Yield Curve Econometric

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August 7, 2019

Abstract

The aim of this document is to provide some technical elements required for the Fundamental Review of the Trading Book and more specifically for the Internal Models Approach of the Yield Curve Econometric in the Historical Simulation framework.

Contents

I Introduction

Defined by the Basel Committee of Banking Supervision, the Fundamental Review of the Trading Book (FRTB) aims to improve the Basel II.5 regulation rules and to build a new market risk framework as a response to the financial crisis. These new standards address a number of both qualitative and quantitative issues such as capital arbitrage between booking and trading books as well as under-capitalization of the trading book.

I.1 Standardised Approach

The Standardised Approach (SA) refers to a set of general risk measurement techniques proposed in the FRTB, giving a market risk overview of all bankink institutions. Needed to be calculated and reported to the relevant supervisor on a monthly basis, this method provide minimum capital requirements as the sum of three metrics :

- the Sensitivities-Based Method (SBM)
- the Default Risk Capital (DRC)
- the Residual Risk Add-On (RRAO)

Through risk agregation rules, the SBM use the sensitivities of financial instruments to a predefined risk factor list for each risk classes (Interest Rate, Credit, Equity, Commodity and Foreign Exchange) to calculate the delta, vega and curvature risk capital requirements. The DRC is intented to capture jump-to-default risk that may not be captured by credit spread shocks under the SBM. To address possible limitations in the SA, the RRAO is introduced to ensure sufficient coverage of market risks.

I.2 Internal Models Approach

In order to reduce capital requirements, banks can also use the Internal Models Approach (IMA) - at trading desk level - for a more accurate measure of their own market risks. This process, however, entails an additional calculation cost and a more complex methodology which is subject to the supervisory authority agreement. As a consequence, the suitability of an internal risk management model is assessed through the two following steps :

- A P&L Attribution test to determine the appropriateness of the choosen risk factors in relation to the material drivers of Actual P&L
- A Backtesting test to determine how well trading desk risks are captured by the risk factors modelling

At bank's level, a third test is apply in order to split eligible trading desks risk factors into the modellable risk factors (MRF) set and the non modelable risk factors (NMRF) set. As in the SA, total capital charge for market risk under the IMA is given by the sum of the following metrics :

- the capital requirement for MRF
- the capital requirement for NMRF
- the Default Risk Capital for internal model
- the standardised capital charge for ineligible trading desks

In order to compare both approachs, banks should perform quantitative studies to measure the impacts on their trading business.

II Yield Curve Econometric

distinguer ce qui est observable (risk factors) de ce que l'on souhaite modeliser par des consideration techniques et functionelle fonction appliquer sur risk factor. contamination d'observabilité sur determination du systeme ???

Hypothèse du zero coupon traitable et observabilité que de quelques instruments systeme sous determiné

Projection des instruments de taux par des raisonement d'arbitrage et des definition propre a chaque banque et algo Diffusion de la courbe de taux

Le vrai risque facteur : courbe des instruments de taux.(justification par donnée reéllement observables).

Interest rate modelling purpose is to describe the random dynamic of a set of zero coupon bond curves through time starting from an initial condition. Yield curve represents the global (majeur le plus important) risk for all interest rate instruments including derivatives by extension.

II.1 Par-Point

In this approach, all pillars of the yield curve are considered independents and the econometrics are directly applied to their facial values. Now, the question is how to compare theses values at different dates. In fact, (selon la date d'observation et la description des instruments (business days vs calendar days), on peut avoir d'es instrument differents)

The first way is to extract zero coupon curves from past dates and use them to value the current yield curve. (ajouter graph)$\times D$

The second way is to consider that rates with sliding maturities such as money market and swap rates, can be compared regardless the date of observation. For other rates induced by rolling securities such as Euribor futures whose maturities are fixed dates, an immediate comparison is impossible. An alternative can be to compute the current value of theses rates from the past yield curves using some interpolation rules.

III Historical Simulation Framework

Historical simulation is a method which assumes that probability law of a set of variable rely on their past evolution, allowing to capture stylized facts of financial time series such as fat tails or volatility clustering without modelling assumptions. For a given time horizion α and a risk factor R defined on a domain D_R and whose spot level at time t - taken to represents one business day- is noted R_t, this translates into

$$R_{t+1}^{s,s+h} = f(R_t, R_{s+h}, R_s), \quad t - \alpha \leq s < t - h$$

where the shock function f specifies how to measure past return between s and $s+h$ and how to apply it between t and $t+1$.

III.1 Shock function caracterisation

Beyond statistical considerations, the following statements gives necessaries criterias to define such functions on a set D :

- The image of f is D.
- A total order exist on D.
- The order relation between R_{s+h} and $R_s$is the same as between $R_{t+1}^{s,s+h}$ and R_t.

Through this caracterisation, it is possible to define major shock functions on their domain of application.

Absolute shock

Absolute return $\delta_A^{s,s+h}$ between s and $s+h$ is defined on $\mathbb{R}$ as

$$\delta_A^{s,s+h} = R_{s+h} - R_s$$

The shocked value using absolute shift is obtened by

$$R_{t+1}^{s,s+h} = R_t + \delta_A$$

Relative shock

The reletive return $\delta_R^{s,s+h}$ between s and $s+h$ is defined on $\mathbb{R}_+^*$ by

$$\delta_R^{s,s+h} = R_t.\frac{(R_{s+h} - R_s)}{R_s}$$

The shocked value using relative shift is obtened by

$$R_{t+1}^{s,s+h} = R_t + \delta_R^{s,s+h}$$

Mixed shock

The most intuitive way to define a mixed -or absolute-relative- return is to use a convexe combination between absolute and relative shocks

$$\forall\ \lambda \in [0,1]\,, \quad \delta_{AR} = \lambda.\delta_A + (1-\lambda).\delta_R$$

It is not clear, however, to find a consistent domain with the above caracterisation (first statement break on $\mathbb{R}_+^*$ and third statement break on $\mathbb{R}$). Using the function ϕ_p defined $\forall\ p \in [0,1]$ by

$$\begin{array}{ccl} \phi_p & : & \mathbb{R}_+^* \to \mathbb{R} \\ x & \to & y = p.x + (1-p).\ln(x) \end{array}$$

and its inverse

$$\begin{array}{rcl}\phi_p^{-1} & : & \mathbb{R} \to \mathbb{R}_+^* \\ y & \to & x = \begin{cases} y, & p = 1 \\ \exp(y), & p = 0 \\ \left(\frac{1-p}{p}\right).W\left(\frac{p}{1-p}.\exp\left(\frac{y}{1-p}\right)\right), & p \in]0,1[\end{cases}\end{array}$$

where W is the Lambert function, it is possible to construct a mixed return function on $\mathbb{R}_+^*$ through the following algorithm :

- Compute R_t^p, R_{s+h}^p, and R_s^p with

$$R^p = \phi_p(R)$$

- Compute R_{t+1}^p using absolute return

$$R_{t+1}^p = R_t^p + R_{s+h}^p - R_s^p$$

- Retrieve R_{t+1} and δ_{AR} by

$$\begin{array}{rcl} R_{t+1} & = & \phi_p^{-1}(R_{t+1}^p) \\ \delta_{AR} & = & R_{t+1} - R_t \end{array}$$

III.2 Bounded risk factors scaling

Another issue that needs to be tackled is related to bounded risk factors. To meet shock function domain requirements, rescaling function of the form

$$\zeta : [m, M] \to \mathbb{K}$$

with $\mathbb{K} = \mathbb{R}$ or $\mathbb{R}_+^*$ can be used. In order to easily retieve risk factor after shock application and to avoid brute force truncature, rescaling function should be at least bijective with analitical inverse function. Noting $f_{\mathbb{K}}$ a suitable shock function on $\mathbb{K}$ and $R_t^\zeta = \zeta(R_{t)}$ the rescaling risk factor at time t, the shocked risk factor is obtained by

$$R_{t+1} = \zeta^{-1} \circ f_{\mathbb{K}}(R_t^\zeta, R_{s+h}^\zeta, R_s^\zeta)$$

To limit composition function impact on the econometric, other features can be take into consideration such that

$$\begin{array}{rcl} \zeta_{\alpha_t,\beta_t,\gamma_t}(R_t) & = & R_t \\ \zeta'_{\alpha_t,\beta_t,\gamma_t}(R_t) & = & 1 \\ \zeta''_{\alpha_t,\beta_t,\gamma_t}(R_t) & = & 0 \end{array}$$