FRTB : Econometric

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Abstract

The aim of this document is to provide some technical elements required for the Fundamental Review of the Trading Book (FRTB) in the historical simulation framework.

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1 Introduction

1.1 Problem formulation

Defined by the Basel Committee of Banking Supervision, the FRTB aims to improve the Basel 2.5 regulation rules and to build a new market risk framework as a response to the financial crisis. These new standards address a number of issues such as capital arbitrage between booking and trading books as well as undercapitalization of the trading book.toto

Minimum capital requirements

- definir les risques factors
- Comment deformer les risk factors (contraintes technique (nb rf) et fonctionelle (pertinence de la defomation intra et cross rf))
- Mesure et application des deformations

However, because of the multitude of financial variables with different properties, define the function f is not an easy task.

how to measure past evolution of risk factors to apply deformations on their current state.

1.2 Notations and Definitions

$D(t,T)$: Stochastic discount factor at time t for the maturity T.

$P(t,T)$: T-maturity zero coupon bond value at time $t \leq T$.

$L(T_R, T_S, T_S + \tau)$: Simply compounded Libor rate resetting at time T_R for the start date T_S and the tenor τ.

$S(T_R, T_S, T_E)$: Swap rate resetting at time T_R for the start date T_S and the end date T_E.

2 Market Risk Modelling

2.1 Interest Rate Securities

La modèlisation du marché des taux d'interets suppose l'existence d'un actif de marché traitable, l'obligation zéro coupon. Les instruments de taux classiques s'en deduisant, la courbe des taux zero coupon permet donc d'en apprehender leur évolution dans le temps. En rèalité,

2.2 Single Rate Derivatives

2.3 Multi Rate Derivatives

3 Risk factors Econometrics

3.1 Yield Curve

As seen in the previous section, the yield curve represents the global risk factor for all interest rate instruments (including derivatives by extension).

3.1.1 Par-Point

In this approach, all pillars of the yield curve are considered independents and the econometrics are directly applied to their facial values. Now, the question is how to compare theses values at different dates. In fact, (selon la date d'observation et la description des instruments (business days vs calendar days), on peut avoir d'es instrument differents)

The first way is to extract zero coupon curves from past dates and use them to value the current yield curve. (ajouter graph)

The second way is to consider that rates with sliding maturities such as money market and swap rates can be compared regardless the date of observation. For other rates induced by rolling securities such as Euribor futures whose maturities are fixed dates, an immediate comparison is impossible. An alternative can be to compute the current value of theses rates from the past yield curves using some interpolation rules.

3.1.2 Forward Rate

3.1.3 Term Structure

3.2 Volatility Smile

3.2.1 SABR

3.2.2 Gaussian Implied volatility

3.3 Correlation Smile

4 Historical Simulation Framework

Historical simulation is a method which assumes that future evolution of a variable rely on its past evolution. In a financial context, for a given time horizion α and a risk factor R whose level at time t is noted R_t, this translates into

$$R_{t+1} = f_R(R_t, R_{s+h}, R_s), \quad t - \alpha \leq s < t - h$$

with respectively $h = 1$ for VaR and $h = 10$ for ES.

However, because of the multitude of financial variables with different properties, define the function f is not an easy task.

(x_1, x_2, x_3) premier critère espace des variable, deuxieme critère fruchard, on reprend les notations et on recherche les endomorphismes et monotonie

4.1 Absolute returns

The absolute return is defined as

$$\begin{aligned} X_{t+1} &= X_t + X_{s+h} - X_s \\ &= X_t + \delta_A \end{aligned}$$

4.2 Relative returns

The reletive return is defined as

$$\begin{aligned} X_{t+1} &= X_t . \left(\frac{X_{s+h}}{X_s} \right) \\ &= X_t + \delta_R, \quad \delta_R = X_t . \frac{(X_{s+h} - X_s)}{X_s} \end{aligned}$$

4.3 Absolute-Relative returns

4.3.1 Convexe combination

The most simple way to define an absolute-relative return is to use a convexe combination between theses two shocks with $\lambda \in [0,1]$

$$\delta_{AR} = \lambda.\delta_A + (1-\lambda).\delta_R$$

4.3.2 p-Space

The Lambert function is the inverse function of

$$y = x.\exp(x) \Longleftrightarrow x = W(y)$$

using this function, it is possible to define a p-variable Y_t by

$$Y_t = p.X_t + (1-p).\ln(X_t)$$

and then

$$X_t = \begin{cases} Y_t, & p = 1 \\ \exp(Y_t), & p = 0 \\ \left(\frac{1-p}{p}\right).W\left(\frac{p}{1-p}.\exp\left(\frac{Y_t}{1-p}\right)\right), & p \in]0,1[\end{cases}$$

$$y_1^{'} = y_1 + y_2 - y_3$$

and then retrieve δ_{AR} by

$$\delta_{AR} = f^{-1}\left(y_1^{'}\right) - x_1$$

4.4 Case of bounded risk factors