

Problem 12.1: Widom insertion method

In this exercise you calculate the chemical potential μ of a system using the Widom insertion method.

- a) Derive Widom's formula:

$$\mu - \mu_{\text{ideal}} = \mu_{\text{ex.}} = -k_{\text{B}}T \ln \left(\int d\mathbf{s}_{N+1} \left\langle \exp\left(-\frac{\Delta U}{k_{\text{B}}T}\right) \right\rangle_N \right), \quad (1)$$

with μ_{ideal} and $\mu_{\text{ex.}}$ the ideal gas and excess chemical potential, respectively, $\langle \cdots \rangle_N$ the canonical average of a system with N particles, T the temperature, k_{B} Boltzmann's constant, \mathbf{s}_{N+1} the position of the trial particle, and ΔU the energy change due to the insertion of the trial particle.

- b) Write a code to calculate the chemical potential of the system that you have worked with in Ex. 10 (two-dimensional WCA potential in the canonical ensemble at $T = 1.0$), but use a smaller density $\rho = 0.3$.
- c) Now change the temperature to $T = 2.0$ and $T = 3.0$. Plot the chemical potential as a function of T . By increasing the temperature, do you expect an increase or a decrease in $\mu_{\text{ex.}}$?