Yield Curve Econometric

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Abstract

The aim of this document is to provide some technical elements required for the Fundamental Review of the Trading Book and more specifically for the Internal Models Approach of the Yield Curve Econometric in the Historical Simulation framework.

Contents

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1 Introduction

Defined by the Basel Committee of Banking Supervision, the Fundamental Review of the Trading Book (FRTB) aims to improve the Basel II.5 regulation rules and to build a new market risk framework as a response to the financial crisis. These new standards address a number of both qualitative and quantitative issues such as capital arbitrage between booking and trading books as well as under-capitalization of the trading book.

1.1 Standardised Approach

The Standardised Approach (SA) refers to a set of general risk measurement techniques proposed in the FRTB, giving a market risk overview of all bankink institutions. Needed to be calculated and reported to the relevant supervisor on a monthly basis, this method provide the minimum capital requirements as the sum of three metrics :

- the Sensitivities-Based Method (SBM)
- the Default Risk Capital (DRC)
- the Residual Risk Add-On (RRAO)

Through risk agregation rules, the SBM uses the sensitivities of financial instruments to a predefined risk factor list for each asset class (Interest Rates, Credit, Equities, Commodities and Foreign Exchange) to calculate the delta, vega and curvature risk capital requirements. The DRC is intented to capture jump-to-default risk that may not be captured by credit spread shocks under the SBM. To address possible limitations in the SA, the RRAO is introduced to ensure sufficient coverage of market risks.

1.2 Internal Models Approach

In order to reduce capital requirements, banks can also use the Internal Models Approach (IMA) - at trading desk level - for a more accurate measure of their own market risks. This process, however, entails an additional calculation cost and a more complex methodology which is subject to the supervisory authority agreement. As a consequence, the suitability of an internal risk management model is assessed through the two following steps :

- A P&L Attribution test to determine the suitability of the chosen risk factors in relation to the material drivers of Actual P&L
- A Backtest to determine how well trading desk risks are captured by the risk factors modelling

At bank's level, a third test is applied in order to split eligible trading desks risk factors into the modellable risk factors (MRF) set and the non modelable risk factors (NMRF) set. As in the SA, the total capital charge for market risk under the IMA is given by the sum of the following metrics :

- the capital requirement for MRF (Expected Shortfall)
- the capital requirement for NMRF (Stress test)
- the Default Risk Capital for internal model
- the standardised capital charge for ineligible trading desks

In order to compare both approaches, banks should perform quantitative studies to measure their trading business impacts.

2 Historical Simulations

Historical simulation is a method which assumes that the distribution of a set of variables relies on their past evolution, allowing to capture stylized facts of financial time series such as fat tails or volatility clustering without modelling assumptions. For a given time horizion α and a risk factor R defined on a domain D_R and whose spot level at time t - taken to represent one business day- is noted R_t, this translates into

$$R_{t+1}^{s,s+h} = f(R_t, R_{s+h}, R_s), \quad t-\alpha \leq s < t-h$$

where the shock function f specifies how to measure past return between s and $s+h$ and how to apply it between t and $t+1$.

2.1 Shock function characterisation

Beyond statistical considerations, the following statements give the necessary criteria to define a shock function on a set D:

- A total order exists on D.
- The image of D^3 under f is D
- $\forall\ (R_t, R_{s+h}, R_s) \in D^3$, the order relationship between R_{s+h} and $R_s$is the same as between $R_{t+1}^{s,s+h}$ and R_t.

Through this caracterisation, it is possible to define major classes of shock functions.

2.1.1 Absolute shock

Absolute return $\delta_A^{s,s+h}$ between s and $s+h$ is defined as

$$\delta_A^{s,s+h} = R_{s+h} - R_s$$

Absolute shock function is relevant for risk factors whose domain is $\mathbb{R}$ and the shocked value is given by

$$R_{t+1}^{s,s+h} = R_t + \delta_A$$

2.1.2 Relative shock

The reletive return $\delta_R^{s,s+h}$ between s and $s+h$ is defined as

$$\delta_R^{s,s+h} = R_t.\frac{(R_{s+h} - R_s)}{R_s}$$

Absolute shock function is relevant for risk factors whose domain is $\mathbb{R}_+^*$ and the shocked value is given by

$$\begin{aligned} R_{t+1}^{s,s+h} &= R_t + \delta_R^{s,s+h} \\ &= R_t.\left(1 + \frac{(R_{s+h} - R_s)}{R_s}\right) \end{aligned}$$

2.1.3 Mixed shock

The most intuitive way to define a mixed return is to use a convex combination between the absolute and relative shocks

$$\forall\ \lambda \in [0,1]\,, \quad \delta_{AR} = \lambda.\delta_A + (1-\lambda).\delta_R$$

It is not clear, however, how to find a consistent domain with the above characterisation ($\mathbb{R}_+^*$ and $\mathbb{R}$ sets are not suitable with regards to the first and third statements).

Using the function ϕ_p defined $\forall\ p \in [0,1[$ by

$$\begin{array}{rcl} \phi_p & : & \mathbb{R}_+^* \to \mathbb{R} \\ x & \to & y = p.x + (1-p).\ln(x) \end{array}$$

and its inverse

$$\begin{array}{rcl} \phi_p^{-1} & : & \mathbb{R} \to \mathbb{R}_+^* \\ y & \to & x = \begin{cases} \exp(y), & p = 0 \\ \left(\frac{1-p}{p}\right) W_0\left(\frac{p}{1-p}.\exp\left(\frac{y}{1-p}\right)\right), & p \in]0,1[\end{cases} \end{array}$$

where W_0 is the principal branch of the Lambert function, it is possible to construct a mixed shock function on $\mathbb{R}_+^*$ through the following steps :

- Compute R_t^p, R_{s+h}^p, and R_s^p with
$$R^p = \phi_p(R)$$
- Apply an absolute shock such that
$$R_{t+1}^p = R_t^p + R_{s+h}^p - R_s^p$$

Mixed return is then given by $\delta_{AR} = R_{t+1} - R_t$ with $R_{t+1} = \phi_p^{-1}(R_{t+1}^p)$

2.2 Scaling function characteriation

Before computing a shock, bounded risk factors must be adapted to the domain requirements of the function used. To achieve this, a composition of scaling and shock functions can be used

$$\begin{array}{rcl} R_{t+1} & = & \tilde{f}_{[m,M]}\left(R_t, R_{s+h}, R_s\right) \\ & = & \eta \circ f_{\mathbb{K}}\left(\ \zeta(R_t),\ \zeta(R_{s+h}),\ \zeta(R_s)\ \right) \end{array}$$

with $\zeta : [m,M] \to \mathbb{K}$, $\eta : \mathbb{K} \to [m,M]$ and $f_{\mathbb{K}}$ a shock function on $\mathbb{K} = \mathbb{R}$ or $\mathbb{R}_+^*$.

Assuming that $\forall\ (R_t, R_{s+h}, R_s) \in [m,M]^3$, $R_{s+h} = R_s$, the third statement implies that to be a shock function on $[m,M]$, $\tilde{f}$ must verify :

$$\begin{array}{rcl} R_t & = & R_{t+1} \\ & = & \tilde{f}_{[m,M]}\left(R_t, R_{s+h}, R_s\right) \\ & = & \eta \circ \zeta(R_t) \end{array}$$

involving that ζ must be an invertble function and $\eta = \zeta^{-1}$. If this condition is fulfilled, then ζ and ζ^{-1} have the same monotonicity, leading to an order-preserving function $\tilde{f}$ on $[m,M]$.

3 Yield Curve Econometric

Generally speaking, knowledge of the current zero coupon curves is a prerequisite for the valuation of interest rate instruments. In practice, only a few bonds are directly observable in the market. On the basis of theoretical and technical considerations - and using a bootstrapping algorithm -, bonds prices can, however, be derived from the fair rates of liquid securities.The benchmark set of rates used in this process, classically composed of K money market rates, M forward rates and N swap rates listed in order of growing maturities, is called yield curve and defines the main risk factor of the interest rate asset class.

3.1 Par-Point Approach

In this approach, all pillars of the yield curve are considered to be independent from each other and (the) shocks are directly applied to their own values. In the case of historical simulations, it is necessary to measure how each elements of the yield curve are affected by the period of observation. The features of rates with sliding maturities such as money market or swap rates are almost identical over (the) time, with a limited impact due to the difference between business and calendar days. For other rates induced by rolling securities whose maturities are fixed dates such as FRAs or bonds, an immediate comparaison is impossible which leads to a specific treatment of the forward rate bucket.

3.1.1 Forward Rates Interpolation

Given a set of forward rates $\left(F_t^k\right)_{k=1\ldots M}$ at time t, one way would be to build synthetic forward rates at time s and $s+h$ with the same maturities $\left(\tau_t^k\right)_{k=1\ldots M}$ using some defined interpolation/extrapolation rules directly applied on the yield curve. Specifically, for an arbitrary date α, synthetic forward rates are obteined as follows

$$\begin{aligned}
F^k_{\alpha,t} &= F^1_\alpha & , \tau^1_\alpha > \tau^k_t \\
F^k_{\alpha,t} &= F^M_\alpha & , \tau^M_\alpha < \tau^k_t \\
F^k_{\alpha,t} &= \frac{\left(\tau^{i+1}_\alpha - \tau^k_t\right).F^i_\alpha + \left(\tau^k_t - \tau^i_\alpha\right).F^{i+1}_\alpha}{\tau^{i+1}_\alpha - \tau^i_\alpha} & , \tau^i_\alpha < \tau^k_t < \tau^{i+1}_\alpha
\end{aligned}$$

wiht $\left(F^i_\alpha\right)_{i=1\ldots M}$ the forward rate values at date α and $\left(\tau^i_\alpha\right)_{i=1\ldots M}$ their maturity. It is also possible to consider the three months money market rate of the yield curve - if available - like/as the first forward rate with a maturity of two business days in order to avoid th flat extrapolation when $\tau^1_\alpha > \tau^k_t$.

Remarque 1 *Being submitted to liquidity and risk management constraints, the yield curve structure rarely allows to find a forward rate proxy - using the swap rate bucket - to avoid the flat extrapolation when* $\tau^M_\alpha < \tau^k_t$.

3.1.2 Zero-Coupon Bonds Interpolation

Another way would be to use past zero coupon curves in order to value the securities of the current yield curve. If a τ_t maturity zero coupon bond is required for a pricing, a log linear interpolation is applied as follows

$$\begin{aligned}
P_\alpha(0,\tau_t) &= P_\alpha(0,\tau^1_\alpha) & , \tau^1_\alpha > \tau_t \\
P_\alpha(0,\tau_t) &= P_\alpha(0,\tau^L_\alpha) & , \tau^L_\alpha < \tau_t \\
P_\alpha(0,\tau_t) &= \exp\left(\frac{\left(\tau^{i+1}_\alpha - \tau_t\right).\ln\left(P_\alpha(0,\tau^i_\alpha)\right) + \left(\tau_t - \tau^i_\alpha\right).\ln\left(P_\alpha(0,\tau^{i+1}_\alpha)\right)}{\tau^{i+1}_\alpha - \tau^i_\alpha}\right) & , \tau^i_\alpha < \tau_t < \tau^{i+1}_\alpha
\end{aligned}$$

with L the index of the last zero coupon bond of the past yield curve.

Remarque 2 *Unlike the first method, the second one is able to carry out interpolations for money market and swap rate buckets One must nevertheless note that bien que plus precises, cette method induit le stripping des courbes de taux sur l'ensemble des dates de l'econometrie*