computational physics

problem 1.2

(a) $f(x) = 3x^4 + 4x^3 - x^2 - 2x$

$= x(3x^3 + 4x^2 - x - 2)$

$\Rightarrow x_0 = 0$

$3(-1)^3 + 4(-1)^2 - (-1) - 2$

$= -3 + 4 + 1 - 2 = 0$

$\Rightarrow x_1 = (-1)$

$(3x^3 + 4x^2 - x - 2) : (x + 1) = 3x^2 + x - 2$

$-(3x^3 + 3x^2)$

$x^2 - x$

$-(x^2 + x)$

$-2x - 2$

$-(2x - 2)$

0

$f(x) = \underbrace{(3x^2 + x - 2)}_{\stackrel{!}{=} 0}(x + 1)\, x$

$x^2 + \frac{1}{3}x - \frac{2}{3} = 0$

$x_{2,3} = -\frac{1}{6} \pm \sqrt{\frac{1}{36} + \frac{24}{36}}$

$= -\frac{1}{6} \pm \frac{5}{6}$

$\Rightarrow x_2 = \frac{4}{6}\ ;\ x_3 = -1$

$\Rightarrow \quad x_1 = x_3 = -1$

non ~~zero~~ 1 multiplicity of this root may cause problems with bisection method.

(b) using numerical methods by hand

bisection method

$5^4 \gg 5^3$, thus beyond $[-5, 5]$ the leading term dominates and f does not change sign;

evaluating $f(x)$ on grid with nodes 0.5 apart in interval $[-5, 5]$ gives:

x	$f(x)$
-5	1360
-4.5	854.44
-4	504
-3.5	273.44
-3	132
-2.5	53.44
-2	16
-1.5	2.44
-1	0
-0.5	0.44
0	0
0.5	-0.56
1	4
1.5	23.44
2	72
2.5	168.44
3	336
3.5	602.44
4	1000
4.5	1565.44
5	2340

this already gives

$f(-1) = 0$

and

$f(0) = 0$

also f changes sign between 0.5 and 1

$f(0.5) < 0$

$f(1) > 0$

$\sim 0.57 = f(0.75) > 0$

$\Rightarrow$ zero between 0.5 and 0.75

$f(0.625) \approx -0.21 < 0$

$\Rightarrow$ zero between 0.625 and 0.75

$f((0.625 + 0.75)/2)$

$= f(0.6875) \approx 0.122 > 0$

$\Rightarrow$ zero between 0.625 and 0.6875

now that we are close to the root we can apply Newtons method from the centre of the interval $([0.625, 0.68751])$ $x_0 = 0.65625$;

five iterations give:

$x_0 = 0.65625$

$x_1 = 0.666972$

$x_2 = 0.666667$

$x_3 = "$

$x_4 = "$

$x_5 = "$

this concludes

$$f(-1) = 0$$
$$f(0) = 0$$
$$f(0.666667) = 0$$

as roots found via numerical methods with pen, paper and pocket calculator;