Abstract Trees

Outline

This topic discusses the concept of an abstract tree:

- Hierarchical ordering
- Description of an Abstract Tree/Hierarchy
- Applications
- Implementation
- Local definitions

Outline

A hierarchical ordering of a finite number of objects may be stored in a tree data structure

Operations on a hierarchically stored container include:

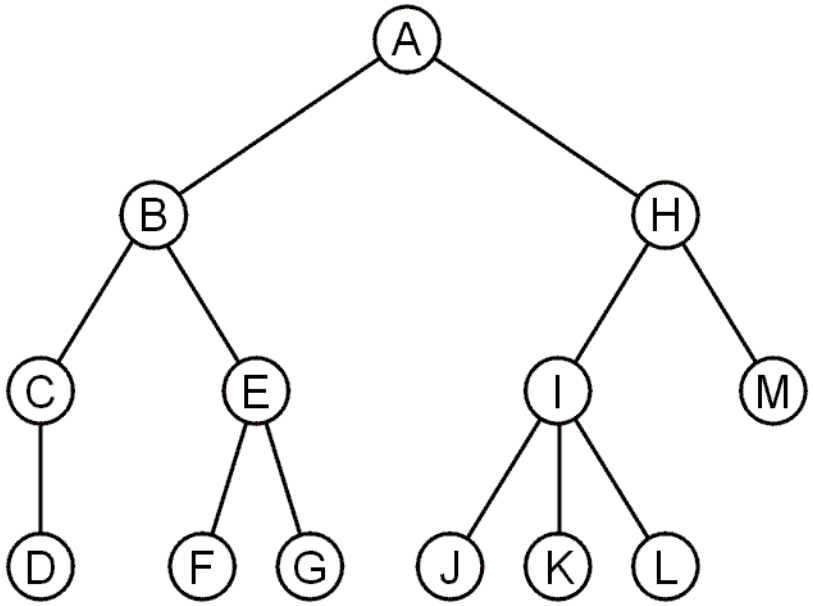
- Accessing the root:
- Given an object in the container:
  - Access the parent of the current object
  - Find the degree of the current object
  - Get a reference to a child,
  - Attach a new sub-tree to the current object
  - Detach this tree from its parent

Abstract Trees

An abstract tree (or abstract hierarchy) does not restrict the number of nodes

- In this tree, the degrees vary:

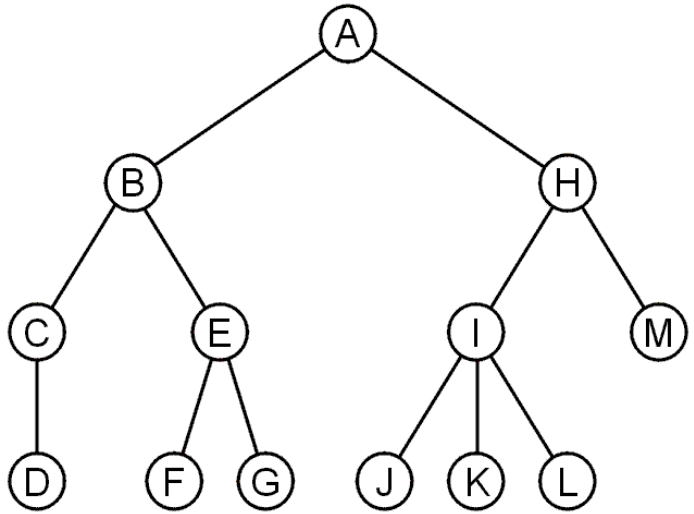
Degree	Nodes
0	D, F, G, J, K, L, M
1	C
2	A, B, E, H
3	I



Abstract Trees: Design

We implement an abstract tree or hierarchy by using a class that:

- Stores a value
- Stores the children in a linked-list



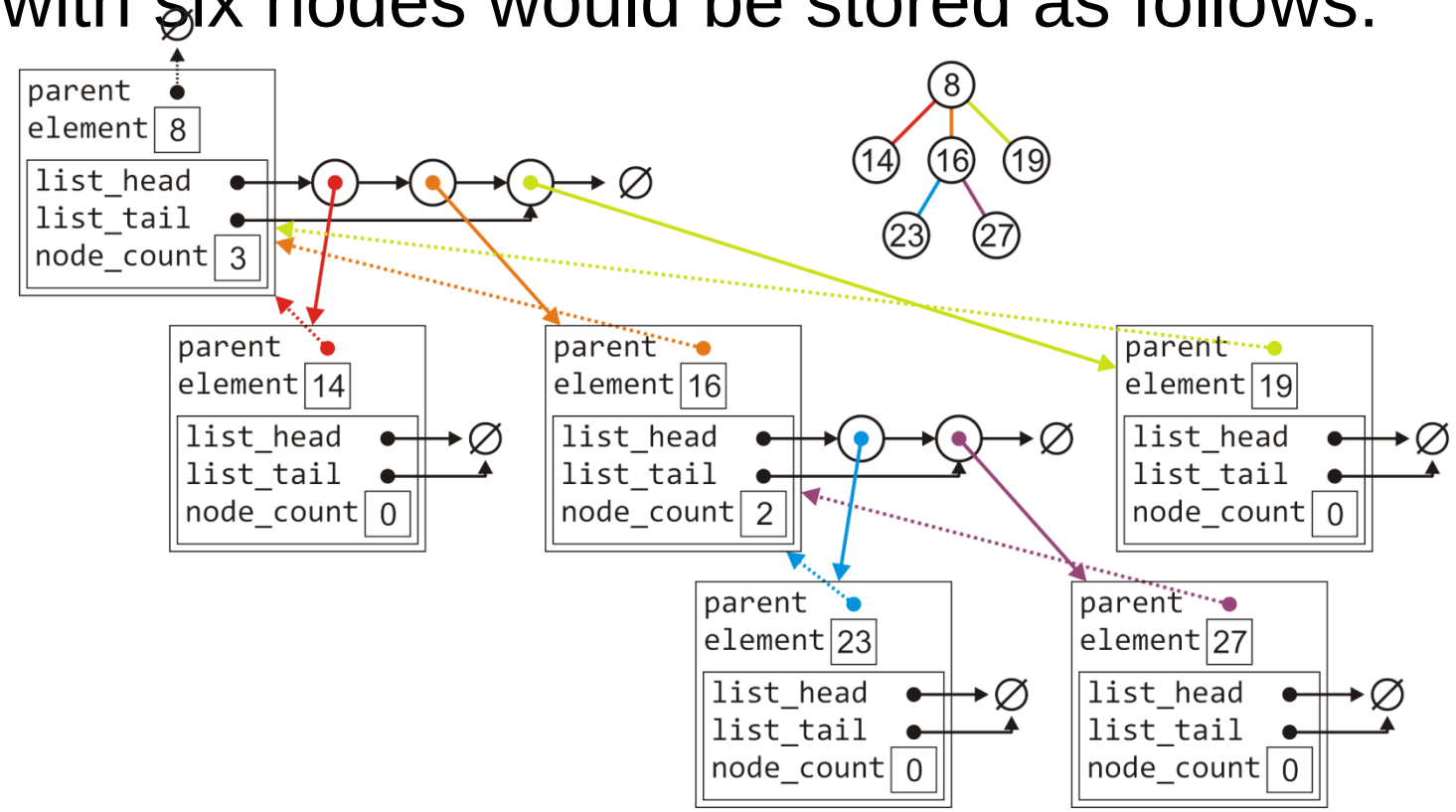
Implementation

The class definition would be:

```
template <typename Type>
class Simple_tree {
    private:
        Type node_value;
        Simple_tree *parent_node;
        Single_list<Simple_tree *> children;

    public:
        Simple_tree( Type const & = Type(), Simple_tree * = nullptr );

        Type value() const;
        Simple_tree *parent() const;
        int degree() const;
        bool is_root() const;
        bool is_leaf() const;
        Simple_tree *child( int n ) const;
        int height() const;

        void insert( Type const & );
        void attach( Simple_tree * );
        void detach();
};
```

Implementation

The tree with six nodes would be stored as follows:



Implementation

Much of the functionality is similar to that of the `Single_list` class:

```
template <typename Type>
Simple_tree<Type>::Simple_tree( Type const &obj, Simple_tree *p ):
node_value( obj ),
parent_node( p ) {
    // Empty constructor
}

template <typename Type>
Type Simple_tree<Type>::value() const {
    return node_value;
}

template <typename Type>
Simple_tree<Type> *Simple_tree<Type>::parent() const {
    return parent_node;
}
```

Implementation

Much of the functionality is similar to that of the `Single_list` class:

```
template <typename Type>
bool Simple_tree<Type>::is_root() const {
    return ( parent() == nullptr );
}

template <typename Type>
int Simple_tree<Type>::degree() const {
    return children.size();
}

template <typename Type>
bool Simple_tree<Type>::is_leaf() const {
    return ( degree() == 0 );
}
```

Implementation

Accessing the n^{th} child requires a for loop ($\Theta(n)$):

```
template <typename Type>
Simple_tree<Type> *Simple_tree<Type>::child( int n ) const {
    if ( n < 0 || n >= degree() ) {
        return nullptr;
    }

    auto *child = children.head();

    // Skip the first n - 1 children
    for ( int i = 1; i < n; ++i ) {
        child = child->next();
    }

    return child->value();
}
```

Implementation

Attaching a new object to become a child is similar to a linked list:

```
template <typename Type>
void Simple_tree<Type>::attach( Type const &obj ) {
    children.push_back( new Simple_tree( obj, this ) );
}
```

Implementation

To detach a tree from its parent:

- If it is already a root, do nothing
- Otherwise, erase this object from the parent's list of children and set the parent pointer to zero

```
template <typename Type>
void Simple_tree<Type>::detach() {
    if ( is_root() ) {
        return;
    }

    parent()->children.erase( this );
    parent_node = nullptr;
}
```

Implementation

Attaching an entirely new tree as a sub-tree, however, first requires us to check if the tree is not already a sub-tree of another node:

- If so, we must detach it first and only then can we add it

```
template <typename Type>
void Simple_tree<Type>::attach( Simple_tree<Type> *tree ) {
    if ( !tree->is_root() ) {
        tree->detach();
    }

    tree->parent_node = this;
    children.push_back( tree );
}
```

Implementation

Suppose we want to find the size of a tree:

- An empty tree has size 0, a tree with no children has size 1
- Otherwise, the size is one plus the size of all the children

```
template <typename Type>
int Simple_tree<Type>::size() const {
    if ( this == nullptr ) {
        return 0;
    }

    int tree_size = 1;

    for ( auto *child = children.begin(); child != children.end(); child = child->next() ) {
        tree_size += child->value()->size();
    }

    return tree_size ;
}
```

Implementation

Suppose we want to find the height of a tree:

- An empty tree has height –1 and a tree with no children is height 0
- Otherwise, the height is one plus the maximum height of any sub tree

```
#include <algorithm>

template <typename Type>
int Simple_tree<Type>::height() const {
    if ( this == nullptr ) {
        return -1;
    }

    int tree_height = 0;

    for ( auto *child = children.begin(); child != children.end(); child = child->next() ) {
        tree_height = std::max( h, 1 + child->value()->height() );
    }

    return tree_height;
}
```

Implementation

Implementing a tree by storing the children in an array is similar,
however, we must deal with the full structure

- A general tree using an array would have a constructor similar to:

```
template <typename Type>
Simple_tree<Type>::Simple_tree( Type const &obj, Simple_tree *p ):
node_value( obj ),
parent_node( p ),
child_count( 0 ),
child_capacity( 4 ),
children( new Simple_tree *[child_capacity] ) {
    // Empty constructor
}
```

Summary

In this topic, we have looked at one implementation of a general tree:

- store the value of each node
- store all the children in a linked list
- not an easy ($\Theta(1)$) way to access children
- if we use an array, different problems...