

$$y = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$$

$$\begin{aligned} x_{n+1} &= x_n + v_n \delta t \\ v_{n+1} &= v_n - x_{n+1} \delta t \end{aligned}$$

$$\begin{pmatrix} x_n \\ \dot{x}_n \end{pmatrix} = y_n$$

$$\begin{aligned} x_{n+1} \delta t + \dot{x}_{n+1} &= x_n + v_n \delta t \\ &= v_n \end{aligned}$$

$$\begin{pmatrix} 1 & 0 \\ \delta t & 1 \end{pmatrix} y_{n+1} = \begin{pmatrix} 1 & \delta t \\ 0 & 1 \end{pmatrix} y_n$$

$$\begin{pmatrix} 1 & 0 \\ \delta t & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\delta t & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow y_{n+1} = \begin{pmatrix} 1 & 0 \\ -\delta t & 1 \end{pmatrix} \begin{pmatrix} 1 & \delta t \\ 0 & 1 \end{pmatrix} y_n$$

$$= \begin{pmatrix} 1 & \delta t \\ -\delta t & -\delta t^2 + 1 \end{pmatrix} y_n$$

$$\begin{vmatrix} 1-\lambda & \delta t \\ -\delta t & -\delta t^2 + 1 - \lambda \end{vmatrix} = \chi(\lambda)$$

$$\begin{aligned} & (1-\lambda)(-\delta t^2 + 1 - \lambda) + \delta t^2 \\ &= -\cancel{\delta t^2} + \underline{1} - \lambda + \lambda \delta t^2 - \lambda + \underline{\lambda^2} + \cancel{\delta t^2} \stackrel{!}{=} 0 \end{aligned}$$

$$= 1 + (\delta t^2 - 2)\lambda \stackrel{!}{=} 0$$

$$\Rightarrow \lambda_{1/2} = \frac{\delta t^2 - 2}{2} \pm \sqrt{\left(\frac{\delta t^2 - 2}{2}\right)^2 + 1}$$

d. (+) - EW ist für $|\delta t| < \sqrt{2}$
~~kleiner~~ kleiner als 1.