

**Chapter 7**

**7-1.** The distribution of means is narrower than the distribution of single results. Hence, the standard error of the mean of 5 measurements is smaller than the standard deviation of a single result. The mean is thus known with more confidence than is a single result

**7-2** (a) Looking at Table 7-1, we find that for  $z = 2.58$ ,  $CL = 99\%$

(b)  $CL = 95\%$

(c)  $CL = 99.9\%$

(d)  $CL = 68\%$

**7-3.** (a) Since  $CI = \bar{x} \pm \frac{z\sigma}{\sqrt{N}}$ , as the standard deviation,  $\sigma$ , increases the confidence interval increases.

(b) As the desired confidence level increases,  $z$  increases and thus the confidence interval increases.

(c). as the sample size,  $N$ , increases the confidence interval decreases.

**7-4.** For Set A

$x_i$	$x_i^2$
2.7	7.29
3.0	9.00
2.6	6.76
2.8	7.84
3.2	10.24
$\Sigma x_i = 14.3$	$\Sigma x_i^2 = 41.13$

mean:  $\bar{x} = 14.3/5 = 2.86$

$$\text{standard deviation: } s = \sqrt{\frac{41.13 - (14.3)^2 / 5}{5 - 1}} = 0.24$$

Since, for a small set of measurements we cannot be certain  $s$  is a good approximation of  $\sigma$ , we should use the  $t$  statistic for confidence intervals. From Table 7-3, at 95% confidence  $t$  for 4 degrees of freedom is 2.78, therefore for set A,

$$\text{CI for } \mu = 2.86 \pm \frac{(2.78)(0.24)}{\sqrt{5}} = 2.86 \pm 0.30$$

Similarly, for the other data sets, we obtain the results shown in the following table:

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>
$\bar{x}$	2.86	0.494	70.19	3.1	0.824	70.53
$s$	0.24	0.016	0.08	0.37	0.051	0.22
CI	$2.86 \pm 0.30$	$0.494 \pm 0.020$	$70.19 \pm 0.20$	$3.1 \pm 0.46$	$0.824 \pm 0.081$	$70.53 \pm 0.34$

The 95% confidence interval is the range within which the population mean is expected to lie with a 95% probability.

**7-5.** If  $s$  is a good estimate of  $\sigma$  then we can use  $z = 1.96$  for the 95% confidence level. For set A, at the 95% confidence,

$$\text{CI for } \mu = 2.86 \pm \frac{(1.96)(0.30)}{\sqrt{5}} = 2.86 \pm 0.26. \text{ Similarly for sets B-F, the limits are:}$$

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>
<b>CI</b>	$2.86 \pm 0.26$	$0.494 \pm 0.013$	$70.19 \pm 0.079$	$3.1 \pm 0.18$	$0.824 \pm 0.009$	$70.53 \pm 0.015$

**7-6.** For set A:  $Q = \frac{|3.2 - 3.0|}{3.2 - 2.6} = 0.33$  and  $Q_{\text{crit}} = 0.710$  for 5 observations at the 95%

confidence level.

Since  $Q < Q_{\text{crit}}$  the outlier value 2.5 cannot be rejected with 95% confidence.

	A	B	C	D	E	F
$Q$	0.33	0.33	0.86	0.60	0.81	0.95
$Q_{\text{crit}}$	0.710	0.710	0.970	0.710	0.829	0.829
Decision	Keep	Keep	Keep	Keep	Keep	Reject

**7-7. (a)** 99% CI =  $18.5 \pm 2.58 \times 3.6 = 18.5 \pm 9.3 \text{ } \mu\text{g Fe/mL}$

95% CI =  $18.5 \pm 1.96 \times 3.6 = 18.5 \pm 7.1 \text{ } \mu\text{g Fe/mL}$

**(b)** 99% CI =  $18.5 \pm \frac{2.58 \times 3.6}{\sqrt{2}} = 18.5 \pm 6.6 \text{ } \mu\text{g Fe/mL}$

95% CI =  $18.5 \pm \frac{1.96 \times 3.6}{\sqrt{2}} = 18.5 \pm 5.0 \text{ } \mu\text{g Fe/mL}$

**(c)** 99% CI =  $18.5 \pm \frac{2.58 \times 3.6}{\sqrt{4}} = 18.5 \pm 4.6 \text{ } \mu\text{g Fe/mL}$

95% CI =  $18.5 \pm \frac{1.96 \times 3.6}{\sqrt{4}} = 18.5 \pm 3.5 \text{ } \mu\text{g Fe/mL}$

**7-8. (a)** 95% CI =  $7.91 \pm 1.96 \times 0.27 = 7.91 \pm 0.53 \text{ } \mu\text{g Cu/mL}$

99% CI =  $7.91 \pm 2.58 \times 0.27 = 7.91 \pm 0.70 \text{ } \mu\text{g Cu/mL}$

**(b)** 95% CI =  $7.91 \pm \frac{1.96 \times 0.27}{\sqrt{4}} = 7.91 \pm 0.26 \text{ } \mu\text{g Cu/mL}$

99% CI =  $7.91 \pm \frac{2.58 \times 0.27}{\sqrt{4}} = 7.91 \pm 0.35 \text{ } \mu\text{g Cu/mL}$

**(c)** 95% CI =  $7.91 \pm \frac{1.96 \times 0.27}{\sqrt{16}} = 7.91 \pm 0.13 \text{ } \mu\text{g Cu/mL}$

99% CI =  $7.91 \pm \frac{2.58 \times 0.32}{\sqrt{16}} = 7.91 \pm 0.17 \text{ } \mu\text{g Cu/mL}$

**7-9.**  $2.2 = \frac{1.96 \times 3.6}{\sqrt{N}}$  For a 95% CI,  $N = 10.3 \cong 11$

$2.2 = \frac{2.58 \times 3.6}{\sqrt{N}}$  For a 99% CI,  $N = 17.8 \cong 18$

**7-10.** (a)  $0.20 = \frac{1.96 \times 0.27}{\sqrt{N}}$  For the 95% CI,  $N = 7$

(b)  $0.20 = \frac{2.58 \times 0.27}{\sqrt{N}}$  For a 99% CI,  $N = 12.13 \cong 13$

**7-11.** For the data set,  $\bar{x} = 3.22$  and  $s = 0.06$

(a) 95% CI =  $3.22 \pm \frac{4.30 \times 0.06}{\sqrt{3}} = 3.22 \pm 0.15$  meq Ca/L

(b) 95% CI =  $3.22 \pm \frac{1.96 \times 0.056}{\sqrt{3}} = 3.22 \pm 0.06$  meq Ca/L

**7-12.** For the data set,  $\bar{x} = 7.24$  and  $s = 0.29$

(a) 90% CI =  $7.24 \pm \frac{2.92 \times 0.29}{\sqrt{3}} = 7.24 \pm 0.49$  % lindane

(b) 90% CI =  $7.24 \pm \frac{1.64 \times 0.28}{\sqrt{3}} = 7.24 \pm 0.27$  % lindane

**7-13.** (a)  $0.3 = \frac{2.58 \times 0.38}{\sqrt{N}}$  For the 99% CI,  $N = 10.7 \cong 11$

(b)  $0.3 = \frac{1.96 \times 0.38}{\sqrt{N}}$  For the 95% CI,  $N = 6.1 \cong 7$

(c)  $0.2 = \frac{1.64 \times 0.38}{\sqrt{N}}$  For the 90% CI,  $N = 9.7 \cong 10$

**7-14.** This is a two-tailed test and from Table 7-1,  $z_{\text{crit}} = 1.96$  for the 95% confidence level.

For carbon,

$$z = \frac{68.5 - 68.8}{0.004 \times 68.8\% / \sqrt{2}} = -1.54 \geq -1.96$$

Systematic error is NOT indicated at 95% confidence level.

$$\text{For hydrogen, } z = \frac{4.882 - 4.953}{0.006 \times 4.953\% / \sqrt{2}} = -3.38 \leq -1.96$$

Systematic error IS indicated at 95% confidence level.

**7-15.** This is a two-tailed test where  $s \rightarrow \sigma$  and from Table 7-1,  $z_{\text{crit}} = 2.58$  for the 99% confidence level.

$$\text{For As: } z = \frac{129 - 119}{9.5 \sqrt{\frac{3+3}{3 \times 3}}} = 1.28 \leq 2.58$$

No significant difference exists at the 99% confidence level .

Proceeding in a similar fashion for the other elements

Element	$z$	Significant Difference?
As	1.28	No
Co	-3.43	Yes
La	2.45	No
Sb	0.20	No
Th	-3.42	Yes

For two of the elements there is a significant difference, but for three there are not. Thus, the defendant might have grounds for claiming reasonable doubt. It would be prudent, however, to analyze other windows and show that these elements are good diagnostics for the rare window.

- 7-16.** The null hypothesis is that  $\mu = 5.0$  ppm dissolved  $O_2$  and the alternative hypothesis is that  $\mu < 5.0$  ppm dissolved  $O_2$ . This is a one-tailed test and from Table 7-1,  $z_{\text{crit}} = 1.64$  for the 95% confidence level.

For the data set,  $\bar{x} = 4.888$  and  $s = 0.40$

$$t = \frac{4.888 - 5.0}{0.40/\sqrt{8}} = -0.79 \geq -1.64 \quad \text{Thus, we must accept the null hypothesis that the mean}$$

dissolved  $O_2$  is 5.0 ppm at the 95% confidence level.

**7-17.**  $Q = \frac{|5.6 - 5.1|}{5.6 - 4.3} = 0.385$  and  $Q_{\text{crit}}$  for 8 observations at 95% confidence = 0.526.

Since  $Q < Q_{\text{crit}}$  the outlier value 5.6 cannot be rejected at the 95% confidence level.

- 7-18.**  $H_0: \mu = 1.0$  ppb for the impurity;  $H_a: \mu < 1.0$  ppb for the impurity. This is a one-tailed test. The type I error for this situation would be that we reject the null hypothesis when, in fact, it is true, i.e. we decide the impurity is  $< 1.0$  ppb at some level of confidence when, in fact, it is not  $< 1.0$  ppb. The type II error would be that we accept the null hypothesis when, in fact, it is false, i.e. we decide the impurity is not  $< 1.0$  ppb at some level of confidence when, in fact, it is  $< 1.0$  ppb.

- 7-19.** The null hypothesis is that for the pollutant the current level = the previous level ( $H_0: \mu_{\text{current}} = \mu_{\text{previous}}$ ). The alternative hypothesis is  $H_a: \mu_{\text{current}} > \mu_{\text{previous}}$ . This would be a one-tailed test. The type I error for this situation would be that we reject the null hypothesis when, in fact, it is true, i.e. we decide the level of the pollutant is  $>$  the previous level at some level of confidence when, in fact, it is not. The type II error would be that we accept the null hypothesis when, in fact, it is false, i.e. we decide the level of the pollutant = the previous level when, in fact, it is  $>$  than the previous level.

- 7-20. (a)**  $H_0: \mu_{\text{ISE}} = \mu_{\text{EDTA}}, H_a: \mu_{\text{ISE}} \neq \mu_{\text{EDTA}}$ . This would be a two-tailed test. The type I error for this situation would be that we decide the methods agree when they do not. The type II error would be that we decide the methods do not agree when they do.
- (b)**  $H_0: \mu = 7.03 \text{ ppm}; H_a: \mu < 7.03 \text{ ppm}$ . This is a one-tailed test. The type I error for this situation would be that we reject  $H_0$  decide that a systematic error exists when it does not. The type II error would be that we accept  $H_0$  decide a systematic error does not exist when it does.
- (c)**  $H_0: \sigma_X^2 = \sigma_Y^2; H_a: \sigma_X^2 < \sigma_Y^2$ . This is a one-tailed test. The type I error would be that we decide that  $\sigma_X^2 < \sigma_Y^2$  when it is not. The type II error would be that we decide that  $\sigma_X^2 = \sigma_Y^2$  when actually  $\sigma_X^2 < \sigma_Y^2$ .
- (d)**  $H_0: \sigma_{\text{AA}}^2 = \sigma_{\text{EC}}^2; H_a: \sigma_{\text{AA}}^2 < \sigma_{\text{EC}}^2$ . This is a one-tailed test. The type I error for this situation would be that we decide AA results are less precise than electrochemistry results, when the precision is the same. The type II error would be that we decide the precision is the same when they electrochemical results are more precise.

- 7-21. (a)** For the Top data set,  $\bar{x} = 26.338$

For the bottom data set,  $\bar{x} = 26.254$

$$s_{\text{pooled}} = 0.1199$$

$$\text{degrees of freedom} = 5 + 5 - 2 = 8$$

For 8 degrees of freedom at 95% confidence  $t_{\text{crit}} = 2.31$

$$t = \frac{26.338 - 26.254}{0.1199 \sqrt{\frac{5+5}{5 \times 5}}} = 1.11 \quad \text{Since } t < t_{\text{crit}}, \text{ we conclude that no significant difference}$$

exists at 95% confidence level.

(b) From the data,  $N = 5$ ,  $\bar{d} = 0.084$  and  $s_d = 0.015166$

For 4 degrees of freedom at 95% confidence  $t = 2.78$

$$t = \frac{0.084 - 0}{0.015 / \sqrt{5}} = 12.52$$

Since  $12.52 > 2.78$ , a significant difference does exist at 95% confidence level.

(c) The large sample to sample variability causes  $s_{\text{Top}}$  and  $s_{\text{Bottom}}$  to be large and masks the differences between the samples taken from the top and the bottom.

**7-22.** (a) A paired  $t$  test should definitely be used in this case due to the large variation in the Cl concentrations resulting from the various contact times and various locations from which the samples were obtained.

(b)  $H_0: \mu_d = 0$ ;  $H_a: \mu_d \neq 0$ , where  $\mu_d$  is the mean difference between the methods

From the data  $N = 8$ ,  $\bar{d} = -0.414$  and  $s_d = 0.32$

For 7 degrees of freedom at 90% confidence level,  $t_{\text{crit}} = 1.90$

$$t = \frac{0.414 - 0}{0.32 / \sqrt{8}} = 3.65$$

Since  $t > t_{\text{crit}}$ , a significant difference is indicated at the 90% confidence level

(c) For 7 degrees of freedom at 95% confidence level,  $t_{\text{crit}} = 2.36$

Therefore, a significant difference in the 2 methods exists at the 95% confidence level.

For 7 degrees of freedom at the 99% confidence level,  $t_{\text{crit}} = 3.50$

Thus, a significant difference is indicated even at the 99% confidence level. The conclusion does not depend on which of the three confidence levels is used.

**7-23.** For the first data set:  $\bar{x} = 2.2978$

For the second data set:  $\bar{x} = 2.3106$



$$s_{\text{pooled}} = 0.0027$$

$$\text{Degrees of freedom} = 4 + 3 - 2 = 5$$

$$t = \frac{2.2978 - 2.3106}{0.0027 \sqrt{\frac{4+3}{4 \times 3}}} = -6.207$$

For 5 degrees of freedom at the 99% confidence level,  $t = 4.03$  and at the 99.9% confidence level,  $t = 6.87$ . Thus, we can be between 99% and 99.9% confident that the nitrogen prepared in the two ways is different. The Excel TDIST(x,df,tails) function can be used to calculate the probability of getting a  $t$  value of  $-6.207$ . In this case we find TDIST(6.207,5,2) = 0.0016. Therefore, we can be 99.84% confident that the nitrogen prepared in the two ways is different. There is a 0.16% probability of this conclusion being in error.

**7-24. (a)**

Source	SS	df	MS	F
Between soils	$0.374 - 0.0972 = 0.2768$	$3 - 1 = 2$	$0.2768/2 = 0.1384$	$0.1384/0.0081 = 17.09$
Within soils	$12 \times 0.0081 = 0.0972$	$15 - 3 = 12$	0.0081	
Total	0.374	$15 - 1 = 14$		

**(b)**  $H_0$ :  $\mu_{\text{samp1}} = \mu_{\text{samp2}} = \mu_{\text{samp3}}$ ;  $H_a$ : at least two of the means differ.

**(c)** From Table 7-4 the  $F$  value for 12 degrees of freedom in the denominator and 2 degrees of freedom in the numerator at the 95% confidence level is 3.89. Since the  $F$  value calculated in the table exceeds  $F$  critical, we reject  $H_0$  and conclude that the phosphorous contents of the soil samples taken from the 3 locations are different.

**7-25 (a)**

Source	SS	df	MS	F
Between juices	$4 \times 7.715 = 30.86$	$5 - 1 = 4$	$0.913 \times 8.45 = 7.715$	8.45
Within juices	$25 \times 0.913 = 22.825$	$30 - 5 = 25$	0.913	
Total	$30.86 + 22.82 = 50.68$	$30 - 1 = 29$		

(b)  $H_0: \mu_{\text{brand1}} = \mu_{\text{brand2}} = \mu_{\text{brand3}} = \mu_{\text{brand4}} = \mu_{\text{brand5}}; H_a$ : at least two of the means differ.

(c) The Excel FINV(prob,df1,df2) function can be used to calculate the  $F$  value for the above problem. In this case we find  $\text{FINV}(0.05,4,25) = 2.76$ . Since  $F$  calculated exceeds  $F$  critical, we reject the null hypothesis and conclude that the average ascorbic acid contents of the 5 brands of orange juice differ at the 95% confidence level.

**7-26. (a)**  $H_0: \mu_{\text{LabA}} = \mu_{\text{LabB}} = \mu_{\text{LabC}} = \mu_{\text{LabD}} = \mu_{\text{LabE}}; H_a$ : at least two of the means differ.

(b) See spreadsheet next page. From Table 7-4 the  $F$  value for 4 degrees of freedom in the numerator and 10 degrees of freedom in the denominator at 95% is 3.48. Since  $F$  calculated exceeds  $F$  tabulated we reject  $H_0$  and conclude that the laboratories differ at 95% confidence. We can also be 99% confident that the laboratories differ, but we cannot be 99.9% confident that the laboratories differ.

(c) Based on the calculated LSD value laboratories A, C and E differ from laboratory D, but laboratory B does not. Laboratories E and A differ from laboratory B, but laboratory C does not. No significant difference exists between laboratories E and A.

**Spreadsheet for Pb. 7-26.**

	A	B	C	D	E	F	G	H	
1	<b>Result No.</b>	<b>Lab A</b>	<b>Lab B</b>	<b>Lab C</b>	<b>Lab D</b>	<b>Lab E</b>			
2	1	10.3	9.5	10.1	8.6	10.6			
3	2	11.4	9.9	10.0	9.3	10.5			
4	3	9.8	9.6	10.4	9.2	11.1			
5									
6	Average	10.50	9.67	10.17	9.03	10.73			
7	St. Dev.	0.818535	0.208167	0.208167	0.37859389	0.321455			
8	Variance	0.67000	0.043333	0.043333	0.143333	0.103333			
9									
10	Grand Mean	10.020			<b>Differences</b>				
11	SSF	5.577333			10.73-9.03=	1.70	<b>Significant difference</b>		
12	SSE	2.006667			10.50-9.03=	1.47	<b>Significant difference</b>		
13	SST	7.584			10.17-9.03=	1.14	<b>Significant difference</b>		
14					9.67-9.03=	0.64	<b>No sig. diff.</b>		
15	MSF	1.394333			10.73-9.67=	1.06	<b>Significant difference</b>		
16	MSE	0.200667			10.50-9.67=	0.83	<b>Significant difference</b>		
17					10.17-9.67	0.5	<b>No sig. diff.</b>		
18	F	6.948505		Labs	10.73-10.17=	0.56	<b>No sig. diff.</b>		
19					10.50-10.17=	0.33	<b>No sig. diff.</b>		
20	LSD	0.816			10.73-10.50=	0.23	<b>No sig. diff.</b>		
21									
22	<b>Spreadsheet Documentation</b>								
23	B6=AVERAGE(B2:B4)								
24	B7=STDEV(B2:B4)								
25	B8=VAR(B2:B4)								
26	B10=AVERAGE(B2:F4)								
27	B11=3*((B6-B10)^2+(C6-B10)^2+(D6-B10)^2+(E6-B10)^2+(F6-B10)^2)								
28	B12=2*SUM(B8:F8)								
29	B13=B11+B12								
30	B15=B11/4								
31	B16=B12/10								
32	B18=B15/B16								
33	B20=2.23*SQRT(2*B16/3)								
34									

**7-27. (a)**  $H_0: \mu_{\text{Analyst1}} = \mu_{\text{Analyst2}} = \mu_{\text{Analyst3}} = \mu_{\text{Analyst4}}$ ;  $H_a$ : at least two of the means differ.

**(b)** See spreadsheet next page. From Table 7-4 the  $F$  value for 3 degrees of freedom in the numerator and 12 degrees of freedom in the denominator at 95% is 3.49. Since  $F$  calculated exceeds  $F$  critical, we reject the null hypothesis and conclude that the analysts differ at 95% confidence. The  $F$  value calculated of 13.60 also exceeds the critical values at the 99% and 99.9% confidence levels so that we can be certain that the analysts differ at these confidence levels.

**(c)** Based on the calculated LSD value there is a significant difference between analyst 2 and analysts 1 and 4, but not analyst 3. There is a significant difference between analyst 3 and analyst 1, but not analyst 4. There is a significant difference between analyst 1 and analyst 4.

**Spreadsheet for Problem 7-27.**

	A	B	C	D	E	F	G
1	<b>Detmn</b>	<b>Analys 1</b>	<b>Analyst 2</b>	<b>Analyst 3</b>	<b>Analyst 4</b>		
2	1	10.24	10.14	10.19	10.19		
3	2	10.26	10.12	10.11	10.15		
4	3	10.29	10.04	10.15	10.16		
5	4	10.23	10.07	10.12	10.10		
6							
7	Mean	10.26	10.09	10.14	10.15		
8	Std. Dev.	0.02646	0.04573	0.03594	0.03742		
9	Variance	0.00070	0.00209	0.00129	0.00140		
10							
11	Grand Mean	10.16					
12	SSF	0.05595		<b>Differences</b>			
13	SSE	0.01645		10.26-10.09=	0.17	Significant difference	
14	SST	0.07240		10.15-10.09=	0.06	Significant difference	
15				10.14-10.09=	0.05	No sig. diff.	
16	MSF	0.01865		10.26-10.14=	0.12	Significant difference	
17	MSE	0.001371		10.15-10.14=	0.01	No sig. diff.	
18				10.26-10.15=	0.11	Significant difference	
19	F	13.60486					
20							
21	LSD	0.057335					
22							
23	<b>Spreadsheet Documentation</b>						
24	B7=AVERAGE(B2:B5)						
25	B8=STDEV(B2:B5)						
26	B9=VAR(B2:B5)						
27	B11=AVERAGE(B2:E5)						
28	B12=4*((B7-B11)^2+(C7-B11)^2+(D7-B11)^2+(E7-B11)^2)						
29	B13=3*SUM(B9:E9)						
30	B14=B12+B13						
31	B16=B12/3						
32	B17=B13/12						
33	B19=B16/B17						
34	B21=2.19*SQRT(2*B17/4)						

**7-28.** (a)  $H_0: \mu_{\text{Des1}} = \mu_{\text{Des2}} = \mu_{\text{Des3}} = \mu_{\text{Des4}}$ ;  $H_a$ : at least two of the means differ.

(b) See Spreadsheet.

	A	B	C	D	E
1	Meas. No.	Design 1	Design 2	Design 3	Design 4
2	1	72	93	96	100
3	2	93	88	95	84
4	3	76	97	79	91
5	4	90	74	82	94
6					
7	Mean	82.75	88.00	88.00	92.25
8	Std. Dev.	10.30776	10.03328	8.75595	6.65207
9	Variance	106.2500	100.6667	76.6667	44.2500
10					
11	Grand mean	87.750			
12	SSF	181.5			
13	SSE	983.5			
14	SST	1165			
15					
16	MSF	60.500			
17	MSE	81.95833			
18					
19	F	0.73818			
20					
21	Spreadsheet Documentation				
22	B7=AVERAGE(B2:B5)				
23	B8=STDEV(B2:B5)				
24	B9=VAR(B2:B5)				
25	B11=AVERAGE(B2:E5)				
26	B12=4*((B7-B11)^2+(C7-B11)^2+(D7-B11)^2+(E7-B11)^2)				
27	B13=3*SUM(B9:E9)				
28	B14=B12+B13				
29	B16=B12/3				
30	B17=B13/12				
31	B19=B16/B17				

From Table 7-4 the  $F$  value for 3 degrees of freedom in the numerator and 12 degrees of freedom in the denominator at 95% is 3.49. Since  $F$  calculated is less than  $F$  critical, we accept the null hypothesis and conclude that 4 flow cell designs give the same results at the 95% confidence level.

(c) No differences were detected.

7-29. (a)  $H_0: \mu_{\text{ISE}} = \mu_{\text{EDTA}} = \mu_{\text{AA}}$ ;  $H_a$ : at least two of the means differ.

(b) See Spreadsheet

	A	B	C	D	E	F	G
1	<b>Repetition</b>	<b>ISE</b>	<b>EDTA</b>	<b>At. Abs.</b>			
2	1	39.2	29.9	44.0			
3	2	32.8	28.7	49.2			
4	3	41.8	21.7	35.1			
5	4	35.3	34.0	39.7			
6	5	33.5	39.1	45.9			
7							
8	Mean	36.52	30.68	42.78			
9	Std. Dev.	3.85707	6.46313	5.49791			
10	Variance	14.877	41.772	30.227			
11							
12	Grand Mean	36.660		<b>Differences</b>			
13	SSF	366.172		42.78-30.68=	12.1	Significant difference	
14	SSE	347.504		36.52-30.68=	5.94	No sig. diff.	
15	SST	713.676		42.78-36.52=	6.26	No sig. diff.	
16							
17	MSF	183.086					
18	MSE	28.95867					
19	F	6.322321					
20	LSD	7.453554					
21							
22	<b>Spreadsheet Documentation</b>						
23	B8=AVERAGE(B2:B6)						
24	B9=STDEV(B2:B6)						
25	B10=VAR(B2:B6)						
26	B12=AVERAGE(B2:D6)						
27	B13=5*((B8-B12)^2+(C8-B12)^2+(D8-B12)^2)						
28	B14=4*SUM(B10:D10)						
29	B15=B13+B14						
30	B17=B13/2						
31	B18=B14/12						
32	B19=B17/B18						
33	B20=2.19*SQRT(2*B18/5)						

From Table 7-4 the  $F$  value for 2 degrees of freedom in the numerator and 12 degrees of freedom in the denominator at 95% is 3.89. Since  $F$  calculated is greater than  $F$  critical, we reject the null hypothesis and conclude that the 3 methods give different results at the 95% confidence level.

(c) Based on the calculated LSD value there is a significant difference between the atomic absorption method and the EDTA titration. There is no significant difference

between the EDTA titration method and the ion-selective electrode method and there is no significant difference between the atomic absorption method and the ion-selective electrode method.

**7-30.** (a)  $Q = \frac{|41.27 - 41.61|}{41.84 - 41.27} = 0.596$  and  $Q_{\text{crit}}$  for 4 observations at 95% confidence = 0.829.

Since  $Q < Q_{\text{crit}}$  the outlier value 41.27 cannot be rejected with 95% confidence.

(b)  $Q = \frac{|7.388 - 7.295|}{7.388 - 7.284} = 0.894$  and  $Q_{\text{crit}}$  for 4 observations at 95% confidence = 0.829.

Since  $Q > Q_{\text{crit}}$  the outlier value 7.388 can be rejected with 95% confidence.

**7-31.** (a)  $Q = \frac{|85.10 - 84.70|}{85.10 - 84.62} = 0.833$  and  $Q_{\text{crit}}$  for 3 observations at 95% confidence = 0.970.

Since  $Q < Q_{\text{crit}}$  the outlier value 85.10 cannot be rejected with 95% confidence.

(b)  $Q = \frac{|85.10 - 84.70|}{85.10 - 84.62} = 0.833$  and  $Q_{\text{crit}}$  for 4 observations at 95% confidence = 0.829.

Since  $Q > Q_{\text{crit}}$  the outlier value 85.10 can be rejected with 95% confidence.

**7-32.**  $Q = \frac{|4.60 - 4.50|}{4.60 - 4.40} = 0.5$  and  $Q_{\text{crit}}$  for 5 observations at 95% confidence = 0.710.

Since  $Q < Q_{\text{crit}}$  the outlier value 4.60 ppm cannot be rejected with 95% confidence.