



## MODEL DOCUMENTATION

*April 1, 2021*

<b>Object</b>	<b>LA18</b>
Direction	Global Market
Author(s)	IR QR
Reference	Model
Diffusion	Restricted
Version	1.0
Complexity	Medium
Proposed tier	1

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# 1 General information

Model Development Information	
Model ID	No reference
Model Name	Strike Equivalent
Business unit	Natixis / Global Market / Trading FI
Model purpose	Model
Name of the model development team	IR QR
Model development start date	
Model development completion date	
Date of the most recent revision	
Model Implementation Information	
Implementation platform	ARM
Integration platform	Summit
Name of model implementation team	DSI
Date of implementation	
Model Governance Information	
Name of the model owner	Pascal Amiel
Date model deployment approved for initial Business use	
Most recent date model approved for continued Business use	
Approved uses	LD-M-FIC4-INFLATION
Model restrictions	Payoff dependent
Approved users	
Name of the model validator lead for the most recent validation	MRM
Date of the most recent prior validation/review	
Type of the most recent prior validation/review	Periodical Review
Model Key Documentation	
Model documentation names	New Livret A pricing
Previous validation report names	
Business requirement documentation name	New Livret A pricing
Specification documentation name	
IT implementation sign off documentation name	
User acceptance report name	
Regulatory references	
Calibration files name	
Back testing documentation name	
Stress testing documentation name	

## 2 Description

### 2.1 Purpose and use

This document aims at describing the pricing methodology of the LA18 option, an inflation-indexed cap floor option, used as a hedging instrument against the interest rate variations of the Livret A, the most popular French saving account.

Without taking convexity adjustments into account, a LA18 option is sensitive to the following risk factors:

Payoff	Risk factors
LA18 Option	CPI curve
	Discount rate curve
	Forecast rate curve
	Forecast rate volatility smile
	YoY index volatility smile
	YoY/YoY correlation
	YoY/Forecast rate correlation
	Forecast rate/Forecast rate correlation

### 2.2 In House or Vendor

The LA18 model is developed by the IR QR team. It is implemented in ARM, the in-house pricing library and used in Summit, the vendor front to back system.

### 2.3 Outputs

The current model is used to calculate the price of the LA18 option. The resulting price is used by summit to compute the greeks by means of bumping the risk factors, repricing and taking the finite difference.

### 2.4 Appropriateness of the combination

The LA18 option cash flows can be seen as a two-dimensional basket option. A moment matching projection is performed to allow the pricing with a bivariate formula. To take into account the volatility smile, a strike equivalent routine is applied. This choice is made to reduce the pricing time.

### 2.5 Pricer use

The LA18 pricer is used by the following stakeholders :

- Model Owner: Pascal Amiel
- Model Users:
  - FO IRD :
    - \* Trading : pricing and hedging
    - \* Structuring : pricing and testing
    - \* Quants : development, testing, support and maintenance
    - \* Sales : pricing
  - DRM : risk monitoring via VaR, CVA and other risk indicators
  - SDR : P&L production and consensus contributions
- Model Developer (Quants): for model development and maintenance
- Model Implementer: DSI

### 3 Payoff

Since February 2020, the Livret A rate is fixed by Banque de France twice a year on January 15 and July 15. These rates are published as the applicable fixing rates for February 1<sup>st</sup> and August 1<sup>st</sup> respectively and are computed as the average of the French inflation and EONIA rates over the same observation window, currently of six months with a 0.5% floor.

#### 3.1 Cash flow

Considering a six month interest period  $[T_{\delta-}, T]$ , a floor rate  $R_F$  of 0.5% and denoting by  $(Y_{T_y^i})_{i=1}^n$ ,  $n$  successive monthly year-on-year (YoY) and  $(F_{T_f^j})_{j=1}^m$ ,  $m$  daily successive EONIA rates over that period, the Livret A rate is obtained in a simple manner at time  $T$  by  $R_T^{LA} = \max(R_F, LA18_T)$  where :

$$LA18_T = \frac{1}{2n} \sum_{i=1}^n \delta_i Y_{T_y^i} + \frac{1}{2m} \sum_{j=1}^m \delta_j F_{T_f^j} \quad (3.1)$$

with  $T_y^1 = T_e^1 = T_{\delta-}$  and  $T_y^n = T_e^m = T$ . The LA18 option enables investors indebted at the Livret A rate to protect themselves by locking the payment at a maximum rate  $K \geq R_F$ . Denoting by  $w$  a weighting coefficient, the  $k$ -th cash flow of the LA18 option payed at date  $T_p^k \geq T^k$  is then given by :

$$C_w^k = \left( \frac{(1-w)}{n} \sum_{i=1}^n Y_{T_y^i, k} + \frac{w}{m} \sum_{j=1}^m F_{T_f^j, k} - K \right) \Big|_{w=\frac{1}{2}} \quad (3.2)$$

**Remark 1** Introducing the generic notation  $C_w$  enables us to define other derivatives. For example, setting  $w = 0$  leads to the definition of the full average inflation option whereas for  $w = 1$ , the LA18 option degenerate into a full average EONIA option.


#### 3.2 Features

In order to give a global description of the LA18, its main features are listed in the table bellow.


	NOTATION	DESCRIPTION	USUAL VALUE
SCHEDULE	$T_0$	asof	
	$T_S$	start date	
	$T_E$	end date	
	$\delta_c$	day count	30/360
	$p_g$	payment gap	
	$p_f$	payment frequency	half yearly
YOY	$r_l^y$	reset lag	-8M
	$r_f^y$	reset frequency	monthly
EONIA	$r_l^e$	reset lag	-7M
	$r_f^e$	reset frequency	daily
OPTION	$N$	notional	
	$K$	strike	0.5
		type	Floor
	$w$	ir weight	0.5
	$n_s$	strike equiv iter	

### 3.3 Example

#### 3.3.1 Summit features

ID: **10756072M** Book **771F** 


**Asset leg**

 **Pay** Ccy **EUR** Start **2/1/2020** Ntl **50,000,000.00**

Index **FORM** End **8/1/2023** Ntl exp

Term  Basis **30/360** Rate  Spread

**Product/Formula**

**OPT\_LA18**  **OPT\_LA18()**

**More...**

**Schedule**

Pay	Freq	Day	Roll	Cal	Reset	Freq	Day	Roll	Cal
<b>S</b>	<b></b>	<b>1</b>	<b>MF</b>	<b>EUR</b>	<b>S</b>	<b></b>	<b>1</b>	<b>MF</b>	<b>INF</b>

**Valuation**

Gross	<b>0.00</b>	Accrued	<b></b>	Net	<b></b>
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**Inflation Index**

Index	<b>[1yEURiff]</b>	<b>f()</b>	Weight Coef	<b>0.50</b>	Source	<b>OATINFL1</b>
Frequency	<b>M</b>		Reset Gap	<b>-8M</b>		
Timing	<b>ARR</b>		Reset Cal	<b>INF</b>		

**Rate Index**

Index	<b>[1dEURonia]</b>	<b>f()</b>	Weight Coef	<b>0.50</b>	Source	<b>T247</b>
Frequency	<b>D</b>		Reset Gap	<b>-7M</b>	AvgBusDay	<b>N</b>
Timing	<b>ARR</b>		Reset Cal	<b>EUR</b>		

**Option**

Type	<b>FLOOR</b>	Strike	<b>0.50</b>	NbIterStkEq	<b>5</b>
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**Roundings**

	<b>Inflation</b>		<b>Rate</b>		<b>Coupon</b>
Round type	<b>NEAR</b>	Round type	<b>NEAR</b>	Round type	<b>NEAR</b>
DP	<b>2</b>	DP	<b>3</b>	DP	<b>4</b>
Timing	<b>AVG_CONV</b>	Timing	<b>CONV_AVG</b>		

### 3.3.2 Schedule

Schedule: Pay Leg #3 - [10756072M] : MAINT

Window Help

App Config

Flows state: **Standard** Trade ID: **10756072M** Notionals: **NTLOUT** Validate

	Start	End	Fixing	Rate	$\Sigma$	Spread	$\Sigma$	Notional	$\Sigma$	Date
▶	2/1/2020	8/1/2020	2/1/2020	0.5				50,000,000.00		8/3/2020
	8/1/2020	2/1/2021	8/1/2020	0.5				50,000,000.00		2/1/2021
	2/1/2021	8/1/2021	2/1/2021	0.5				50,000,000.00		8/2/2021
	8/1/2021	2/1/2022	8/1/2021					50,000,000.00		2/1/2022
	2/1/2022	8/1/2022	2/1/2022					50,000,000.00		8/1/2022
	8/1/2022	2/1/2023	8/1/2022					50,000,000.00		2/1/2023
	2/1/2023	8/1/2023	2/1/2023					50,000,000.00		8/1/2023

Figure 1: Fixed leg

Infla Rates IR Rates Price

	Date	Rate	$\Sigma$
▶	7/1/2019	-0.3660000000	
	7/2/2019	-0.3670000000	
	7/3/2019	-0.3680000000	
	7/4/2019	-0.3670000000	
	7/5/2019	-0.3670000000	
	7/6/2019	0.0000000000	
	7/7/2019	0.0000000000	
	7/8/2019	-0.3680000000	
	7/9/2019	-0.3680000000	
	7/10/2019	-0.3680000000	
	7/11/2019	-0.3660000000	
	7/12/2019	-0.3660000000	
	7/12/2019	0.0000000000	

Figure 2: EONIA

Infla Rates IR Rates Price

	Pay Date	Fixing	Rate	$\Sigma$	Aux Fixing	Aux Rate	$\Sigma$
▶	8/3/2020	7/1/2019	103.9100000000		7/1/2018	102.9600000000	
	8/3/2020	8/1/2019	104.4000000000		8/1/2018	103.4800000000	
	8/3/2020	9/1/2019	104.0400000000		9/1/2018	103.2500000000	
	8/3/2020	10/1/2019	103.9900000000		10/1/2018	103.3700000000	
	8/3/2020	11/1/2019	103.9200000000		11/1/2018	103.1400000000	
	8/3/2020	12/1/2019	104.3900000000		12/1/2018	103.1600000000	
	2/1/2021	1/1/2020	103.9400000000		1/1/2019	102.6700000000	
	2/1/2021	2/1/2020	103.9300000000		2/1/2019	102.7300000000	
	2/1/2021	3/1/2020	103.8500000000		3/1/2019	103.4300000000	
	2/1/2021	4/1/2020	103.8100000000		4/1/2019	103.7600000000	
	2/1/2021	5/1/2020	103.9500000000		5/1/2019	103.8600000000	

Figure 3: YoY

## 4 Diffusion model x numerical method description

### 4.1 Framework

Let  $(\Omega, \mathcal{F}_t, \mathbb{P})$  be a probability space, where  $\mathbb{P}$  is the historical probability and  $(\mathcal{F}_t)_{t \geq 0}$  the natural filtration generated by a multidimensional brownian motion  $(W_t^l)_{l=1}^d$  with  $d \geq n + m$  such that

- $\forall i, j = 1, \dots, n$ :  $d \langle W_t^i, W_t^j \rangle = \theta_{ij}^Y dt$
- $\forall i, j = 1, \dots, m$ :  $d \langle W_t^{n+i}, W_t^{n+j} \rangle = \theta_{ij}^F dt$
- $\forall i = 1, \dots, n$  and  $\forall j = 1, \dots, m$ :  $d \langle W_t^i, W_t^{n+j} \rangle = \rho_{ij} dt$

We suppose moreover that the market is arbitrage-free and introduce the risk neutral probability measure  $\mathbb{Q}$ , defined by the riskless savings account numeraire  $\beta$ . According to these definitions, the standard valuation formula of the LA18 option at time  $t \leq T_S$  is then given by

$$V_t = \mathbb{E}_t^{\mathbb{Q}} \left[ \sum_{k=1}^N \frac{\beta_t}{\beta_{T_p^k}} C_k \right] = \sum_{k=1}^N P(t, T_p^k) \mathbb{E}_t^{\mathbb{Q}_{T_p^k}} [C_k] \quad (4.1)$$

with  $P(t, T_p^k)$ , the  $T_p^k$ - maturity zero coupon bond value at time  $t$  and  $\mathbb{Q}_{T_p^k}$  the  $T_p^k$  forward measure. Omitting the  $k$ - subscript, the real challenge is to estimate the following basket option

$$P_{\alpha, \beta}(t, T, K) = \mathbb{E}_t^{\mathbb{Q}_{T_p}} \left[ \left( \sum_{i=1}^n \alpha_i Y_{T_i} + \sum_{j=1}^m \beta_j F_{T_j} - K \right)_+ \right] \quad (4.2)$$

where as shown before,  $T = T_n = T_m$  is the last fixing date.

### 4.2 Moment matching

We denote  $W_t^{T_p}$  the brownian motion under  $\mathbb{Q}_{T_p}$  and assume a shifted lognormal dynamic for the pay lagged forwards defined by  $\tilde{Y}_t^T = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [Y_T]$  and  $\tilde{F}_t^T = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [F_T]$  such that

$$d\tilde{Y}_t^{T_i} = \mu_i^* \left( \tilde{Y}_t^{T_i} + s_i \right) dW_t^{T_p, i}, \quad \forall i = 1, \dots, n \quad (4.3)$$

$$d\tilde{F}_t^{T_j} = \nu_j^* \left( \tilde{F}_t^{T_j} + r_j \right) dW_t^{T_p, n+j}, \quad \forall j = 1, \dots, m \quad (4.4)$$

with  $(s_i, \mu_i^*)_{i=1}^n$  and  $(r_j, \nu_j^*)_{j=1}^m$  the shifts and the strike equivalent volatilities of YoY and EONIA forward rates respectively. The basket option value rewrites as

$$\tilde{P}_{\alpha, \beta}(t, T, \tilde{K}) = \mathbb{E}_t^{\mathbb{Q}_{T_p}} \left[ \left( \tilde{Y}_T + \tilde{F}_T - \tilde{K} \right)_+ \right] \quad (4.5)$$

with  $\tilde{Y}_T = \sum_{i=1}^n \alpha_i \left( \tilde{Y}_{T_i}^{T_i} + s_i \right)$ ,  $\tilde{F}_T = \sum_{j=1}^m \beta_j \left( \tilde{F}_{T_j}^{T_j} + r_j \right)$  and  $\tilde{K} = K + \sum_{i=1}^n \alpha_i + \sum_{j=1}^m \beta_j$ .



In order to price  $\tilde{P}_{\alpha,\beta}$  quickly, we take as a proxy of  $\tilde{Y}_T$  and  $\tilde{F}_T$  the correlated processes  $\bar{Y}_T$  and  $\bar{F}_T$  defined by

$$\begin{aligned} d\bar{Y}_t &= \mu_{\bar{Y}} \bar{Y}_t dW_t^{\bar{Y}} \\ d\bar{F}_t &= \nu_{\bar{F}} \bar{F}_t dW_t^{\bar{F}} \end{aligned}$$

with  $(W_t^{\bar{Y}}, W_t^{\bar{F}})$  two brownian motions under  $\mathbb{Q}_{T_p}$  such that  $d\langle W_t^{\bar{Y}}, W_t^{\bar{F}} \rangle = \rho dt$  and where the parameters  $(\bar{Y}_t, \bar{F}_t, \mu_{\bar{Y}}, \nu_{\bar{F}}, \rho)$  have to be determined so that the first two moments of  $\tilde{Y}_T$  and  $\tilde{F}_T$  and their covariance are exactly reproduced:

$$\left\{ \begin{array}{l} \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\bar{Y}_T] = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\tilde{Y}_T] \\ \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\bar{F}_T] = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\tilde{F}_T] \\ \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\bar{Y}_T^2] = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\tilde{Y}_T^2] \\ \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\bar{F}_T^2] = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\tilde{F}_T^2] \\ \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\bar{Y}_T \bar{F}_T] = \mathbb{E}_t^{\mathbb{Q}_{T_p}} [\tilde{Y}_T \tilde{F}_T] \end{array} \right. \iff \left\{ \begin{array}{l} \bar{Y}_t = \sum_{i=1}^n \alpha_i \left( \tilde{Y}_t^{T_i} + s_i \right) \\ \bar{F}_t = \sum_{j=1}^m \beta_j \left( \tilde{F}_t^{T_j} + r_j \right) \\ \mu_{\bar{Y}}^2 = \ln \left( \frac{\sum_{i,j=1}^n \alpha_i \alpha_j \tilde{Y}_t^{T_i} \tilde{Y}_t^{T_j} e^{\mu_i \mu_j \theta_{ij}^Y}}{\sum_{i,j=1}^n \alpha_i \alpha_j \tilde{Y}_t^{T_i} \tilde{Y}_t^{T_j}} \right) \\ \nu_{\bar{F}}^2 = \ln \left( \frac{\sum_{i,j=1}^m \beta_i \beta_j \tilde{F}_t^{T_i} \tilde{F}_t^{T_j} e^{\nu_i \nu_j \theta_{ij}^F}}{\sum_{i,j=1}^m \beta_i \beta_j \tilde{F}_t^{T_i} \tilde{F}_t^{T_j}} \right) \\ \rho = \frac{1}{\mu_{\bar{Y}} \nu_{\bar{F}}} \ln \left( \frac{\sum_{i=1}^n \sum_{j=1}^m \alpha_i \beta_j \tilde{Y}_t^{T_i} \tilde{F}_t^{T_j} e^{\mu_i \nu_j \rho_{ij}}}{\sum_{i=1}^n \sum_{j=1}^m \alpha_i \beta_j \tilde{Y}_t^{T_i} \tilde{F}_t^{T_j}} \right) \end{array} \right.$$

Finally, the pricing is obtained using the bilog formula

$$\tilde{P}_{\alpha,\beta}(t, T, \tilde{K}) = \text{Bilog} \left( t, T, \tilde{K}, \bar{Y}_t, \bar{F}_t, \mu_{\bar{Y}}, \nu_{\bar{F}}, \rho \right)$$

### 4.3 Strike equivalent method

#### 4.3.1 Purpose

In reality, the market option quotes provide volatility smiles  $(\mu_i(k), \nu_j(k))$  for the basket assets  $(\tilde{Y}_t^{T_i})_{i=1}^n$  and  $(\tilde{F}_t^{T_j})_{j=1}^m$ . In consequence, these random variables aren't strictly lognormal. To apply the moment matching method, a strike choice  $(k_i^*, k_j^*)$  is needed such that

$$\begin{aligned}\mu_i^* &= \mu_i(k_i^*) \\ \nu_j^* &= \nu_j(k_j^*)\end{aligned}$$

The strike equivalent method relies on the assumption that the price of a basket option at a strike  $K$  should be determined from the most likely configuration of its assets conditional on arriving at this level at maturity. Starting from arbitrariable standard deviations  $(\tilde{\mu}_i, \tilde{\nu}_j)$  and denoting by  $M$  the set of points on the hypersurface  $\Gamma_{\tilde{K}}$  such that

$$\Gamma_{\tilde{K}} = \left\{ (x, y), \quad \sum_{i=1}^n \tilde{\alpha}_i e^{\mu_i x_i} + \sum_{j=1}^m \tilde{\beta}_j e^{\nu_j y_j} - \tilde{K} = 0 \right\} \quad (4.6)$$

with  $\tilde{\alpha}_i = \alpha_i (\tilde{Y}_t^{T_i} + s_i) e^{\frac{-\tilde{\mu}_i^2}{2}}$  and  $\tilde{\beta}_j = \beta_j (\tilde{F}_t^{T_j} + r_j) e^{\frac{-\tilde{\nu}_j^2}{2}}$ , the mathematical description of this method is given by

$$\left\{ (p_i)_{i=1}^n, (q_j)_{j=1}^m \right\} = \arg \min_{(x, y) \in M} \left( \sum_{i,j=1}^n \Omega_{i,j}^{-1} x_i x_j + \sum_{i,j=1}^m \Omega_{i,j}^{-1} y_i y_j + 2 \sum_{i=1}^n \sum_{j=1}^m \Omega_{i,j}^{-1} x_i y_j \right) \quad (4.7)$$

where  $\Omega = \begin{pmatrix} \theta^Y & \rho \\ \rho & \theta^F \end{pmatrix} \in \mathcal{M}((n+m)^2, [-1, 1])$  is the correlation matrix defined in the previous section. At the end of the procedure, the equivalent strikes are then given by

$$\begin{cases} \forall i = (1, \dots, n), & k_i^* = \frac{\tilde{\alpha}_i}{\alpha_i} e^{\tilde{\mu}_i p_i} \\ \forall j = (1, \dots, m), & k_j^* = \frac{\tilde{\beta}_j}{\beta_j} e^{\tilde{\nu}_j p_j} \end{cases} \quad (4.8)$$

#### 4.3.2 Resolution

To resolve this minimization problem, we use the method of Lagrange multipliers. Introducing the variable  $(z_k)_{k=1}^{n+m}$ ,  $(\gamma_k)_{k=1}^{n+m}$  and  $(\sigma_k)_{k=1}^{n+m}$  such that

- $\forall k = 1, \dots, n$

$$\begin{aligned}z_k &= p_k \\ \gamma_k &= \tilde{\alpha}_k \\ \sigma_k &= \mu_k\end{aligned}$$

- $\forall l = 1, \dots, m$

$$\begin{aligned}z_{n+l} &= q_l \\ \gamma_{n+l} &= \tilde{\beta}_l \\ \sigma_{n+l} &= \nu_l\end{aligned}$$

the Lagrange function  $H$  is define by

$$H_\lambda = \left( \sum_{k,l=1}^{n+m} \Omega_{k,l}^{-1} z_k z_l \right) + \lambda \left( \sum_{k=1}^{n+m} \gamma_k e^{\sigma_k z_k} - \tilde{K} \right), \quad \lambda \in \mathbb{R}$$

An admissible solution is found by solving the system :

$$\begin{cases} \partial_\lambda H(\lambda) = \sum_{k=1}^{n+m} \gamma_k e^{\sigma_k z_k} - \tilde{K} = 0 \\ \partial_{z_k} H(\lambda) = \sum_{l=1}^{n+m} \Omega_{k,l}^{-1} z_l + \sigma_k \gamma_k \lambda e^{\sigma_k z_k} = 0, \quad \forall k = 1, \dots, n+m \end{cases}$$

We assume that the solution is small enough to guarantee

$$\forall k, \quad e^{\sigma_k z_k(\lambda)} \simeq 1 + \sigma_k z_k(\lambda)$$

This linearization forces us to handle the strike equivalent method as an iterative process but enables to write the solution in a simple manner :

$$z^* = \lambda^* \Lambda \Gamma \quad \text{with} \quad \lambda^* = \frac{\tilde{K} - \sum_{k=1}^{n+m} \gamma_k}{\Gamma \Lambda \Gamma} \quad (4.9)$$

where the matrix  $\Lambda$  and the vector  $\Gamma$  are defined  $\forall k, l = 1, \dots, n+m$  by

$$\begin{aligned} \Lambda_{kl}^{-1} &= \Omega_{k,l}^{-1} + \delta_{kl} \gamma_k \sigma_k^2 \\ \Gamma_k &= -\sigma_k \gamma_k \end{aligned} \quad (4.10)$$

We sum up the several steps of the computation of the strike equivalent method:

- we start by assigning the Atm volatility to each asset  $\left\{ (\mu_i^0)_{i=1}^n, (\nu_j^0)_{j=1}^m \right\}$
- we determine the induced decomposition of the strike  $K = \sum_{i=1}^n \alpha_i k_i^0 + \sum_{j=1}^m \beta_j k_j^0$
- we iterate the process by assuming new volatilities  $(\mu_i^1)_{i=1}^n = (\mu_i(k_i^0))_{i=1}^n$  and  $(\nu_j^1)_{j=1}^m = (\nu_j(k_j^0))_{j=1}^m$
- this provides another decomposition  $K = \sum_{i=1}^n \alpha_i k_i^1 + \sum_{j=1}^m \beta_j k_j^1$  and define a sequence of strike equivalent set. The algorithm stops at the  $n_s$ -th iteration.

$$\left( \begin{pmatrix} \tilde{\alpha}_i^0 \\ \tilde{\beta}_j^0 \end{pmatrix}_{i,j=1}^n, \begin{pmatrix} \mu_i^0 \\ \nu_j^0 \end{pmatrix}_{i,j=1}^n \right) \rightarrow \left( \begin{pmatrix} k_i^0 \\ k_j^0 \end{pmatrix}_{i,j=1}^n, \begin{pmatrix} \mu_i^1 \\ \nu_j^1 \end{pmatrix}_{i,j=1}^n \right) \rightarrow \dots \rightarrow \left( \begin{pmatrix} k_i^{n_s} \\ k_j^{n_s} \end{pmatrix}_{i,j=1}^n, \begin{pmatrix} \mu_i^{n_s+1} \\ \nu_j^{n_s+1} \end{pmatrix}_{i,j=1}^n \right)$$

## 5 Model development tests

### 5.1 Scope of the tests

Realized tests are split into three main categories:

- Consistency tests: the objective of these tests is to verify that instruments prices are coherent and respect parities and absence of arbitrage conditions.
- Numerical method tests : the objective of these tests is to verify the behaviour and the convergence of the numerical methods used for the pricing.
- Sensitivities tests: by analysing different sensitivities of the products, we seek to demonstrate and verify that the results are in line with what is expected.

Each single test was conducted over a wider range of products parametrization, but for each section we chose to demonstrate the results in the present document only over a reduced subset of products for concision.

### 5.2 Consistency tests

In this part, we provide some consistency tests to check the coherence of the LA18 option pricing. All tables of this section are related to the following reference product:

	FEATURES	VALUE
SCHEDULE	asof	01/03/2021
	start date	variable
	end date	start date + 6M
	day count	30/360
	payment gap	0
	payment frequency	half yearly
YOY	reset lag	-8M
	reset frequency	monthly
EONIA	reset lag	-7M
	reset frequency	daily
OPTION	notional	10000
	strike	variable
	type	variable
	ir weight	0.5
	strike equiv iter	10

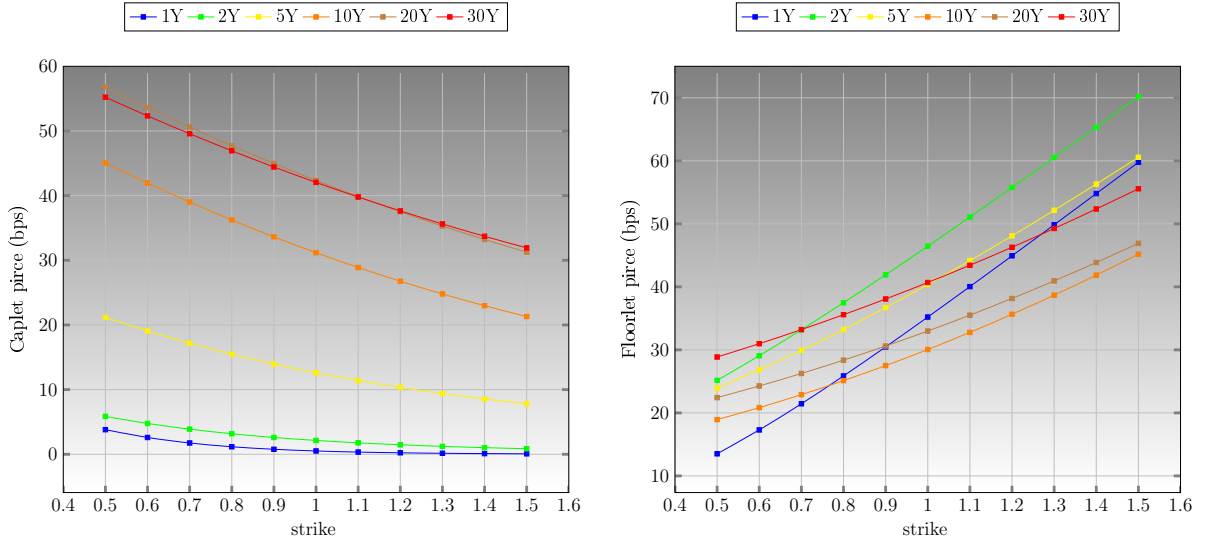
CAPLET PRICE											
Expiry \ Strike	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
1Y	3.81	2.60	1.75	1.16	0.77	0.51	0.34	0.23	0.15	0.10	0.07
2Y	5.86	4.77	3.89	3.17	2.60	2.14	1.77	1.47	1.22	1.02	0.86
5Y	21.17	19.06	17.17	15.47	13.96	12.61	11.41	10.34	9.39	8.55	7.81
10Y	45.05	41.95	39.01	36.23	33.62	31.17	28.89	26.77	24.80	22.98	21.3
20Y	56.78	53.63	50.61	47.72	44.97	42.35	39.86	37.52	35.30	33.22	31.27
30Y	55.22	52.33	49.57	46.94	44.43	42.04	39.78	37.64	35.62	33.71	31.92

FLOORLET PRICE											
Expiry \ Strike	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
1Y	13.5	17.29	21.44	25.86	30.47	35.21	40.04	44.92	49.85	54.8	59.77
2Y	25.16	29.07	33.19	37.48	41.91	46.45	51.07	55.77	60.53	65.33	70.17
5Y	23.93	26.82	29.93	33.23	36.71	40.36	44.16	48.1	52.15	56.31	60.56
10Y	18.93	20.83	22.89	25.12	27.5	30.06	32.77	35.65	38.68	41.86	45.18
20Y	22.42	24.27	26.25	28.36	30.61	32.99	35.51	38.16	40.95	43.86	46.91
30Y	28.86	30.98	33.22	35.58	38.07	40.69	43.43	46.29	49.26	52.36	55.56

LA18 PRICE											
Expiry \ Strike	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
1Y	3.81	2.60	1.75	1.16	0.77	0.51	0.34	0.23	0.15	0.10	0.07
2Y	5.86	4.77	3.89	3.17	2.60	2.14	1.77	1.47	1.22	1.02	0.86
5Y	21.17	19.06	17.17	15.47	13.96	12.61	11.41	10.34	9.39	8.55	7.81
10Y	45.05	41.95	39.01	36.23	33.62	31.17	28.89	26.77	24.80	22.98	21.3
20Y	56.78	53.63	50.61	47.72	44.97	42.35	39.86	37.52	35.30	33.22	31.27
30Y	55.22	52.33	49.57	46.94	44.43	42.04	39.78	37.64	35.62	33.71	31.92

FIXED LEG PRICE											
Expiry \ Strike	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
1Y	13.5	17.29	21.44	25.86	30.47	35.21	40.04	44.92	49.85	54.8	59.77
2Y	25.16	29.07	33.19	37.48	41.91	46.45	51.07	55.77	60.53	65.33	70.17
5Y	23.93	26.82	29.93	33.23	36.71	40.36	44.16	48.1	52.15	56.31	60.56
10Y	18.93	20.83	22.89	25.12	27.5	30.06	32.77	35.65	38.68	41.86	45.18
20Y	22.42	24.27	26.25	28.36	30.61	32.99	35.51	38.16	40.95	43.86	46.91
30Y	28.86	30.98	33.22	35.58	38.07	40.69	43.43	46.29	49.26	52.36	55.56

### 5.2.1 Strike effect



As expected, as the strike goes up, the value of the caplet goes to 0. In an opposite way, as expected, as the strike goes down, the floorlet value goes to 0. The price convexity is also respected

### 5.2.2 Call-Put parity

The next tables show the results of the Call-Put parity relation:

$$C(t, T, K) = F(t, T, K) + \text{LA18}_t^T - K$$

PARITY												
Expiry \ Strike	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	
1Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The formula described above looks valid all pairs of (Strike, Expiry).

### 5.3 Price sensitivity to market data

In this tests we focus now on the behavior of a LA18 Caplet option via their Greeks. We mainly focus on the Delta, Vega and Cega which represents the risks to which Cap/Floor are sensitive. All results of this section are related to the following reference product:

	FEATURES	VALUE
SCHEDULE	asof	01/03/2021
	start date	01/03/2031
	end date	01/09/2031
	day count	30/360
	payment gap	0
	payment frequency	half yearly
YOY	reset lag	-8M
	reset frequency	monthly
EONIA	reset lag	-7M
	reset frequency	daily
OPTION	notional	50000000
	strike	0.5
	type	floor
	ir weight	0.5
	strike equiv iter	10

Please note that we consider here a high Notional of 50m€ simply to emphasize the impacts. We analyze the dispersion of risk over market inputs to check their coherence.

## 5.4 Delta

The cash flows are discounted with the CSA curve, thus we have risk on the three following curves

CSA	-102.7	EONIA	-462.36	CPI	-451.88
2D	-0.05	2D	0	01/12/2020	0
1W	0	1W	0	.	.
1M	0	2W	0	.	.
2M	0	1M	0	.	.
3M	0	2M	0	.	.
4M	0	3M	0	01/11/2022	0
5M	0	4M	0	2Y	0
6M	0	5M	0	3Y	0
9M	0	6M	0	4Y	0
1Y	-0.01	7M	0	5Y	0
18M	0	8M	0	6Y	0
2Y	-0.01	9M	0	7Y	0
3Y	-0.01	10M	0	8Y	497.38
4Y	-0.04	11M	0.01	9Y	2908.65
5Y	0.04	1Y	0.27	10Y	-3785.8
6Y	-0.29	15M	0.02	11Y	-72.12
7Y	0.92	18M	-0.08	12Y	0
8Y	-3.64	21M	0.31	13Y	0
9Y	13.51	2Y	0.16	14Y	0
10Y	-61.67	3Y	1.53	15Y	0
11Y	-61.91	4Y	-1.29	16Y	0
12Y	13.26	5Y	10.38	17Y	0
13Y	-3.55	6Y	-31.73	18Y	0
14Y	0.91	7Y	126.37	19Y	0
15Y	-0.17	8Y	-466.82	20Y	0
20Y	0	9Y	4217.77	21Y	0
25Y	0	10Y	-3437.17	22Y	0
30Y	0	11Y	-1118.29	23Y	0
40Y	0	12Y	299.72	24Y	0
50Y	0	13Y	-80.31	25Y	0
		14Y	20.6	26Y	0
		15Y	-3.89	27Y	0
		20Y	0.11	28Y	0
		25Y	-0.03	29Y	0
		30Y	0.01	30Y	0
		40Y	0	40Y	0
		50Y	0	50Y	0

As expected all the sensitivities are negatives and the most important impact comes from the Forecast curve.

- The sensitivity to the CSA curve is concentrated between the 10Y maturity and the 11Y maturity which is coherent with the paiement date year fraction  $\simeq 10.5Y$ .
- The EONIA risk is concentrated between the 9Y maturity and the 10Y maturity which is coherent with the start date year fraction  $\simeq 10Y$  and the end date year fraction  $\simeq 10.5Y$  minus the lag of seven month. Residual risk can be observed on other maturities in a small scale but enough to show a sensitivity. This is due to the Stripping method which smooths the forward curve.
- The CPI risk is concentrated between the 8Y maturity and the 10Y maturity which is coherent with the start date year fraction  $\simeq 10Y$  and the end date year fraction  $\simeq 10.5Y$  minus the lag of seven month and the natural YoY lag of one year.



5.5 Vega

5.6 Cega

## 5.7 Convergence of the numerical method

## **6 Implementation**

### **6.1 Implementation description**

Waiting for DSI.

### **6.2 Implementation test(s)**

Waiting for DSI.

## **7 Model management**

### **7.1 Maintenance of the model**

As model developer and implementer, the quantitative team and the MOA are responsible for the maintenance of the model when it is needed.

### **7.2 Process for ongoing monitoring**

According to the market evolution, the model users are able to raise the alarm if they point out any inconsistency of the model.

### **7.3 Change control management**

The model owner is responsible for any change in the control management.

## **References**