks totaling \$796.32.
57. The range of altitude for each continent is Africa, 19,852 ft; Antarctica, 25,191 ft; Asia, 30,340 ft; Australia, 7362 ft; Europe, 18,602 ft; North America, 20,602 ft; and South America, 22,965 ft. The smallest range is in Australia. It means it is a rather flat continent.
59. The depth of the ocean floor at Mauna Kea is -19,680 ft, or 19,680 ft below sea level.

61. The difference in temperatures recorded at 11:00 P.M. on day 3 and day 5 is -15°C.
63. The difference between the highest and lowest temperatures recorded during the 5 days is 105°C.
65. Thomas's bank balance is -\$6.41, or \$6.41 overdrawn.
67. Janna wrote checks totaling \$477.14.
69. Gina's sales total -\$164.45, or a loss of \$164.45.
71. The difference in coal imports from 1980 to 1990 is -0.31 quadrillion BTU. The difference in coal imports from 1990 to 2004 is 2.24 quadrillion BTU. The difference in coal imports from 1980 to 2004 is 1.93 quadrillion BTU.
73. On a number line, start at zero and move 12 units to the right, positive. Now if we were adding -8, we would move 8 units to the left of +12, but because we are subtracting we reverse the direction and move an additional 8 units to the right, ending at +20.
75. The first problem is addition, and the second is subtraction. The answer to the two problems is the same, -3.
77. -12.68 79. -41 83. 546 85. 1,731,084
87. The area of the square is 278.89 m².
89. The area of the triangle is 56.16 m².
91. The number of tiles needed is 896.

Section 8.4

1. -8 3. -10 5. -80 7. -77 9. 11
11. -306 13. -255 15. -9 17. -350
19. $-\frac{1}{4}$ 21. 3 23. 28 25. 99 27. 60
29. -11 31. 210 33. 5.4 35. 24.2
37. $\frac{1}{6}$ 39. 1.645 41. -2520 43. 465
45. 7.14 47. $-\frac{3}{10}$ 49. -14.035 51. 0.5423
53. 0 55. 21 57. -240 59. $\frac{1}{3}$
61. The Celsius measure is -15°C.
63. Ms. Riles's loss expressed as a signed number is -20.8 lb.
65. The Dow Jones loss over 12 days is -20.88.
67. -49 69. -42 71. -252
73. Safeway's loss expressed as a signed number is -\$65.62.
75. Winn Dixie's loss expressed as a signed number is -\$31.38.
77. Safeway's loss expressed as a signed number is -\$250.79.
79. The trader paid \$6257.80 for the stock. She received \$4321.50 when she sold the stock. She lost \$1936.30, or -\$1936.30, on the sale of the stock. 81. Asia
83. An even number of negatives may be grouped in pairs. Because the product of each pair of negatives is positive, the next step is multiplying all positive numbers. So the final product is positive.
85. -70 87. 56 91. 68 93. 268
95. The piecework wage is \$2.60 per piece.
97. The tip of the hand moves 9.42 in.
99. The cost is \$0.5625 per square foot per year.

Section 8.5

1. -2 3. -4 5. -3 7. -8 9. 8
11. -6 13. -2.02 15. -35 17. -3
19. -1.2 21. 2 23. 3 25. 7 27. 6
29. -5 31. 7 33. 8 35. 4.04 37. $\frac{1}{2}$

39. 5 41. -45 43. 116 45. -4.5
47. 0 49. -0.0026 51. -452 53. 16

55. Each member's share of the loss is -\$125.65.
57. Mr. Harkness loses an average of -4.6 lb per week.
59. The average daily loss for the stock is -3.82 points.
61. Answers will vary.
63. The continent is Australia.

65. The division rules are the same as a consequence of the fact that division is the inverse of multiplication. Every division fact can be rewritten as a multiplication fact.

So we have: $a \div b = a \times \frac{1}{b}$.

67. -10 69. $-\frac{8}{5}$ 71. -1.4 73. 7 75. 18

77. The volume of the cylinder is about 4823.04 in³.

79. The tank will hold approximately 3805 gallons.

81. There will be 12 truckloads.

Section 8.6

1. -24 3. 19 5. -6 7. -16 9. -6
11. 8 13. 4 15. 13 17. 11 19. 8
21. 1 23. -22 25. -6 27. 40 29. -38
31. -8 33. 8 35. 0 37. -24 39. 64
41. -11 43. -15 45. 144 47. -12

49. The average temperature recorded during day 5 is -56°C.

51. The average high temperature recorded for the 5 days is -34°C.

53. 77 55. -41 57. $-\frac{62}{3}$ 59. 1

61. -2065 63. -1594 65. -16

67. Keshia pays a total of \$800 for the TV set.

69. K-Mart has a loss on the fishing poles of -\$9.

71. The average of the highest and lowest points are Africa, 9414 ft; Antarctica, 4268.5 ft; Asia, 13,858 ft; Australia, 3629 ft; Europe, 9209 ft; North America, 10,019 ft; and South America, 11,351.5 ft. The largest average is in Asia. The smallest average is in Australia.

73. The three cities are Athens, Bangkok, and New Delhi.

75. The balance at the end of the month is \$3226.

77. At the end of the trip you are 78 miles north and 91 miles west of the starting point.

79. The problem is worked according to the order of operations. So we work inside the brackets first, $3 + 5(-4)$. This is where the error has been made. We must multiply first, so $2[3 + 5(-4)] = 2[3 + (-20)]$

$$= 2[-17] \\ = -34$$

81. Yes, if a different operation is inside parentheses. For example, in $(3 - 6)^2$ the subtraction is done first. So, $(3 - 6)^2 = (-3)^2 = 9$.

83. -11 87. -17.09 89. 16.1 91. -40,920
93. -7366 95. The stock gains 0.438 point for the week.

Section 8.7

1. $x = 7$ 3. $x = 5$ 5. $y = -5$ 7. $a = 7$
9. $x = 7$ 11. $x = 10$ 13. $x = -3$ 15. $x = -6$
17. $x = 0$ 19. $x = 3$ 21. $y = 7$ 23. $a = 0$
25. $x = -0.5$ 27. $x = -25$ 29. $x = -16$

31. $y = -4$ 33. $a = \frac{4}{5}$

35. The number is 28. 37. The number is 8.

39. The acceleration is 33.

41. There will be 14 payments.

43. The equation is $7310 = 4L + 12,558$, where L represents the lowest point. Asia's lowest point fits the description.

45. The equation is $-35,840 = 2D - 2000$, where D represents the deepest part. The Ionian Basin's deepest part fits this description.

47. $z = 7$ 49. $z = 6$

- 51.

$$\begin{array}{r} -3x + 10 = 4 \\ \hline -10 \quad -10 \\ \hline -3x = -6 \\ \hline -3x = -6 \\ \hline -3 \quad -3 \\ x = 2 \end{array}$$

Add -10 to both sides.
Simplify.
Divide both sides by -3.
Simplify.

53. $x = -4$ 55. $x = -2$

Chapter 8 Review Exercises

Section 8.1

1. 39 3. 0.91 5. 16.5 7. 71 9. -6.4

Section 8.2

11. -98 13. 52 15. -3.2 17. -0.53
19. Yes, the Bears gained 10 yd.

Section 8.3

21. 22 23. 91 25. -7.81 27. 29
29. Maria wrote checks totaling \$685.90.

Section 8.4

31. -66 33. -4.08 35. 51 37. 8.463
39. Kroger's loss expressed as a signed number is -\$284.40.

Section 8.5

41. -3 43. 5 45. -16 47. -56.2
49. The average daily loss of the stock is -3.875 points.

Section 8.6

51. -56 53. -50 55. -24 57. 17
59. -20.72

Section 8.7

61. $x = -5$ 63. $x = 25$ 65. $x = 0$

67. $a = -13$

69. The Fahrenheit temperature is -7.6°F .

Chapter 8 True/False

Concept Review

1. true; 2. false; the opposite of a positive number is negative.
3. false; the absolute value of a nonzero number is always positive.
4. true; 5. false; the sum of two signed numbers can be positive, negative, or zero.
6. false; the sum of a positive signed number and a negative signed number may be positive, negative, or zero.
7. true 8. false; to subtract two signed numbers, add the opposite of the number to be subtracted.
9. true
10. true 11. false; the sign of the product of a positive number and a negative number is negative.
12. true 13. true 14. true

Chapter 8 Test

1. -33
2. -35
3. $-\frac{5}{4}$
4. $\frac{2}{15}$
5. -5
6. 1.15
7. a. 17
- b. 33
8. -33
9. -2
10. 2
11. 96
12. -21
13. -108.9
14. 40
15. -31.2
16. $-\frac{1}{6}$
17. 14
18. -80
19. -86
20. $-\frac{3}{10}$
21. -68
22. -5
23. $x = -6$
24. $x = 7$
25. $a = -3.5$
26. Ms. Rosier lost -16.8 lb.
27. The drop in temperature is -21°F .
28. The closing price on Friday is 17.97 .
29. The Fahrenheit temperature is 14°F .
30. The average is -4 .

Chapters 1–4 Midterm Examination

1. thousand 2. $84,183$
3. sixty-seven thousand, five hundred nine
4. 2341 5. 27,605 6. 45,760 7. 720,000;
727,518 8. 213 9. 1592 R 30 10. 13
11. 74 12. 173
13. average, 81; median, 73; mode, 64
14. a. Civic has the highest sales. b. There are 5 more Accords sold than Passports.
15. 120 16. It is divisible by 3 and 5.
17. prime number 18. yes
19. 41, 82, 123, 164, 205
20. 1, 2, 4, 8, 16, 19, 38, 76, 152, 304
21. $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 = 2^2 \cdot 3^3 \cdot 5$
22. $11\frac{6}{7}$
23. $\frac{123}{8}$
24. $\frac{3}{4}, \frac{7}{8}$
25. $\frac{5}{9}, \frac{7}{12}, \frac{5}{8}, \frac{2}{3}$
26. $\frac{4}{5}$
27. $\frac{5}{72}$
28. $46\frac{2}{7}$
29. 1
30. $\frac{8}{19}$

31. $\frac{97}{90}$ or $1\frac{7}{90}$
32. $18\frac{13}{24}$
33. $5\frac{3}{8}$
34. $17\frac{34}{63}$
35. $7\frac{3}{4}$
36. $\frac{4}{15}$
37. 80,240.122
38. 6.7, 6.74, 6.736
39. $\frac{13}{50}$
40. 1.02, 1.034, 1.044, 1.07, 1.094, 1.109
41. 173.76
42. 2,537
43. 164.294
44. 4,600
45. 0.000902
46. 5.8×10^{-4}
47. 56.8
48. 0.529
49. average, 6.4; median, 6.9
50. 1.62
51. The perimeter is 60 cm.
52. The area is $63\frac{4}{5} \text{ ft}^2$.
53. A total of 6950 chinook were counted.
54. Maria received \$107.20 as a dividend.
55. The bananas cost \$0.74 per pound.

Chapters 1–8 Final Examination

1. $\frac{17}{16}$ or $1\frac{1}{16}$
2. 630
3. 35.73
4. 54,363
5. $\frac{3}{2}$ or $1\frac{1}{2}$
6. 130
7. 199
8. 38.33
9. $9\frac{1}{12}$
10. 945
11. 246.71
12. 47.212
13. $\frac{16}{25}$
14. $10\frac{17}{30}$
15. $x = \frac{22}{3}$ or $7\frac{1}{3}$
16. ten-thousandths
17. $\frac{7}{10}, \frac{3}{4}, \frac{4}{5}$
18. 92%
19. 17.23
20. 0.515
21. 6000.015
22. 8.09
23. 0.57625
24. 50
25. 452%
26. 0.284
27. 59.86
28. 0.6435
29. The cost of 25 lb of strawberries is \$46.25.
30. The percent of discount is about 18.8%.
31. 51,102, 153, 204, 255
32. eight thousand, thirty-seven and thirty-seven thousandths
33. false
34. $2 \cdot 2 \cdot 2 \cdot 5 \cdot 17 = 2^3 \cdot 5 \cdot 17$
35. $\frac{7}{10}$
36. $\frac{189}{200}$
37. $42\frac{5}{8}$
38. $\frac{2}{15}$
39. $\frac{21}{250}$
40. Of the customers surveyed, 54% ordered a Big Mac.
41. no
42. $13\frac{13}{15}$
43. $\frac{239}{11}$
44. 2.299, 2.32, 2.322, 2.332
45. 1, 2, 3, 4, 6, 8, 12, 17, 24, 34, 51, 68, 102, 136, 204, 408
46. 0.00045893
47. $2\frac{14}{15}$
48. $\frac{17}{100}$

- 49.** It will cost Mildred \$1455 to drive 7500 miles.
50. Billy earns \$17,873 for the month.
51. The sales tax rate is 8.6%.
52. The final cost of the vacuum cleaner is \$183.02.
53. 39 **54.** 66.7 **55.** 8 hr 24 min 20 sec
56. 1 m 83 cm **57.** \$50 per kilogram

- 58.** The perimeter is 183 ft. **59.** 26.46 m^2
60. 62.13 yd^2 **61.** 1932 in^3 **62.** 21.37
63. The hypotenuse of the triangle is about 35.5 cm.
64. -16 **65.** 22 **66.** -105 **67.** 3.1 **68.** 2
69. $a = -8$
70. The Fahrenheit temperature is 5°F.

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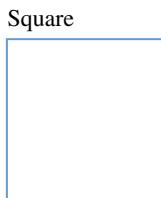
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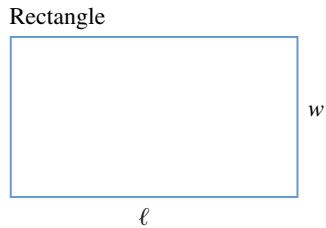
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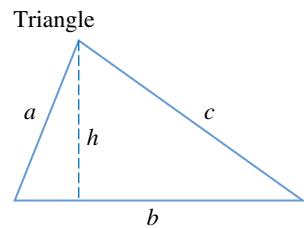
Perimeter and Area



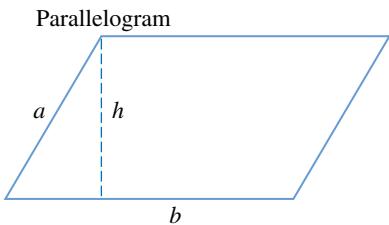
Perimeter: $P = 4s$
Area: $A = s^2$



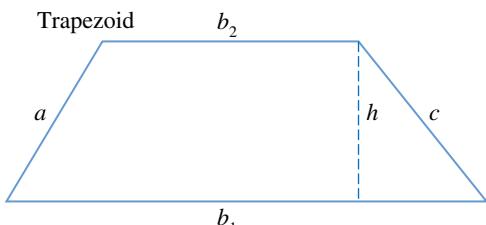
Perimeter: $P = 2\ell + 2w$
Area: $A = \ell w$



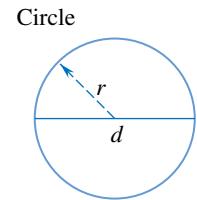
Perimeter: $P = a + b + c$
Area: $A = \frac{bh}{2}$



Perimeter: $P = 2a + 2b$
Area: $A = bh$

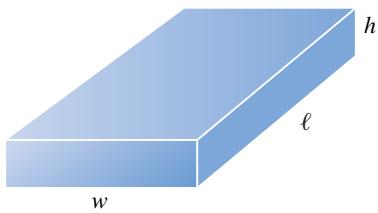


Perimeter: $P = a + b_1 + c + b_2$
Area: $A = \frac{1}{2}(b_1 + b_2) \cdot h$

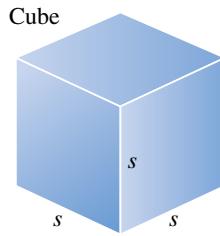


Circumference: $C = 2\pi r$
 $C = \pi d$
Area: $A = \pi r^2$

Volume



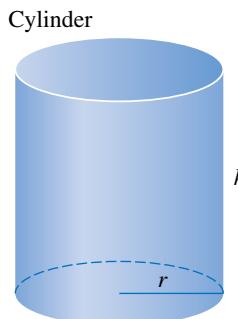
$$V = \ell w h$$



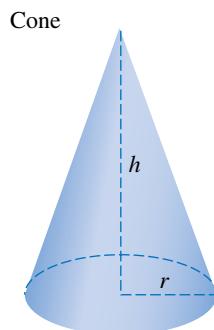
$$V = s^3$$



$$V = \frac{4}{3} \pi r^3$$



$$V = \pi r^2 h$$



$$V = \frac{1}{3} \pi r^2 h$$

Measures (English, Metric, and Equivalents)

English Measures and Equivalents	
Length	Time
12 inches (in.) = 1 foot (ft)	60 seconds (sec) = 1 minute (min)
3 feet (ft) = 1 yard (yd)	60 minutes (min) = 1 hour (hr)
5280 feet (ft) = 1 mile (mi)	24 hours (hr) = 1 day
Liquid Volume	Weight
3 teaspoons (tsp) = 1 tablespoon (tbs)	16 ounces (oz) = 1 pound (lb)
2 cups (c) = 1 pint (pt)	2000 pounds (lb) = 1 ton
2 pints (pt) = 1 quart (qt)	
4 quarts (qt) = 1 gallon (gal)	

Metric Measures and Equivalents						
Kilo-	Hecto-	Deka-	Base unit	Deci-	Centi-	Milli-
(1000x)	(100x)	(10x)		$\left(\frac{1}{10}x\right)$	$\left(\frac{1}{100}x\right)$	$\left(\frac{1}{1000}x\right)$
Length (Base unit is 1 meter)			Weight* (Base unit is 1 gram)		Liquid Volume (Base unit is 1 liter)	
1 meter (m) = 1000 millimeters (mm)	1 gram (g) = 1000 milligrams (mg)		1 liter (l) = 1000 milliliters (ml)			
1 meter (m) = 100 centimeters (cm)	1 gram (g) = 100 centigrams (cg)		1 liter (l) = 100 centiliters (cl)			
1 kilometer (km) = 1000 meters (m)	1 kilogram (kg) = 1000 grams (g)		1 kiloliter (kl) = 1000 liters (l)			
	1 metric ton = 1000 kilograms (kg)					

System-to-system Equivalents	
English → Metric Conversions	Metric → English Conversions
1 inch = 2.54 centimeters	1 centimeter ≈ 0.3937 inch
1 foot ≈ 0.3048 meter	1 meter ≈ 3.2808 feet
1 yard ≈ 0.9144 meter	1 meter ≈ 1.0936 yards
1 mile ≈ 1.6093 kilometers	1 kilometer ≈ 0.6214 mile
1 quart ≈ 0.9464 liter	1 liter ≈ 1.0567 quarts
1 gallon ≈ 3.7854 liters	1 liter ≈ 0.2642 gallon
1 ounce ≈ 28.3495 grams	1 gram ≈ 0.0353 ounce
1 pound ≈ 0.4536 kilogram	1 kilogram ≈ 2.2046 pounds
Area Conversions	
1 ft ² = 144 in ²	1 cm ² = 100 mm ²
1 yd ² = 9 ft ²	1 m ² = 10,000 cm ²
1 mi ² = 3,097,600 yd ²	1 km ² = 1,000,000 m ²
1 in ² = 6.4516 cm ²	1 cm ² ≈ 0.1550 in ²
1 ft ² ≈ 0.0929 m ²	1 m ² ≈ 10.7636 ft ²
1 mi ² ≈ 2.5898 km ²	1 km ² ≈ 0.3861 mi ²

*These units are technically reserved for measuring mass. We use them for weight or mass. The difference between weight and mass is covered in science classes.