Heinrich–Heine–Universität Düsseldorf
Institut für Theoretische Physik II
Computational Physics
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Problem 9.1: **Monte-Carlo Integration**

Integrals can be computed using the Monte-Carlo method. Here in this exercise we calculate the area of a circle in 2D. Consider a circle of radius $r = 0.5$ embedded in a square of length $l = 1$. The constant π is equal to the ratio between the area of the circle A_c and the area of the square A_s

$$\frac{A_c}{A_s} = \frac{\pi}{4} \tag{1}$$

The ratio in Eq. (1) can be approximated by sampling uniformly distributed random points (x, y) in the square, $x \in [-0.5, 0.5]$ and $y \in [-0.5, 0.5]$, using the equation

$$\frac{A_c}{A_s} = \frac{N_c}{N} \ , \tag{2}$$

where N and N_c correspond to the total number of sampling points and the number of points inside the circle, respectively. A trial point contribute to N_c if $x^2 + y^2 < r^2$ with $r = 0.5$.

a) Plot the value of the estimated π as a function of N; try $N = 10$, $N = 100$, $N = 1000$, and $N = 10000$.

b) Repeat a), but now using the Markov chain sampling.

c) Find the optimum value of the maximum number displacement δ, i.e. the value of δ for which the convergence is faster (see lecture for a definition of δ).

Hints: To generate uniform random numbers $u_n \in [0, 1]$ can use the recursive equation

$$x_n = \mathrm{mod}(a \cdot x_{n-1}, b)$$
$$u_n = \frac{x_n}{c} \ ,$$

with $a = 16807$, $b = 2147483648$, $c = b + 1$ and $x_0 \in [0, 1]$.

Problem 9.2: **The volume of a hyper-sphere**

The shape of a circle (defined in 2D by $x^2 + y^2 < R^2$) can be generalized to three dimensions by $x^2 + y^2 + z^2 < R^2$ (referred to as sphere). One can continue the generalization to higher dimensions although they are difficult to imagine.

a) Find, in the literature, the general definition of a sphere in n-dimensions.

b) By extending your code Exercise 9.1, find the volume of a sphere in 5 dimensions.