**Chapter 6**

**6-1.** **(a)** The *standard error of the mean* is the standard deviation of the mean and is given by the standard deviation of the data set divided by the square root of the number of measurements.

**(b)** The *coefficient of variation* is the percent relative standard deviation or ( $s / \bar{x}$ ) × 100%.

**(c)** The *variance* is the square of the standard deviation.

**(d)** *Significant figures* are all the digits in a number that are known with certainty plus the first uncertain digit.

**6-2.** **(a)** The term *parameter* refers to quantities such as the mean and standard deviation of a population or distribution of data. The term *statistic* refers to an estimate of a parameter that is made from a sample of data.

**(b)** The population mean is the true mean for the population of data. The sample mean is the arithmetic average of a limited sample drawn from the population.

**(c)** *Random errors* result from uncontrolled variables in an experiment while *systematic errors* are those that can be ascribed to a particular cause and can usually be determined.

**(d)** *Accuracy* represents the agreement between an experimentally measured value and the true or accepted value. *Precision* describes the agreement among measurements that have been performed in exactly the same way.

**6-3.** **(a)** The *sample standard deviation s* is the standard deviation of a sample drawn from the population. It is given by $s = \sqrt{\dfrac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$, where $\bar{x}$ is the sample mean.

The *population standard deviation* $\sigma$ is the standard deviation of an entire population

given by $\sigma = \sqrt{\dfrac{\sum_{i=1}^{N}\left(x_i - \mu\right)^2}{N}}$, where $\mu$ is the population mean.

**(b)** In statistics, a sample is a small set of replicate measurements. In chemistry, a sample is a portion of a material that is taken for analysis.

**6-4.** The standard error of a mean, $s_m$, is the standard deviation of the set of data, $s$, divided by the square root of the number of data in the set, i.e. $s_m = s/\sqrt{N}$. The standard error of the mean $s_m$ is lower than the standard deviation of the data points in a set $s$ because a set of data made up of means will have less spread than a set of data made up of data points. In the equations for $s_m$, the denominator ($\sqrt{N}$) always has a value greater than 1 so that $s_m$ will always be less than $s$.

**6-5**. Since the probability that a result lies between -1σ and +1σ is 0.683, the probability that a result will lie between 0 and +1σ will be half this value or 0.342. The probability that a result will lie between +1σ and +2σ will be half the difference between the probability of the result being between -2σ and +2σ, and -1σ and +1σ, or ½ (0.954-0.683) = 0.136.

**6-6**. Since the probability that a result lies between -2σ and +2σ is 0.954, the probability that a result will lie outside this range is (1 – 0.954) = 0.046. The probability that a result will be more negative than -2σ will be half this value, or 0.023.

**6-7.** Listing the data from Set A in order of increasing value:

| $x_i$ | $x_i^2$ |
|---|---|
| 9.5 | 90.25 |
| 8.5 | 72.25 |
| 9.1 | 82.81 |
| 9.3 | 86.49 |
| 9.1 | 82.81 |
| $\Sigma x_i = 45.5$ | $\Sigma x_i^2 = 414.61$ |

**(a)** mean: $\bar{x} = 45.5/5 = 9.1$

**(b)** median = 9.1

**(c)** spread: $w = 9.5 - 8.5 = 1.0$

**(d)** standard deviation: $s = \sqrt{\dfrac{414.61 - (45.5)^2/5}{5-1}} = 0.37$

**(e)** coefficient of variation: CV = (0.37/9.1) × 100% = 4.1%

Results for Sets A through F, obtained in a similar way, are given in the following table.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $\bar{x}$ | 9.1 | 55.29 | 0.650 | 5.1 | 20.61 | 0.958 |
| median | 9.1 | 55.32 | 0.653 | 5.0 | 20.64 | 0.954 |
| $w$ | 1.0 | 0.15 | 0.108 | 1.5 | 0.14 | 0.049 |
| $s$ | 0.37 | 0.08 | 0.056 | 0.6 | 0.07 | 0.02 |
| CV, % | 4.1 | 0.14 | 8.5 | 12.2 | 0.32 | 2.1 |

**6-8.** For Set A, $E = 9.1 - 9.0 = 0.1$

$E_r = (0.1/9.0) \times 1000$ ppt = 11.1 ppt

Set B, $E = -0.040$ $E_r = -0.7$ ppt

Set C $E = 0.0195$ $E_r = 31$ ppt

Set D $E = -0.34$ $E_r = -63$ ppt

Set E $E = 0.03$ $E_r = 1.3$ ppt

Set F $E = -0.007$ $E_r = -6.8$ ppt

**6-9.** **(a)** $s_y = \sqrt{(0.03)^2 + (0.001)^2 + (0.001)^2} = 0.030$

CV = (0.03/–2.082) × 100% = –1.4%

$y$ = –2.08(±0.03)

**(b)** $s_y = \sqrt{(0.04)^2 + (0.0001)^2 + (0.08)^2} = 0.089$

CV = (0.089/19.1637) × 100% = 0.46%

$y$ = 19.16(±0.09)

**(c)** $\frac{s_y}{y} = \sqrt{\left(\frac{0.3}{29.2}\right)^2 + \left(\frac{0.02\times10^{-17}}{2.034\times10^{-17}}\right)^2} = 0.01422$

CV = (0.0142) × 100% = 1.42%

$s_y = (0.0142) \times (5.93928 \times 10^{-16}) = 0.08446 \times 10^{-16}$

$y = 5.94(\pm0.08) \times 10^{-16}$

**(d)** $\frac{s_y}{y} = \sqrt{\left(\frac{1}{326}\right)^2 + \left(\frac{2}{740}\right)^2 + \left(\frac{0.006}{1.964}\right)^2} = 0.00510$

CV = (0.00510) × 100% = 0.510%

$s_y$ = (0.00510) × (122830.9572) = 626

$y = 1.228(\pm0.006) \times 10^5$

**(e)** $s_{num} = \sqrt{(6)^2 + (3)^2} = 6.71$ $\qquad y_{num} = 187 - 89 = 98$

$s_{den} = \sqrt{(1)^2 + (8)^2} = 8.06$ $\qquad y_{den} = 1240 + 57 = 1297$

$\frac{s_y}{y} = \sqrt{\left(\frac{6.71}{98}\right)^2 + \left(\frac{8.06}{1297}\right)^2} = 0.0688$

CV = (0.0688) × 100% =6.88%

$s_y$ = (0.0688) × (0.075559) = 0.00520

$y = 7.6(\pm 0.5) \times 10^{-2}$

**(f)** $$\frac{s_y}{y} = \sqrt{\left(\frac{0.01}{3.56}\right)^2 + \left(\frac{3}{522}\right)^2} = 0.006397$$

CV = (0.006397) × 100% = 0.6397%

$s_y = (0.006397) \times (6.81992 \times 10^{-3}) = 4.36 \times 10^{-5}$

$y = 6.82(\pm 0.04) \times 10^{-3}$

**6-10.** **(a)** $$s_y = \sqrt{(0.02\times10^{-8})^2 + (0.2\times10^{-9})^2} = 2.83\times10^{-10}$$

$y = 1.02\times10^{-8} - 3.54\times10^{-9} = 6.66\times10^{-9}$

$$\text{CV} = \frac{2.83\times10^{-10}}{6.66\times10^{-9}}\times100\% = 4.25\%$$

$y = 6.7 \pm 0.3 \times 10^{-9}$

**(b)** $$s_y = \sqrt{(0.08)^2 + (0.06)^2 + (0.004)^2} = 0.10$$

$y = 90.31 - 89.32 + 0.200 = 1.190$

$$\text{CV} = \frac{0.10}{1.190}\times100\% = 8.41\%$$

$y = 1.2(\pm 0.1)$

**(c)** $$\frac{s_y}{y} = \sqrt{\left(\frac{0.0005}{0.0040}\right)^2 + \left(\frac{0.02}{10.28}\right)^2 + \left(\frac{1}{347}\right)} = 0.1250$$

CV = (0.1250) × 100% = 12.5%

$y = 0.0040 \times 10.28 \times 347 = 14.27$

$s_y = (0.125) \times (14.27) = 1.78$

$y = 14(\pm 2)$

**(d)** $$\frac{s_y}{y} = \sqrt{\left(\frac{0.03\times10^{-14}}{223\times10^{-14}}\right)^2 + \left(\frac{0.04\times10^{-16}}{1.47\times10^{-16}}\right)^2} = 0.0272$$

CV = $(0.027) \times 100\% = 2.7\%$

$y = 1.63\times10^{-14} / 1.03\times10^{-16} = 1.517 \times 10^4$

$s_y = (0.0272) \times (1.517 \times 10^4) = 0.0413 \times 10^4$

$y = 1.52(\pm 0.04) \times 10^4$

**(e)** $$\frac{s_y}{y} = \sqrt{\left(\frac{1}{100}\right)^2 + \left(\frac{1}{2}\right)^2} = 0.500$$

CV = $(0.500) \times 100\% = 50.0\%$

$y = 100 / 2 = 50.0$

$s_y = (0.500) \times (50.0) = 25$

$y = 50(\pm 25)$

**(f)** $$s_{\text{num}} = \sqrt{\left(0.02 \times 10^{-2}\right)^2 + \left(0.06 \times 10^{-3}\right)^2} = 2.09 \times 10^{-4}$$

num = 0.0149 – 0.00497 = 0.00993

$$s_{\text{den}} = \sqrt{\left(0.7\right)^2 + \left(0.08\right)^2} = 0.704$$

den = 27.1 + 8.99 = 36.09

$$\frac{s_y}{y} = \sqrt{\left(\frac{0.000209}{0.00993}\right)^2 + \left(\frac{0.704}{36.09}\right)^2} = 0.0287$$

CV = (0.0287) × 100% = 2.87%

$y$ = 0.00993/36.09 = 2.751 × $10^{-4}$

$s_y$ = (0.0287) × (2.751 × $10^{-4}$) = 7.899 × $10^{-6}$

$y$ = 2.75(± 0.08) × $10^{-4}$

**6-11. (a)** $y$ = log(2.00 × $10^{-4}$) = –3.6989 $\quad s_y = \dfrac{(0.434)(0.03\times10^{-4})}{(2.00\times10^{-4})} = 6.51\times10^{-3}$

$y$ = –3.699 ± 0.0065

CV = (0.0065/3.699) × 100% = 0.18%

**(b)** As in part (a): $y$ = 37.645 ±0.001

CV = 0.003%

**(c)** $y$ = antilog(1.200) = 15.849 $\quad \dfrac{s_y}{y} = (2.303)(0.003) = 0.0069$

$s_y$ = (0.0069)(15.849) = 0.11 $\quad y$ = 15.8 ±0.1

CV = (0.11/15.8) × 100% = 0.69%

**(d)** As in part (c): $y$ = 3.5(±0.3) × $10^{49}$

CV = 9.2%

**6-12.** **(a)** $y = (4.17 \times 10^{-4})^3 = 7.251 \times 10^{-11}$ $\qquad \dfrac{s_y}{y} = 3\left(\dfrac{0.03\times10^{-4}}{4.17\times10^{-4}}\right) = 0.0216$

$s_y = (0.0216)(7.251 \times 10^{-11}) = 1.565 \times 10^{-12}$ $\quad y = 7.3(\pm0.2) \times 10^{-11}$

CV = $(1.565 \times 10^{-12}/7.251 \times 10^{-11}) \times 100\% = 2.2\%$

**(b)** As in part (a): $y = 1.3090(\pm0.0002)$

CV = 0.02%

**6-13.** From the equation for the volume of a sphere, we have

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{2.15}{2}\right)^3 = 5.20\text{ cm}^3$$

Hence, we may write

$$\frac{s_V}{V} = 3\times\frac{s_d}{d} = 3\times\frac{0.02}{2.15} = 0.0279$$

$s_V = 5.20 \times 0.0279 = 0.145$

$V = 5.2(\pm0.1)\text{ cm}^3$

**6-14.** The mean diameter of the tank is $\bar{d} = \dfrac{5.2+5.7+5.3+5.5}{4} = 5.425\text{ m}$ The standard deviation of the diameter is $s_d = \sqrt{\dfrac{\sum_{i=1}^{N}(x_i-\bar{x})^2}{N-1}}$ = 0.222. The standard deviation of the mean diameter is $s_d/\sqrt{4}$ = 0.111.

The mean height of the tank is $\bar{h} = \dfrac{7.9+7.8+7.6}{3} = 7.767\text{ m}$ and $s_h$ = 0.153. The standard deviation of the mean height is $s_h/\sqrt{3}$ = 0.0883

The volume of the tank is given by

$$V = h \times \pi\left(\frac{d}{2}\right)^2 = 7.767(\pm 0.0883) \times \pi \times \left(\frac{5.425(\pm 0.111)}{2}\right)^2$$

The error in the 3rd term is given by

$$\frac{s_y}{y} = \sqrt{\left(\frac{0.111}{5.425}\right)^2} = 0.02046 \qquad y = ½\,(5.425) = 2.7125 \quad s_y = 0.02046 \times 2.7125 = 0.0555$$

$V = 7.767(\pm 0.0883) \times (3.14159) \times (2.7125(\pm 0.0555))^2$

$$\frac{s_y}{y} = 2\left(\frac{0.0555}{2.7125}\right) = 0.0409 \qquad y = (2.7125)^2 = 7.358 \quad s_y = (0.0409)(7.358) = 0.301$$

$V = 7.767(\pm 0.0883) \times (3.14159) \times 7.358(\pm 0.301)$

Next, we propagate the error in volume by assuming the error in pi is negligible.

$$\frac{s_y}{y} = \sqrt{\left(\frac{0.0883}{7.767}\right)^2 + \left(\frac{0.301}{7.358}\right)^2} = 0.04246 \qquad y = 7.767 \times 3.14159 \times 7.358 = 179.5$$

$s_y = (0.04246)(179.5) = 7.62$

$y = 180(\pm 8)\ \mathrm{m}^3$

Converting to liters, we have

$$V = 180(\pm 8)\ \mathrm{m}^3 \times \frac{1000L}{m^3} = 1.8(\pm 0.08) \times 10^5\ L$$

**6-15.** Since the titrant volume equals the final buret reading minus the initial buret reading, we can introduce the values given into the equation for %A.

%A = [9.26(±0.03) – 0.19(±0.02)] × equivalent mass × 100/[45.0(±0.2)]

Obtaining the value of the first term and the error in the first term

$$s_y = \sqrt{(0.03)^2 + (0.02)^2} = 0.0361 \qquad y = 9.26 - 0.19 = 9.07$$

We can now obtain the relative error of the calculation

$$\frac{s_{\%A}}{\%A} = \sqrt{\left(\frac{0.036}{9.07}\right)^2 + \left(\frac{0.2}{45.0}\right)^2} = 0.00596$$

The coefficient of variation is then

CV = (0.00596) × 100% = 0.596% or 0.6%

**6-16.** To obtain a CV in $S$ of 1% or less,

$$\frac{s_S}{S} \le 0.01 = \sqrt{\left(\frac{s_{k'}}{k'}\right)^2 + \left(\frac{s_{e^{-E/kT}}}{e^{-E/kT}}\right)^2}$$

Since $k'$ is a constant, the first term is zero resulting in:

$$0.01 = \frac{s_{e^{-E/kT}}}{e^{-E/kT}} \qquad \text{From Table 6-4} \qquad \frac{s_{e^{-E/kT}}}{e^{-E/kT}} = s_{-E/kT}$$

$s_{-E/kT}$ can be determined by evaluation of the errors in each of the numbers.

$$\frac{s_{-E/kT}}{-E/kT} = \sqrt{\left(\frac{s_E}{E}\right)^2 + \left(\frac{s_k}{k}\right)^2 + \left(\frac{s_T}{T}\right)^2}$$

Since both $E$ and $k$ have no uncertainty, this equation reduces to:

$$s_{-E/kT} = \left(\frac{s_T}{T}\right) \times \frac{-E}{kT} = \left(\frac{s_T}{6500}\right)\left(\frac{-6.12\times10^{-19}}{(1.3807\times10^{-23})(6500)}\right) = 0.01$$

Solving for $s_T$ gives:

$s_T \le 9.5$ K

**6-17.** We first calculate the mean transmittance and the standard deviation of the mean.

mean T = $\left(\frac{0.213 + 0.216 + 0.208 + 0.214}{4}\right) = 0.2128$

$s_T = 0.0034$

**(a)** $$c_X = \left(\frac{-\log T}{\varepsilon b}\right) = \frac{-\log(0.2128)}{3312} = 2.029\times10^{-4}\text{ M}$$

**(b)** For $-\log T$, $s_y = (0.434)s_T/T = 0.434 \times (0.0034/0.2128) = 0.00693$

$-\log(0.2128) = 0.672$

$$c_X = \frac{-\log T}{\varepsilon b} = \frac{0.672 \pm 0.00693}{3312 \pm 12}$$

$$\frac{s_{c_X}}{c_X} = \sqrt{\left(\frac{0.00693}{0.672}\right)^2 + \left(\frac{12}{3312}\right)^2} = 0.0109$$

$$s_{c_X} = (0.0109)(2.029\times10^{-4}) = 2.22\times10^{-6}$$

**(c)** CV = $(2.22 \times 10^{-6}/2.029 \times 10^{-4}) \times 100\% = 1.1\%$

**6-18.** **(a)** and **(b)**

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Problem 6-18** | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | **Sample** | 1 | $(x_i-x_{ave})^2$ | 2 | $(x_i-x_{ave})^2$ | 3 | $(x_i-x_{ave})^2$ | 4 | $(x_i-x_{ave})^2$ | 5 | $(x_i-x_{ave})^2$ | |
| 4 | | | | | | | | | | | | **No. Sets** |
| 5 | | 6.02 | 0.0031 | 7.48 | 0.0044 | 3.90 | 0.0090 | 4.48 | 0.0060 | 5.29 | 0.0067 | 5 |
| 6 | | 6.04 | 0.0058 | 7.47 | 0.0032 | 3.96 | 0.0012 | 4.65 | 0.0086 | 5.13 | 0.0061 | |
| 7 | | 5.88 | 0.0071 | 7.29 | 0.0152 | 4.16 | 0.0272 | 4.68 | 0.0150 | 5.14 | 0.0046 | |
| 8 | | 6.06 | 0.0092 | | | 3.96 | 0.0012 | 4.42 | 0.0189 | 5.28 | 0.0052 | |
| 9 | | 5.82 | 0.0207 | | | | | | | 5.20 | 0.0001 | |
| 10 | | | | | | | | | | | | |
| 11 | **mean** | 5.964 | | 7.413 | | 3.995 | | 4.558 | | 5.208 | | **Total** |
| 12 | ***s*** | 0.107 | | 0.107 | | 0.114 | | 0.127 | | 0.075 | | |
| 13 | ***N*** | | 5 | | 3 | | 4 | | 4 | | 5 | 21 |
| 14 | $\Sigma(x_i-x_{ave})^2$ | | 0.0459 | | 0.0229 | | 0.0387 | | 0.0485 | | 0.0227 | 0.17864 |
| 15 | | | | | | | | | | | | |
| 16 | $s_{pooled}$ | 0.11 | | | | | | | | | | |
| 17 | | | | | | | | | | | | |
| 18 | **Spreadsheet Documentation** | | | | | | | | | | | |
| 19 | | | | | | | | | | | | |
| 20 | B11=AVERAGE(B5:B9) | | | | | | | | | | | |
| 21 | B12=STDEV(B5:B9) | | | | | | | | | | | |
| 22 | C5=(B5-$B$11)^2 | | | | | | | | | | | |
| 23 | C13=COUNT(C5:C9) | | | | | | | | | | | |
| 24 | C14=SUM(C5:C9) | | | | | | | | | | | |
| 25 | L13=SUM(C13:K13) | | | | | | | | | | | |
| 26 | L14=SUM(C14:K14) | | | | | | | | | | | |
| 27 | B16=SQRT(L14/(L13-L5)) | | | | | | | | | | | |

**(c)** Pooling the variations in %K from the five samples gives a better estimate of $\sigma$ because several data sets are used. Thus the pooled standard deviation uses a larger number of data points and can reflect variations that arise because of sample selection and sample preparation.

**6-19.**

| | A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Problem 6-19** | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | |
| 3 | **Sample** | 1 | $(x_i\text{-}x_{ave})^2$ | 2 | $(x_i\text{-}x_{ave})^2$ | 3 | $(x_i\text{-}x_{ave})^2$ | 4 | $(x_i\text{-}x_{ave})^2$ | 5 | $(x_i\text{-}x_{ave})^2$ | 6 | $(x_i\text{-}x_{ave})^2$ |
| 4 | | | | | | | | | | | | | |
| 5 | | 1.02 | 0.0049 | 1.13 | 0.0020 | 1.12 | 0.0071 | 0.77 | 0.0100 | 0.73 | 0.0144 | 0.73 | 0.0008 |
| 6 | | 0.84 | 0.0121 | 1.02 | 0.0042 | 1.32 | 0.0135 | 0.58 | 0.0081 | 0.92 | 0.0049 | 0.88 | 0.0150 |
| 7 | | 0.99 | 0.0016 | 1.17 | 0.0072 | 1.13 | 0.0055 | 0.61 | 0.0036 | 0.90 | 0.0025 | 0.72 | 0.0014 |
| 8 | | | | 1.02 | 0.0042 | 1.20 | 0.0000 | 0.72 | 0.0025 | | | 0.70 | 0.0033 |
| 9 | | | | | | 1.25 | 0.0021 | | | | | | |
| 10 | | | | | | | | | | | | | |
| 11 | **mean** | 0.950 | | 1.085 | | 1.204 | | 0.670 | | 0.850 | | 0.758 | |
| 12 | ***s*** | 0.096 | | 0.077 | | 0.084 | | 0.090 | | 0.104 | | 0.083 | |
| 13 | ***N*** | | 3 | | 4 | | 5 | | 4 | | 3 | | 4 |
| 14 | $\Sigma(x_i\text{-}x_{ave})^2$ | | 0.0186 | | 0.0177 | | 0.0281 | | 0.0242 | | 0.0218 | | 0.0205 |
| 15 | | | | | | | | | | | | | |
| 16 | $s_{pooled}$ | 0.088 | | | | | | | | **No. Sets** | 6 | | |
| 17 | | | | | | | | | | | | | |
| 18 | **Spreadsheet Documentation** | | | | | | | | | $N_{Total}$ | 23 | | |
| 19 | | | | | | | | | | $\Sigma(x_i\text{-}x_{ave})^2$ | 0.1309 | | |
| 20 | **B11=AVERAGE(B5:B9)** | | | | | | | | | | | | |
| 21 | **B12=STDEV(B5:B9)** | | | | | | | | | | | | |
| 22 | **C5=(B5-$B$11)^2** | | | | | | | | | | | | |
| 23 | **C13=COUNT(C5:C9)** | | | | | | | | | | | | |
| 24 | **C14=SUM(C5:C9)** | | | | | | | | | | | | |
| 25 | **K18=SUM(C13:M13)** | | | | | | | | | | | | |
| 26 | **K19=SUM(C14:M14)** | | | | | | | | | | | | |
| 27 | **B16=SQRT(K19/(K18-K16))** | | | | | | | | | | | | |

**(a)** The standard deviations are $s_1 = 0.096$, $s_2 = 0.077$, $s_3 = 0.084$, $s_4 = 0.090$, $s_5 = 0.104$, $s_6 = 0.083$

**(b)** $s_{pooled} = 0.088$ or 0.09

**6-20.**

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **Problem 6-20** | | | | | |
| 2 | | | | | | |
| 3 | **Sample** | $x_1$ | $x_2$ | mean | $(x_1-x_{ave})^2$ | $(x_2-x_{ave})^2$ |
| 4 | 1 | 2.24 | 2.27 | 2.255 | 0.00022 | 0.00023 |
| 5 | 2 | 8.4 | 8.7 | 8.55 | 0.02250 | 0.02250 |
| 6 | 3 | 7.6 | 7.5 | 7.55 | 0.00250 | 0.00250 |
| 7 | 4 | 11.9 | 12.6 | 12.25 | 0.12250 | 0.12250 |
| 8 | 5 | 4.3 | 4.2 | 4.25 | 0.00250 | 0.00250 |
| 9 | 6 | 1.07 | 1.02 | 1.045 | 0.00063 | 0.00062 |
| 10 | 7 | 14.4 | 14.8 | 14.6 | 0.04000 | 0.04000 |
| 11 | 8 | 21.9 | 21.1 | 21.5 | 0.16000 | 0.16000 |
| 12 | 9 | 8.8 | 8.4 | 8.6 | 0.04000 | 0.04000 |
| 13 | | | | | | |
| 14 | ***N*** | 18 | | **Total** | 0.39085 | 0.39085 |
| 15 | **No. of Sets** | 9 | | | | |
| 16 | $s_{pooled}$ | 0.29 | | | | |
| 17 | | | | | | |
| 18 | **Spreadsheet Documentation** | | | | | |
| 19 | | | | | | |
| 20 | D4=AVERAGE(B4:C4) | | | | | |
| 21 | E4=(B4-$D$4)^2 | | | | | |
| 22 | F4=(C4-$D4)^2 | | | | | |
| 23 | B14=COUNT(B4:C12) | | | | | |
| 24 | E14=SUM(E4:E12) | | | | | |
| 25 | B16=SQRT((E14+F14)/(B14-B15)) | | | | | |

**6-21.**

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **Problem 6-21** | | | | | | | |
| 2 | | | | | | | | |
| 3 | **Sample** | 1 | $(x_i-x_{ave})^2$ | 2 | $(x_i-x_{ave})^2$ | 3 | $(x_i-x_{ave})^2$ | |
| 4 | | | | | | | | **No. Sets** |
| 5 | | 13 | 0.06 | 42 | 2.78 | 29 | 5.76 | 3 |
| 6 | | 19 | 39.06 | 40 | 0.11 | 25 | 2.56 | |
| 7 | | 12 | 0.56 | 39 | 1.78 | 26 | 0.36 | |
| 8 | | 7 | 33.06 | | | 23 | 12.96 | |
| 9 | | | | | | 30 | 11.56 | |
| 10 | | | | | | | | |
| 11 | **mean** | 12.75 | | 40.33 | | 26.60 | | **Total** |
| 12 | ***s*** | 4.92 | | 1.53 | | 2.88 | | |
| 13 | ***N*** | | 4 | | 3 | | 5 | 12 |
| 14 | $\Sigma(x_i-x_{ave})^2$ | | 72.75 | | 4.67 | | 33.20 | 110.62 |
| 15 | | | | | | | | |
| 16 | $s_{pooled}$ | 3.5 | | | | | | |
| 17 | | | | | | | | |
| 18 | **Spreadsheet Documentation** | | | | | | | |
| 19 | | | | | | | | |
| 20 | B11=AVERAGE(B5:B9) | | | | | | | |
| 21 | B12=STDEV(B5:B9) | | | | | | | |
| 22 | C5=(B5-$B$11)^2 | | | | | | | |
| 23 | C13=COUNT(C5:C9) | | | | | | | |
| 24 | C14=SUM(C5:C9) | | | | | | | |
| 25 | H13=SUM(C13:G13) | | | | | | | |
| 26 | H14=SUM(C14:G14) | | | | | | | |
| 27 | B16=SQRT(H14/(H13-H5)) | | | | | | | |

**6-22.**

| | **Excel value** | **NIST value** |
|---|---|---|
| **Mean** | 2.001856000000000 | 2.001856000000000 |
| **SD** | 0.000429123454003085 | 0.000429123454003053 |

The means are the same in both cases. There is some difference, however, in the standard deviations in the last two digits. The differences could arise because of the different algorithms used to calculate the SD and to roundoff errors in the calculations. The first 16 digits are identical.