NATIXIS
BEYOND BANKING

SHOCKS METHODOLOGY FOR SHORT TERM INTEREST RATES FUTURES

Type of document	**Methodological note**
Projet :	Shock
Reference	RMD_FRTB_MN_ShocksForFutures
Sub project	Futures
Department :	Market Risk Department
From :	Risk Model Design
Date :	13/2/2018
Owner :	DRM / RMD

Executive Summary

Futures prices are generally collected as settlement prices. Because the contracts mature and because multiple contracts usually trade simultaneously for a given underlier, future prices do not form continuous time series. Instead, they form multiple time series corresponding to different contracts. Each one begins when the corresponding contract strat to trade and ends when that contract expires. Techniques of time series analysis require data to be in continual time series. This notes describes the contruction of time series based on a constant maturity price series. This constant maturity prices approach id challenged by a nearby approach based on interpolation/extrapolation.

Document version

Version	Date	Author	Step
V0.1	16/02/2018	A.Givi & N. Lehdili	Creation
V0.2	16/02/2018	P.Oswald	Review

Table of contents

1. Preamble

Interest rates risk factors will be presented in FRTB market risk measures as prices of market instruments, unlike the zero-coupon format currently used in the VaR Monte Carlo simulation.When market instruments as defined with sliding dates (like "2Y") such as money market rates and swaps, past shocks on those instruments associated to their historical data can be directly applied to the current rates. However, market instruments that roll through time, like euribor futures, whose maturities are defined by fixed dates (like "March 2018") require a specific treatment.to define from a past strip of futures a shock on the current strip. Indeed, as multiple future contracts usually trade for a given underlying (like Euribor 3M), futures prices may form time series of historical data with missing data. Rather, they form multiple overlapping time series corresponding to different contracts. This can be illustrated in the following table where the historical quotations of 8 Euribor 3M futures contacts are displayed.

	Euribor 3M	mars-12	juin-12	sept-12	déc-12	mars-13	juin-13	sept-13	déc-13	mars-14
10/03/2012	0,894	99,155	99,305	99,340	99,320	99,290	99,240	99,185	99,105	
11/03/2012	0,894	99,155	99,305	99,340	99,320	99,290	99,240	99,185	99,105	
12/03/2012	0,884	99,155	99,300	99,340	99,320	99,295	99,245	99,190	99,115	
13/03/2012	0,876	99,155	99,300	99,335	99,310	99,280	99,225	99,170	99,090	
14/03/2012	0,871	99,145	99,280	99,295	99,260	99,220	99,150	99,075	98,975	
15/03/2012	0,862	99,155	99,290	99,290	99,245	99,195	99,120	99,035	98,930	
16/03/2012	0,853	99,150	99,300	99,285	99,225	99,160	99,065	98,965	98,845	
17/03/2012	0,853	99,150	99,300	99,285	99,225	99,160	99,065	98,965	98,845	
18/03/2012	0,853	99,150	99,300	99,285	99,225	99,160	99,065	98,965	98,845	
19/03/2012	0,842	99,158	99,345	99,330	99,260	99,185	99,085	98,975	98,850	
20/03/2012	0,832		99,350	99,340	99,275	99,205	99,110	99,005	98,875	98,760
21/03/2012	0,824		99,365	99,360	99,305	99,245	99,160	99,060	98,930	98,815
22/03/2012	0,817		99,355	99,355	99,310	99,260	99,185	99,095	98,975	98,860
23/03/2012	0,808		99,360	99,355	99,315	99,270	99,200	99,115	98,995	98,885
24/03/2012	0,808		99,360	99,355	99,315	99,270	99,200	99,115	98,995	98,885
25/03/2012	0,808		99,360	99,355	99,315	99,270	99,200	99,115	98,995	98,885

Table 1.1 : Daily historical quotations of futures prices "Mars12", "Juin-12", "Sept-12", "December-12", "Mars-13", "Juin-13", "Sept-13", "December-13", "Mars -214". Period : 10/3/12 – 25/3/2012. Source: Bloomberg

As expected, multiple overlapping time series of futures prices poses a difficulty for the effective implementation of the historical value-at-risk. More precisely, a constant-maturity future price time series must be built in ordre to calculate the relevant historical shocks.

2. Constant-maturityfutures prices and shocks calculation

Fixed rate market instruments are considered as terms structures defined by the maturity date. For money market futures, the maturity date is the start date of a 3M money market forward rate. A constant-maturity future price series, indicates, for each time t, an interpolated price reflecting a specific time-to-expiration that is constant over time.

2.1 An introductive example

Some historical data of the year 2012 have been chosen to highlight different aspects of the methodology which would not have been material for the recent past. For instance, Mars-12 denotes19 of Mars 2012. For data fixed on 10/3/12, that money market future Mars-12 has residual maturity equal to 9 calendar.

	Euribor 3M	19/3/12	18/6/12	17/9/12	17/12/12	18/3/13	17/6/13	16/9/13	16/12/13	17/3/14
10/03/2012	2	9	100	191	282	373	464	555	646	737
11/03/2012	2	8	99	190	281	372	463	554	645	736
12/03/2012	2	7	98	189	280	371	462	553	644	735
13/03/2012	2	6	97	188	279	370	461	552	643	734
14/03/2012	2	5	96	187	278	369	460	551	642	733
15/03/2012	2	4	95	186	277	368	459	550	641	732
16/03/2012	2	3	94	185	276	367	458	549	640	731
17/03/2012	2	2	93	184	275	366	457	548	639	730
18/03/2012	2	1	92	183	274	365	456	547	638	729
19/03/2012	2	0	91	182	273	364	455	546	637	728
20/03/2012	2		90	181	272	363	454	545	636	727
21/03/2012	2		89	180	271	362	453	544	635	726
22/03/2012	2		88	179	270	361	452	543	634	725
23/03/2012	2		87	178	269	360	451	542	633	724
24/03/2012	2		86	177	268	359	450	541	632	723
25/03/2012	2		85	176	267	358	449	540	631	722

Table 2.1 Residual maturities of futures prices "Mars12", "Juin-12", "Sept-12", "December-12", "Mars-13", "Juin-13", "Sept-13", "December-13", "Mars -214". Period : 10/3/12 – 25/3/2012. Source: Bloomberg.

Now if the As of date is fixed at 26/3/12 and by reference of the Table 1, the market instruments used to compute the historical value-at-risk at 26/3/12 are given as follows:

Instruments	Futur Label	Future expiry	Clandar Days
Fut1	juin-12	18/06/2012	84
Fut2	sept-12	17/09/2012	175
Fut3	Dec-12	17/12/2012	266
Fut4	mars-13	18/03/2013	357
Fut5	juin-13	17/06/2013	448
Fut6	sept-13	16/09/2013	539
Fut7	Dec-13	16/12/2013	630
Fut8	mars-14	17/03/2014	721

Table 2.2 *Constant-maturity futures instruments used to computed the VaR as of date 26/3/12*

For example, the market prices of Mars-12 and Juin-12 will be interpolated to reconstitute the 84-day constant-maturity futures prices (Fut1) during the period from 10/3/12 to 19/3/12. In the same way, the future prices derived from Euribor 3M and market prices of Juin-12 will be interpolated to compute 84-day constant-maturity futures prices (Fut1) during the period from 20/3/12 to 25/3/12. More precisely, the methodology used to reconstitute the MM futures prices with fixed maturities during a given period will be detailed in the following subsection.

2.2 Pricing methodology of constant-maturity futures

In order to introduce the pricing methodology, note t the time as of date.The maturity of the futrue is noted T. The residual maturity which is kept constant in the historical pricings is then $T-t$. The steps of historical shocks calculation are :

1. Consider the market conditions at time h in the past and suppose that at this time there are N futures available in the market. Their expiry dates are noted $T_1^h, \dots T_N^h$ and their prices are $F_1^h, \dots F_N^h$. The calculation of the price of the synthetic future with constant-maturity future $T-t$ is carried out using the following Extrapolation/Interpolation :

➔ If $(T-t) \leq (T_1^h - h)$ then $F_{synthetic}^h = F_1^h$

➔ If there is an integer i such that $\left(T_i^h - h\right) \leq (T-t) \leq \left(T_{i+1}^h - h\right)$then

$$F_{synthetc}^h = \frac{\left(T_{i+1}^h - h - T + t\right)F_i^h + \left(T - t - T_i^h + h\right)F_{i+1}^h}{T_{i+1}^h - T_i^h}$$

➔ If $(T_N^h - h) \leq (T-t)$ then $F_{synthetc}^h = F_N^h$.

2. Reprice the future with residual maturity $T-t$ with the market conditions prevailing at time $h+1$. That means exatly going through the last step. So if at time $h+1$ there are M futures[1] with expiry dates $T_1^{h+1}, \dots T_M^{h+1}$ and prices $F_1^{h+1}, \dots F_M^{h+1}$, the future prices $F_{synthetic}^{h+1}$ is calculated by :

$$F_{synthetic}^{h+1} = \begin{cases} F_1^{h+1} & if \quad T-t \leq T_1^{h+1} - h - 1 \\ \frac{\left(T_{i+1}^{h+1} - h - 1 - T + t\right)F_i^{h+1} + \left(T - t - T_i^{h+1} + h + 1\right)F_{i+1}^{h+1}}{T_{i+1}^{h+1} - T_i^{h+1}} & if \ \exists\, i \ t.q. \quad T_i^{h+1} - h - 1 \leq T - t \leq T_{i+1}^{h+1} - h - 1 \\ F_M^{h+1} & if \quad T_M^{h+1} - h - 1 \leq T - t \end{cases}$$

3. The historical shock is then calculated as the difference :

$$shock_{histo} = F_{synthetic}^{h+1} - F_{synthetic}^h. \tag{2.1}$$

[1] If h is a roll date then the number of market instruments diminishes and $M = N - 1$ if not $M = N$.

Remark 1: at any time the fixing of the 3M Money Market rate can be used as an extra information and calculate a future with maturity 2 days and price equal to (100 – 3M Money Market rate). That means for any time h : $T_1^h = h + 2d$and $F_1^h = 100 - 3M\ Money\ Market\ rate$.

Remark 2: An alternative approach to that described above consists in considering the same futures and in calculating their variations between h and $h+1$. Then the variation of the synthetic future is obtained by Extrapolating/Interpolating thr variations of these same futures. Obviously if time h is a roll date, this method poses a problem : the first future does not exist any more at time (h+1). The 3M MM rate can be used to overcome this difficulty. So if h is a roll date $F_1^{h+1} = 100 - 3M\ Money\ Market\ rate$. In this case, when $\left(T_i^h - h\right) \leq (T - t) \leq \left(T_{i+1}^h - h\right)$, the historical choc is then defined by:

$$Alternative\ shock_{histo} = \frac{\left(T_{i+1}^h - h - T + t\right)\left(F_i^{h+1} - F_i^h\right) + \left(T - t - T_i^h + h\right)\left(F_{i+1}^{h+1} - F_{i+1}^h\right)}{T_{i+1}^h - T_i^h}$$

This alternative approach seems to be suggested in the intial version of the business requirement document (BRD) ([2]). This can be questionable if the time horizon for VaR is different of one day and in most cases the shock level is slightly different of that of (2.1).
This shock level is given by

$$Alternative\ choc_{histo} = shock_{histo} - \frac{F_{i+1}^{h+1} - F_i^{h+1}}{T_{i+1}^h - T_i^h}$$

In case of steep curves the difference can become material.

3. Numerical examples

3.1 Application to a period with a roll date

In this section, the methodology will be applied to the market data given in Table 1.1. The methodology was applied for 4 synthetic futures with residual maturities of 15,40,95 and 200 days.The period running from 10 March 2012 to 25 March 2012 has been chosen for 2 reasons. First, because of the European Sovereign Debt crisis rates have been moving enough during this period to illustrate our methodology. Second, the period contains a roll day, this allows us to show how the methodology deals with rolling futures.

Four different constant residual maturities (15,40,95 and 200 days) have been chosen to show all the different configurations which can happen while implementing the methodology. In the following tables, for each synthetic future we exhibit the futures which are used for the interpolations (their prices are in grey and their residual matrities are reminded as Res Mat 1 ans Res Mat 2).

The result of the pricing methodology of synthetic futures are exhibited in the last column of the following tables.

	2d Synthetic future	mars-12	juin-12	Res Mat 1	Res Mat 2	Pricing
10/03/2012	99,106	99,155	99,305	9	100	99,165
11/03/2012	99,106	99,155	99,305	8	99	99,167
12/03/2012	99,116	99,155	99,300	7	98	99,168
13/03/2012	99,124	99,155	99,300	6	97	99,169
14/03/2012	99,129	99,145	99,280	5	96	99,160
15/03/2012	99,138	99,155	99,290	4	95	99,171
16/03/2012	99,147	99,150	99,300	3	94	99,170
17/03/2012	99,147	99,150	99,300	2	93	99,171
18/03/2012	99,147	99,150	99,300	2	92	99,169
19/03/2012	99,158	99,158	99,345	2	91	99,185
20/03/2012	99,168		99,350	2	90	99,195
21/03/2012	99,176		99,365	2	89	99,204
22/03/2012	99,183		99,355	2	88	99,209
23/03/2012	99,192		99,360	2	87	99,218
24/03/2012	99,192		99,360	2	86	99,218
25/03/2012	99,192		99,360	2	85	99,218

Table 3.1 : 15d synthetic future pricing.

	2d Synthetic future	mars-12	juin-12	Res Mat 1	Res Mat 2	Pricing
10/03/2012	99,106	99,155	99,305	9	100	99,206
11/03/2012	99,106	99,155	99,305	8	99	99,208
12/03/2012	99,116	99,155	99,300	7	98	99,208
13/03/2012	99,124	99,155	99,300	6	97	99,209
14/03/2012	99,129	99,145	99,280	5	96	99,197
15/03/2012	99,138	99,155	99,290	4	95	99,208
16/03/2012	99,147	99,150	99,300	3	94	99,211
17/03/2012	99,147	99,150	99,300	2	93	99,213
18/03/2012	99,147	99,150	99,300	2	92	99,212
19/03/2012	99,158	99,158	99,345	2	91	99,238
20/03/2012	99,168		99,350	2	90	99,247
21/03/2012	99,176		99,365	2	89	99,259
22/03/2012	99,183		99,355	2	88	99,259
23/03/2012	99,192		99,360	2	87	99,267
24/03/2012	99,192		99,360	2	86	99,268
25/03/2012	99,192		99,360	2	85	99,269

Table 3.2 : 40d synthetic future pricing

For this example, the historical shocks associated to 15d and 40d futures from 10/3/2012 to 25/3/2012 are reconstituted in the following way : the synthetic futures are priced using mars-12 and juin-12 contracts from 10/3/2012 to17/3/2012 while the 3m Eribor contracts and juin-12 contracts are used from 18/3/2012 to 25/3/2012.

	mars-12	juin-12	septmbre-2012	Res Mat 1	Res Mat 2	Pricing
10/03/2012	99,155	99,305	99,340	9	100	99,297
11/03/2012	99,155	99,305	99,340	8	99	99,298
12/03/2012	99,155	99,300	99,340	7	98	99,295
13/03/2012	99,155	99,300	99,335	6	97	99,297
14/03/2012	99,145	99,280	99,295	5	96	99,279
15/03/2012	99,155	99,290	99,290	4	95	99,290
16/03/2012	99,15	99,300	99,285	94	185	99,300
17/03/2012	99,15	99,300	99,285	93	184	99,300
18/03/2012	99,15	99,300	99,285	92	183	99,300
19/03/2012	99,158	99,345	99,330	91	182	99,344
20/03/2012		99,350	99,340	90	181	99,349
21/03/2012		99,365	99,360	89	180	99,365
22/03/2012		99,355	99,355	88	179	99,355
23/03/2012		99,360	99,355	87	178	99,360
24/03/2012		99,360	99,355	86	177	99,360
25/03/2012		99,360	99,355	85	176	99,359

Table 3.3 : 95d synthetic future pricing

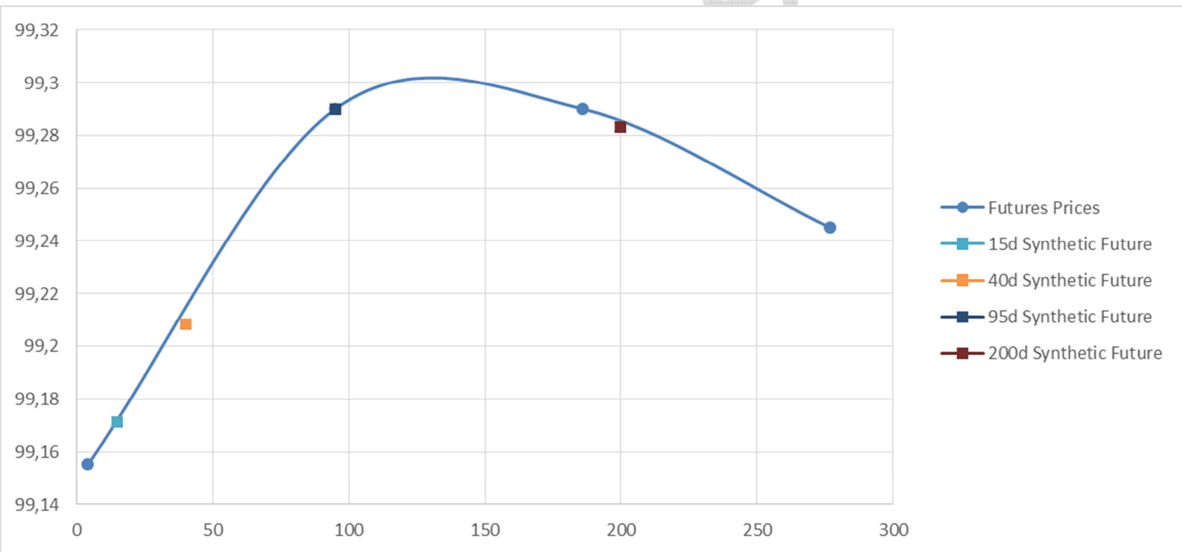
In the same way, the historical shocks associated with the 95d synthetic future from 10/3/2012 to 25/3/2012 are reconstituted in the following way : the synthetic futures is priced using mars-12 and juin-12 contracts from 10/3/2012 to15/3/2012 while juin-2012 and septembre-2012 contracts are used from 16/3/2012 to 25/3/2012.

	juin-12	septembre-12	dec-2012	Res Mat 1	Res Mat 2	Pricing
10/03/2012	99,305	99,340	99,320	191	282	99,338
11/03/2012	99,305	99,340	99,320	190	281	99,338
12/03/2012	99,300	99,340	99,320	189	280	99,338
13/03/2012	99,300	99,335	99,310	188	279	99,332
14/03/2012	99,280	99,295	99,260	187	278	99,290
15/03/2012	99,290	99,290	99,245	186	277	99,283
16/03/2012	99,300	99,285	99,225	185	276	99,275
17/03/2012	99,300	99,285	99,225	184	275	99,274
18/03/2012	99,300	99,285	99,225	183	274	99,274
19/03/2012	99,345	99,330	99,260	182	273	99,316
20/03/2012	99,350	99,340	99,275	181	272	99,326
21/03/2012	99,365	99,360	99,305	180	271	99,348
22/03/2012	99,355	99,355	99,310	179	270	99,345
23/03/2012	99,360	99,355	99,315	178	269	99,345
24/03/2012	99,360	99,355	99,315	177	268	99,345
25/03/2012	99,360	99,355	99,315	176	267	99,344

Table 3.4 : 200d synthetic future pricing

Finally, the historical shocks associated with the 200d synthetic future from 10/3/2012 to 25/3/2012 are reconstituted in the following way : the synthetic futures is priced using septembre-12 and decembre-12 contracts from 10/3/2012 to 25/3/2012

As an example the figure 3.1 gives the prices of the relevant futures and shows the required interpolations to price the synthetic 15d, 40d,95d and 200d futures as of 15/3/2012



Graph 3.1 :Pricing of synthetic futures as of 15/3/2012

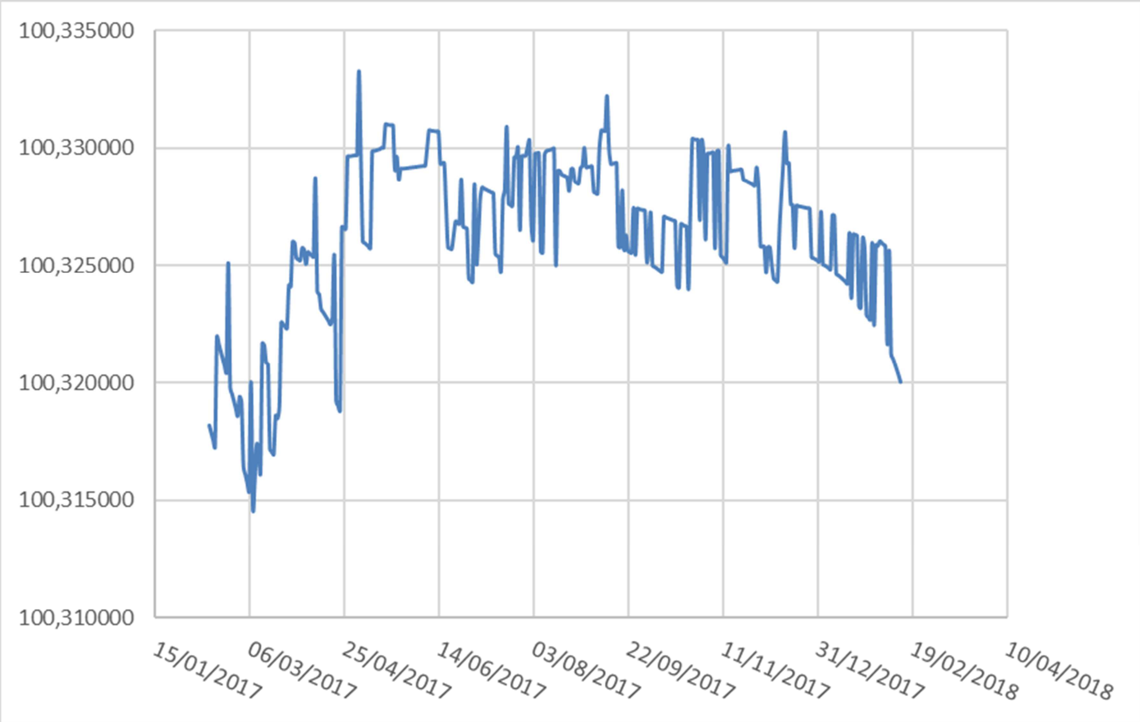
This illustates the result of required interpolations for the considered synthetic futures as of 15/3/12.

3.2 Examples of shocks calculation over 1y

As of 13/2/18 the residual maturity of the first future was 34d. So the calculation of the synthetic future with a constant residual maturity of 34d is carried out over 1y. This was done for 2 periods.

The first period is the recent past but the shocks inferred don't show all the different aspects of the methodology because not much have happenned in the euribor market last year. That is why a second period from 13/2/2007 to 13/2/2008 has been cosidered as well to hifghlight different configurations of the curve. Indeed euribor market has been moving much more during this second period.

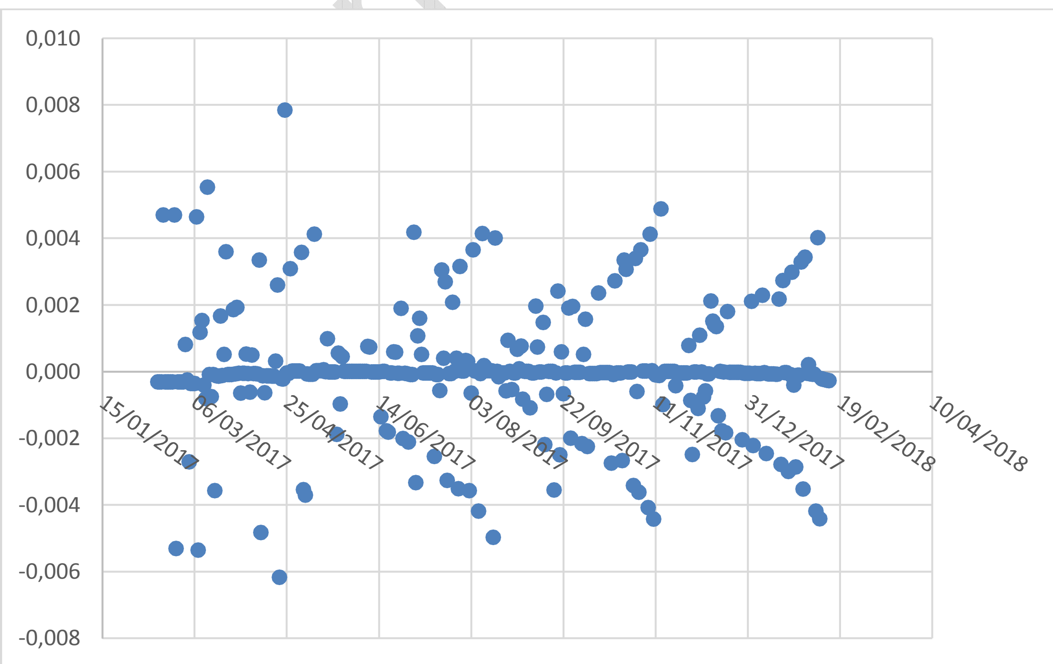
3.2.1 Pricing of the synthetic future from 13/2/2017 to 13/2/2018



Graph 3.2 : 34d future from 13/2/2017 to 13/2/2018

During this period the 3m Euribor rate has not much changed and this is reflected in the Graph 3.2 which exhibites an alomost constant future price (difference between the extrema is around 2 bps)

The daily shocks (to be used for the VaR for example) inferred from these pricings over 1y are given in the graph 3.3



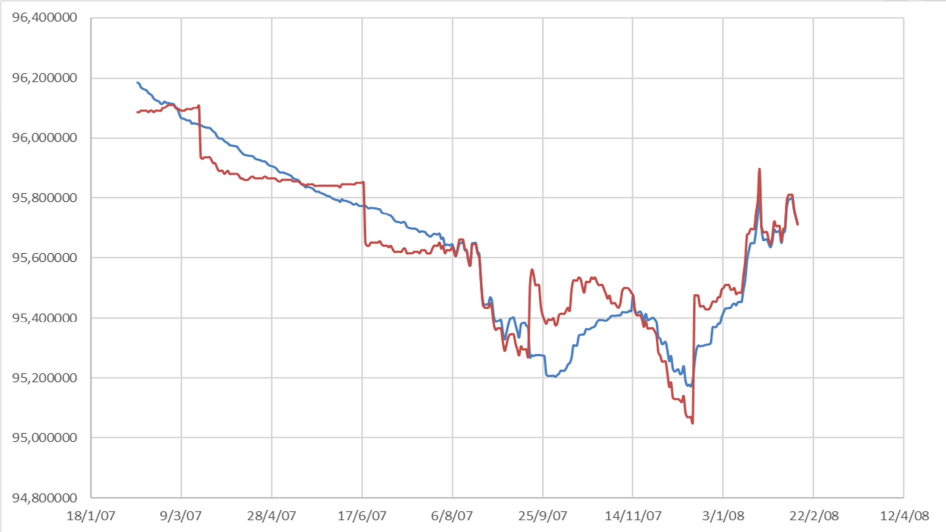
Graph 3.3 : Historical shocks

As can be seen not much has happenned to the Euribor market within this period. That is why the we same study was carried out ver a second period. Notice that the shocks levels exhibited in Graph 3.3 are applicable to futures prices.

3.2.2 Pricing of the synthetic future from 13/2/2007 to 13/2/2008

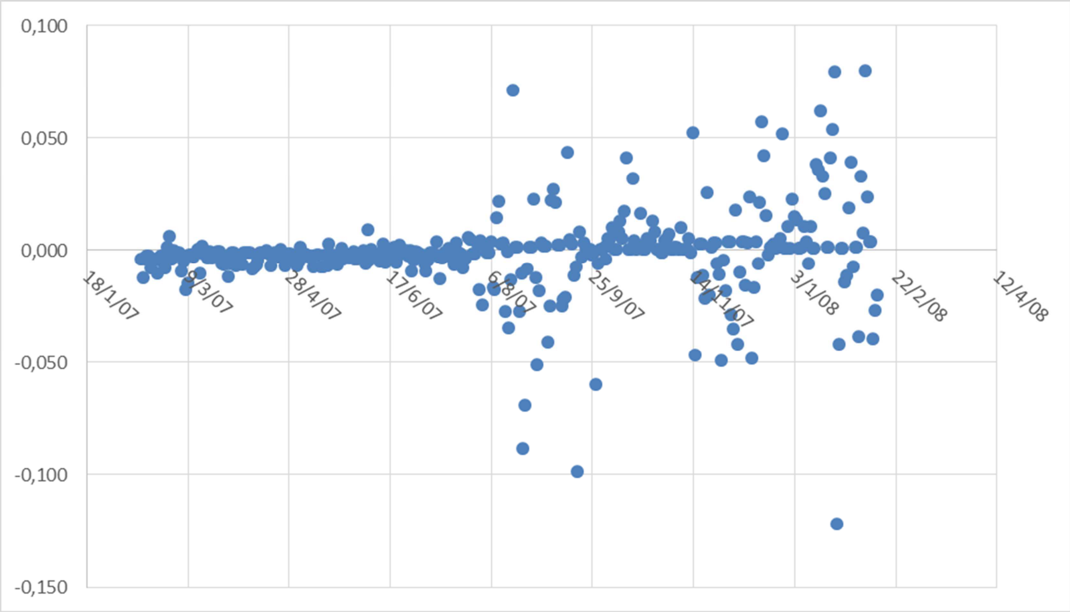
Unlike the last numerical example, in this section an example is given for a period where the short interest rates volatility is material. In this purpose the same synthetic future (with a constant maturity of 34d) has been priced according to the proposed methodology from 13/2/2007 to 13/2/2008 The results are exhibited in the graphs 3.4 and 3.5.

It must be emphasized here that a naïve approach, which consists in considering the future with the closest maturity in ordre to calculate the shocks, could be misleading regarding the VaR calculation. In the Graph 3.4 the historical value of the closest future are shown over time and compared with the price of the 34d synthetic future.



Graph 3.4 : Synthetic 34d future (blue) versus the future with the closestmaturity (in red) from 13/2/2007 to 13/2/2008

Obviously it will be a mistake to calculate the historical shocks using the red curve. The rolling of the maturity brings a distorition which ha no financial signification since shocks in this case would be calculated in a non consistent way.In a nutshell the proposed methodology allows to retreat data in ordre to eliminate the effects of rolling and changing maturities in the process of historical shocks calculation. In the Graph 3.5 historical shocks inferred from the appropriate curve (blue one in Graph 3.4) are shown.



Graph 3.5 : Historical shocks

Unlike the period 13/2/2017 to 13/2/2018, the Graph 3.5 shows shocks levels computed with the proposed approach (formula (2.1)) which are material for the period 13/2/2007 to 13/2/2008.

4. Alternative technics for calculating syntetic futures

There is at least one alternative to the methodology we described for calculating the price of a synthetic future. Given the market conditions at time h , any FRA (Forward Rate Agrrement) can be calculated with no difficulty but. Some additional information are needed to calculate the price of the future because of the margin call mechanism. That is provided by the convexity adjustment. In this method the collection of zero coupons and convexity spreads is an other representation of the market conditions. However, the methodology proposed in this note directly uses futures prices and does not need the convexity adjustment spreads.

5. References

[1] IR Risk factors: ZC or market, FRTB-Methodological note DRM, Natixis April 2017

[2] Interest rates curves shock definition and application, Business requirement document (BRD) DRM, Natixis, Sept 2017