IEOR 242: Applications in Data Analysis, Fall 2021

Homework Assignment #2

September 25, 2021

Problem 0: (0 points) Please self-grade Problems 1 and 2 from Homework 1. Please use the following rubric to assign yourself a score of $0 - 2$ for each part of the two problems:

1. 0 points = Did not attempt or very wrong
2. 1 point = Showed something non-trivial, but some mistakes or some missing justification
3. 2 points = 100% correct

The GSIs will make an annoucement concerning how to submit your self-grade scores on Gradescope.

Problem 1: (20 points)

Consider the problem of predicting whether a team of students will receive a "good" or "poor" grade on the final project for IEOR 242 based on the number of hours that they spend working on the project and whether they use Python or R. We will consider three different approaches for addressing this problem. First we define variables:

1. $Y \in \{0, 1\}$ is defined by:

$$Y = \begin{cases} 1 & \text{if the team receives a "good" grade} \\ 0 & \text{if the team receives a "poor" grade} \end{cases}$$

2. $Z \in \{0, 1\}$ is defined by:

$$Z = \begin{cases} 1 & \text{if the team uses Python} \\ 0 & \text{if the team uses R} \end{cases}$$

3. $X \geq 0$ is the total number of hours that the team spends working on the project.

Please answer the following questions.

a) (5 points) First consider a logistic regression model:

$$\Pr(Y = 1 \mid X, Z) = \frac{1}{1 + \exp(-(\beta_0 + \beta_1 X + \beta_2 Z))}.$$

Suppose that the coefficients were estimated based on data from a prior semester. The coefficient estimates are $\hat{\beta}_0 = -3.50, \hat{\beta}_1 = 0.18, \hat{\beta}_2 = 1.24$. Using these coefficient estimates, give a prediction of the probability that a team that spends 30 hours working on the project and uses Python will receive a good grade.

b) (15 points) Now consider a different type of logistic model:

$$\Pr(Y = 1 \mid X, Z) = Z\left(\frac{1}{1 + \exp(-(\alpha_0 + \alpha_1 X))}\right) + (1 - Z)\left(\frac{1}{1 + \exp(-(\beta_0 + \beta_1 X))}\right)$$

(a) (2 points) Carefully explain the logic of how this model makes predictions of the probability that a team will receive a good grade.

(b) (5 points) Given a training dataset consisting of teams from prior semesters of IEOR 242, how would you fit the above model to the training data? You may assume that you have access to a subroutine that can fit a standard logistic regression model to any appropriate dataset.

(c) (4 points) Suppose that we fit this new model to the previously mentioned training dataset and the coefficient estimates are $\hat{\alpha}_0 = -4.02, \hat{\alpha}_1 = 0.24, \hat{\beta}_0 = -2.85, \hat{\beta}_1 = 0.15$. Using these coefficient estimates, give a prediction of the probability that a team that spends 30 hours working on the project and uses Python will receive a good grade.

(d) (2 points) Suppose that there are 90 total teams in the training data, of which 50 used Python and 40 used R. Additionally, of the 50 teams that used Python, 30 received a good grade and 20 did not. Of the 40 teams that used R, 21 received a good grade and 19 did not. Based on this information, which logistic model – the model from part (*a*) or the model from part (*b*) – do you expect to make more accurate probability predictions in future semesters?

(e) (2 points) Now suppose that there are 15 total teams in the training data, of which 10 used Python and 5 used R. Of the 10 teams that used Python, 8 received a good grade and 2 did not. Of the 5 teams that used R, 4 received a good grade and 1 did not. Based on this information, which logistic model – the model from part (*a*) or the model from part (*b*) – do you expect to make more accurate probability predictions in future semesters?

Problem 2: (10 points)

Consider a binary classification problem where $X \in \mathbb{R}^p$ and $Y \in \{0, 1\}$. For a fixed $x \in \mathbb{R}^p$, suppose that $\Pr(Y = 1 \mid X = x) = p$ for some $p \in [0, 1]$. Consider a prediction problem where there is a loss $L_{FN} > 0$ associated with predicting $Y = 0$ when the actual outcome is $Y = 1$, and another loss $L_{FP} > 0$ associated with predicting $Y = 1$ when the actual outcome is $Y = 0$. There is no loss associated with true positives or true negatives (i.e., a correct answer). Show that there is a threshold value $\bar{p}$ such that following the expected loss criterion for making a prediction is equivalent to predicting $Y = 1$ if $p \geq \bar{p}$ and predicting $Y = 0$ otherwise. What is the value of $\bar{p}$?

Problem 3: Framingham Heart Study (Adapted from Bertsimas Chapter 7) (70 points)

Heart disease is one of the leading causes of death worldwide. Over 8 million people died from coronary heart disease (CHD) in 2019, which was the leading cause of death that year. (For 2020, it is estimated that COVID-19 has overtaken heart disease as the leading cause of death globally.)

In the late 1940s, the U.S. government took steps to study cardiovascular disease. In order develop high quality data for their study, they decided to track a large cohort of initially-healthy people over time. The town of Framingham, Massachusetts (a suburb of Boston) was selected as the site for the study, which commenced in 1948. The study enrolled 5,209 participants aged 30-62. Participants were given a questionnaire and a medical exam every two years. They also collected data on the participants' physical characteristics and behavioral characteristics, in addition to the medical test data. Over the years, the study has expanded to include multiple generations and has collected many more factors including genetic information. This data is now famously known and is simply called the Framingham Heart Study.

In this exercise, you are asked to build models using Framingham Heart Study data in order to predict CHD and to make recommendations to better prevent heart disease. There are 3,658 total observations in our data, with each observation representing the data from a particular study participant. There are 16 variables in the dataset, which are described in Table 1. You will be asked to predict `TenYearCHD` (whether the patient experiences coronary heart disease within 10 years of their first examination). As a consequence of your modeling efforts, you should be able to identify *risk factors*, which are the variables that increase the risk of CHD.

a) (40 points) To lower the risk of CHD, physicians can prescribe preventive medication such as blood-pressure-lowering or cholesterol-lowering medications. Many policy makers, when recommending certain preventive medications to patients at risk of developing CHD, rely on evidence-based analysis that weighs the pros and cons of such interventions. Health economic evaluation is a commonly applied methodology for decision-making that takes both medical costs and health benefits (a monetized version of improved life longevity) into consideration. In fact, many countries establish clinical practice guidelines using such formalized health economic evaluation methodologies (the National Institute for Health and Clinical Excellence in England, for example).

As prior work, let us suppose that a colleague of yours has completed a health economics study analyzing the costs and benefits of a recently approved medication aimed at preventing CHD. The colleague determined that patients who experience CHD within the next 10 years are expected to incur a lifetime cost of \$700,000 associated with the disease; this cost includes both the costs of treatment for CHD, \$200,000, as well as a cost intended to capture the decreased quality and length of life experienced by patients with CHD, which is \$500,000. Also, your colleague has determined that patients who take the preventative medicine being studied will have their probability of developing CHD within the next 10 years reduced by 85%; in other words, if their current 10-year risk (probability) of developing CHD is p without taking the medication, then their 10-year risk (probability) with the medicine would instead be $(0.15 * p)$. Regardless of whether a patient eventually develops CHD, there is a \$75,000 cost associated with taking this recently approved medication. A decision tree capturing your colleague's analysis is shown in Figure 1 (below).

Using all of the provided independent variables, build a logistic regression model to predict

Table 1: Variables in the dataset `framingham.csv`.

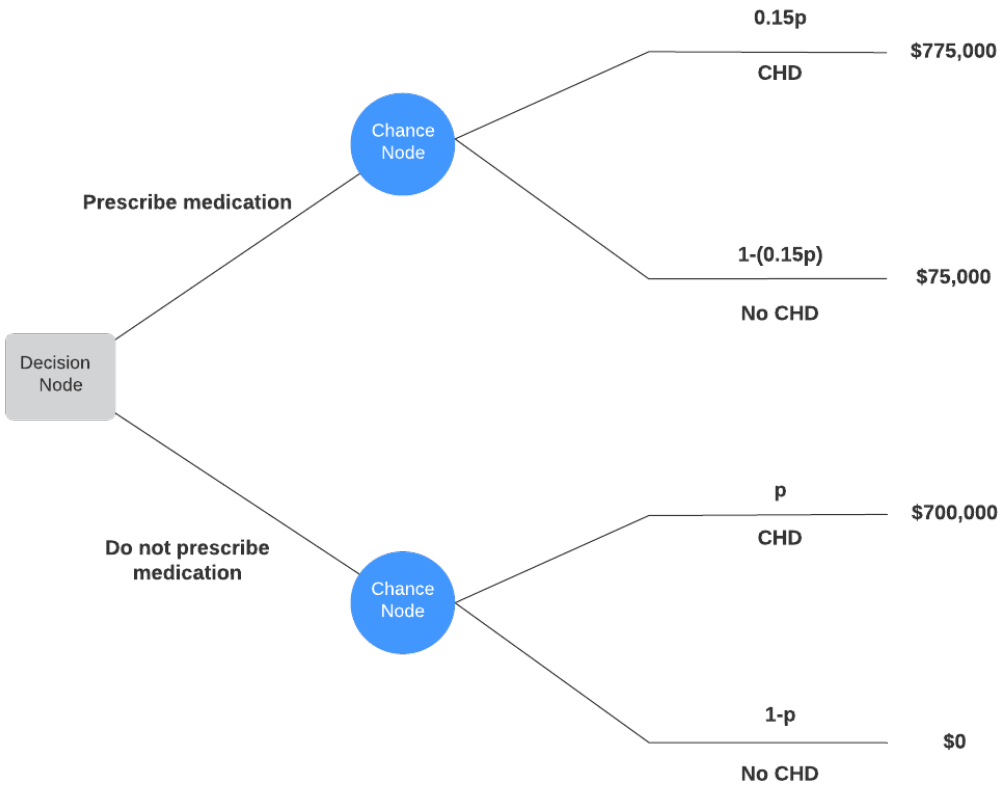
Variable	Description
`male`	Is biological sex male
`age`	Age (in years) at first examination
`education`	Some high school, high school/GED, some college/vocational school, college
`currentSmoker`	Is a current smoker
`cigsPerDay`	Number of cigarettes per day
`BPMeds`	Is on blood pressure medication at time of first examination
`prevalentStroke`	Previously had a stroke
`prevalentHyp`	Currently hypertensive
`diabetes`	Currently has diabetes
`totChol`	Total cholesterol (mg/dL)
`sysBP`	Systolic blood pressure
`diaBP`	Diastolic blood pressure
`BMI`	Body Mass Index, weight (kg)/height $(\text{m})^2$
`heartRate`	Heart rate (beats/minute)
`glucose`	Blood glucose level (mg/dL)
`TenYearCHD`	Experienced coronary heart disease within 10 years of first examination

the probability that a patient will experience CHD within the next 10 years. Use dataset `framingham_train.csv` to train your model. This training set has 2560 data points, which are randomly selected from the original `framingham.csv` dataset (around 70%). Use dataset `framingham_test.csv` to test your model. This test set has the remaining 1098 data points. Please answer the following questions concerning your model:

i) What is the fitted logistic regression model? Do not simply copy the results of your code, but instead state the equation used by the model to make predictions. **Use all features from Table 2 to build your model.**

ii) What are the most important risk factors for 10-year CHD risk identified by the model? Pick one of these variables and describe its impact on a patient's predicted odds of developing CHD in the next 10 years.

Figure 1: Decision tree for prescribing the approved medication to prevent CHD. The leaf nodes represent cost values.



iii) Suppose that you wish to determine the optimal strategy for assigning which new patients receive the medication. Given your colleague's analysis of the costs and benefits associated with the recently approved treatment, identify a threshold value of p, call it $\bar{p}$, such that it is optimal to prescribe the medication to a patient if and only if their 10-year CHD risk exceeds $\bar{p}$.

iv) Describe the test set performance of the logistic regression model, using the threshold identified in part (*iii*) to separate patients into those who are at high risk for CHD (risk exceeding the threshold $\bar{p}$) and those who are at low risk for CHD (risk below the threshold $\bar{p}$). State the model's accuracy, True Positive Rate (TPR), and False Positive Rate (FPR), and briefly describe these three metrics in a way that is accessible to a non-technical audience.

v) If patients are prescribed the medication using the strategy implied by the model, use the test set data to provide an estimate(s) for the expected economic cost per patient. You should first report your estimate assuming that the CHD outcomes in the test set are not affected by the treatment decision. Is this assumption reasonable? You should then adjust your estimate in a way that takes into account the fact that the treatment decision impacts a patient's risk of developing CHD. (Hint: keep in mind that this dataset was collected before the option of prescribing the medication was even considered.)

vi) Consider a simple baseline model that predicts none of the patients are at high risk for CHD and therefore does not recommend treatment for any of the patients. Describe the test set performance of the baseline model in terms of accuracy, TPR, and FPR, as well as expected economic cost per patient.

vii) Use an example to explain how to use the model in a real clinical setting. Suppose a new patient arrives, and the physician accesses the patient's electronic medical records and retrieves the following about the patient:

```
Female, age 45, college education, currently a smoker with an average of
9 cigarettes per day.  Currently on blood pressure medication, has had stroke
but not hypertensive.  Currently diagnosed with diabetes; total Cholesterol
at 220.  Systolic/diastolic blood pressure at 140/100, BMI at 33, heart rate
at 69, glucose level at 74.
```

What is the predicted probability that this patient will experience CHD in the next ten years? Based on your calculated $\bar{p}$ threshold from part (*iii*) from the decision tree, should the physician prescribe the preventive medication for this patient?

b) (15 points) Show the ROC curve for your logistic regression model on the test set and describe how this curve may be helpful to decision-makers looking to further study the medication you have considered so far in this homework as well as other possible medications for preventing CHD. Describe one interesting observation implied by examining the ROC curve. What is the area under the curve (AUC) for your model in the test set?

c) (10 points) Rather than explicitly dictating which patients should receive the medication, let us consider letting patients decide for themselves. Suppose that if a patient has health insurance, the treatment costs for CHD (including the proposed medication) will be covered by their insurance company. However, a patient will still incur an equivalent cost of $500,000 for decreased quality of life if they develop CHD. Disregarding other factors such as side effects of the medication, if there were no insurance co-payment then it should be clear that every patient would always choose to receive the medication because it would cost them nothing and it would lower their risk of CHD. Thus let us consider setting a co-payment value C – the amount that each patient would have to pay in order to receive the medication – in order to provide an incentive for some patients to forego the treatment while others would choose to receive the treatment. What value of C should the insurance company charge as a co-payment for the medication in order that the patients would "self select" in a manner that is consistent with the previously examined "optimal strategy" discussed in part *(a)* above?

d) (5 points) Are there any aspects of the analysis performed thus far that raise ethical concerns? If so, suggest at least one way that this analysis could be changed to address such concerns.