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Statistical Theory – Formative Assessment 2

Exercise 3.49: Prove that $\sum_{j=1}^{N}\left(X_j - 1\right)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$

Proof: Consider the notation symbol $\sum_{j=1}^{N}(X_j)$ denoted as the summation notation for the expression X_j

Thus, the sum of all X_j exists from $j = 1$ to $j = N$ and mathematically to be described as:

$$\sum_{j=1}^{N} X_j = X_1 + X_2 + X_3 + \cdots + X_N$$

Replace the parameter X_j as $\left(X_j - 1\right)^2$ in the summation notation, consequently, this describes that the sum of $\left(X_j - 1\right)^2$ also exists from $j = 1$ to $j = N$

$$\sum_{j=1}^{N}\left(X_j - 1\right)^2 = (X_1 - 1)^2 + (X_2 - 1)^2 + (X_3 - 1)^2 + \cdots + (X_N - 1)^2$$

Algebraically, the square of the binomial in the form $(A - B)^2$ always generate a trinomial form of $(A^2 - 2AB + B^2)$; similarly, we can simplify the expression $\left(X_j - 1\right)^2$ as $\left(X_j^2 - 2X_j + 1\right)$

$$\sum_{j=1}^{N}\left(X_j^2 - 2X_j + 1\right) = (X_1^2 - 2X_1 + 1) + (X_2^2 - 2X_2 + 1) + (X_3^2 - 2X_3 + 1) + \cdots + (X_N^2 - 2X_N + 1)$$

Since each term of $\left(X_j^2 - 2X_j + 1\right)$ are in terms of additions and subtractions being added to another next term of $\left(X_j^2 - 2X_j + 1\right)$. Notice that whenever we change which one is being added or subtracted first, the overall summation remains the same and equivalent, therefore, this long thread of summation satisfies the law of associativity, therefore we can regroup into their common polynomial degree:

$$\sum_{j=1}^{N}\left(X_j^2 - 2X_j + 1\right) = (X_1^2 + X_2^2 + X_3^2 + \cdots + X_N^2) + (-2X_1 - 2X_2 - 2X_3 - \cdots - 2X_N) + (1 + 1 + 1 + \cdots + 1)$$

The new expression in the right-hand of equation implies that each grouped terms also satisfies the summation notations definitions:

1. $(X_1^2 + X_2^2 + X_3^2 + \cdots + X_N^2)$ defines the sum of all X_j^2's from $j = 1$ to $j = N$ thus $(X_1^2 + X_2^2 + X_3^2 + \cdots + X_N^2) = \sum_{j=1}^{N}\left(X_j^2\right)$

2. Since -2 is the common factor in the expression $(-2X_1 - 2X_2 - 2X_3 - \cdots - 2X_N)$ then $-2(X_1 + X_2 + X_3 + \cdots + X_N)$ which implies the sum of all X_j's indeed made existence from $j = 1$ to $j = N$; $-2(X_1 + X_2 + X_3 + \cdots + X_N) = -2\sum_{j=1}^{N} X_j$
3. Finally, the expression $(1 + 1 + 1 + \cdots + 1)$ defines the sum of all 1, from $j = 1$ unto N times, making it $(1 + 1 + 1 + \cdots + 1) = \sum_{j=1}^{N}(1) = N$

$$\sum_{j=1}^{N}(X_j^2 - 2X_j + 1) = \sum_{j=1}^{N}(X_j^2) - 2\sum_{j=1}^{N} X_j + N$$

Recall $\left(X_j^2 - 2X_j + 1\right)$ can be rewritten as $\left(X_j - 1\right)^2$

$$\sum_{j=1}^{N}\left(X_j - 1\right)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$$

$$Q.E.D$$

Exercise 3.51: Two variables *U* and *V* and assume their values as follows:

U	U_1	3
	U_2	-2
	U_3	5
V	V_1	-4
	V_2	-1
	V_3	6

Calculate the following:

$\sum(UV)$	$\sum(UV) = U_1V_1 + U_2V_2 + U_3V_3$ $\Rightarrow (3)(-4) + (-2)(-1) + (5)(6) \Rightarrow (-12) + (2) + (30) \Rightarrow 20$
$\sum(U+3)(V-4)$	$\sum(U+3)(V-4) = (U_1+3)(V_1-4) + (U_2+3)(V_2-4) + (U_3+3)(V_3-4)$ $\Rightarrow \left((3)+3\right)\left((-4)-4\right) + \left((-2)+3\right)\left((-1)-4\right) + \left((5)+3\right)\left((6)-4\right)$ $\Rightarrow (6)(-8) + (1)(-5) + (8)(2) \Rightarrow (-48) + (-5) + (16)$ $\Rightarrow -37$
$\sum(V^2)$	$\sum(V^2) = V_1^2 + V_2^2 + V_3^2$ $\Rightarrow (-4)^2 + (-1)^2 + (6)^2 \Rightarrow (16) + (1) + (36) \Rightarrow 53$

$\sum(U)(\sum V)^2$	$\sum(U)(\sum V)^2 = (U_1 + U_2 + U_3)(V_1 + V_2 + V_3)^2$ $\Rightarrow ((3) + (-2) + (5))((-4) + (-1) + (6))^2 \Rightarrow (6)(1)^2 \Rightarrow 6$
$\sum(UV^2)$	$\sum(UV^2) = U_1V_1^2 + U_2V_2^2 + U_3V_3^2$ $\Rightarrow (3)(-4)^2 + (-2)(-1)^2 + (5)(6)^2 \Rightarrow (3)(16) + (-2)(1) + (5)(36) \Rightarrow 226$
$\sum(U^2 - 2V + 2)$	$\sum(U^2 - 2V + 2) = \sum(U) - 2\sum(V) + \sum(2)$ $\Rightarrow [(3) + (-2) + (5)] - 2[(-4) + (-1) + (6)] + [2 + 2 + 2]$ $\Rightarrow [6] - 2[1] + [6] \Rightarrow 10$
$\sum\left(\frac{U}{V}\right)$	$\sum\left(\frac{U}{V}\right) = \frac{U_1}{V_1} + \frac{U_2}{V_2} + \frac{U_3}{V_3}$ $\Rightarrow \left(\frac{3}{-4}\right) + \left(\frac{-2}{-1}\right) + \left(\frac{5}{6}\right) \Rightarrow \left(-\frac{9}{12}\right) + \left(\frac{24}{12}\right) + \left(\frac{10}{12}\right) \Rightarrow \frac{25}{12}$

Exercise 3.90: Find the geometric mean of the sets A and B

$$A = \{3,5,8,3,7,2\}$$

Consider the Geometric Mean of set A to be defined as $G_A = \sqrt[N]{\prod_{j=1}^{N}(A_j)}$; since there are total of 5 distinct elements within the set (the element 3 has been duplicated). Let $N = 5$:

$$G_A = \sqrt[5]{\prod_{j=1}^{5}(A_j)} = \sqrt[5]{(3)(5)(8)(7)(2)} = \sqrt[5]{1680}$$

Apply natural logarithm for G_A:

$$\ln(G_A) = \ln\left(\sqrt[5]{1680}\right) = \frac{1}{5}\ln(1680) = \frac{1}{5}(7.43\ldots) \approx 1.49$$

Apply the inverse of natural logarithm – exponential – for $\ln(G_A)$ and simplify:

$$\mathbf{\exp(\ln(G_A)) = G_A = e^{1.49} \approx 4.44}$$

$$B = \{28.5, 73.6, 47.2, 31.5, 64.8\}$$

Consider the Geometric Mean of set B to be defined as $G_B = \sqrt[N]{\prod_{j=1}^{N}(B_j)}$; since there are 5 elements within the set. Let $N = 5$

$$G_B = \sqrt[N]{\prod_{j=1}^{N}(B_j)} = \sqrt[5]{(28.5)(73.6)(47.2)(31.5)(64.8)}$$

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Apply the natural logarithm then its natural exponential for the G_B

$$\ln(G_B) = \ln\left(\sqrt[5]{(28.5)(73.6)(47.2)(31.5)(64.8)}\right) = \frac{1}{5}\ln\big((28.5)(73.6)(47.2)(31.5)(64.8)\big)$$

$$\ln(G_B) = \frac{1}{5}(\ln(28.5) + \ln(73.6) + \ln(47.2) + \ln(31.5) + \ln(64.8))$$

$$\ln(G_B) \approx \frac{1}{5}(3.35 + 4.30 + 3.85 + 3.45 + 4.17) = \frac{1}{5}(19.12) = \frac{478}{125}$$

$$\boldsymbol{e^{\ln(G_B)} = G_B = e^{\frac{478}{125}} \approx 45.79}$$